The proton mass from first principles: lattice QCD at the physical point

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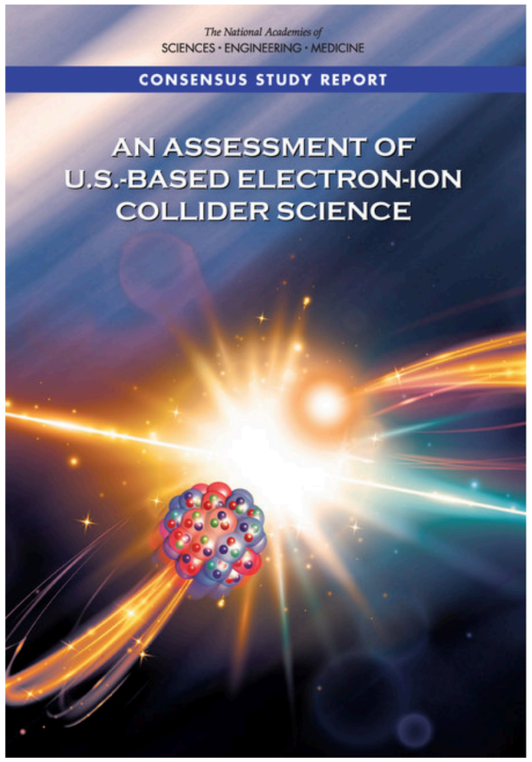
Temple University

In collaboration with: C. Alexandrou, K. Hadjiyiannakou

3rd Proton Mass Workshop; Origin and Perspective

January 14, 2021

Proton Mass



Main Pillar of NAS
Assessment report for EIC

Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

Lattice QCD can provide valuable input in understanding the proton mass decomposition from *first principles*

… before experimental EIC data are available

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- ★ **In rest-frame, the mass is related to the matrix elements of EMT**

$$\frac{\langle T^{\mu}_{\mu}\rangle}{\langle N|N\rangle} = M\,, \qquad \frac{\langle T^{00}\rangle}{\langle N|N\rangle} = M$$

Proton Mass Decomposition

Based on sum rules

(not unique)

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see, e.g., [M. Shifman et al., Phys. Lett. 78B (1978); D. Kharzeev, Proc. Int. Sch. Phys. Fermi 130 (1996)]

★ **Decomposition of T^{00} in trace and traceless parts in rest frame**
[X.D. Ji, Phys. Rev. Lett. 74, 1071 (1995); X. D. Ji, Phys. Rev. D 52, 271 (1995)]

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Once the EMT is decomposed into components, renormalization of the latter is necessary

Ji's Decomposition

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★ **Traceless** $(\overline{T}^{\mu\nu})$ **& trace** $(\hat{T}^{\mu\nu})$ **parts of EMT:**

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}\,, \qquad T_{q,g}^{\mu\nu} = \overline{T}_{q,g}^{\mu\nu} + \hat{T}_{q,g}^{\mu\nu}$$

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★ $\langle T^{00}\rangle$ **has for contributions from** $\langle \hat{T}_q^{00}\rangle\,,\ \langle \overline{T}_q^{00}\rangle\,,\ \langle \hat{T}_g^{00}\rangle\,,\ \langle \overline{T}_g^{00}\rangle$

★ **energy density component gives a decomposition for the mass:**

$$M = \frac{\langle N|T^{00}|N\rangle}{\langle N|N\rangle} = M_m \ + \ M_q \ + \ M_g \ + \ M_a$$

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$\langle \hat{T}_q^{00}\rangle$ **quark mass**

$\langle \overline{T}_q^{00}\rangle$ **quark energy**

gluon energy $\langle \overline{T}_g^{00}\rangle$

trace anomaly $\langle \hat{T}_g^{00}\rangle$

Question:

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Answer:

Components associated with operators calculable in lattice QCD

- ★ **Quark mass** $$M_m = \sum_q \sigma_q$$
- ★ **Quark energy** $$M_q = \frac{3}{4}\left(M\sum_q \langle x\rangle_q - \sum_q \sigma_q\right)$$
- ★ **Gluon energy** $$M_g = \frac{3}{4}M\langle x\rangle_g$$
- ★ **Trace anomaly** $$M_a = \frac{\gamma_m}{4}\sum_q \sigma_q - \frac{\beta(g)}{4g}(E^2 + B^2)$$

σ_q: sigma-terms

$\langle x\rangle_q$: Quark momentum fraction

$\langle x\rangle_g$: Gluon momentum fraction

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Results at the physical point

[C. Alexandrou et al., PRD 102, 054517 (2020), arXiv:1909.00485]

[C. Alexandrou et al., PRD 101, 094513 (2020), arXiv:2003.08486]

See Alexandrou's talk

Results @ physical pion mass

$\overline{MS}(2\mathrm{GeV})$

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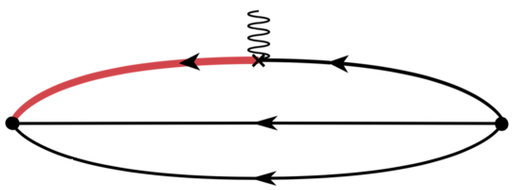
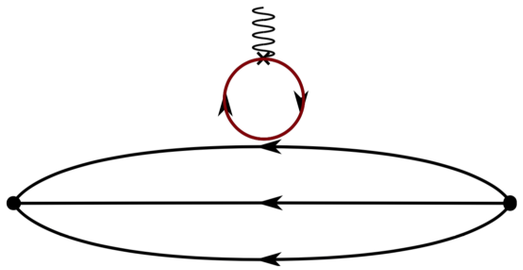
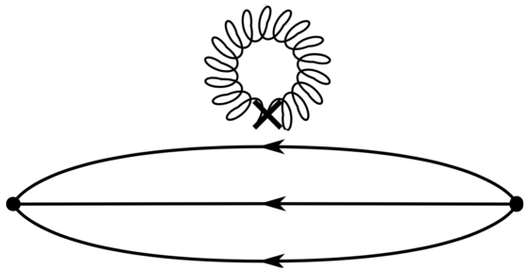
PRD 102, 054517 (2020)
PRD 101, 094513 (2020)







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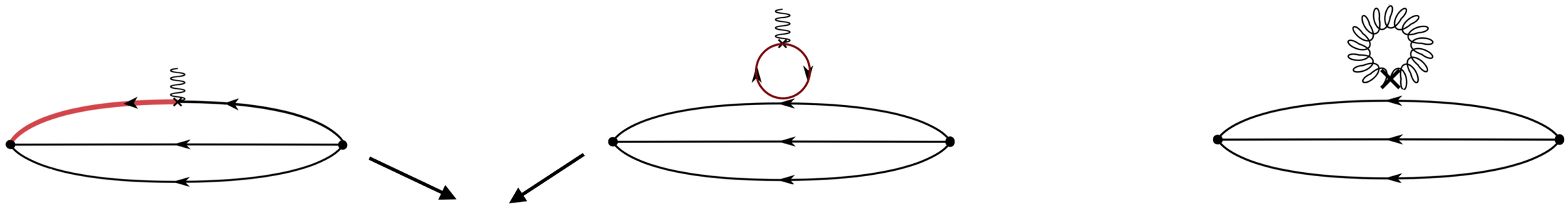

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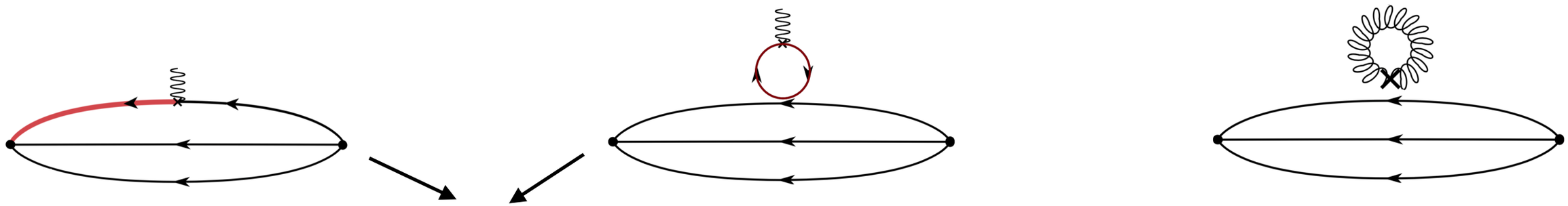

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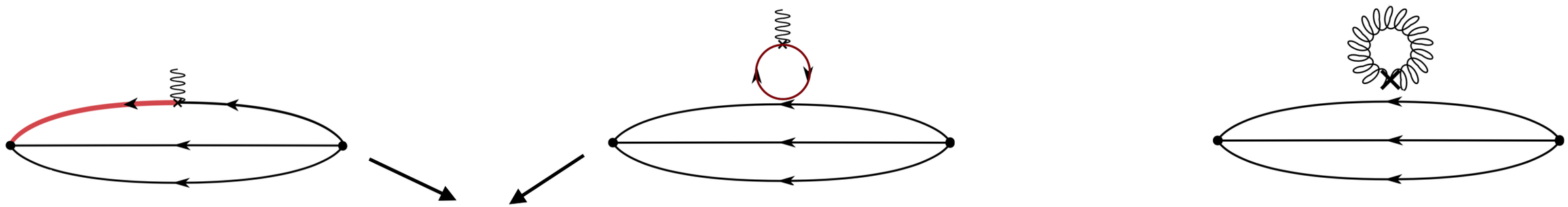

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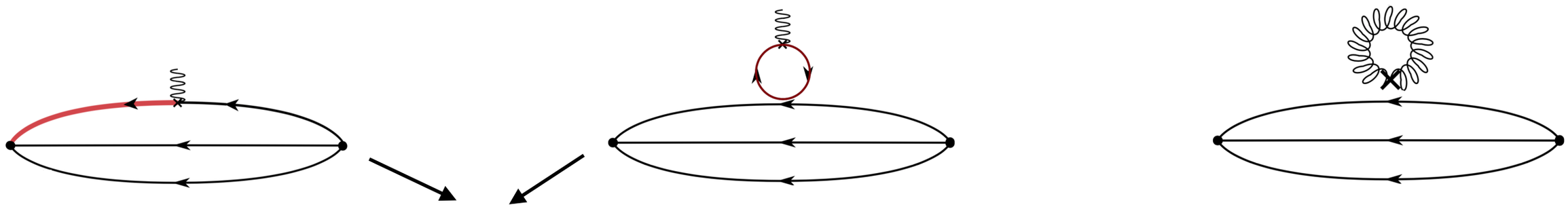

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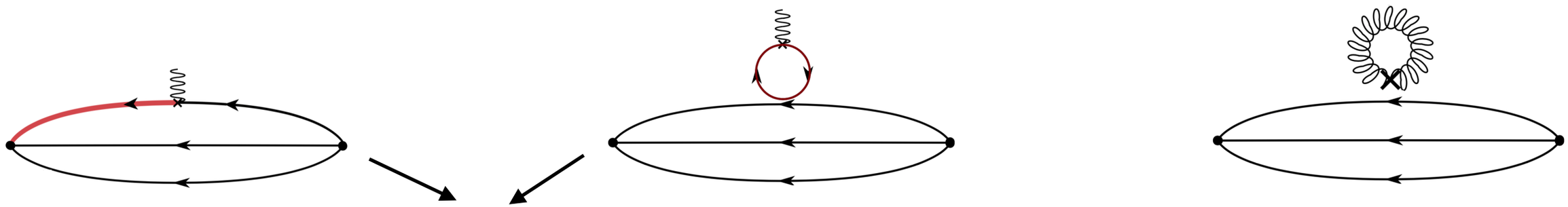

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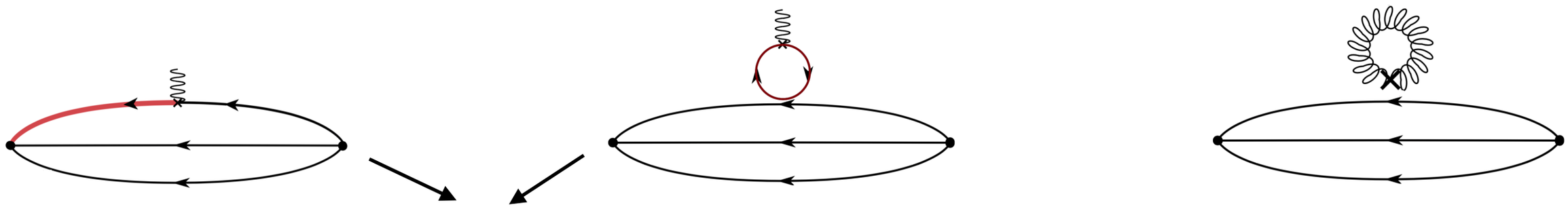

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Momentum sum rule satisfied!

Ji's Decomposition

Proton Mass Budget

★ **Available contributions:**

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Proton Mass Budget

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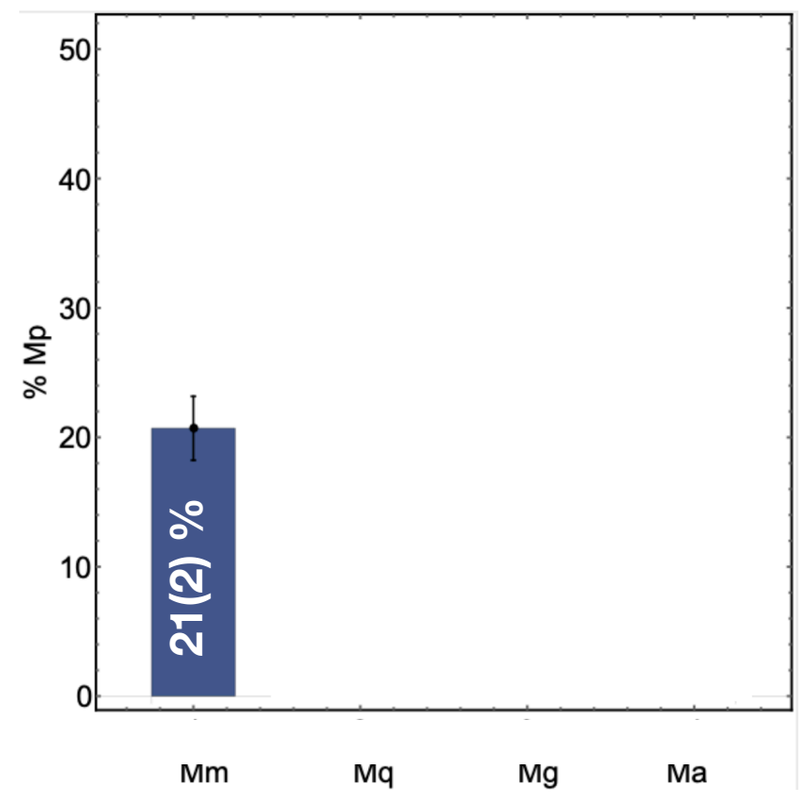
- ▶ **quark mass** (σ-terms)



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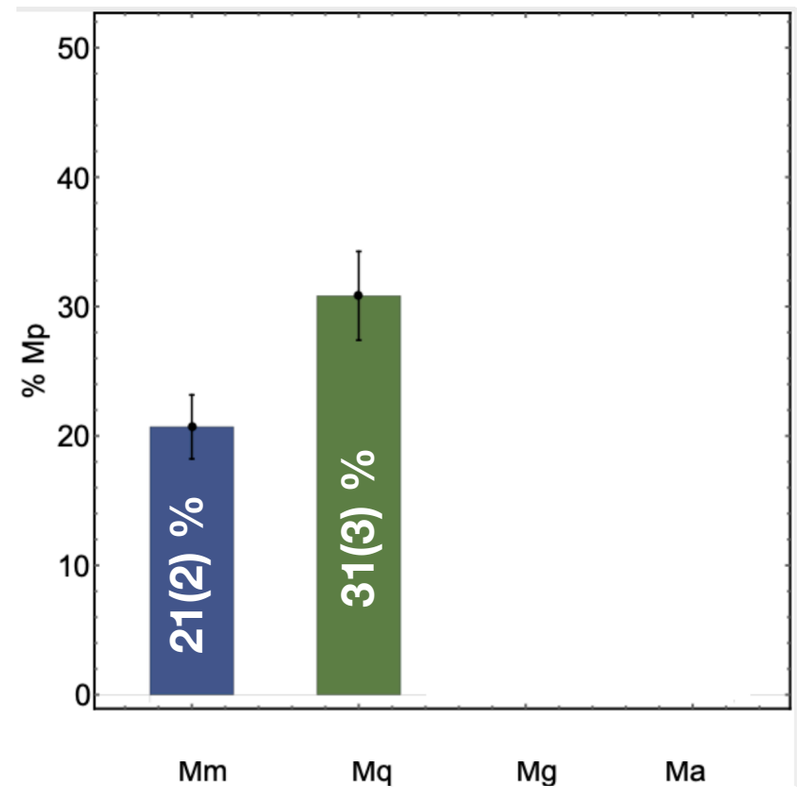

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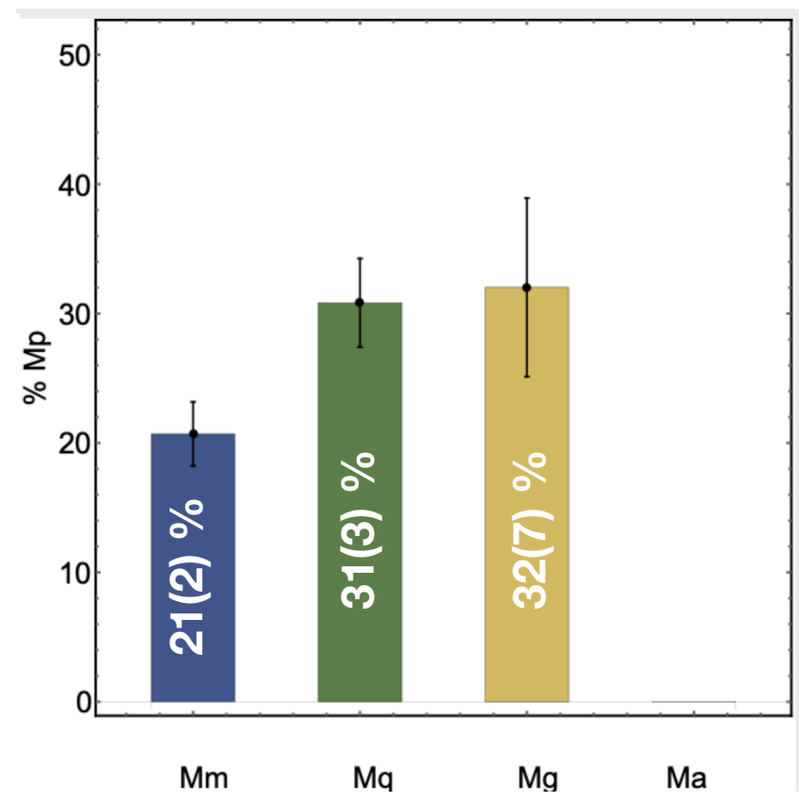

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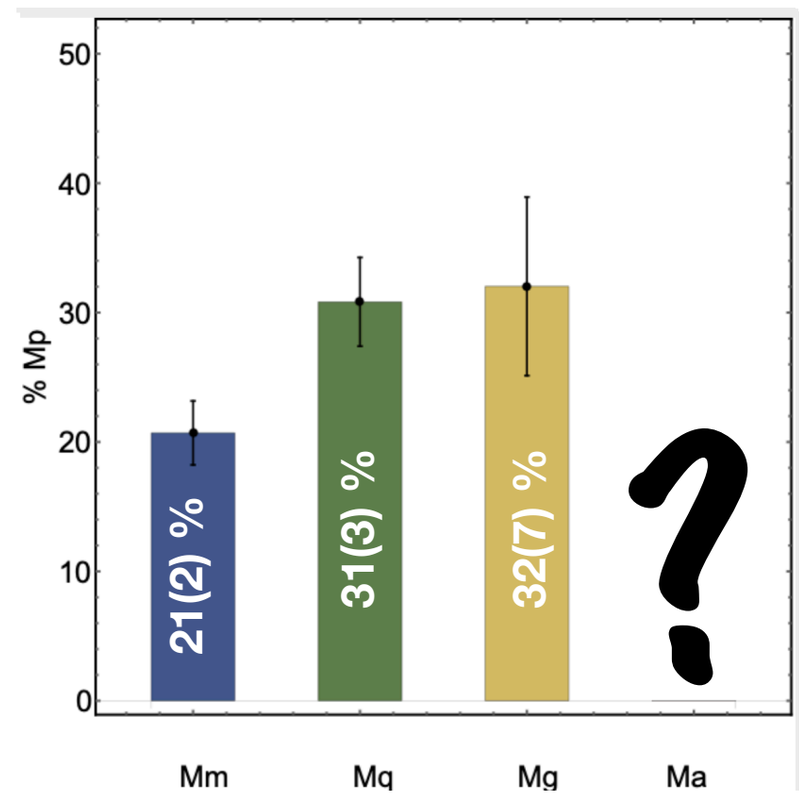

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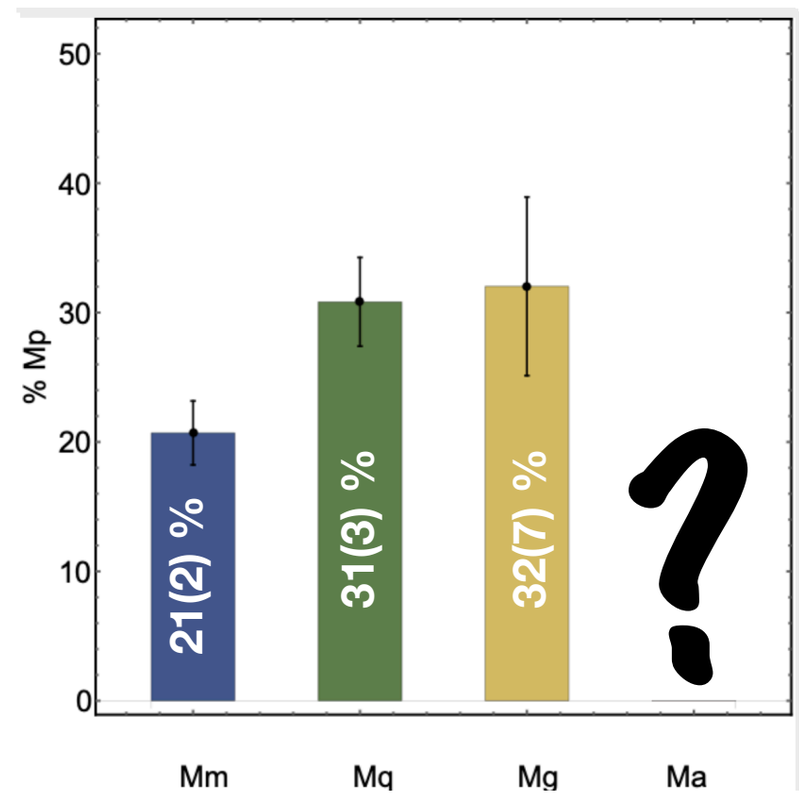
Currently not available



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Theoretical & Technical challenges

- ★ **Disconnected contribution (signal-to-noise ratio suppressed)**

- ★ **Presence of mixing with operators that are BRST variations and that vanish by the e.o.m.**
  (Full EMT: 10 renormalization functions)
  (Trace: 3-operator mixing under renormalization in the continuum)

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Answer:

- ★ **Direct calculation of trace anomaly not available**
- ★ **Possibility to access trace anomaly indirectly from sum rules**

$$M_a = \frac{M}{4} - \sum_q \frac{\sigma_q}{4} \qquad M_a = M - \sum_{i=m,q,g} M_i$$

Ji's Decomposition

Proton Mass Budget

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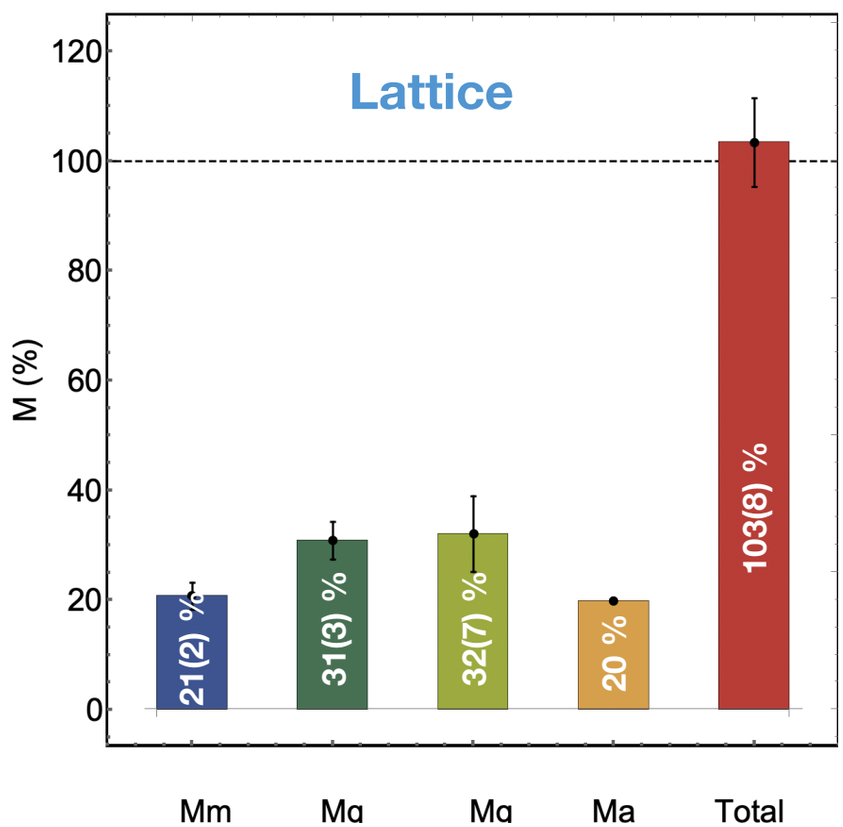
Approach A

$$M_a = \frac{M_p}{4} - \sum_q \frac{\sigma_q}{4} \sim 19.83(0.07)\,\%$$

$$M_p = M_m + M_q + M_g + M_a = 103.39(8.09)\,\%$$



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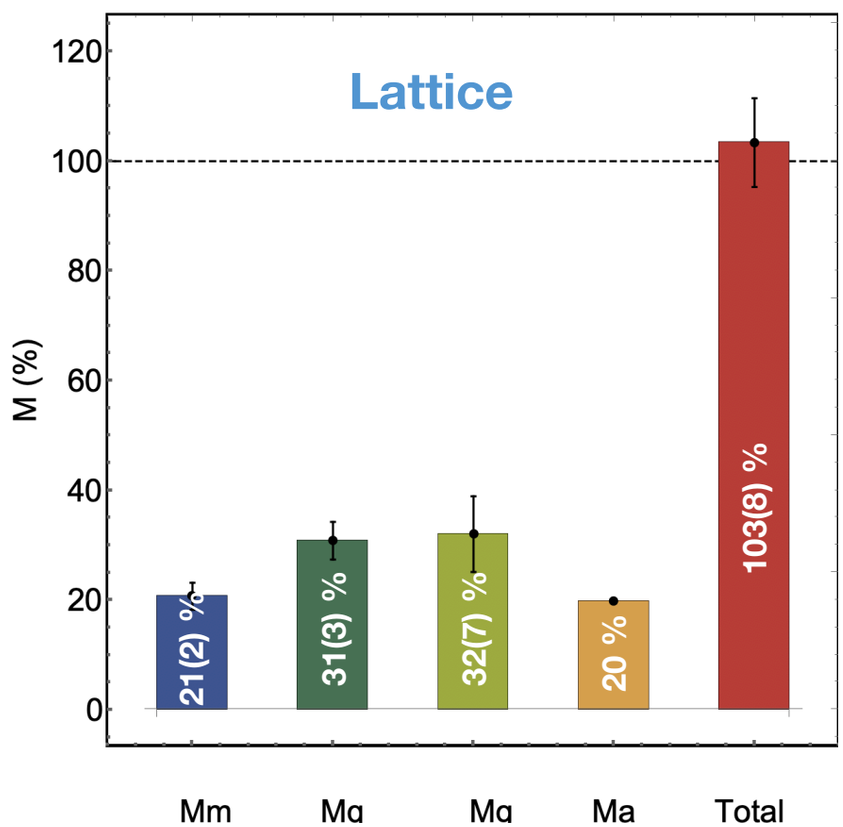
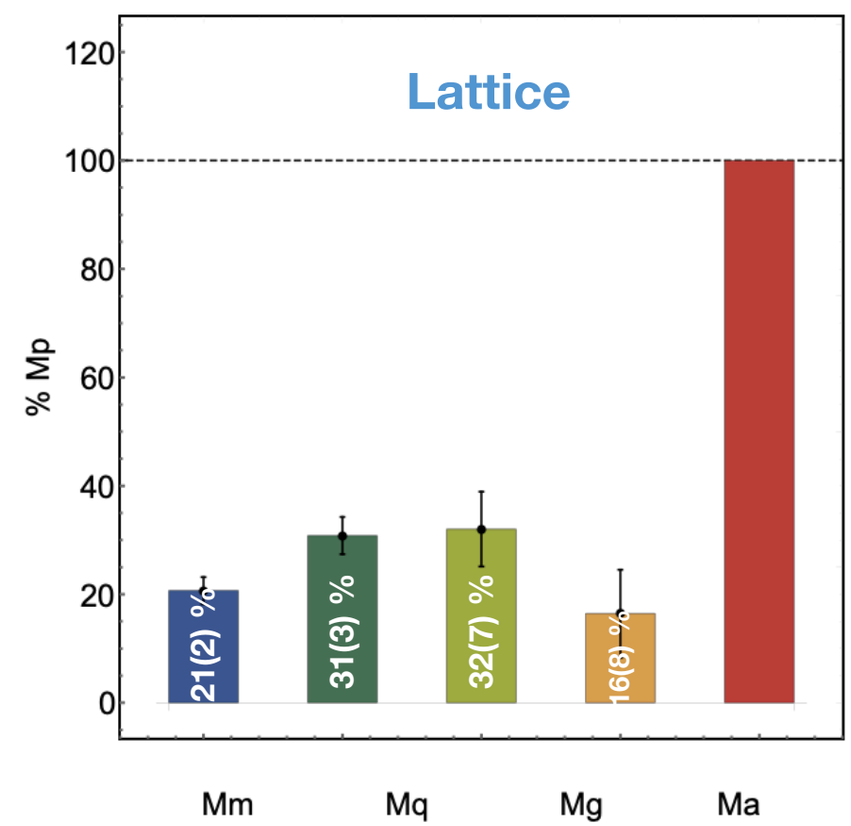


Approach B

$$M_a = M_p - \sum_{i=m,q,g} M_i \sim 16.45(8.09)\,\%$$

M_p : sum rule enforced



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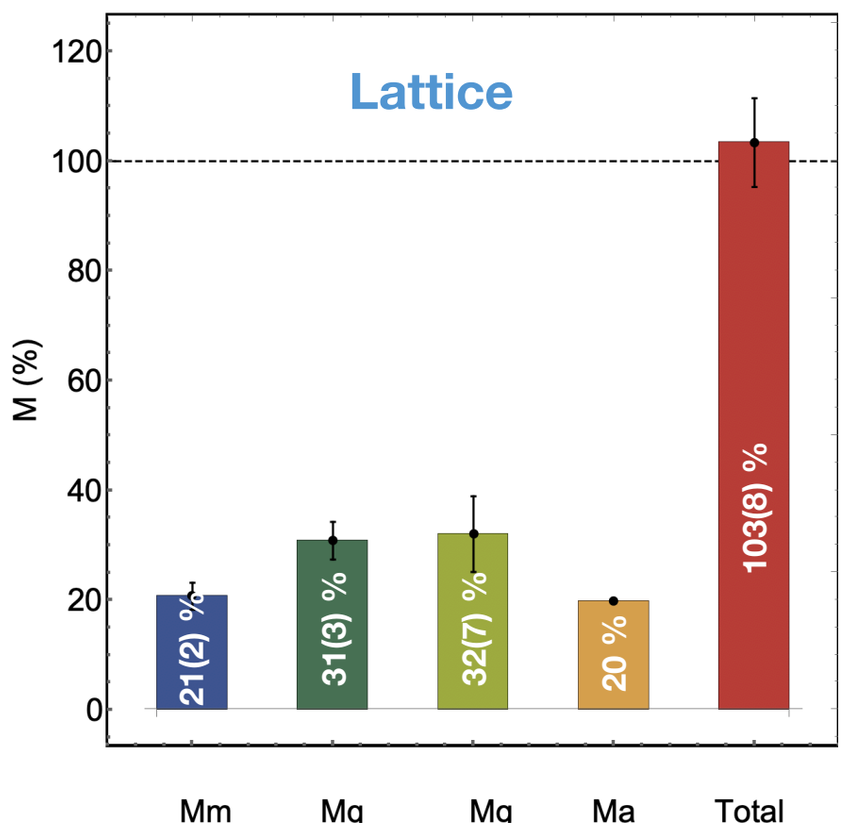
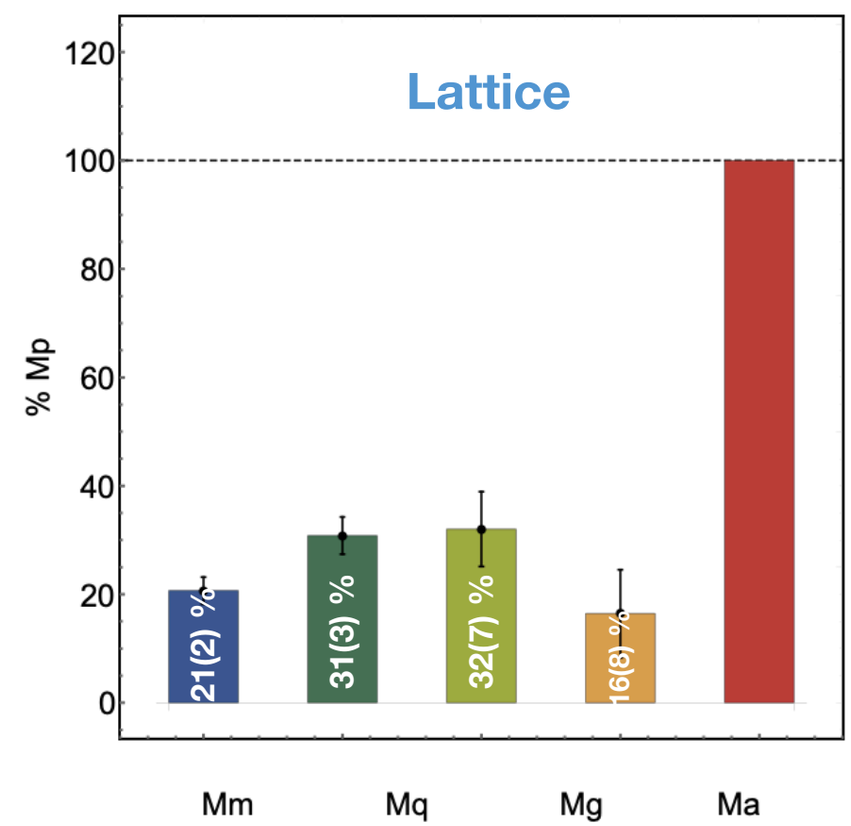


Approach B

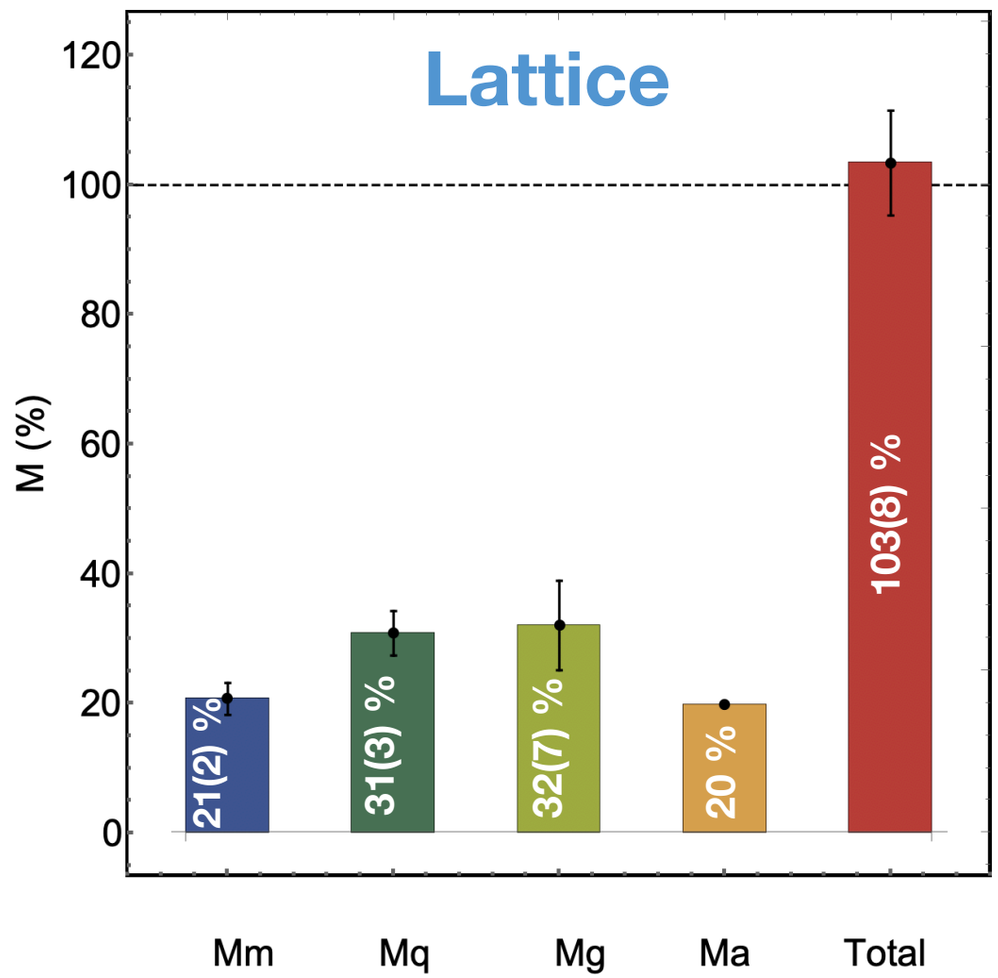
$$M_a = M_p - \sum_{i=m,q,g} M_i \sim 16.45(8.09)\,\%$$

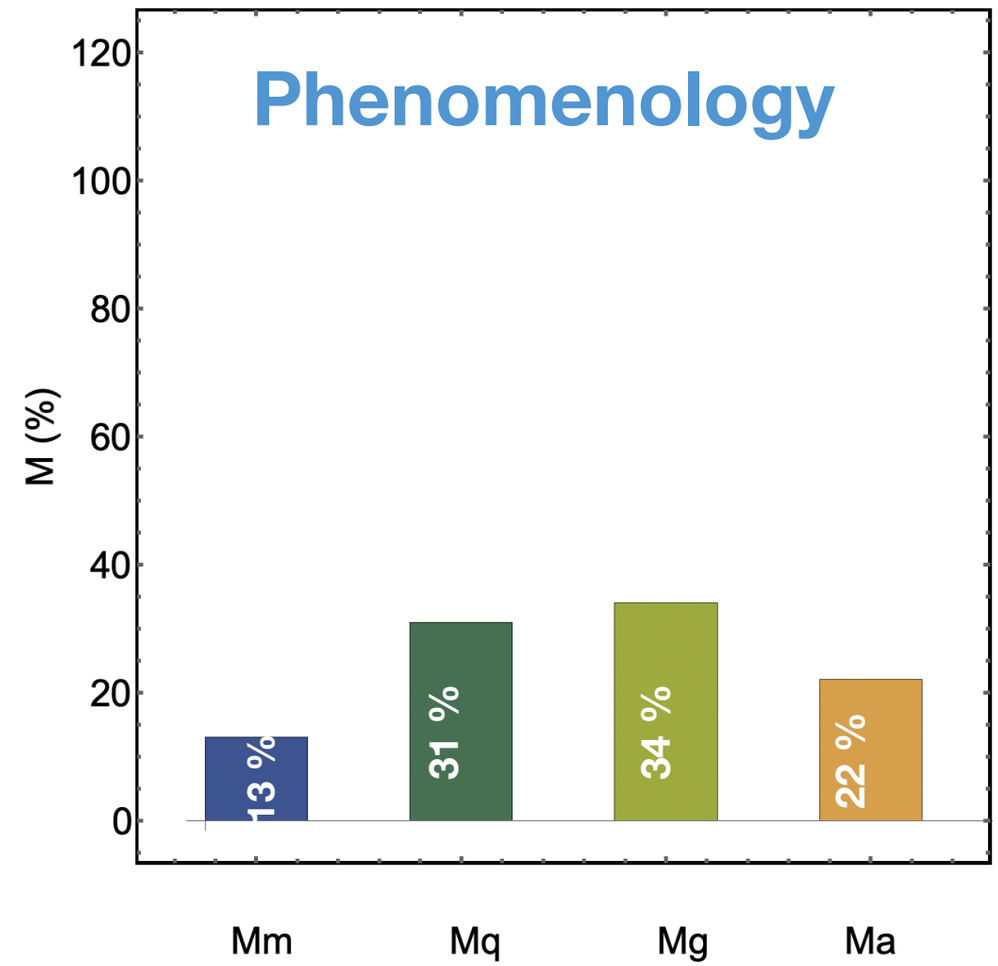
M_p : sum rule enforced



- ★ M_a compatible but different systematic uncertainties
- ★ Uncertainties of trace anomaly term depend on the sum rule

Ji's Decomposition





[C. Lorce´, EPJ. C78 (2018) 2]
[L. Harland-Lang et al., EPJ. C 75 (2015)]
[M. Hoferichter et al., PRL 115 (2015)]

- ★ Lattice and pheno data give similar picture
- ★ The tension in the sigma terms affects M_m
- ★ Contributions are of similar order

Lorcé's Decompositions

$$\frac{\langle T^{00} \rangle}{\langle N | N \rangle} = M$$

Decomposition with Pressure effects

[C. Lorce´, Eur. Phys. J. C78 (2018) 2, arXiv:1706.05853]

★ **Consider a EMT decomposition:** $T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$

$$T_q^{\mu\nu} = \frac{1}{4}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi \qquad T_g^{\mu\nu} = -F^{\mu\alpha}F_\alpha^\nu + \frac{g^{\mu\nu}}{4}F^{\alpha\beta}F_{\alpha\beta}$$

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★ **Matrix elements of each component in the fwd limit:**

$$\langle N | T_{q,g}^{\mu\nu}(0) | N \rangle = 2P^\mu P^\nu A_{q,g}(0) + 2M^2 g^{\mu\nu}\overline{C}_{q,g}(0)$$

$$\left(\langle T^{\mu\nu} \rangle = 2P^\mu P^\nu \right)$$

Sum rules

$$A_q(0) + A_g(0) = 1$$

$$\bar{C}_q(0) + \bar{C}_g(0) = 0$$

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[C. Lorce´, Eur. Phys. J. C78 (2018) 2, arXiv:1706.05853]

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Sum rules

$$A_q(0) + A_g(0) = 1$$
$$\bar{C}_q(0) + \bar{C}_g(0) = 0$$

★ **The EMT FFs can be related to the components of Ji's picture:**

$$A_q(0) = \sum_f \langle x \rangle_f, \quad A_g(0) = \langle x \rangle_g, \quad \bar{C}_q(0) = (1+\gamma_m)\frac{\sigma_{u+d+s+c}}{4M} - \frac{\langle x \rangle_{u+d+s+c}}{4}, \quad \bar{C}_g(0) = -\bar{C}_q(0)$$

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$$A_q(0) + A_g(0) = 1 \checkmark$$

Decomposition with Pressure effects

[C. Lorce´, Eur. Phys. J. C78 (2018) 2, arXiv:1706.05853]

★ **Thermodynamic potentials using the energy component $T^{00}_{q,g}$:**

(effective coupled two-fluid picture, combinations of internal energies and pressure-volume works

U**: internal energy (kinetic & potential)**

W**: pressure-volume work)**

$$U_{q,g} = M\left[A_{q,g}(0) + \overline{C}_{q,g}(0)\right], \quad W_{q,g} = -M\,\overline{C}_{q,g}(0)$$

energy density and pressure kept separately

Sum rules

$$M = U_q + U_g$$

$$W_q + W_g = 0$$

Decomposition with Pressure effects

[C. Lorce´, Eur. Phys. J. C78 (2018) 2, arXiv:1706.05853]

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$$M = U_q + U_g$$

$$W_q + W_g = 0$$

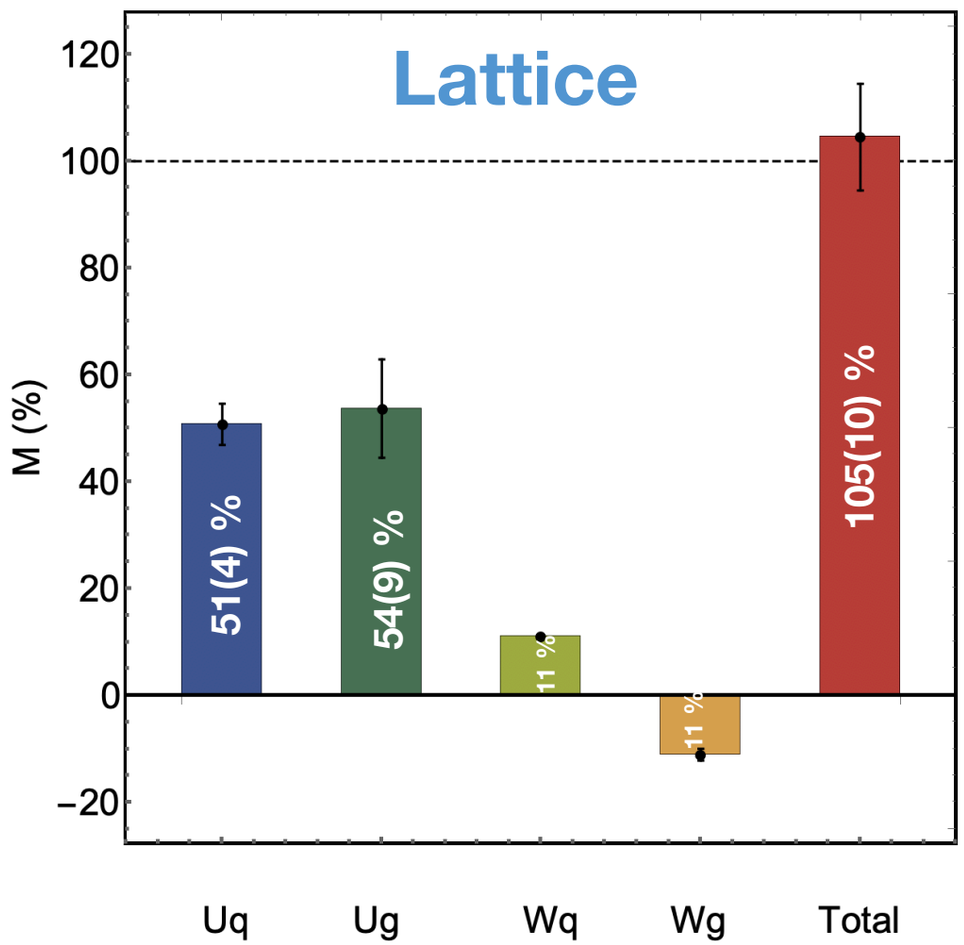
★ **Ji's components ($T^{00}_{q,g} = \hat{T}^{00}_{q,g} + \overline{T}^{00}_{q,g}$) interpreted as internal energy**

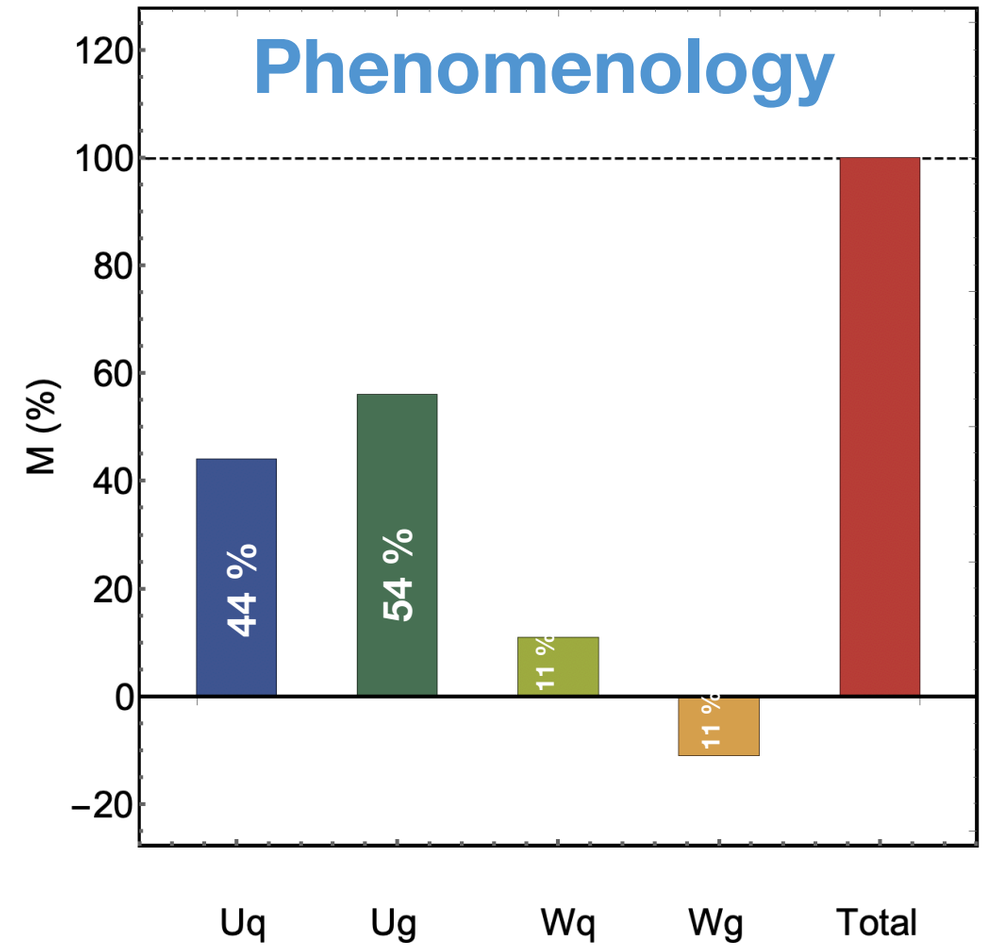
(effective coupled four-fluid picture)

$$\mathscr{W}_m = -M_m, \quad \mathscr{W}_q = \frac{1}{3}\left(M_q + \frac{12M_m}{4+\gamma_m}\right), \quad \mathscr{W}_g = \frac{M_g}{3}, \quad \mathscr{W}_a = -M_a$$

Lorce's Pressure-volume work decomposition

two-fluid picture





[C. Lorce´, EPJ. C78 (2018) 2]
[L. Harland-Lang et al., EPJ. C 75 (2015)]
[M. Hoferichter et al., PRL 115 (2015)]

- ★ Decomposition of lattice data gives the same picture as phenomenology
- ★ Equal contributions to the mass from the internal quark and gluon energies
- ★ U_g, W_g use sum rule:

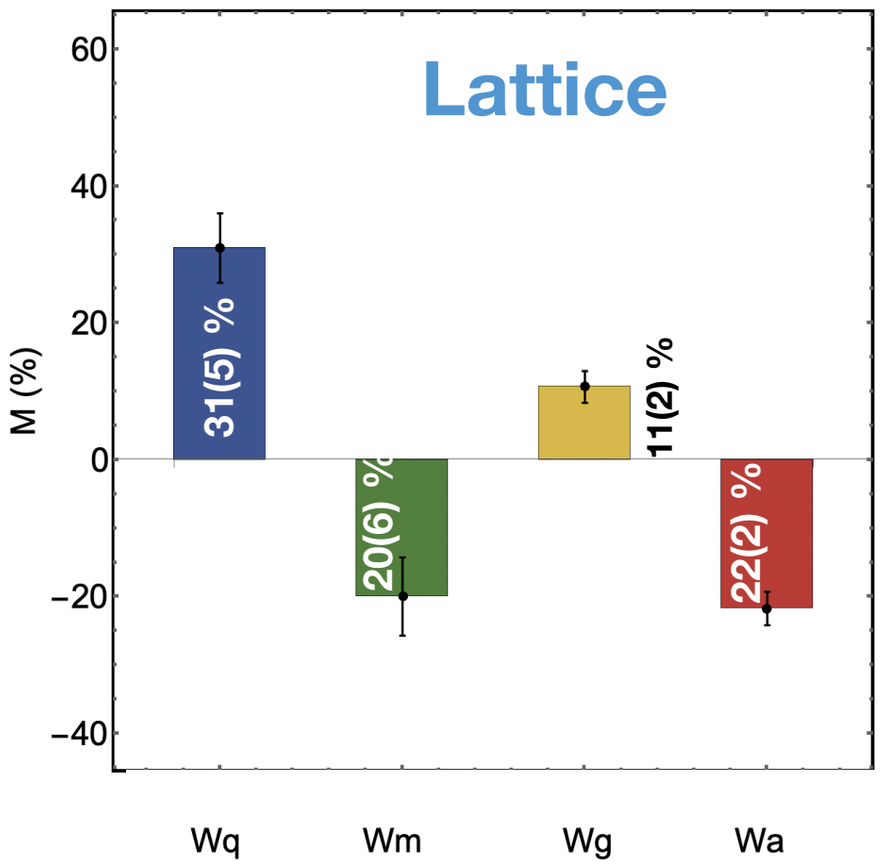
$$\overline{C}_g(0) + \overline{C}_q(0) = 0$$

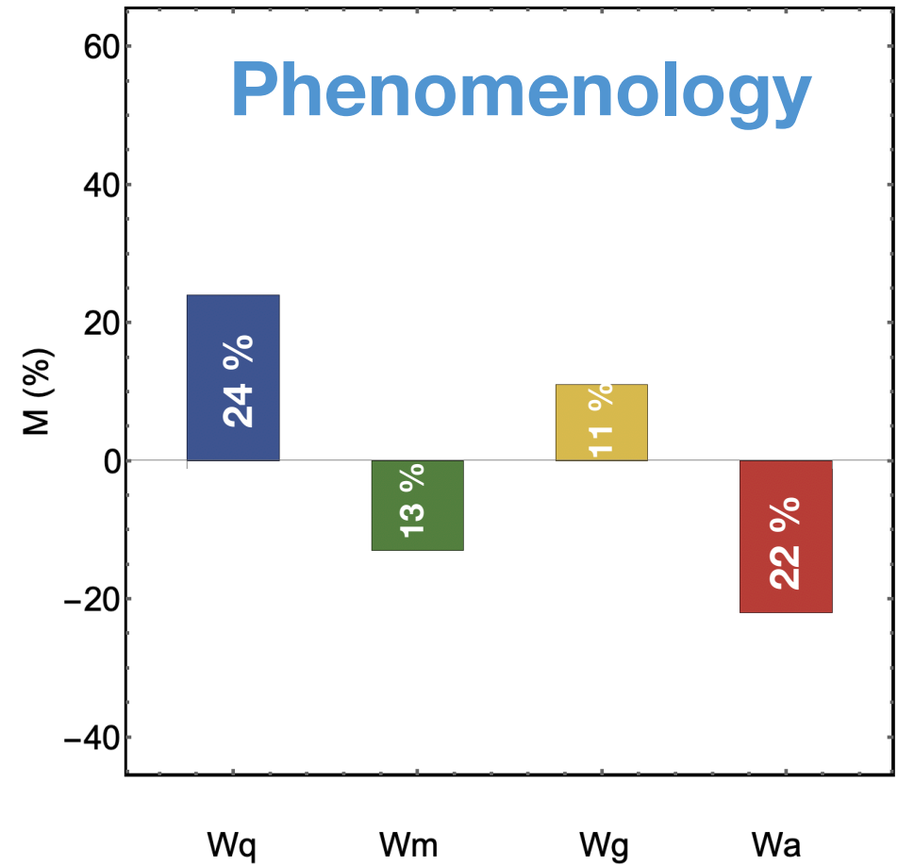
		$O(\alpha_s^1)$	$O(\alpha_s^2)$	$O(\alpha_s^3)$
Scenario A	U_q	0.384 ± 0.035	0.383 ± 0.036	0.384 ± 0.036
	U_g	0.554 ± 0.035	0.556 ± 0.036	0.555 ± 0.036
Scenario B	U_q	0.420 ± 0.016	0.420 ± 0.017	0.421 ± 0.017
	U_g	0.518 ± 0.016	0.518 ± 0.017	0.517 ± 0.017

[A. Metz, B. Pasquini, S. Rodini, arXiv:2006.11171]

Ji's Pressure-volume work decomposition

four-fluid picture





- ★ Lattice and pheno data give similar picture
- ★ Total "quark" contribution (W_g, W_m) similar to total "gluon" contribution (W_a, W_g)
- ★ W_a, W_g receive input from sum rule
- ★ $\Sigma W_q = 0$ by construction

[C. Lorce´, EPJ. C78 (2018) 2]
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Hatta-Rajan-Tanaka Decomposition

$$\frac{\langle T^{\mu}_{\mu} \rangle}{\langle N | N \rangle} = M$$

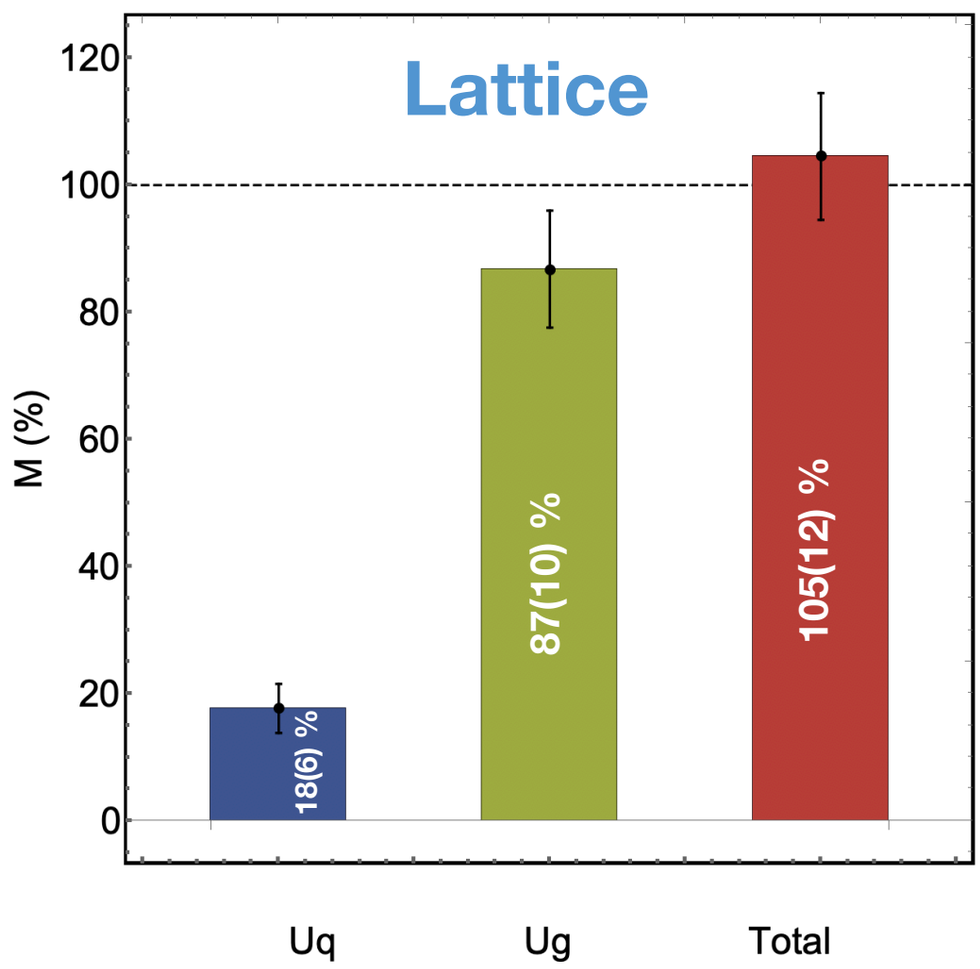
HRT Decomposition

★ [Y. Hatta, A. Rajan, K. Tanaka, JHEP 12, 008 (2018) arXiv:1810.05116; K. Tanaka, JHEP 01, 120 (2019), arXiv:1811.07879]

★ **Separation of quark and gluon parts of trace part of EMT,** $T^{\mu}_{\mu;\,q,g}$

$$T^{\mu}_{\mu;\,q} = (1 + c_1^{\rm MS})\, m\overline{\psi}\psi + c_2^{\rm MS}\, F^{\alpha\beta}F_{\alpha\beta}$$

$$T^{\mu}_{\mu;\,q} = (\gamma_\mu - c_1^{\rm MS})\, m\overline{\psi}\psi + \left(\frac{\beta(g)}{2g} - c_2^{\rm MS}\right) F^{\alpha\beta}F_{\alpha\beta}$$



$$U_{q,g} = M\left[A_{q,g}(0) + 4\overline{C}_{q,g}(0)\right]$$

Sum rules

$$M = U_q + U_g$$

★ **Gluon contributions is dominant (in support of the argument that gluons are responsible for the mass)**

Concluding Remarks

Concluding Remarks

"Can lattice calculate the mass distribution in the nucleon?"

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DOE Early Career Award (NP)
Grant No. DE-SC0020405

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Concluding Remarks

"Can lattice calculate the mass distribution in the nucleon?"

- ★ **Mass components are local 2-parton operators calculable in LQCD**
- ★ **Successful calculation of nucleon sigma terms, quark and gluon momentum fractions**

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"Can one calculate the anomaly contribution on the lattice?"

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- ★ **Mass components are local 2-parton operators calculable in LQCD**
- ★ **Successful calculation of nucleon sigma terms, quark and gluon momentum fractions**

"Can one calculate the anomaly contribution on the lattice?"

- ★ **Well-defined operator, but complicated renormalization pattern, and suppressed signal-to-noise ratio**
- ★ **Sum rules very useful to extract trace anomaly indirectly**

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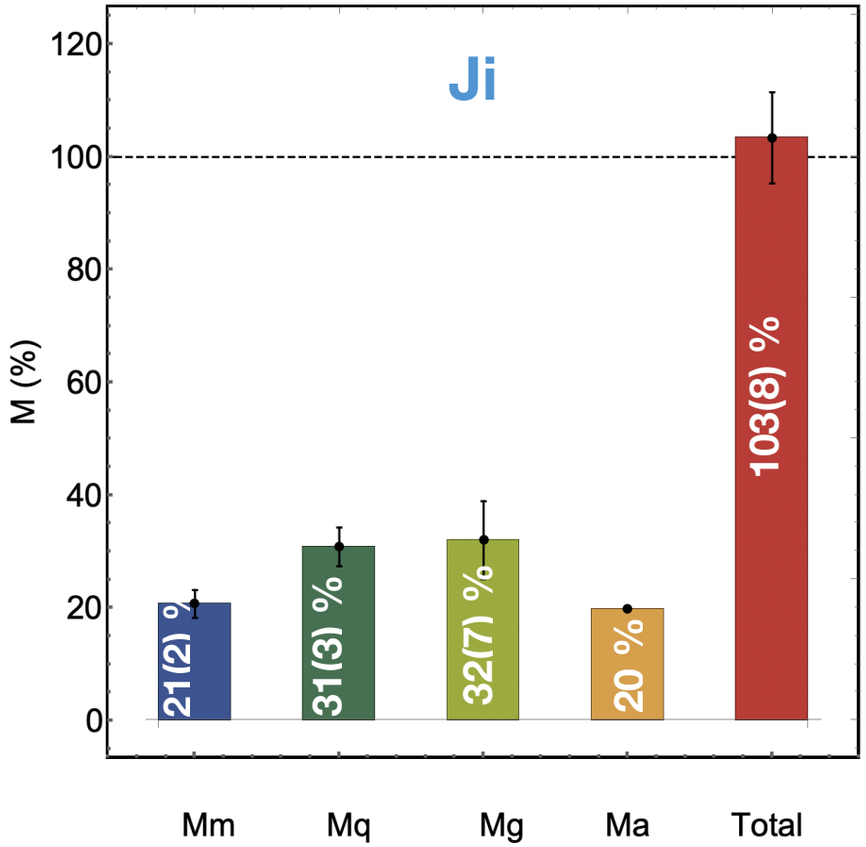
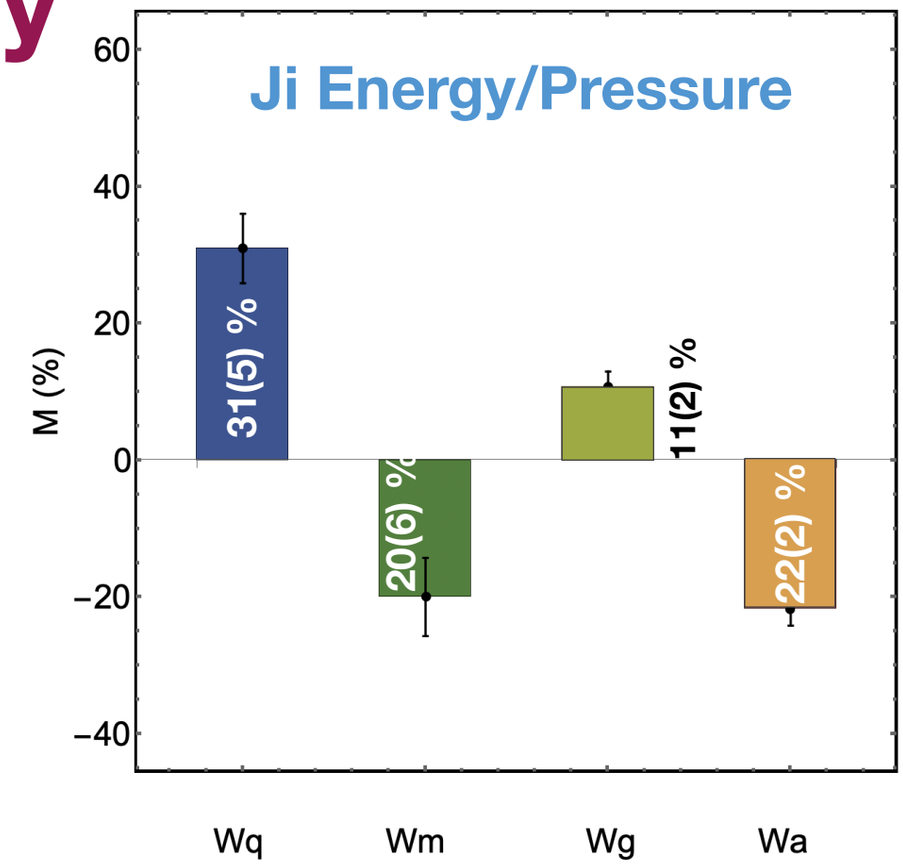
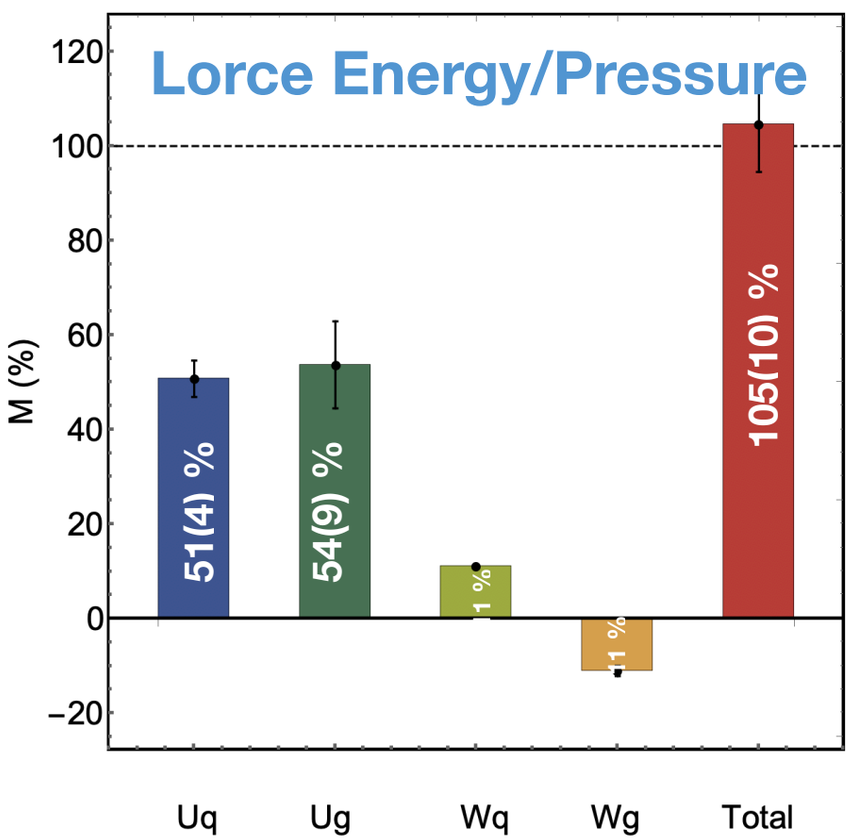
Summary







Thank you