Chiral Perturbation for Large Momentum Effective Field Theory

Wei-Yang. Liu[1] Juinn-Wei. Chen[2]

[1]Stony Brook University, New York

[2]National Taiwan University, Taiwan

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Outline

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1. Large Momentum Effective Field Theory (LaMET)
2. Finite Volume Extrapolation
3. The framework to apply Chiral Perturbation Theory (ChPT)
4. Results
5. Conclusion

LaMET

Lattice QCD provides a tool to study nonperturbative properties in QCD, e.g. parton physics.

- partonic observables: nonlocal in light front direction, e.g. parton distribution function
- light cone condition only gives a trivial solution in Euclidean Space

$$x^2 = 0 \tag{1}$$

- LaMET: quasi observables introduced

LaMET

- Factorization between finite momentum and infinite momentum.

$$\langle\tilde{\mathcal{O}}\rangle(\Lambda, P_z, \Lambda_{QCD}) = Z(\Lambda, P_z, \mu) \otimes \langle\mathcal{O}\rangle(\mu, \Lambda_{QCD}) + \mathcal{O}(1/P_z^2) \quad (2)$$

$$\Lambda_{QCD} \ll \mu \ll P_z \ll \Lambda \quad (3)$$

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- Effective field theory (EFT): Chiral Perturbation Theory

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- Operator product expansion (OPE) of non-local quark bilinear operators in the basis of local operators.

$$\lambda_\mu \bar{\psi}(z)\, \Gamma^\mu W[z,0]\psi(0) \approx \sum_{n=0}^{\infty} \frac{(iz)^n}{n!} \lambda_{\mu_1}\lambda_{\mu_2}\cdots\lambda_{\mu_n}\mathcal{O}^{\mu\mu_1\cdots\mu_n} \tag{4}$$

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- Only leading twist operators considered $\Rightarrow$ traces suppressed by higher power of momentum $\mathcal{O}(M^{2n}/P_z^{2n})$

$$\mathcal{O}^{\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\Gamma^{(\mu} iD^{\mu} iD^{\mu_1}\cdots iD^{\mu_n)}\psi \tag{5}$$

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- Finite volume effect between local operator basis.

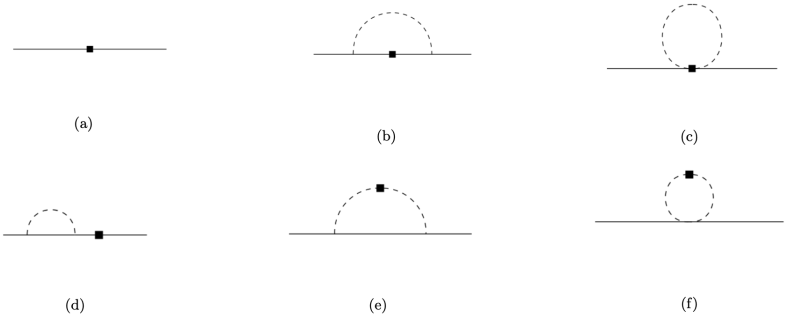
ChPT to LaMET

For example, $\Gamma^\mu = \gamma^\mu$ in isosinglet case

$$\mathcal{O}_{u-d}^{\mu\mu_1\mu_2\cdots\mu_n} = c_1 \bar{N} v^{(\mu} v^{\mu_1} v^{\mu_2} \cdots v^{\mu_n)} (u\tau^3 u^\dagger + u^\dagger \tau^3 u) N$$
$$+ \tilde{c}_1 \bar{N} S^{(\mu} v^{\mu_1} v^{\mu_2} \cdots v^{\mu_n)} (u\tau^3 u^\dagger - u^\dagger \tau^3 u) N + \cdots$$

- Matching quark level operators to hadronic level operators with symmetries preserved.
- The theory further separated by the chiral symmetry breaking scale Λ_χ

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{tr}(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \eta \mathrm{tr}(M\Sigma^\dagger + M^\dagger \Sigma) + \bar{N} iv \cdot DN + 2g_A \bar{N} S \cdot AN + \cdots \quad (6)$$



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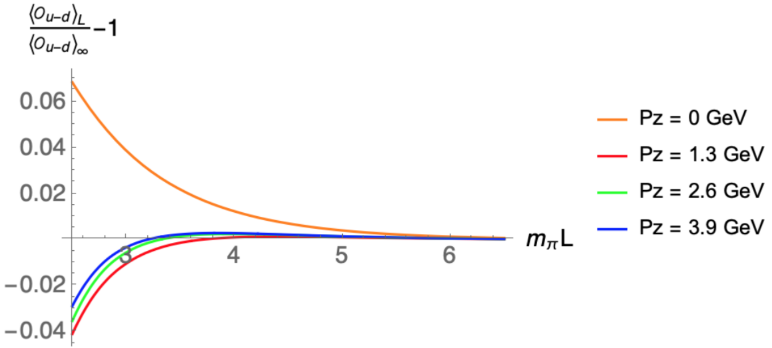
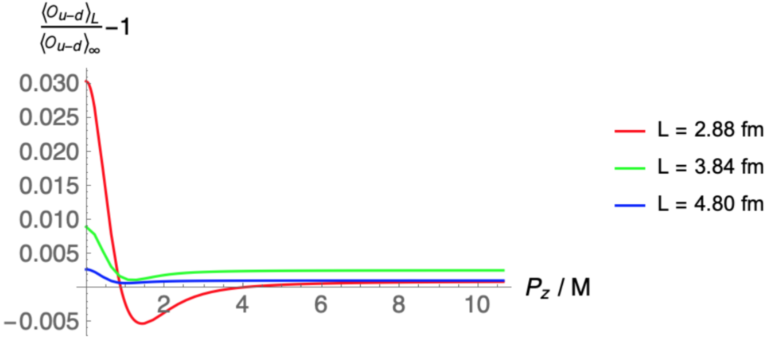
$$\frac{\langle \mathcal{O}_{u-d}^{\mu\mu_1\cdots\mu_n}\rangle_L}{\langle \mathcal{O}_{u-d}^{\mu\mu_1\cdots\mu_n}\rangle_\infty}-1=-(1-\delta_{0n})\frac{m_\pi^2}{4\pi^2F_\pi^2}\sum_{\vec{n}\neq 0}\left[\frac{K_1(nm_\pi L)}{nm_\pi L}+3g_A^2 J(nm_\pi L,\frac{\vec{n}\cdot\vec{P}}{n\ M})\right] \tag{7}$$


⟨O_{u−d}⟩_L / ⟨O_{u−d}⟩_∞ −1
0.06
0.04
0.02
−0.02
−0.04
3
4
5
6
m_π L
Pz = 0 GeV
Pz = 1.3 GeV
Pz = 2.6 GeV
Pz = 3.9 GeV

(b)


$\frac{\langle O_{u-d}\rangle_L}{\langle O_{u-d}\rangle_\infty} - 1$
0.030
0.025
0.020
0.015
0.010
0.005
−0.005
2
4
6
8
10
P_z / M
L = 2.88 fm
L = 3.84 fm
L = 4.80 fm

(a)

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- Model independent formula for extrapolations
- Lorentz contraction of the nucleon size

Q & A

Thanks for Your Listening