

Quark Masses

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Proton Mass

- We QCD folks like to say that 98% of the nucleon mass and, hence, that of everyday objects is the strong interaction.
- The rest comes from quark masses and, in the Standard Model, that means from quarks burrowing their way through the Higgs field.



up, down, strange



charm, bottom



top

What's a Quark Mass?

- You can't put a quark on a scale and weigh it.
- Need definition, preferably regularization-independent, in QFT.
- Natural candidate is the “perturbative pole mass.” Alas, ambiguous:
 - physics—infrared gluons need to find a sink;
 - mathematics—obstruction to Borel summation of perturbative series;
 - numbers: $m_{b,\text{pole}}/\bar{m}_b = (1, 1.093, 1.143, 1.183, 1.224)$,
 $\bar{m}_h \equiv m_{h,\overline{\text{MS}}}(\bar{m}_h)$;
 - nonsense: pole mass makes little sense for a light quark, m_l , because the natural scale for self-energy contributions is m_l itself.

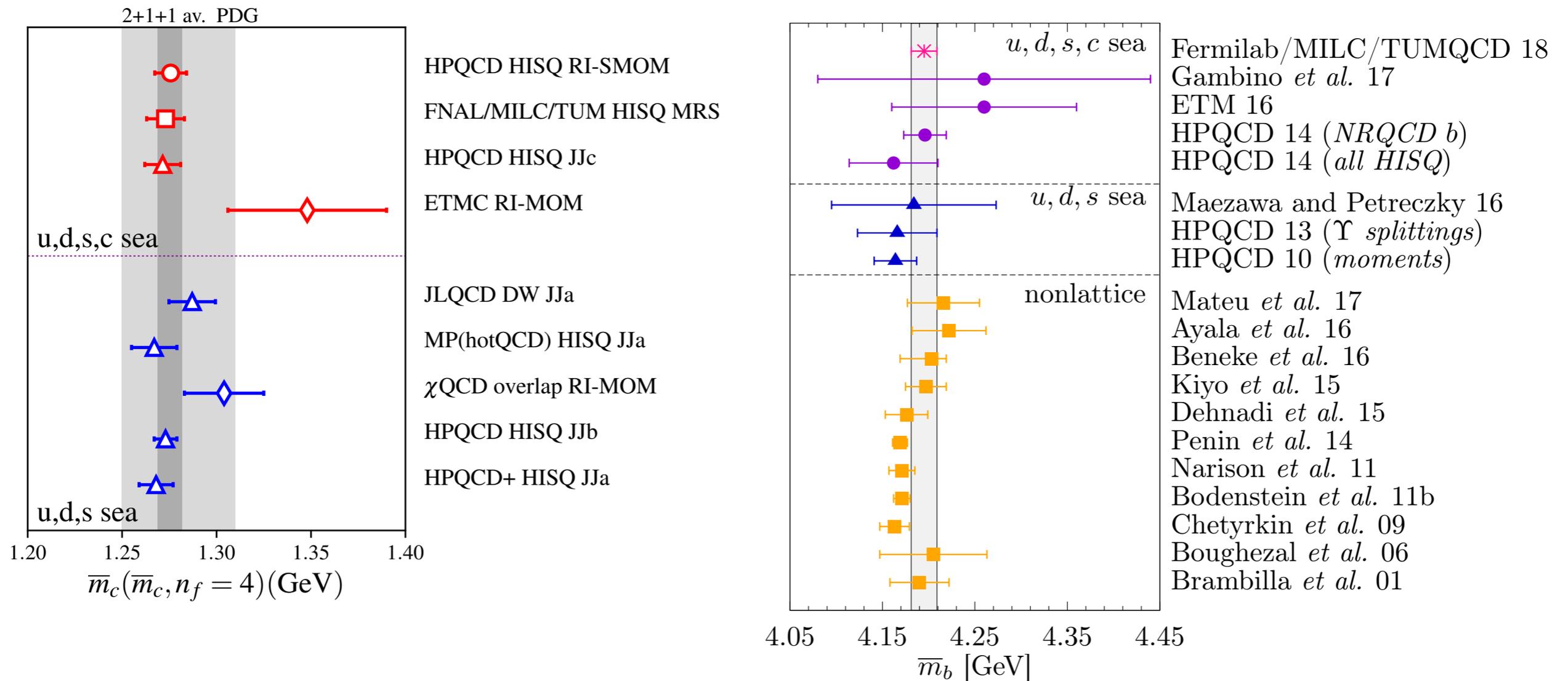
Precise Results from Lattice QCD

- Numerical results [quoting [arXiv:1802.04248](https://arxiv.org/abs/1802.04248)]:

α_s parametric not
PT truncation

- Masses:
 - $m_{l,\overline{\text{MS}}}(2 \text{ GeV}) = 3.404(14)_{\text{stat}}(08)_{\text{syst}}(19)_{\alpha_s}(04)_{f_{\pi,\text{PDG}}} \text{ MeV}$
 - $m_{u,\overline{\text{MS}}}(2 \text{ GeV}) = 2.118(17)_{\text{stat}}(32)_{\text{syst}}(12)_{\alpha_s}(03)_{f_{\pi,\text{PDG}}} \text{ MeV}$
 - $m_{d,\overline{\text{MS}}}(2 \text{ GeV}) = 4.690(30)_{\text{stat}}(36)_{\text{syst}}(26)_{\alpha_s}(06)_{f_{\pi,\text{PDG}}} \text{ MeV}$
 - $m_{s,\overline{\text{MS}}}(2 \text{ GeV}) = 92.52(40)_{\text{stat}}(18)_{\text{syst}}(52)_{\alpha_s}(12)_{f_{\pi,\text{PDG}}} \text{ MeV}$
 - $m_{c,\overline{\text{MS}}}(3 \text{ GeV}) = 984.3(4.2)_{\text{stat}}(1.6)_{\text{syst}}(3.2)_{\alpha_s}(0.6)_{f_{\pi,\text{PDG}}} \text{ MeV}$
 - $m_{b,\overline{\text{MS}}}(m_{b,\overline{\text{MS}}}) = 4203(12)_{\text{stat}}(1)_{\text{syst}}(8)_{\alpha_s}(1)_{f_{\pi,\text{PDG}}} \text{ MeV}$
- Mass ratios:
 - $m_c/m_s = 11.784(11)_{\text{stat}}(17)_{\text{syst}}(00)_{\alpha_s}(08)_{f_{\pi,\text{PDG}}}$
 - $m_b/m_s = 53.93(7)_{\text{stat}}(8)_{\text{syst}}(1)_{\alpha_s}(5)_{f_{\pi,\text{PDG}}}$
 - $m_b/m_c = 4.577(5)_{\text{stat}}(7)_{\text{syst}}(0)_{\alpha_s}(1)_{f_{\pi,\text{PDG}}}$

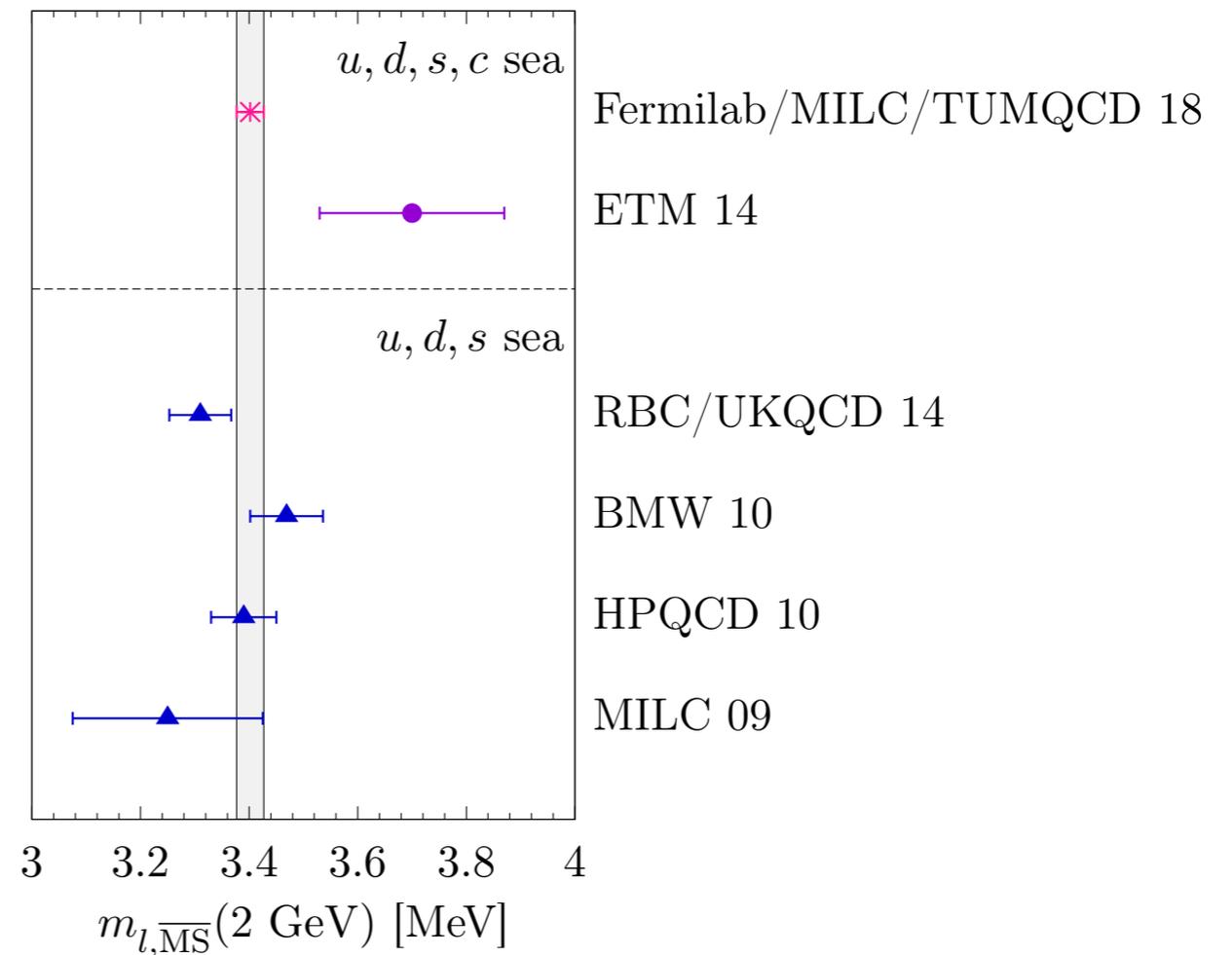
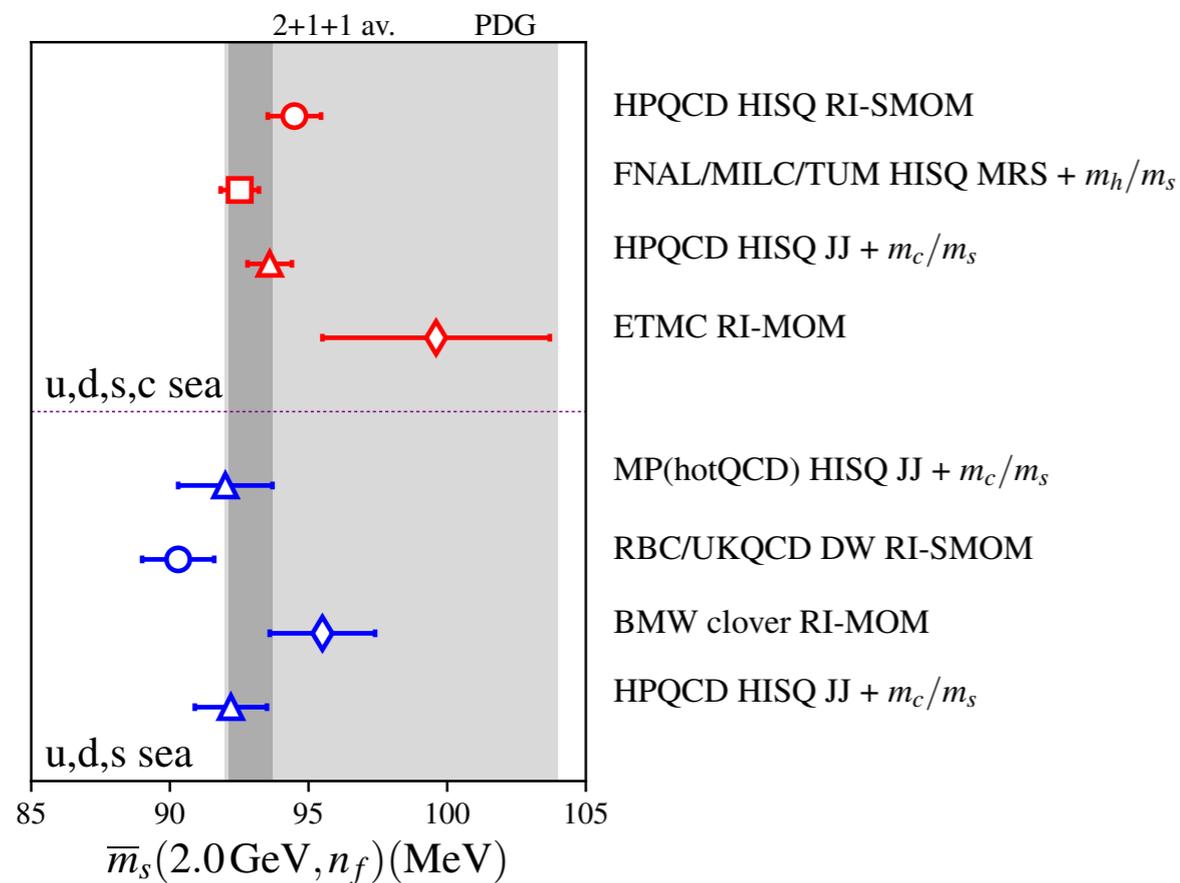
Heavy Comparisons



- Precision: 0.3% for bottom to 0.5% for charm.

plots from [arXiv:1802.04248](https://arxiv.org/abs/1802.04248), [arXiv:1805.06225](https://arxiv.org/abs/1805.06225)

Light Comparisons



- 0.75% for strange quark.

- 2% for up quark.

plots from [arXiv:1802.04248](https://arxiv.org/abs/1802.04248), [arXiv:1805.06225](https://arxiv.org/abs/1805.06225)

$$M_p - 2m_u - m_d = 0.9905M_p$$

$$M_n - 2m_d - m_u = 0.9878M_n$$

Synopsis

- Precise results for all quarks but top (use pQCD + Tevatron, LHC, ILC).
- Good agreement—plots not from FLAG 2019, but results shown are all highly rated.
- Several different methods: RI-SMOM, correlator moments, HQET+MRS.
- Common features:
 - adjust bare lattice mass until chosen hadron mass agrees with PDG;
 - compute a regulator-independent renormalized mass;
 - convert this mass to \overline{MS} with (multi-loop) perturbation theory.
- Precise results for charm & bottom \Rightarrow light-quark masses via mass ratios.

Outline

- Introduction: precise results for all quarks but top.
- What's a quark mass?
- What does it mean that the quoted up-quark mass has a 2% uncertainty?
Or what does “50 sigma from zero” say about the strong CP problem?
- Requires discussion of renormalization:
 - most of which you know;
 - pay attention to additional additive effects lying beyond perturbation theory.

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Unambiguous Definitions

- All come from quantum field theory:
 - bare mass of a cutoff Lagrangian, *e.g.*, lattice gauge theory;
 - renormalized masses—
 - based on a simple physical observable, *e.g.*, correlator moment, quarkonium mass as computed in perturbation theory, ...;
 - Ward identities;
 - regulator-independent via momentum-space subtraction;
 - computationally simple, *e.g.*, (modified) minimal subtraction in dimensional regularization.

Quark Propagator

- Consider quark propagator FT $[q(x)\bar{q}(0)]$.
- The quark field is a $\mathbf{3}$, so have to choose a (covariant) gauge. Then,

$$\text{FT} [q(x)\bar{q}(0)] = \frac{i}{\not{p} - m_0 - \Sigma(p^2; m_0)}$$

$$\Sigma(p^2; m_0) = \not{p}A(p^2; m_0) - C(p^2; m_0)$$

where m_0 is chosen to absorb UV divergences not compensated w/ Z_q .

- The second term could have additive renormalization:

$$C(p^2; m_0) = m_0^* + (m_0 - m_0^*)B(p^2; m_0) + \mathcal{O}(\Lambda_{\text{QCD}}) + \mathcal{O}(m_d m_s / \Lambda_{\text{QCD}})$$

linear UV

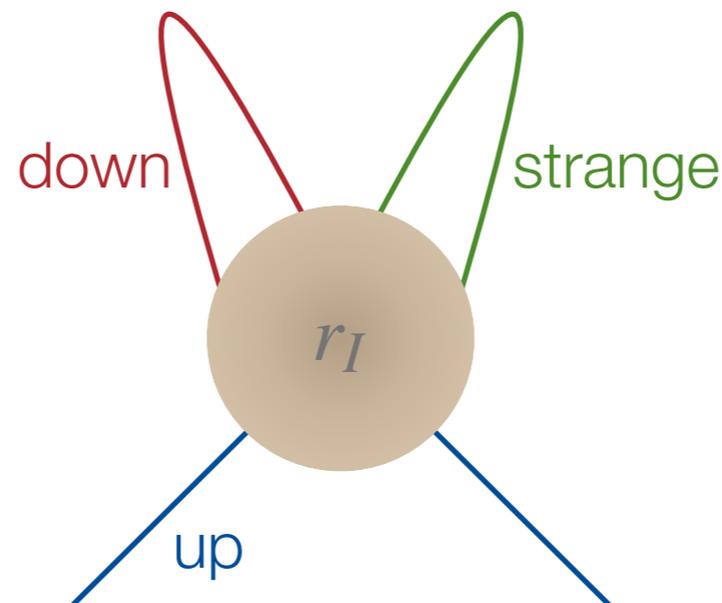
condensates, renormalons

instantons

Instantons?

- Here “instanton” is any gauge-field configuration with nonzero topological charge Q .

- Consider $Q = 1$:



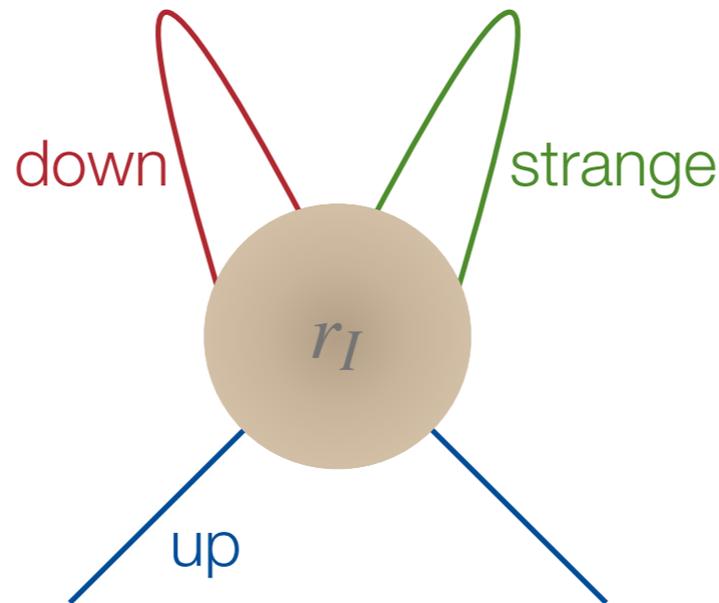
Georgi, McArthur (1981)
 Choi, Kim, Sze
 Kaplan, Manohar
 Banks, Nir, Seiberg
 Cohen, Kaplan, Nelson
 Creutz
 Srednicki
 Bardeen (2018)

- Zero mode: $(\text{Det} \times S_{\text{up}}) r_I e^{-S_I} = \frac{(m_u + \lambda)(m_d + \lambda)(m_s + \lambda)}{(m_u + \lambda)} r_I e^{-S_I}$
 $= \frac{m_u m_d m_s}{m_u} r_I \exp \left[-\frac{2\pi}{\alpha_s(1/r_I)} \right]$
 $\sim \frac{m_d m_s}{\Lambda_{\text{QCD}}} \quad \text{when } r_I \Lambda_{\text{QCD}} \sim 1$

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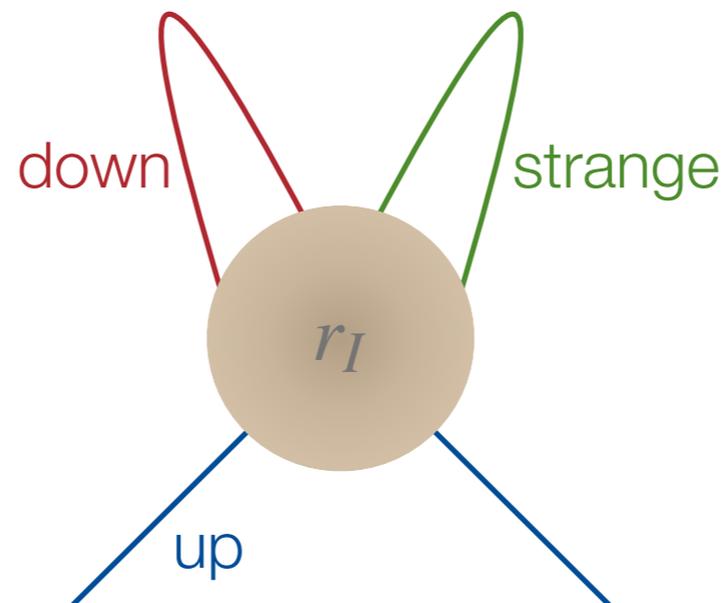
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 $= \frac{m_u m_d m_s}{m_u} r_I (r_I \Lambda_{\text{QCD}})^{18}$
 $\sim \frac{m_d m_s}{\Lambda_{\text{QCD}}}$ when $r_I \Lambda_{\text{QCD}} \sim 1$

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 $\sim \frac{m_d m_s}{\Lambda_{\text{QCD}}}$ when $r_I \Lambda_{\text{QCD}} \sim 1$ 2 MeV?

Pole Mass

- The pole mass is defined via

$$m_{\text{pole}} = \lim_{p^2 \rightarrow m_{\text{pole}}^2} \frac{m_0 - C(p^2; m_0)}{1 - A(p^2; m_0)}$$

- If we “knew” m_{pole} , m_0 would be chosen to absorb UV divergences and any additive terms.
- Perturbation theory with a chirally symmetric UV regulator:
 - $C(p^2; m_0) = m_0 B(p^2; m_0)$ and the “nonperturbative” terms are lost;
 - develop asymptotic expansion in α_s for A & B and obtain m_{pole} order-by-order using iteration;
 - infrared finite & gauge independent at every order of perturbation theory.

Pole Mass II

- The natural scale for perturbation theory is of order m_{pole} :
 - fine for heavy quarks with $m \gg \Lambda_{\text{QCD}}$, modulo renormalons;
 - for light quarks with $m \lesssim \Lambda_{\text{QCD}}$, even the perturbative loops are long-distance contributions.
- Self energy could be calculated in lattice gauge theory (Landau gauge), establishing a curve on the (m_{pole}, m_0) plane, but with no prospect of converting m_{pole} to anything useful.
- Renormalization is not just supposed to make quantities UV finite:
 - to gain the full power of the renormalization group, one wants to separate long- and short-distance quantities.

Mass-independent Renormalization

- For light quarks, the conceptually (and computationally) cleanest schemes are mass independent.
- (Modified) minimal subtraction MS ($\overline{\text{MS}}$) is the best known example; limited to dimensional regularization and, thus, perturbation theory.
- Regulator-independent momentum-subtraction:

$$m(\mu) = \frac{m_0 - C(-\mu^2; m_0)}{1 - A(-\mu^2; m_0)} \quad Z_q(\mu) = 1 - A(-\mu^2)$$

- Renormalized mass $m(\mu)$ vanishes when the numerator vanishes, *i.e.*, at the self-consistent of $m_0^*(\mu) = C(-\mu^2; m_0^*(\mu))$.
- Additive contributions to C (at $p^2 = -\mu^2$) is absorbed into $m_0^*(\mu)$?

Additive Corrections

- With a large, space-like momentum routed through the quark, only short-distance contributions matter:
 - renormalons disappear;
 - instanton effects now of order $m_d m_s \Lambda^{18} \mu^{-19}$;
 - but now the OPE tells us that condensates appear [Politzer, 1976; Pascual, de Rafael, 1982], e.g., $\langle \bar{q}q \rangle / \mu^2$ —
 - because of fixed gauge, icky condensates like $\langle A^2 \rangle$ can also arise.
- It makes more sense to omit these contributions from the renormalized mass: then a purely perturbative conversion to $\overline{\text{MS}}$ is well-defined.

Mass Ratios

- In particular, fitting away the condensates means

$$\frac{m_{Rb}(\mu_R)}{m_{Rc}(\mu_R)} = \frac{m_{\overline{\text{MS}}b}(\mu_{\overline{\text{MS}}})}{m_{\overline{\text{MS}}c}(\mu_{\overline{\text{MS}}})} = \frac{m_{0b}}{m_{0c}} + \mathcal{O}(a^2)$$

in any “mass independent” scheme.

- Last equality holds if the lattice fermions have some chiral symmetry (staggered, overlap)—
 - more work for other lattice fermions (Wilson, domain wall) needed.
- The precise results use staggered fermions.
- Perturbative conversion under best control for heavy quarks; use the ratios to get the up, down, and strange masses.

Methods for Heavy Quarks

RI/SMOM

[hep-lat/9411010](#), [arXiv:0712.1061](#), [arXiv:1306.3881](#), [arXiv:1805.06225](#)

- Here, the bare charm mass is fixed to a meson mass.
- The renormalization constant is computed via $Z_m Z_S = 1$, renormalizing the scalar density in a scheme similar to that outlined above:
 - fit away (milder) condensates by varying μ ;
 - extrapolating valence mass to zero to make Z_m mass independent;
 - convert to $\overline{\text{MS}}$ with perturbation theory at ~ 5 GeV.
- Adjust light bare masses to further meson masses (one-to-one).
- Use ratios of bare masses to obtain light $\overline{\text{MS}}$ masses.

Quarkonium Moments

[hep-lat/9505025](#), [arXiv:0805.2999](#), [arXiv:1408.4169](#), [arXiv:1901.06424](#)

- Here, the bare charm (bottom) mass is fixed to a meson mass.
- A physical renormalized charm mass is defined via time moments of the Euclidean correlation function:
 - natural scale is $2m_c$ ($2m_b$), so perturbation theory should work;
 - fit away (mild) condensates by varying m_h in $m_c < m_h < m_b$;
 - analyze moments with $\overline{\text{MS}}$ perturbation theory at $\sim 3\text{--}10$ GeV.
- Adjust light bare masses to further meson masses (one-to-one).
- Use ratios of bare masses to obtain light $\overline{\text{MS}}$ masses.

HQET \oplus MRS

arXiv:1701.00347, arXiv:1712.04983, arXiv:1802.04248

- Here, the bare charm (bottom) mass is fixed to a heavy-light meson mass.
- The leading renormalon is removed from the pole mass, called minimal renormalon subtraction (MRS):

$$m_{\text{MRS}} \equiv \bar{m} \left(1 + \sum_{n=0}^{\infty} [r_n - R_n] \alpha_g^{n+1}(\bar{m}) \right) + \mathcal{J}_{\text{MRS}}(\bar{m})$$

- the $r_n - R_n$ are very small; $\mathcal{J}_{\text{MRS}}(\bar{m})$ is known exactly and can be computed via a convergent series in $1/\alpha_s(\bar{m})$.
- fit the binding energy away by varying m_h in $m_c < m_h < m_b$:

$$M = m_{\text{MRS}} + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2(\bar{m})}{2m_{\text{MRS}}} + \dots$$

- Use ratios of bare masses to obtain light $\overline{\text{MS}}$ masses.

Summary, Outlook

Summary

- Precise bottom and charm masses are determined via three distinct methods with very different systematics:
 - heavy-quark scale makes a clean separation of short- and long-distance contributions possible (OPE, EFT);
 - nonperturbative short-distance contributions are fit away or tiny;
 - tests of reliability of conversion to $\overline{\text{MS}}$.
- Precise light masses are obtained from these via mass ratios that are the same in all mass-independent schemes.
- Precise results from MILC's 2+1+1 HISQ (or 2+1 asqtad) ensembles, *i.e.*, with staggered quarks.

HISQ Ensembles: 2+1+1

MILC, [arXiv:1212.4768](https://arxiv.org/abs/1212.4768), [arXiv:1712.09262](https://arxiv.org/abs/1712.09262)

a (fm)	size	$am_l/am'_l/am'_c$	# confs	# sources	notes
≈ 0.15	$16^3 \times 48$	0.0130/0.065/0.838	1020	4	
≈ 0.15	$24^3 \times 48$	0.0064/0.064/0.828	1000	4	
≈ 0.15	$32^3 \times 48$	0.00235/0.0647/0.831	1000	4	physical
≈ 0.12	$24^3 \times 64$	0.0102/0.0509/0.635	1040	4	
≈ 0.12	$32^3 \times 64$	0.00507/0.0507/0.628	1020	4	also $24^3, 40^3$
≈ 0.12	$48^3 \times 64$	0.00184/0.0507/0.628	999	4	physical
≈ 0.12	$24^3 \times 64$	0.0102/0.03054/0.635	1020	4	$m'_s < m_s$
≈ 0.12	$24^3 \times 64$	0.01275/0.01275/0.640	1020	4	$m'_s = m_l$
≈ 0.12	$32^3 \times 64$	0.00507/0.0304/0.628	1020	4	$m'_s < m_s$
≈ 0.12	$32^3 \times 64$	0.00507/0.022815/0.628	1020	4	$m'_s < m_s$
≈ 0.12	$32^3 \times 64$	0.00507/0.012675/0.628	1020	4	$m'_s \ll m_s$
≈ 0.12	$32^3 \times 64$	0.00507/0.00507/0.628	1020	4	$m'_s = m_l$
≈ 0.12	$32^3 \times 64$	0.0088725/0.022815/0.628	1020	4	$m'_s < m_s$
≈ 0.09	$32^3 \times 96$	0.0074/0.037/0.440	1005	4	
≈ 0.09	$48^3 \times 96$	0.00363/0.0363/0.430	999	4	
≈ 0.09	$64^3 \times 96$	0.0012/0.0363/0.432	484	4	physical
≈ 0.06	$48^3 \times 144$	0.0048/0.024/0.286	1016	4	
≈ 0.06	$64^3 \times 144$	0.0024/0.024/0.286	572	4	
≈ 0.06	$96^3 \times 192$	0.0008/0.022/0.260	842	6	physical
≈ 0.042	$64^3 \times 192$	0.00316/0.0158/0.188	1167	6	
≈ 0.042	$144^3 \times 288$	0.000569/0.01555/0.1827	429	6	physical
≈ 0.03	$96^3 \times 288$	0.00223/0.01115/0.1316	724	4	

Remark

- Idea that $m_u = 2 \text{ MeV}$ could come from instantons seems implausible:
 - such contributions to the self energy enter into pole mass of light quarks, which is not portable;
 - thus, not what lattice QCD does;
 - in RI/(S)MOM scheme this contribution is clearly tiny;
 - similarly for the bare lattice mass, which probably somehow includes effects for r_I in $(250 \text{ GeV})^{-1} \ll r_I < a$;
 - starting from heavy-quark masses, large instantons are not probed.
- Hence, $m_u = 2 \text{ MeV}$ stems from Yukawa coupling at 250 GeV .