



Topological Origin of Hadronic Mass

I. Zahed

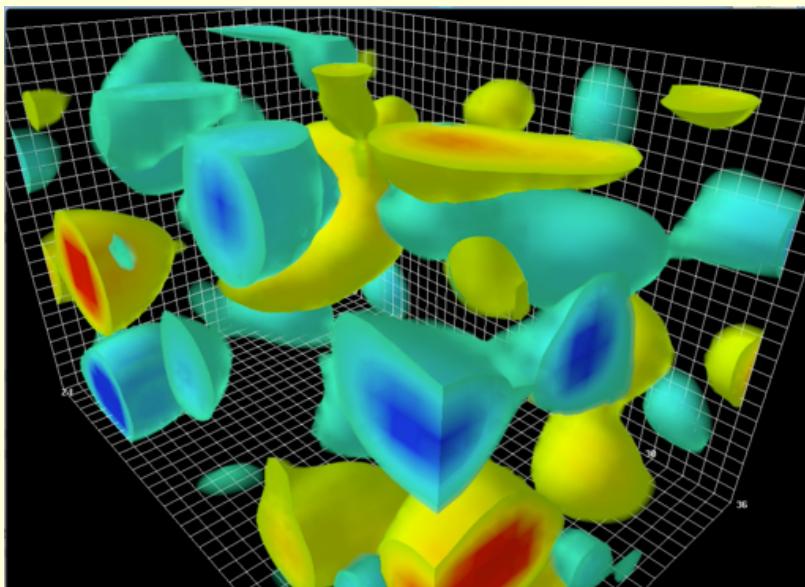
Argonne 21'

Outline

- YM vacuum
- Instantons, CSB and mass
- Ji-Femtography
- P-vortices and strings
- AdS-Holography
- Summary

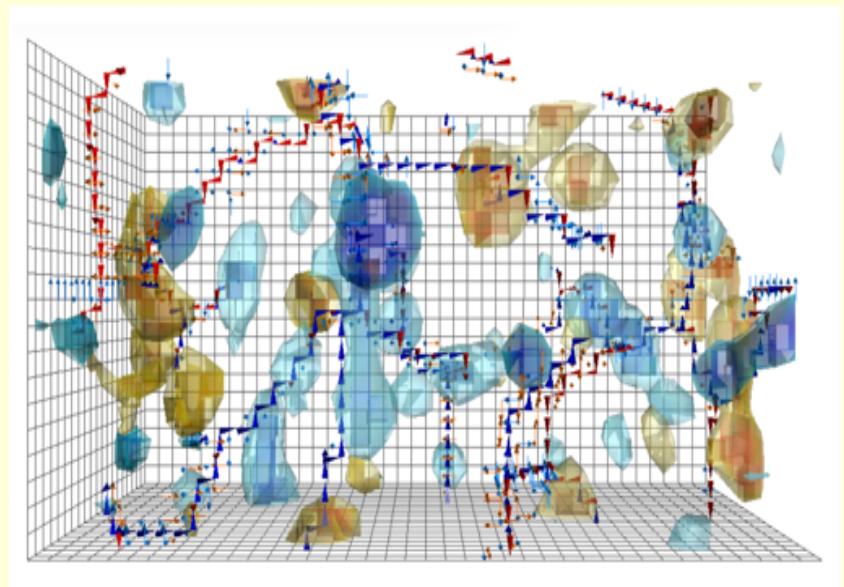
Cooled YM vacuum filled with topological gauge fields with large actions threaded by P-vortices

Instantons and anti-instantons



Leinweber 03'

Instantons and anti-instantons and P-vortices



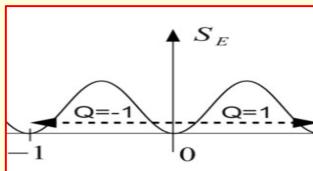
Biddle-Kamleh-Leinweber 20'

Cooled YM vacuum as a strongly inhomogeneous ensemble of instantons and anti-instantons (RIV)

One:

$$g_s A_\mu^a(x) = U_b^a \bar{\eta}_{\mu\nu} \frac{2\rho^2(x_\nu - z_\nu)}{(x-z)^2((x-z)^2 + \rho^2)}$$

Belavin et al. 75'



$$3_c : 11 \quad 2_c : \frac{22}{3}$$

↑

$$\text{RIV} : d[\rho] \approx \frac{(\rho\Lambda)^{\beta_0}}{\rho^5} e^{-2\pi\sigma\rho^2}$$

$$E = B = \frac{\sqrt{48}}{\rho^2} \approx 2.5 \text{ GeV}^2 \quad q_I = \frac{6}{\pi^2 \rho^4}$$

$$S_I = \frac{8\pi^2}{g_s^2} = \frac{2\pi}{\alpha_s} \approx 6\pi \gg 1 \quad Q_I = \frac{\alpha_s S_I}{2\pi} = 1$$

Many:

$$n_{I+\bar{I}} \approx 1 \text{ fm}^{-4}, \quad \rho \sim 1/3 \text{ fm} \sim 1/(0.6 \text{ GeV})$$

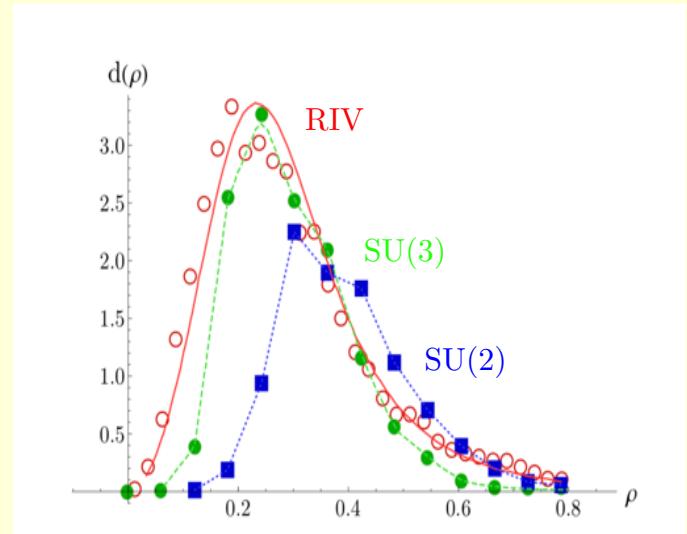
Shuryak 82'

Small expansion parameter: diluteness factor!

$$\kappa \equiv \alpha^2 \rho^4 = [n_{I+\bar{I}} \rho^4] \left[\frac{1}{2N_c} \right]_U = \frac{1}{3^4} \frac{1}{6} \approx 10^{-3}$$

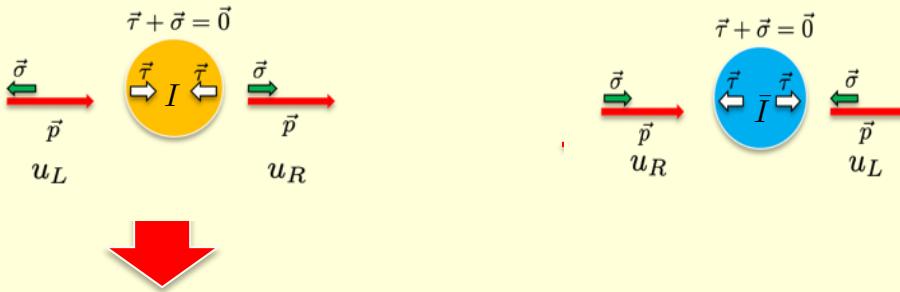


instanton packing fraction



Michael-Spencer 95'
Schafer-Shuryak 98'

Topological origin of mass: zero modes



$$(i\partial + A_I)\psi_I = 0 \quad 't Hooft 76' Atiyah-Patodi-Singer 63'$$

$$\psi_{iI}^\alpha(p) = \sqrt{2}\varphi'(p)(\hat{p}\epsilon U)_i^\alpha$$

$$\varphi'(p) = \pi\rho^2 \left(I_0 K_0 - I_1 K_1 \right)' (z = \rho p/2)$$

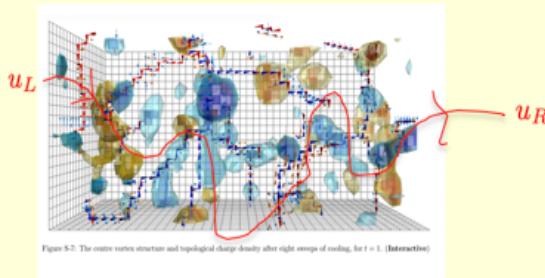
$$\int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \left[(2\pi)^4 \delta^4(p_1 - p_2) \right]_Z \left[\frac{n_I}{2} \right]_\rho \left\langle u_R^\dagger(p_2) p_2 \left[\sqrt{2} \varphi'_2 \hat{p}_2 \epsilon U \right] \frac{1}{m} \left[\sqrt{2} \varphi'_1 U^\dagger \epsilon \hat{p}_1 \right] p_1 u_L(p_1) \right\rangle_U + L \leftrightarrow R$$

$$M(p) = \kappa \frac{|p\varphi'(p)|^2}{mp^4}$$

too singular!

Disordering of the zero modes: Banks Casher

Biddle-Kamleh-Leinweber 20'



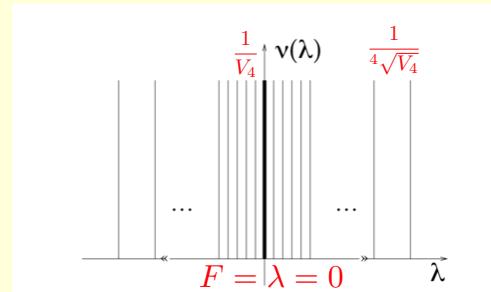
Random instanton vacuum (RIV)



Zero-modes \rightarrow Quasi-zero-modes

$$iD[A]\psi_n[A] = \lambda_n[A]\psi_n[A]$$

$$\nu(\lambda) = \lim_{m \rightarrow 0} \lim_{V_4 \rightarrow \infty} \frac{1}{V_4} \left\langle \sum_n \delta(\lambda - \lambda_n[A]) \right\rangle_A \equiv \frac{1}{V_4} \frac{1}{\Delta\lambda}$$



$$\sigma = -\langle \bar{q}q \rangle = \pi\nu(0)$$

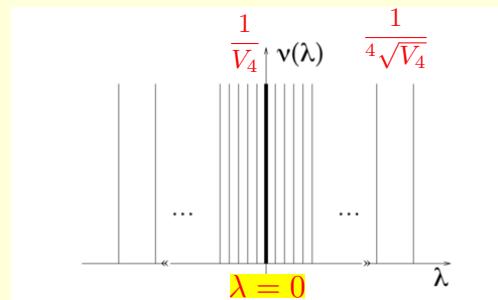
Banks-Casher 80'

YM vacuum is metallic: Spontaneous CSB!

Universal conductance fluctuations : Many “Banks-Casher”

Many ”Banks-Casher”!

$$\sigma_n = \langle [\bar{q}q]^n \rangle_c$$

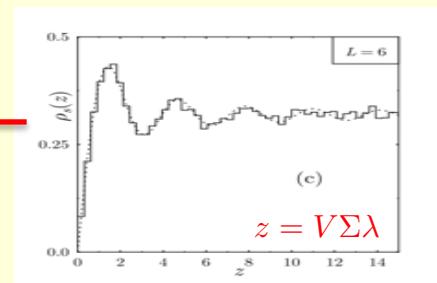


Conductance fluctuations are fingerprints of the topological origin of CSB!

$$\rho_s(z = N\lambda) = \frac{z}{2} \left(J_{\nu+N_f}^2(z) - J_{N_f+\nu+1}(z)J_{N_f+\nu-1}(z) \right)$$

Universal accumulation: chi-GUE random matrix theory

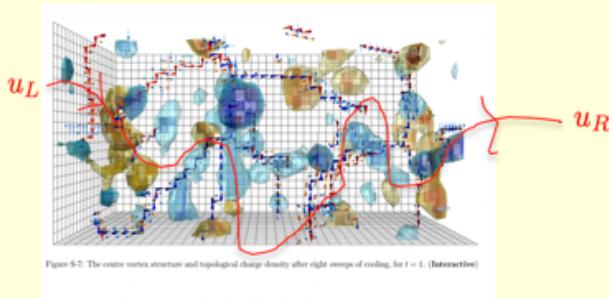
Verbaarschot-Zahed 93'



Confirmed by lattice SU(3)

Gockeler et al. 98'

Topologically induced quark-mass in metallic regime



SIA: IR unsafe!

$$M(p) = \kappa \frac{|p\varphi'(p)|^2}{m\rho^4}$$



Metal: IR safe!

$$M(0) = \frac{\sqrt{\kappa}}{\sqrt{2}\rho^2} \frac{|p\varphi'(p)|_0^2}{\|q\varphi'^2\|^2} \approx 400 \text{ MeV}$$

$$\alpha \approx \sqrt{\kappa} \approx \sqrt{10^{-3}}$$

$$\frac{M(p)}{M(0)} = |p\varphi'(p)|^2 = |z(I_0(z)K_0(z) - I_1(z)K_1(z))'|_{z=\rho p/2}^2$$

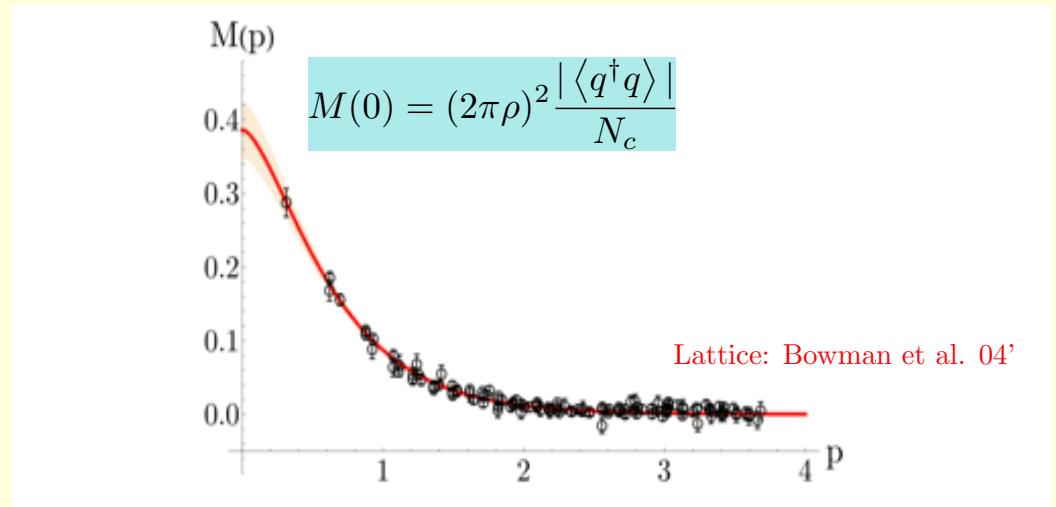
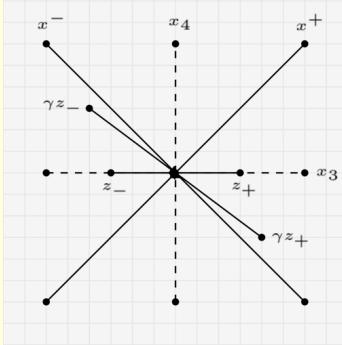


FIG. 2: Momentum dependence of the instanton induced effective quark mass in singular gauge (13) at LO (solid-curves), compared to the effective quark mass measured on the lattice in Coulomb gauge [21] (open-circles). The unit scale is GeV. We obtain a fitted parameter intervals $M(0) = 383 \pm 39$ MeV and $\rho = 0.313 \pm 0.016$ fm.

Diakonov-Petrov 86'
Shuryak 88'
Nowak-Verbaarschot-Zahed 88'
Pobylitsa 89'
....
Kock-Liu-Zahed 20'

Ji-Femtography: Pion Quasi-PDF

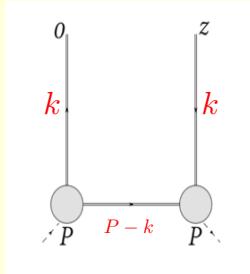
Ji 03'



$$\begin{aligned}\tilde{\psi}_\pi(x, P_z) &= \int \frac{dz}{2\pi} e^{-\frac{i}{2}(x-\bar{x})zP_z} \langle \pi(P) | \psi^\dagger(z_-) \gamma^z [z_-, z_+] \psi(z_+) | \pi(P) \rangle \\ &\approx \lim_{P^2 \rightarrow 0} \frac{P^4}{g_\pi^2} \int \frac{dz}{2\pi} e^{-\frac{i}{2}(x-\bar{x})zP_z} \langle O_5(-P) \psi^\dagger(z_-) \gamma^z [z_-, z_+] \psi(z_+) O_5(P) \rangle\end{aligned}$$

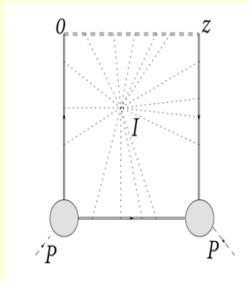


$$O_5(P, k) = \gamma^5 + \frac{\varphi'(k)\varphi'(k-P)|k||k-P|}{\sigma_{00}^2} \int \frac{d^4 p}{(2\pi)^4} \sum_{I, \bar{I}} \left(\psi_{I, \bar{I}}^\dagger(p) \psi_{I, \bar{I}}(p - P) \frac{1 \mp \gamma_5}{2} \right) O_5(P, k) + \mathcal{O}(\alpha)$$



order : $\alpha^0 \sim \kappa^0$

$$\begin{aligned}\tilde{\psi}_\pi(x, P_z) &\approx \\ &- \lim_{P^2 \rightarrow 0} \frac{P^4}{g_\pi^2 P_z} \int \frac{d^4 k}{(2\pi)^4} \delta\left(x - \frac{1}{2} - \frac{k_z}{P_z}\right) \text{Tr}_C \left(\gamma^z \frac{1}{k_1} \gamma^5 F_5(P, k) \frac{1}{k_2} \gamma^5 F_5(P, k) \frac{1}{k_1} \right) + \text{cross} \\ &+ \lim_{P^2 \rightarrow 0} \frac{P^4}{g_\pi^2 \sigma_{00}^2 P_z} \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \delta\left(x - \frac{1}{2} - \frac{k_z}{P_z}\right) \text{Tr}_C \left(\gamma^z \psi_{0I}(k_1) \psi_{0I}^\dagger(q_1) \gamma^5 F_5(P, q) \delta G_I(q_2, p_2) \gamma^5 F_5(P, p_2) \right) \\ &+ \lim_{P^2 \rightarrow 0} \frac{P^4}{g_\pi^2 \sigma_{00}^2 P_z} \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \delta\left(x - \frac{1}{2} - \frac{k_z}{P_z}\right) \text{Tr}_C \left(\gamma^z \psi_{0I}(k_1) \psi_{0I}^\dagger(q_1) \gamma^5 F_5(P, q) \psi_{0I}(q_1) \psi_{0I}^\dagger(p_2) \gamma^5 F_5(P, p_2) \right) \\ &+ \lim_{P^2 \rightarrow 0} \frac{P^4}{g_\pi^2 \sigma_{00}^2 P_z} \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \delta\left(x - \frac{1}{2} - \frac{k_z}{P_z}\right) \text{Tr}_C \left(\gamma^z \delta G_I(k_1, q_1) \gamma^5 F_5(P, q) \psi_{0I}(q_2) \psi_{0I}^\dagger(p_2) \gamma^5 F_5(P, p_2) \right)\end{aligned}$$



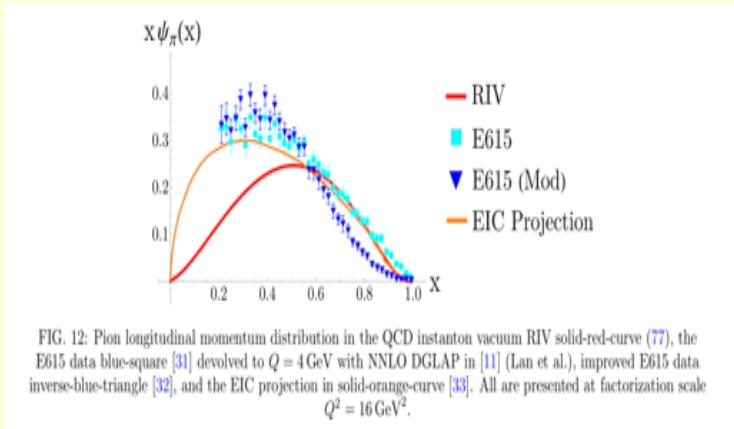
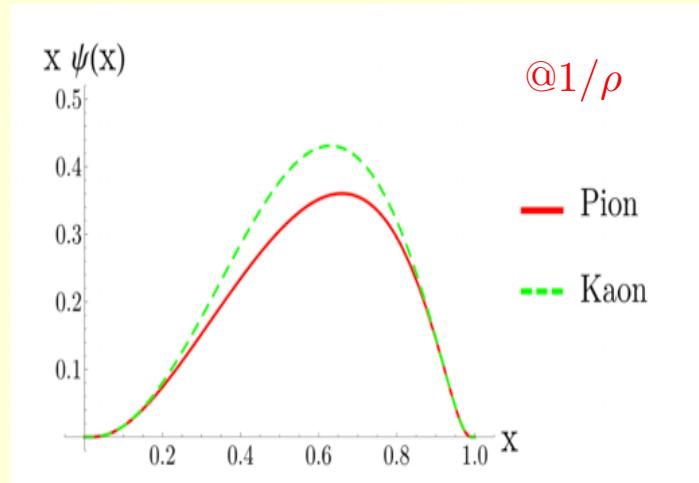
Kock-Liu-Zahed 20'

Pion and Kaon PDF from Quasi-PDF

RIV: Kock-Liu-Zahed 20'

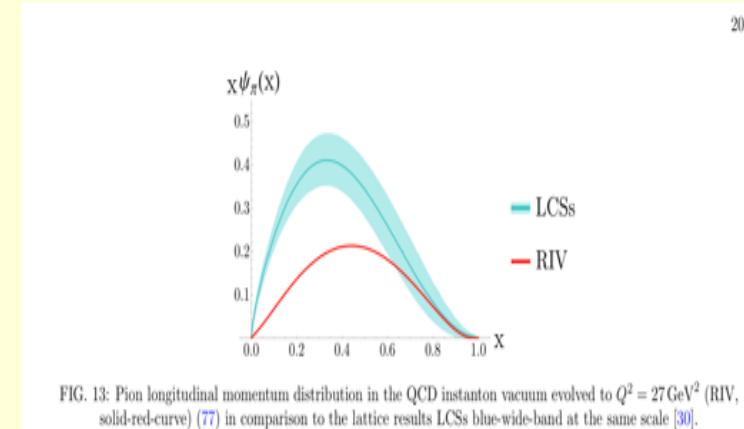
$$\psi_{f/P}^0(x) \rightarrow \frac{2N_c}{f_P^2} \int_{k_\perp \geq M(0, m_f)} \frac{d^2 k_\perp}{(2\pi)^3} \frac{\theta(x\bar{x}) k_\perp^2}{(k_\perp^2 - x\bar{x}m_P^2)^2} M^2(k_\perp/\lambda_P \sqrt{x\bar{x}})$$

order : $\alpha^0 \sim \kappa^0$



E615: Conway et al. 89'

EIC: Agilar et al. 19'



LCS: Sufian et al. 20'

Pion & Kaon TMD

@1/ ρ

21

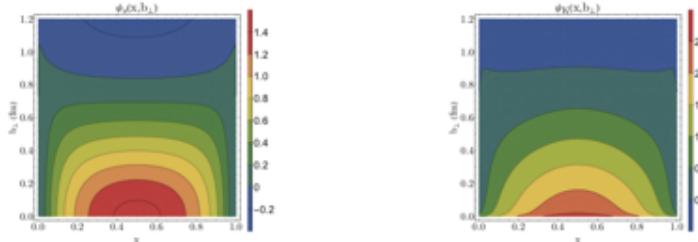


FIG. 14: Pion and Kaon transverse spatial distribution from the QCD instanton vacuum (87) with physical masses and at renormalization scale $Q_0 = 631$ MeV.

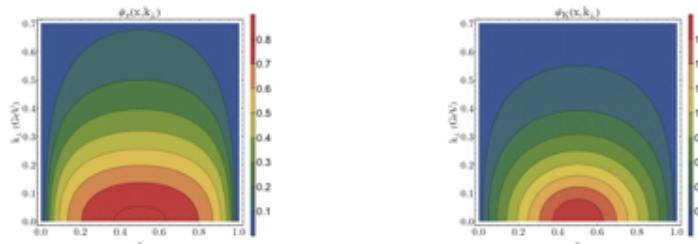


FIG. 15: Pion and Kaon transverse momentum distribution from the QCD instanton vacuum (87) with physical masses and at renormalization scale $Q_0 = 631$ MeV.

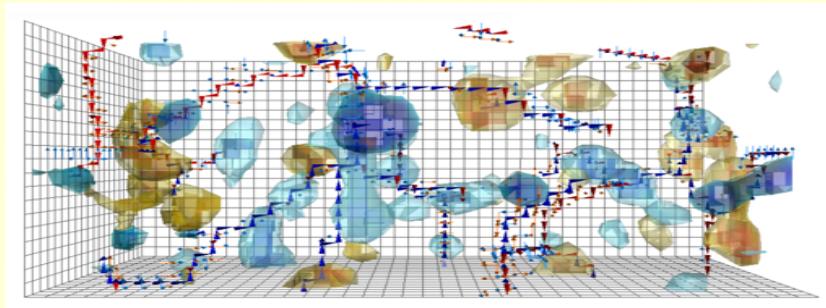
$$\text{order : } \alpha^0 \sim \kappa^0$$

$$\psi_\pi^0(x, k_\perp) \rightarrow \frac{2N_c}{f_\pi^2} \frac{1}{(2\pi)^3} \frac{\theta(x\bar{x}) (k_\perp^2 + M^2(0))}{(k_\perp^2 + M^2(0) - \bar{x}xm_\pi^2)^2} M^2 \left(\frac{\sqrt{k_\perp^2 + M^2(0)}}{\lambda\sqrt{x\bar{x}}} \right)$$

Kock-Liu-Zahed 20'

What about Confinement? Localized or screened fields weakly sensitive

YM in 1+3: localized fields!



Biddle-Kamleh-Leinweber 20'

QCD in 1+1: screened fields!

$$T^{\mu\nu} = \frac{1}{2}g^{\mu\nu}E^aE^a + \frac{1}{2}\bar{\psi}\gamma^{[\mu}i\overleftrightarrow{D}^{\nu]}+\psi \rightarrow T_\mu^\mu = E^aE^a + m\bar{\psi}\psi$$

QCD 1+1 non-conformal

$$\left\{ \begin{array}{l} D^\mu F_{\mu\nu}^a = \bar{\psi}\gamma_\nu T^a\psi \\ D_\mu^{ab} \left(\bar{\psi}\gamma^\mu \gamma^5 T^b\psi \right) = -\frac{g_0}{4\pi} \epsilon_{\mu\nu} F^{a\mu\nu} = -\frac{g_0}{2\pi} E^a \end{array} \right.$$

color anomaly

$$\left(\partial^2 + \frac{m_0^2}{N_c} \right) (E^a E^a) = 2g_0^2 \left(\bar{\psi}\gamma^\mu \gamma^5 T^b\psi \right)^2 \quad m_0^2 = \frac{g_0^2 N_c}{\pi} = \frac{\lambda}{\pi}$$



EE shorter range by screening!

Ji-Liu-Zahed 20'

What about Trace anomaly? Cheshire cat strikes back!

QCD in 1+1: Meson masses!

$$M_n^2 = \langle P, n | \frac{1}{2} T_\mu^\mu | P, n \rangle = \langle P, n | \left[\frac{1}{2} E^a E^a \right] = \frac{2\lambda}{N_c} \frac{1}{(\partial^2 + m_0^2/N_c)} (\bar{\psi} \gamma^\mu \gamma^5 T^a \psi)^2 \Big] | P, n \rangle + \frac{1}{2} \langle P, n | m \bar{\psi} \psi | P, n \rangle$$



$$M_n^2 = \left\langle P, n \left| \frac{2\lambda}{m_0^2} \left[(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma^5 \psi)^2 - \frac{2}{N_c} (\bar{\psi} \gamma^\mu \psi)^2 \right] \right| P, n \right\rangle + \frac{1}{2} \langle P, n | m \bar{\psi} \psi | P, n \rangle$$

EE-gluon = "tHooft-like-vertex" = dual mass sum rule!



Ji-Liu-Zahed 20'

measurable through dilaton FF!

$$\Theta(q^2) = \frac{\langle p_2, n | 2T_\mu^\mu | p_1, n \rangle}{4M_n^2 - q^2}$$

2M scalar-meson t-exchange!

Pion as a would be Goldstone boson in $1/N_c$ GOR

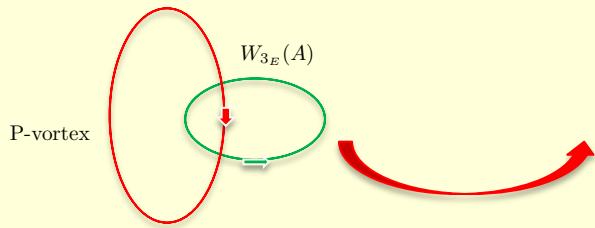
$$M_0^2 = \pi \times 0 - \frac{1}{2} \left\langle \left[\frac{Q_5}{f_0}, \left[\frac{Q_5}{f_0}, \bar{\psi} \psi \right] \right] \right\rangle = -2m \frac{\langle \bar{\psi} \psi \rangle}{f_0^2}$$

The pion is not empty of "gluons"!

The "gluons" in the pion are not distinguishable from those in the vacuum!

$$f_0 = \frac{\sqrt{N_c}}{\pi}$$

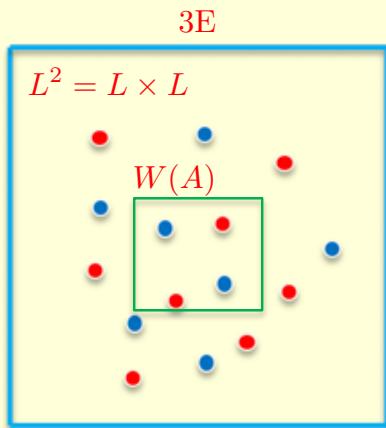
What about the P-vortices? Emergent strings



QCD ($N_c=2$) in 3E : $L \times L \times L$

$$Z_\mu(x) = \text{sgn} \text{Tr}[U_\mu(x)] \rightarrow Z_2 = \pm 1$$

$$W_{3E}(A) \rightarrow (Z_2 = -1) \times W_{3E}(A)$$



Emergent string behavior!

$$\begin{aligned} \langle W_{3E}(A) \rangle &= \sum_{n=1}^N (e^{i\pi})^n p_n(A) \\ &= \sum_{n=1}^N (-1)^n \left[C_N^n \left(\frac{A}{L^2} \right)^n \left(1 - \frac{A}{L^2} \right)^{N-n} \right] \\ &= \left(1 - \frac{\rho A}{N} \right)^N \rightarrow e^{-\sigma_T A} \end{aligned}$$

$$\rho = \frac{N_+ + N_-}{L^2} = \frac{2N}{L^2}$$

$$\rho = \frac{2N}{L^2} \equiv \sigma_T = \frac{1}{2\pi l_s^2} \approx 4 \text{ fm}^{-2}$$

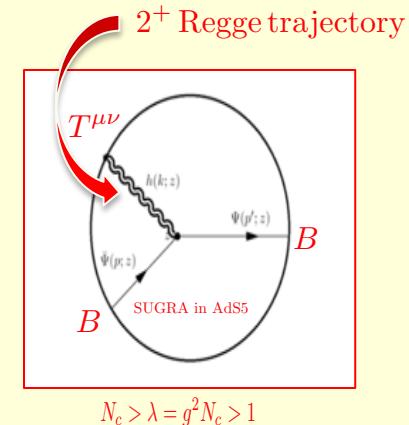
Holography provides a string based approach dual to YM $N_c > \lambda = g^2 N_c > 1$

Holographic EMT Resonance Dual Model redux

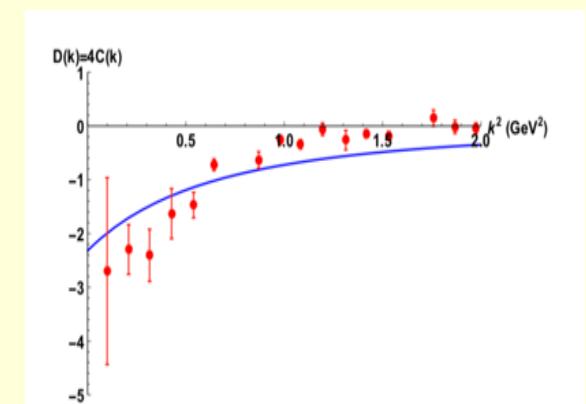
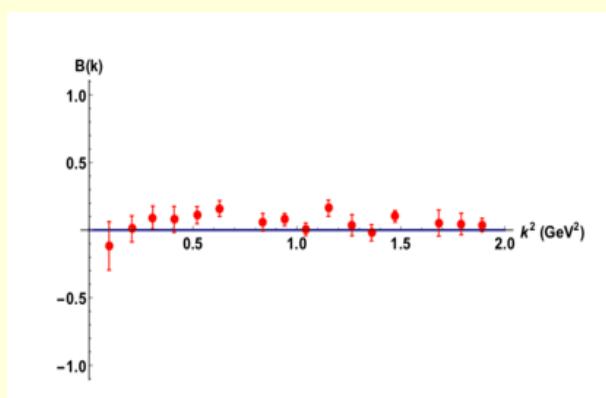
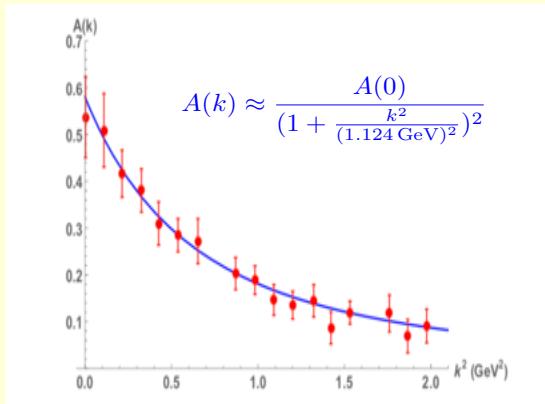
$$\langle p_2 | T^{\mu\nu}(0) | p_1 \rangle = \bar{u}(p_2) \left(A(k) \gamma^{(\mu} p^{\nu)} + B(k) \frac{i p^{(\mu} \sigma^{\nu)\alpha} k_\alpha}{2m_N} + C(k) \frac{k^\mu k^\nu - \eta^{\mu\nu} k^2}{m_N} \right) u(p_1),$$

$$A(K) = A(0) \left((1 - 2a_K)(1 + a_K^2) + a_K(1 + a_K)(1 + 2a_K^2) \left(\psi\left(\frac{1 + a_K}{2}\right) - \psi\left(\frac{a_K}{2}\right) \right) \right)$$

$$a_K = \frac{K^2}{8\kappa_N^2}$$



$$N_c > \lambda = g^2 N_c > 1$$



Mamo-Zahed 19'

Lattice: Shanahan-Detmold 19'

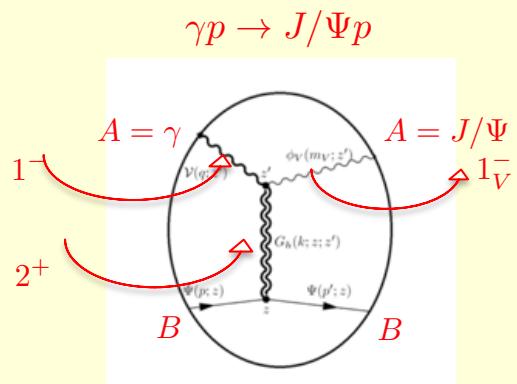
$$\text{PT : } A_g(0) = \frac{4C_F}{4C_F + N_F} = \frac{16}{25} = 0.64$$

Measurable? Diffractive photo-production

$$\gamma p \rightarrow J/\Psi p \quad [4\pi^2/N_c^2]/2/[12\pi^2/N_c]^2$$

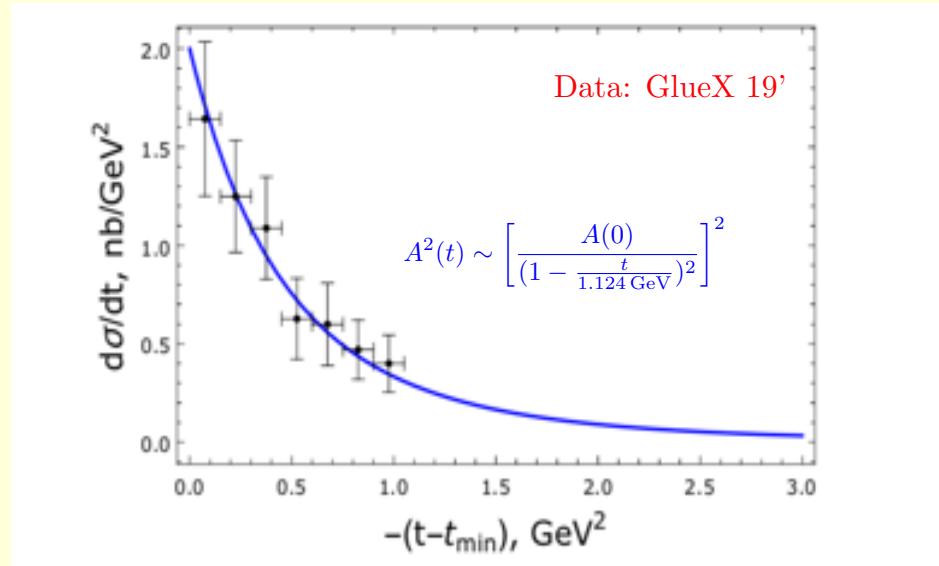
$$\left(\frac{d\sigma}{dt} \right) = \frac{e^2}{64\pi(s - m_N^2)^2} \times \left[\frac{\kappa^2}{2g_5^4} \mathbb{V}_{hAA}^2 \right] \times \left[\frac{A^2(K)}{4m_N^2} \times F(s, t = -K^2, M_V, m_N) \times (2K^2 + 8m_N^2) \right]$$

non-universal!



Exchange dominated by graviton!

Note: no VMD used!



Mamo-Zahed 19'
see also Hatta-Yang 18'

See also Kharzeev-Satz-Zinovjev 99'!

Summary I

- The YM vacuum is populated by strongly inhomogeneous topological gauge fields with large actions.
- A systematic way to treat these topological fields in the context of semi-classics is the dilute RIV.
- The spontaneous breaking of chiral symmetry (CSB), the origin of mass and π^N femtography are mostly due to these topological fields.
- The strong localization (screening) of these fields make the essentials of CSB less sensitive to the long distance confining P-vortices structures.

Summary II

- The P-vortices confine large W-loops, with the emergence of a string at larger distances.
- A systematic framework to treat the stringy effects is holography in the double limit $N_c > g^2 N_c \gg 1$.
- Holography brings the pre-QCD resonance dual model in the framework of QCD.