2D Models for QCD

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STRINGS MICHIGAN THEORY

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"Millennium Madness"

Physics Problems for the Next Millennium

The best 10 problems were selected at the end of the conference by a selection panel consisting of:

- Michael Duff (University of Michigan)
- David Gross (Institute for Theoretical Physics, Santa Barbara)
- Edward Witten (Caltech & Institute for Advanced Studies)

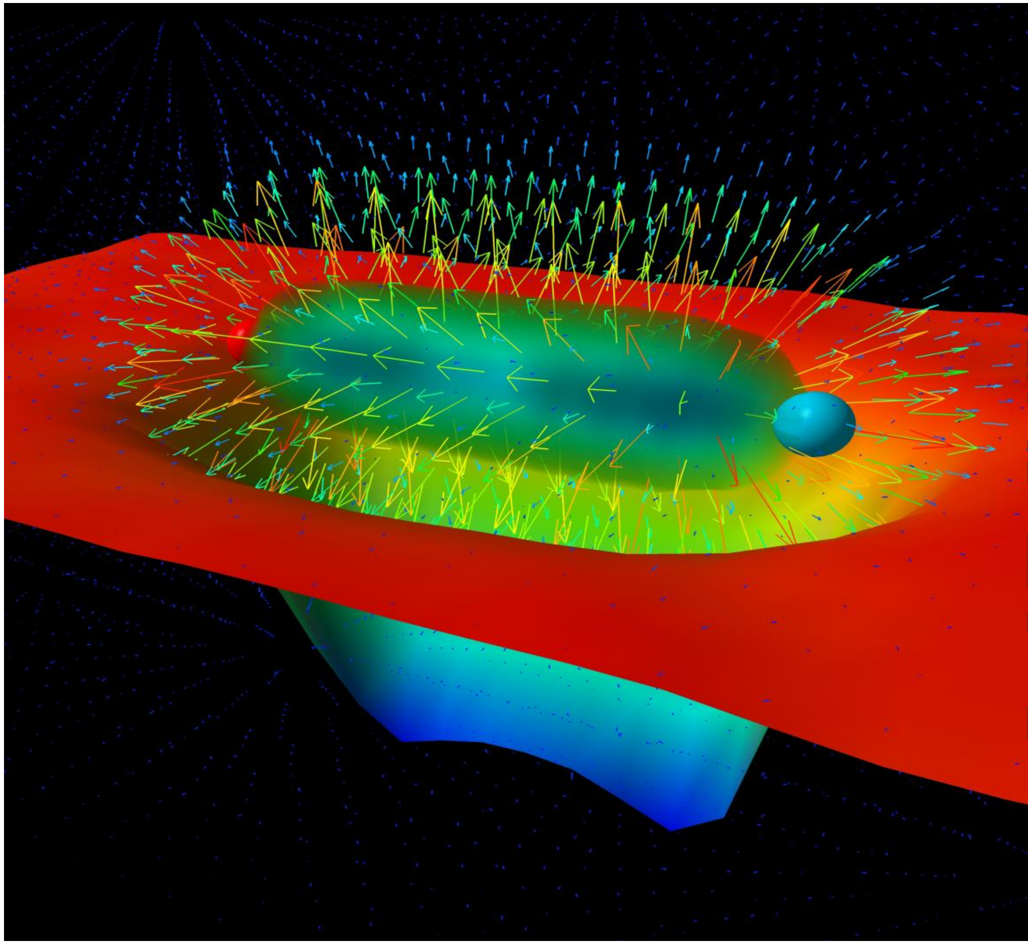
10. Can we quantitatively understand quark and gluon confinement in Quantum Chromodynamics and the existence of a mass gap?
    Igor Klebanov, Princeton University
    Oyvind Tafjord, McGill University

Introduction

- The problem of Color Confinement in QCD is indeed among the deepest in modern Theoretical Physics.
- While there has been great progress in Lattice Gauge Theory, there is no quantitative analytic understanding yet of the mass gap and confinement, even in the large N pure glue theory in 2+1 or 3+1 dimensions.
- At the same time, there have been tantalizing experimental discoveries of exotic states (for example, the XYZ mesons) which put new focus on the physics of strong interactions.

QCD and Strings

- At distances much smaller than 1 fm, the quark-antiquark potential is nearly Coulombic.
- At larger distances the potential should be linear (Wilson) due to formation of confining flux tubes. Their dynamics is approximately described by the Nambu-Goto area action. So, strings have been observed, at least in numerical simulations of Yang-Mills theory.



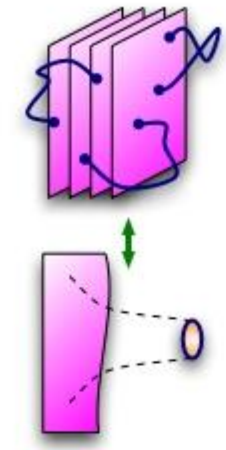
Large N Yang-Mills Theories

- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling fixed:

$$\lambda = g_{\mathrm{YM}}^2 N$$

- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.

D-Branes vs. Geometry



- Dirichlet branes led string theory back to gauge theory in the mid-90's. Polchinski
- A stack of N Dirichlet 3-branes realizes $\mathcal{N}$=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

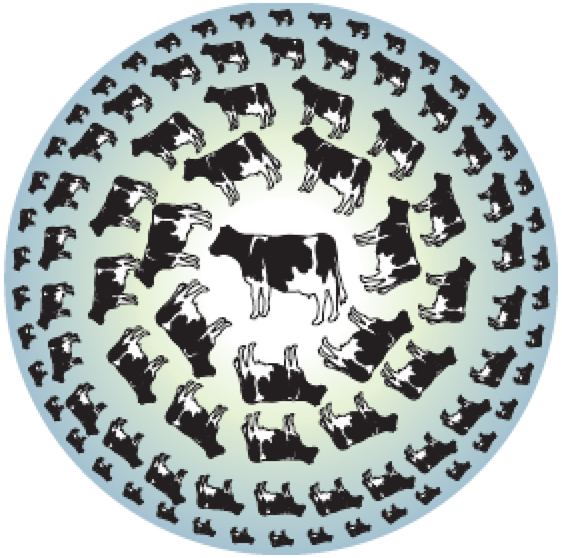
which for small r approaches $AdS_5 \times \mathbf{S}^5$

whose radius is related to the coupling by $L^4 = g_{\mathrm{YM}}^2 N \alpha'^2$

The AdS/CFT Duality

Maldacena; Gubser, IRK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}=4$ SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The AdS space-time is a generalized hyperboloid. It has negative curvature.



- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g_{\rm YM}^2 N}$

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of

$$\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$$

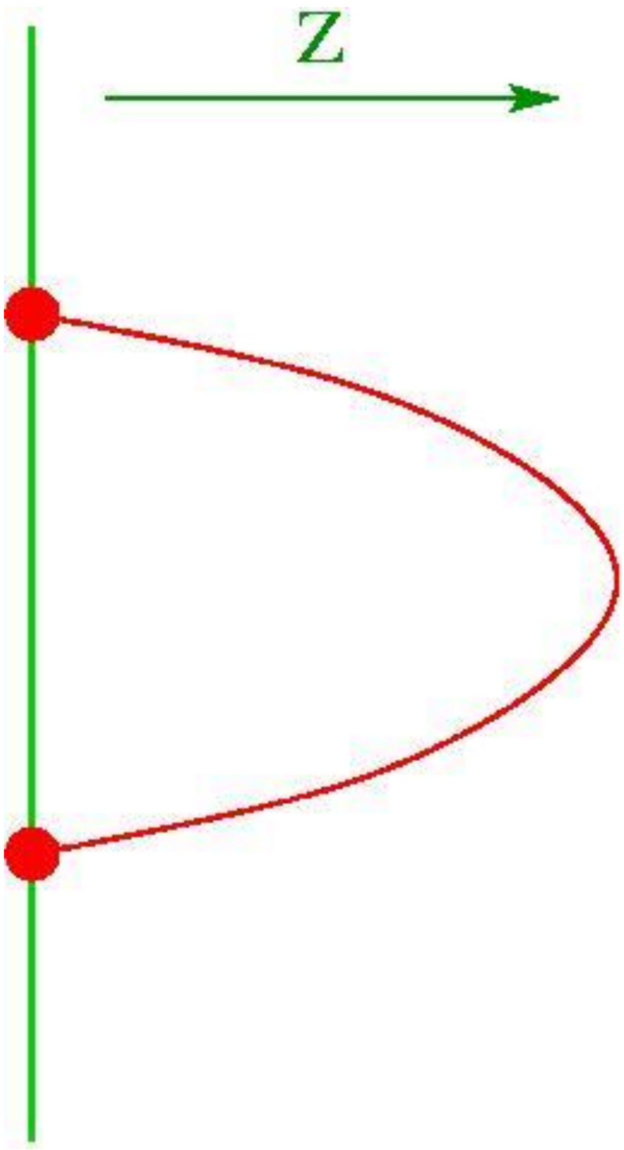
- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

The quark anti-quark potential

- The z-direction of AdS is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.

- The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential Maldacena; Rey, Yee

$$V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r}$$



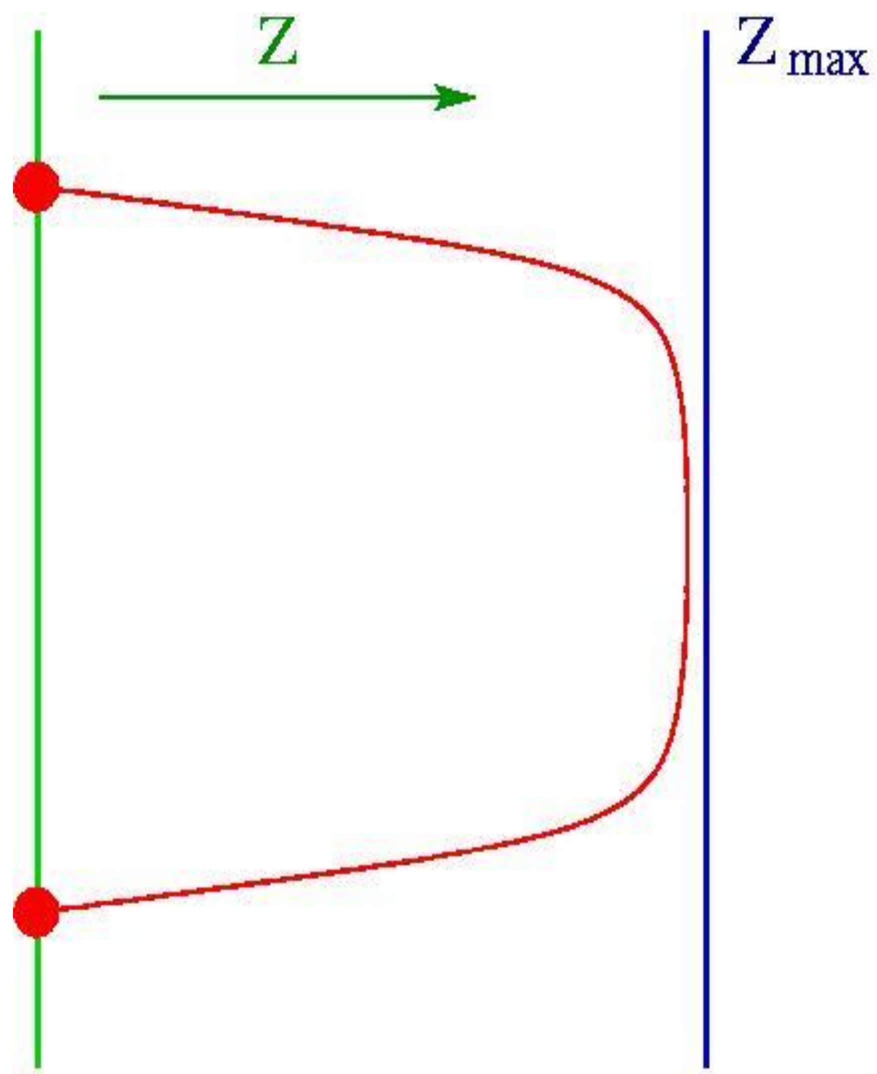
Color Confinement

- The quark anti-quark potential is linear at large distances but nearly Coulombic at small distances.
- The 5-d metric should have a warped form Polyakov

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

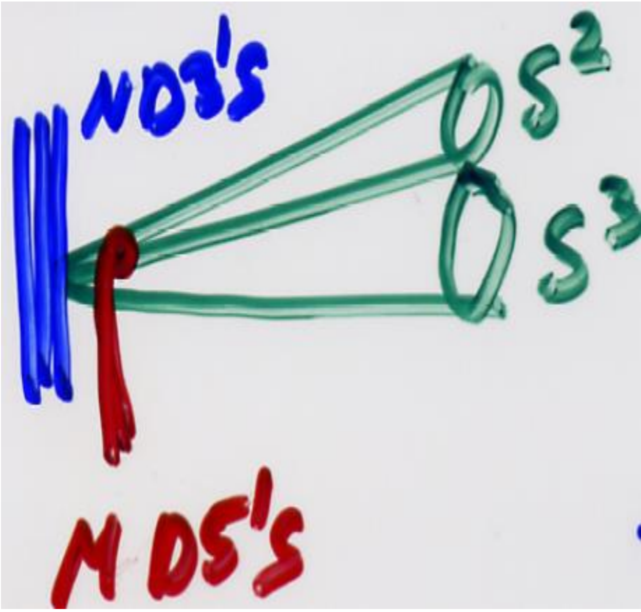
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is

$$\frac{a^2(z_{\max})}{2\pi\alpha'}$$



Confinement and Warped Throat

- To break conformal invariance, change the gauge theory: add to the N D3-branes M D5-branes wrapped over the sphere at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold IRK, Strassler (2000)



$$ds_{10}^2 = h^{-1/2}(y)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(y)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD (graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm).

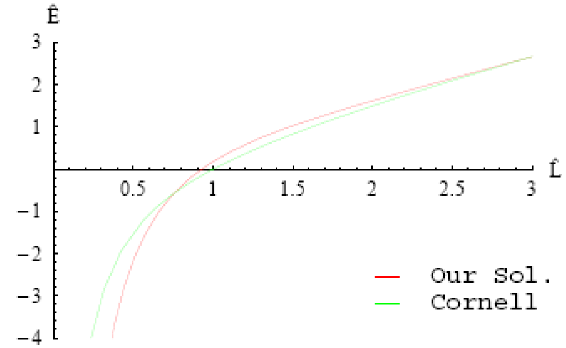
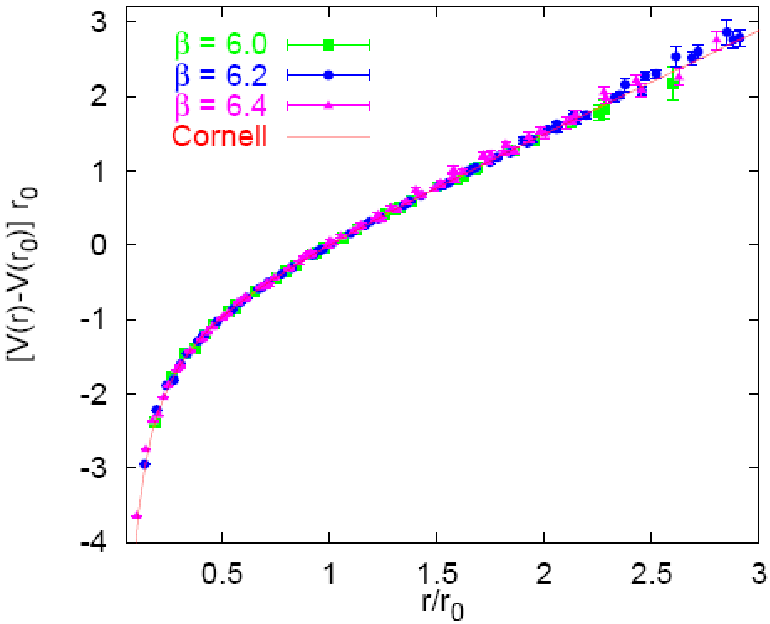


Figure 11: Comparison to the Cornell model

- Normal modes of the warped throat correspond to glueball-like bound states in the gauge theory.
- Their spectra have been calculated using standard methods of (super)gravity.
- The warped deformed conifold incorporates Dimensional Transmutation.



- The gauge/string duality has provided us with a “physicists’s proof of confinement” in some exotic minimally supersymmetric gauge theories.
- Yet, we still don’t have a quantitative handle on the asymptotically free theories in 3+1 dimensions.
- Let us consider some 1+1 dimensional gauge theories, which can hopefully give some intuition about aspects of the higher dimensional dynamics.

The ‘t Hooft Model

- 2d SU(N) gauge theory coupled to N_f fermions in the fundamental representation.
- Exactly solvable in the large N limit using the light-cone gauge.
- Find a single Regge trajectory of mesons whose masses are obtained by solving an integral equation.
- This beautiful toy model may be too simple to teach us much about the higher dimensional gauge dynamics. In particular, it is missing dynamical degrees in the adjoint representation of SU(N).

2D QCD with Adjoint Matter

- Not exactly solvable at large N, but numerically tractable using Discretized Light-Cone Quantization (DLCQ). Dalley, IRK (1992)
- The model with an adjoint Majorana fermion (a toy gluino) has particularly nice properties. The mass is protected against renormalization by a discrete chiral Z_2 symmetry

$$S_{\mathrm{f}} = \int d^2x \, \mathrm{Tr} \left[i\Psi^T \gamma^0 \gamma^\alpha D_\alpha \Psi - m\Psi^T \gamma^0 \Psi - \frac{1}{4g^2} F_{\alpha\beta} F^{\alpha\beta} \right]$$

- In the large N limit we can focus on the string-like single trace gluinoball states

$$|\Phi_{\rm b}(P^+)\rangle = \sum_{j=1}^{\infty} \int_0^{P^+} dk_1 \ldots dk_{2j}\, \delta\Big(\sum_{i=1}^{2j} k_i - P^+\Big)$$
$$f_{2j}(k_1, k_2, \ldots, k_{2j}) N^{-j}\, {\rm Tr}\, [b^\dagger(k_1)\ldots b^\dagger(k_{2j})]|0\rangle$$

- Remarkably, for m=0 some of these states appear to be "threshold bound states" of other states. Gross, Hashimoto, IRK
- This is due to the current algebra module structure of the m=0 theory. Kutasov, Schwimmer; Dempsey, IRK, Pufu
- The massless theory appears to be in the screening rather than confining phase. Gross, IRK, Matytsin, Smilga; Komagodski et al.
- It is also a "gapped topological phase."

DLCQ

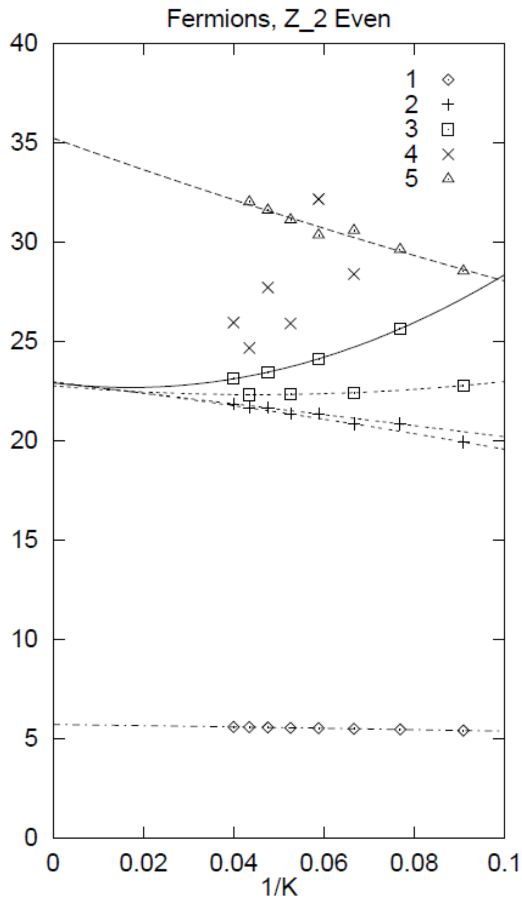
- Make one of the light-cone directions compact. Brodsky, Hornbostel, Pauli
- Anti-periodic boundary conditions

$$\psi_{ij}(x^-) = -\psi_{ij}(x^- + 2\pi L) \qquad\qquad P^+ = K/(2L)$$

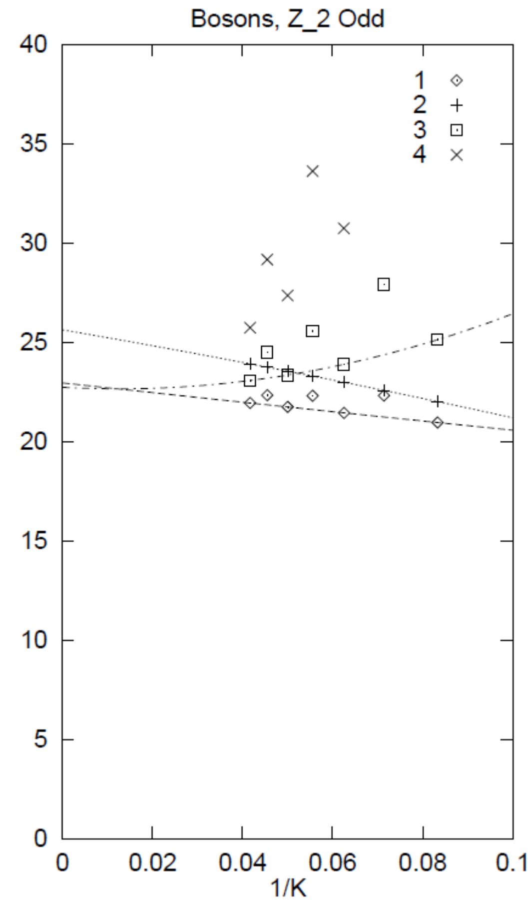
- K is an integer.

$$\psi_{ij}(x) = \frac{1}{\sqrt{2\pi L}} \sum_{\text{odd } n>0} \left(B_{ij}(n) e^{-in\frac{x}{2L}} + B^\dagger_{ji}(n) e^{in\frac{x}{2L}} \right)$$

- Some single-trace gluinoball states exhibit exact degeneracies with multi-trace states!



$5.72 g^2 N/\pi$

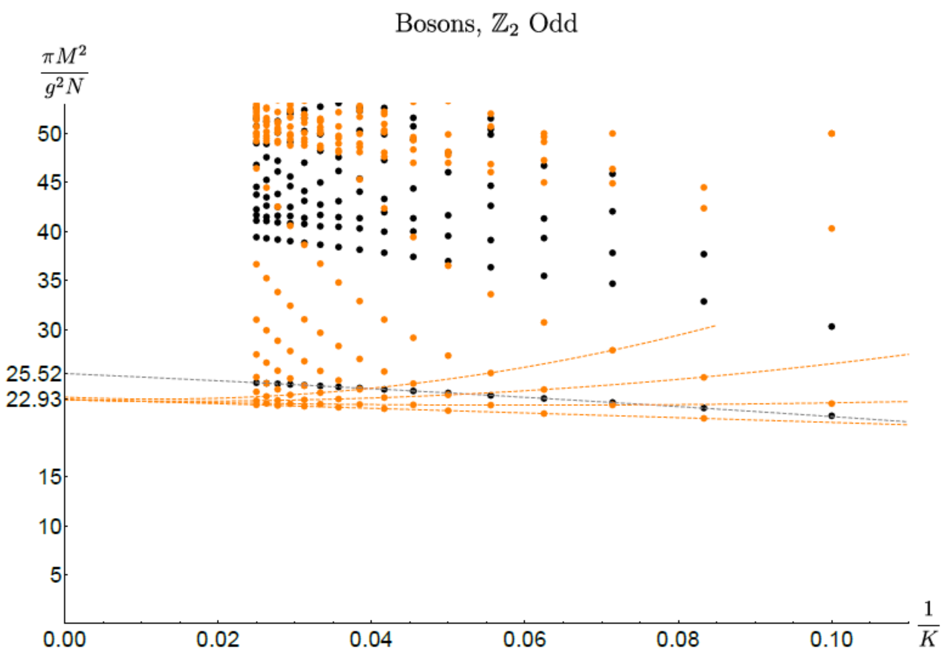


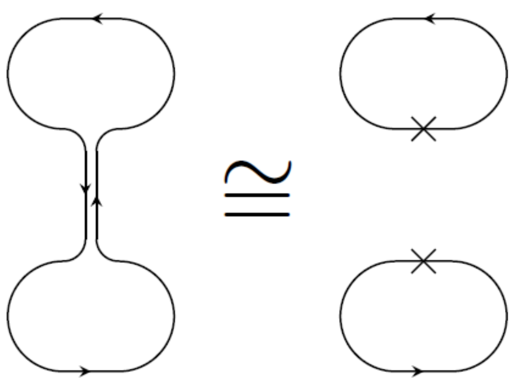
$$M^2 = K\left(\frac{M_{F1}^2(n)}{n} + \frac{M_{F1}^2(K-n)}{(K-n)}\right)$$

- The threshold is at 4 times the lightest mass-squared. Gross, Hashimoto, IRK (1997)

Exact Degeneracies

- In the work with Ross Dempsey and Silviu Pufu (paper will appear tonight) we obtained a better understanding of the exactly degenerate states marked with orange dots.





A New 2D Model for Mesons

- If we add N_f fundamental Dirac fermions to the adjoint Majorana, we find a model which contains both gluinoballs and mesons:

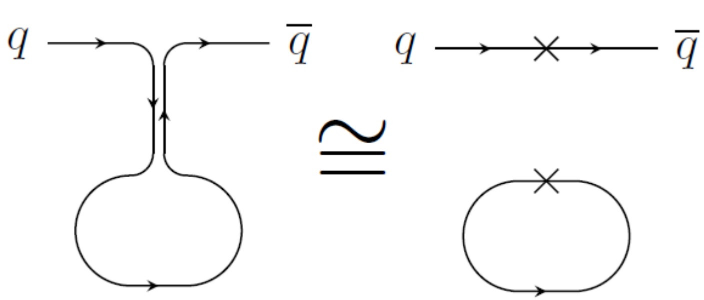
$$S = \int d^2x \left[\mathrm{tr}\left(-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \overline{\Psi} \not{D} \Psi - \frac{m_{\mathrm{adj}}}{2} \overline{\Psi} \Psi \right) + i \sum_{\alpha=1}^{N_f} \left(\overline{q}_\alpha \not{D} q_\alpha - m_{\mathrm{fund}} \overline{q}_\alpha q_\alpha \right) \right]$$

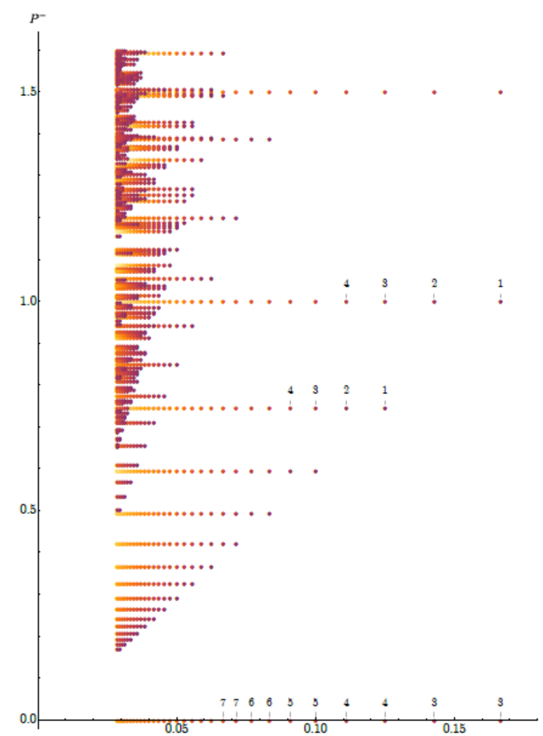
- The mesons are more complicated than in the 't Hooft model, since they also contain the adjoint quanta. There are now multiple Regge trajectories of mesons, which can be bosonic or fermionic.

- The large N meson light-cone wave functions

$$|\{g\}_{\alpha\beta}; P^+\rangle = \frac{(P^+)^{(n-1)/2}}{N^{(n-1)/2}} \sum_n \int_0^1 dx_1 \cdots dx_n \, \delta\left(\sum_{i=1}^n x_i - 1\right) \times g_n(x_1, \ldots, x_n) c_\alpha^\dagger(k_1) b^\dagger(k_2) \cdots b^\dagger(k_{n-1}) d_\beta^\dagger(k_n) \, |0\rangle$$

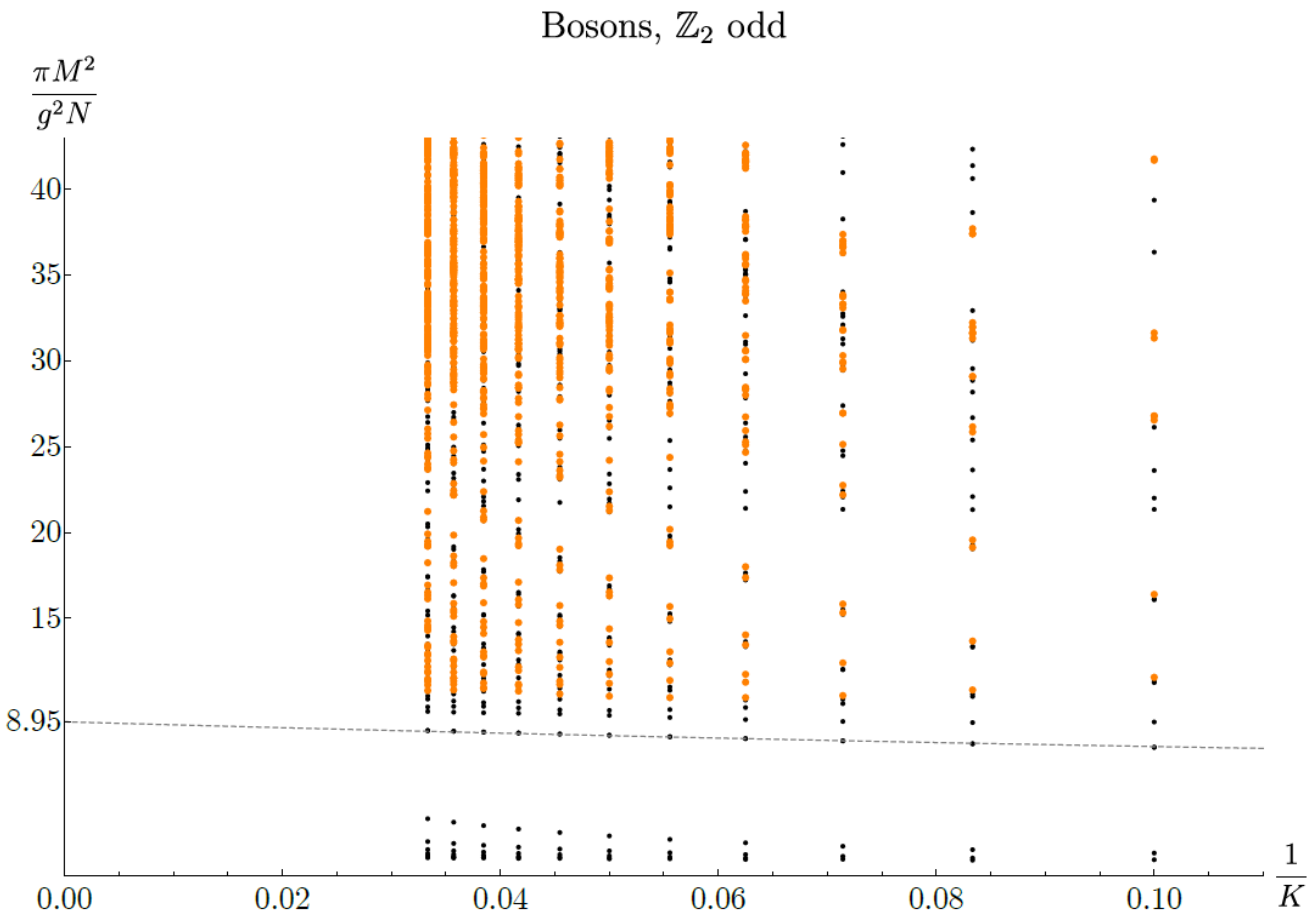
- The mesons exhibit interesting patterns of DLCQ degeneracies!





Massive quarks

- DLCQ degeneracies are present as long as the adjoint fermion is massless.



Conclusions

- Throughout its history, string theory has been intertwined with the theory of strong interactions.
- The Anti-de Sitter/Conformal Field Theory correspondence makes this connection precise. It makes many dynamical statements about strongly coupled conformal gauge theories.
- Extensions of AdS/CFT provide a new geometrical understanding of color confinement and other strong coupling phenomena.
- 2D gauge theories with adjoint matter are other interesting laboratories. They lead to appearance of threshold bound states in some limits.
- Any connection with the seemingly “molecular” structure of some XYZ mesons?