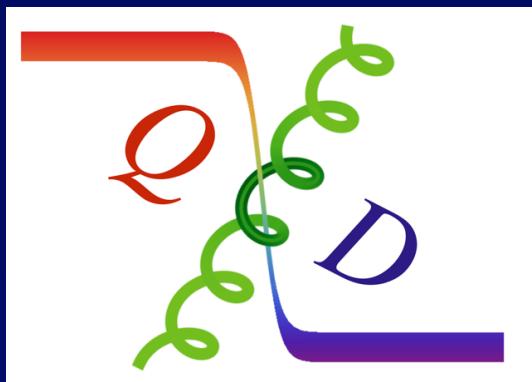


Decomposition of Proton Mass and Rest Energy

- $E = mc^2$
- Hadron Mass and Trace Anomaly
- Rest Energy Decomposition from Gravitational Form Factor of Energy-Momentum Tensor
- Hamiltonian

χ QCD Collaboration



3rd Proton Mass Workshop
Jan. 11, 2021

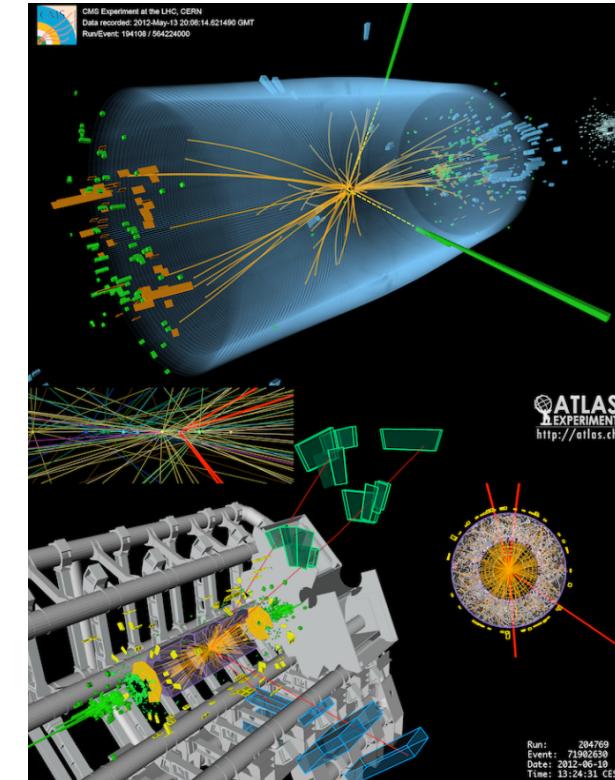
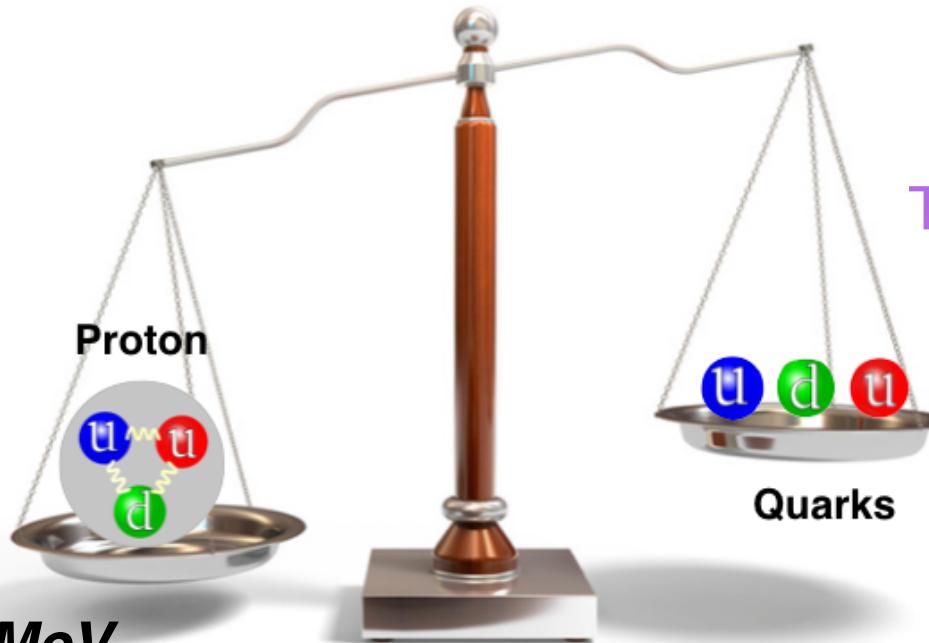
Motivation

Where does the proton mass come from, and how ?

But the mass of the proton is

938.272046(21) MeV.

~100 times of the sum of the quark masses!



The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$$m_u = 2.08(9) \text{ MeV}$$
$$m_d = 4.73(12) \text{ MeV}$$

Laiho, Lunghi, & Van de Water,
Phys.Rev.D81:034503,2010

References

- Xiangdong. Ji, arXiv:hep-ph/9603249.
- D. Kharzeev, H. Satz, A. Syamtomov and G.~Zinovjev, arXiv:hep-ph/9901375
- Maxim Polyakov and Peter Schweitzer, arXiv:1805.06596
- Cedric Locre, arXiv:1706.05853, 1811.02803
- Yoshitaka Hatta, A. Rajan and K. Tanaka, arXiv:1810.05116.
- Andreas Metz, Barbara Pasquini and Simone Rodini, arXiv:2006.11171
- Tie-Juinn Hou (CT18), arXiv:1912.10053
- Lattice calculations

Mass and Rest Energy

- $E = m c^2 \longrightarrow m$ increases with E ? converting mass to energy?
- $E_0 = m c^2$ (Einstein 1905, $m^2 = E^2 - p^2$) - L. Okun
- $e^+ e^- \rightarrow \gamma\gamma$ ($m_{\gamma\gamma} = 2m_e$)
- E and p are additive, not mass
- In general relativity, the gravitational field is coupled to the EMT.
- In non-relativistic limit, Newton's law of force and universal gravitational involves E_0 or mass.
- Inertial mass and gravitational mass are the same mass.
- Relativistic mass is a misnomer, rest mass is redundant.
- -- L.B. Okun doi:10.1134/1.1358478

Quark and Glue Components of Hadron Mass and Rest Energy

■ Energy momentum tensor

$$T_{\mu\nu} = \frac{1}{4} \bar{\psi} \gamma_{(\mu} \vec{D}_{\nu)} \psi + G_{\mu\alpha} G_{\nu\alpha} - \frac{1}{4} \delta_{\mu\nu} G^2$$

■ Mass from trace of EMT – scalar, frame independent, components are scale invariant

$$T_\mu^\mu = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g) \bar{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta} \right]$$

$$\langle P | T_\mu^\mu | P \rangle = 2(E^2 - P^2) = 2M_N^2$$

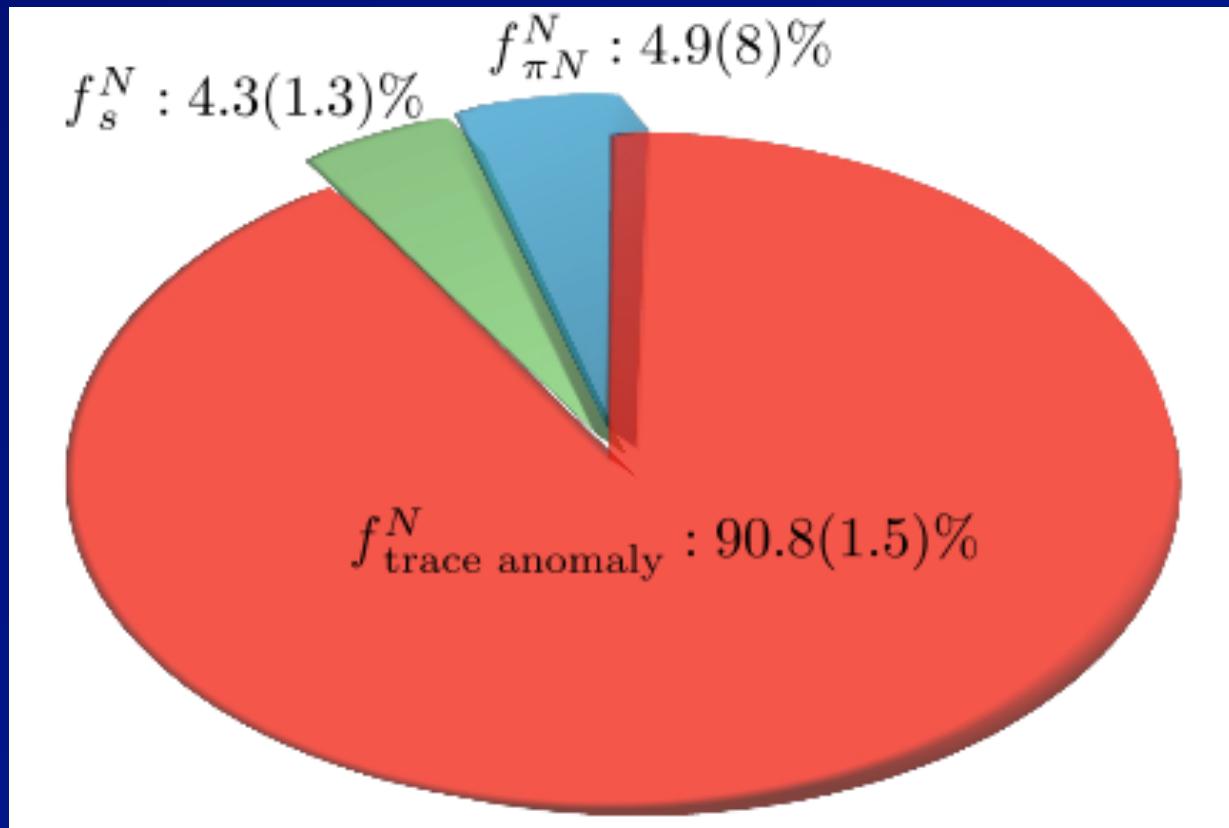
$$(T^{\mu\nu})_R = T^{\mu\nu}, \quad \partial_\nu T^{\mu\nu} = 0$$

■ Rest energy from EMT – vector, frame dependent, components are scale dependent

$$\langle P | T^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$

Mass from Trace of EMT

- Lattice calculation of quark condensate
 - Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]
 - Overlap fermion ($Z_m Z_s = 1$)
 - 3 lattices (one at physical m_π), systematics (volume, continuum)

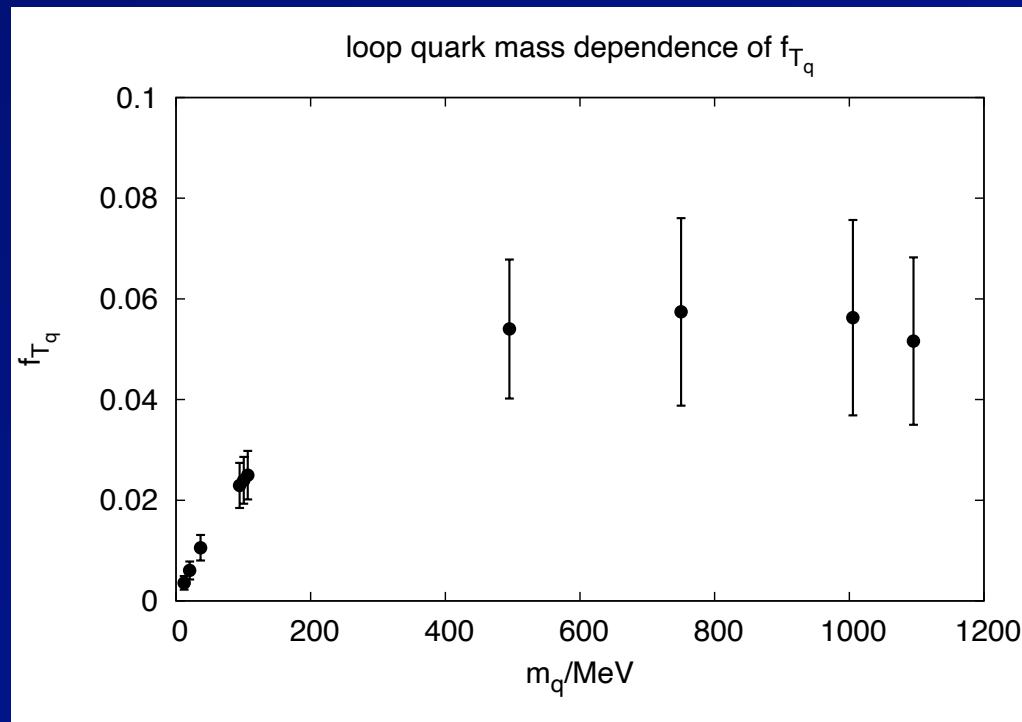


Hadronic scale
~ 1 GeV

$$f_f^N = \frac{m_f \langle N | \bar{\psi}_f \psi_f | N \rangle}{M_N}$$

Mass from Trace of EMT

- At electroweak scale, the standard model includes Higgs, t, b, c quarks in external states
 - M. Gong et al (χ QCD) [arXiv: 1304.1191]



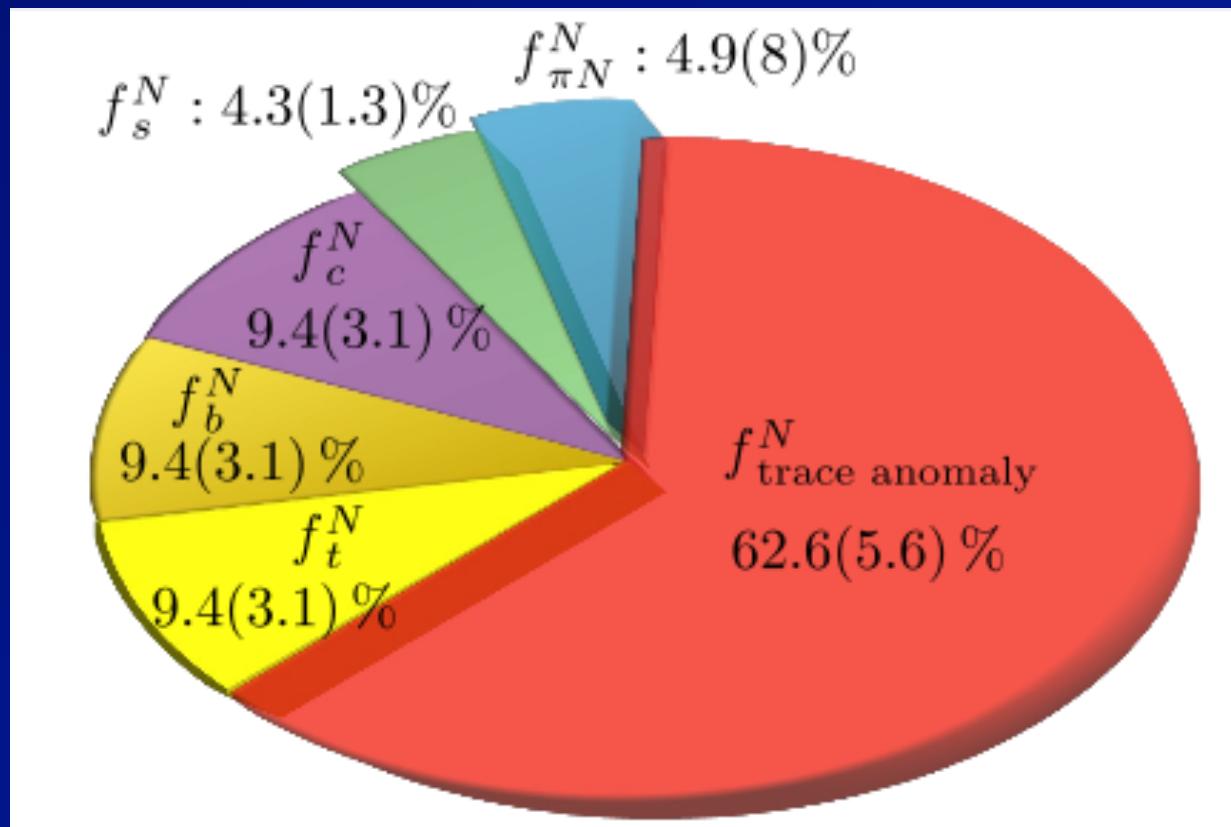
For $m_q > \sim 500 \text{ MeV}$, $m_f \langle N | \bar{\psi}_f \psi_f | N \rangle \sim \text{constant}$

Mass from Trace of EMT

- M. A. Shifman, A. Vainshtein, and V. I. Zakharov, Phys.Lett. B 78, 443 (1978) -- heavy quark expansion

$$m_h \langle N | \bar{\psi}_h \psi_h | N \rangle \sim -\frac{n_f}{3} \frac{\alpha_s}{4\pi} \langle N | G^2 | N \rangle + \mathcal{O}(1/m_h)$$

$$\frac{\beta(g)}{2g} = -\frac{\beta_0}{2} \left(\frac{\alpha_s}{4\pi}\right) - \frac{\beta_1}{2} \left(\frac{\alpha_s}{4\pi}\right)^2 - \frac{\beta_2}{2} \left(\frac{\alpha_s}{4\pi}\right)^3 + \dots \quad \beta_0 = 11 - \frac{2}{3} n_f$$



Rest Energy from Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$\begin{aligned}\langle P' | (T_{q,g}^{\mu\nu})_R(\mu) | P \rangle / 2M_N &= \bar{u}(P') [T_{1_{q,g}}(q^2, \mu) \gamma^{(\mu} \bar{P}^{\nu)} + T_{2_{q,g}}(q^2, \mu) \frac{\bar{P}^{(\mu} i\sigma^{\nu)\alpha} q_{\alpha}}{2M_N} \\ &+ D_{q,g}(q^2, \mu) \frac{q^{\mu} q^{\nu} - g^{\mu\nu} q^2}{M_N} + \bar{C}_{q,g}(q^2, \mu) M_N \eta^{\mu\nu}] u(P)\end{aligned}$$

– T_1 and T_2

$$T_{1_{q,g}}(0) = \langle x \rangle_{q,g}(\mu); \quad \langle x \rangle_q(\mu) + \langle x \rangle_g(\mu) = 1$$

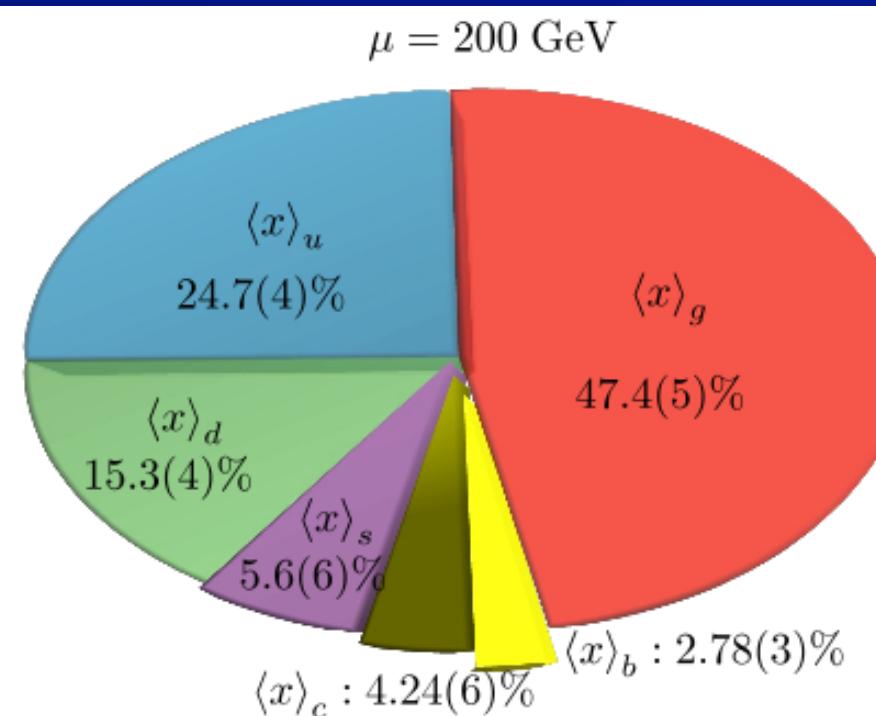
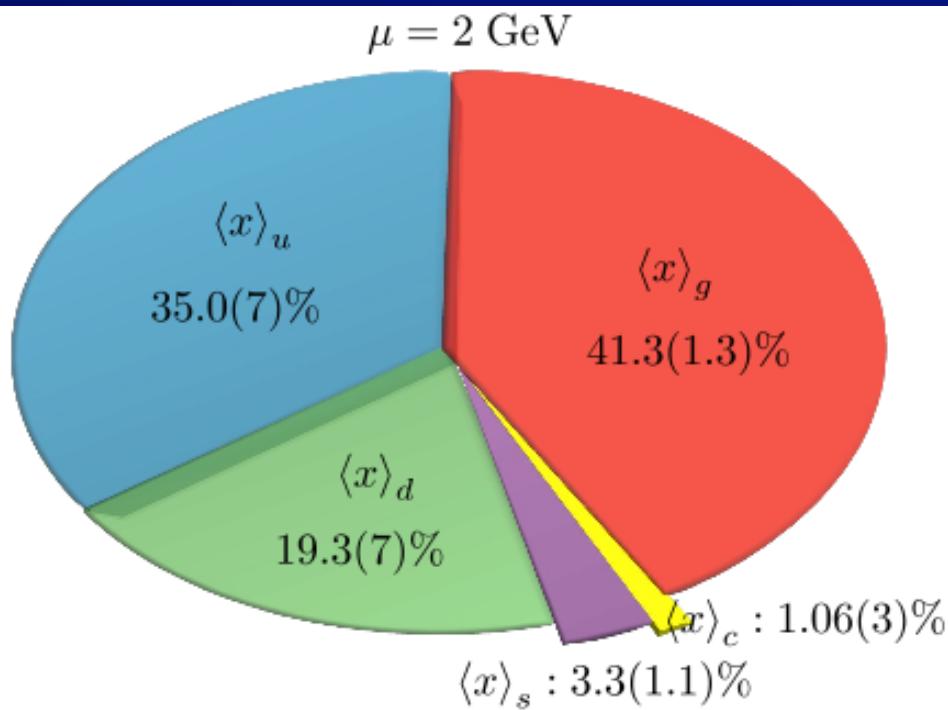
$$T_{1_{q,g}}(0) + T_{2_{q,g}}(0) = 2J_{g,g}(\mu); \quad 2J_q(\mu) + 2J_g(\mu) = 1$$

- D term: deformation of space = elastic property - [Polyakov & Schweitzer]
- C term: pressure-volume work - [Lorce]

$$\bar{C}_q + \bar{C}_g = 0, \quad \partial_{\nu} T^{\mu\nu} = 0$$

Rest Energy from Gravitational FF

- Momentum fractions from CT18



Rest Energy from Gravitational FF

- What are \bar{C}_q and \bar{C}_g ?

$$\langle P | (T_{q,g}^{00})_{RM}(\mu) | P \rangle / 2M_N = \langle x \rangle_{q,g}(\mu) M_N + \bar{C}_{q,g}(0, \mu) M_N,$$

$$\langle P | (T_{q,g}^{ii})_{RM}(\mu) | P \rangle / 2M_N = 3\bar{C}_{q,g}(0, \mu) M_N$$

Note: Being scale dependent, separate quark and glue T^{00} are renormalized and mixed.

$$3\bar{C}_{q,g}(0, \mu) M_N = [\langle P | \eta_{\mu\nu} (T_{q,g}^{\mu\nu})_{RM} | P \rangle - \langle P | (T_{q,g}^{00})_{RM}(\mu) | P \rangle] / 2M_N$$

$$\eta_{\mu\nu} (T_q^{\mu\nu})_R = (\sum_f m_f \bar{\psi}_f \psi_f)_R + \frac{\alpha_s}{4\pi} \left[\frac{n_f}{3} (F^2)_R + \frac{4C_F}{3} (\sum_f m_f \bar{\psi}_f \psi_f)_R \right].$$

$$\eta_{\mu\nu} (T_g^{\mu\nu})_R = \frac{\alpha_s}{4\pi} \left[-\frac{11C_A}{6} (F^2)_R + \frac{14C_F}{3} (\sum_f m_f \bar{\psi}_f \psi_f)_R \right].$$

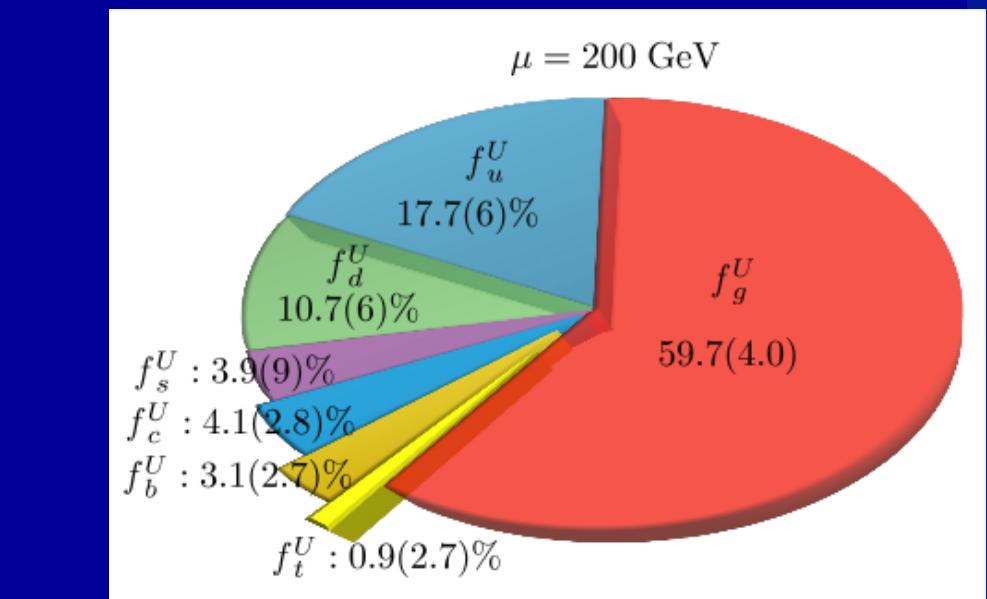
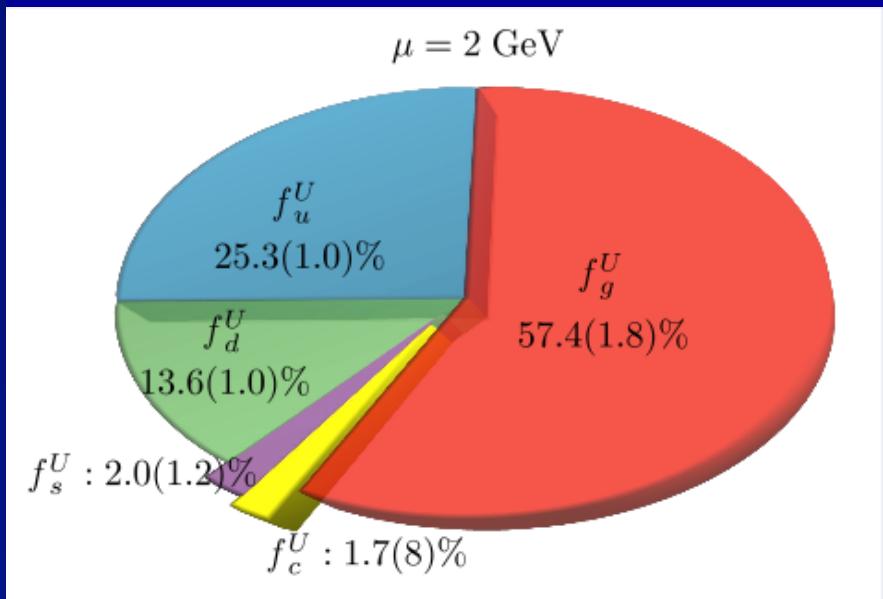
Rest Energy from Gravitational FF

- Decomposition of rest energy in quark and glue components

$$U_{q_f} = \langle x \rangle_{q_f}(\mu) + \bar{C}_{q_f}(0, \mu) = \frac{3}{4} \langle x \rangle_q(\mu) + \frac{1}{4} \left\{ f_f^N + \frac{\alpha_s}{4\pi} \left[\frac{\langle (F^2)_R \rangle}{3M_N} + \frac{4C_F}{3} f_f^N \right] \right\},$$

$$U_g = \langle x \rangle_g(\mu) + \bar{C}_g(0, \mu) = \frac{3}{4} \langle x \rangle_g(\mu) + \frac{1}{4} \left\{ \frac{\alpha_s}{4\pi} \left[-\frac{11C_A}{6} \frac{\langle (F^2)_R \rangle}{M_N} + \frac{14C_F}{3} \sum_f f_f^N \right] \right\}.$$

- $U_{q,g}$ – Internal Energy, $-\bar{C}_{q,g}$ -- pressure-volume work (C. Lorcé),



Rest Energy from Hamiltonian

- With equation of motion

$$H = H_m + H_E(\mu) + H_g(\mu) + \frac{1}{4} H_a$$

Ji

$$H_m = \int d^3x \sum_f m_f \bar{\psi}_f \psi_f,$$

$$H_E(\mu) = \int d^3x \left[\sum_f (T_{q_f}^{00} - \text{trace})_{RM} - H_m \right] -- \text{quark kinetic and potential energy}$$

$$H_g(\mu) = \int d^3x \frac{1}{2} (B^2 + E^2)_{RM} -- \text{glue field energy.}$$

$$f_f^E = \langle H_{E_f}(\mu) \rangle / M_N = \frac{3}{4} (\langle x \rangle_{q_f} - f_f^N)$$

$$f_g^E = \langle H_g \rangle / M_N = \frac{3}{4} \langle x \rangle_g.$$

A Demonstration of Hadron Mass Origin from QCD Trace Anomaly

Fangcheng He¹, Peng Sun², Yi-Bo Yang^{1,3,4}



(χ QCD Collaboration)

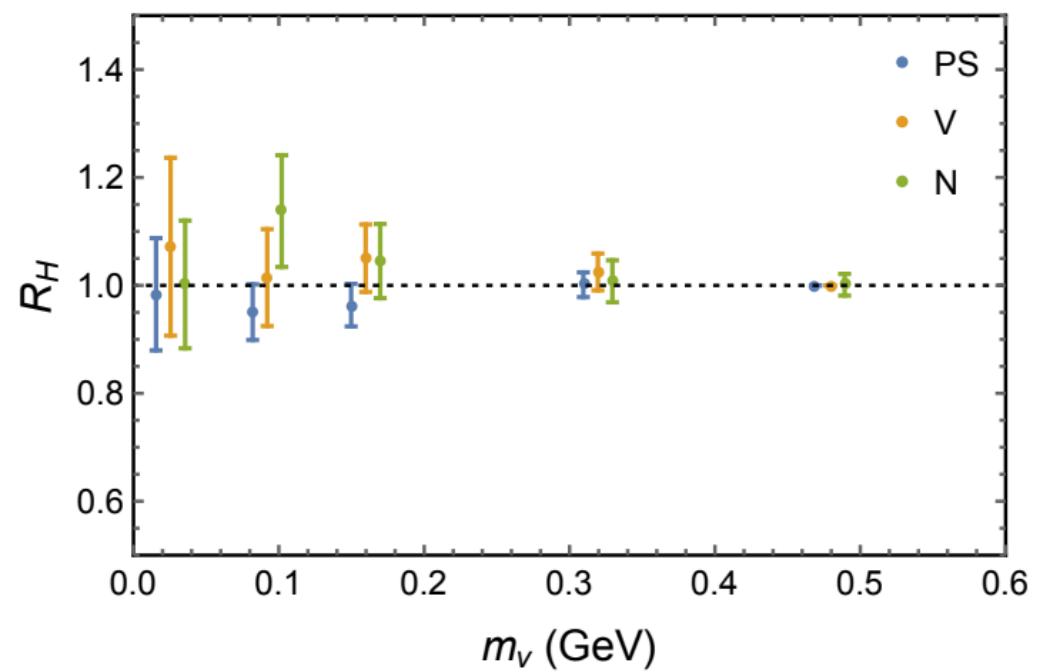
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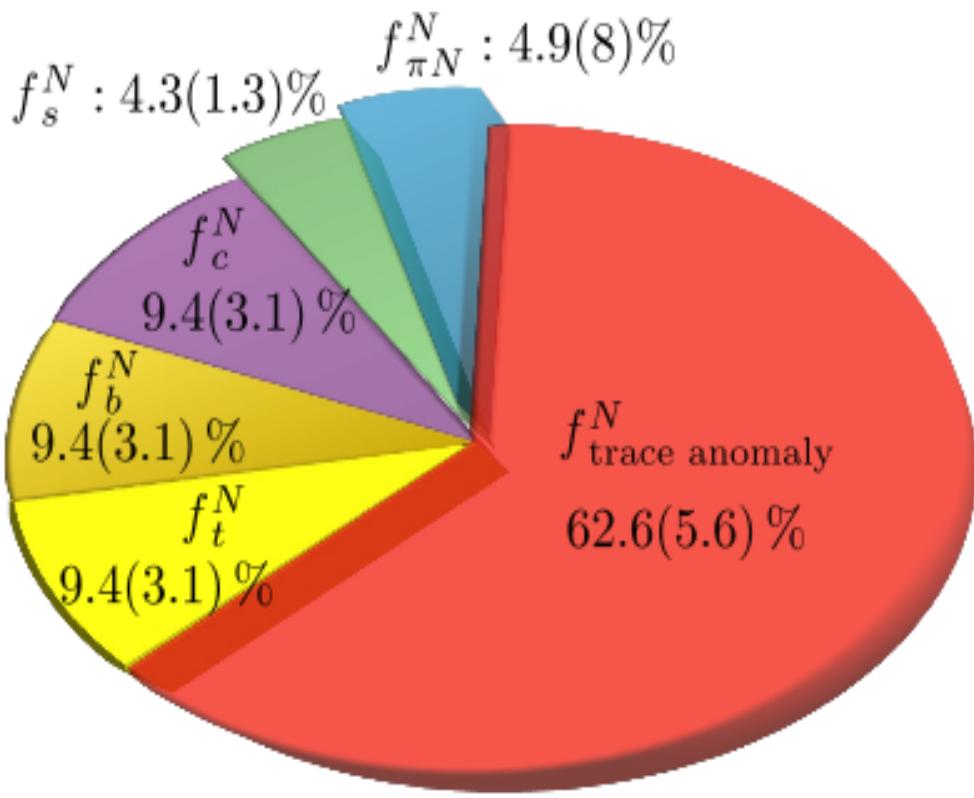
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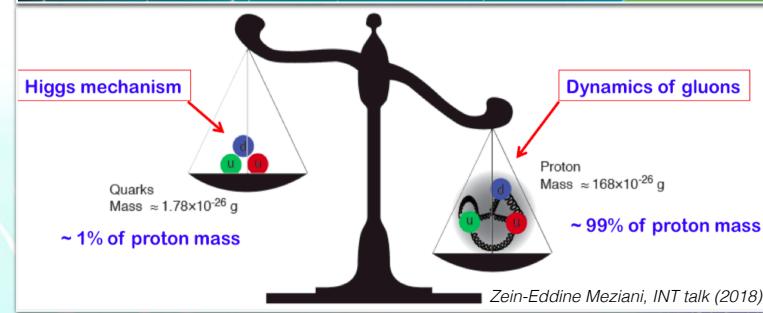
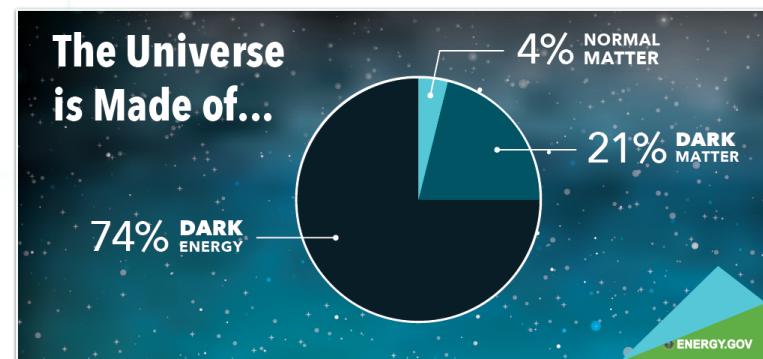
$$R_H(m_v) = \frac{(1 + \gamma_m)(\langle H_m \rangle_H + \frac{\beta}{2g} \langle F^2 \rangle_H)}{M_H},$$



Where does the proton mass come from?



Proton mass



Summary and Challenges

- There is a unique decomposition of proton mass (scale independent).
- Rest energy components (scale dependent):
 - Gravitational form factors (related to momentum fraction)
 - Internal energies
 - Hamiltonian (quark condensate and KE+PE, glue field energy and anomaly)
- $m_q \leftarrow$ Higgs mechanism
- Quark condensate \leftarrow chiral symmetry breaking
- Trace anomaly \leftarrow conformal symmetry breaking
- Conformal window with multi-flavors or different gauge group
- Finite temperature and density
 - A. Alexandru and I. Horvath [arXiv:1906.08047]
 - Nuclei