3rd Proton Mass Workshop; Origin and Perspective
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Model studies of energy-momentum tensor (EMT) and mechanical properties

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Outline

- **EMT form factors**
  - → brief overview
  - → D-term last global unknown
- **interpretation, insights, lessons**
  - → mass decomposition, spin decomposition
  - → matter distribution, spin distribution
  - → visualization of forces
- **models**
  - → predict, sharpen intuition, set expectations
  - → some selected results
- **conclusions**

Definition EMT form factors, spin $\frac{1}{2}$ (Kobzarev, Okun 1962, Pagels 1966)

$$\langle p'|\hat{T}^{a}_{\mu\nu}|p\rangle = \bar{u}(p')\Bigg[\; A^a(t,\mu^2)\,\frac{\gamma_\mu P_\nu+\gamma_\nu P_\mu}{2} + B^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu-g_{\mu\nu}\Delta^2}{4M} + \bar{c}^a(t,\mu^2)\,M\,g_{\mu\nu}\Bigg]u(p)$$

- $\hat{T}^a_{\mu\nu}$ symmetric, gauge invariant, total EMT $\partial_\mu\hat{T}^{\mu\nu}=0$, $\bar{u}u=2M$
- $\sum_a\bar{c}^a(t,\mu^2)=0$, $A(t)=\sum_a A^a(t,\mu^2)$, $B(t)$, $D(t)$ scale invariant
- constraints: **mass** $\Leftrightarrow$ $A(0)=1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ $B(0)=0$ $\Leftrightarrow$ anomalous gravitomagnetic moment of nucleon = 0 *

  D-term $\Leftrightarrow$ $D(0)\equiv D$ $\rightarrow$ unconstrained! **Last global unknown!**

$$2P=(p'+p)$$
$$\Delta=(p'-p)$$
$$t=\Delta^2$$

notation: $2\,J^q(t)=A^q(t)+B^q(t)$

$D^q(t)=\frac{4}{5}\,d_1^q(t)=\frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t)=M_2^q(t)$

* equivalent to $2J(0)=A(0)+B(0)=1$ $\Leftrightarrow$ spin of nucleon $\frac{1}{2}$

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$$2P = (p'+p) \qquad \Delta = (p'-p) \qquad t = \Delta^2$$

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non-symmetric EMT, additional form factor related to $G_A(t)$ → Cédric Lorcé

Definition EMT form factors, spin 0 (Kobzarev, Okun 1962, Pagels 1966)

$$\langle p'|\hat{T}^{a}_{\mu\nu}|p\rangle = 2\,M\left[\,A^a(t,\mu^2)\,\frac{P_\mu P_\nu}{M}\right.$$

$$+ \text{ no spin-related structure}$$

$$\left.+\,D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M} + \bar{c}^a(t,\mu^2)\,M\,g_{\mu\nu}\right]$$

- $\hat{T}^a_{\mu\nu}$ symmetric, gauge invariant, total EMT $\partial_\mu \hat{T}^{\mu\nu} = 0$
- $\sum_a \bar{c}^a(t,\mu^2) = 0$, $A(t) = \sum_a A^a(t,\mu^2)$, $D(t)$ scale invariant
- constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ corresponding structure and form factor absent *

  D-term $\Leftrightarrow$ $D(0) \equiv D$ $\rightarrow$ unconstrained! **Last global unknown!**

$$2P = (p' + p)$$
$$\Delta = (p' - p)$$
$$t = \Delta^2$$

notation:
$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$
$A^q(t) = M_2^q(t)$

* e.g. pion

D-term on same footing as mass, spin, charge, ...

$|N\rangle$ = **strong**–interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\mathbf{em}} = 0$	$\langle N'\vert J^\mu_{\mathbf{em}}\vert N\rangle$	$\longrightarrow$	$G_E(t),\ G_M(t)$	$\longrightarrow$	$\boldsymbol{Q},\ \boldsymbol{\mu}$
weak:	PCAC	$\langle N'\vert J^\mu_{\mathbf{weak}}\vert N\rangle$	$\longrightarrow$	$G_A(t),\ G_P(t)$	$\longrightarrow$	$\boldsymbol{g_A},\ \boldsymbol{g_p}$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N'\vert T^{\mu\nu}_{\mathbf{grav}}\vert N\rangle$	$\longrightarrow$	$A(t),\ J(t),\ D(t)$	$\longrightarrow$	$\boldsymbol{M},\ \boldsymbol{J},\ \boldsymbol{D}$

global properties:

$$
\begin{aligned}
Q_{\text{prot}} &= 1.602176487(40)\times 10^{-19}\text{C}\\
\mu_{\text{prot}} &= 2.792847356(23)\mu_N\\
g_A &= 1.2694(28)\\
g_p &= 8.06(0.55)\\
M &= 938.272013(23)\,\text{MeV}\\
J &= \tfrac{1}{2}\\
\boldsymbol{D} &= \mathbf{?}
\end{aligned}
$$

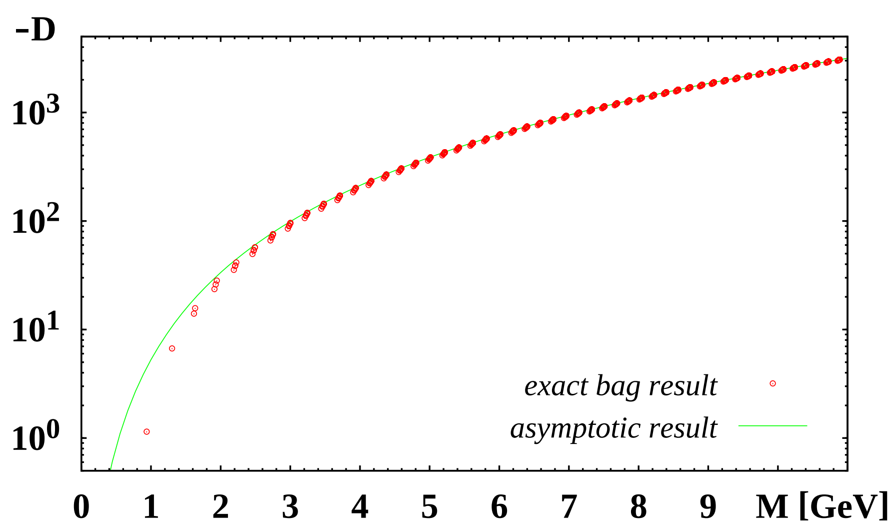
$\hookrightarrow$ $D =$ **"last" global unknown**

Results for D-term from theory (incomplete selection, apologies for omissions)

particle	D-term	method	reference
free spin-0 free spin-$\frac{1}{2}$	-1 0	free Klein-Gordon free Dirac theory	Pagels 1966; Hudson, PS 2017 Donoghue et al 2002, Hudson, PS 2018
pion	-1	soft pion theorem	Novikov, Shifman 1980 Voloshin, Zakharov 1980 Polyakov, Weiss 1999
π K η	-0.97 -0.77 -0.69	χPT to $\mathcal{O}(E^4)$ χPT to $\mathcal{O}(E^4)$ χPT to $\mathcal{O}(E^4)$	Donoghue, Leutwyler 1991
σ	-2.27	NJL model	Clöet, Freese 2019
nucleon	-1.5 (q) -1.0 (q) -3.4 -3.6 -1.1	dispersion relations, at 4 GeV^2 lattice $m_\pi = 450\,$MeV; 4 GeV^2 chiral quark soliton Skyrme model bag model	Pasquini et al 2014 Göckeler et al 2004 Petrov et al 1998 Cebulla et al 2007 Ji et al 1997, Neubelt et al 2019
Roper	-6.7	bag model	Neubelt et al 2019
Δ	-2.6	Skyrme model	Perevalova et al 2016
nucleus	$-0.2 \times A^{7/3}$	liquid drop model	Polyakov 2002
^{16}O ^{40}Ca ^{90}Zr ^{208}Pb	-58 -610 -3300 -19700	Walecka model	Guzey, Siddikov 2006
Q-balls excited N^{th} Q-cloud limit	$-(90-\infty)$ $-$ const N^8 $-$ const$/\varepsilon^2$	non-topolog. soliton radial excitations limit $\varepsilon \to 0$	 Mai, PS 2012 Cantara, Mai, PS 2016

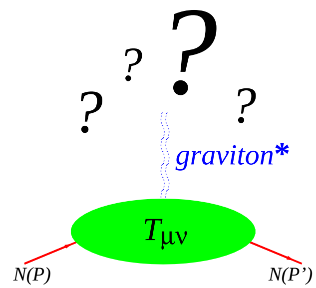
Notable insights

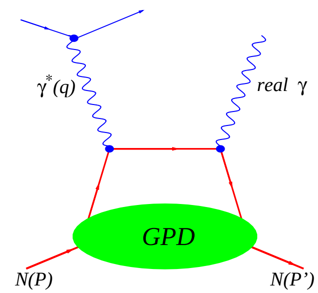
- free spin-$\frac{1}{2}$ Dirac equation $\boldsymbol{D = 0}$ Donoghue et al (2002), Hudson, PS 2018
  D-terms of fermions of dynamical origin! (analog to prediction $g = 2$)
- Goldstone bosons of chiral symmetry breaking $\boldsymbol{D = -1}$
  in chiral limit (pion very close to it, kaon and η deviate more
- D-term always negative! **Why?**
  except for non-interacting fermions
- D-terms of nuclei grow as $A^{7/3}$ with mass number A
  DVCS amplitude $\sim A^{4/3}$, to be tested in experiment
- even stronger growth $D \sim \text{const} \times M^{8/3}$ with mass M for excited states
  in Q-ball system and bag model; can be tested in other models
- D-term sensitive to dynamics $\rightarrow$ interesting to study!

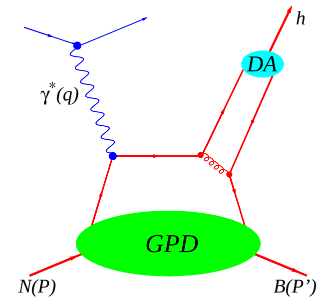


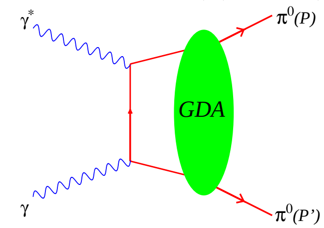
$(-D)$ vs M for the first 4000 states with $J^P = \frac{1}{2}^+$ made of u- and d-quarks in bag model. Neubelt et al, 2019

How to measure?









- direct probe: **graviton** (in principle)
- indirect probe: **photon** (in practice)
  $\to$ generalized parton distributions **GPDs**
  $\to$ generalized distribution amplitudes **GDAs**

- **polynomiality**
  $\int \mathrm{d}x\; x\, H^q(x,\xi,t) = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$
  $\int \mathrm{d}x\; x\, E^q(x,\xi,t) = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$

- **extracting** $A^q(t)$, $B^q(t)$ **difficult**: GPD convoluted in "Compton form factors:"

$$\mathcal{H}(\xi,t,\mu^2) = \sum_q e_q^2 \int \mathrm{d}x \left[\frac{1}{x-\xi-i\varepsilon} - \frac{1}{x+\xi-i\varepsilon}\right] H^q(x,\xi,t,\mu^2) \quad \text{in LO}$$

- **extracting** $\boldsymbol{D(t)}$ more direct through **dispersion relations**
  beam-spin asymmetry in DVCS $\rightsquigarrow$ $\mathcal{I}m\,\mathcal{H}$ JLab, EIC $\to$ Q^2-leverage
  unpolarized DVCS cross section $\rightsquigarrow$ $\mathcal{R}e\,\mathcal{H}$

$$\Re\mathrm{e}\,\mathcal{H}(\xi,t,\mu^2) = \frac{1}{\pi}\,\mathrm{PV}\int \mathrm{d}x \left[\frac{1}{\xi-x} - \frac{1}{\xi+x}\right] \mathcal{I}m\,\mathcal{H}(x,t,\mu^2) - \Delta(t,\mu^2)$$

$$\Delta(t,\mu^2) = 4\sum_q e_q^2 \left[d_1^q(t,\mu^2) + d_3^q(t,\mu^2) + d_5^q(t,\mu^2) + \dots \right]$$

$$\lim_{\mu\to\infty} d_1^Q(t,\mu^2) = d_1(t)\,\frac{N_f}{N_f+4C_F} \qquad \frac{4}{5}\,d_1(t) = D(t) \qquad C_F = \frac{N_c^2-1}{2N_c}$$

$$\lim_{\mu\to\infty} d_1^g(t,\mu^2) = d_1(t)\,\frac{4C_F}{N_f+4C_F}$$

$$\lim_{\mu\to\infty} d_i^a(t,\mu^2) \to 0 \quad \text{for } i = 3,5,\dots$$

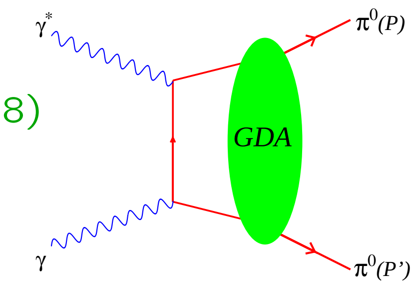
Teryaev hep-ph/0510031
Anikin, Teryaev, PRD76 (2007)
Diehl and Ivanov, EPJC52 (2007)
Radyushkin, PRD83, 076006 (2011)
M.V.Polyakov, PLB 555 (2003) small x

first insights from experiment

- **π^0:** $\gamma\gamma^* \to \pi^0\pi^0$ in e^+e^- Bell data: Masuda et al, PRD 93 (2016)

  $D^Q_{\pi^0} \approx -0.7$ at $\langle Q^2\rangle = 16.6$ GeV^2 Kumano, Song, Teryaev, PRD97 (2018)

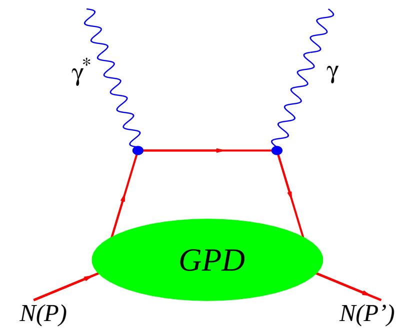
  chiral symmetry: total $D_{\pi^0} \approx -1$ (gluons contribute the rest)

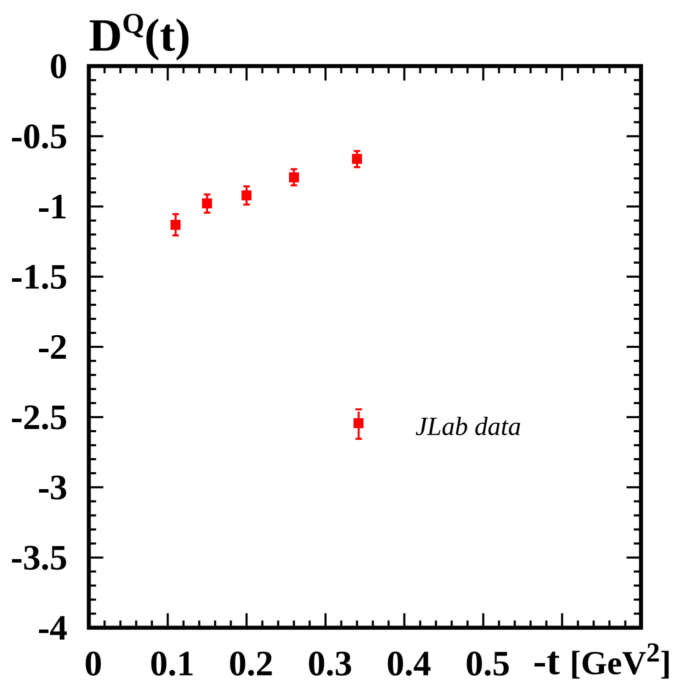


- **proton:** Burkert, Elouadrhiri, Girod, **Nature** 557, 396 (2018)

  JLab data: PRL100 (2008) & PRL115 (2015)

  beam–spin asym. $\to \mathcal{I}m\,\mathcal{H}$ unpol. cross sect. $\to \mathcal{R}e\,\mathcal{H}$





$\Delta(t,\mu^2) \to D^Q(t)$ model-dependent (very first attempt)
K. Kumerički, **Nature** 570, 7759 (2019)
proof of principle: method works

scale dependence of $\Delta(t,\mu^2) \to D^Q(t,\mu^2)$
explore Q^2 **range at EIC**

What will we learn from $D(t)$**?**

first insights from experiment

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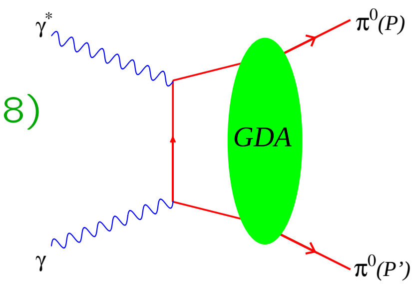

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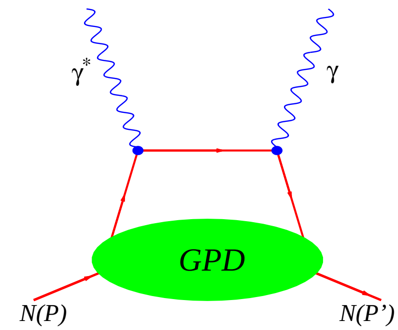

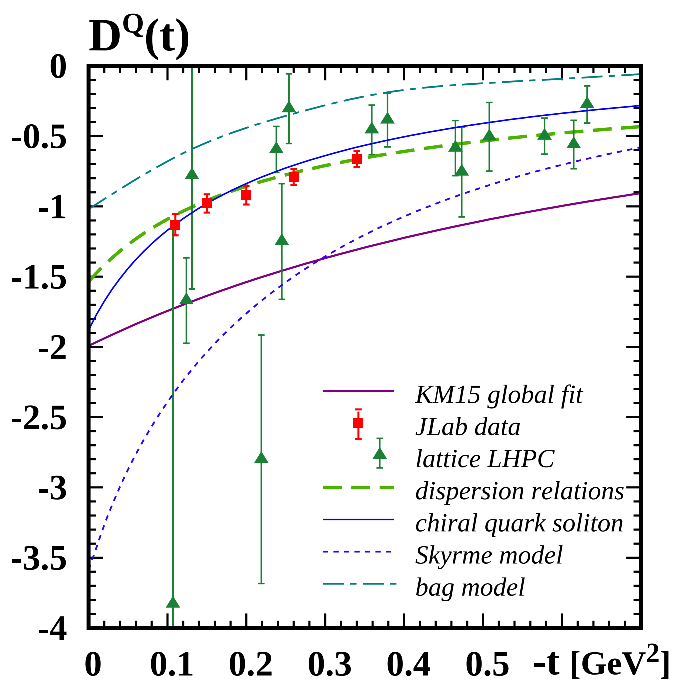

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What will we learn from $D(t)$?

what can we learn from EMT form factors?

- 3D density interpretation in Breit frame $\Delta^\mu = (0, \vec{\Delta})$ and $t = -\vec{\Delta}^2$

- **static EMT** $T_{\mu\nu}(\vec{r}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{-i\vec{\Delta}\vec{r}} \langle P'|\hat{T}_{\mu\nu}|P\rangle$ M.V.Polyakov, PLB 555 (2003) 57

- analog to electric form factor, with the same reservation Sachs, PR126 (1962) 2256

$$G_E(t) = \int d^3\vec{r}\, \rho_E(r)\, e^{-i\vec{\Delta}\vec{r}} \quad \text{for proton}$$

$$= 1 + \tfrac{1}{6}\, t \underbrace{\langle r_{ch}^2\rangle}_{\approx(0.8\ldots\,\text{fm})^2} + \ldots \rightarrow \text{mean square charge radius } \langle r_{ch}^2\rangle = \int d^3\vec{r}\, r^2 \rho_E(\vec{r}) = 6\, G_E'(0)$$

- important: we cannot measure the charge (or other) density inside the nucleon
  we can measure form factors(!) and we can interpret them(!)

- reservation:
  2D densities: exact partonic probability densities, Burkardt 2000, for all particles
  3D densities: not exact, mechanical response functions ($\neq$ probabilities!)
  valid for $r \gtrsim \lambda_{\text{Compt}} = \frac{\hbar}{mc}$, relativistic corrections

reservation known since Sachs (1962). Discussed in detail e.g. in:
Belitsky & Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2
X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not a big effect)
G. Miller, PRC80 (2009) 045210 (toy model, dramatic effect)
Hudson, PS PRD **96** (2017) 114013 (not a big effect)
Jaffe, e-Print: 2010.15887 (most recent)

illustration of reservation

$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2} m^2 \Phi^2$ free point-like spin-0 particle Hudson, PS 2017

- $\boldsymbol{A(t) = -D(t) = 1}$ $\rightarrow$ energy density:

$$\boldsymbol{T_{00}(\vec{r})} = m^2 \int \frac{d^3\Delta}{E(2\pi)^3} \, e^{-i\vec{\Delta}\vec{r}} \left[A(t) - \frac{t}{4m^2}(A(t) + D(t)) \right] = \frac{m}{\sqrt{1 - \vec{\nabla}^2/(4m^2)}} \, \delta^{(3)}(\vec{r})$$

in Breit frame $E = E' = \sqrt{m^2 + (\vec{\Delta}/2)^2}$

- expected result $\boldsymbol{T_{00}(\vec{r})} = m\, \delta^{(3)}(\vec{r})$ for $m \to \infty$ ... m large with respect to what?

- let's give particle a finite size R: $\boldsymbol{T_{00}(\vec{r})_{\rm true}} \overset{\text{e.g.}}{=} m \, \frac{e^{-r^2/R^2}}{\pi^{3/2} R^3}$ (i.e. "smeared out" δ-function)

$\langle r_E^2 \rangle = \boldsymbol{\langle r_E^2 \rangle_{\rm true}} \left(1 + \boldsymbol{\delta_{\rm rel}} \right)$ with $\boldsymbol{\delta_{\rm rel}} = \boldsymbol{\frac{1}{2m^2R^2}} \ll \boldsymbol{1}$ (it is $\langle r_E^2 \rangle_{\rm true} = \frac{3}{2} R^2$ here)

numerically $\underbrace{\text{pion}}_{220\,\%}$, $\underbrace{\text{kaon}}_{25\,\%}$, $\underbrace{\text{nucleon}}_{3\,\%}$, $\underbrace{\text{deuterium}}_{1\times10^{-3}}$, $\underbrace{^4\text{He}}_{5\times10^{-4}}$, $\underbrace{^{12}\text{C}}_{3\times10^{-5}}$, $\underbrace{^{20}\text{Ne}}_{6\times10^{-6}}$, $\underbrace{^{56}\text{Fe}}_{5\times10^{-7}}$, $\underbrace{^{132}\text{Xe}}_{6\times10^{-8}}$, $\underbrace{^{208}\text{Pb}}_{2\times10^{-8}}$

- for nucleon in large-N_c limit ($M \sim N_c$, $R \sim N_c^0$) $\rightarrow$ $\boldsymbol{\delta_{\rm rel}} \sim \frac{1}{N_c^2} \overset{!!}{\ll} 1$

"$1/N_c$ only small parameter in QCD at all energies" (S. Coleman, Aspects of Symmetry)

$\Rightarrow$ formulae correct, interpretation subject to small corrections

static EMT $T_{\mu\nu}(\vec{r}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{-i\vec{\Delta}\,\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$

$$\int d^3r\; T_{00}(\vec{r}) = M \qquad \text{known}$$

$$\int d^3r\; \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(\vec{r},\, \vec{s}) = \frac{1}{2} \qquad \text{known}$$

$$-\frac{2}{5}\, M \int d^3r\, \left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right) T_{ij}(\vec{r}) \equiv D \qquad \textbf{new!}$$

- **stress tensor** $T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}$

  $s(r)$ related to distribution of *shear forces*
  $p(r)$ distribution of *pressure* inside hadron
  $\Big\} \longrightarrow$ **"mechanical properties"**

- **relation to stability**: EMT conservation $\Leftrightarrow$ $\partial^\mu \hat{T}_{\mu\nu} = 0$ $\Leftrightarrow$ $\nabla^i T_{ij}(\vec{r}) = 0$

  $\hookrightarrow$ necessary condition for stability $\int_0^\infty dr\; r^2\, p(r) = 0$ (von Laue, 1911)

  $$D = -\frac{16\pi}{15}\, m \int_0^\infty dr\; r^4 s(r) = 4\pi m \int_0^\infty dr\; r^4\, p(r) \rightarrow \text{related to internal forces}$$

consequences from EMT conservation

- EMT conservation $\partial^\mu \hat{T}_{\mu\nu} = 0 \;\Rightarrow\;$ static EMT $\nabla^i T_{ij} = 0$

  $\rightarrow\;\; \frac{2}{3}\, s'(r) + \frac{2}{r}\, s(r) + p'(r) = 0$

- interesting insight: imagine we would have $s(r) = 0$

  $\rightarrow$ then $p'(r) = 0$ and $p(r) =$ constant (boring situation)

  $s(r)$ is responsible for structure, important(!) Polyakov, Lorcé

- integrate $\int_0^\infty dr\; r^3 \left(\frac{2}{3}\, s'(r) + \frac{2}{r}\, s(r) + p'(r)\right) = 0$

  $\rightarrow\;\; \int_0^\infty dr\; r^2 p(r) = 0$ von Laue condition 1911

mechanical radius

- $T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}$ = symmetric 3×3 matrix $\to$ diagonalize:

  $\frac{2}{3}\,s(r) + p(r) =$ normal force (eigenvector $\vec{e}_r$)

  $-\frac{1}{3}\,s(r) + p(r) =$ tangential force ($\vec{e}_\theta$, $\vec{e}_\phi$, degenerate for spin 0 and $\frac{1}{2}$)

- mechanical stability $\Leftrightarrow$ normal force directed towards outside

  $\Leftrightarrow T^{ij} e_r^j\,dA = \underbrace{[\frac{2}{3}\,s(r) + p(r)]}_{>0} e_r^i\,dA \quad \Rightarrow \quad D < 0$ **(proof!)** Perevalova et al (2016)

- define: $\langle r^2 \rangle_{\rm mech} = \frac{\int d^3r\ r^2[\frac{2}{3}\,s(r) + p(r)]}{\int d^3r\ [\frac{2}{3}\,s(r) + p(r)]} = \frac{6D(0)}{\int_{-\infty}^{0}{\rm d}t\,D(t)}$ "anti-derivative" vs $\langle r^2_{\rm ch} \rangle_p = \frac{6\,G'_E(0)}{G_E(0)}$

- advantages:

  in chiral limit $\langle r^2 \rangle_{\rm mech}$ finite vs $\langle r^2_{\rm ch} \rangle$ divergent (better concept)

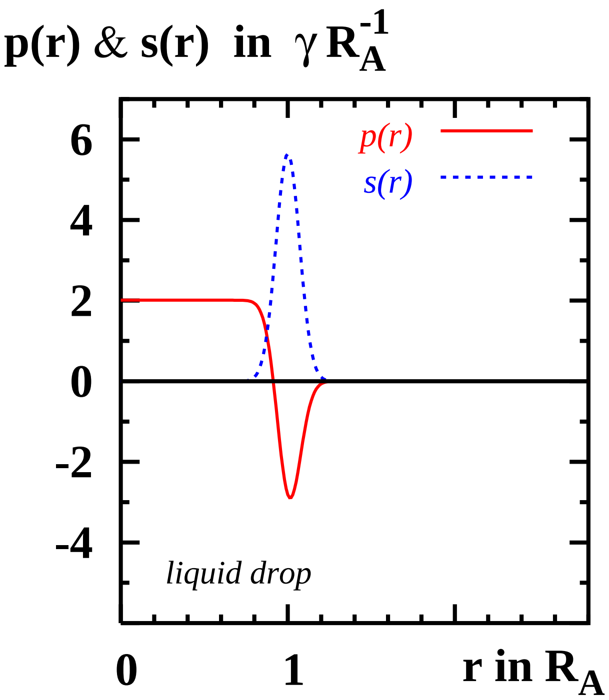
  neutron $\langle r^2 \rangle_{\rm mech}$ same as proton(!) vs $\langle r^2_{\rm ch} \rangle = -0.11\,{\rm fm}^2 \neq$ neutron size

  unknown

- prediction: nucleon $\langle r^2 \rangle_{\rm mech} \approx 0.75\,\langle r^2_{\rm ch} \rangle$ in chiral quark soliton model

visualization of concepts in models

liquid drop model of nucleus



radius $R_A = R_0\, A^{1/3}$, $m_A = m_0\, A$

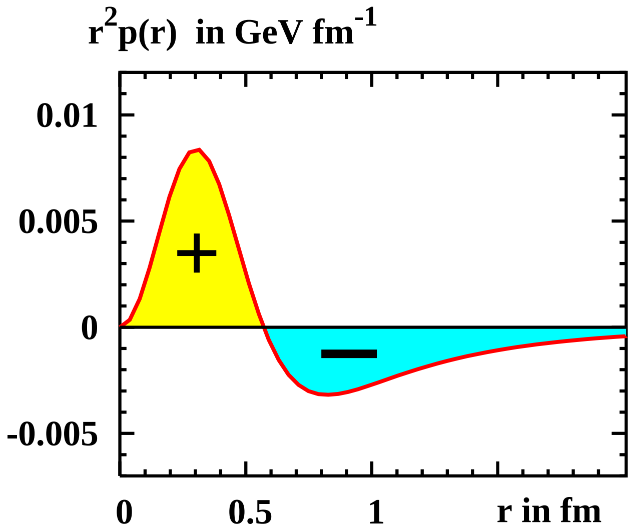
surface tension $\gamma = \frac{1}{2} p_0 R_A$, $s(r) = \gamma\, \delta(r - R_A)$

pressure $p(r) = p_0\, \Theta(R_A - r) - \frac{1}{3} p_0 R_A\, \delta(r - R_A)$

D-term $\boldsymbol{D = -\frac{4\pi}{3}\, m_A\, \gamma\, R_A^4 \approx -0.2\, A^{7/3}}$

M.V.Polyakov PLB555 (2003)

chiral quark soliton model of nucleon



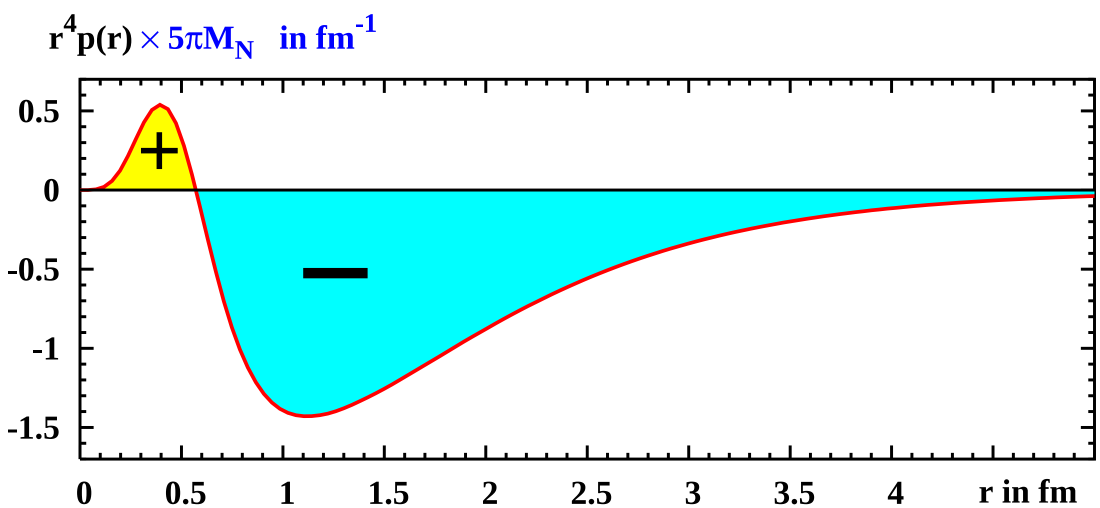
- $p(0) = 0.23\,\mathrm{GeV/fm^3} \approx 4 \times 10^{34}\,\mathrm{N/m^2}$

  $\gtrsim$ 10-100×(pressure in center of neutron star)

- $p(r) = 0$ at $r = 0.57\,\mathrm{fm}$ change of sign in pressure

- $p(r) = \left(\frac{3g_A^2}{8\pi f_\pi}\right)^2 \frac{1}{r^6}$ at large r in chiral limit $m_\pi \to 0$

  Goeke et al, PRD75 (2007) 094021



recall: $\int_0^\infty \mathrm{d}r\, \boldsymbol{r^2\, p(r)} = 0$

$\boldsymbol{D} = 4\pi M \int_0^\infty \mathrm{d}r\, \boldsymbol{r^4\, p(r)} < 0$

$\rightarrow$ **negative sign of D** $\Leftrightarrow$ **stability** (necessary condition)

balance of forces

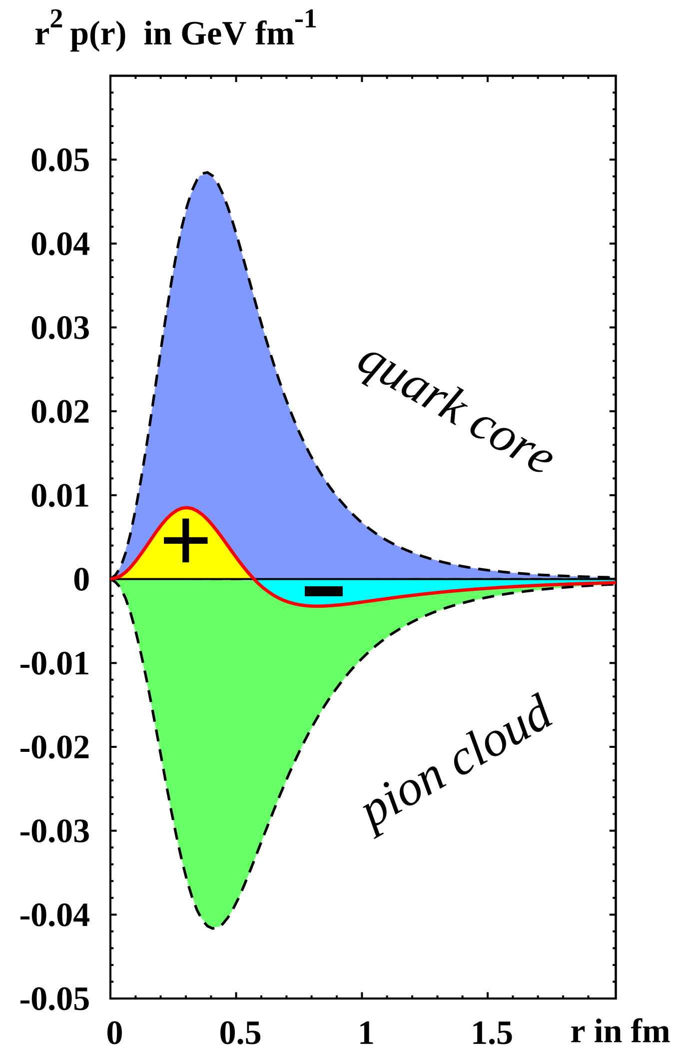
- question: how do the forces balance inside the nucleon?
- answer in **model:** strong cancellation of repulsive forces due to quark core, and attractive forces from pion cloud (soliton field)

  compare to $V_{\text{conf}}(r) \approx k\,r$ with $k \approx 1\,\text{GeV/fm}$
  forces inside nucleon $\ll$ string tension
- in principle answer from **QCD:** forces due to quarks and gluons

  experiment (JLab, EIC)

  lattice Shanahan, Detmold 2019
  $\mu = 2\,\text{GeV}$, $m_\pi = 450(5)\,\text{MeV}$

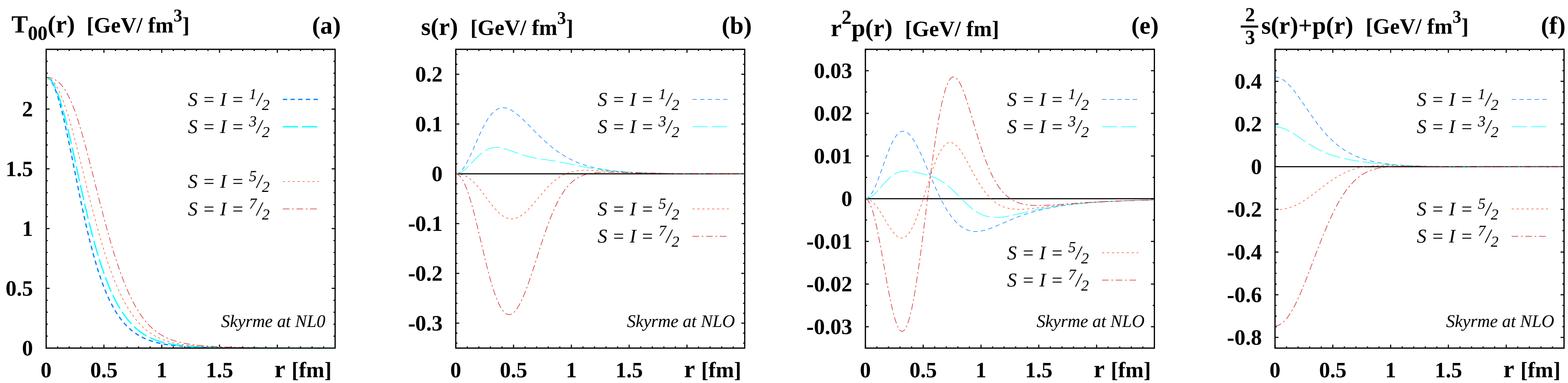
  account for role of $\bar{c}^a(t,\mu)$
  Polyakov, Son JHEP 09 (2018) 156



in chiral quark soliton model
chiral symmtry breaking ✓
realization of QCD in large-N_c ✓
good model (but it is a model)
Goeke et al, PRD75 (2007)

Skyrme model nucleon, Δ, large-N_c artifacts Witten 1979

- in large N_c baryons = rotational excitations of soliton with $S = I = \underbrace{\frac{1}{2}, \frac{3}{2}}_{\text{observed}}, \underbrace{\frac{5}{2}, \dots}_{\text{artifacts}}$



$$M_B = M_{\text{sol}} + \frac{S(S+1)}{2\Theta}$$

nucleon $s(r) \neq \gamma\delta(r-R)$
Δ much more diffuse

$$\int_0^\infty dr\, r^2 p(r) = 0$$

stability requires:
$p(r) > 0$ in center,
negative outside
okay for nucleon, Δ
$\Longrightarrow$ implies $D < 0$

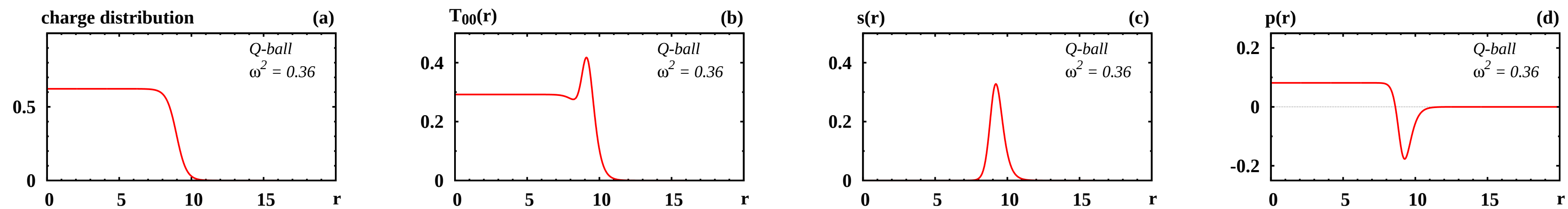
mechanical stability
$T^{ij}\, da^j \geq 0$
$\Leftrightarrow \quad \frac{2}{3}\, s(r) + p(r) \geq 0$
artifacts do not satisfy!
$\Rightarrow$ have positive D-term!
So do not exist!
dynamical understanding
Perevalova et al (2016)

$\Rightarrow$ particles with positive D unphysical!!!

Q-balls $\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V, \; V = A\,(\Phi^*\Phi) - B\,(\Phi^*\Phi)^2 + C\,(\Phi^*\Phi)^3$

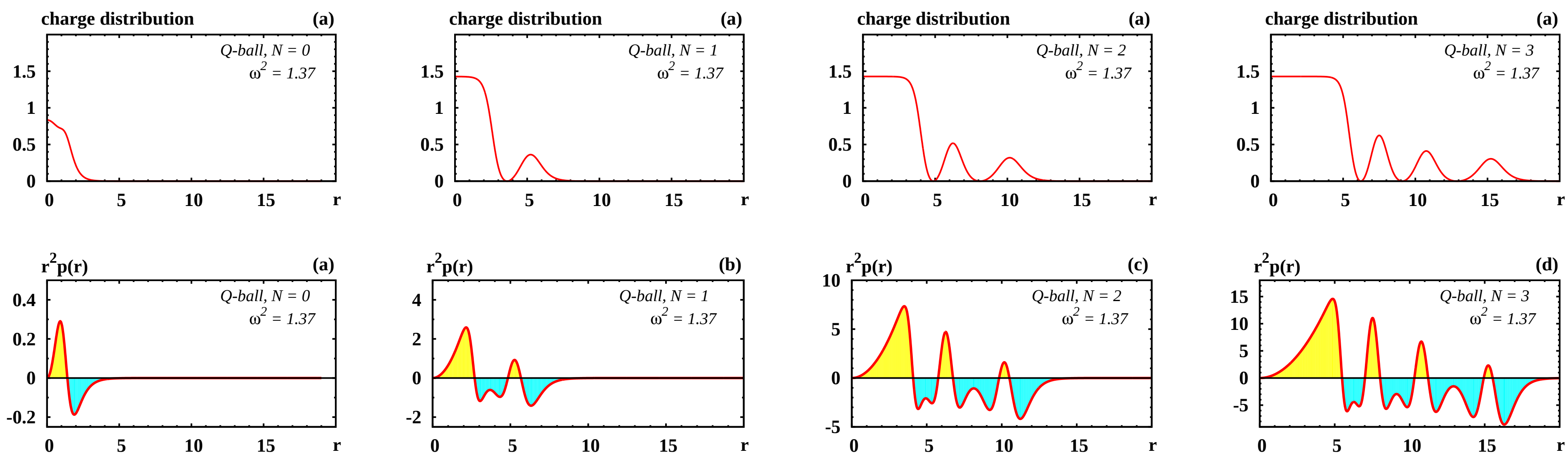
global U(1) symmetry, solution $\Phi(t,\vec{r}) = e^{i\omega t}\,\phi(r)$

- ground state properties for large Q-ball



- excitations: $N = 0$ ground state, $N = 1$ first excited state, etc Volkov, Wohnert 2002; Mai, PS 2012

charge density exhibits N shells, $p(r)$ exhibits $(2N+1)$ zeros

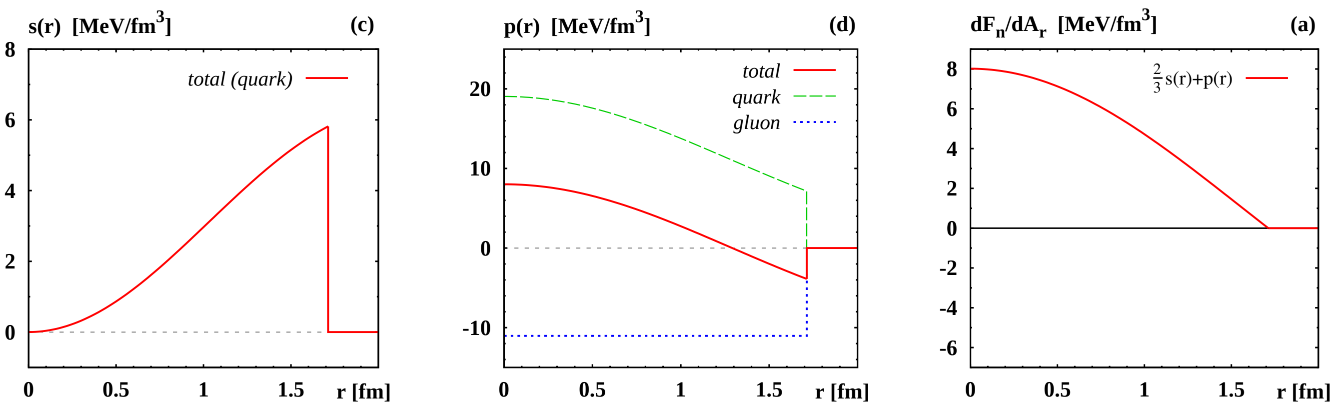


excited states unstable, but $\int_0^\infty \mathrm{d}r\; r^2 p(r) = 0$ always valid, and D-term always negative!

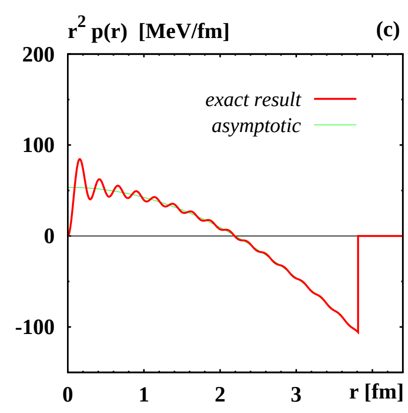
bag model Neubelt, Sampino, Hudson, Tezgin, PS, PRD101 (2020) 034013

- free quarks + boundary condition, formulated in large-N_c
- $T^{\mu\nu}(r) = T^{\mu\nu}_{\text{quarks}}(r) + T^{\mu\nu}_{\text{bag}}(r)$

  $T^{\mu\nu}_{\text{bag}}(r) = B\,\Theta(R-r)\,g^{\mu\nu}$ binding effect ("mimics gluons" Jaffe & Ji 1991)
- all densities defined with Θ-functions, assume non-zero values at $r = R$



- only exception:
  the normal force = $\frac{2}{3}\,s(r) + p(r) > 0$ for $r < R$, becomes exactly zero at $r = R$
- this is how one determines the radius of a neutron star:
  solve Tolman-Oppenheimer-Volkoff equations with an "equation of state"
  where "radial pressure" $\frac{2}{3}\,s(r) + p(r)$ turns negative, define "end of the system"
- excitated states different pattern than Q-balls:
  $p(r)$ has one node (here 3163th excited state)
  but $D \sim \text{const} \times M^{8/3}$ bag & Q-balls
  deeper reason?



Summary & Outlook

- **GPDs, GDAs** $\rightarrow$ form factors of **energy momentum tensor**
- **D-term**: last unknown global property. Important to know!
- D-term of fermions: generated dynamically (free Dirac theory $D = 0$)
- theory: D negative (Goldstone bosons, models, lattice, disersion relations)
- early phenomenological results: proton (JLab) DVCS, π^0 (Belle $\gamma^*\gamma \to \pi^0\pi^0$)
- interpretation: pressure, forces (and more)
- application: **visualization of forces!**
- mechanical radius: true size of hadrons (especially neutron!)
- proof that $D < 0$ (based on mechanical concepts)
- connection to thermodynamics, pressure, temperature, transport phenomena?
- only small selection of topics

Thank you!