3rd Proton Mass Workshop: Origin and Perspective

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J/ψ photoproduction near threshold and the proton mass distribution

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What is the origin of the proton mass?

Image: CERN

How is the mass distributed inside the proton?

Is it associated with quarks ("visible matter") or with gluons ("dark matter")?

How can we measure the mass distribution?

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Outline

• Gravitational formfactors and the mass distribution

- Scale invariance and scale anomaly in QCD
- The origin of the proton mass

 Charmonium photoproduction near the threshold: measuring the mass distribution inside the proton
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The mass distribution is encoded in the gravitational formfactors.

For the spin ¹/₂ nucleon, 3 formfactors appear:

H. Pagels '66, A. Pais, S. Epstein '49

$$\langle \mathbf{p}_{1} | \theta_{\mu\nu} | \mathbf{p}_{2} \rangle = \left(\frac{M^{2}}{p_{01} \ p_{02}} \right)^{1/2} \frac{1}{4M} \ \bar{u}(p_{1}, s_{1}) \left[G_{1}(q^{2})(p_{\mu}\gamma_{\nu} + p_{\nu}\gamma_{\mu}) + G_{2}(q^{2}) \frac{p_{\mu}p_{\nu}}{M} + G_{3}(q^{2}) \frac{(q^{2}g_{\mu\nu} - q_{\mu}q_{\nu})}{M} \right] u(p_{2}, s_{2})$$

 $\partial^{\mu}\theta_{\mu\nu} = 0$

Energy-momentum conservation:

Satisfied for on-shell nucleons (use Dirac equation)

$$p_1^2 = p_2^2 = M^2$$

 $q^{\mu} \langle \mathbf{p}_1 | \theta_{\mu\nu} | \mathbf{p}_2 \rangle = 0;$

For the spin ½ nucleon, 3 formfactors appear: (no G₁ for spin 0)

$$\langle \mathbf{p}_1 | \theta_{\mu\nu} | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01} p_{02}} \right)^{1/2} \frac{1}{4M} \bar{u}(p_1, s_1) \left[G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \right] u(p_2, s_2)$$

Compare to
the macroscopic energy-momentum
tensor in relativistic hydrodynamics:
The Thermodynamics of Irreversible Processes
III. Relativistic Theory of the Simple Fluid
CARL ECKART
Ryerson Physical Laboratory, University of Chicago, Chicago, Illinois
(Received September 26, 1940)

C. Eckart, 1940

 $u_
u$ - matter velocity

For the spin ½ nucleon, 3 formfactors appear: (no G₁ for spin 0)

$$\langle \mathbf{p}_1 | \theta_{\mu\nu} | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01} \ p_{02}} \right)^{1/2} \frac{1}{4M} \ \bar{u}(p_1, s_1) \left[G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \right] u(p_2, s_2)$$

Zero momentum transfer $q \rightarrow 0$:

$$\langle \mathbf{p} | \theta_{\mu\nu} | \mathbf{p} \rangle = \left(\frac{M^2}{p_0^2} \right)^{1/2} \bar{u}(p,s) u(p,s) \frac{p_\mu p_\nu}{M^2} \left[G_1(0) + G_2(0) \right]$$

(no "stress" G₃)

In the rest frame of the nucleon:

the Hamiltonian

$$H = \int d^3x \theta_{00}(x)$$

$$\langle \mathbf{p} = 0 | \theta_{00} | \mathbf{p} = 0 \rangle = M$$
$$\bigcup$$
$$G_1(0) + G_2(0) = M.$$

Formfactor of the trace of the energy-momentum tensor

Let us call it "scalar gravitational formfactor", as it would be a gravitational formfactor in a scalar model of gravity: Nords

Nordstrom 1912 Einstein 1913

$$\langle \mathbf{p}_1 | \theta | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01} \ p_{02}} \right)^{1/2} \ \bar{u}(p_1, s_1) u(p_2, s_2) \ G(q^2)$$

Scalar gravitational formfactor:

 $\theta \equiv \theta^{\mu}$

$$G(q^2) = G_1(q^2) + G_2(q^2) \left(1 - \frac{q^2}{4M^2}\right) + G_3(q^2) \frac{3q^2}{4M^2}$$

In the rest frame of the nucleon:

How to define the mass distribution in the nucleon?

- At small momentum transfer $|q^2| \ll M^2$,
- the formfactor of θ_{00} and the scalar gravitational formfactor coincide, thus the scalar gravitational formfactor can be used to extract the mass distribution at distances $r \gg \frac{1}{M} \simeq 0.2$ fm; usual definition: $\langle R_s^2 \rangle = 6 \frac{dG}{dt} \Big|_{t=0}$
- In the relativistic region (mass -> energy), it is natural to consider the scalar gravitational formfactor, as θ is the Lorentz scalar
- The trace of the energy-momentum tensor also plays a special role it is a generator of dilatations. Its formfactor thus carries information about the Renormalization Group (RG) evolution inside the nucleon.

Scale invariance

Scale transformations (dilatations) are defined by

$$x \to e^{\lambda} x$$

the corresponding dilatational current is

$$s^{\mu} = x_{\nu} \ \theta^{\mu\nu}$$



Hermann Weyl (1885-1955)

It is conserved (a theory is scale-invariant) if the energy-momentum is traceless:

$$\partial_{\mu}s^{\mu} = \theta^{\mu}_{\mu}$$

Scale invariance

A scale-invariant theory cannot contain massive particles, all particles must be massless

For example, in Maxwell electrodynamics with action

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

the energy-momentum is traceless: $~\theta^{\mu}_{\mu}=0~$ (massless photons)

Note: because of this, in scalar gravity (e.g. Einstein, 1913) there would be no light bending by massive bodies!

Scale invariance and matrix element of θ

$$\begin{split} & \text{In a macroscopic theory,} & \text{energy} \\ & \text{density pressure} \\ & \bar{\theta} \equiv \langle \text{matter} | \theta | \text{matter} \rangle = \epsilon - 3p \\ & \text{Ideal massless gas:} \quad \epsilon = 3p \\ & \bar{\theta} \neq 0 \quad \text{interaction measure, deviation from "conformality"} \\ & \text{Usually, one expects} \quad \bar{\theta} \geq 0 \quad \text{See e.g. Landau, Lifshitz, v. 2, sect. IV} \\ & \text{Can } \bar{\theta} \text{ be negative? Yes! May happen in the interior of } \\ & \text{Zel'dovich 1961} \quad \text{neutron stars} \\ & \text{THE EQUATION OF STATE AT ULTRAHIGH DENSITIES AND ITS RELATIVISTIC} & e.g. D.Podkowka et al \\ & \text{LIMITATIONS} \\ & \text{Submitted to JETP editor May 31, 1961} \\ & \text{J. Exptl. Theorer. Phys. (U.S.S.R.) 41, 1609-1616 (November, 1961)} \\ \end{split}$$

Scale invariance in QCD

The trace of the energy-momentum tensor in QCD (computed in classical field theory) is

$$\Theta_{\alpha}^{\alpha} = \sum_{l=u,d,s} m_l \ \bar{q}_l q_l + \sum_{h=c,b,t} m_h \ \bar{q}_h q_h$$

Two problems:

- 1. Potentially large contribution from heavy quarks to the masses of light hadrons
- 2. If we forget about heavy quarks, all hadron masses must be equal to zero in the chiral limit 12

Scale anomaly in QCD

The quantum effects (loop diagrams) modify

the expression for the trace of the energy-momentum tensor:

$$\Theta^{\alpha}_{\alpha} = \frac{\beta(g)}{2g} G^{\alpha\beta a} G^a_{\alpha\beta} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l + \sum_{h=c,b,t} m_h (1+\gamma_{m_h}) \bar{Q}_h Q_h,$$

Running coupling -> dimensional transmutation -> mass scale

Gross, Wilczek;
$$eta(g)=-brac{g^3}{16\pi^2}+...,\ b=9-rac{2}{3}n_h,$$
Politzer

Ellis, Chanowitz; Crewther; Collins, Duncan, Joglecar; ...

 $\theta'_{\mathsf{had}\,\mu}$

At small momentum transfer, heavy quarks decouple:

$$\begin{split} &\sum_{h} m_{h} \bar{Q_{h}} Q_{h} \rightarrow -\frac{2}{3} n_{h} \frac{g^{2}}{32\pi^{2}} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \dots \\ &\text{so only light quarks enter the final expression} \\ &\Theta^{\alpha}_{\alpha} = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \sum_{l=u,d,s} m_{l} \bar{q_{l}} q_{l}, \end{split}$$

The proton mass

At zero momentum transfer, the matrix element of the trace of the energy-momentum tensor defines the mass of the proton:

$$\langle \mathbf{p} = 0 | \theta | \mathbf{p} = 0 \rangle = \langle \mathbf{p} = 0 | \theta_{00} | \mathbf{p} = 0 \rangle = M$$

$$\Theta^{\alpha}_{\alpha} = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \sum_{l=u,d,s} m_{l} \bar{q}_{l} q_{l},$$

In the chiral limit, the entire mass is from gluons!

The proton mass

At finite quark mass, contribution from "sigma-terms"

$$\Sigma_{\pi N} = \hat{m} \langle p | \bar{u}u + \bar{d}d | p \rangle$$

can be extracted from pion-nucleon scattering or measured on the lattice e.g. Y.-B.Yang et al arXiv:1511.09089

Sometimes interpreted as either

- 1. Contribution from quark masses
- or
- 2. Contribution from chiral symmetry breaking

But the interpretation is more subtle

The proton mass

The matrix elements over a hadron state have to be understood as the **difference** of the value of the measured quantity in the hadron and in the vacuum, e.g.

$$\langle P|\bar{q}q|P\rangle = \langle P|\int d^3x \ \bar{q}(x)q(x)|P\rangle - \langle 0|\bar{q}q|0\rangle V_P$$

This difference results from the partial **restoration** of spontaneously broken chiral symmetry inside the hadron e.g., Donoghue, Nappi '86

Partial restoration of chiral symmetry inside the nucleon



6 4.10^{-4} 4 2 ≥ 0 $2 \cdot 10^{-4}$ -2 -4 -6 0.10^{0} 2 4 6 -6 0 X

Significant suppression of the local quark condensate by the confining flux tube!

> Iritani, Cossu, Hashimoto arXiv:1502.04845 PRD¹⁷

Partial restoration of chiral symmetry inside the nucleon



The proton mass as a result of the vacuum polarization induced by the presence of the proton

$$\Theta^{\alpha}_{\alpha} = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \sum_{l=u,d,s} m_{l} \bar{q}_{l} q_{l},$$

Polarization of the gluon field;

~ 90% of the proton's mass ?



Polarization of the quark condensate;

numerically, ~80 MeV using

Y.-B.Yang et al arXiv:1511.09089

How to measure the mass distribution inside the proton?

No dilatons available... next best thing: a heavy quarkonium

QCD multipole expansion:

Voloshin '78; Appelquist, Fischler '78; Gottfried '78; Peskin '79; Novikov, Shifman '81; Leutwyler '81, ...





M.B. Voloshin 1953-2020

$$g^{2}\mathbf{E}^{a2} = \frac{g^{2}}{2}(\mathbf{E}^{a2} - \mathbf{B}^{a2}) + \frac{g^{2}}{2}(\mathbf{E}^{a2} + \mathbf{B}^{a2})$$
$$= -\frac{1}{4}g^{2}G^{a}_{\alpha\beta}G^{a\alpha\beta} + g^{2}(-G^{a}_{0\alpha}G^{a\alpha}_{0} + \frac{1}{4}g_{00}G^{a}_{\alpha\beta}G^{a\alpha\beta}) = \frac{8\pi^{2}}{b}\theta^{\mu}_{\mu} + g^{2}\theta^{(G)}_{00}$$

$$\theta^{\mu}_{\mu} \equiv \frac{\beta(g)}{2a} G^{a\alpha\beta} G^a_{\alpha\beta} = -\frac{bg^2}{32\pi^2} G^{a\alpha\beta} G^a_{\alpha\beta} \ , \quad \theta^{(G)}_{\mu\nu} \equiv -G^a_{\mu\alpha} G^{a\alpha}_{\nu} + \frac{4}{4} g_{\mu\nu} G^a_{\alpha\beta} G^{a\alpha\beta}_{\alpha\beta} G^{a\alpha\beta}_{\alpha\beta} \ , \quad \theta^{(G)}_{\mu\nu} \equiv -G^a_{\mu\alpha} G^{a\alpha}_{\nu} + \frac{4}{4} g_{\mu\nu} G^a_{\alpha\beta} G^{a\alpha\beta}_{\alpha\beta} G^{a\alpha\beta}_{\alpha\beta} \ , \quad \theta^{(G)}_{\mu\nu} \equiv -G^a_{\mu\alpha} G^{a\alpha}_{\nu} + \frac{4}{4} g_{\mu\nu} G^a_{\alpha\beta} G^{a\alpha\beta}_{\alpha\beta} G^{a\alpha\beta}_{\alpha\beta} \ , \quad \theta^{(G)}_{\mu\nu} \equiv -G^a_{\mu\alpha} G^{a\alpha}_{\nu} + \frac{4}{4} g_{\mu\nu} G^a_{\alpha\beta} G^{a\alpha\beta}_{\alpha\beta} \ .$$

Quarkonium interactions at low energy

Colonna -

Perturbation theory:

at large distances, the Casimir-Polder interaction (retardation)

Bhanot, Peskin '78

$$V^{\text{pt}}(R) = -g^4 \left(\bar{d}_2 \frac{a_0^2}{\epsilon_0} \right)^2 \frac{23}{8\pi^3} \frac{1}{R^7};$$

Fujii, DK '99
$$23 = 15 + 8 \underbrace{53}_{\text{scalar 0++}} \underbrace{15}_{\text{tensor 2++}} \underbrace{15}_{\text{tensor 2++} \underbrace{15}_{\text{tensor 2++}} \underbrace{15}_{\text{tensor 2++}} \underbrace{15}_{\text{tensor 2++}} \underbrace{15}_{\text{tensor 2++} \underbrace{15}_{\text{tensor 2++}} \underbrace{15}_{\text{tensor 2++} \underbrace{15}_{\text{tensor 2++}} \underbrace{15}_{\text{tensor 2++} \underbrace{15}_{\text{tensor 2++} \underbrace{15}_{\text{tensor 2++} \underbrace{15}_{\text{tensor 2++} \underbrace{15}_{\text{tensor 2++} \underbrace{15}_{\text{tensor 2++} \underbrace{15}_{\text{tensor 2$$

Beyond perturbation theory, scalar is strongly enhanced due to scale anomaly Quarkonium interactions at low energy and the scale anomaly

But, at very large distances, the interaction must be dominated by the lightest physical states - pions



conversion of gluons to pions is a (hopeless?) non-perturbative problem

...but, can use scale anomaly matching!

Voloshin,2Zakharov '80

Quarkonium interactions at low energy and the scale anomaly

Use RG invariance to match the EMT computed in QCD and in the chiral theory:

$$\theta^{\mu}_{\mu} = -2 \ \frac{f_{\pi}^2}{4} \ \mathrm{tr} \ \partial_{\mu} U \partial^{\mu} U^{\dagger} \ - \ m_{\pi}^2 f_{\pi}^2 \ \mathrm{tr} \left(U + U^{\dagger} \right)$$

to lowest order in the pion field

$$\theta^{\mu}_{\mu} = -\partial_{\mu}\pi^{a}\partial^{\mu}\pi^{a} + 2m_{\pi}^{2}\pi^{a}\pi^{a} + \cdots$$

In the chiral limit scale anomaly yields:

$$\langle \pi^+ \pi^- | \theta^\mu_\mu | 0 \rangle = q^2$$
²³

Quarkonium interactions at low energy and the scale anomaly

The result (long distances):

$$V^{\pi\pi}(R) \to -\left(\bar{d}_2 \frac{a_0^2}{\epsilon_0}\right)^2 \left(\frac{4\pi^2}{b}\right)^2 \frac{3}{2} (2m_\pi)^4 \frac{m_\pi^{1/2}}{(4\pi R)^{5/2}} e^{-2m_\pi R}.$$

Fujii, DK, PRD'99



 Not a Yukawa potential (retardation)
 The QCD coupling has disappeared at large distance (but not b from the beta-function)

3. Entirely due to scalar 0⁺⁺ exchange

Probing the proton mass

The quarkonium-proton scattering amplitude

$$\begin{split} F_{\Phi h} &= r_0^3 \epsilon_0^2 \sum_{n=2}^{\infty} d_n \langle h | \frac{1}{2} G_{0i}^a (D^0)^{n-2} G_{0i}^a | h \rangle \\ & \text{Wilson coefficients} \\ d_n^{(1S)} &= \left(\frac{32}{N}\right)^2 \sqrt{\pi} \frac{\Gamma(n+\frac{5}{2})}{\Gamma(n+5)} & \text{M.Peskin '78} \\ d_n^{(2S)} &= \left(\frac{32}{N}\right)^2 4^n \sqrt{\pi} \frac{\Gamma(n+\frac{5}{2})}{\Gamma(n+7)} (16n^2+56n+75) \\ d_n^{(2P)} &= \left(\frac{15}{N}\right)^2 4^n 2 \sqrt{\pi} \frac{\Gamma(n+\frac{7}{2})}{\Gamma(n+6)} & \text{DK, '96}_{\text{nucl-th/9601029}} \\ \end{split}$$

Quarkonium-proton interaction

$$F_{\Phi h} = r_0^3 \epsilon_0^2 \sum_{n=2}^{\infty} d_n \langle h | \frac{1}{2} G_{0i}^a (D^0)^{n-2} G_{0i}^a | h \rangle$$

1. Interaction is attractive (VdW force of QCD)

S.Brodsky, I.Schmidt, G. de Teramond '90

 For n=2 (low energy) the amplitude is proportional to the trace of the energy-momentum tensor

M.Luke, A.Manohar, M.Savage '92



Near threshold, dominance of $g^2 \mathbf{E}^{a2} = \frac{8\pi^2}{b} \theta^{\mu}_{\mu} + g^2 \theta^{(G)}_{00}$

Assuming the validity of vector meson dominance, can relate photoproduction to quarkonium scattering amplitude and probe the mass of the proton

DK, Satz, Syamtomov, Zinovjev '99

Other approaches to threshold photoproduction: Hatta, Yang '18; Hatta, Rajan, Yang '19; Mamo, Zahed '19



$$t_{min} = -\frac{M_{\psi}^2 M}{M_{\psi} + M} \simeq -2.23 \text{ GeV}^2 \simeq -(1.5 \text{ GeV})^2$$

-> VDM questionable.
 but, scanning the energy range near the threshold,
 we measure the scalar gravitational formfactor –
 can extract the proton mass distribution!





The scalar operator dominates for small velocity of heavy quarkonium;

Limiting $V_{J/\psi} < 0.2$, (corrections ~ $v_{J/\psi}^2$) the optimal kinematical region is:

$$E_{cm} < 4.25 \text{ GeV}$$

 $E_{\gamma} < 9.2 \text{ GeV}$
-t < 6 GeV²

Editors' Suggestion

First Measurement of Near-Threshold J/ψ Exclusive Photoproduction off the Proton

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(GlueX Collaboration)



Need to focus on the threshold region!

 $E_{cm} < 4.25 \text{ GeV}$ $E_{\gamma} < 9.2 \text{ GeV}$

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Threshold photoproduction of quarkonium: the effect of the scalar gravitational formfactor

The scalar gravitational formfactor can be constrained theoretically by using:

- i) dispersion relations;
- ii) low-energy theorems of broken scale invariance;

iii) experimental data on $\pi\pi$ phase shifts and scalar mesons

See e.g. However, as a first step, can try a simple Fujii, DK'99 : 0.1 dipole formfactor of the type used for

$$G(t) = rac{M}{\left(1 - t/M_s^2\right)^2}$$
 radius $\langle R_s^2 \rangle = 6 rac{dG}{dt} \Big|_{t=0}$

Dipole formfactor was also used for 2-gluon coupling in perturbative models See e.g. See e.g. ³² Frankfurt, Strikman '02

The cross section is sensitive to the formfactor due to the energy dependence of t_{min}

Differential cross section

Differential cross section

The proton mass radius

My estimate of the r.m.s. "proton mass radius" from GlueX data:

$$ar{
m R}_{
m m}\equiv \sqrt{R_s^2}=0.7\pm 0.2~{
m fm}$$
 DK, to appear

Compare to the proton charge radius:

$$\bar{\mathrm{R}}_{\mathrm{c}} \equiv \sqrt{R_c^2} = 0.8409 \pm 0.0004 ~\mathrm{fm}$$

Perhaps, a hint for a more compact mass distribution? Need more data!

VALUE (fm)		DOCUMENT ID		TECN	COMMENT
0.8409 ± 0.0004	OUR A	VERAGE			
0.833 ±0.010	1	BEZGINOV	2019	LASR	2S-2P transition in H
$0.831 \pm 0.007 \pm 0.012$	2	XIONG	2019	SPEC	$e \ p \rightarrow ep$ form factor
$0.84087 \pm 0.00026 \pm 0.00029$		ANTOGNINI	2013	LASR	μp -atom Lamb shift
••• We do not use the following data for averages, fits, limits, etc. •••					
0.877 ±0.013	3	FLEURBAEY	2018	LASR	1S-3S transition in H
0.8335 ±0.0095	4	BEYER	2017	LASR	2S-4P transition in H
0.8751 ± 0.0061		MOHR	2016	RVUE	2014 CODATA value
$0.895 \pm 0.014 \pm 0.014$	5	LEE	2015	SPEC	Just 2010 Mainz data
0.916 ±0.024		LEE	2015	SPEC	World data, no Mainz
0.8775 ± 0.0051		MOHR	2012	RVUE	2010 CODATA, ep data
$0.875 \pm 0.008 \pm 0.006$		ZHAN	2011	SPEC	Recoil polarimetry
$0.879 \pm 0.005 \pm 0.006$		BERNAUER	2010	SPEC	$e \ p \rightarrow ep$ form factor

2020 Review of Particle Physics.

P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Some day: p MASS RADIUS in PDG?

Summary

- The proton mass to large extent originates from quantum anomalies
- The threshold photoproduction of J/ψ probes the mass distribution inside the proton; current data and a simple dipole model favor

$$ar{\mathrm{R}}_\mathrm{m}\equiv\sqrt{R_s^2}=0.7\pm0.2~\mathrm{fm}$$

• We need a quantitative theory of the scalar gravitational formfactor and precise data at $E_{cm} < 4.3$ GeV to understand the mass distribution inside the proton, and the origin of the proton mass!