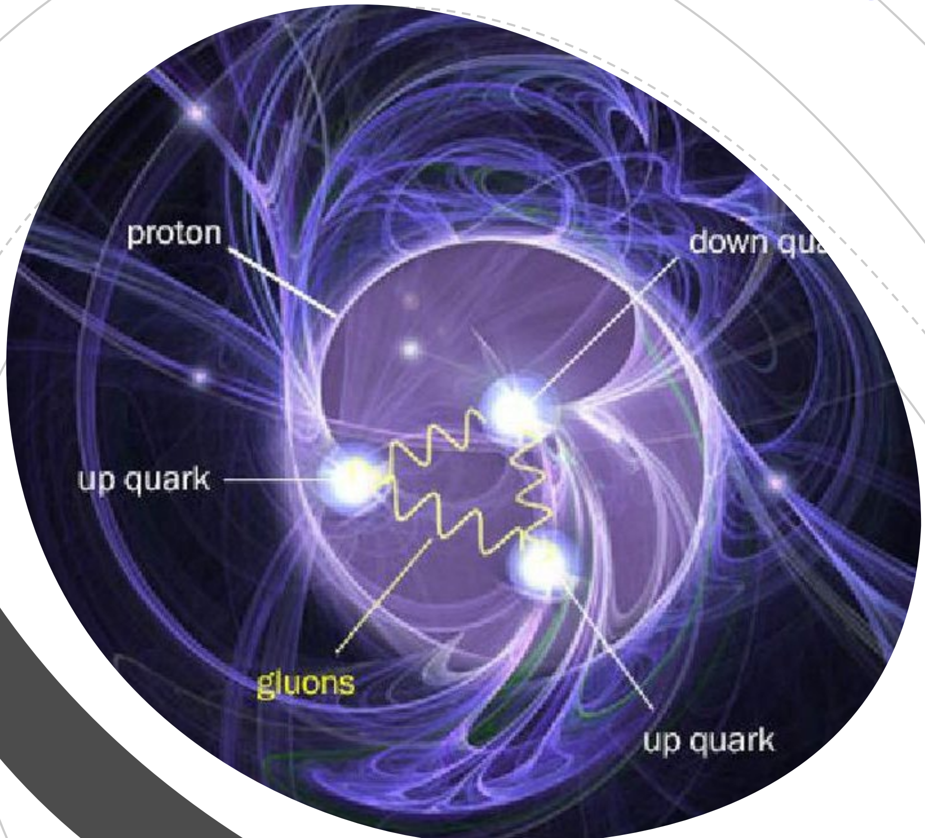
Questions about the nucleon mass



Xiangdong Ji
CNF, SURA, DC
& U. Maryland

3rd Proton Mass Workshop:
Origin and Perspective
Argonne National Laboratory,
January 14-16, 2021

The Sub-atomic Particles

Relative size	Name	Mass (Kg)	Charge (C)
	Proton	1.67×10^{-27}	$+1.602 \times 10^{-19}$
	Neutron	1.67×10^{-27}	0
	Electron	9.11×10^{-31}	-1.602×10^{-19}


QUESTION
EVERYTHING

Categories of questions

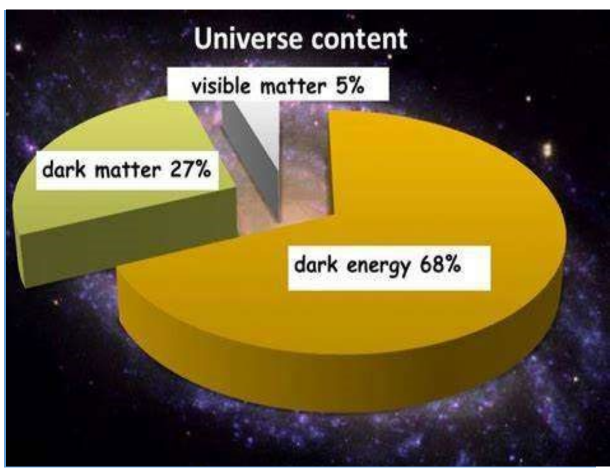
- Questions related to broader fields (astronomy, particle & nuclear physics, condensed matter, chemistry & biology...)
- Questions related to fundamentals of QCD
- Questions related to lattice calculations
- Questions related to experiments
- …

In hope to find yet more, and the most important one!

Questions related to the broader fields

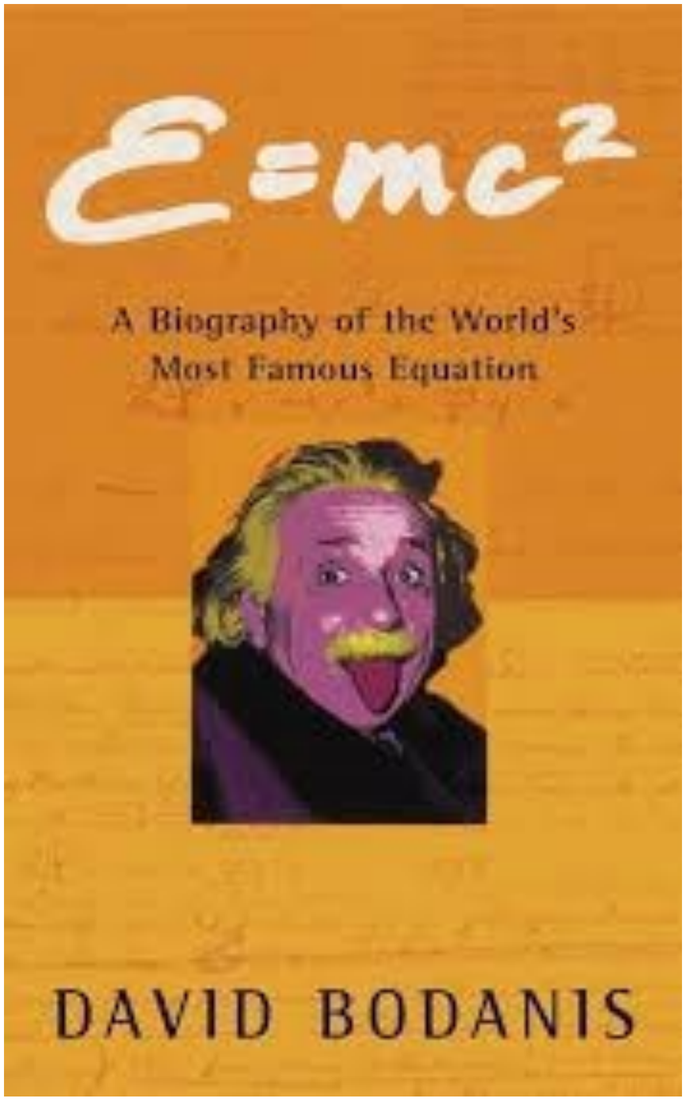
Nucleon mass:
“charges” of stellar dynamics &
“mother of all energies”

Nucleon mass in astrophysics and cosmology



- Proton and neutron masses account for 5% of the energy of the Universe
- The mass is the gravity charge which determines the stellar formation dynamics: **supernova, neutron stars, blackholes**

Biggest revelation about the mass



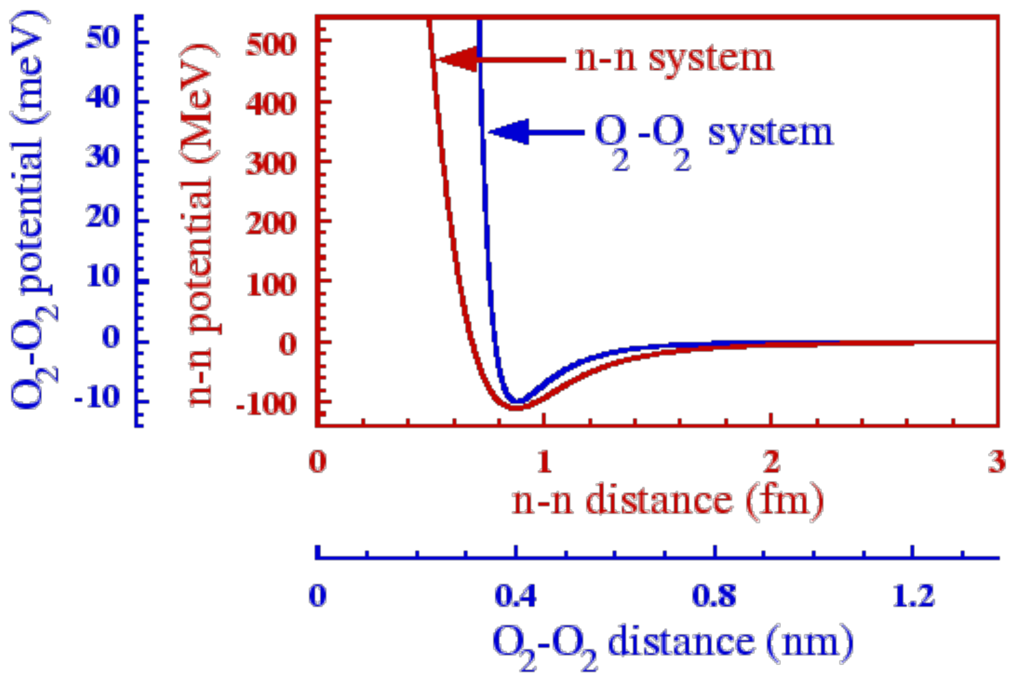
- "It appears far more natural to consider every inertial mass as a store of energy" ,

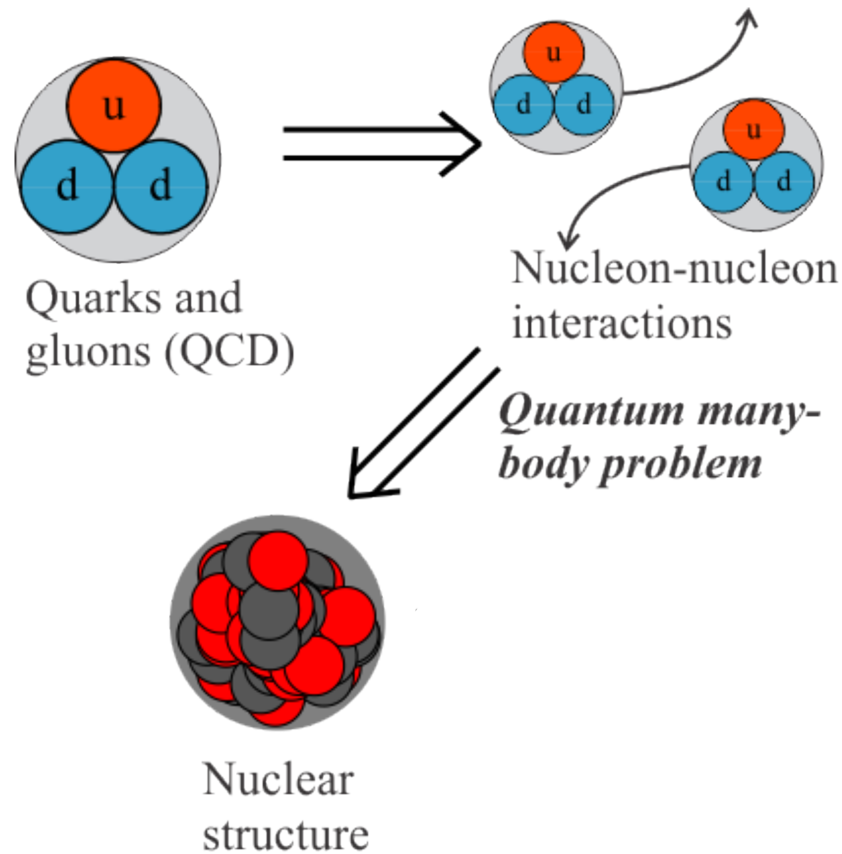
  Dec. 1907, A. Einstein

Saving energy in the nucleon mass

- As the Universe expands and cools, the only way to store the hot plasma energy is to form color-neutral droplets (nucleons) of quark and gluons, locking their kinetic energies inside.

Squeezing the energy out of the nucleon mass

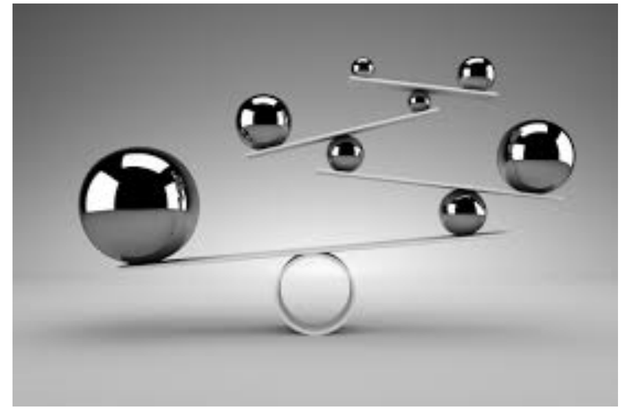




Mother of all energies

- Through gravity contraction, the quark and gluon energy inside the nucleon mass can be unleashed by forming heavier nuclei.
- Conspiracy between gravity and other forces

Mass scales in QCD

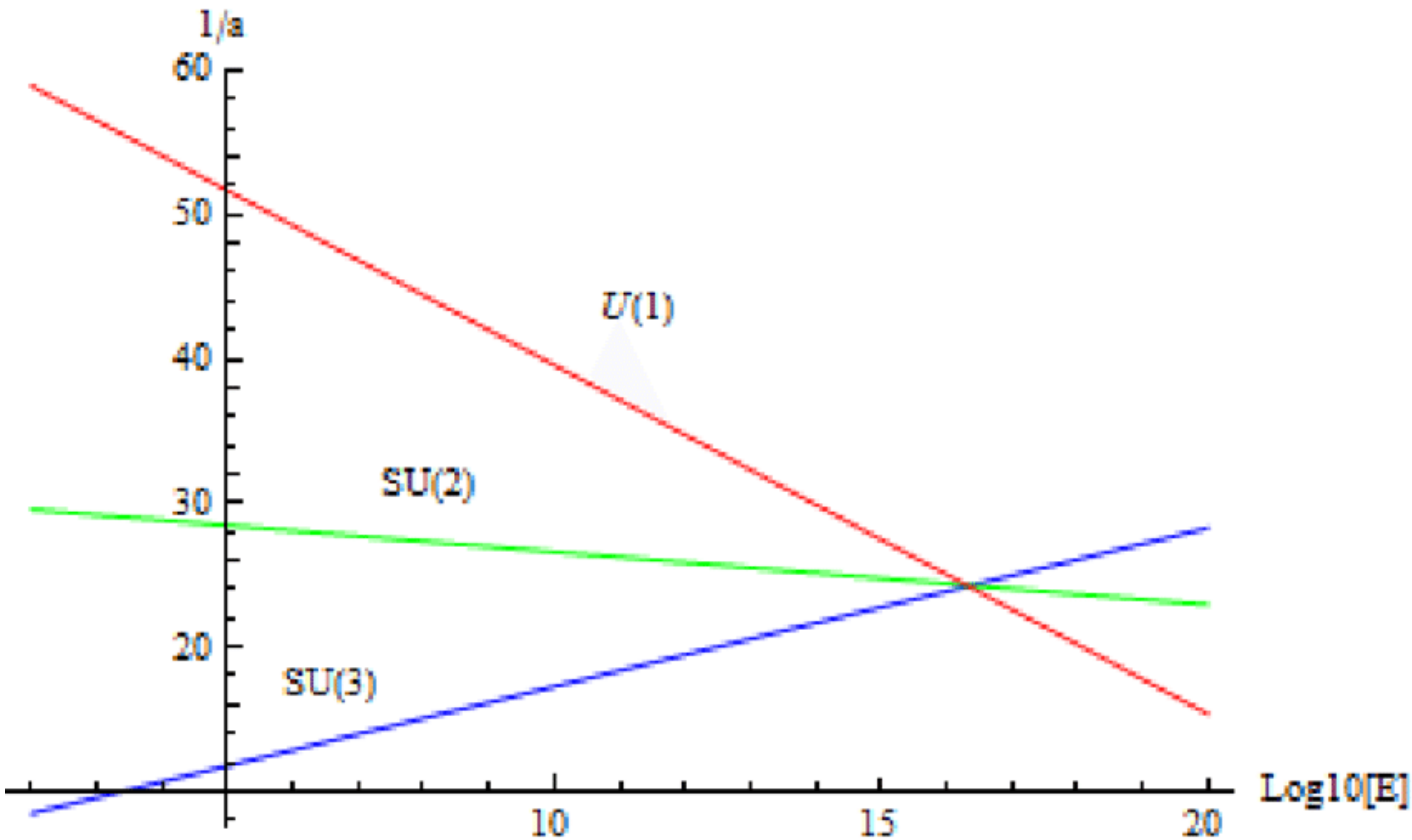


- The nucleon mass is determined by two different mass scales:
- **Quark masses**
  - Just like the electron mass in atomic physics, determined by Higgs mechanism
  - Electroweak symmetry breaking scale.
- **QCD scale Λ_{QCD}**
  - QCD scale Λ_{QCD} does not appear directly in the lagrangian: dimensional transmutation
  - Free parameter

Changing the QCD scale

- What happens if we can change Λ_{QCD} by a factor of 10 or 1/10? How will the world change?
  - The Earth may be closer to or further from the Sun, may rotate faster and slower around it?
  - The neutron can be lighter than the proton?
  - Nuclear energy production and details of star evolution will be very different?
  - Atoms and molecules will remain the similar size?
  - Feeling of hot and cold might be different?
  - Superconductivity phenomena might enhanced or decreased?
  - Change of gravity may affect biology evolution?

What decides the QCD scale? Running of the couplings



Questions related to QCD fundamentals

Mass from QCD dynamics

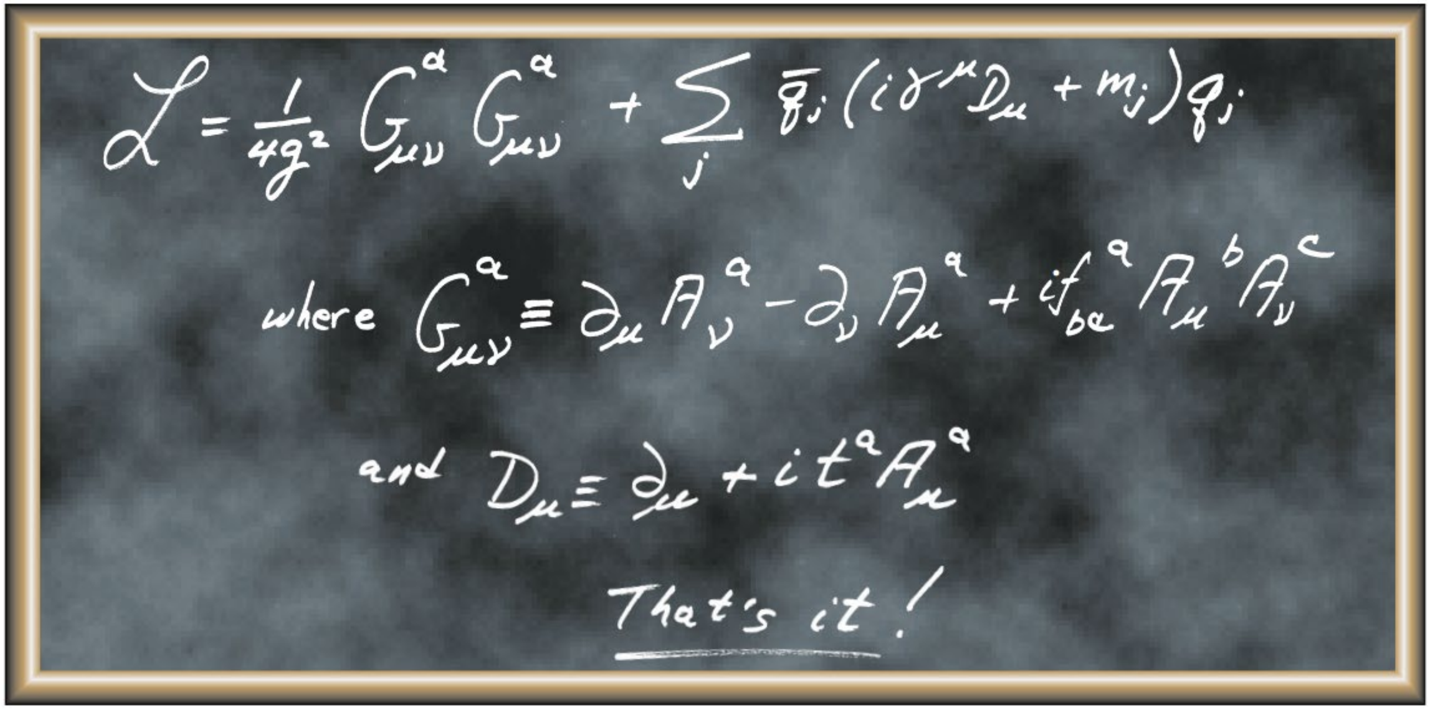
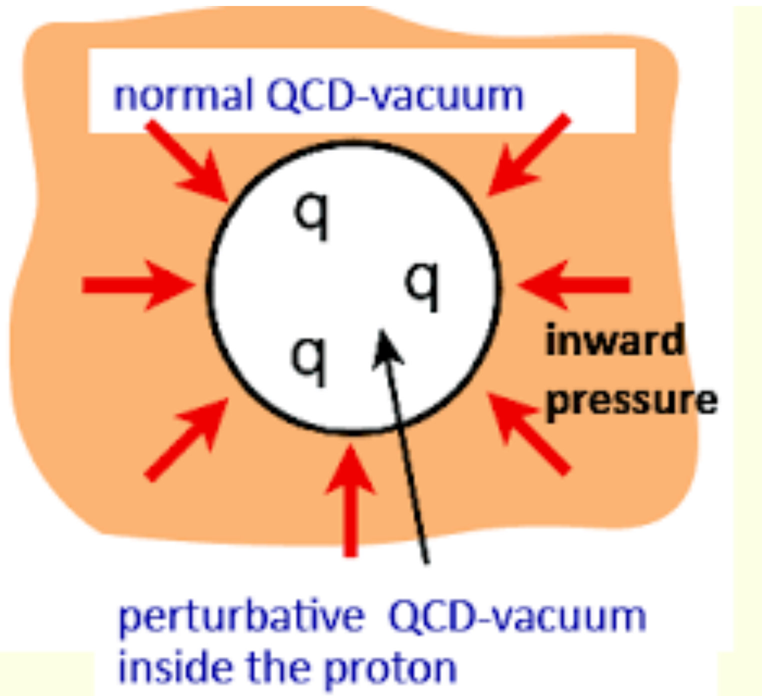


FIGURE 1. THE QCD LAGRANGIAN $\mathcal{L}$ displayed here is, in principle, a complete description of the strong interaction. But, in practice, it leads to equations that are notoriously hard to solve. Here m_j and q_j are the mass and quantum field of the quark of jth flavor, and A is the gluon field, with spacetime indices μ and ν and color indices a, b, c. The numerical coefficients f and t guaran-

$$M_N = \alpha\Lambda_{QCD} + \sum_q \beta_q m_q$$

Role of color confinement



M.I.T. Bag Model

- The boundary condition generates discrete energy eigenvalues.

$$\varepsilon_n = \frac{x_n}{R}$$

R - radius of the Bag

x_1=2.04

$$E_{kin}(R) = N_q \frac{x_n}{R}$$

N_q = # of quarks inside the bag

$$E_{pot}(R) = \frac{4}{3}\pi R^3 B$$

B – bag constant that reflects the bag pressure

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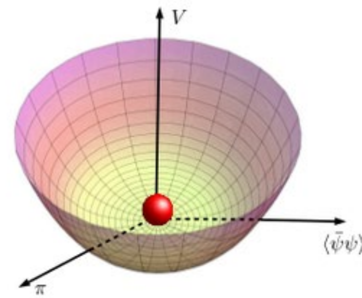
Mass = quark kinetic energy + B(scalar-field condensate)

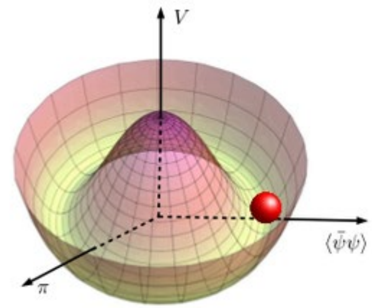
Role of chiral symmetry breaking

- SSB of chiral symmetry

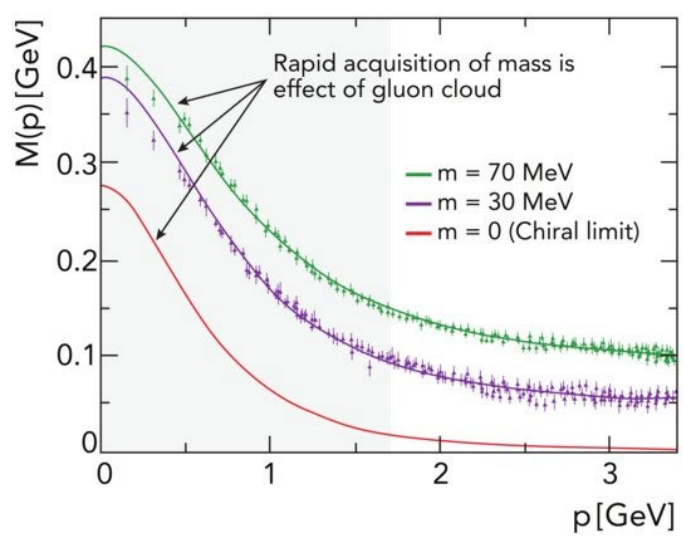
  Goldstone bosons: π, K

  Chiral condensate $\langle \bar{\psi}\psi \rangle \neq 0$





- Quarks acquire an effective mass? NJL models etc.



Mass of the nucleon

- Internal mass as a store of energy

$$mc^2 = \langle N | \hat{H}_{QCD} | N \rangle$$

- For any relativistic system, the Hamiltonian can be separated into two terms,

$$\hat{H}_{QCD} = \hat{H}_T + \hat{H}_S$$

This separation is a fundamental property of relativity and both parts are scale invariant

Tensor and scalar energies

- Tensor energy $E_T = \langle H_T \rangle$ is related to the usual kinetic and potential energy sources
- Scalar energy $E_S = \langle H_S \rangle$ is related to related to **scale-breaking properties of the theory**, as such as quark mass m_q and trace anomaly:
  - In the massless limit, the classical theory is scale-invariant.
  - Due to UV divergences, the scale invariance is broken, the trace of EMT is now zero, $T^{\mu}_{\mu} \neq 0.$ Composite scalar fields which could have scale-breaking vacuum expectation values

$$\langle \bar{\psi}\psi \rangle, \quad \langle F^2 \rangle$$

Relativistic virial theorem

- As an important feature of relativity, one can show

  $$E_T = 3E_S \qquad \text{(virial theorem)}$$

  3 is the dimension of space.

- Scalar energy sets the scale of the tensor (kinetic and potential energies of the system). But it is not the only energy ($\mathrm{M_N} = 4E_S$)

- In non-relativistic limit of QED & gravity, it reduces

  $$\langle V \rangle = -2\langle T \rangle$$

  kinetic energy sets the scale for potential energy!

Scalar energy in QCD

- What is the scalar Hamiltonian in QCD?

$$H_S = H_m + H_a$$

$$H_m = \frac{1}{4}\int d^3x\, m\bar{\psi}\psi$$

$$H_a = \frac{1}{4}\int d^3\vec{x}\left(\frac{\beta(g_0)}{2g_0}F^2 + m_0\gamma_m\bar{\psi}\psi\right)$$

- Quantum anomaly contribution: due to breaking of scale symmetry from renormalization.
- In the massless limit, $H_a \sim F^2$

Quantum Anomalous Energy (QAE)

- There is an anomalous scalar contribution to the nucleon mass

$$E_a \sim \langle N|F^2|N\rangle \sim \frac{1}{4} m_N c^2$$

- This particular contribution comes from the scalar response to the presence of the quarks (bag constant). Lts contribution is also similar to Higgs mechanism in electroweak theory, with

$$\phi = F^2 - \langle 0|F^2|0\rangle$$

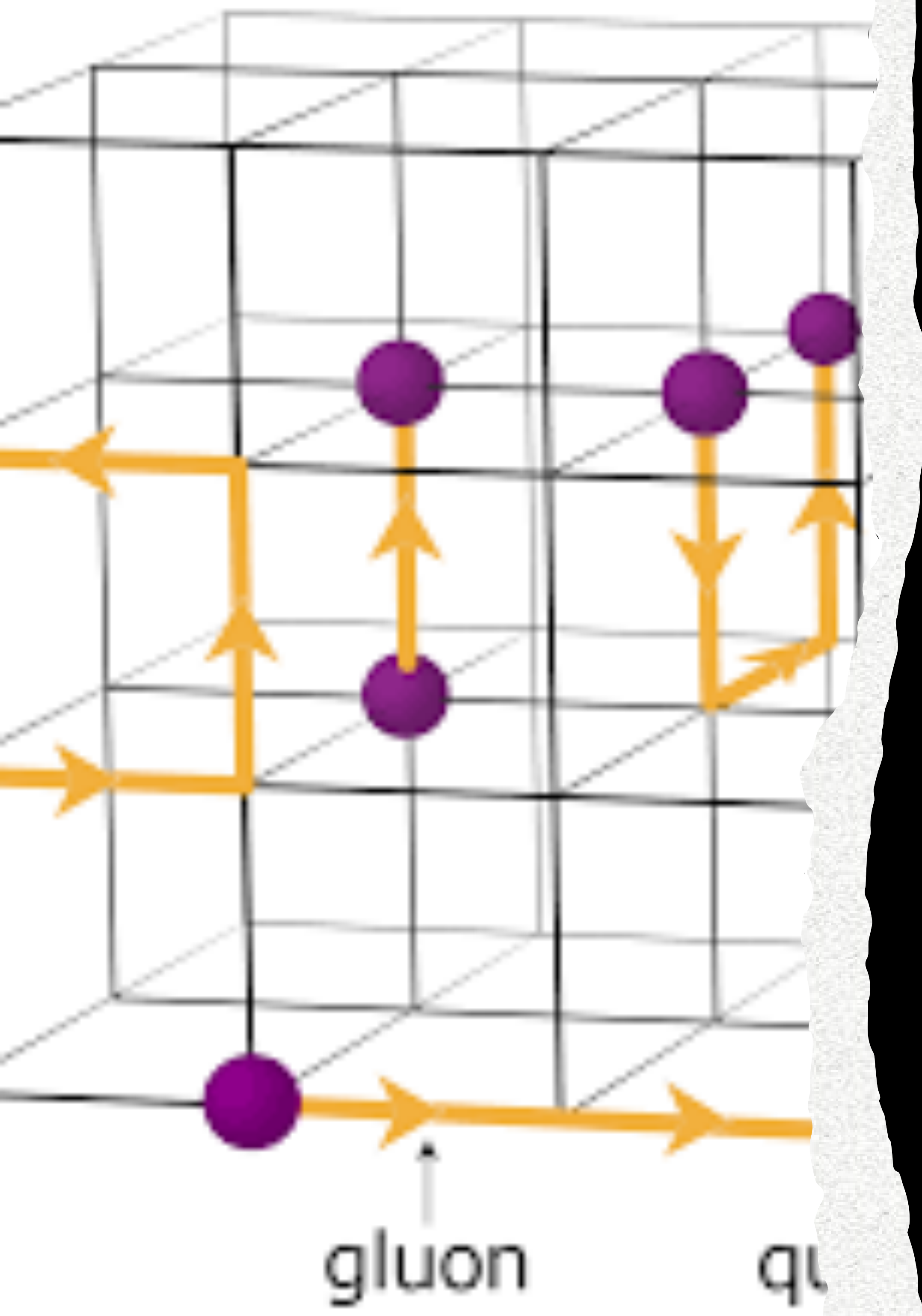
as a dynamical Higgs field.

X. Ji & Y. Liu, e-Print: 2101.04483

Questions related to QCD fundamentals

- What is the role of confinement in the proton mass?
- What is the role of chiral symmetry breaking in the proton mass?
- Does the relativistic virial theorem tell us something deep about the mass?
- What is the role of quantum anomalous energy (QAE) in the nucleon mass?
- Why are the nucleon resonances separated by large mass gap? (why quark model works?)
- Why the nucleon-nucleon potential is attractive (one pion change)?

Questions related to lattice QCD



Lattice is the only
first-principles
approach
to understand
non-perturnative
QCD dynamics

Splitting the QCD energy sources

- Four different type energies (X. Ji, PRL, 1995)

$$H_{\rm QCD} = H_q + H_m + H_g + H_a .$$

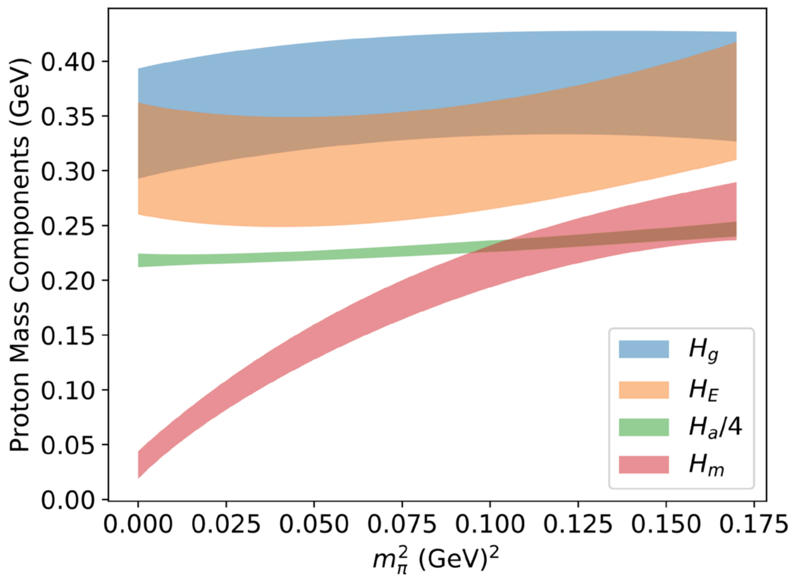
$$H_q = \int d^3\vec{x}\ \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi,$$ ← Quark energy

$$H_m = \int d^3\vec{x}\ \bar{\psi}m\psi,$$ ← Quark mass

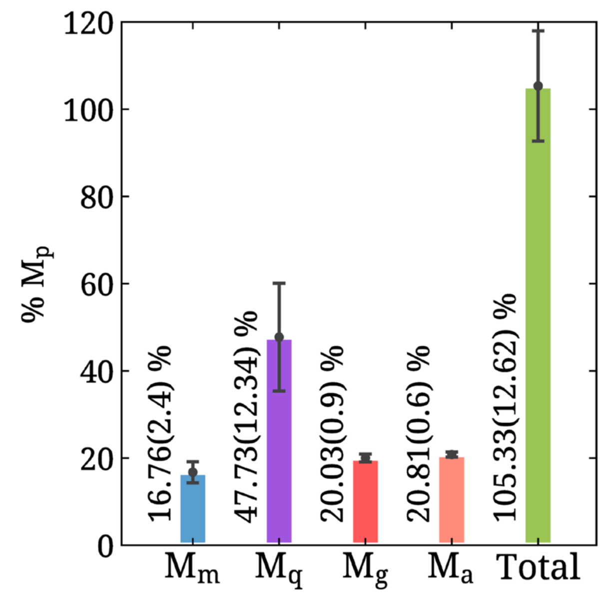
$$H_g = \int d^3\vec{x}\ \frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2),$$ ← Gluon energy

$$H_a = \int d^3\vec{x}\ \frac{9\alpha_s}{16\pi}(\mathbf{E}^2-\mathbf{B}^2).$$ ← Quantum Anomalous Energy (QAE)

Proton mass on the lattice



Y.-B. Yang *et al.*, (χQCD), PRL 121, 212001 (2018)
Y.-B. Yang *et al.*, (χQCD), PRD94,054503 (2016)



C. Alexandrou et al., (ETMC), PRL 119, 142002 (2017)
C. Alexandrou et al., (ETMC), PRL 116, 252001 (2016)

- Trace anomaly: 2101.04942 by Chi-QCD
- B^2 is negative

Quark mass contribution

- Lattice QCD calculations of the pion-N sigma term and strange quark contribution

$$\sum_q m_Q\langle\bar{q}q\rangle = 80 - 90\ MeV$$

about 10% of the total.

- Strange quark contribution is not as big as from chiral perturbation theory.
- 1 MeV accuracy?

QAE contribution to confinement

H. J. Rothe, Phys. Lett. B 346, 227 (1995); 355, 260 (1995)

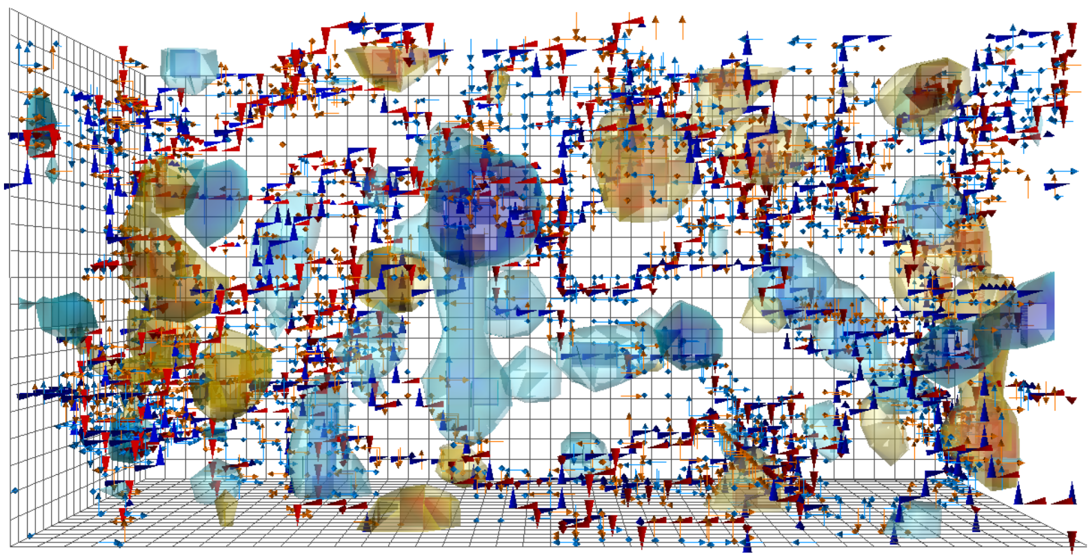
- Heavy quark potential between QQ-bar

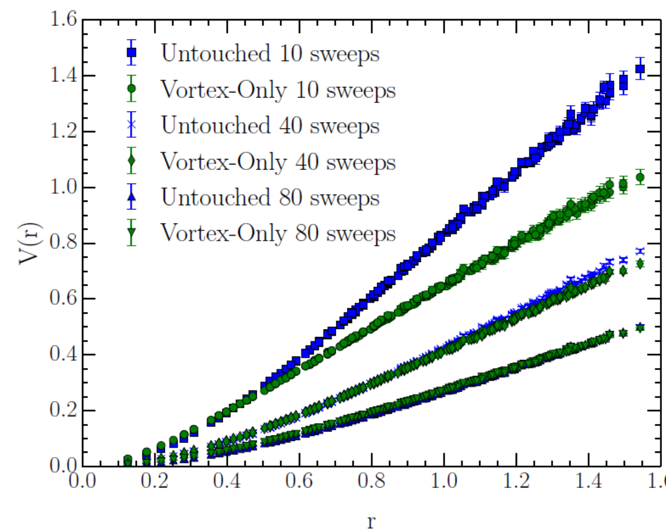
$$V(r) = \sigma r$$

- Contribution to the potential from the anomalous energy is

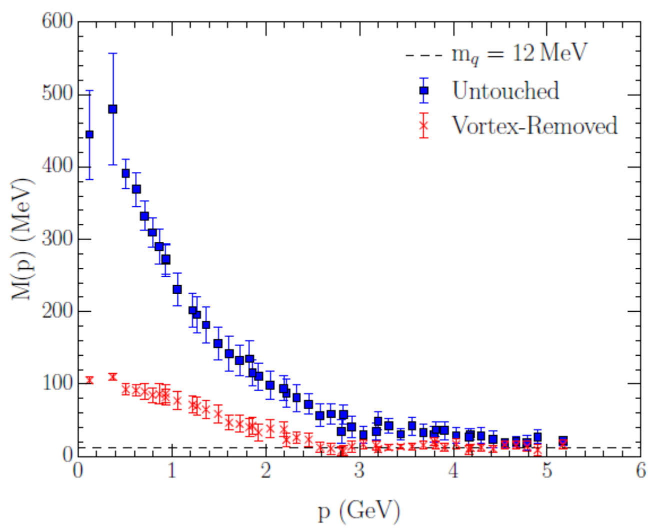
$$E_a(r) = \frac{1}{4}\left(V(r) + rV'(r)\right)$$
$$= \frac{1}{2}V(r)$$

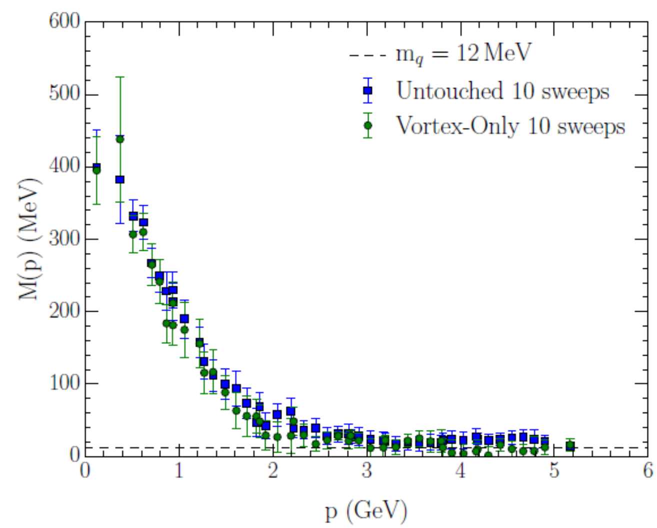
Center-vertices: confinement & chiral symmetry breaking



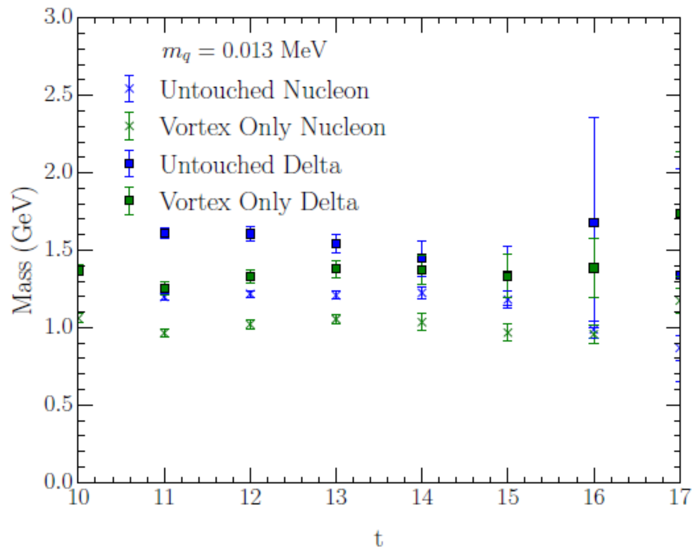


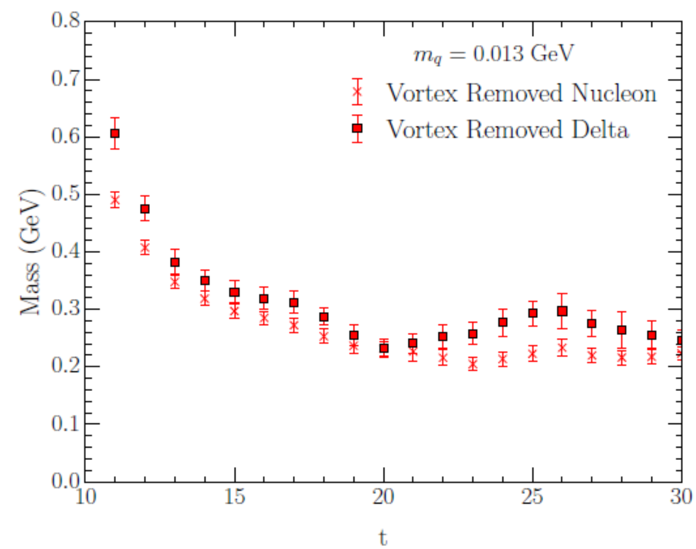
Leinweber et al.
J.Phys.G 44 (2017) 12, 125002

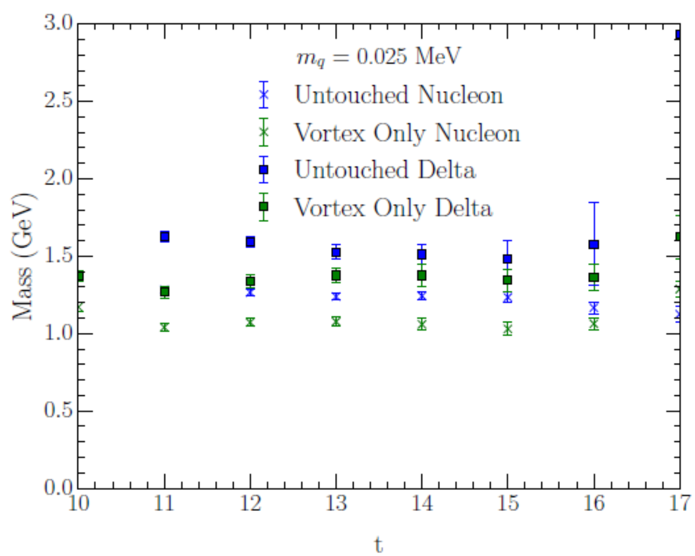


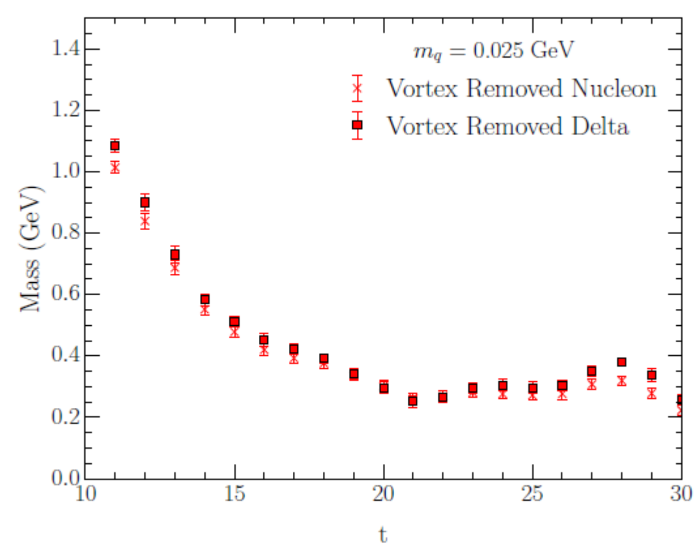


Effects of center vertices on the nucleon & Delta mass









Questions related to lattice QCD

- Precision quark mass contributions to the proton mass from lattice QCD?
- What is the best way to compute he QAE contribution to the mass (bag constant)
- How to separate influences of chiral symmetry breaking from color confinement on the mass?
- Can one reproduce the dynamical Higgs mechanism for the gluon scalar contribution?
- Why is the color magnetic field depleted inside the nucleon?

Questions related to experiments

Quark mass contribution

- Up & down quark mass contribution

  Pi-N scattering σ term:

- Strange quark contribution

  χ-PT from baryon spectrum

  Meissner's talk

- Any other useful data to fix them?

Quark and gluon energy: PDFs ji,1995

- Statistical model: C. Bourrely, F. Buccella, J.-C.Peng, arXiv:2008.05703

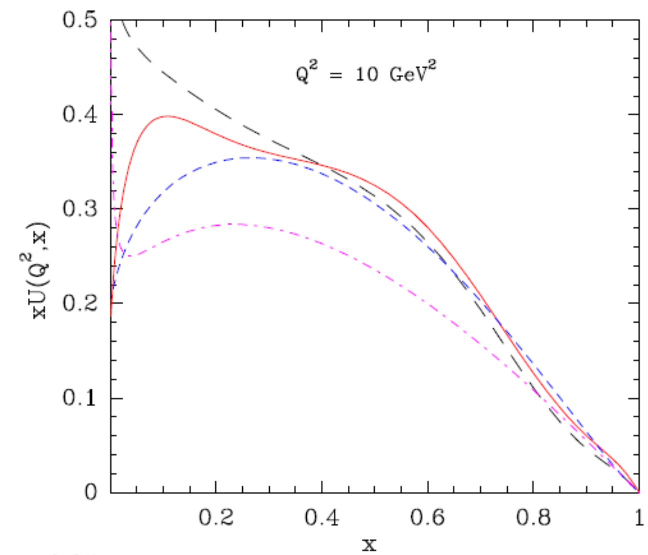
$$xU(x) = xD(x) = \frac{A_U X_U x^{b_U}}{\exp[(x - X_U)/\bar{x}] + 1} + \frac{\tilde{A}_U x^{\tilde{b}_U}}{\exp(x/\bar{x}) + 1}. \quad (7)$$

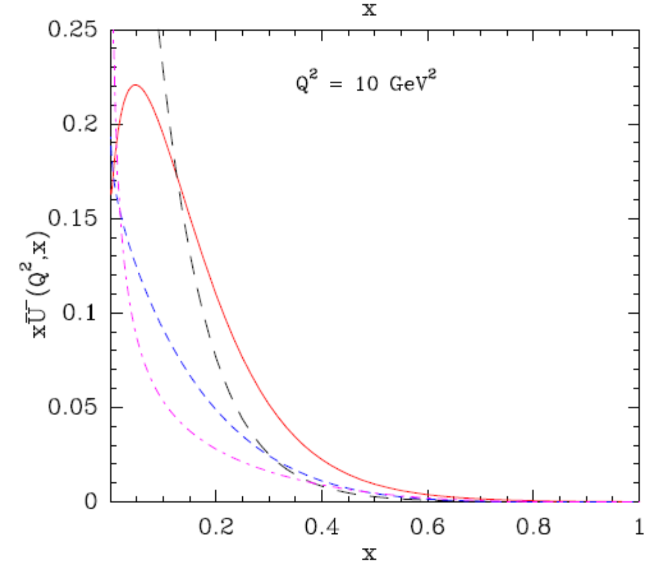
$$x\bar{U}(x) = x\bar{D}(x) = \frac{A_U (X_U)^{-1} x^{b_U}}{\exp[(x + X_U)/\bar{x}] + 1} + \frac{\tilde{A}_U x^{\tilde{b}_U}}{\exp(x/\bar{x}) + 1}. \quad (8)$$

$$A_U = 0.80536 \pm 0.10 \qquad b_U = 0.5161 \pm 0.02$$
$$X_U = 0.7551 \pm 0.01 \qquad \bar{x} = 0.10614 \pm 0.004$$
$$\tilde{A}_U = 2.2773 \pm 0.324 \qquad \tilde{b}_U = 0.4911 \pm 0.0092$$
$$A_G = 31.0019 \pm 1.68 \qquad b_G = 1 + \tilde{b}_U\ .$$

T=100 MeV

TMDPDF?

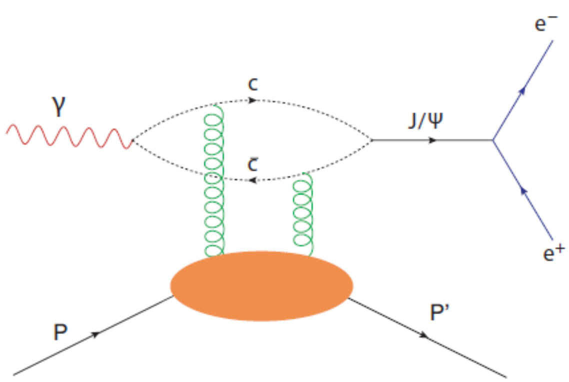




$\langle p|F^2|p\rangle$: heavy quarkonium production

D. Kharzeev, Proc. Int. Sch. Phys. Fermi 130, 105 (1996)

- Using a color dipole to measure the scalar field static response. Voloshin (1978), ….
- This naturally leads to the "probe" through photo or electro-production of heavy quarkonium.
- Precision data is needed + VDM



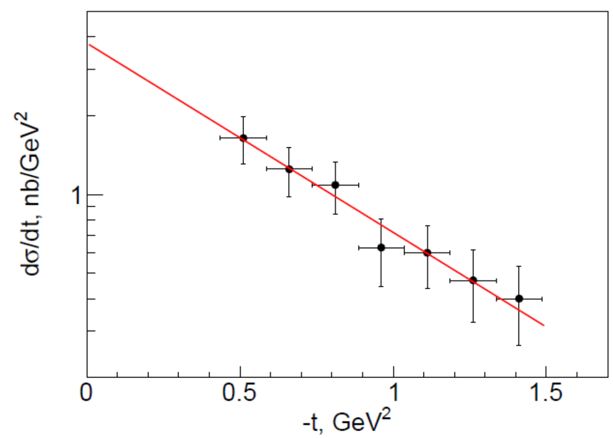
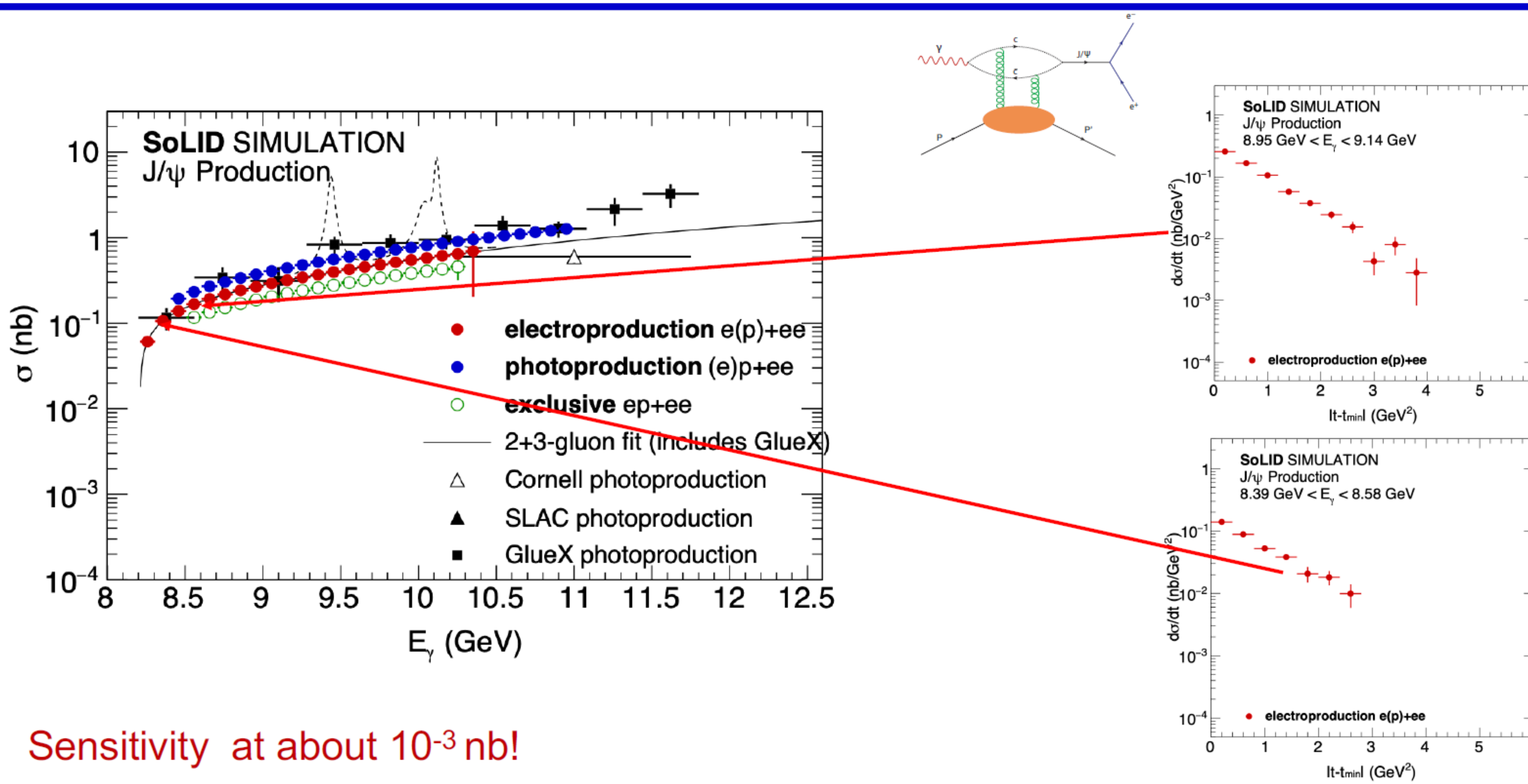


Fig. 1. The differential cross section of J/Ψ photo-production near threshold as measured by the GlueX Collaboration [22]. Only statistical uncertainties are shown.

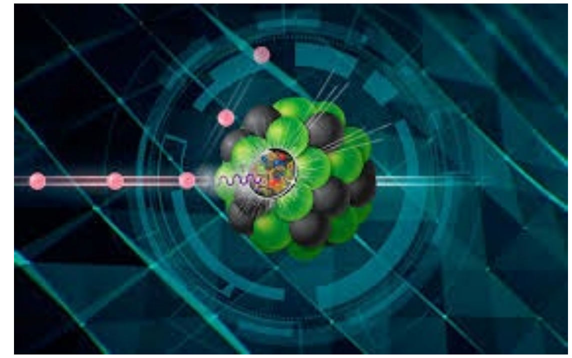
J/Psi Experiment E12-12-006 @ SoLID



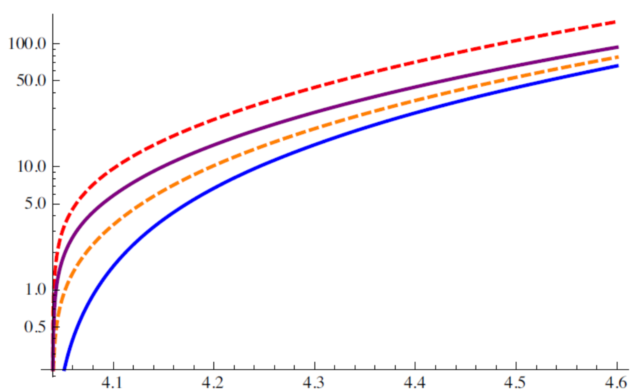
Sensitivity at about 10^{-3} nb!

Meziani's talk

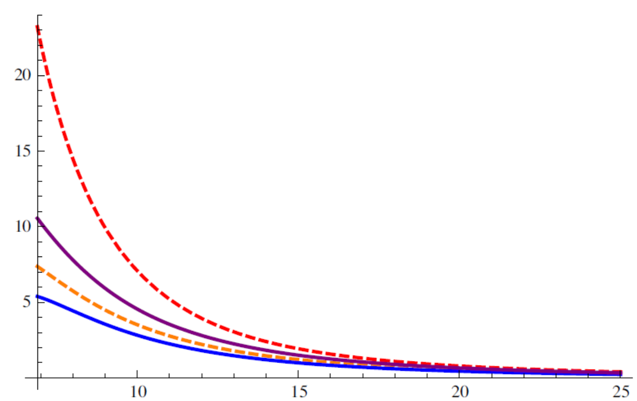
Higher Q^2: Looking to EIC



- Higher Q^2 provides a more rigorous connection between cross section and the matrix element.
  (Boussarie & Hatta et al, Phys.Rev.D 101 (2020) 11, 114004)
- J/psi & Υ production



(a) Total cross section (in fb) as a function of W (in GeV).



(b) Differential cross section (in fb/GeV^2) at $W = 4.4$ GeV as a function of $|t|$ (in GeV^2).

FIG. 1. J/ψ total and differential cross sections at $Q^2 = 64$ GeV^2. The upper and lower dashed curves correspond to Case 1 with $D_g = 0$ and $D_g = -7.2$, respectively. The upper and lower solid curves correspond to Case 2, $D_g = -7.2$, with $b = 1$ and $b = 0$, respectively.

Mass form-factor A(Q^2), $\bar{C}$(Q^2)

- Form factors of energy-momentum tensor

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M]U(P),$$

A.M.

pressure (Polyakov)

X. Ji, PRL 1996

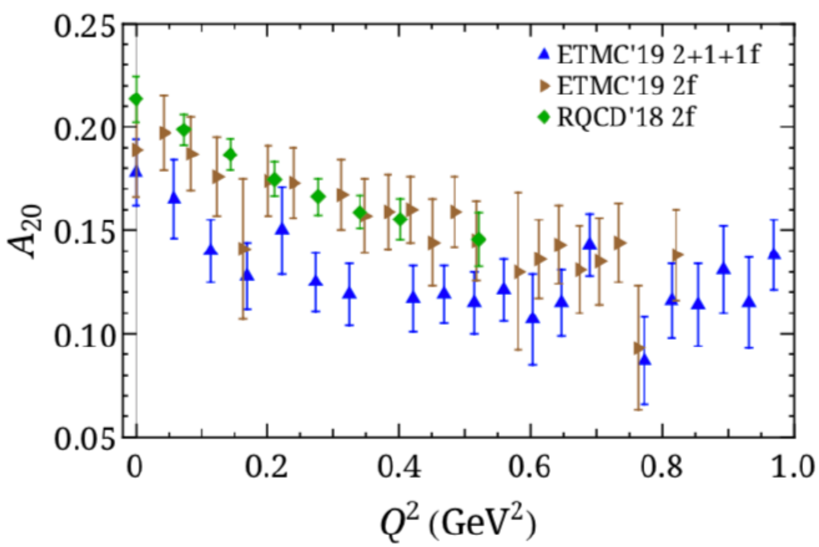
- Mass form factors:

A(Q^2) and C-bar

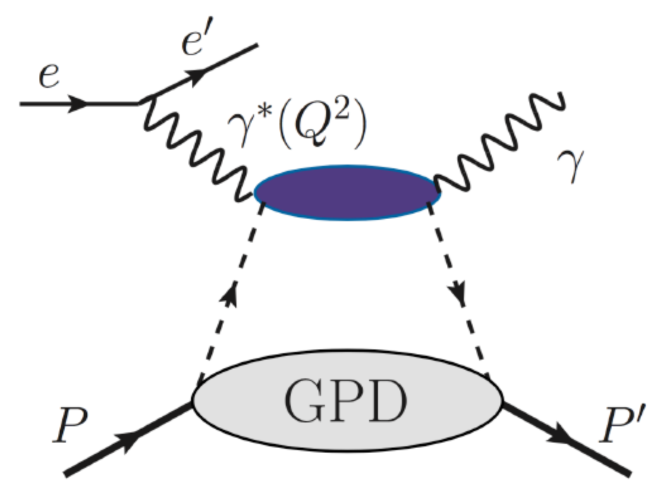
C-bar is a twist-4 contribution

(Ji, Melnitchouk, Song, 1997, Hatta, Rajan, Tanaka, 2018, .)

Mass form factor: lattice QCD and DVCS



Deeply Virtual Compton Scattering



$$\int_{-1}^{1} dx\ xH(x,\xi,t) = A(t) + \xi^2\ C(t)\ ,$$

$$\int_{-1}^{1} dx\ xE(x,\xi,t) = B(t) - \xi^2\ C(t)\ .$$

(Ji, Melnitchouk, Song, 1997)

Questions related to experiments and observables

- How to improve the precision in measuring and extracting the quark mass contribution experimentally?
- How to reliably measure and interpret the QAE contribution from J/psi, upsilon threshold production?
- How well are the mass distributions in the nucleon measured from DVCS and similar processes?
- What is the most important question that EIC can answer about the nucleon mass?

Conclusions

- The nucleon mass is one of the most important physical quantities in our universe.
- Many interesting questions can be asked and answered by models, lattice QCD calculations, and experiments.
- More questions to be asked and answered through this workshop.