

Mechanical properties of particles

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- Interaction of the nucleon with gravity, gravitational ffs
- Three global fundamental mechanical particle properties: mass, spin & D (related to shear and pressure, *Druck-term*)
- Effective chiral theory in presence of gravity
- Normal and tangential forces inside large N_c nucleon. Stability conditions.
- First experimental results on gravitational form factors
- Conclusion and outlook.

based on:

MVP, PLB555 (2003)

HD Son, MVP, JHEP09 (2018)

Alharazin, Djukanovic, Gegelia, MVP, PRD102 (2020)

for a review see: P. Schweitzer, MVP [1805.06596](#)

Interaction of a hadron with gravity

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} L_M \quad \text{/Hilbert '1915/}$$

Let us change the metric in long wave, static way

$$g^{\mu\nu}(x) = \eta^{\mu\nu} + \delta g^{\mu\nu}(\vec{r}) \quad \lambda_{\text{grav}} \gg \frac{1}{M_N} \quad T_{\mu\nu}(x) = \frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}(x)}$$

Then the response of the nucleon to the static change of the space-time metric can be characterised by static EMT (Breit frame): MVP '2003

$$T_{\mu\nu}(\vec{r}) = \int \frac{d^3\Delta}{(2\pi)^3 2E} e^{-i\vec{r}\vec{\Delta}} \langle p' | T_{\mu\nu}(0) | p \rangle,$$

Particular way to localise a hadron. Other possibilities and discussion of static EMT in terms of phase-space Wigner distribution see in /Lorce et al. '18/

A location \vec{r} is uncertain by $\delta r \sim \frac{\hbar}{Mc}$ Compton wave length or amplitude of Zitterbewegung

For e.m. ffs fully relativistic and model independent interpretation in terms of charge distributions can be given within a phase space approach /Lorce, PRL 125 (2020)/. Analogous interpretation of EMT ffs is possible.

Zitterbewegung is irrelevant here:

- For Integrated quantities (charges) like $\int d^3r T_{\mu\nu}(\vec{r}) \{1, r^i, r^i r^k\}$
- For large distance asymptotic $r \gg 1/M$
- For heavy particles, e.g. for nucleon in the large N_c limit, nuclei, quarkonia
- Everything can be reformulated using phase space Wigner picture /Lorce, PRL 125 (2020)/
- Interesting task: to define static EMT in IMF= light cone pressure and shear forces?
(for charge distributions see /M. Burkardt '00, G.A. Miller '07/)

EMT form factors for the nucleon

$$T_q^{\mu\nu} = \frac{1}{4} \bar{\psi}_q \left(-i \overleftarrow{\mathcal{D}}^\mu \gamma^\nu - i \overleftarrow{\mathcal{D}}^\nu \gamma^\mu + i \overrightarrow{\mathcal{D}}^\mu \gamma^\nu + i \overrightarrow{\mathcal{D}}^\nu \gamma^\mu \right) \psi_q - g^{\mu\nu} \bar{\psi}_q \left(-\frac{i}{2} \overleftarrow{\mathcal{D}} + \frac{i}{2} \overrightarrow{\mathcal{D}} - m_q \right) \psi_q,$$

$$T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,\eta\nu} + \frac{1}{4} g^{\mu\nu} F^{a,\kappa\eta} F^{a,\kappa\eta}.$$

$$\langle p' | T_{\mu\nu}^a(0) | p \rangle = \bar{u}' \left[A^a(t) \frac{P_\mu P_\nu}{M_N} + J^a(t) \frac{i P_{\{\mu} \sigma_{\nu\} \rho} \Delta^\rho}{2M_N} + D^a(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4M_N} + M_N \bar{c}^a(t) g_{\mu\nu} \right] u$$

Kobzarev, Okun '1962 , Pagels '1966

$$P = (p' + p)/2, \Delta = p' - p.$$

The name “**D-term**” is rather technical, it can be traced back to more or less accidental notations chosen in /Weiss, MVP '99/. Nowadays, given more clear physics meaning of this quantity, we might call this term as “**Druck-term**” derived from German word for pressure

$$A = A^{Ji}$$

$$J = \frac{1}{2} (A^{Ji} + B^{Ji})$$

$$D = 4C^{Ji}$$

$$\bar{c} = \bar{c}^{Ji}$$

Relations to X.-D. Ji's ABC notations

Interaction of the nucleon with gravity

$$\langle p' | T_{\mu\nu}^a(0) | p \rangle = \bar{u}' \left[A^a(t) \frac{P_\mu P_\nu}{M_N} + J^a(t) \frac{i P_{\{\mu} \sigma_{\nu\} \rho} \Delta^\rho}{2M_N} + D^a(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4M_N} + M_N \bar{c}^a(t) g_{\mu\nu} \right] u$$

$a = g, Q$ (gluon or quark parts)

δg^{00} δg^{0i} δg^{ij} non-conservation of EMT pieces

Mass Spin deformation of space = elastic properties of N

$\sum_a A^a(0) = 1$ $\sum_a J^a(0) = \frac{1}{2}$ $\sum_a \bar{c}^a(t) = 0$

Elasticity stress tensor (Landau & Lifshitz vol. 7)

Shear forces distribution (pressure anisotropy)

Pressure distribution

MVP '2003

$$T_{ij}^a(\vec{r}) = \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) s^a(r) + \delta_{ij} p^a(r)$$

$$s^a(r) = -\frac{1}{4M_N} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}^a(r), \quad p^a(r) = \frac{1}{6M_N} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}^a(r) - M_N \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\vec{\Delta}\vec{r}} \bar{c}^a(-\vec{\Delta}^2).$$

$$\tilde{D}^a(r) = \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\vec{\Delta}\vec{r}} D^a(-\vec{\Delta}^2)$$

O.Teryaev called it nucleon "cosmological term" $\delta p^a = -\delta \rho_E^a$

- a) related to forces between quark and gluon subsystems /HDSon,MVP'18/
- b) contribute to "gluon" and "quark" parts of energy density (mass decomposition) /Lorce '18/
- c) instanton contribution to nucleon $\bar{c}^q(0) \approx +1.4 \cdot 10^{-2}$ /HDSon,MVP'18/
- d) $\bar{c}^q(0) = 0$ for Goldstone bosons /Schweitzer, MVP'19/

Three global fundamental mechanical properties of hadrons: M, J, D

MVP '2003

Two the most important particle properties:

$$M = \int d^3\mathbf{r} \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{00}(\vec{r})} \Big|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

$T^{00}(\vec{r})$

$$J^i = \epsilon^{ikl} \int d^3\mathbf{r} r^k \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{0l}(\vec{r})} \Big|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

M, J, D are independent of way we localise hadron

Variations of (00) and (0l) components of metric can be such that the Riemann tensor =0, e.g. just going to non-inertial reference frame
That is why we can measure the mass by the shape of particle track in external em field, or Earth angular velocity with help of Foucault pendulum

Unexplored property:

$$D = -\frac{M}{2} \int d^3\mathbf{r} \left(r^i r^k - \frac{1}{3} r^2 \delta^{ik} \right) \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{ik}(\mathbf{r})} \Big|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

Variation of (ik) components necessarily leads to **non-zero Riemann tensor** =“true gravity”
Probably that is why D-term, being as fundamental as M & J, escaped attention of the community

MVP '2003

D-term is a global and fundamental quantity related to the distribution of strong forces (pressure and shear) inside a hadron

$$D = M \int d^3r r^2 p(r) = -\frac{4M}{15} \int d^3r r^2 s(r)$$

Effective chiral action for nucleon and pions in external grav. field

based on /Alharazin, Djukanovic, Gegelia, MVP '20/

Contains known LECs /UG Meissner et al. '00/

$$\begin{aligned}
 S_{\pi N} = & \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \bar{\Psi} i e_a^\mu \gamma^a \nabla_\mu \Psi - \frac{1}{2} \nabla_\mu \bar{\Psi} i e_a^\mu \gamma^a \Psi - m \bar{\Psi} \Psi + \frac{g_A}{2} \bar{\Psi} e_a^\mu \gamma^a \gamma_5 u_\mu \Psi \right. \\
 & + c_1 \langle \chi_+ \rangle \bar{\Psi} \Psi - \frac{c_2}{8m^2} g^{\mu\alpha} g^{\nu\beta} \langle u_\mu u_\nu \rangle (\bar{\Psi} \{ \nabla_\alpha, \nabla_\beta \} \Psi + \{ \nabla_\alpha, \nabla_\beta \} \bar{\Psi} \Psi) + \frac{c_3}{2} g^{\mu\nu} \langle u_\mu u_\nu \rangle \bar{\Psi} \Psi \\
 & + \frac{i c_4}{4} \bar{\Psi} e_a^\mu e_b^\nu \sigma^{ab} [u_\mu, u_\nu] \Psi + c_5 \bar{\Psi} \hat{\chi}_+ \Psi + \frac{c_6}{8m} \bar{\Psi} e_a^\mu e_b^\nu \sigma^{ab} F_{\mu\nu}^+ \Psi + \frac{c_7}{8m} \bar{\Psi} e_a^\mu e_b^\nu \sigma^{ab} \langle F_{\mu\nu}^+ \rangle \Psi \\
 & \left. + \frac{c_8}{8} R \bar{\Psi} \Psi + \frac{i c_9}{m} R^{\mu\nu} (\bar{\Psi} e_\mu^a \gamma_a \nabla_\nu \Psi - \nabla_\nu \bar{\Psi} e_\mu^a \gamma_a \Psi) \right\}
 \end{aligned}$$

D-term is related to interaction with space-time curvature

New LECs: interaction with curvature
/Alharazin, Djukanovic, Gegelia, MVP '20/

$$\frac{D(0)}{m_N} = c_8 + \frac{g_A^2}{16\pi F^2} M_\pi + \frac{-3g_A^2/m_N + 2(-4c_1 + c_2 + 2c_3)}{8\pi^2 F^2} M_\pi^2 \ln \left(\frac{M_\pi}{m_N} \right) + \mathcal{O}(M_\pi^3)$$

Computed previously /Belitsky, XD Ji '02/

Chiral expansion of D-term
/Alharazin, Djukanovic, Gegelia, MVP '20/

Computed previously /Diehl, Manashov, Schafer '06/
we corrected their mistake. New result might be important
to revisit chiral extrapolation of lattice data!

Effective chiral action for nucleon and pions in external grav. field

based on /Alharazin, Djukanovic, Gegelia, MVP '20/

Im part of $D(t)$. Is useful for dispersion relations analysis

$$\begin{aligned} \text{Im}D(t+i0) = & -\frac{3g_A^2 m_N (t-2M_\pi^2)(t+4M_\pi^2)}{64\pi F^2 t^{3/2}} \arctg\left(2m_N \frac{\sqrt{t-4M_\pi^2}}{t-2M_\pi^2}\right) \\ & + \frac{m_N \sqrt{t-4M_\pi^2}}{40\pi F^2 t^{3/2}} [20c_1 M_\pi^2 (t+2M_\pi^2) + c_2 (t^2 - 3M_\pi^2 t - 4M_\pi^4) + 5c_3 (t^2 - 4M_\pi^4) \\ & + \frac{5}{16} \frac{g_A^2}{m_N} (t^2 + 20M_\pi^2 t - 12M_\pi^4)]. \end{aligned}$$

E. g. strong forces in nucleon periphery (chiral limit example):

$$\begin{aligned} p(r) &= -\frac{3g_A^2}{64\pi^2 F^2} \frac{1}{r^6} + \frac{(5g_A^2/m_N + 4(c_2 + 5c_3))}{16\pi^3 F^2} \frac{1}{r^7} + O\left(\frac{1}{r^8}\right), \\ s(r) &= \frac{9g_A^2}{64\pi^2 F^2} \frac{1}{r^6} - \frac{21(5g_A^2/m_N + 4(c_2 + 5c_3))}{128\pi^3 F^2} \frac{1}{r^7} + O\left(\frac{1}{r^8}\right). \end{aligned}$$

Useful for derivation general stability conditions and inequalities for strong forces.

Also it is important to describe large distance interaction of quarkonia with the nucleon. Due to the trace anomaly the pressure here plays important role
See [talk by D. Kharzeev](#) today.

Note that at nucleon periphery

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) \geq 0$$

Many other applications:

- Soft hadron reactions in grav. field, e.g. pion gravitoproduction
- Maybe relevant for physics of LIGO mergers
- Low energy gravitoproduction can be probed in a lab, e.g. non-diagonal DVCS (ongoing analysis at CLAS12, plans for EIC)

.....

Total $p(r)$ and $s(r)$, normal and tangential forces, stability conditions

The force acting on the area element $d\vec{S} = dS_r \vec{e}_r + dS_\theta \vec{e}_\theta + dS_\phi \vec{e}_\phi$ $(dF_i = T_{ij} dS_j)$

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r), \quad \frac{dF_\theta}{dS_\theta} = \frac{dF_\phi}{dS_\phi} = -\frac{1}{3}s(r) + p(r).$$

Normal forces
Tangential forces

Eigenvalues of stress tensor

Stability condition
(spatial trace of EMT does not contribute to the mass)

$$\int d^3r p(r) = 0 \quad \text{von Laue '1911}$$

Local stability condition

(Conjecture /Perevalova, Schweitzer, MVP' 17/
similar conjectures in astrophysics /Zeldovich, Novikov' 62, Herrera' 98/)

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) \geq 0$$

D-term $D(0) = -\frac{4m}{15} \int d^3r r^2 s(r) = m \int d^3r r^2 p(r) \leq 0$

All calculations of the D-term in various approaches give negative value for it. Talk by P. Schweitzer

For some systems the D-term is fixed by general principles:

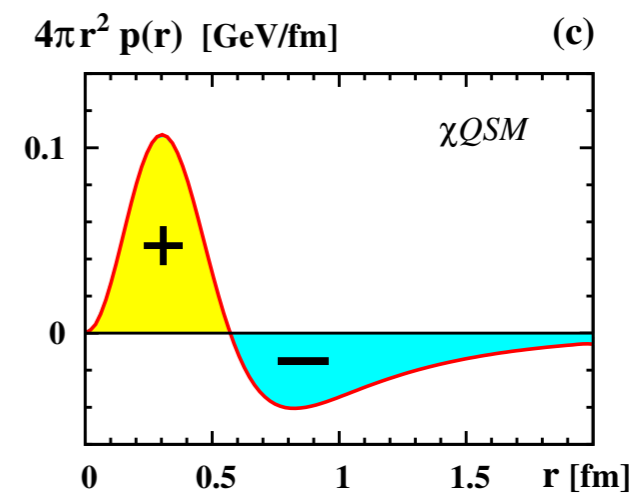
$D(0) = -1$ Goldstone bosons (pions etc.) Novikov, Shifman '1980

$D(0) = 0$ Free fermions Voloshin, Zakharov' 1980

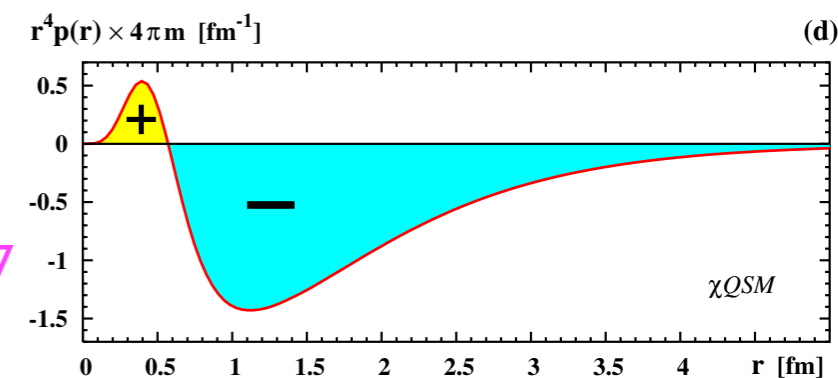
Donoghue et al.' 02, Hudson, Schweitzer '17

$D(0) = -\frac{4\pi}{3} \gamma M_A R_A^4 \sim -A^{7/3}$ Heavy nuclei MVP '2003

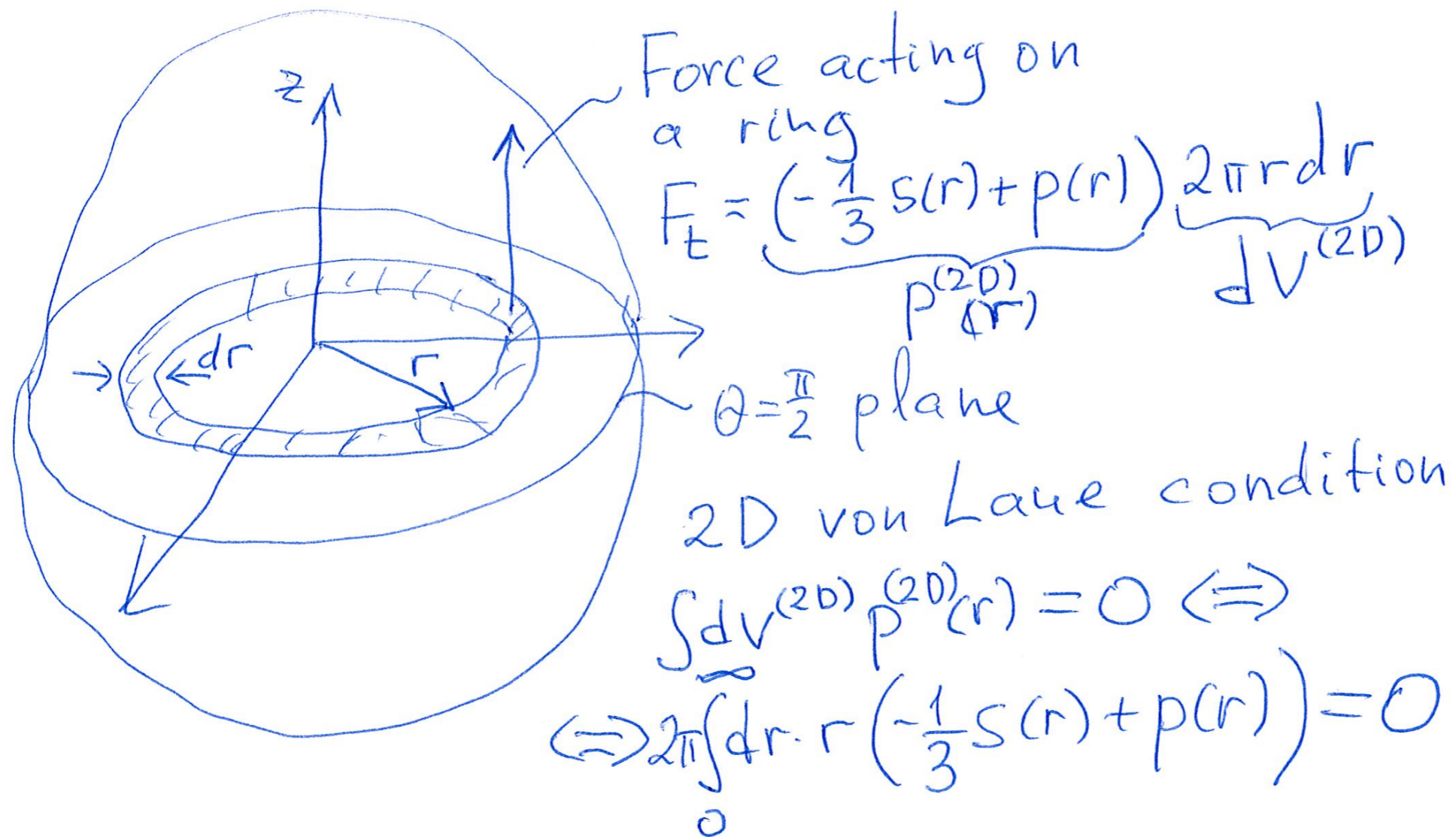
Surface tension coeff in Weizsäcker mass formula $\sim 1 \text{ MeV/fm}^2$



Goeke et al. '2007



Stability conditions for low dimensional sub-systems in the nucleon



Generically, the dimensional reduction:

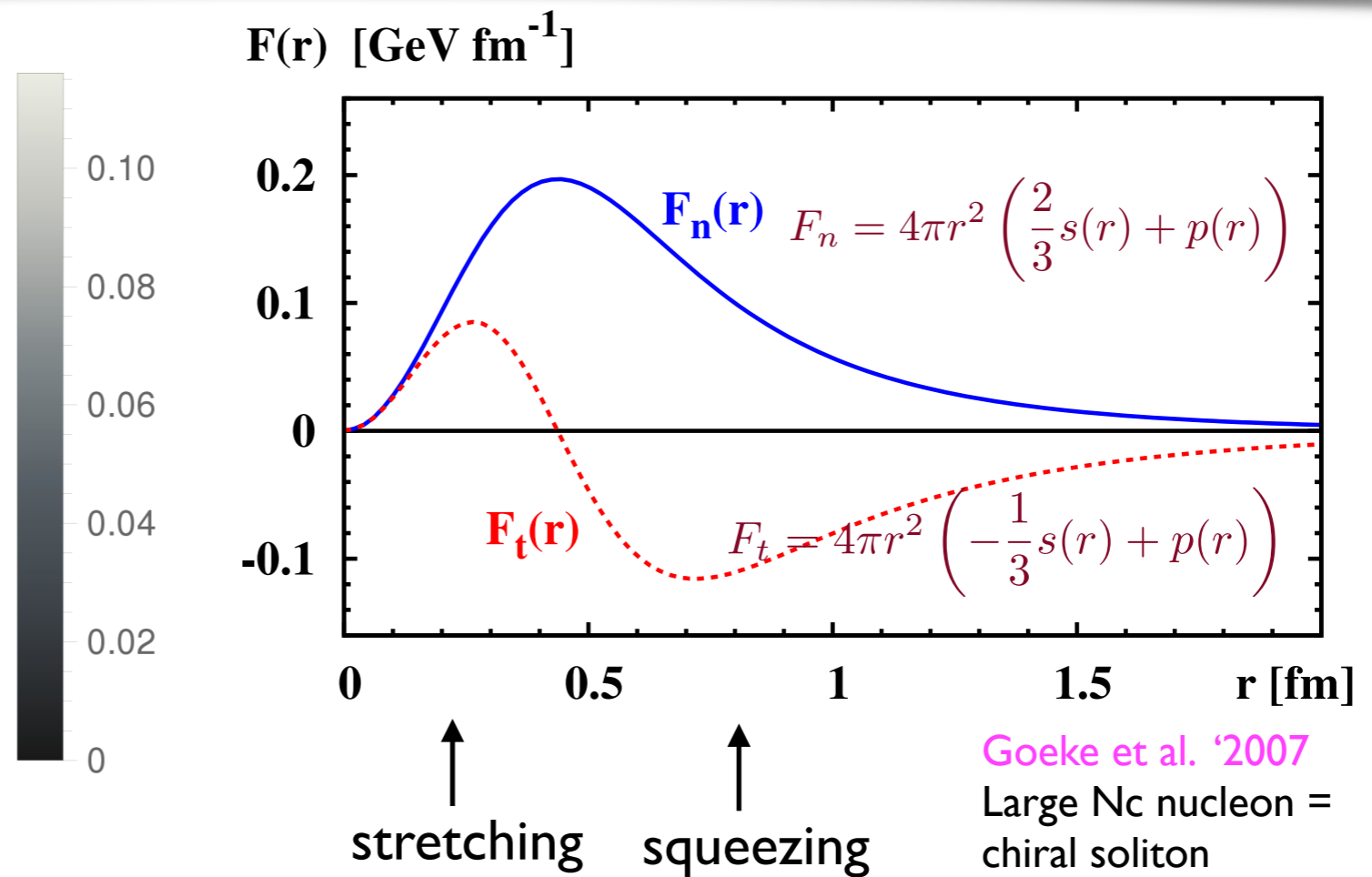
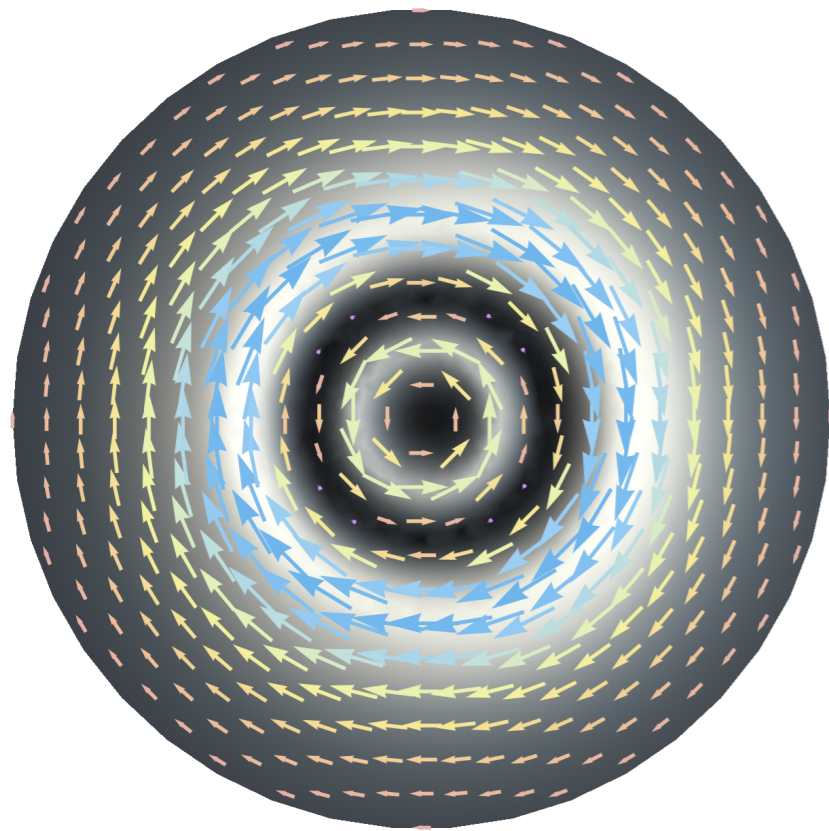
$$p^{(n-1)D}(r) = -\frac{1}{n} s^{(nD)}(r) + p^{(nD)}(r), \quad s^{(n-1)D}(r) = \frac{n-1}{n-2} s^{(nD)}(r)$$

The von Laue stability conditions for n-dimensional subsystem is: $\int dV^{(nD)} p^{(nD)}(r) = 0$

We can view the nucleon as a dimensional reduction of an object in higher dimensions!

AdS/QCD ? We found that the pressure in 2D subsystem of chiral soliton is governed by 2D baby Skyrmions. Relations to exactly solvable 2D models?

Size of the forces in the nucleon. Comparison with confinement forces



Goeke et al. '2007
Large Nc nucleon =
chiral soliton
Detailed discussion in
P. Schweitzer talk today

Compare with the linear potential force of $\sim 1 \text{ GeV/fm}$!

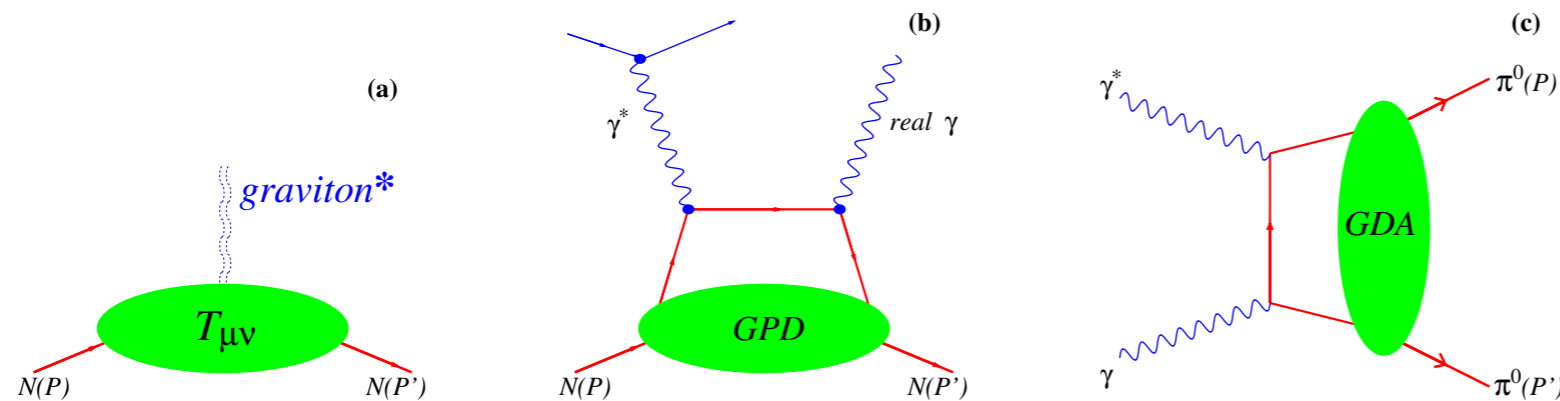
What does it imply for pictures of the confinement?

Values of D-term for the nucleon:

$-2 \geq D(0) \geq -4$ Chiral Quark Soliton model Boffi, Radici, Schweitzer '2001 Goeke et al. '2007

$D^Q(0) \approx -1.56$ at $\mu = 4 \text{ GeV}^2$ Dispersion relations Pasquini, Vanderhaeghen, MVP '2014

Accessing D(t) in hard exclusive processes



$$\int_{-1}^1 dx \, x \, H^a(x, \xi, t) = A^a(t) + \xi^2 D^a(t), \quad \int_{-1}^1 dx \, x \, E^a(x, \xi, t) = 2J^a(t) - A^a(t) - \xi^2 D^a(t).$$

X. D. Ji '96

Unfortunately the Mellin moments are not observable in model independent way. However, D(t) is related to subtraction constant in dispersion relations for amplitudes (observables!)

$$\mathcal{H}(\xi, t) = \int_{-1}^1 dx \left(\frac{1}{\xi - x - i0} - \frac{1}{\xi + x - i0} \right) H(x, \xi, t) \quad \text{DVCS amplitude at LO (directly measurable!)}$$

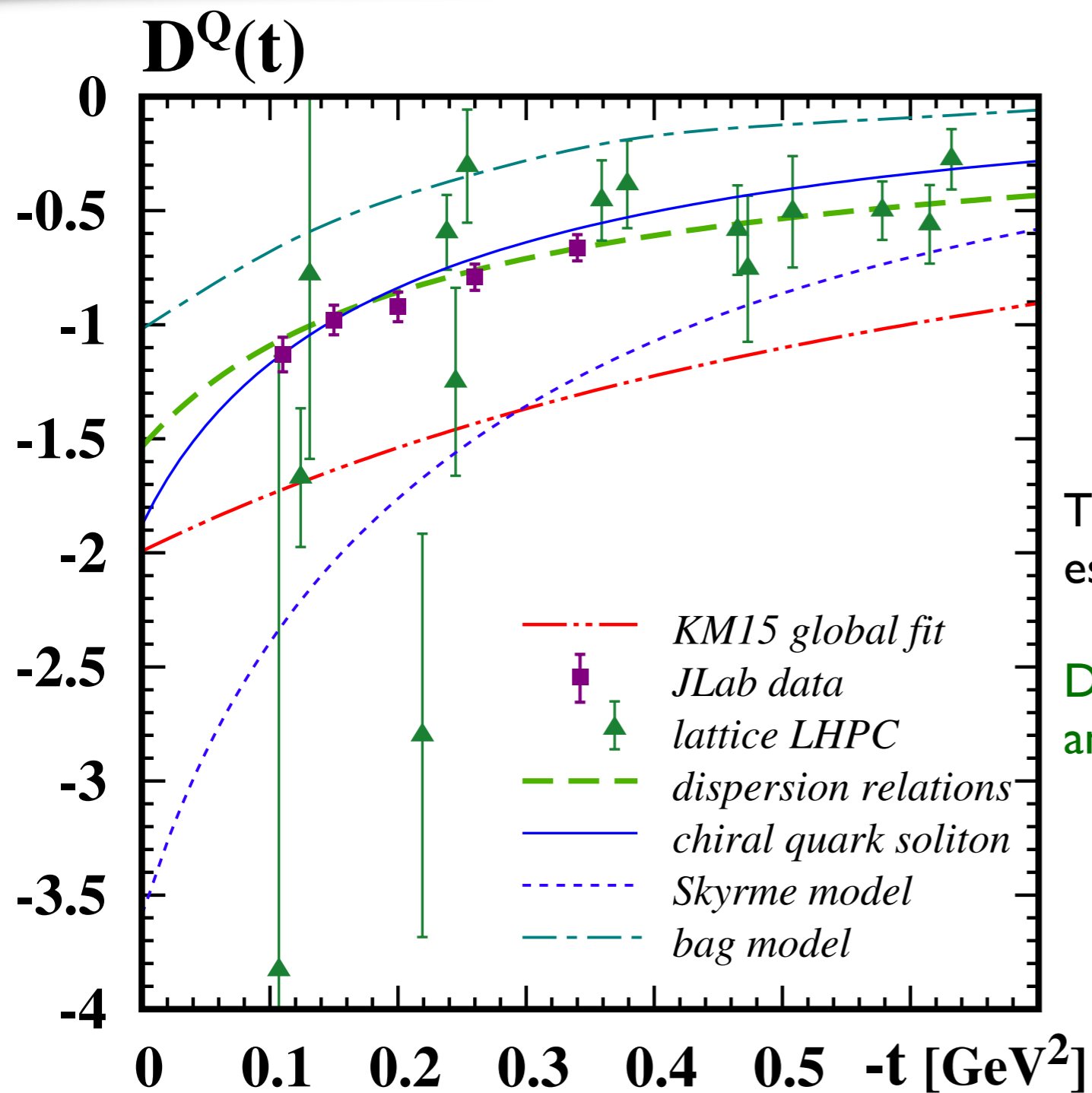
$$\text{Re}\mathcal{H}(\xi, t) = \Delta(t) + \frac{1}{\pi} \text{vp} \int_0^1 d\xi' \, \text{Im}\mathcal{H}(\xi', t) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right)$$

MVP '2003 (small-x DR)
Teryaev '2005
Anikin, Teryaev '2007
Diehl, Ivanov '2007

$$\Delta(t) = \frac{4}{5} \sum_q e_q^2 D^q(t) + \sum_q e_q^2 d_3^q(t) + \dots$$

D(t) is more easy access than J(t). It is possible model independent extraction of D(t) in contrast to J(t)

Accessing $D(t)$ in hard exclusive processes



Recent analysis of CLAS data

Burkert, Elouadrhiri, Girod, *Nature* 557, 396 (2018)

- 1) D-term negative and sizeable
- 2) Agrees with chiral quark soliton model
DR calculations

The systematic uncertainty needs more detailed estimate!

Details in [K. Kumericki, *Nature* 570 (2019)]
and in [Dutrieux, Lorce et al. (2021)]

Conclusions

- gravitational D-form factor is related to “elastic properties” of the nucleon, and gives access to details of strong forces inside the nucleon.
- $D(0)$ (the D-term) is the last unexplored global (in the same sense as mass and spin) property of the nucleon
- First experimental results for $D(t)$ of the nucleon and of the pion are obtained. It is negative, as expected from stability conditions.
- $C_{\text{bar}}(t)$ (“nucleon cosmological term”) FF is important to understand forces between quark and gluon subsystems inside hadrons. Instanton picture of QCD vacuum predicts small positive value of the FF. That corresponds to compression forces experienced by quark subsystem (at variance with lattice results)

Outlook

- knowledge of the D-term can be important to understand hadron interaction in gravitational field relevant to BH or NS mergers (LIGO events). *Gegelia, MVP in preparation*
- the pressure distribution inside hadrons important to understand the physics of quarkonia interaction with the nucleon and the physics of hadro-charmonia (LHCb pentaquarks, tetraquarks with hidden charm)
Eides, Petrov, MVP '2016, Perevalova, Schweitzer, MVP '2017, Panteleeva, Perevalova, Schweitzer, MVP '2018
- several theoretical issues - relation between pressure and energy density (elastic waves in hadrons?), analogies with cosmology, hadrons as projection of higher dimensional objects, relations to exactly solvable 2D models, etc.

Backup slides

Mechanical radius and surface tension

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) \geq 0$$

Positive quantity - allows to define the mechanical radius

$$\langle r^2 \rangle_{\text{mech}} = \frac{\int d^3r \, r^2 \left[\frac{2}{3}s(r) + p(r) \right]}{\int d^3r \left[\frac{2}{3}s(r) + p(r) \right]} = \frac{6D(0)}{\int_{-\infty}^0 dt \, D(t)}$$

Note that mech radius is NOT the slope of D(t)

For a liquid drop

$$p(r) = p_0 \theta(r - R) - \frac{p_0 R}{3} \delta(r - R), \quad s(r) = \gamma \delta(r - R),$$

$p_0 = 2\gamma/R$ Relation between pressure in the drop and the surface tension
Lord Kelvin '1858

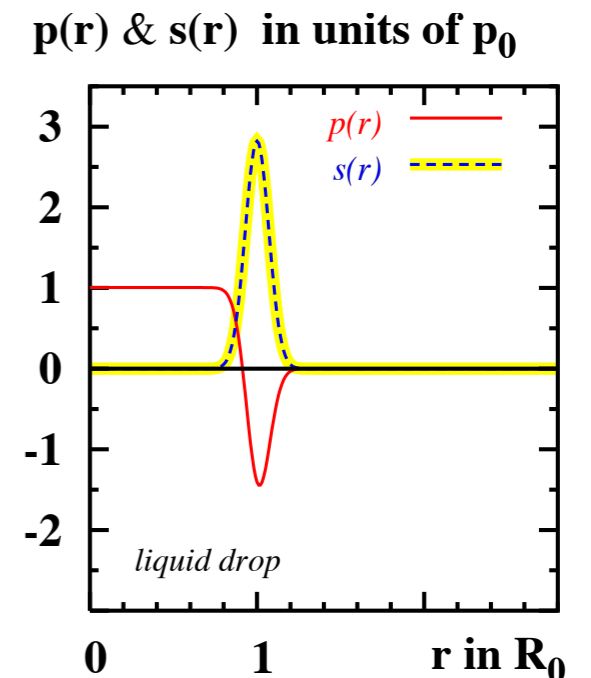
Hence for a liquid drop $\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) = p_0 \theta(r - R)$

mechanical radius has the intuitive clear value

For general systems one can obtain the generalisation of the Kelvin relation

$$p(0) = \int_0^\infty dr \, \frac{2s(r)}{r}$$

$s(r)$ can be called surface tension for the system



Mechanical radius and surface tension

The surface tension energy $\int d^3r s(r) = -\frac{3}{8m} \int_{-\infty}^0 dt D(t)$

This energy must be less than the total energy of the system $\int d^3r s(r) \leq m$ this implies

$\langle r^2 \rangle_{\text{mech}} \geq -9D/(4m^2)$ we checked that for stable systems (stable solitons) is always satisfied.
Violated for unstable systems!

$\langle r^2 \rangle_{\text{mech}} \approx 0.75 \langle r^2 \rangle_{\text{charge}}$ in chiral soliton picture of the nucleon

Shear forces distribution $s(r)$ is important for forming the shape of the hadron.
For $s(r)=0$ the hadron corresponds to homogeneous, isotropic fluid. Hence has infinite mechanical radius. Non-zero $s(r)$ is responsible for *hadron structure formation*!

Interestingly the pressure anisotropy (shear forces distribution) plays an essential role in astrophysics, see the review [\[Herrera:1997plx\]](#) on the role of pressure asymmetry for self-gravitating systems in astrophysics and cosmology.

Nucleon “cosmological term”.

Interaction of the gluon and quark subsystems inside the nucleon

$$\langle p' | T_{\mu\nu}^a(0) | p \rangle = \bar{u}' \left[A^a(t) \frac{P_\mu P_\nu}{M_N} + J^a(t) \frac{i P_{\{\mu} \sigma_{\nu\} \rho} \Delta^\rho}{2M_N} + D^a(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4M_N} + \underbrace{M_N \bar{c}^a(t) g_{\mu\nu}} \right] u$$

$$\Delta^\beta M_N \bar{c}^Q(t) \bar{u}' u = \langle p' | i g \bar{\psi} G^{\beta\alpha} \gamma_\alpha \psi | p \rangle$$

In QCD:

$$\partial_\mu T_{\mu\nu}^Q = -g \bar{\psi} G_{\mu\nu} \gamma_\mu \psi \quad \partial_\mu T_{\mu\nu}^g = -\frac{1}{2} \text{tr} (G_{\nu\alpha} [\mathcal{D}^\sigma, G_{\sigma\alpha}])$$

$$\partial_\mu (T_{\mu\nu}^Q + T_{\mu\nu}^g) = 0 \text{ due to EOM} \quad [\mathcal{D}^\sigma, G_{\sigma\alpha}] = j_\alpha^a t^a \quad \text{with} \quad j_\alpha^a = -g \bar{\psi} \gamma_\alpha t^a \psi$$

$$\partial_\mu T_{\mu\nu}^Q = G_{\mu\nu}^a j_\mu^a \longleftarrow \text{Expression for the Lorentz force experienced by a quark in external gluon field. We may expect that } \bar{C}(t) \text{ is related to forces between quark and gluon subsystems.}$$

Interaction of the gluon and quark subsystems inside the nucleon

$$\frac{\partial T_{ij}^Q(\mathbf{r})}{\partial r_j} + f_i(\mathbf{r}) = 0. \quad (5.5)$$

This equation can be interpreted (see e.g §2 of [28]) as equilibrium equation for quark internal stress and external force (per unit of the volume) $f_i(\mathbf{r})$ from the side of the gluons. This gluon force can be computed in terms of EMT form factor $\bar{c}^Q(t)$ as:

$$f_i(\mathbf{r}) = M_N \frac{\partial}{\partial r_i} \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta \mathbf{r}} \bar{c}^Q(-\Delta^2) \quad (5.6)$$

H.-D. Son, MVP '2018

For $\bar{c}^Q(0) > 0$ the corresponding force is directed towards the nucleon centre, therefore we call it squeezing (compression) force. For opposite sign the corresponding force is stretching.

The total squeezing gluon force acting on quarks in the nucleon is equal to

$$F_{\text{total}} = \frac{2M_N}{\pi} \int_{-\infty}^0 \frac{dt}{\sqrt{-t}} \bar{c}^Q(t).$$

$\bar{c}^Q(t)$ FF important to know what are (compressing or stretching) forces experienced by quarks from side of gluons inside the nucleon. Size of this forces?

The nucleon's “cosmological term” from instantons.

Instantons form a dilute liquid in the QCD vacuum. They provide a mechanism of spontaneous breakdown of chiral symmetry in QCD.

Shuryak '1982
Diakonov, Petrov '1983

$$\Delta^\beta M_N \bar{c}^Q(t) \bar{u}'u = \langle p' | ig\bar{\psi} G^{\beta\alpha} \gamma_\alpha \psi | p \rangle$$



Computed in QCD vacuum using the method of Diakonov, Weiss, MVP '1996

Balla, Weiss, MVP '1997

We found a strong suppression by the instanton packing fraction

$$\bar{c}^Q(t) = \frac{\bar{c}_{\text{quark}}}{(1 - t/\Lambda^2)^2} \quad \bar{c}_{\text{quark}} \sim \frac{1}{6} \frac{\bar{\rho}^4}{\bar{R}^4} \ln \left(\frac{\bar{R}}{\bar{\rho}} \right) \quad \bar{c}^Q(0) = \bar{c}_{\text{quark}} \simeq 1.4 \cdot 10^{-2}.$$

H.-D. Son, MVP '2018

We obtained small and *positive* value at a low normalisation point of $\sim 0.5 \text{ GeV}^2$.

This corresponds to rather *small* compression forces experienced by quarks!

$$F_{\text{total}} = \bar{c}_{\text{quark}} M_N \Lambda \simeq 5.9 \cdot 10^{-2} \frac{\text{GeV}}{\text{fm}} \quad \text{it looks like the two systems almost decouple.}$$

Justification of Teryaev's equipartition conjecture ?

We estimate that the contribution of $\bar{c}^Q(t)$ to the pressure distribution inside the nucleon is in the range of 1 – 20% relative to the contribution of the quark D -term.

↙ negative

No Goldstone boson “cosmological term”

Schweitzer, MVP: 1812.06143

EMT ffs for Goldstone boson (pion in the chiral limit):

$$\langle p' | \Theta_a^{\mu\nu}(0) | p \rangle = 2 P^\mu P^\nu A^a(t) + \frac{1}{2} (\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2) D^a(t) + 2 f_\pi^2 \eta^{\mu\nu} \bar{c}^a(t)$$

Soft Goldstone (pion in the chiral limit) theorem:

$$\lim_{p'^\mu \rightarrow 0} \langle p' | \Theta_Q^{\mu\nu}(x) | p \rangle = 0. \quad 0 = \frac{1}{2} p^\mu p^\nu (A^Q(0) + D^Q(0)) + 2 f_\pi^2 \eta^{\mu\nu} \bar{c}^Q(0).$$

This equation is satisfied if the EMT form factors of massless Goldstone boson are related other by:

$$D^Q(0) = -A^Q(0), \quad \bar{c}^Q(0) = 0.$$

Accessing $p(r)$ and $s(r)$ in hard exclusive processes

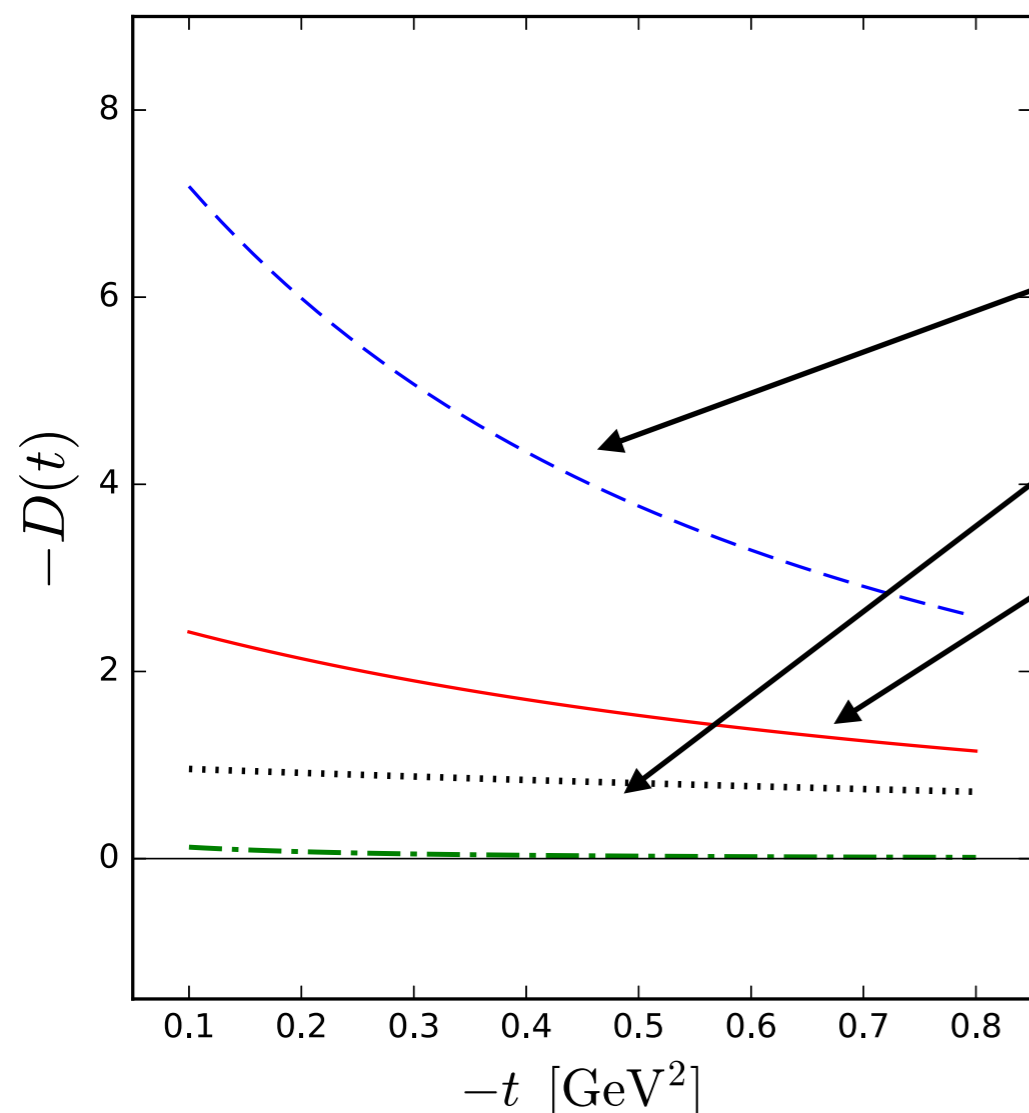
Simplifying assumptions (for present state of art of the experiment):

- 1) $d3(t)$, $d5(t)$, ... much smaller than $D(t)$. It is so at large normalisation scale.
- 2) Flavour singlet $D(t)$ is dominant. Justified in large N_c limit. Can be relaxed for more precise data.

Under these assumptions we obtain:

$$D^Q(t) = \frac{4}{5} \frac{1}{2(e_u^2 + e_d^2)} \Delta(t) = \frac{18}{25} \Delta(t).$$

The first determination of $D(t)$ from DVCS
Kumericki, Mueller Nucl. Phys. B841 (2010) I



KM10, statistical accuracy 60%

KM12, statistical accuracy 50%

KM15, statistical accuracy 20%

The D-term is *negative*, statistical accuracy is increasing with new data added.

The systematic uncertainty remains unestimated !
Details in K. Kumericki paper Nature 570 (2019)

