Higher-order QCD corrections to SIDIS

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CTEQ, 03/22/2022

Semi-inclusive DIS $\ell p \to \ell\, h\, X$

- probe of nucleon / nuclear structure
  (flavor separation, polarized PDFs, TMDs)
- source of information on fragmentation fcts.
  (and thus hadronization)
- testbed for QCD calculations

Today's talk:

QCD corrections for ”basic collinear” SIDIS

with Maurizio Abele and Daniel de Florian, PRD104 (2021) & 2203.07928
earlier work: D. Anderle, F. Ringer, WV, PRD87 (2013)

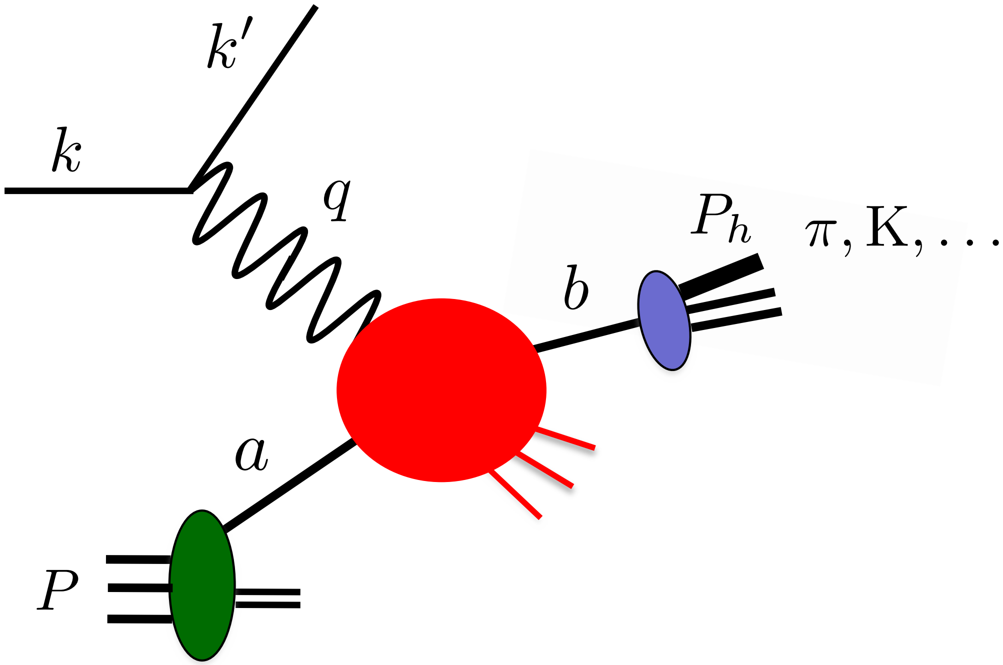
implications for fragmentation functions

with Ignacio Borsa, Daniel de Florian, Rodolfo Sassot, Marco Stratmann
2202.05060

Outline:

- Basics of perturbation theory for SIDIS
- Threshold resummation and expansion
- Phenomenology
- Fragmentation functions
- Conclusions

Basics of perturbation theory for SIDIS



$$x = \frac{Q^2}{2P\cdot q}$$

$$z = \frac{P\cdot P_h}{P\cdot q}$$

$$\frac{d^3\sigma^h}{dx\,dy\,dz} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}F_T^h(x,z,Q^2) + \frac{1-y}{y}F_L^h(x,z,Q^2)\right]$$

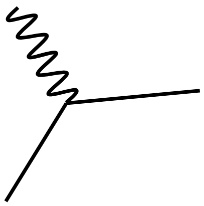
$$F_T^h(x,z,Q^2) = \sum_{a,b}\int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\, f_a\left(\frac{x}{\hat{x}},Q^2\right)\,\omega_{ab}\left(\hat{x},\hat{z},\alpha_s\right)\,D_b^h\left(\frac{z}{\hat{z}},Q^2\right) + \mathrm{P.C.}$$

perturbative

$$\hat{x} = \frac{Q^2}{2p_a\cdot q} \qquad \hat{z} = \frac{p_a\cdot p_b}{p_a\cdot q}$$

$$\omega_{ab} = \omega_{ab}^{(0)} + \frac{\alpha_s}{2\pi}\omega_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \omega_{ab}^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \omega_{ab}^{(3)} + \mathcal{O}(\alpha_s^4)$$

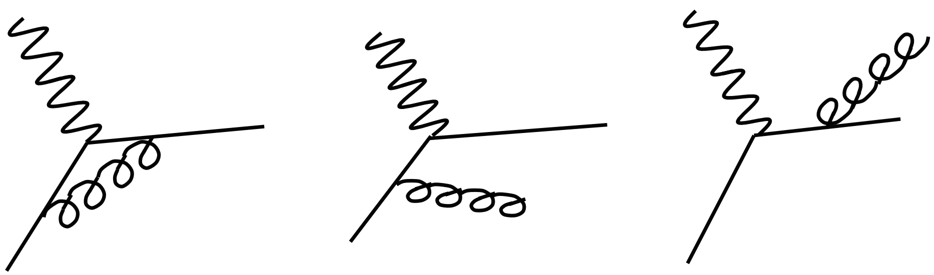
LO:



$$\omega_{qq}^{(0)}(\hat{x}, \hat{z}) = e_q^2\, \delta(1-\hat{x})\, \delta(1-\hat{z})$$

$$\Rightarrow\ F_T^h(x, z, Q^2) = \sum_q e_q^2\, f_q\left(x, Q^2\right) D_q^h\left(z, Q^2\right)$$

NLO:



$+ \ldots$

Altarelli et al.; de Florian, Stratmann, WV

NNLO: so far unknown

(Daleo, Garcia Canal, Sassot; Anderle, de Florian, Habarnau)

→ can one obtain approximate results?

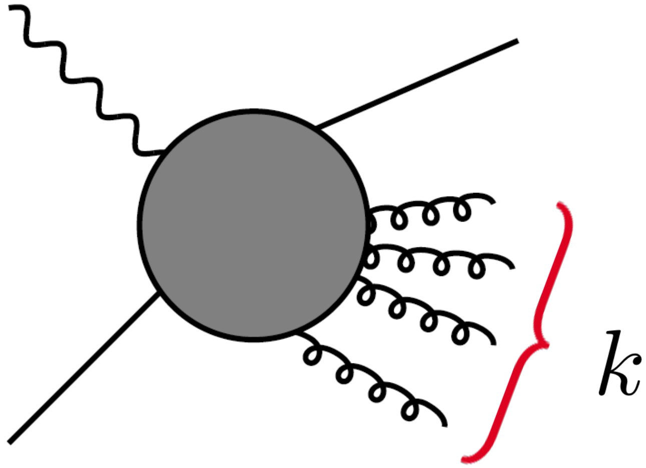
LO: $$\omega_{qq}^{(0)}(\hat{x},\hat{z}) = e_q^2\,\delta(1-\hat{x})\,\delta(1-\hat{z})$$

NLO, as $\hat{x},\hat{z} \longrightarrow 1$:

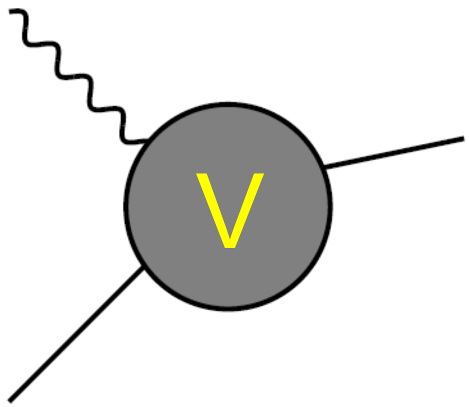
$$\omega_{qq}^{(1)}(\hat{x},\hat{z}) = e_q^2 C_F \left[2\delta(1-\hat{x}) \left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + 2\delta(1-\hat{z}) \left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ \right.$$
$$\left. + \frac{2}{(1-\hat{x})_+(1-\hat{z})_+} - 8\delta(1-\hat{x})\,\delta(1-\hat{z}) + \ldots \right]$$

k[th] order of perturbation theory:

$$\alpha_s^k\,\omega_{qq}^{(k)}(\hat{x},\hat{z}) \sim \alpha_s^k \left[\delta(1-\hat{x}) \left(\frac{\ln^{2k-1}(1-\hat{z})}{1-\hat{z}}\right)_+ + \delta(1-\hat{z}) \left(\frac{\ln^{2k-1}(1-\hat{x})}{1-\hat{x}}\right)_+ \right.$$
$$\left. + \frac{1}{(1-\hat{x})_+} \left(\frac{\ln^{2k-2}(1-\hat{z})}{1-\hat{z}}\right)_+ + \frac{1}{(1-\hat{z})_+} \left(\frac{\ln^{2k-2}(1-\hat{x})}{1-\hat{x}}\right)_+ + \ldots \right]$$



$$(1-\hat{x}) + (1-\hat{z}) \approx \frac{2k_0}{Q}$$



$$\hat{x} = \hat{z} = 1$$

- real and virtual contributions “imbalanced”
- logs can be resummed to all orders: threshold resummation

  Sterman; Catani,Trentadue; …
- use to determine dominant parts of NNLO, N^{3}LO corrections

Threshold resummation

$$F_T^h(x,z,Q^2) = \sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \, f_a\left(\frac{x}{\hat{x}}, Q^2\right) \, \omega_{ab}\left(\hat{x}, \hat{z}, \alpha_s\right) \, D_b^h\left(\frac{z}{\hat{z}}, Q^2\right)$$

Mellin moments:

$$\tilde{F}_T^h(N,M,Q^2) \equiv \int_0^1 dx\, x^{N-1} \int_0^1 dz\, z^{M-1}\, F_T^h(x,z,Q^2)$$

$$= \sum_{a,b} \tilde{f}_a(N,Q^2)\, \tilde{\omega}_{ab}(N,M,\alpha_s)\, \tilde{D}_b^h(M,Q^2)$$

where

$$\tilde{f}_a(N,Q^2) \equiv \int_0^1 dx\, x^{N-1} f_a(x,Q^2)$$

$$\tilde{D}_b^h(M,Q^2) \equiv \int_0^1 dz\, z^{M-1} D_b^h(z,Q^2)$$

$$\tilde{\omega}_{ab}(N,M,\alpha_s) \equiv \int_0^1 d\hat{x}\, \hat{x}^{N-1} \int_0^1 d\hat{z}\, \hat{z}^{M-1}\, \omega_{ab}(\hat{x},\hat{z},\alpha_s)$$

Note: similarity to Drell Yan with rapidity: Owens, Westmark, …

Recall, NLO:

$$\omega_{qq}^{(1)}(\hat{x},\hat{z}) \;=\; e_q^2 C_F \Bigg[2\delta(1-\hat{x}) \left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + 2\delta(1-\hat{z}) \left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \frac{2}{(1-\hat{x})_+(1-\hat{z})_+} - 8\delta(1-\hat{x})\,\delta(1-\hat{z}) + \ldots \Bigg]$$

Large $\hat{x}, \hat{z}$ $\leftrightarrow$ large N, M :

$$\omega_{qq}^{(1)}(N,M) \;=\; e_q^2 C_F \left[\left(\ln \bar{N} + \ln \bar{M}\right)^2 - 8 + \frac{\pi^2}{3} + \ldots \right]$$

$$\bar{N} \equiv N e^{\gamma_E}, \; \bar{M} \equiv M e^{\gamma_E}$$

k[th] order: corrections

$$\alpha_s^k \left(\ln \bar{N} + \ln \bar{M}\right)^{2k} + \ldots$$

$$L \equiv \ln \bar{N} + \ln \bar{M}$$

Fixed Order

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^kLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

$$L \equiv \ln \bar{N} + \ln \bar{M}$$

Fixed Order →

Resummation ↓

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

LL NLL NNLL

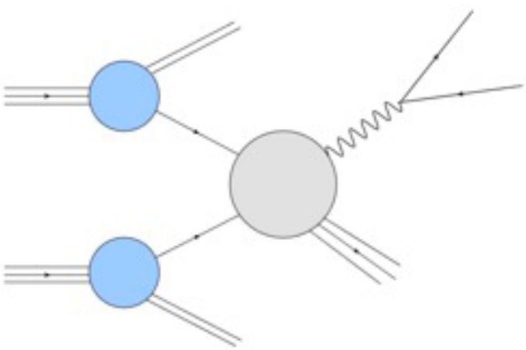
to NLL:

Sterman, WV; Anderle, Ringer, WV

$$\tilde{\omega}_{qq}^{\rm res}(N, M, \alpha_s)$$

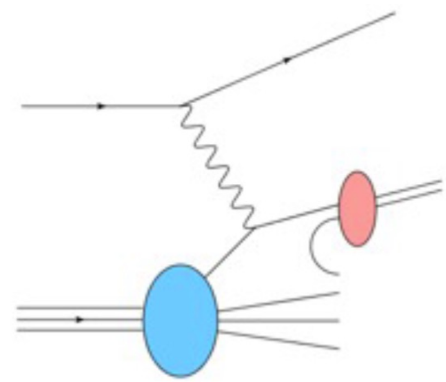
$$\propto \exp\left[\int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q(\alpha_s(k_\perp^2))\left\{\int_{\frac{k_\perp^2}{Q}}^{Q} \frac{dk^+}{k^+}\left[{\rm e}^{-(Nk^+ + Mk^-)/Q} - 1\right] + \ln \bar{N} + \ln \bar{M}\right\}\right]$$

cf. inclusive Drell-Yan



$N \to \sqrt{NM}$

SIDIS



$$\int_{\frac{k_\perp^2}{Q}}^{Q} \frac{dk^+}{k^+}\left[{\rm e}^{-N(k^+ + k^-)/Q} - 1\right] + 2\ln \bar{N}$$

$$\approx 2\left[K_0\left(N\frac{2k_\perp}{Q}\right) + \ln\left(\frac{k_\perp}{Q}\bar{N}\right)\right]$$

$$\int_{\frac{k_\perp^2}{Q}}^{Q} \frac{dk^+}{k^+}\left[{\rm e}^{-(Nk^+ + Mk^-)/Q} - 1\right] + \ln \bar{N} + \ln \bar{M}$$

$$\approx 2\left[K_0\left(\sqrt{NM}\frac{2k_\perp}{Q}\right) + \ln\left(\frac{k_\perp}{Q}\sqrt{\bar{N}\bar{M}}\right)\right]$$

Full resummed formula becomes (to *all* log order!):

$$\tilde{\omega}_{qq}^{\rm res}(N,M,\alpha_s) \;=\; e_q^2\, H_{qq}(\alpha_s)\, \widehat{C}_{qq}(\alpha_s)$$

$$\times\, \exp\left\{ \int_{Q^2/(\bar{N}\bar{M})}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A_q(\alpha_s(\mu)) \ln\left(\frac{\mu^2 \bar{N}\bar{M}}{Q^2}\right) - \frac{1}{2}\widehat{D}_q(\alpha_s(\mu)) \right] \right\}$$

$$H_{qq}(\alpha_s) \;=\; 1 + \frac{\alpha_s}{\pi}\, H_{qq}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 H_{qq}^{(2)} + \ldots$$

hard virtual corr.

$$\widehat{C}_{qq}(\alpha_s) \;=\; 1 + \frac{\alpha_s}{\pi}\, \widehat{C}_{qq}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \widehat{C}_{qq}^{(2)} + \ldots$$

N,M-independent corr. in resummation

Catani,de Florian,Grazzini,Nason
Hinderer,Ringer,Sterman,WV

$$A_q(\alpha_s) \;=\; \frac{\alpha_s}{\pi}\, A_q^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_q^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 A_q^{(3)} + \ldots$$

$$\widehat{D}_q(\alpha_s) \;=\; \left(\frac{\alpha_s}{\pi}\right)^2 \widehat{D}_q^{(2)} + \ldots$$

Kodaira, Trentadue; …
Moch, Vermaseren, Vogt;
Catani, de Florian, Grazzini

Hard factor may be determined from spacelike quark form factor:

Catani, Cieri, de Florian, Ferrera, Grazzini

quark form factor

2 loops:

Gehrmann, Huber, Maitre
Moch, Vermaseren, Vogt

3 loops:

Gehrmann, Glover, Huber,
Ikizlerli, Studerus

(4 loops:

Lee , von Manteuffel,
Schabinger, Smirnov,
Smirnov, Steinhauser)

$$H_{qq}^{(1)} = C_F \left(-4 - \frac{\pi^2}{6} \right)$$

$$\begin{aligned} H_{qq}^{(2)} &= C_F^2 \left(-\frac{15\zeta(3)}{4} + \frac{61\pi^2}{48} + \frac{511}{64} - \frac{\pi^4}{60} \right) \\ &+ C_F C_A \left(\frac{7\zeta(3)}{4} + \frac{3\pi^4}{80} - \frac{1535}{192} - \frac{403\pi^2}{432} \right) \\ &+ C_F N_f \left(\frac{\zeta(3)}{2} + \frac{29\pi^2}{216} + \frac{127}{96} \right) \end{aligned}$$

$$
\begin{aligned}
H_{qq}^{\mathrm{SIDIS},(3)} &= C_F^3\left(\frac{\zeta(3)^2}{2}+\frac{25\pi^2\zeta(3)}{12}-\frac{115\zeta(3)}{16}+\frac{83\zeta(5)}{4}\right.\\
&\qquad\left.+\frac{1937\pi^6}{136080}-\frac{5599}{384}-\frac{181\pi^4}{960}-\frac{4729\pi^2}{1152}\right)\\
&+ C_F^2C_A\left(\frac{37\zeta(3)^2}{12}-\frac{571\pi^2\zeta(3)}{216}-\frac{8653\zeta(3)}{432}-\frac{689\zeta(5)}{72}\right.\\
&\qquad\left.+\frac{2603\pi^4}{38880}+\frac{93581\pi^2}{10368}+\frac{74321}{2304}-\frac{227\pi^6}{17010}\right)\\
&+ C_FC_A^2\left(-\frac{25\zeta(3)^2}{12}+\frac{571\pi^2\zeta(3)}{288}+\frac{82385\zeta(3)}{5184}-\frac{51\zeta(5)}{16}\right.\\
&\qquad\left.+\frac{41071\pi^4}{311040}-\frac{51967\pi^2}{10368}-\frac{125\pi^6}{27216}-\frac{1505881}{62208}\right)\\
&+ C_F^2N_f\left(-\frac{1}{27}7\pi^2\zeta(3)+\frac{869\zeta(3)}{216}-\frac{19\zeta(5)}{18}-\frac{421}{192}-\frac{1363\pi^2}{1296}-\frac{157\pi^4}{4860}\right)\\
&+ C_FC_AN_f\left(-\frac{1}{72}5\pi^2\zeta(3)-\frac{94\zeta(3)}{81}-\frac{\zeta(5)}{8}+\frac{10595\pi^2}{7776}+\frac{110651}{15552}-\frac{1259\pi^4}{77760}\right)\\
&+ C_FN_f^2\left(-\frac{79\zeta(3)}{324}-\frac{\pi^4}{3888}-\frac{307\pi^2}{3888}-\frac{7081}{15552}\right)\\
&+ C_FN_{f,V}\left(\frac{C_A^2-4}{C_A}\right)\left(\frac{7\zeta(3)}{48}-\frac{5\zeta(5)}{6}+\frac{5\pi^2}{96}+\frac{1}{8}-\frac{\pi^4}{2880}\right)\\
&+ \left[C_FC_A\left(\frac{7\zeta(3)}{2}+\frac{3\pi^4}{40}-\frac{1535}{96}-\frac{403\pi^2}{216}\right)+C_FN_f\left(\zeta(3)+\frac{29\pi^2}{108}+\frac{127}{48}\right)\right.\\
&+ C_FC_AN_f\left(\frac{\zeta(3)}{3}+\frac{767\pi^2}{1296}-\frac{\pi^4}{80}+\frac{853}{144}\right)\\
&+ \left.C_F^2\left(-\frac{15\zeta(3)}{2}+\frac{61\pi^2}{24}-\frac{\pi^4}{30}+\frac{511}{32}\right)\right]\pi b_0\ln\frac{\mu_R^2}{Q^2}\\
&+ C_F\left(-4-\frac{\pi^2}{6}\right)\pi^2b_1\ln\frac{\mu_R^2}{Q^2}+C_F\left(-4-\frac{\pi^2}{6}\right)\pi^2b_0^2\ln^2\frac{\mu_R^2}{Q^2}\,.
\end{aligned}
$$

Expansion to NNLL or N^{3}LL:

$$\lambda = b_0 \, \alpha_s \, \frac{1}{2}(\ln \bar{N} + \ln \bar{M})$$

LL NLL

$$\tilde{\omega}_{qq}^{\rm res}(N, M, \alpha_s) = e_q^2 \, H_{qq}(\alpha_s) \, \widehat{C}_{qq}(\alpha_s) \, \exp\left\{ \frac{\lambda}{b_0 \alpha_s} h_q^{(1)}(\lambda) + h_q^{(2)}(\lambda) + \alpha_s \, h_q^{(3)}(\lambda) + \alpha_s^2 \, h_q^{(4)}(\lambda) + \ldots \right\}$$

NNLL N^{3}LL

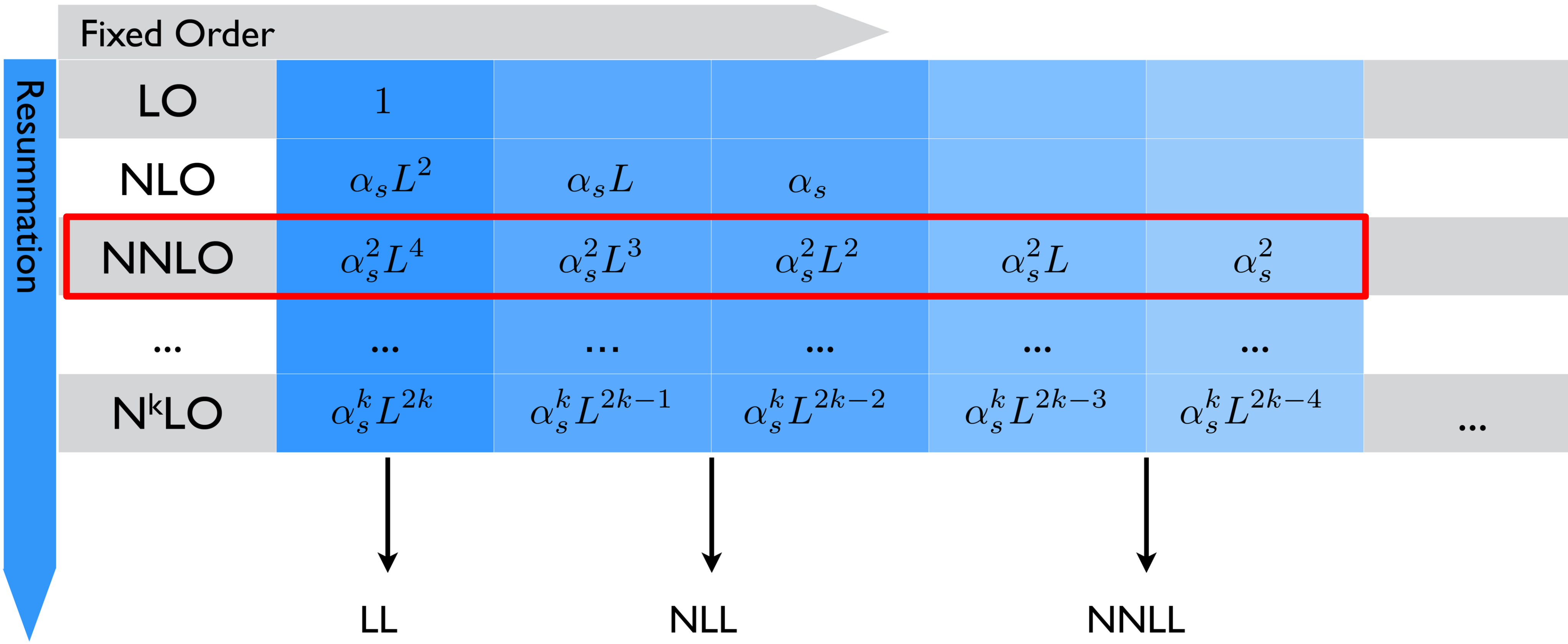
$$h_q^{(1)}(\lambda) = \frac{A_q^{(1)}}{\pi b_0 \lambda} \left[2\lambda + (1 - 2\lambda) \ln(1 - 2\lambda) \right]$$

$$h_q^{(2)}(\lambda) = \ldots$$

$$h_q^{(3)}(\lambda) = \ldots$$

$$
\begin{aligned}
h_q^{(4)}(\lambda) &= \frac{1}{(1-2\lambda)^2}\left(\frac{A_q^{(2)} b_1^2}{\pi^2 b_0^4}\left[-\frac{8}{3}\lambda^3 - \lambda^2 + \lambda + \frac{1}{2}\ln^2(1-2\lambda) + \frac{1}{2}\ln(1-2\lambda)\right]\right. \\
&+ \frac{A_q^{(2)} b_2}{\pi^2 b_0^3}\frac{8}{3}\lambda^3 + \frac{A_q^{(1)} b_1^3}{\pi b_0^5}\left[\frac{8}{3}\lambda^3 + 2\lambda^2\ln(1-2\lambda) - \frac{1}{6}\ln^3(1-2\lambda)\right] \\
&+ \frac{A_q^{(1)} b_1 b_2}{\pi b_0^4}\left[-\frac{16}{3}\lambda^3 + 3\lambda^2 - \lambda - 4\lambda^2\ln(1-2\lambda) + 2\lambda\ln(1-2\lambda) - \frac{1}{2}\ln(1-2\lambda)\right] \\
&+ \frac{A_q^{(1)} b_3}{\pi b_0^3}\left[\frac{8}{3}\lambda^3 - 3\lambda^2 + \lambda + 2\lambda^2\ln(1-2\lambda) - 2\lambda\ln(1-2\lambda) + \frac{1}{2}\ln(1-2\lambda)\right] \\
&+ \frac{A_q^{(3)} b_1}{\pi^3 b_0^3}\left[\frac{8}{3}\lambda^3 - \lambda^2 - \lambda - \frac{1}{2}\ln(1-2\lambda)\right] + \frac{A_q^{(4)}}{\pi^4 b_0^2}\left[2\lambda^2 - \frac{8}{3}\lambda^3\right] \\
&+ \left.\frac{\widehat{D}_q^{(2)} b_1}{\pi^2 b_0^2}\left[\lambda - \lambda^2 + \frac{1}{2}\ln(1-2\lambda)\right] + \frac{\widehat{D}_q^{(3)}}{\pi^3 b_0}\left[\lambda^2 - \lambda\right]\right)
\end{aligned}
$$

$$L \equiv \ln \bar{N} + \ln \bar{M}$$



Finally, further expansion to NNLO and N^{3}LO:

$$\tilde{\omega}_{qq}(N,M,\alpha_s) = 1 + \frac{\alpha_s}{\pi}\,\tilde{\omega}_{qq}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \tilde{\omega}_{qq}^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \tilde{\omega}_{qq}^{(3)} + \mathcal{O}(\alpha_s^4)$$

$$\mathcal{L} \equiv \frac{1}{2}\left(\ln(\bar{N}) + \ln(\bar{M})\right)$$

dominant subleading terms

$$\tilde{\omega}_{qq}^{(1)}(N,M) = e_q^2 C_F\left[2\mathcal{L}^2 + \frac{\pi^2}{6} - 4\right] \quad + e_q^2 C_F \mathcal{L}\left(\frac{1}{N} + \frac{1}{M}\right)$$

$$\frac{1}{e_q^2}\,\tilde{\omega}_{qq}^{(2)}(N,M) = 2C_F^2\mathcal{L}^4 + \frac{4\pi b_0 C_F}{3}\mathcal{L}^3 + C_F\mathcal{L}^2\left[C_F\left(-8 + \frac{\pi^2}{3}\right) + \left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{5}{9}N_f\right]$$

$$+ C_F\mathcal{L}\left[\left(\frac{101}{27} - \frac{7}{2}\,\zeta(3)\right)C_A - \frac{14}{27}\,N_f\right] + C_F^2\left[\frac{511}{64} - \frac{\pi^2}{16} - \frac{\pi^4}{60} - \frac{15}{4}\,\zeta(3)\right]$$

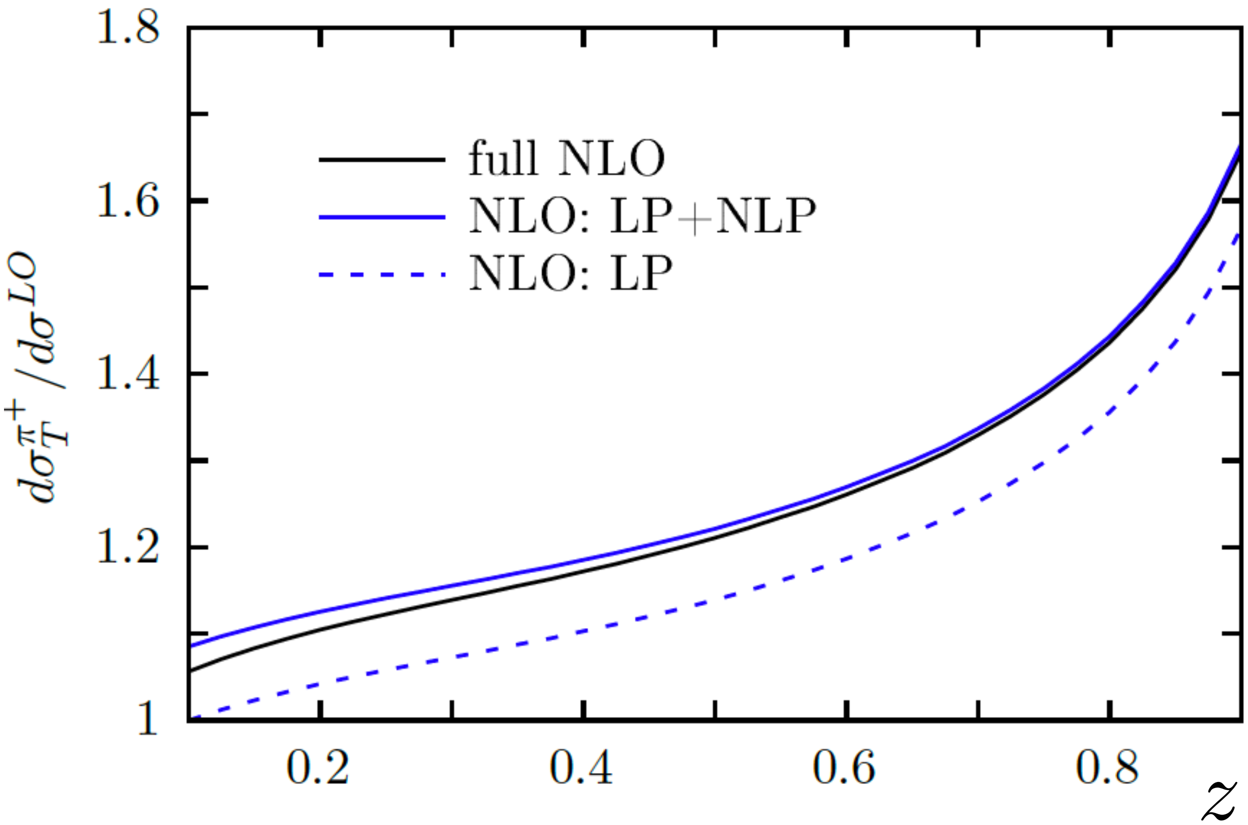
$$+ C_F C_A\left[-\frac{1535}{192} - \frac{5\pi^2}{16} + \frac{7\pi^4}{720} + \frac{151}{36}\,\zeta(3)\right] + C_F N_f\left[\frac{127}{96} + \frac{\pi^2}{24} + \frac{\zeta(3)}{18}\right]$$

$$+ 2\,C_F^2\,\mathcal{L}^3\left(\frac{1}{N} + \frac{1}{M}\right)$$

$$
\begin{aligned}
\frac{1}{e_q^2}\,\tilde{\omega}_{qq}^{T,(3)}(N,M,1,1) \;=\;& \frac{4}{3}C_F^3\mathcal{L}^6+\frac{8}{3}C_F^2\pi b_0\mathcal{L}^5+\mathcal{L}^4\Bigg[C_F^3\left(-8+\frac{\pi^2}{3}\right)-\frac{11}{27}C_FC_AN_f\\
+\;& C_F^2C_A\left(\frac{67}{9}-\frac{\pi^2}{3}\right)+\frac{121}{108}C_FC_A^2-\frac{10}{9}C_F^2N_f+\frac{1}{27}C_FN_f^2\Bigg]\\
+\;& \mathcal{L}^3\Bigg[C_FC_AN_f\left(\frac{\pi^2}{27}-\frac{289}{162}\right)+C_F^2C_A\left(-7\zeta(3)+\frac{11\pi^2}{54}+\frac{70}{27}\right)\\
& +C_FC_A^2\left(\frac{445}{81}-\frac{11\pi^2}{54}\right)+C_F^2N_f\left(-\frac{\pi^2}{27}-\frac{17}{54}\right)+\frac{10}{81}C_FN_f^2\Bigg]\\
+\;& \mathcal{L}^2\Bigg[C_F^3\left(\frac{511}{32}-\frac{15}{2}\zeta(3)-\frac{\pi^2}{8}-\frac{\pi^4}{30}\right)+C_FC_AN_f\left(\frac{5\pi^2}{54}-\frac{2051}{648}\right)\\
& +C_F^2C_A\left(\frac{151}{18}\zeta(3)+\frac{143\pi^2}{216}-\frac{\pi^4}{120}-\frac{8893}{288}\right)+C_F^2N_f\left(\frac{10}{9}\zeta(3)+\frac{67}{18}-\frac{\pi^2}{108}\right)\\
& +C_FC_A^2\left(\frac{11\pi^4}{360}-\frac{11}{2}\zeta(3)-\frac{67\pi^2}{108}+\frac{15503}{1296}\right)+\frac{25}{162}C_FN_f^2\Bigg]\\
+\;& \mathcal{L}\Bigg[C_F^2C_A\left(14\zeta(3)-\frac{1}{12}7\pi^2\zeta(3)+\frac{101\pi^2}{162}-\frac{404}{27}\right)+C_FN_f^2\left(\frac{\zeta(3)}{9}+\frac{58}{729}\right)\\
& +C_FC_A^2\left(\frac{11\pi^2}{36}\zeta(3)-\frac{1541}{108}\zeta(3)+6\zeta(5)-\frac{11\pi^4}{720}-\frac{799\pi^2}{1944}+\frac{297029}{23328}\right)\\
& +C_F^2N_f\left(\frac{19}{18}\zeta(3)+\frac{\pi^4}{180}+\frac{3}{32}-\frac{7\pi^2}{81}\right)+C_FC_AN_f\left(\frac{113}{108}\zeta(3)+\frac{103\pi^2}{1944}-\frac{\pi^4}{360}-\frac{31313}{11664}\right)\Bigg]\\
& +C_F^3\left(\frac{\zeta(3)^2}{2}+\frac{5\pi^2}{6}\zeta(3)-\frac{115}{16}\zeta(3)+\frac{83}{4}\zeta(5)+\frac{761\pi^6}{136080}+\frac{37\pi^4}{2880}-\frac{5599}{384}-\frac{1663\pi^2}{1152}\right)\\
& +C_F^2C_A\left(\frac{37}{12}\zeta(3)^2-\frac{119\pi^2}{72}\zeta(3)-\frac{12877}{432}\zeta(3)-\frac{689}{72}\zeta(5)+\frac{40223\pi^2}{10368}+\frac{74321}{2304}-\frac{149\pi^6}{27216}-\frac{1147\pi^4}{38880}\right)\\
& +C_F^2N_f\left(\frac{1181}{216}\zeta(3)-\frac{19}{18}\zeta(5)-\frac{421}{192}-\frac{559\pi^2}{1296}-\frac{29\pi^4}{9720}\right)+C_FN_f^2\left(\frac{\zeta(3)}{324}-\frac{23\pi^2}{432}-\frac{7081}{15552}-\frac{17\pi^4}{19440}\right)\\
& +C_FC_A^2\left(-\frac{25}{12}\zeta(3)^2+\frac{569\pi^2}{864}\zeta(3)+\frac{139345}{5184}\zeta(3)-\frac{51}{16}\zeta(5)+\frac{17\pi^6}{34020}+\frac{3103\pi^4}{311040}-\frac{93889\pi^2}{31104}\right.\\
& \left.-\frac{1505881}{62208}\right)+C_FC_AN_f\left(\frac{\pi^2}{216}\zeta(3)-\frac{383}{81}\zeta(3)-\frac{\zeta(5)}{8}+\frac{469\pi^4}{77760}+\frac{6493\pi^2}{7776}+\frac{110651}{15552}\right)\\
& +C_FN_{f,V}\frac{(C_A^2-4)}{C_A}\left(\frac{7\zeta(3)}{48}-\frac{5\zeta(5)}{6}+\frac{5\pi^2}{96}+\frac{1}{8}-\frac{\pi^4}{2880}\right)+2\,C_F^3\,\mathcal{L}^5\left(\frac{1}{N}+\frac{1}{M}\right)
\end{aligned}
$$

Phenomenology

EIC $e^- p \to e^- \pi^+ X$



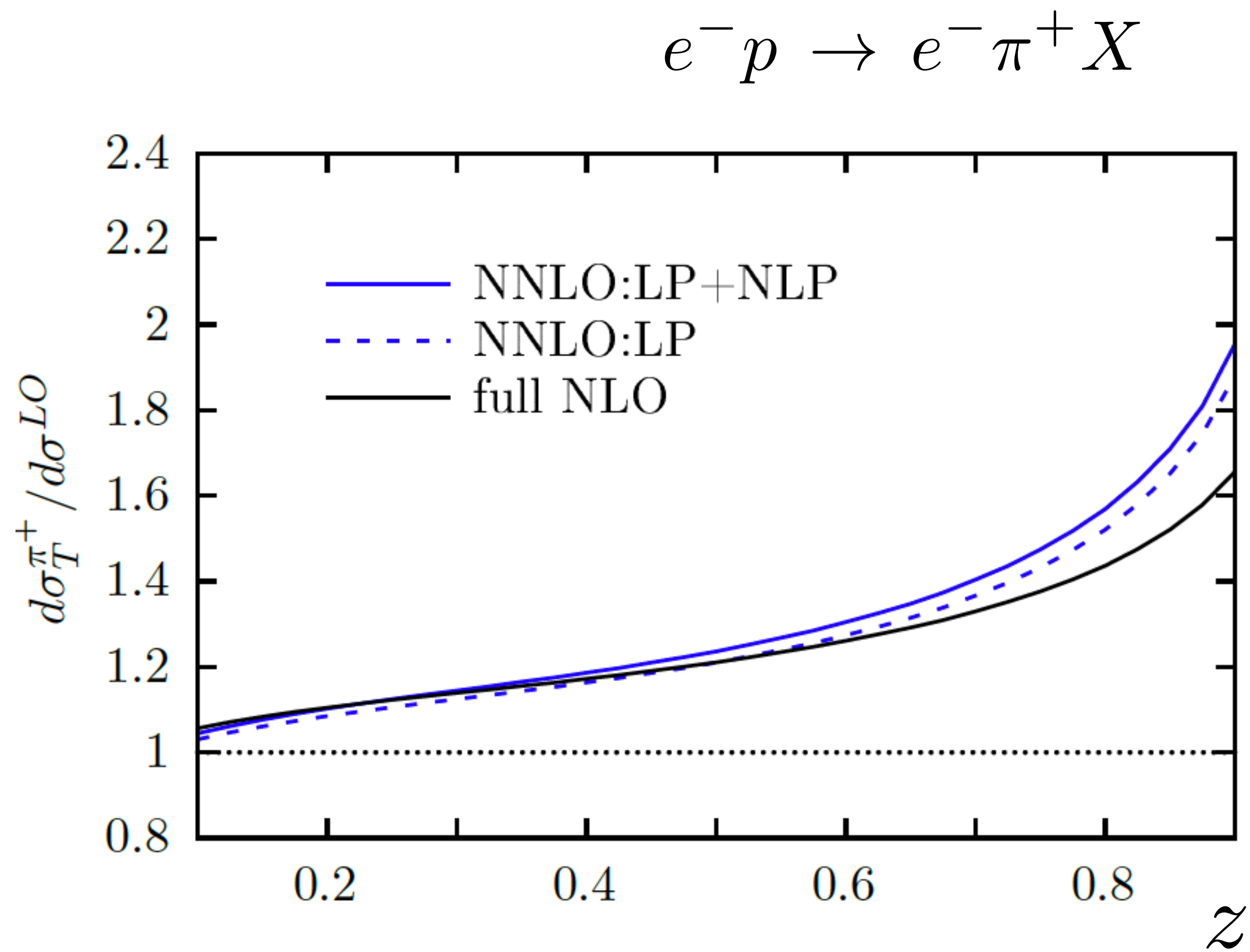
$\sqrt{s} = 100$ GeV

$x \in [0.1, 0.8]$

$y \in [0.1, 0.9]$

(normalized to LO cross section)

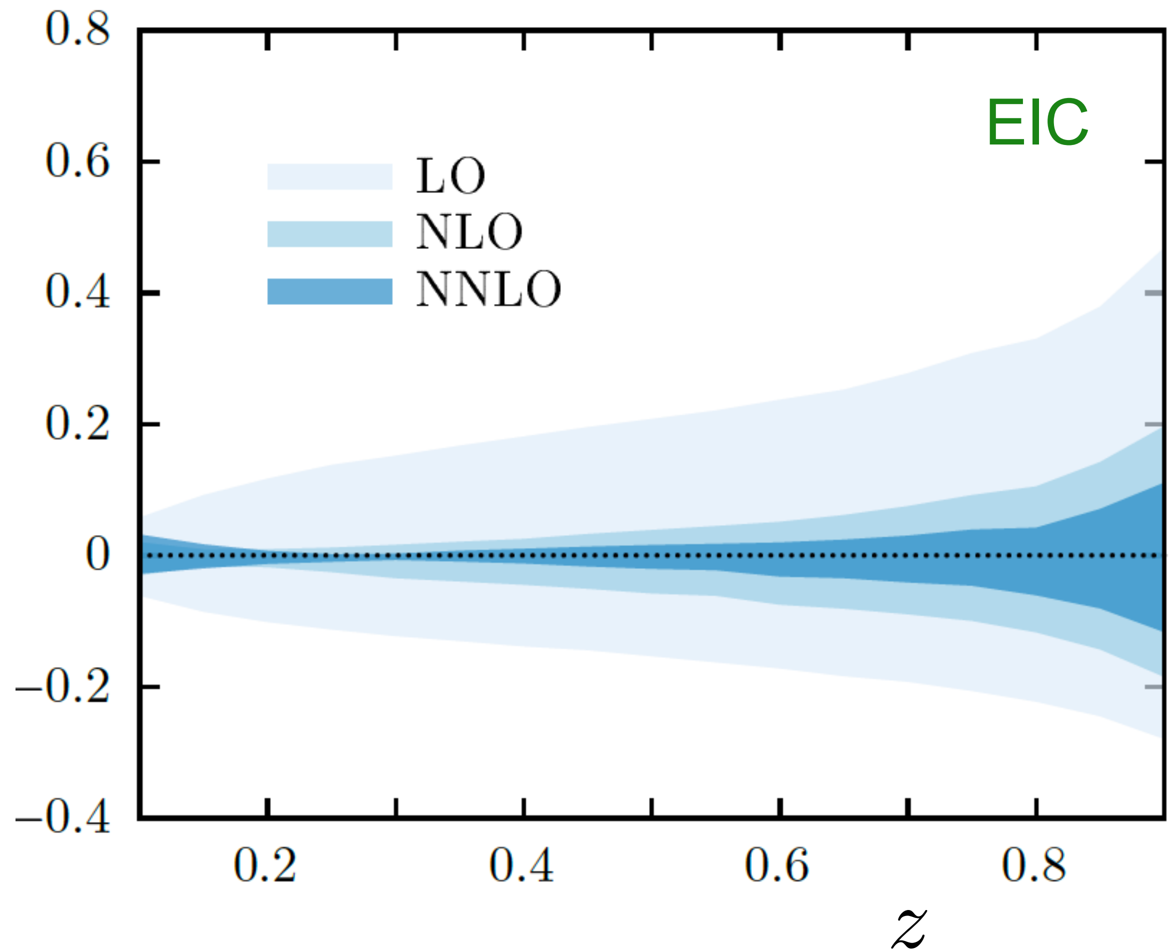
PDFs: CT18, FFs: Anderle, Kaufmann, Ringer, Stratmann

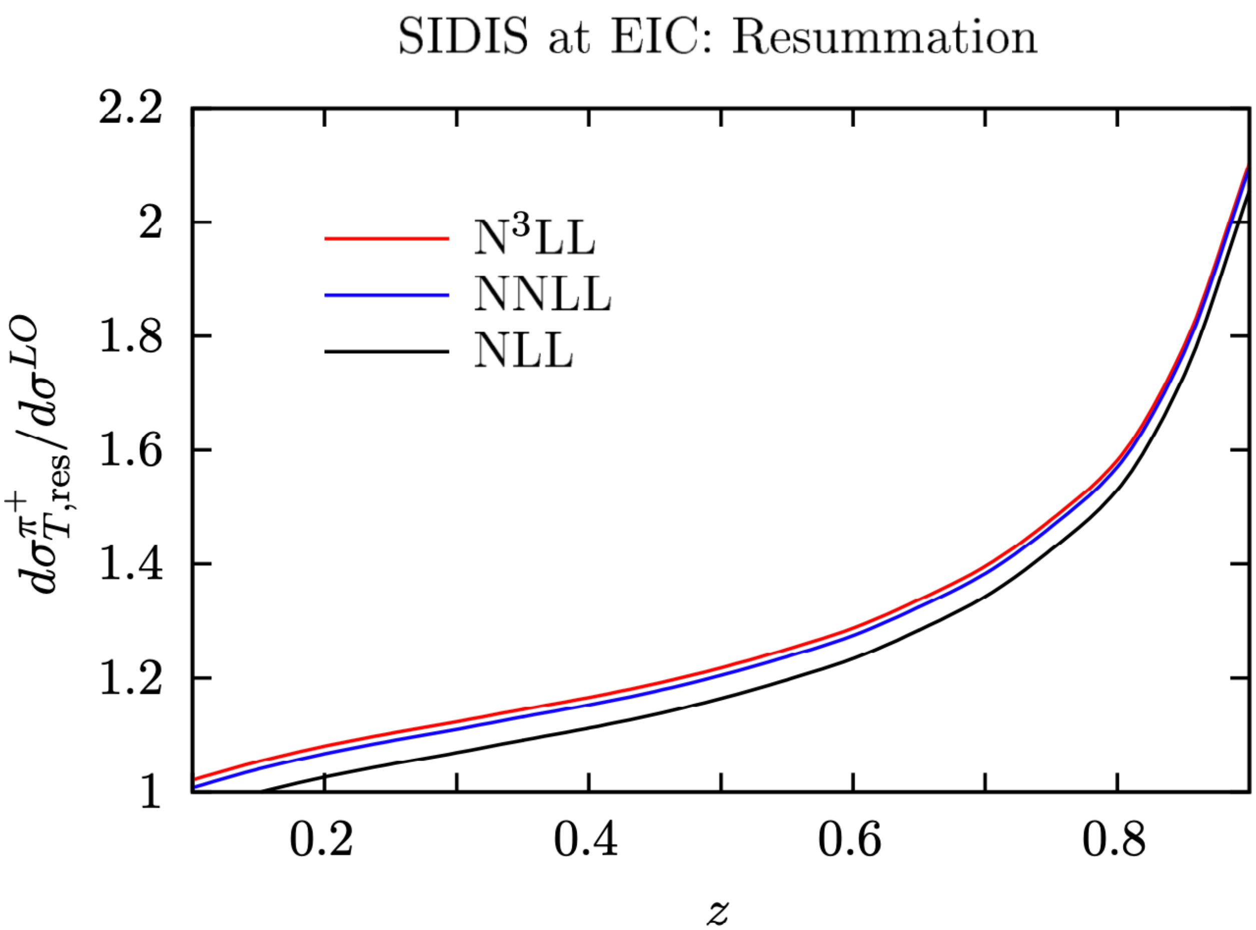


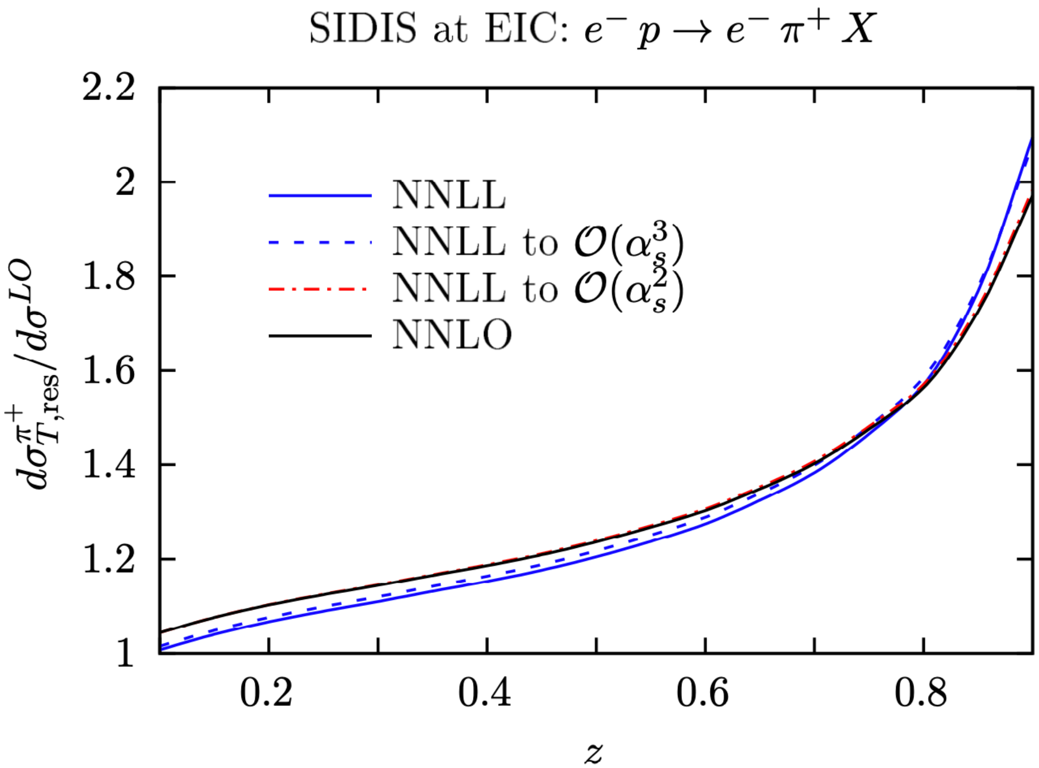
$e^- p \to e^- \pi^+ X$
NNLO:LP+NLP
NNLO:LP
full NLO
$d\sigma_T^{\pi^+}/d\sigma^{LO}$
z
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.2
0.4
0.6
0.8

scale dependence: $Q/2 \le \mu_{R,F} \le 2Q$

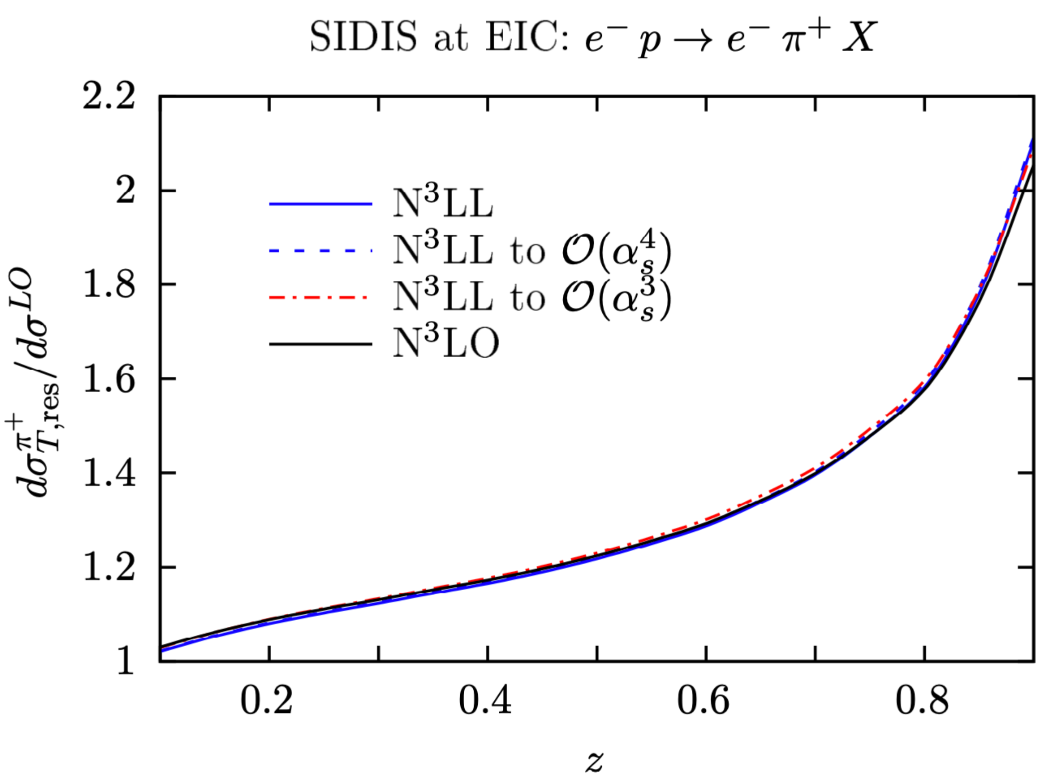
$$\frac{\sigma(\mu) - \sigma(Q)}{\sigma(Q)}$$




SIDIS at EIC: Resummation
$d\sigma^{\pi^+}_{T,\mathrm{res}}/d\sigma^{LO}$
z
N^{3}LL
NNLL
NLL
2.2
2
1.8
1.6
1.4
1.2
1
0.2
0.4
0.6
0.8



SIDIS at EIC: $e^- p \to e^- \pi^+ X$
NNLL
NNLL to $\mathcal{O}(\alpha_s^3)$
NNLL to $\mathcal{O}(\alpha_s^2)$
NNLO
$d\sigma^{\pi^+}_{T,\mathrm{res}}/d\sigma^{LO}$
z



SIDIS at EIC: $e^- p \to e^- \pi^+ X$
$\mathrm{N}^3\mathrm{LL}$
$\mathrm{N}^3\mathrm{LL}$ to $\mathcal{O}(\alpha_s^4)$
$\mathrm{N}^3\mathrm{LL}$ to $\mathcal{O}(\alpha_s^3)$
$\mathrm{N}^3\mathrm{LO}$
$d\sigma^{\pi^+}_{T,\mathrm{res}}/d\sigma^{LO}$
z

resummation “schemes”:

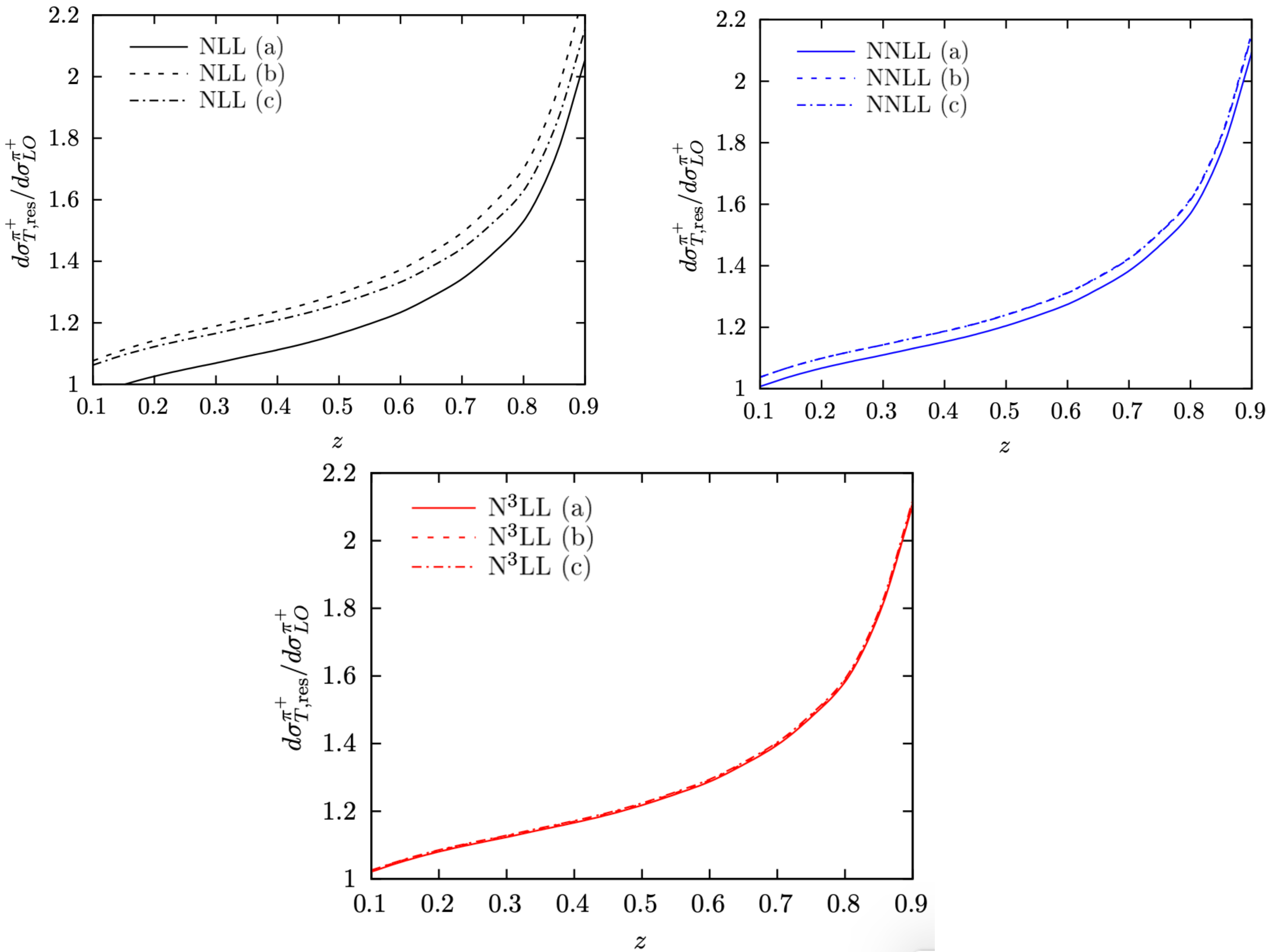
$$\tilde{\omega}_{qq}^{\rm res}(N,M,\alpha_s) \;=\; e_q^2\, H_{qq}(\alpha_s)\, \widehat{C}_{qq}(\alpha_s) \times \exp\left\{ \int_{Q^2/(\bar{N}\bar{M})}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A_q(\alpha_s(\mu)) \ln\left(\frac{\mu^2 \bar{N}\bar{M}}{Q^2}\right) - \frac{1}{2}\widehat{D}_q(\alpha_s(\mu)) \right] \right\}$$

$$\left(1+\frac{\alpha_s}{\pi} H_{qq}^{\rm SIDIS,(1)}+\ldots\right)\left(1+\frac{\alpha_s}{\pi}\widehat{C}_{qq}^{(1)}+\ldots\right)$$

vs. $$1+\frac{\alpha_s}{\pi}\left(H_{qq}^{\rm SIDIS,(1)}+\widehat{C}_{qq}^{(1)}\right)+\ldots$$

$$\bar{N} \equiv N{\rm e}^{\gamma_E},\; \bar{M} \equiv M{\rm e}^{\gamma_E}$$

$$\lambda = \frac{b_0\alpha_s}{2}\left(\ln\bar{N}+\ln\bar{M}\right) = \frac{b_0\alpha_s}{2}\left(\ln N+\ln M\right) + \alpha_s b_0 \gamma_E$$

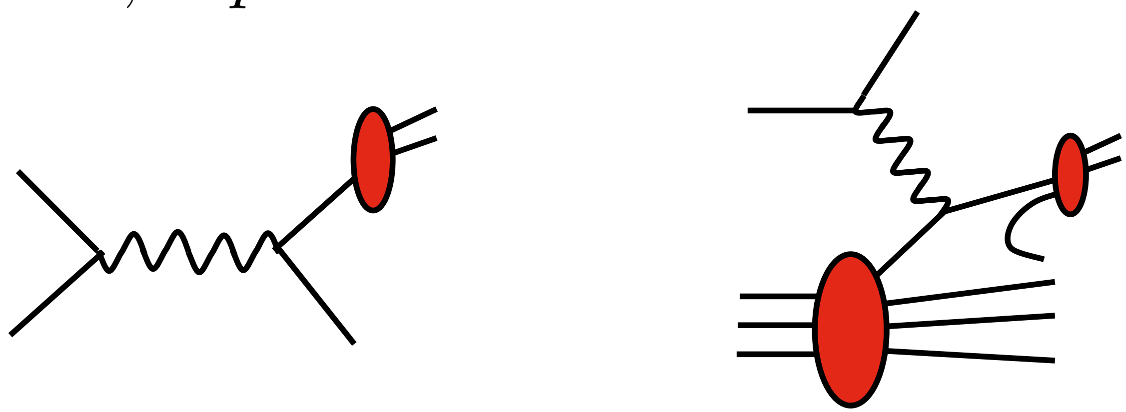

NLL (a)
NLL (b)
NLL (c)
$d\sigma^{\pi^+}_{T,\mathrm{res}}/d\sigma^{\pi^+}_{LO}$
z
NNLL (a)
NNLL (b)
NNLL (c)
$d\sigma^{\pi^+}_{T,\mathrm{res}}/d\sigma^{\pi^+}_{LO}$
z
N^{3}LL (a)
N^{3}LL (b)
N^{3}LL (c)
$d\sigma^{\pi^+}_{T,\mathrm{res}}/d\sigma^{\pi^+}_{LO}$
z

Fragmentation functions

Borsa, De Florian, Sassot, Stratmann, WV

- global analysis of fragmentation functions at "nearly NNLO"

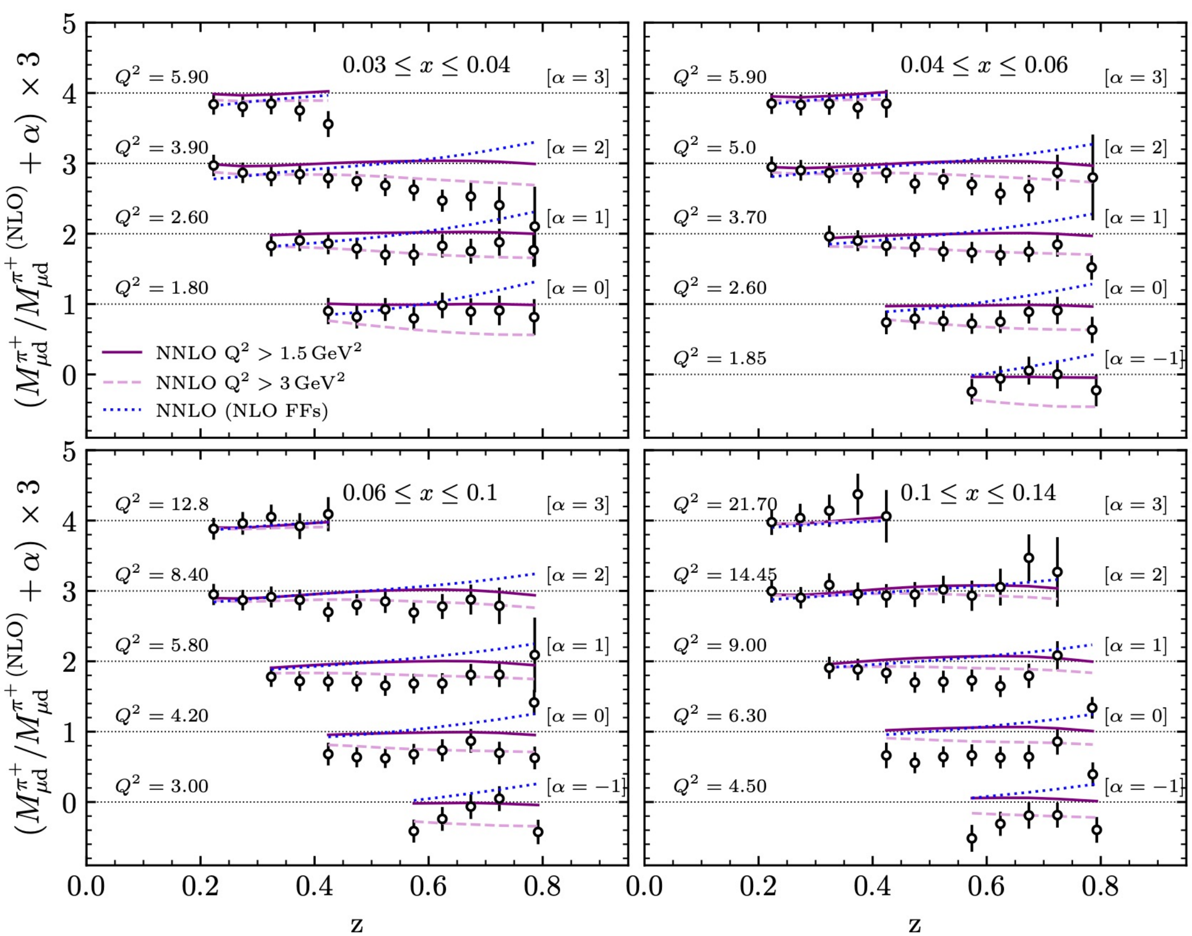
$$e^+e^- \rightarrow hX, \quad ep \rightarrow hX$$

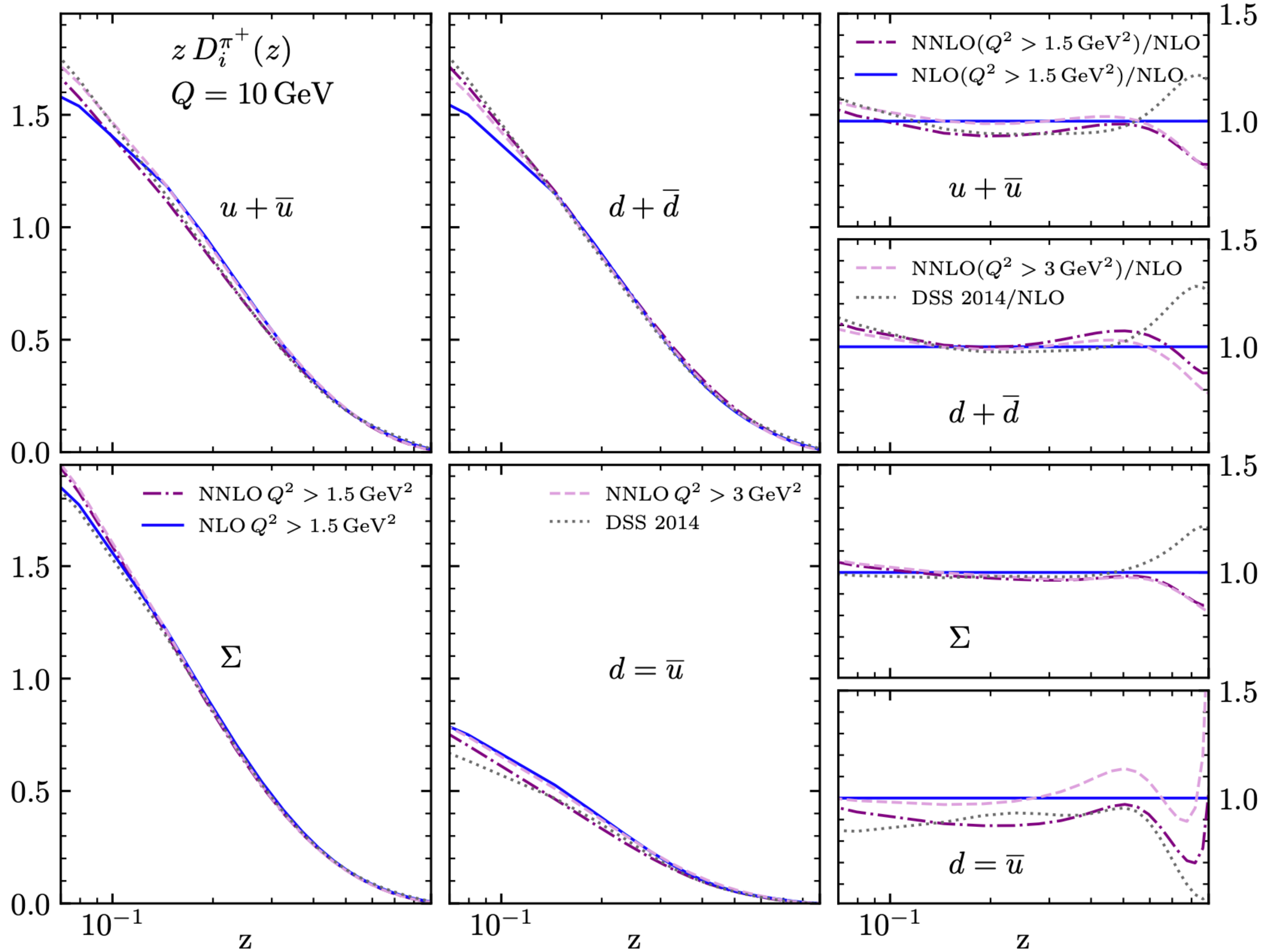


- NNLO for e^+e^-: Anderle, Ringer, Stratmann
  Abdolmaleki et al.
  Salajeghhej, Kniehl, et al.

$\chi^2/\#\text{data points}$

Experiment	$Q^2 \geq 1.5\,\text{GeV}^2$			$Q^2 \geq 2.0\,\text{GeV}^2$			$Q^2 \geq 2.3\,\text{GeV}^2$			$Q^2 \geq 3.0\,\text{GeV}^2$		
	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO
SIA	288	1.05	0.96	288	0.91	0.87	288	0.90	0.91	288	0.93	0.86
COMPASS	510	0.98	1.14	456	0.91	1.04	446	0.91	0.92	376	0.94	0.93
HERMES	224	2.24	2.27	160	2.40	2.08	128	2.71	2.35	96	2.75	2.26
TOTAL	1022	1.27	1.33	904	1.17	1.17	862	1.17	1.13	760	1.16	1.07





$z\,D_i^{\pi^+}(z)$
$Q = 10\,\mathrm{GeV}$
$u+\overline{u}$
$d+\overline{d}$
Σ
$d=\overline{u}$
NNLO $Q^2 > 1.5\,\mathrm{GeV}^2$
NLO $Q^2 > 1.5\,\mathrm{GeV}^2$
NNLO $Q^2 > 3\,\mathrm{GeV}^2$
DSS 2014
NNLO($Q^2 > 1.5\,\mathrm{GeV}^2$)/NLO
NLO($Q^2 > 1.5\,\mathrm{GeV}^2$)/NLO
NNLO($Q^2 > 3\,\mathrm{GeV}^2$)/NLO
DSS 2014/NLO
z

Conclusions:

- generally modest beyond-NLO effects, but non-negligible towards high-z. Mostly captured by NNLO.

- reduction in scale dependence

- may be used for early NNLO phenomenology of PDFs and FFs

- benchmark for future NNLO calculations

- further extensions: F_L, qg contribution, interplay with power corrections