The partonic structure of nucleons and nuclei from lattice QCD

Phiala Shanahan, MIT

Image Credit: 2018 EIC User's Group Meeting

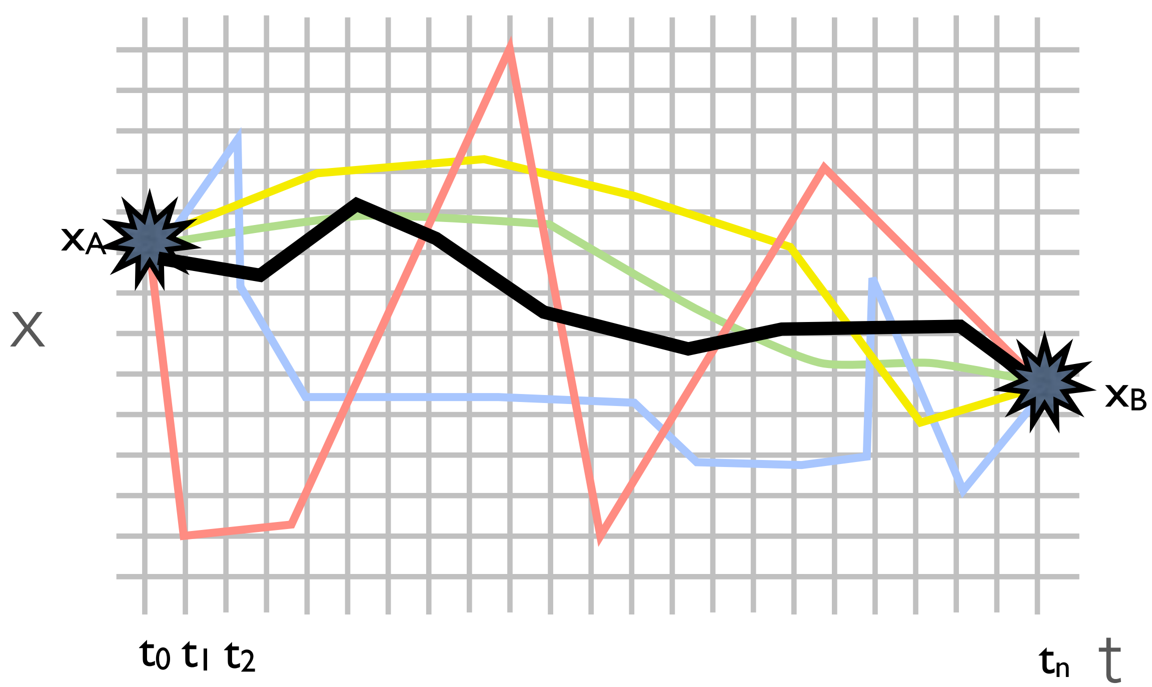
2022 CTEQ Spring Meeting

Massachusetts Institute of Technology

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- QCD equations ⟷ integrals over the values of quark and gluon fields on each site/link (QCD path integral)
- ~10^{12} variables (for state-of-the-art)

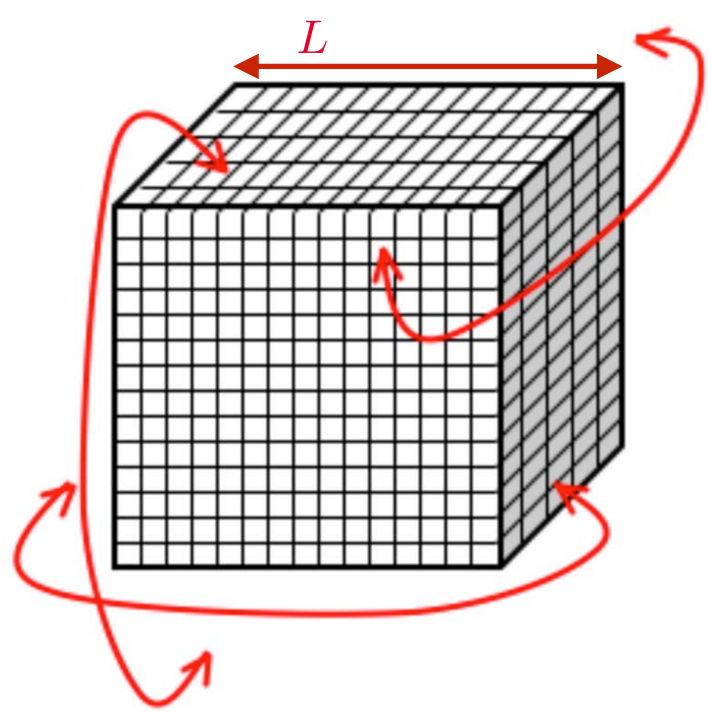


- Evaluate by importance sampling
- Paths near classical action dominate
- Calculate physics on a set (ensemble) of samples of the quark and gluon fields

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Euclidean space-time
  - Non-zero lattice spacing
  - Finite volume
- Some calculations use larger-than-physical quark masses (cheaper)



Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A, \overline{\psi}\psi]} \rightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\mathrm{conf}}} \sum_{i}^{N_{\mathrm{conf}}} \mathcal{O}([U^i])$$

with field configurations U^i distributed according to $e^{-S[U]}$

Lattice QCD

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Numerical first-principles approach to
non-perturbative QCD
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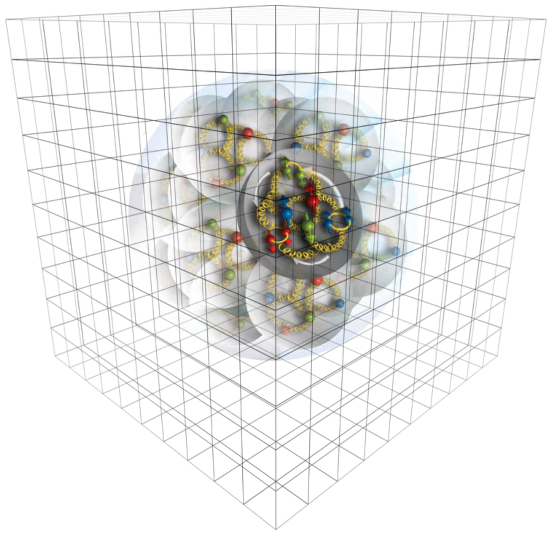
INPUT

Lattice QCD action has same free parameters as QCD: quark masses, α_S

- Fix quark masses by matching to measured hadron masses, e.g., π, K, D_s, B_s for u, d, s, c, b
- One experimental input to fix lattice spacing in GeV (and also α_S), e.g., $2S$-$1S$ splitting in Υ, or f_π or Ω mass

OUTPUT

Calculations of all other quantities are QCD predictions



Lattice QCD

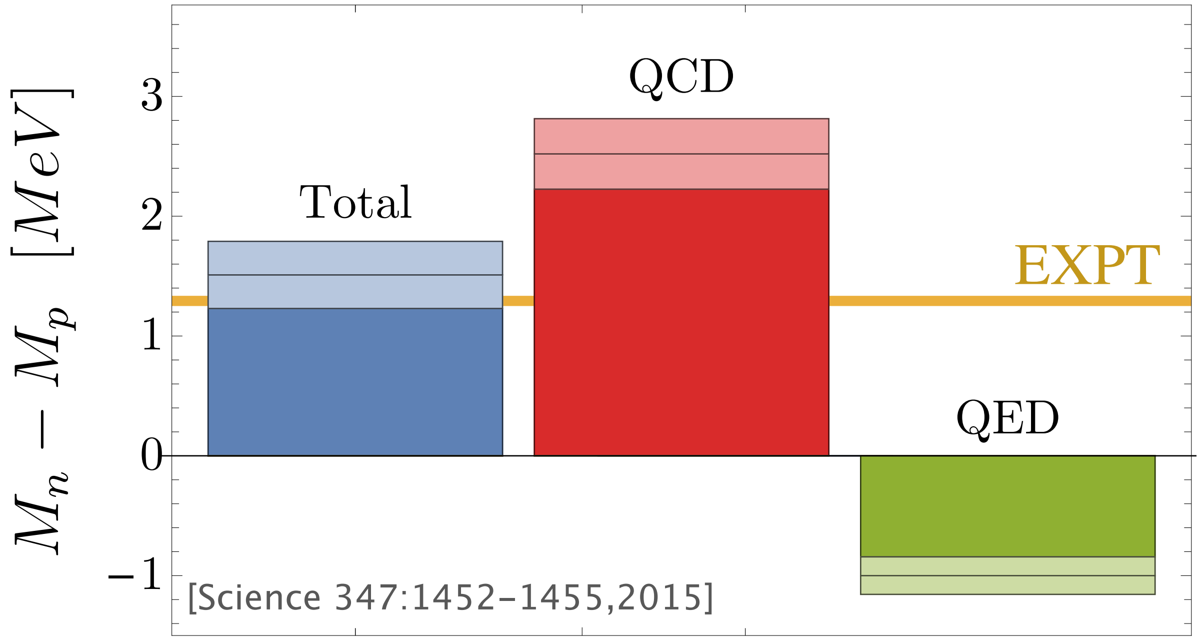
Numerical first-principles approach to non-perturbative QCD

Calculations use world's largest computers

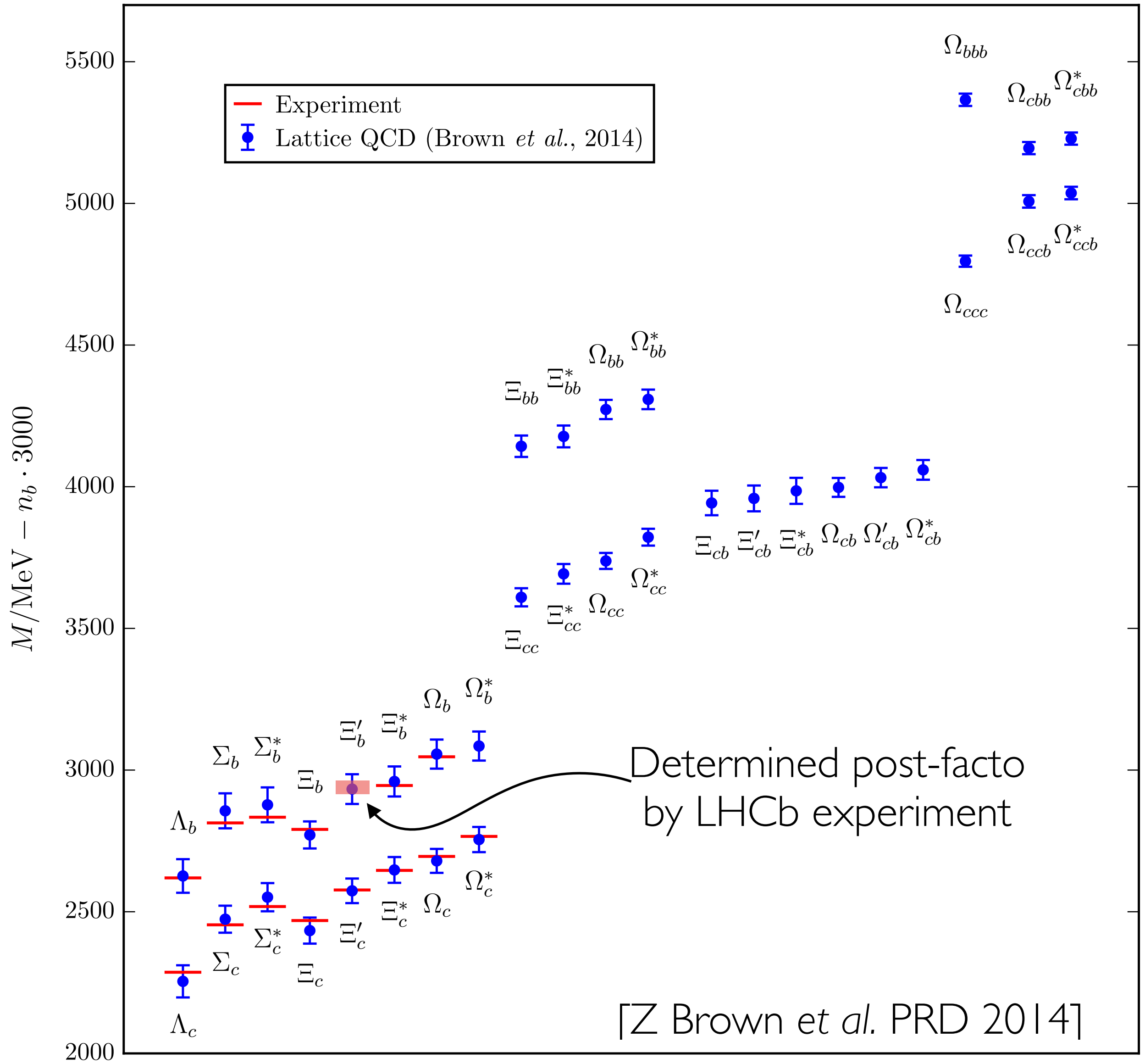
- Many millions of CPU/GPU hours

Lattice QCD works

- Ground state hadron spectrum reproduced
- p-n mass splitting reproduced
- …



- Predictions for new states with controlled uncertainties



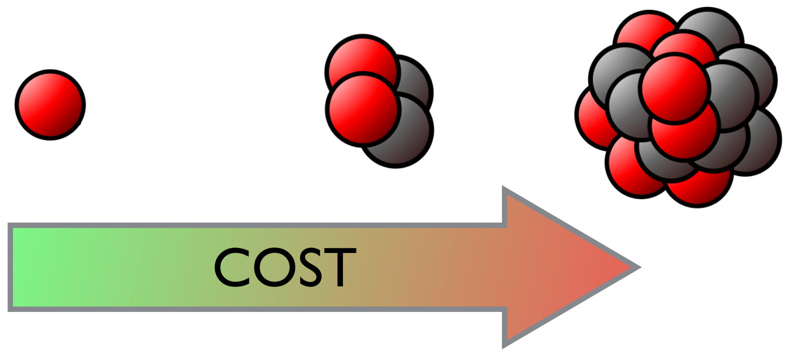
Phiala Shanahan, MIT

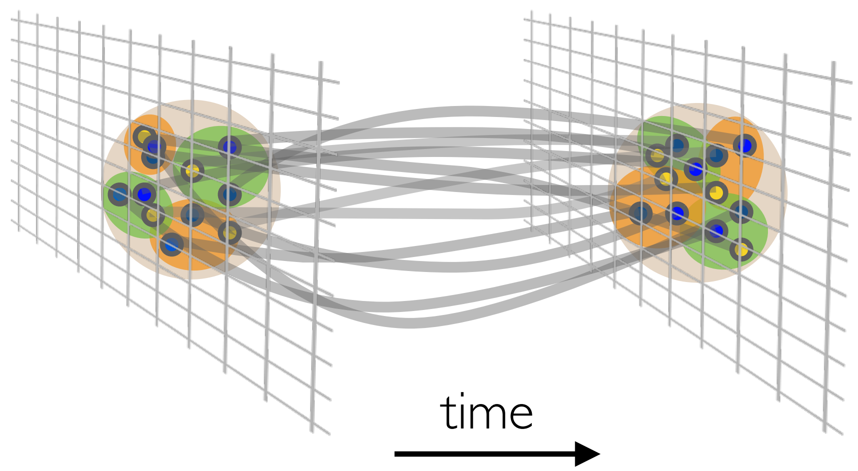
Nuclear physics from lattice QCD

Nuclei on the lattice are HARD

- **Noise:**
  Statistical uncertainty grows exponentially with number of nucleons

- **Complexity:**
  Number of contractions grows factorially



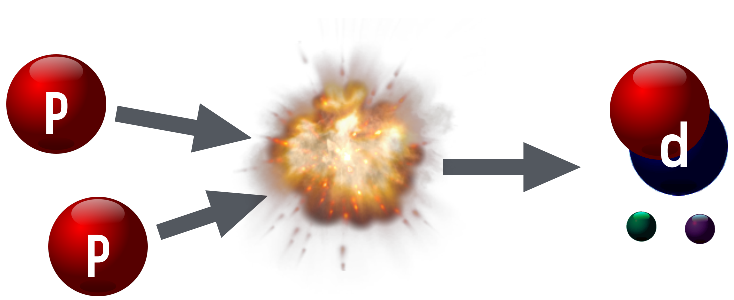


Calculations possible for A<5

Nuclear physics from lattice QCD

"Nuclear physics from LQCD Collaboration"
NPLQCD

- Nuclei with A<5 unphysical quark masses
- Physical-mass calculations begun 2021

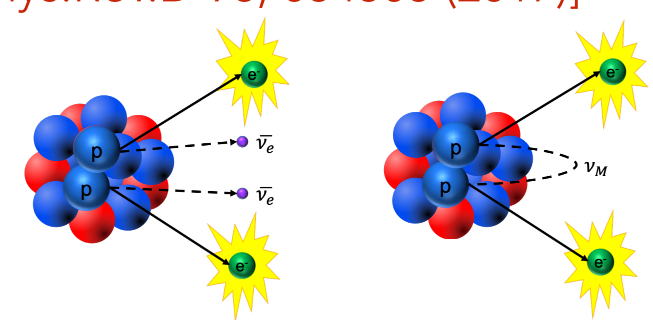


Proton-proton fusion and tritium β-decay
[Phys.Rev.Lett. 119, 062002 (2017)]

Scalar, axial, tensor matrix elements
[Phys.Rev.Lett. 120 (2018), Phys.Rept. 900 (2021), Phys.Rev.D 103, 074511(2021)]

Baryon-baryon interactions, incl. QED
[Phys.Rev.D 96, 114510 (2017), Phys.Rev.D 103, 054504 (2021), Phys.Rev.D 103, 054508 (2021), 2108.10835 (2021)]

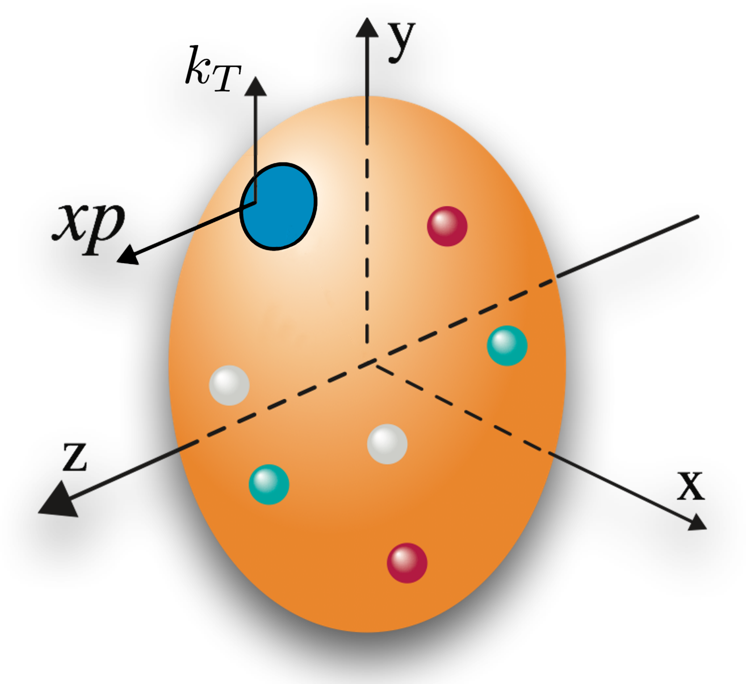
Double β-decay
[Phys.Rev.Lett. 119, 062003 (2017), Phys.Rev.D 96, 054505 (2017)]



Nuclear parton distribution functions
[Phys.Rev.D 96, 094512 (2017), Phys.Rev.Lett. 126, 202001 (2021)]

Parton physics from Lattice QCD

Understanding the quark and gluon structure of matter



Three-dimensional partonic structure of the proton

PDF
$f_{q/H}(x)$
longitudinal

TMD
$f_{q/H}(x, k_T)$
+ transverse

$f_{q/H}(x, b_T)$
+ impact parameter

Fourier transform
$\xi = 0$

GPD
$H_{q/H}(x, \xi, \Delta_T^2)$

Community overview:
Constantinou et al.,
"2020 PDFLattice Report"
arXiv:2006.08636

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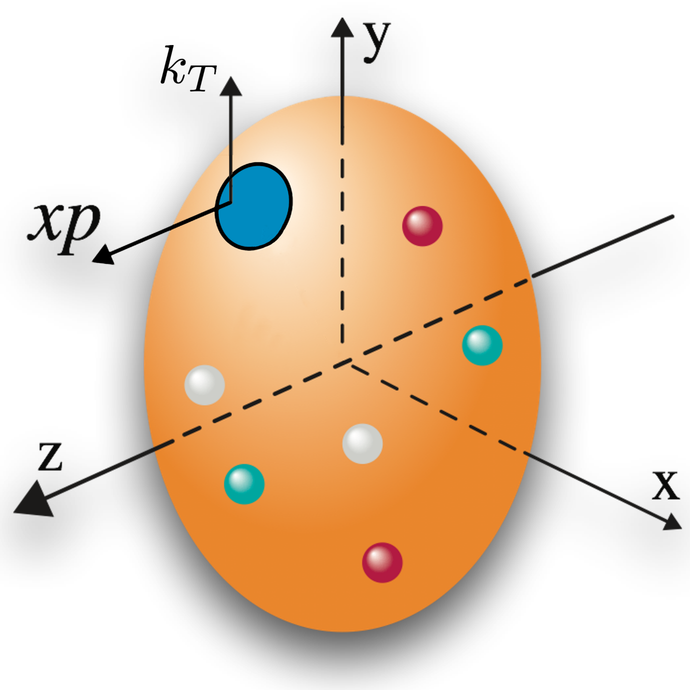

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Decomposition of proton momentum

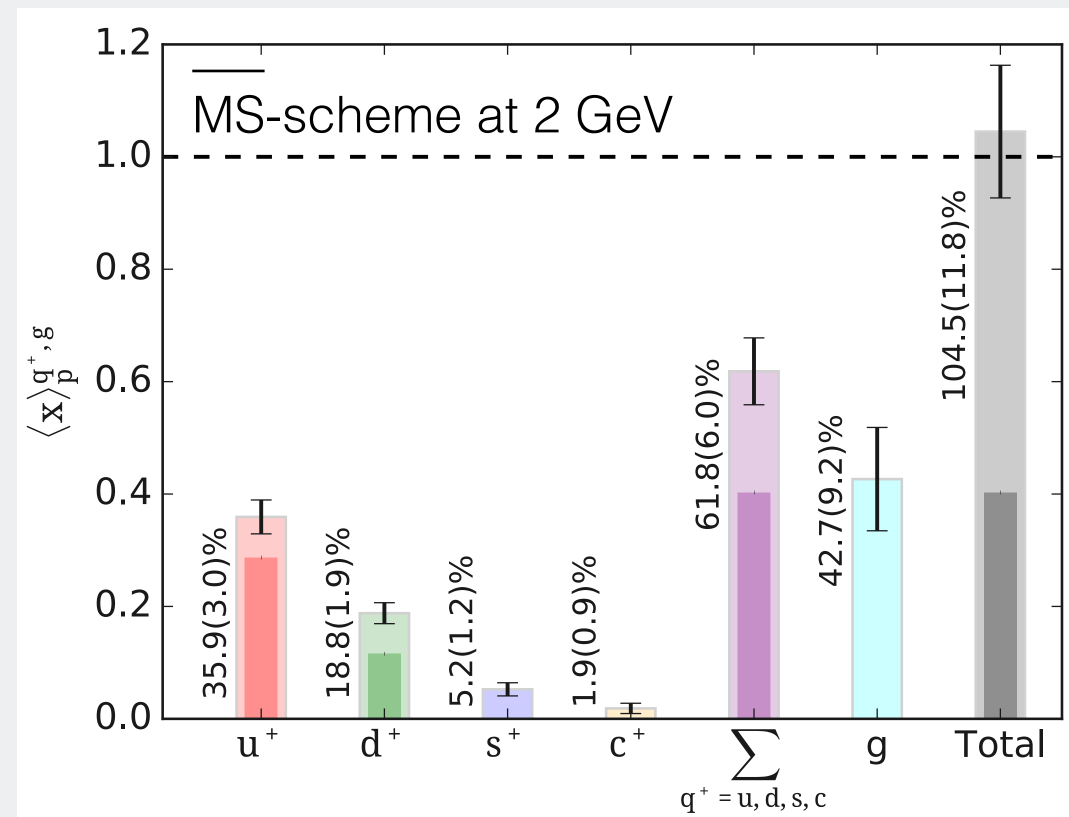
Moments of PDFs encode key aspects of hadron structure

$$\int_0^1 dx\, x^n f(x,\mu^2) = \langle x^n \rangle_f (\mu^2)$$

- Lattice QCD can cleanly access low moments of PDFs (n ≲ 3)
- Massive community efforts to study x-dependence from LQCD + perturbative matching and other approaches
  [Ji, PRL 110 (2013) 262002, Radyushkin, PRD 96 (2017) 034025, Ma & Qiu, PRL 120 (2018) 022003, Braun & Müller, EPJ C55 (2008) 349; Chambers et al., PRL 118 (2017) 242001, Detmold & Lin, PRD 73 (2006) 014501, Liu & Dong, PRL 72 (1994) 1790+many more]

Example: Lowest moment defines contribution of each type of parton to the hadron momentum

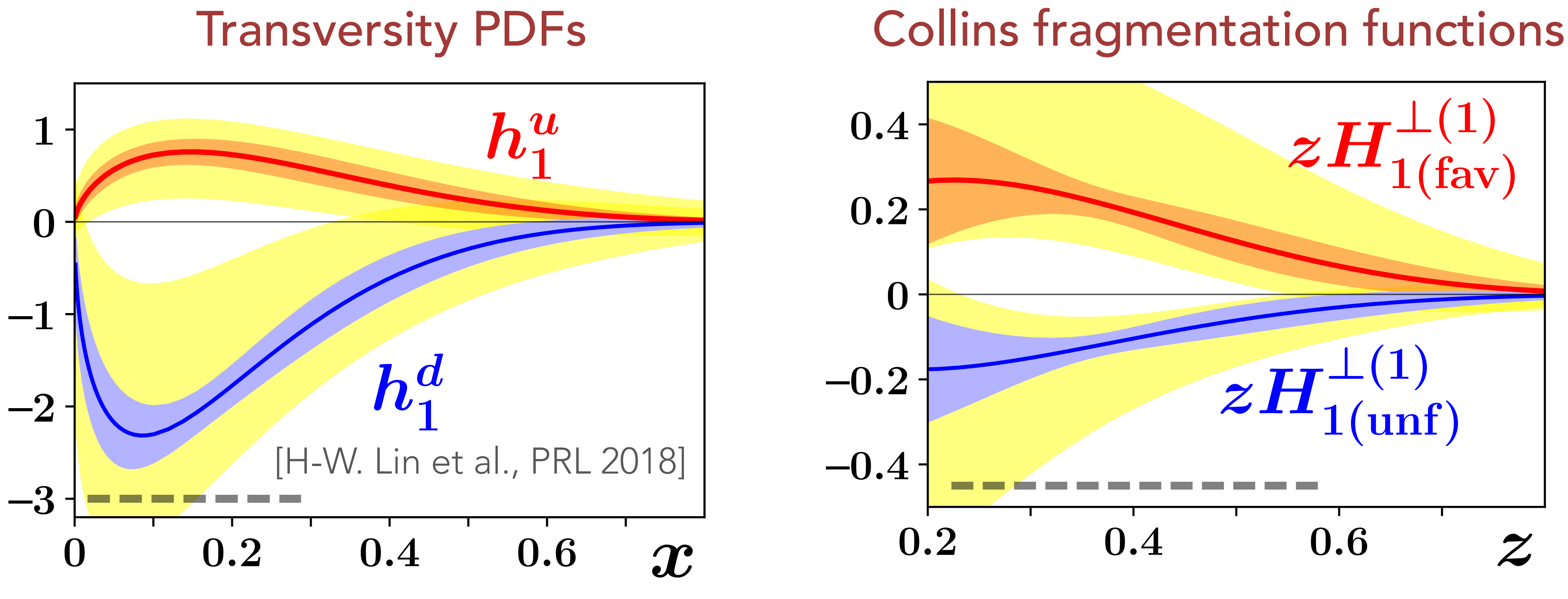
2020 Highlight:
Proton momentum decomposition



[C. Alexandrou et al., PRD 101 (2020)]

Constraints on global PDF fits

- Including lattice QCD results for moments in global PDF fits can yield significant improvements
- Community white paper (LQCD + phenomenologists) assessed potential impacts [Lin et al., Prog. Part. Nucl. Phys 100 (2018), 107]

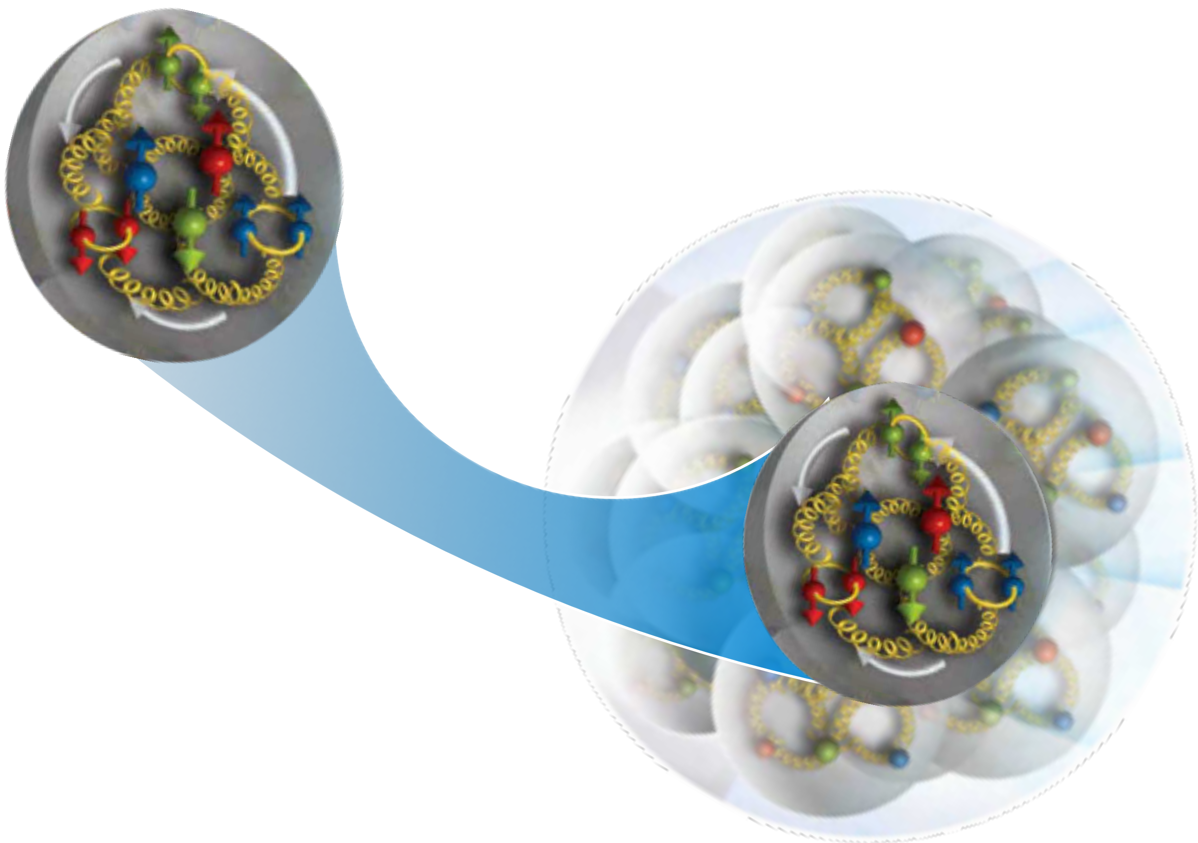


Yellow: SIDIS data only: direct constraints in region indicated by dashes
Blue/Red: SIDIS + lattice QCD for tensor charge (zeroth moment)

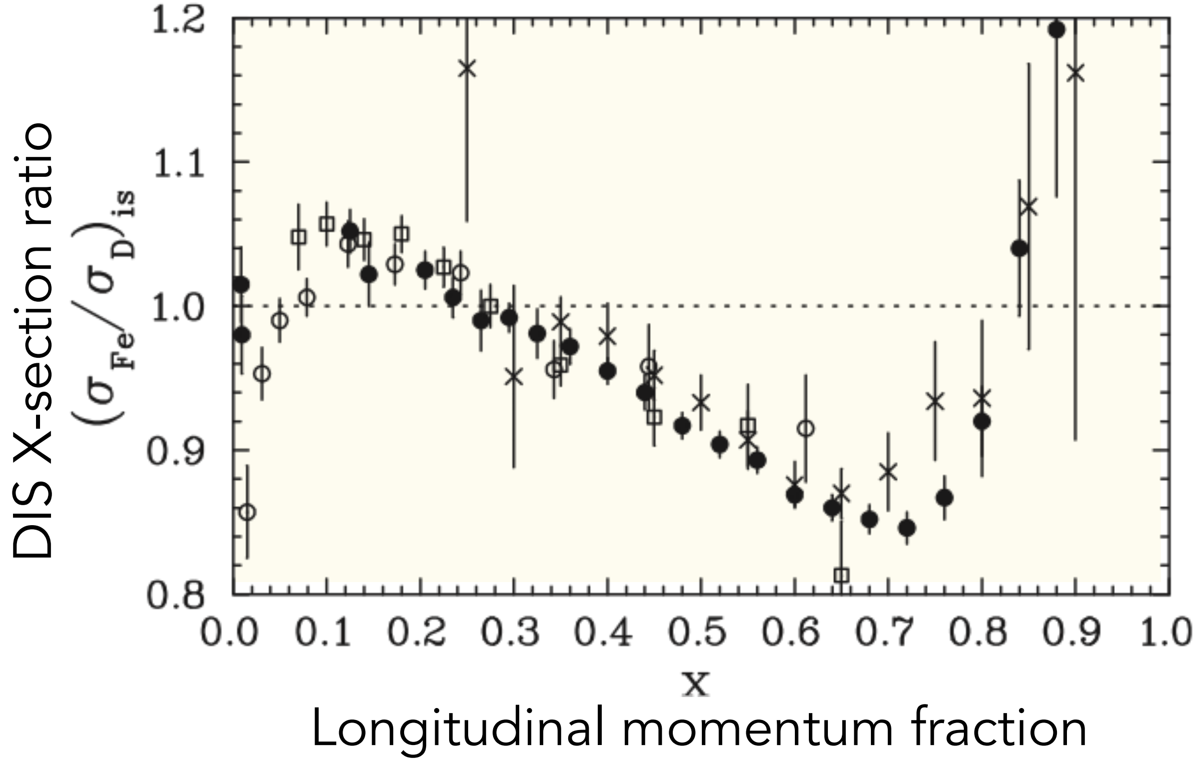
The structure of matter

Understanding the quark and gluon structure of matter

How is the partonic structure of nuclei different from that of nucleons?



Encoded in EMC-type effects



(EMC: Aubert et al., 1983)

EMC-type effects from Lattice QCD

Understanding the quark and gluon
structure of matter

Many aspects of EMC effects will be accessible at a future Electron-Ion Collider

Cover image from EIC whitepaper arXiv::1212.1701

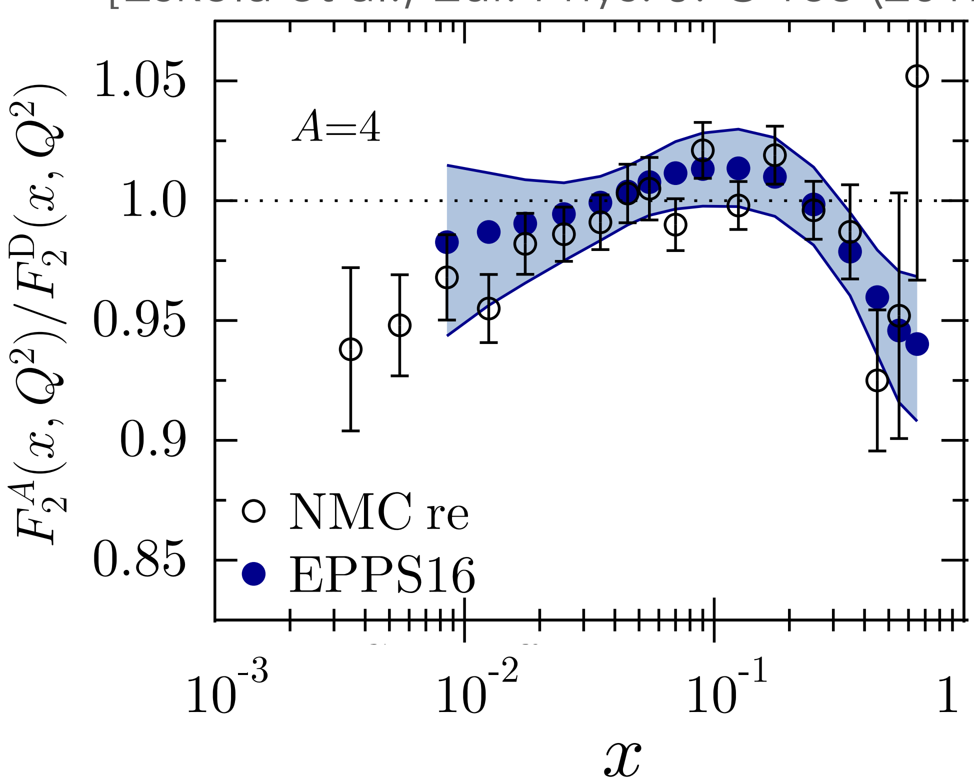
- Polarised EMC (polarised light ions)
- Isovector EMC (SIDIS)
- Gluon EMC (quarkonium production)
- LQCD will make predictions!

EMC effects in Mellin moments

First investigation of EMC-type effects from LQCD:
Nuclear effects in Mellin moments of PDFs

- Calculable from local operators
- BUT EMC effects in moments are very small

[Eskola et al., Eur. Phys. J. C 163 (2017)]



Classic EMC effect is defined in F_2:

$$F_2(x,Q^2) = \sum_{q=u,d,s\ldots} x\, e_q^2\, [q(x,Q^2) + \overline{q}(x,Q^2)]$$

Number density of partons of flavour q

→ x-integrals of numerator and denominator $\int_0^1 dx\, x^n q(x,Q^2)$

EMC effects in Mellin moments

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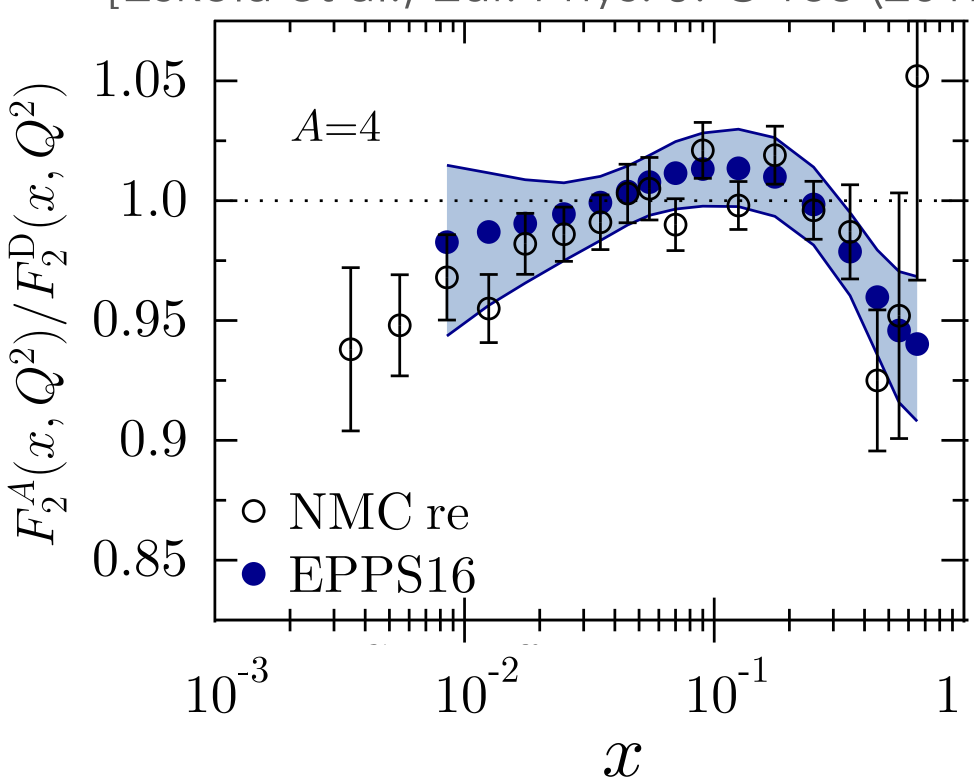
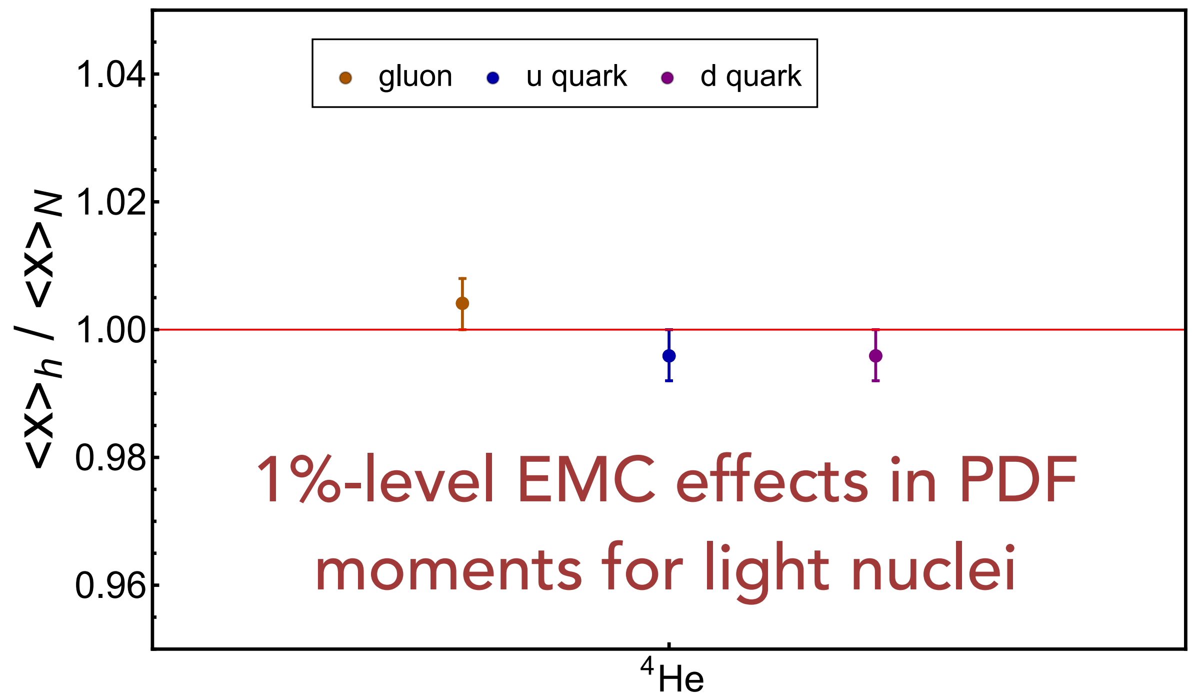
[Eskola et al., Eur. Phys. J. C 163 (2017)]





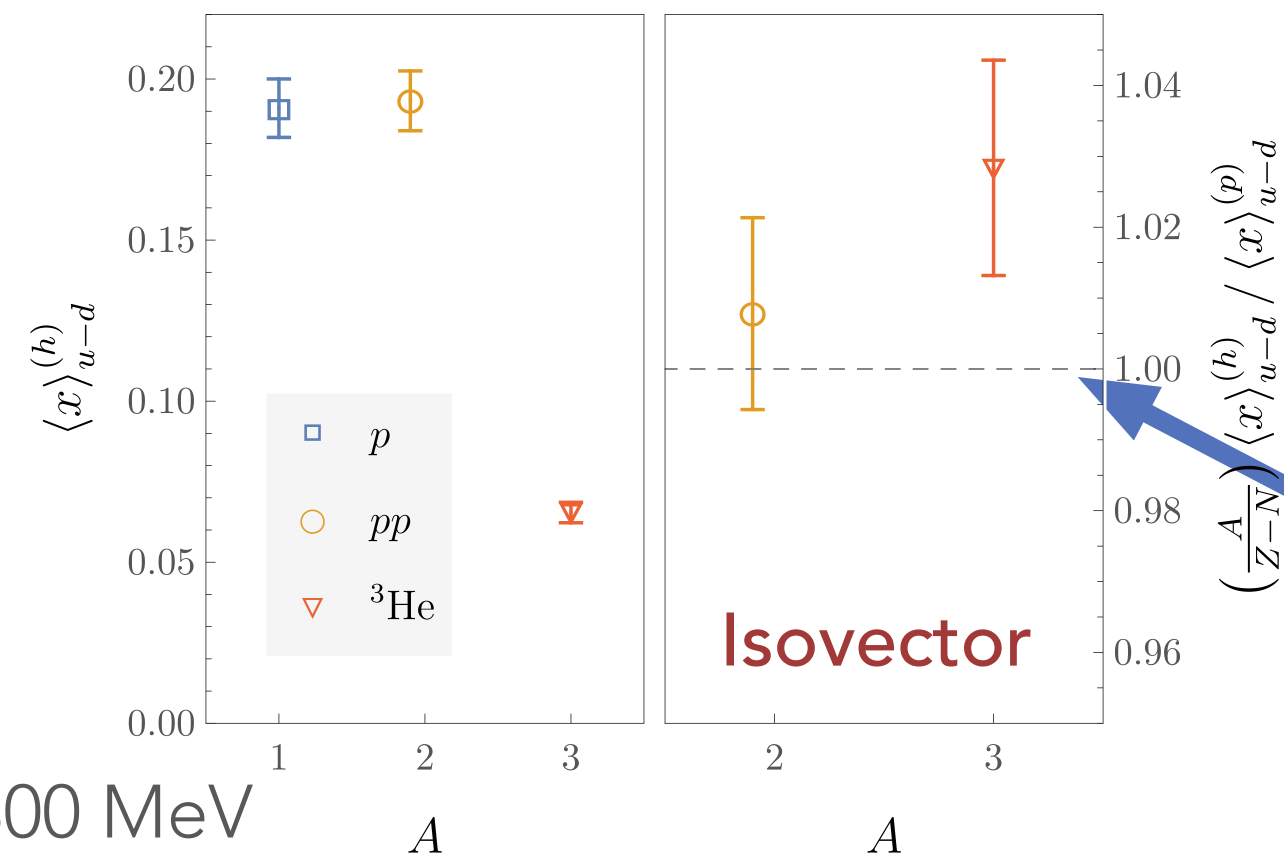
Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

→ **first QCD determination of momentum fraction of nuclei**

- Bounds on EMC effect in moments at ~few percent level, consistent with phenomenology [2009.05522 [hep-lat] (2021)]

Ratio of quark momentum fraction in nucleus to nucleon



Normalised to proton result

- No mixing
- No sum rule constraint

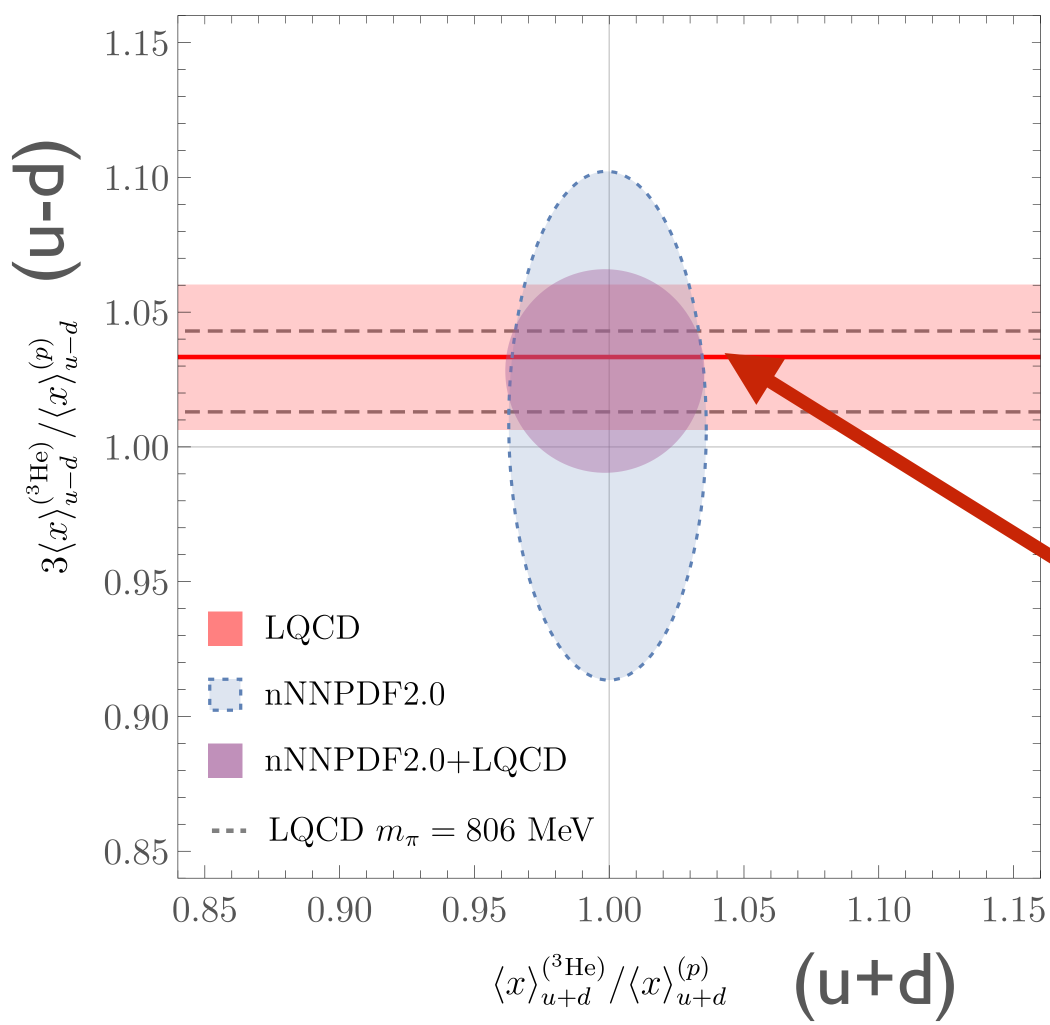
m_π ~800 MeV

[NPLQCD PRL 126, 202001 (2021) [2009.05522]]

Momentum fraction of ^{3}He

Matrix elements of the Energy-Momentum Tensor in light nuclei
→ **first QCD determination of momentum fraction of nuclei**

Ratio of ^{3}He to proton momentum fractions



- Match isovector (u-d quark combination) momentum fraction to low-energy constants of effective field theory, extrapolate to physical quark masses
- Include into nNNPDF global fits of experimental lepton-nucleus scattering data

Blue → Purple:
Improvement using theory constraints

[NPLQCD PRL 126, 202001 (2021) [2009.05522]]

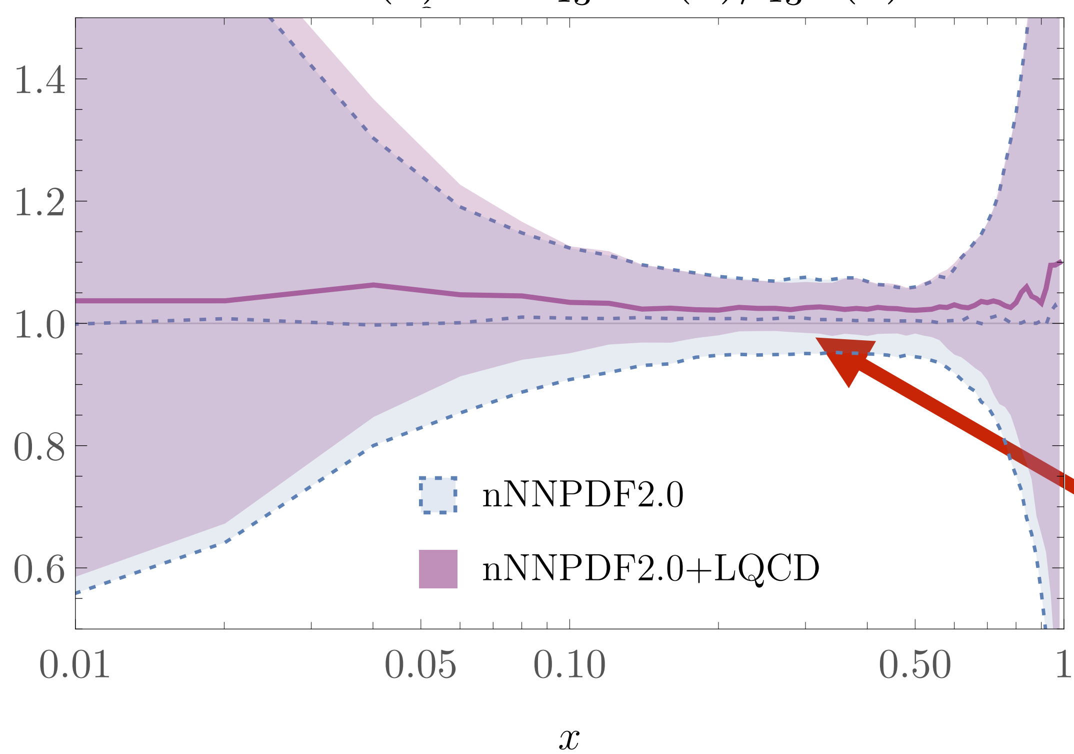
Momentum fraction of ^{3}He

Matrix elements of the Energy-Momentum Tensor in light nuclei

→ **first QCD determination of momentum fraction of nuclei**

Ratio of ^{3}He to proton parton distributions

$$R^{(^3\mathrm{He})}(x) = 3q_3^{(^3\mathrm{He})}(x)/q_3^{(p)}(x)$$



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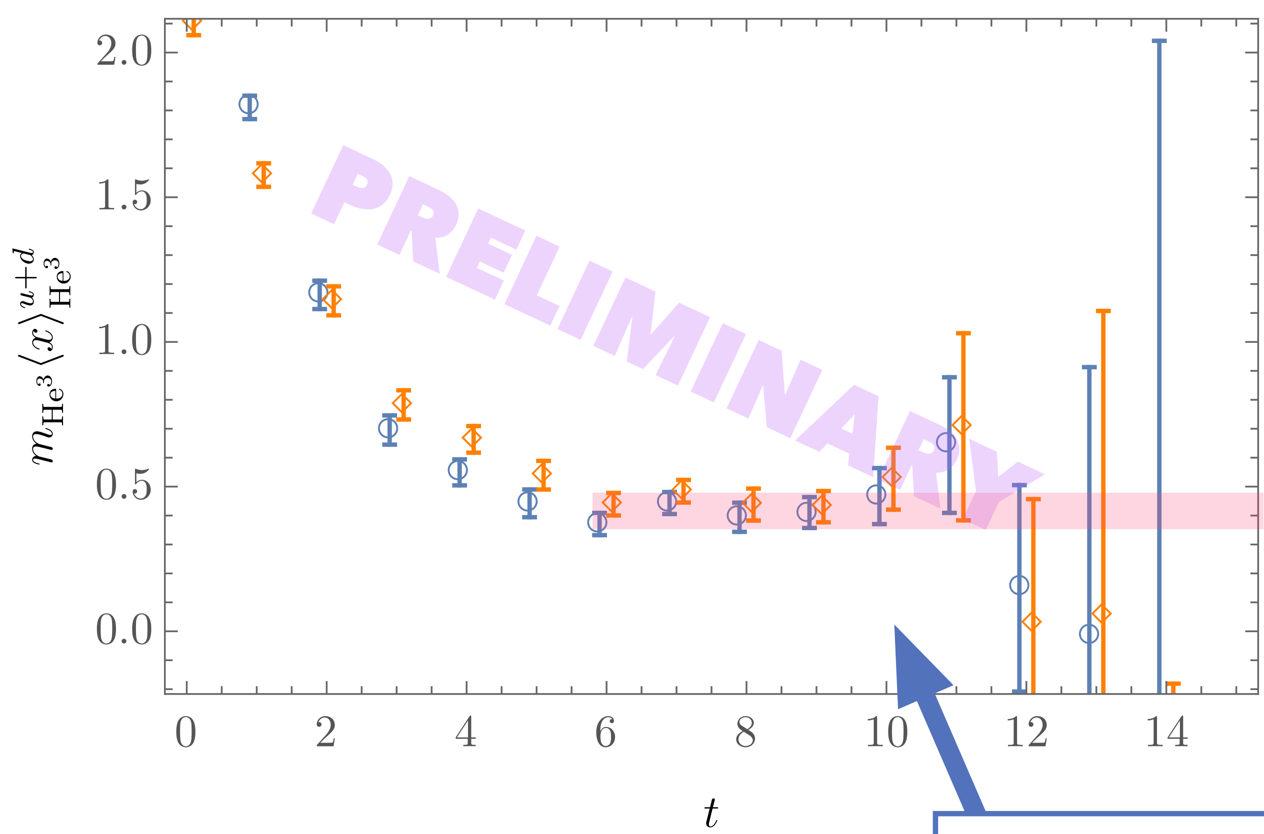
Improvement using theory constraints

[NPLQCD PRL 126, 202001 (2021) [2009.05522]]

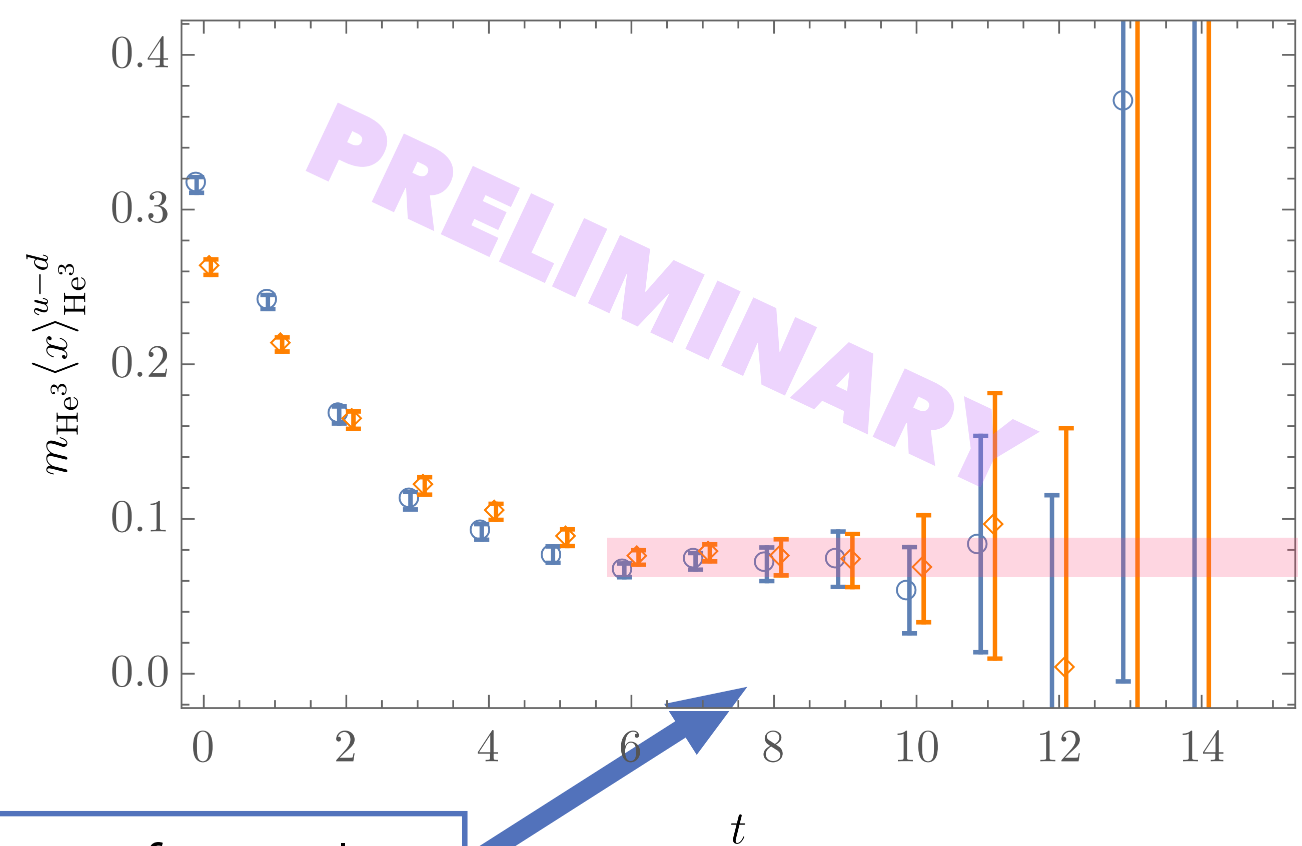
Momentum fraction of ^{3}He

- Work in progress at close-to-physical values of the quark masses

Isoscalar



Isovector



First evidence of signals in physical-point data

- Polarised PDFs also accessible from moments

Exotic glue in the deuteron

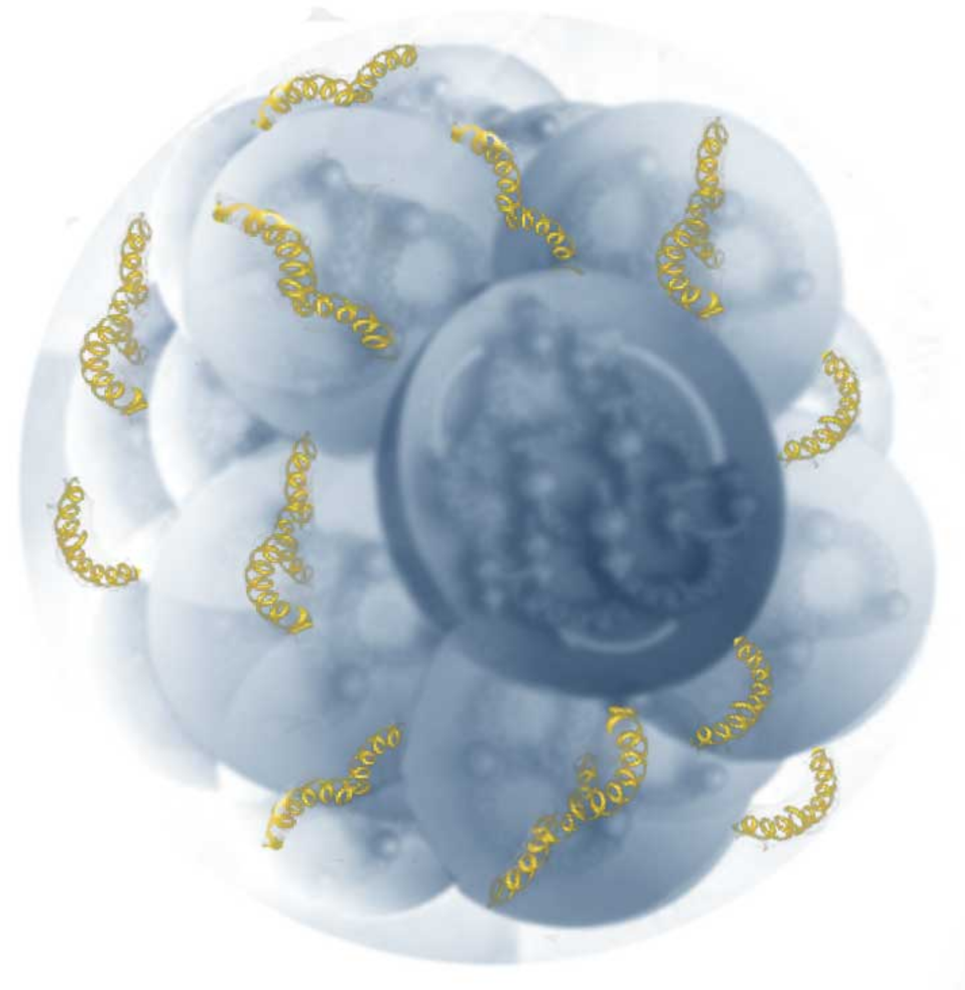
a "pure" EMC-type effect

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

Exotic glue operator:

nucleon $\langle p|\mathcal{O}|p\rangle = 0$

nucleus $\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$

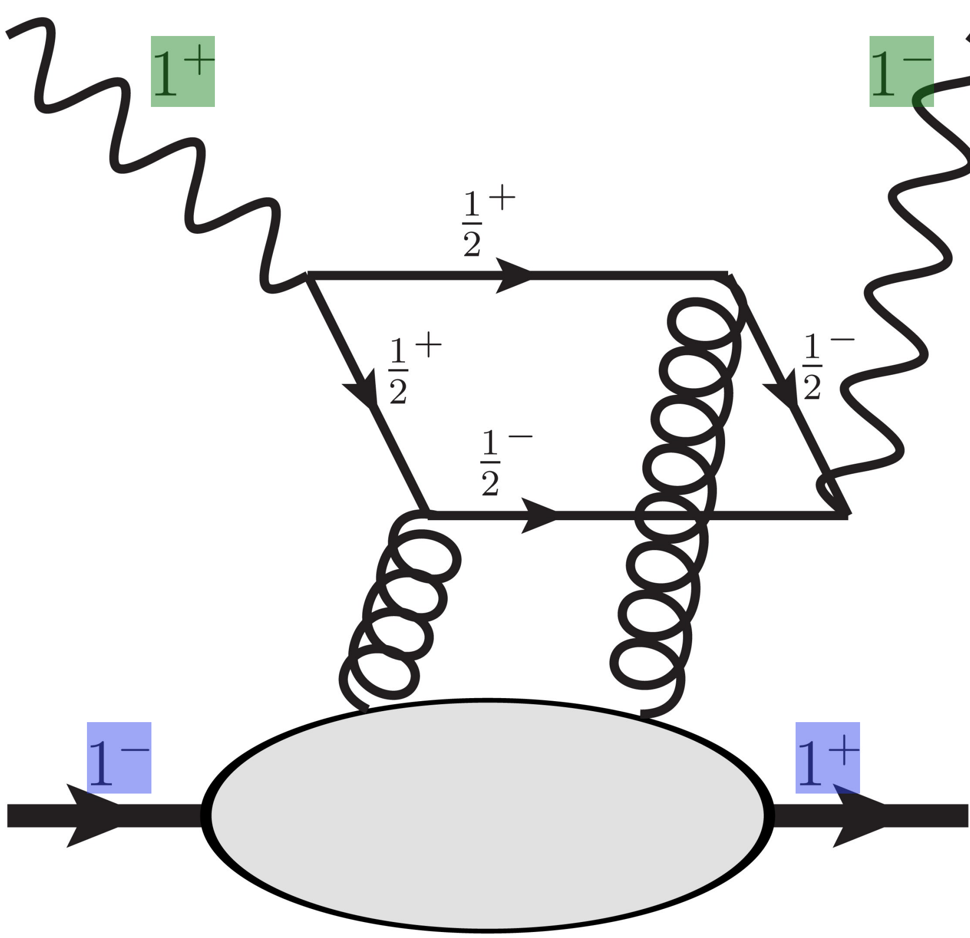


Jaffe and Manohar, "Nuclear Gluonometry"
Phys. Lett. B223 (1989) 218

Exotic glue in the deuteron

a "pure" EMC-type effect

Double helicity flip structure function $\Delta(x,Q^2)$:
changes both photon and target helicity by 2 units



- **Unambiguously gluonic:** no analogous quark PDF at twist-2
- Non-vanishing in forward limit for targets with spin≥1
- **Experimentally measurable**
  - Unpolarised electron DIS on polarised target: JLab LoI 2015, Polarised nuclei at EIC
  - Proton-deuteron Drell-Yan at FNAL
  - J/ψ production at NICA
- **Moments calculable in LQCD**

Exotic glue in the deuteron

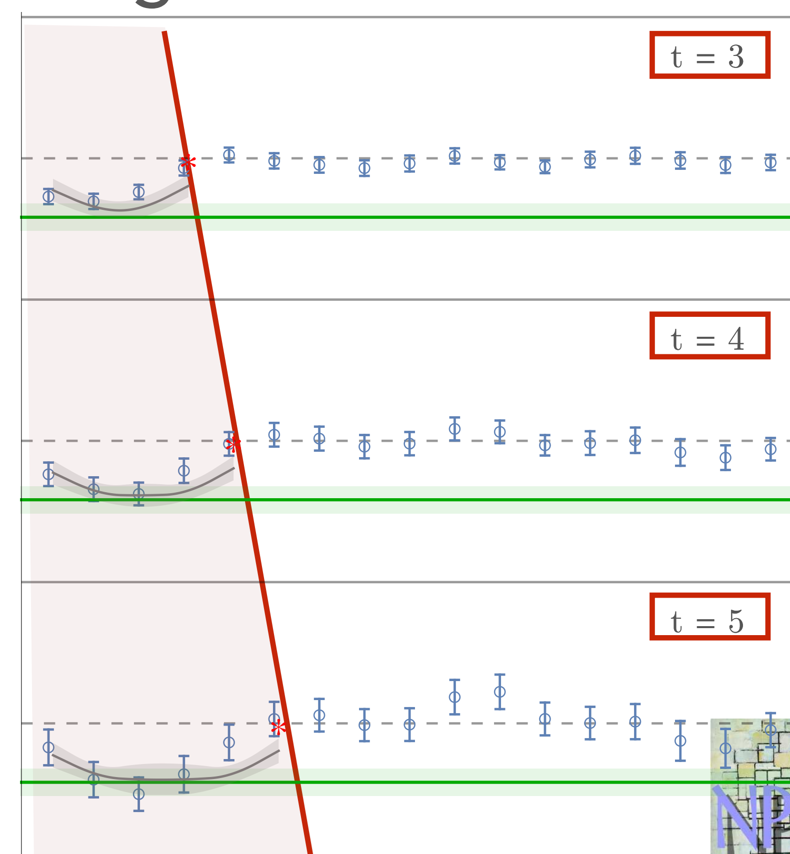
Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

nucleon: $\langle p|\mathcal{O}|p\rangle = 0$

nucleus: $\langle N, Z|\mathcal{O}|N, Z\rangle \neq 0$

- First moment of gluon transversity distribution in the deuteron
  [Jaffe, Manohar PLB223 (1989) 218]
- First evidence for non-nucleonic gluon contributions to nuclear structure: LQCD with m_π ~800 MeV [NPLQCD PRD96 (2017)]
- Magnitude relative to momentum fraction as expected from large-N_c

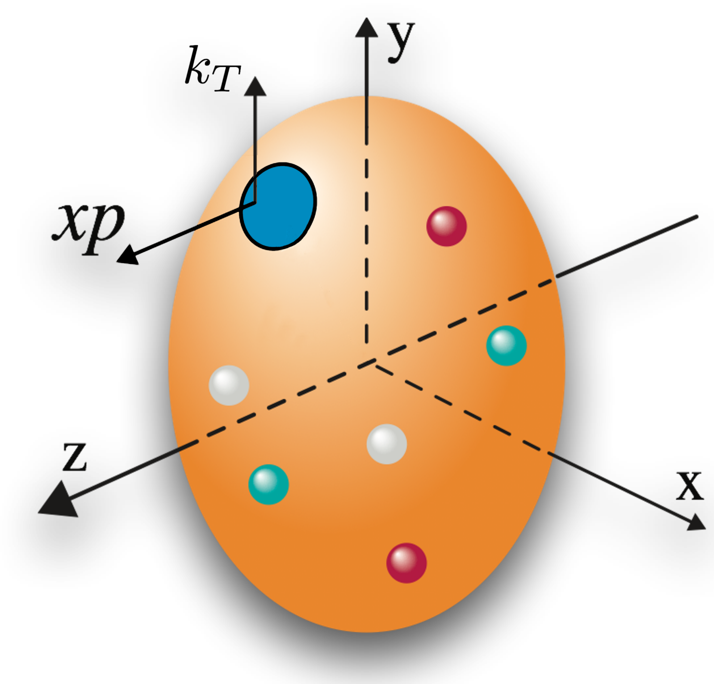
Signal in LQCD data



Phiala Shanahan, MIT

Parton physics from Lattice QCD

Understanding the quark and gluon
structure of matter



Three-dimensional partonic structure of the proton

PDF
$f_{q/H}(x)$
longitudinal

TMD
$f_{q/H}(x, k_T)$
+ transverse

$f_{q/H}(x, b_T)$
+ impact parameter

Fourier transform
$\xi = 0$

GPD
$H_{q/H}(x, \xi, \Delta_T^2)$

Community overview:
Constantinou et al.,
"2020 PDFLattice Report"
arXiv:2006.08636

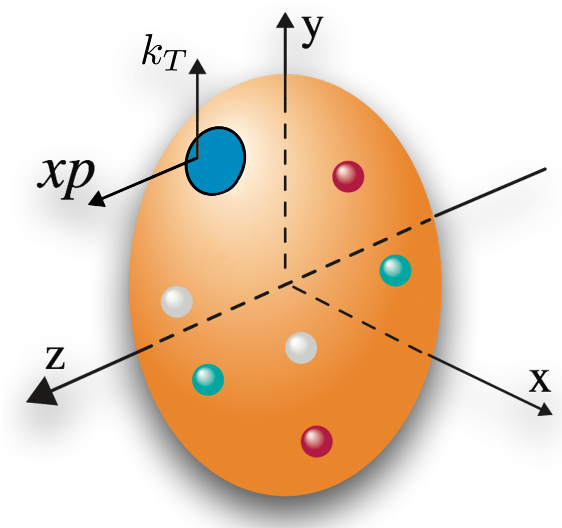
Collins-Soper evolution kernel

- More detailed picture of nucleon including transverse structure

PDF
$f_{q/P}(x)$
longitudinal

→

TMD
$f_{q/P}(x, k_T)$
+ transverse



Collins-Soper Evolution Kernel

$$\gamma_\zeta^q(\mu, b_T) = \zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta)$$

- Governs TMD evolution
- Needed to match quasi-TMD (lattice QCD) to physical TMD

- Perturbative at short distances $\mu, b_T^{-1} \gg \Lambda_{\mathrm{QCD}}$
- Non-perturbative for $b_T^{-1} \lesssim \Lambda_{\mathrm{QCD}}$ → lattice QCD?
- Independent of hadron

Collins-Soper evolution kernel

Estimates of size of nonperturbative contributions to CS kernel

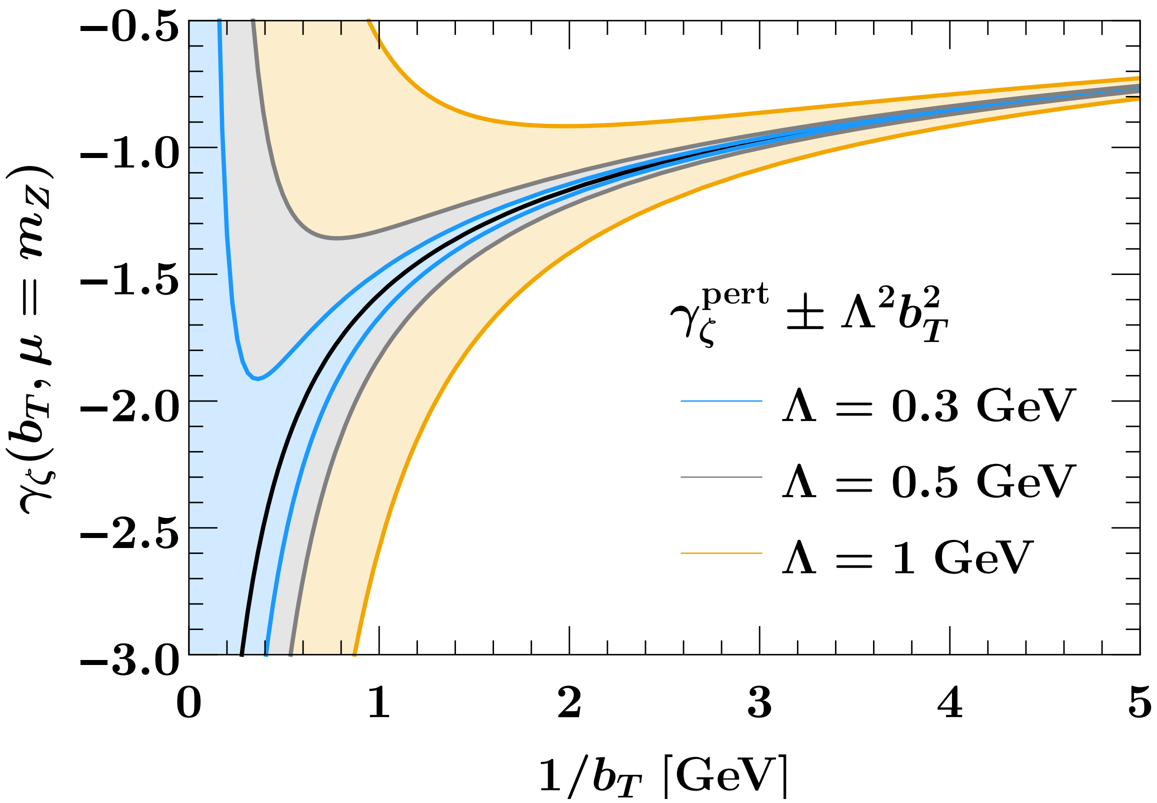


Figure: Iain Stewart

- Large uncertainties in estimates of the size of nonperturbative contributions to the CS kernel

CS kernel from phenomenological fits

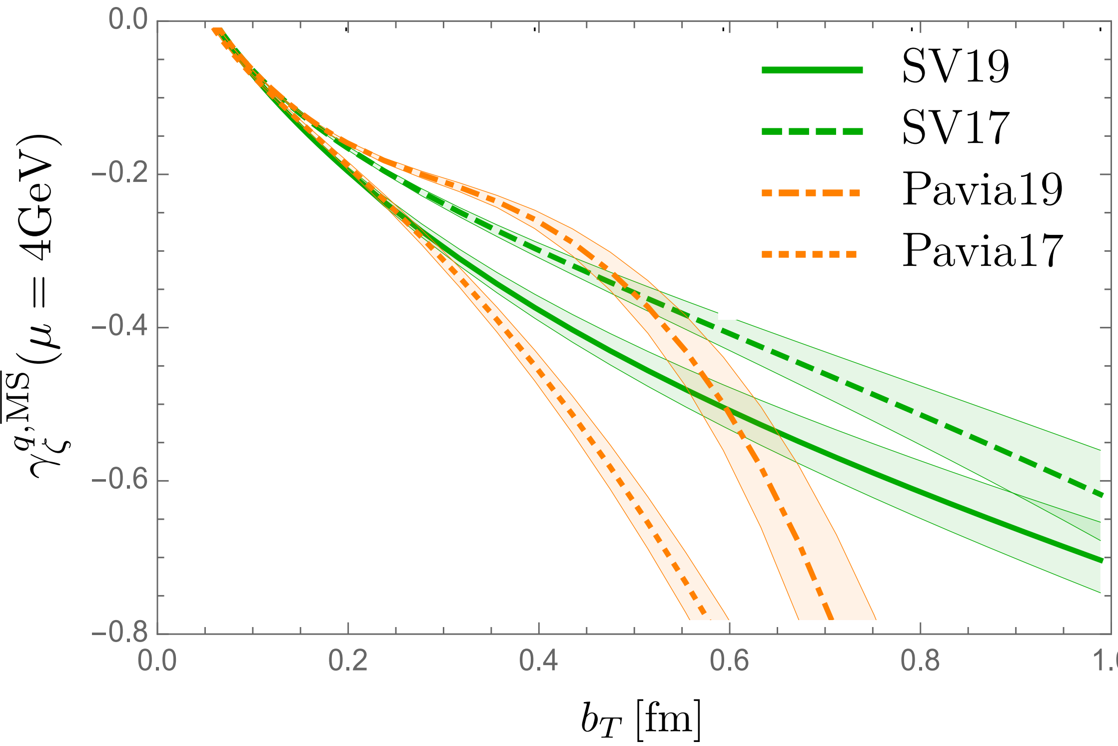


Figure adapted from Vladimirov [2003.02288]
[see also e.g., Kang, Prokudin, Sun,Yuan Phys.Rev.D 93 (2016), Collins, Rogers Phys.Rev.D 91 (2015), Sun, Yuan Phys.Rev.D 88 (2013)]

- LQCD constraints in nonperturbative region with ~10% uncertainties would contribute to model differentiation

TMDPDFs from lattice QCD

PDF
$f_{q/H}(x)$
longitudinal

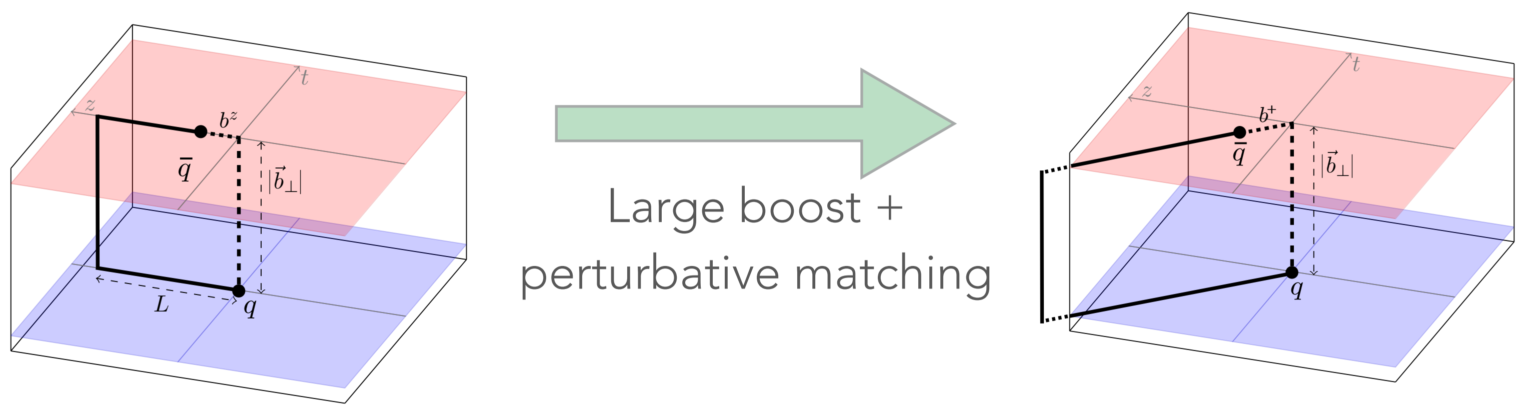
→

TMD
$f_{q/H}(x, k_T)$
+ transverse

TMD Collaboration

- TMDPDF defined by matrix element of non-local light-cone quark bilinear operator
  - Calculate via quasi-TMD prescription
    [Ji, Sun, Xiong,Yuan '14, Ji, Jin, Yuan, Zhang, Zhao, PRD99 (2019), Ebert, Stewart, Zhao, PRD99 (2019), JHEP09 (2019), Ji, Liu, Liu, [1911.03840] (2019), Nucl.Phys.B (2020), Vladimirov, Schäfer, Phys.Rev.D 101 (2020)]

Spatial staple calculable in LQCD



$$\tilde{f}_{u\text{-}d}(x, \vec{b}_T, \mu, P^z) = C^{\text{TMD}}_{u\text{-}d}(\mu, xP^z)\, g^S_q(b_T, \mu) \exp\left[\frac{1}{2}\gamma^q_\zeta(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_{u\text{-}d}(x, \vec{b}_T, \mu, \zeta)$$

quasi-TMD from LQCD | **perturbative matching** | **non-pert. soft factor** | **Collins-Soper evolution kernel** | **desired TMDPDF**

TMDPDFs from lattice QCD

Collins-Soper Evolution Kernel

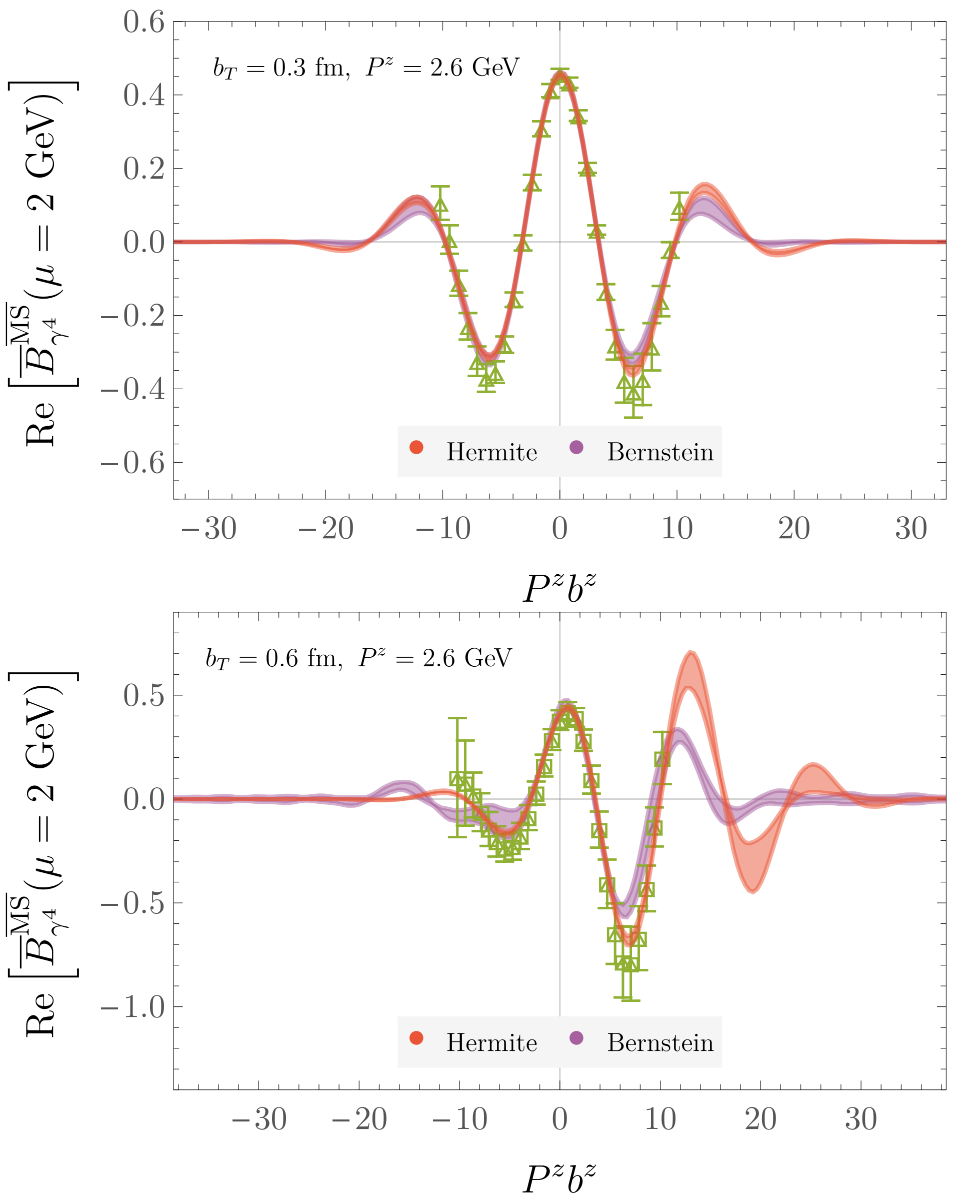
$$\gamma_\zeta^q(\mu, b_T) = \zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta)$$

- Governs TMD evolution
- Needed to match quasi-TMD (lattice QCD) to physical TMD

- Can be accessed via ratio of non-local MEs in LQCD [Ebert, Stewart, Zhao, PRD99 (2019)]

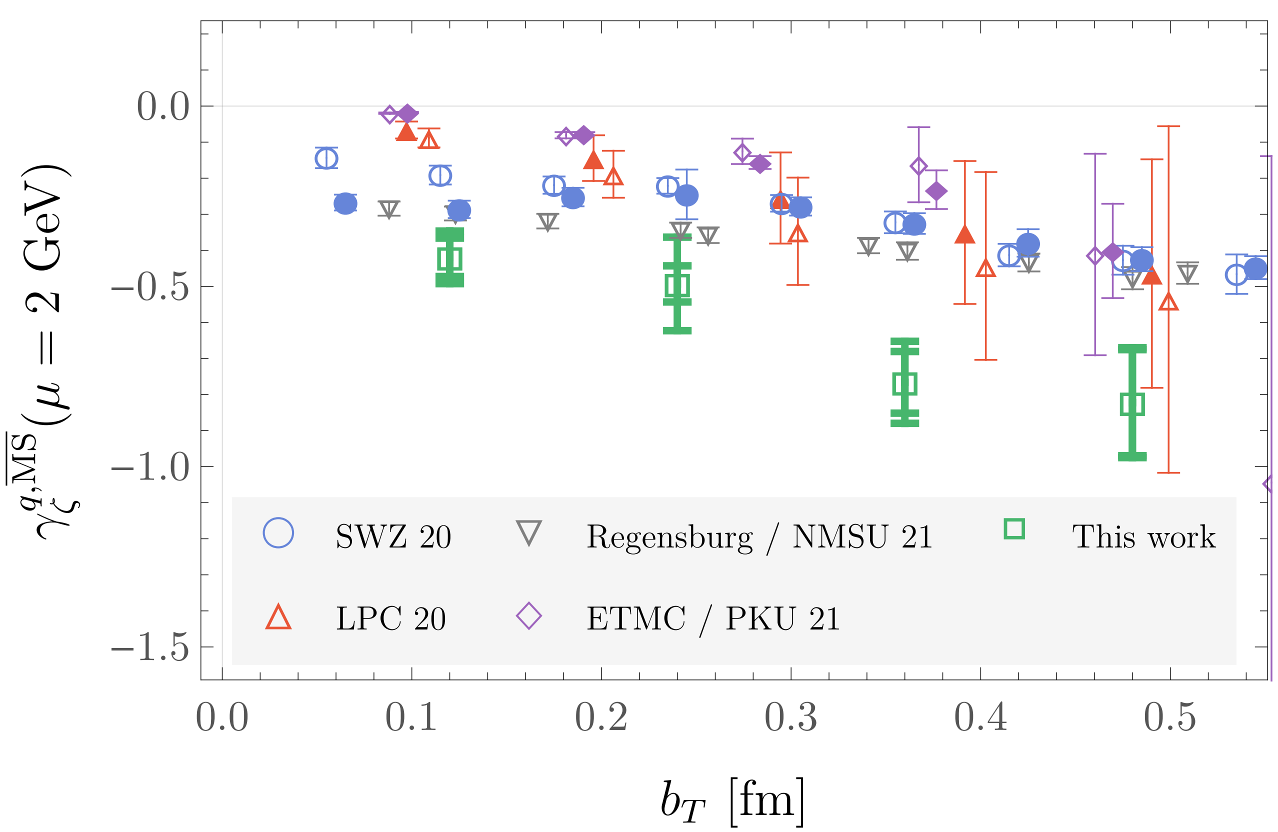
$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)} \times \frac{\int db^z e^{-ib^z x P_1^z} P_1^z \lim_{\substack{a\to 0\\ \eta\to\infty}} B_{\gamma^4}^{\overline{\rm MS}}(\mu, b^z, \vec{b}_T, a, \eta, b_T^R, P_1^z)}{\int db^z e^{-ib^z x P_2^z} P_2^z \lim_{\substack{a\to 0\\ \eta\to\infty}} B_{\gamma^4}^{\overline{\rm MS}}(\mu, b^z, \vec{b}_T, a, \eta, b_T^R, P_2^z)}\right]$$

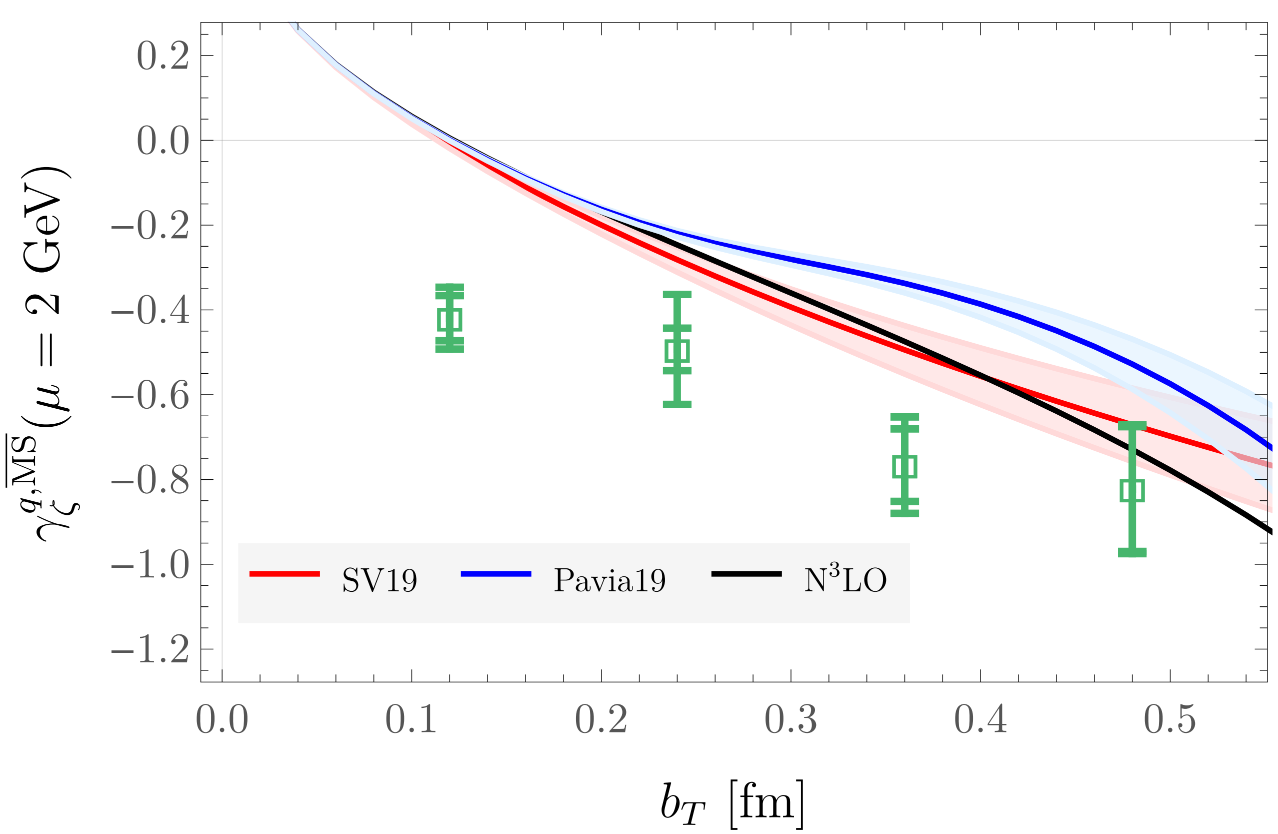
- First (quenched) calculation in 2020 [PES, Wagman, Zhao PRD102, 014511 (2020)]
- CS-kernel independent of state: study unphysically-heavy pion with no systematic bias



Collins Soper kernel from lattice QCD

- First dynamical results 2021
- Currently: Significant spread in results from different LQCD studies from analysis systematics (e.g., NLO matching is important)
- Fully-controlled results at similar precision would already impact phenomenology
- Will require more robust treatment of power corrections





[LPC 2005.14572; Regensburg/NMSU 2103.16991; ETMC/PKU 2106.13027; SWZ 2003.06063, 2107.11930]

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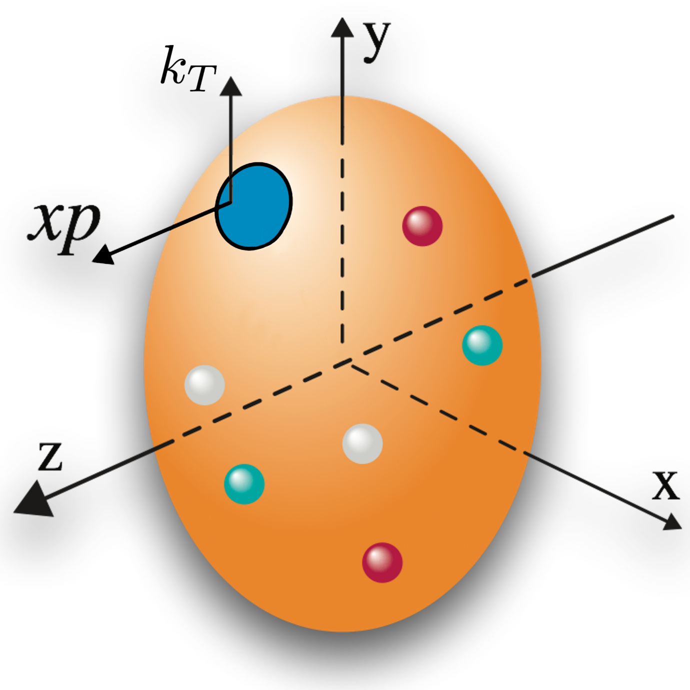

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Community overview:
Constantinou et al.,
"2020 PDFLattice Report"
arXiv:2006.08636

Moments of GPDs

- Moments of GPDs define generalised form factors which encode e.g., Energy-Momentum Tensor
- Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G^{a}_{\{\mu\alpha} G^{a\alpha}{}_{\nu\}} | p, s \rangle = \bar{U}(p', s') \left(A_g(t)\, \gamma_{\{\mu} P_{\nu\}} + B_g(t)\, \frac{i\, P_{\{\mu} \sigma_{\nu\}\rho} \Delta^{\rho}}{2M_N} + D_g(t)\, \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Gluon field-strength tensor

Generalised gluon form factors

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = \Delta^2$

- Sum rules of gluon and quark GFFs in forward limit
  - Momentum fraction $A_a(0) = \langle x \rangle_a$ → $\sum_{a=q,g} A_a(0) = 1$
  - Spin $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ → $\sum_{a=q,g} J_a(0) = \frac{1}{2}$
  - D-terms $D_a(0)$ unknown but equally fundamental!
    - $D_a(t)$ GFFs encodes pressure and shear distributions

D-term from JLab DVCS

Experimental determination of DVCS D-term and extraction of proton pressure distribution
[Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

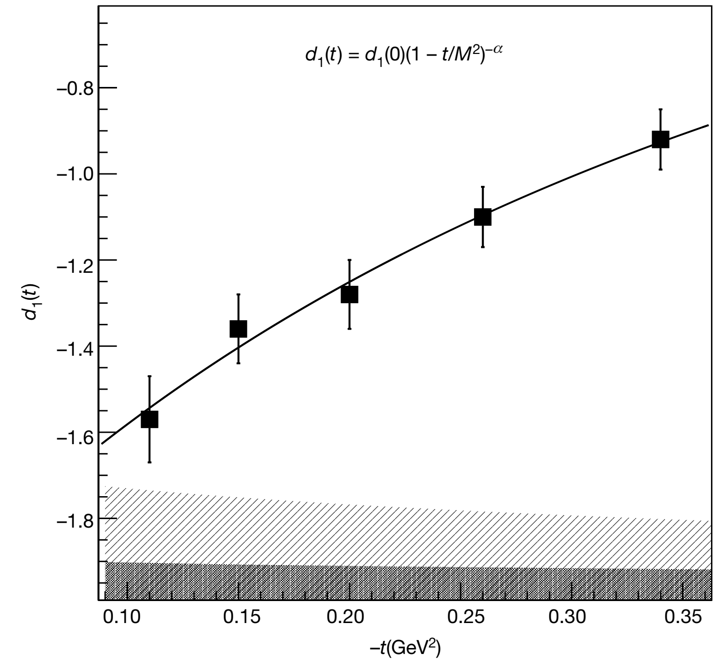
- Peak pressure near centre ~10^{35} Pascal, greater than pressure estimated for neutron stars
- Key assumptions: gluon D-term same as quark term, tripole form factor model, $D_u(t,\mu) = D_d(t,\mu)$

EXP + LQCD

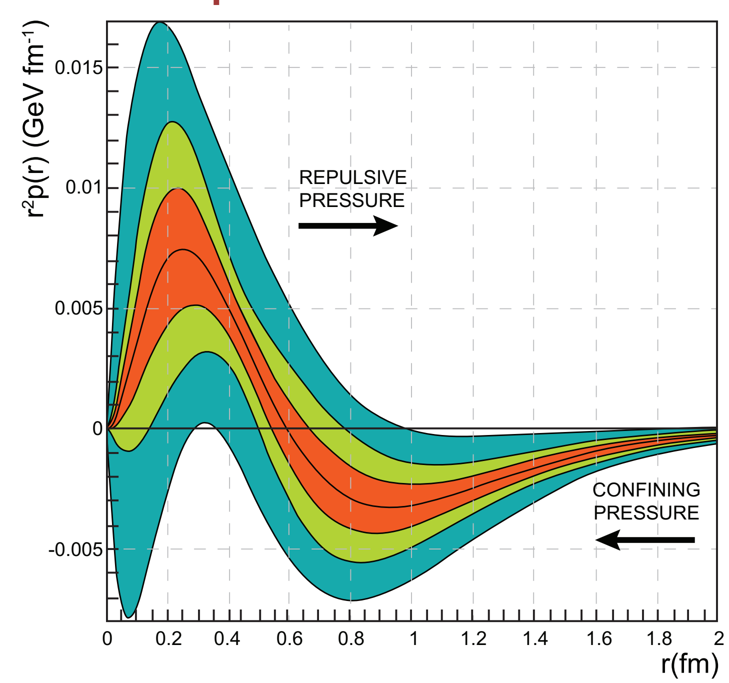
first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

DVCS (quark) D-term



Radial pressure distribution



Nucleon D-term GFFs from LQCD

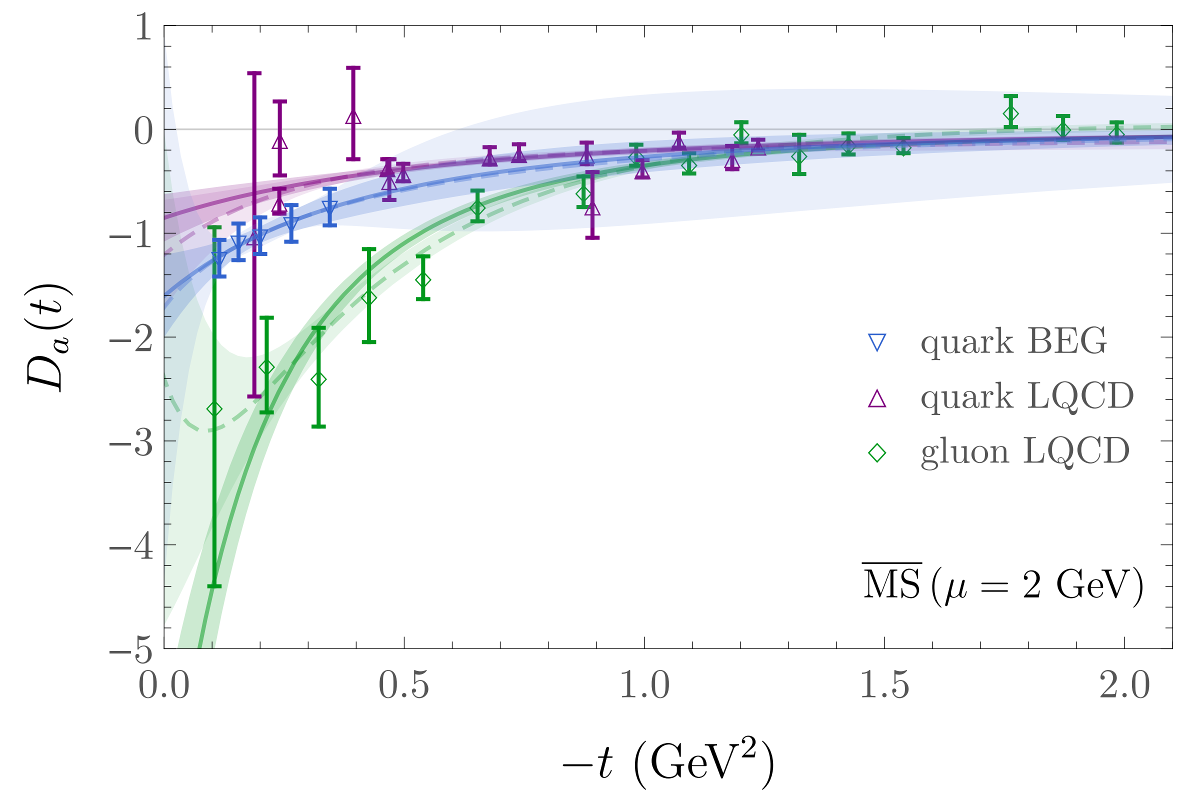
EXP + LQCD

first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

Key assumptions in pressure extraction from DVCS

- Gluon D-term same as quark term in magnitude and shape
  Factor of ~2 difference in magnitude, somewhat different t-dependence
- Tripole form factor model
  LQCD results consistent with ansatz, but more general form is less well constrained
- Isovector quark D-term vanishes
  $D_{u-d}(t) \sim 0$ *from other LQCD studies*



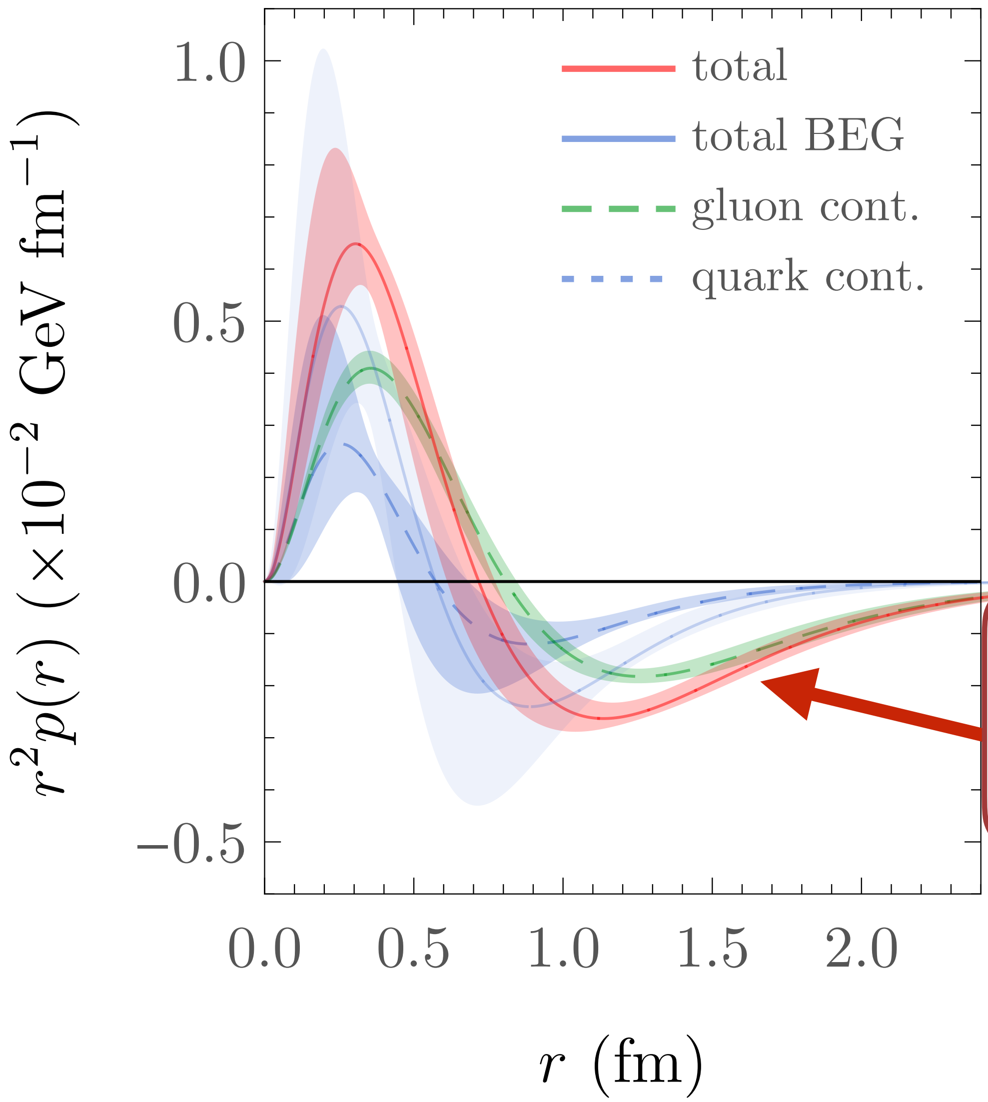
Gluon GFFs: Shanahan, Detmold, PRD 99, 014511
Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

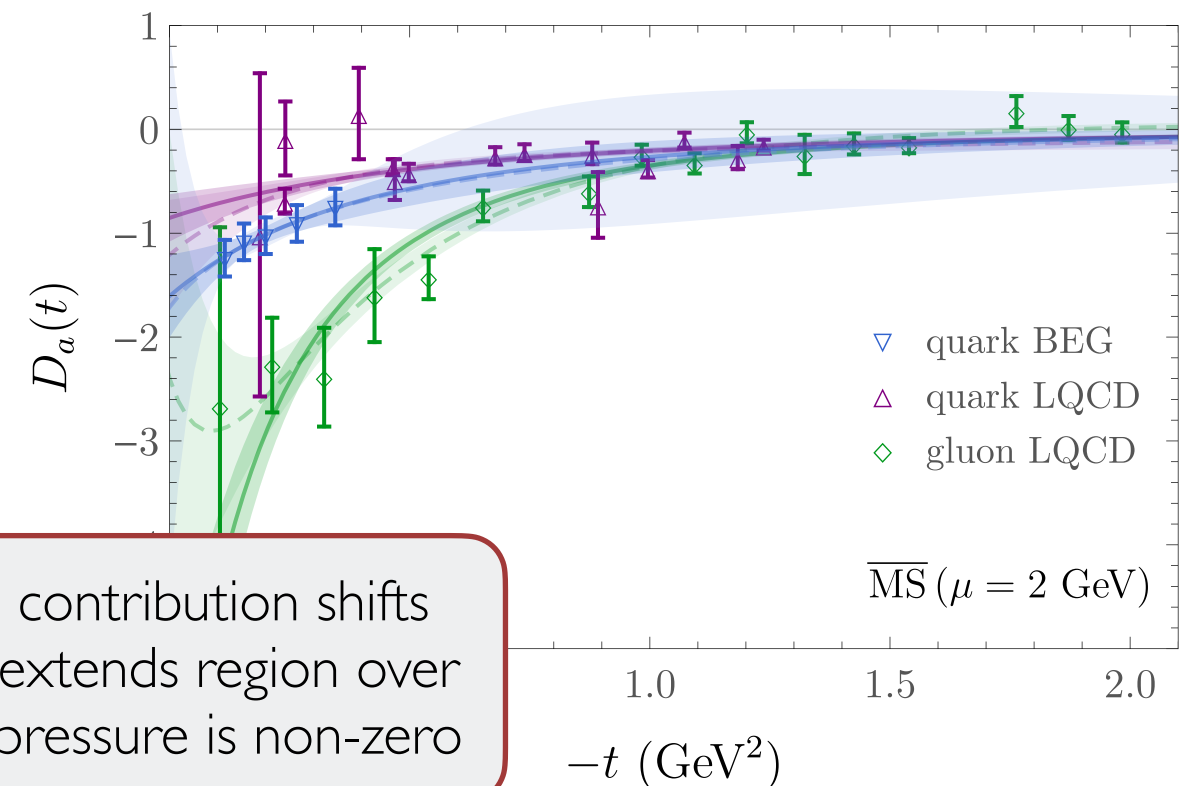
Proton pressure from LQCD

EXP + LQCD

first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]





gluon contribution shifts peaks, extends region over which pressure is non-zero

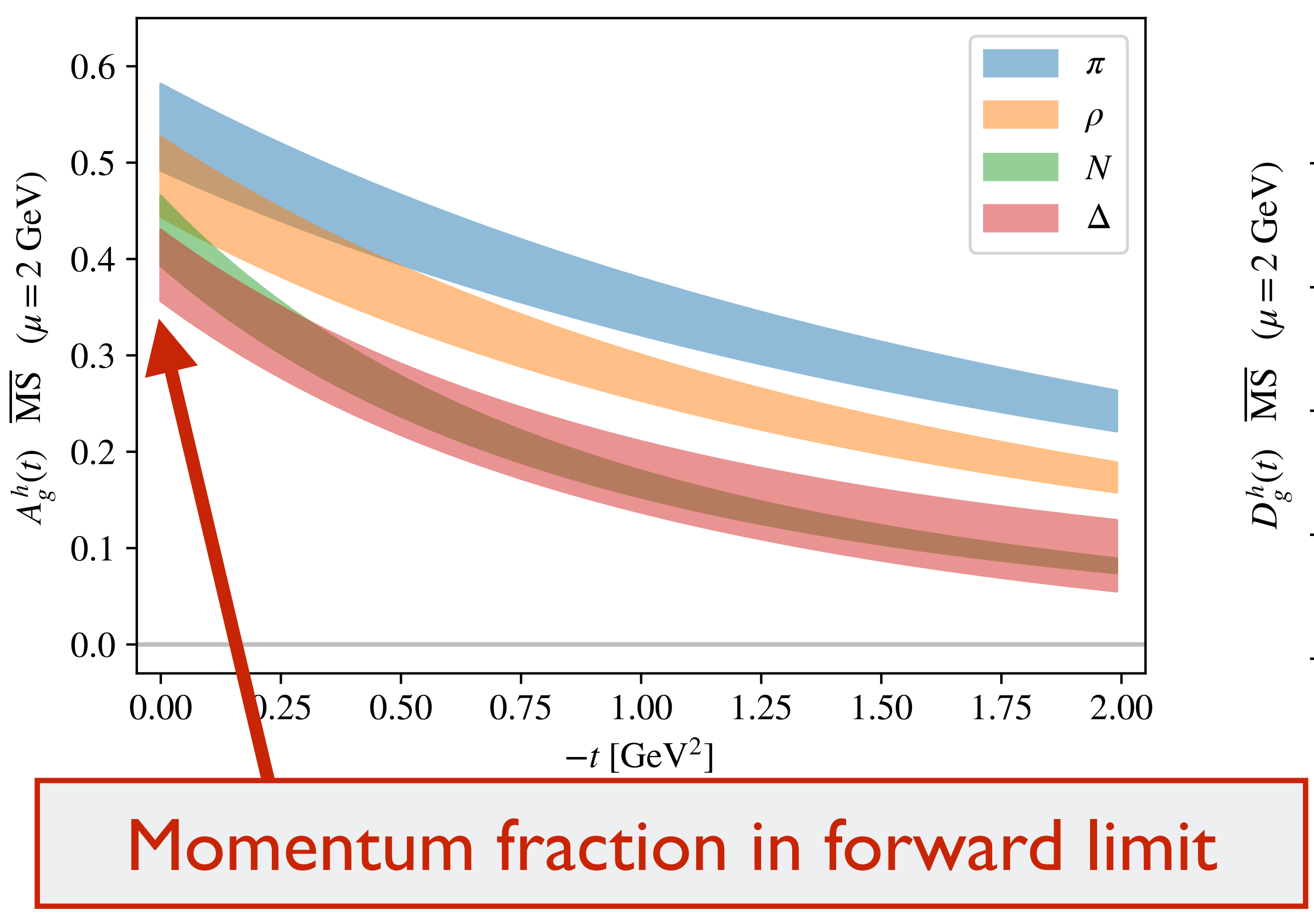
Gluon GFFs: Shanahan, Detmold, PRD 99, 014511

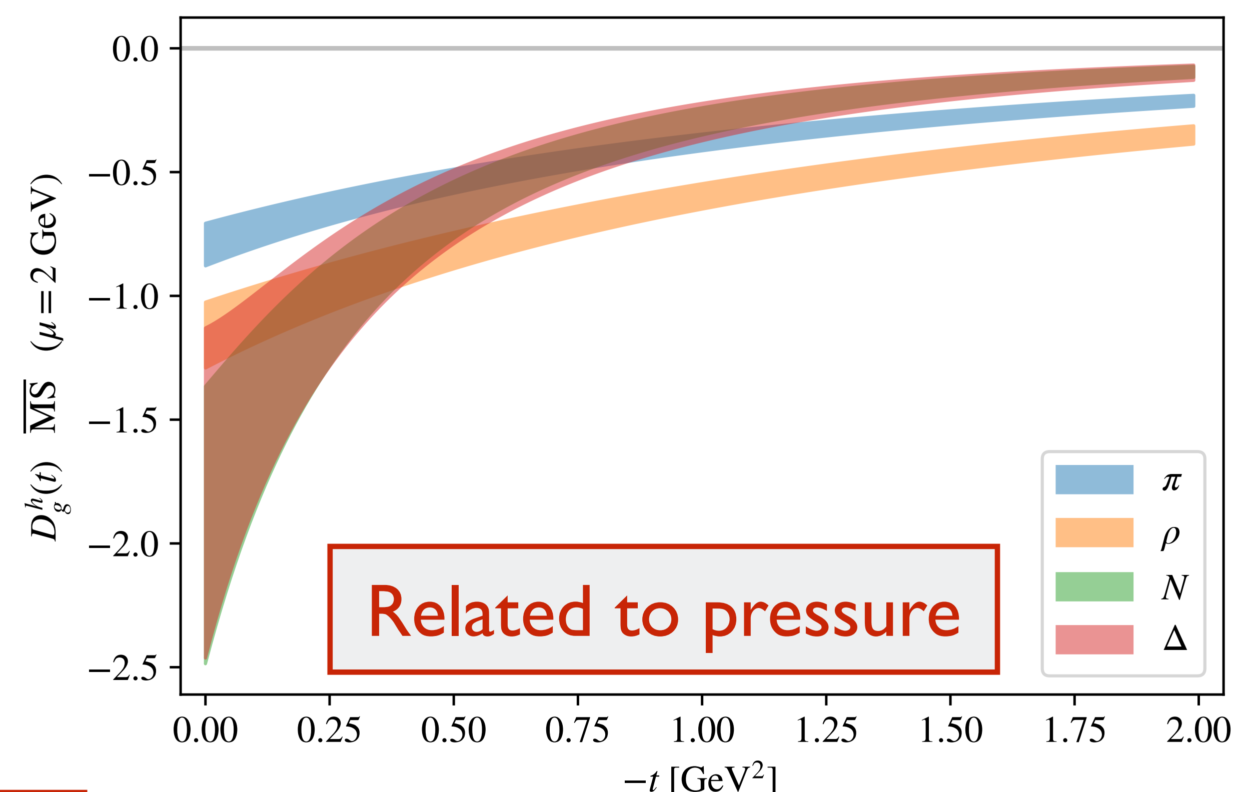
Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

Pressure distribution in other hadrons

- First comparisons of generalised gluon form factors (related to Fourier transforms of pressure distribution) for hadrons of different spin
- Qualitative differences between meson and baryon form factors
- Lattice QCD calculation with unphysically-heavy pion mass





[2107.10368 (2021)]

Parton structure from LQCD

Future colliders will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei

- Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)

- First lattice QCD constraints on nuclear PDFs through their moments
- First lattice QCD calculations of the Collins-Soper kernel for TMD rapidity evolution
- First determination of gluon contributions to shear and pressure distributions in the proton and other hadrons

Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature

Image Credit: 2018 EIC User's Group Meeting