Insights into hadron structure from superconformal light-front holographic QCD



Stan Brodsky, Hans G. Dosch, Alexandre Deur, Tianbo Liu, Raza Sabbir Sufian, Marina Nielsen and Liping Zou

Motivation: "While there has been great progress in Lattice Gauge Theory, there is no quantitative analytic understanding yet of the mass gap and confinement" From Igor Klebanov's talk

- The light-front (LF) frame independent Hamiltonian based on Dirac's front form of relativistic dynamics is a rigorous formalism for computing bound states in QCD: Is is the natural formalism for describing partonic physics and also maps directly to AdS and superconformal eigenvalue equations
- Recent insights into the nonperturbative structure of QCD based on LF quantization and its holographic embedding have lead to effective semiclassical bound-state equations for arbitrary spin: LFHQCD
- Confinement potential for baryons, mesons and tetraquarks are uniquely determined by an underlying superconformal algebraic structure leading to unsuspected connections across the entire mass spectrum of hadrons
- Breaking of scale invariance in the LF Hamiltonian which emerges from the superconformal structure leads simultaneously to a massless pion <u>and</u> a massive proton
- Phenomenological extension incorporates the structure of pre-QCD Generalized Veneziano models useful to describe form factors and parton distributions including the sea component of the proton

Review in:

- S. J. Brodsky, GdT, H.G. Dosch, J. Erlich, Phys. Rept. 584, 1 (2015) [hep-ph/9705477]
- S. J. Brodsky, GdT, H. G. Dosch, arXiv:2004.07756 [hep-ph]

Contents

1	Half-integer-spin wave equations in AdS and LF holographic embedding	4
2	Superconformal algebraic structure and emergence of a mass scale	7
3	Light-front mapping and baryons	10
4	Superconformal meson-baryon-tetraquark symmetry	11
5	Longitudinal dynamics in LFHQCD	13
6	Form factors, parton distributions and intrinsic quark sea	17
7	Outlook	25

1 Half-integer-spin wave equations in AdS and LF holographic embedding

[J. Polchinski and M. J. Strassler, JHEP 0305, 012 (2003)]
[GdT and S. J. Brodsky, PRL 94, 201601 (2005)]
[Z. Abidin and C. E. Carlson, PRD 79, 115003 (2009)]
[GdT, H.G. Dosch and S. J. Brodsky, PRD 87, 075004 (2013)]

• AdS₅ is a 5-dim space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space



$$ds^2\!=\frac{R^2}{z^2}\left(dx_\mu dx^\mu \!-dz^2\right)$$

- Isomorphism of SO(4,2) conformal group with the group of isometries of AdS $_5$
- Physical interpretation: holographic variable z inverse of energy scale of a probe
- To describe the proton and its excited states we start from the effective AdS action for a half-integer spin $J = T + \frac{1}{2}$ RS spinor $\Psi_{N_1 \cdots N_T}$ with AdS mass μ and effective potential $\rho(z)$

$$S_{eff} = \frac{1}{2} \int d^{d}x \, dz \, \sqrt{g} \, g^{N_{1} \, N_{1}'} \cdots g^{N_{T} \, N_{T}'} \\ \left[\overline{\Psi}_{N_{1} \cdots N_{T}} \left(i \, \Gamma^{A} \, e^{M}_{A} \, D_{M} - \mu - \rho(z) \right) \Psi_{N_{1}' \cdots N_{T}'} + h.c. \right]$$
(1)

where $\sqrt{g} = (R/z)^{d+1}$ and the covariant derivative D_M includes the affine and spin connection • e_A^M is the vielbein, $e_A^M = (z/R)\delta_A^M$, and Γ^A tangent space Dirac matrices $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$ • In holographic QCD a nucleon is described by a *z*-dependent wave function $\psi(z)$ and a plane wave and spinors along Minkowski coordinates

$$\Psi^T(x,z) = e^{iP \cdot x} u_{\nu_1 \cdots \nu_T}(P) \left(\frac{R}{z}\right)^{T-d/2} \psi(z)$$

with invariant mass $P_{\mu}P^{\mu}=M^2$

• From the RS action in AdS (1) follows the coupled equations for the chiral components $\psi_{\pm}(z)$

$$-\frac{d}{dz}\psi_{-} - \frac{\nu + \frac{1}{2}}{z}\psi_{-} - V(z)\psi_{-} = M\psi_{+}$$

$$\frac{d}{dz}\psi_{+} - \frac{\nu + \frac{1}{2}}{z}\psi_{+} - V(z)\psi_{+} = M\psi_{-}$$
(2)

with $|\mu R|=\nu+\frac{1}{2}$ and $V(z)=\frac{R}{z}\rho(z)$

• Non-trivial geometry of AdS encodes the higher-spin kinematics, including the constraints required to eliminate lower-spin from the symmetric RS spinor $\Psi_{N_1\cdots N_T}$ with $J = T + \frac{1}{2}$

Light-front mapping

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

• LF Hamiltonian from Poincaré group invariant $P^2 = P_{\mu}P^{\mu} = P^-P^+ - \mathbf{P}_{\perp}^2$ $P^2 |\psi(P)\rangle = M^2 |\psi(P)\rangle, \qquad |\psi\rangle = \sum \psi_n |n\rangle$

becomes in QCD a relativistic semiclassical LF wave equation in the limit $m_q
ightarrow 0$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

where ζ is the impact invariant transverse variable $\zeta^2 = x(1-x) \mathbf{b}_{\perp}^2$

• Mapping to the light front $z \to \zeta$, system of linear Eqs in AdS (2) is equivalent to second order Eqs:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U^+(\zeta)\right)\psi_+ = M^2\psi_+$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4(L+1)^2}{4\zeta^2} + U^-(\zeta)\right)\psi_- = M^2\psi_-$$
(3)

the semiclassical QCD LF WE with ψ_+ and ψ_- corresponding to LF orbital L and L+1 with

$$U^{\pm}(\zeta) = V^{2}(\zeta) \pm V'(\zeta) + \frac{1+2L}{\zeta}V(\zeta), \qquad L = \nu,$$

• $V(\zeta)$ plays the role of a superpotential and its form is completely fixed by superconformal QM

2 Superconformal algebraic structure and emergence of a mass scale

[V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A 34, 569 (1976)]
[S. Fubini and E. Rabinovici, NPB 245, 17 (1984)]
[S. J. Brodsky, GdT and H. G. Dosch, Phys. Lett. B 729, 3 (2014)]
[GdT, H. G. Dosch and S. J. Brodsky, PRD 91, 045040 (2015)]
[H. G. Dosch, GdT, and S. J. Brodsky, PRD 91, 085016 (2015)]

• Superconformal algebra underlies in LFHQCD the scale invariance of the QCD Lagrangian: Its breaking leads to the emergence of a scale in the Hamiltonian keeping the action conformal invariant





Supersymmetric Quantum Mechanics

[E. Witten, NPB 188, 513 (1981)]

• SUSY QM contains two fermionic generators Q and Q^\dagger and a bosonic generator, the Hamiltonian H

 $\frac{1}{2} \{Q, Q^{\dagger}\} = H$ $\{Q, Q\} = \{Q^{\dagger}, Q^{\dagger}\} = 0, \quad [Q, H] = [Q^{\dagger}, H] = 0$

which closes under the graded algebra ${\it sl}(1/1)$

- Since $[Q^{\dagger}, H] = 0$, the states $|E\rangle$ and $Q^{\dagger}|E\rangle$ for $E \neq 0$ are degenerate, but for E = 0 we can have the trivial solution $Q^{\dagger}|E = 0\rangle = 0$ (the pion)
- Matrix representation of SUSY generators Q, Q^{\dagger} and H

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \qquad Q^{\dagger} = \begin{pmatrix} 0 & 0 \\ q^{\dagger} & 0 \end{pmatrix}, \qquad H = \frac{1}{2} \begin{pmatrix} q q^{\dagger} & 0 \\ 0 & q^{\dagger} q \end{pmatrix}$$

• For a conformal theory (f dimensionless)

$$q = -\frac{d}{dx} + \frac{f}{x}, \qquad q^{\dagger} = \frac{d}{dx} + \frac{f}{x}$$

• Conformal graded-Lie algebra has in addition to the Hamiltonian H and supercharges Q and Q^{\dagger} , a new operator S related to the generator of conformal transformations $\frac{1}{2}\{S, S^{\dagger}\} = K$

$$S = \left(\begin{array}{cc} 0 & x \\ 0 & 0 \end{array}\right), \qquad S^{\dagger} = \left(\begin{array}{cc} 0 & 0 \\ x & 0 \end{array}\right)$$

and leads to a conformal enlarged algebra [Haag, Lopuszanski and Sohnius (1974)]

• Following Fubini and Rabinovici we define the fermionic generator $R = Q + \lambda S$, $[\lambda] = \text{GeV}^2$, $\{R_\lambda, R_\lambda^\dagger\} = G_\lambda$ $\{R_\lambda, R_\lambda\} = \{R_\lambda^\dagger, R_\lambda^\dagger\} = 0$, $[R_\lambda, G_\lambda] = [R_\lambda^\dagger, G_\lambda] = 0$

which also closes under the graded algebra sl(1/1):

• In a 2×2 matrix representation the Hamiltonian equation $G |\phi\rangle = E |\phi\rangle$ leads to the equations

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f - \lambda + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2}\right)\phi_1 = E\phi_1$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f + \lambda + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2}\right)\phi_2 = E\phi_2$$
(4)

Light-front mapping and baryons 3

[GdT, H.G. Dosch and S. J. Brodsky, PRD 91, 045040 (

 Upon LF mapping to the superconformal eigenval equations (4)

$$x \mapsto \zeta, \quad E \mapsto M^2, \quad f \mapsto L + \frac{1}{2}$$

$$\phi_1 \mapsto \psi_-, \phi_2 \mapsto \psi_+$$

we recover the nucleon bound-state equations (3)

$$\begin{array}{l} \begin{array}{c} \begin{array}{c} mapping and baryons \\ and S. J. Brodsky, PRD 91, 045040 (2015)] \\ g \text{ to the superconformal eigenvalue} \\ \phi & E \mapsto M^2, \quad f \mapsto L + \frac{1}{2} \\ -, \quad \phi_2 \mapsto \psi_+ \\ \text{ucleon bound-state equations (3)} \end{array} \qquad \begin{array}{c} \begin{array}{c} & & & & & n = 2 \\ & & & n = 3 \\ & & & n = 2 \\ \end{array} \qquad \begin{array}{c} n = 3 \\ & & n = 2 \\ & & & n = 1 \\ \end{array} \qquad \begin{array}{c} & & n = 0 \\ & & & n = 2 \\ \end{array} \qquad \begin{array}{c} & & n = 1 \\ & & & n = 0 \\ & & & n = 3 \\ \end{array} \qquad \begin{array}{c} & & n = 2 \\ & & & n = 1 \\ \end{array} \qquad \begin{array}{c} n = 0 \\ & & & n = 3 \\ \end{array} \qquad \begin{array}{c} & & n = 2 \\ & & & n = 1 \\ \end{array} \qquad \begin{array}{c} & n = 0 \\ & & & n = 3 \\ \end{array} \qquad \begin{array}{c} & & n = 2 \\ & & n = 1 \\ \end{array} \qquad \begin{array}{c} & n = 0 \\ & & n = 3 \\ \end{array} \qquad \begin{array}{c} & & n = 2 \\ & & n = 1 \\ \end{array} \qquad \begin{array}{c} & n = 0 \\ & & n = 3 \\ \end{array} \qquad \begin{array}{c} & n = 2 \\ \end{array} \qquad \begin{array}{c} & n = 1 \\ & & n = 0 \\ \end{array} \qquad \begin{array}{c} & & n = 0 \\ & & n = 3 \\ \end{array} \qquad \begin{array}{c} & n = 2 \\ \end{array} \qquad \begin{array}{c} & n = 1 \\ & & n = 0 \\ \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \begin{array}{c} & & n = 3 \\ \end{array} \qquad \begin{array}{c} & n = 2 \\ \end{array} \qquad \begin{array}{c} & n = 1 \\ \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \end{array} \qquad \begin{array}{c} & n = 0 \\ \end{array} \qquad \end{array} \qquad \end{array} \qquad \begin{array}{c} & n$$

with
$$U^+ = \lambda^2 \zeta^2 + 2\lambda (L+1) ~~ {\rm and} ~~ U^- = \lambda^2 \zeta^2 + 2\lambda L$$

• Eigenvalues

$$M^2 = 4\lambda(n+L+1)$$

Eigenfunctions

$$\psi_{+}(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\lambda\zeta^{2}/2} L_{n}^{L}(\lambda\zeta^{2}), \quad \psi_{-}(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\lambda\zeta^{2}/2} L_{n}^{L+1}(\lambda\zeta^{2})$$

4 Superconformal meson-baryon-tetraquark symmetry

[H.G. Dosch, GdT, and S. J. Brodsky, PRD 91, 085016 (2015)]

• Upon substitution in the superconformal equations (4)

$$x \mapsto \zeta, \quad E \mapsto M^2,$$

$$\lambda \mapsto \lambda_B = \lambda_M, \quad f \mapsto L_M - \frac{1}{2} = L_B + \frac{1}{2}$$

$$\phi_1 \mapsto \phi_M, \quad \phi_2 \mapsto \phi_B$$

we find the LF bound-state equations

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M (L_M - 1)\right)\phi_M = M^2 \phi_M \\ \left(-\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B (L_N + 1)\right)\phi_B = M^2 \phi_B$$
(6)

 $M^2/4\lambda$

 $N^{\frac{5}{2}}$

b₁

 $N^{\frac{3}{2}}$

π

 $N^{\overline{2}}$

 $N^{\frac{3}{2}}$

 π_2

b₃

 π_4

- Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $L_M = L_B + 1$
- L_M is the LF angular momentum between the quark and antiquark in the meson and L_B between the active quark and spectator cluster in the baryon: effective quark-diquark approximation

• Special role of the pion as a unique state of zero energy

$$R^{\dagger}|M,L\rangle = |B,L-1\rangle, \quad R^{\dagger}|M,L=0\rangle = 0$$

- Hadron quantum numbers determined from the pion
- Spin-dependent Hamiltonian for mesons and baryons with internal spin ${\cal S}$

[S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé (2016)]

$$G = \{R_{\lambda}^{\dagger}, R_{\lambda}\} + 2\lambda S \qquad S = 0, 1$$

• Supersymmetric 4-plet

$$M_M^2 = 4\lambda (n + L_M) + 2\lambda S$$

$$M_B^2 = 4\lambda (n + L_B + 1) + 2\lambda S$$

$$M_T^2 = 4\lambda (n + L_T + 1) + 2\lambda S$$

• Expected accuracy $1/N_C^2 \sim 10\%$





5 Longitudinal dynamics in LFHQCD

• BdT LFWF ansatz [S. J. Brodsky, GdT (2009)]

$$\psi_{\overline{q}q/\pi}(x,\mathbf{k}_{\perp}) \sim \frac{1}{\sqrt{x(1-x)}} e^{-\frac{1}{2\lambda}\frac{\mathbf{k}_{\perp}^2}{x(1-x)}} \to \frac{1}{\sqrt{x(1-x)}} e^{-\frac{1}{2\lambda}\left(\frac{\mathbf{k}_{\perp}^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

• Quark masses treated as effective masses in LF Hamiltonian ($m_{u,d} = 46 \text{ MeV}, m_s = 357 \text{ MeV}$) [S. J. Brodsky, GdT, H. G. Dosch, J. Erlich (2015)]

$$\Delta M^2[m_1, \cdots, m_n] = \frac{\lambda^2}{F} \frac{\mathrm{d}F}{\mathrm{d}\lambda}$$

with

$$F[\lambda] = \int_0^1 \cdots \int_0^1 \mathrm{d}x_1 \cdots \mathrm{d}x_n \, e^{-\frac{1}{\lambda} \left(\sum_{i=1}^n \frac{m_i^2}{x_i}\right)} \delta(\sum_{i=1}^n x_i - 1)$$

Hadron scale determined from different light hadron channels including all radial and orbital excitations
 [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé (2016)]

0.7
$$\sqrt{\lambda}$$
 GeV
0.5 $\sqrt{\lambda} = 0.523 \pm 0.024$ GeV
N Δ Λ Σ $\Sigma^* \Xi$ $\Xi^* \Omega^- \pi \rho K K^* \phi$
0.3

Longitudinal dynamics

- [G. 't Hooft, NPB B 75, 461 (1974)]
- [S. S. Chabysheva and J. R. Hiller, Annals Phys 337, 143 (2013)]
- [A. P. Trawiński, S. D. Głazek, S. J. Brodsky, GdT and H. G. Dosch, PRD 90, 074017 (2014)]
- [Y. Li, P. Maris, X. Zhao and J. P. Vary, PLB 758, 118 (2016)]
- [A. B. Sheckler and G. A. Miller, [arXiv:2101.00100 [hep-ph]]
- LMZV longitudinal potential constrained to recover spherical symmetry of LF transverse oscillator in the non-relativistic limit of heavy quark masses

$$\left[-\partial_x \left(x(1-x)\partial_x\right) - \gamma\right] + \frac{1}{4} \left(\frac{\alpha^2}{x} + \frac{\beta^2}{1-x}\right)\right] \chi = \nu^2 \chi$$

where $\alpha = \frac{2m_1(m_1+m_2)}{\lambda}, \ \beta = \frac{2m_2(m_1+m_2)}{\lambda}, \ \gamma = \frac{(m_1+m_2)^2}{\lambda}, \ \nu = \frac{M_L(m_1+m_2)}{\lambda}$

• Eigenvalues

$$\nu_{\alpha,\beta,\ell}^2 = -\gamma + \left(\frac{1}{2}(\alpha+\beta) + \ell\right) \left(\frac{1}{2}(\alpha+\beta) + \ell + 1\right)$$

and eigenfunctions

$$\chi_{\ell}^{\alpha,\beta}(x) = N_{\alpha,\beta} \, x^{\beta/2} (1-x)^{\alpha/2} P_{\ell}^{\alpha,\beta}(2x-1)$$

with normalization $\int_0^1 dx \, \chi_\ell^{\alpha,\beta}(x) \chi_m^{\alpha,\beta}(x) = \delta_{\ell m}$

• We introduce the constant γ -term to enforce the chiral symmetry limit: u o 0 for $m_q o 0$

• Following C&H we expand the BdT exponential ansatz in terms of Jacobi polynomials

$$\mathcal{N}e^{-\frac{1}{2\lambda}\left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)} = \sum_{\ell} C_{\ell} \chi_{\ell}(x), \quad \sum_{\ell} |C_{\ell}|^2 = 1$$

• Only first few modes in the expansion are relevant

	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$C(u\overline{d})$	0.998	0	-0.027	0	-0.029
$C(u\overline{s})$	0.895	-0.431	-0.029	-0.108	0.012
$C(c\overline{c})$	0.999	0	-0.029	0	0.020



Extension to heavy-light, heavy-heavy, isoscalar and exotic sectors

- Scale dependence of hadronic scale λ from HQET
- Extension to the heavy-light hadronic sector: [H. G. Dosch, GdT, S. J. Brodsky (2015, 2017)]
- Extension to the double-heavy hadronic sector:

[M. Nielsen, S. J. Brodsky, GdT, H. G. Dosch, F. S. Navarra (2018)]

• Extension to the isoscalar hadronic sector:

[L. Zou, H. G. Dosch, GdT, S. J. Brodsky (2018)]

• Tetraquarks and Exotic States:

[H. G. Dosch, S. J. Brodsky, GdT, M. Nielsen, L. Zou (2020)]



6 Form factors, parton distributions and intrinsic quark sea

- Form factors and parton distributions from phenomenological extension of LF holographic framework to arbitrary Regge trajectories incorporating the analytic structure of Veneziano amplitudes
- Study of strange and charm quark-sea in the proton from combined LQCD and holographic methods
- Nucleon Form Factors:

[R. S. Sufian, GdT, S. J. Brodsky, A . Deur, H. G. Dosch (2017)]

• Generalized quark distributions:

[GdT, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, A. Deur (HLFHS Collaboration (2018)]

• Strange-quark sea in the nucleon

[R. S. Sufian, T Liu, GdT, H. G. Dosch, S. J. Brodsky, A. Deur, M. T. Islam, B-Q. Ma (2018)]

• Unified description of polarized and unpolarized quark distributions:

[T Liu, R. S. Sufian, GdT, H. G. Dosch, S. J. Brodsky, A. Deur (HLFHS Collaboration (2019)]

• Intrinsic-charm content of the proton:

[R. S. Sufian, T. Liu, A. Alexandru, S. J. Brodsky, GdT, H. G. Dosch, T. Draper, K. F. Liu and Y. B. Yang (2020)]

Form factors

• Hadron form factor expressed as a sum from the Fock expansion of states

$$F(t) = \sum_{\tau} c_{\tau} F_{\tau}(t)$$

where the c_{τ} are spin-flavor expansion coefficients for different twist

• $F_{\tau}(t)$ in LFHQCD has the Euler's Beta function structure

$$F_{\tau}(t) = \frac{1}{N_{\tau}} B\big(\tau - 1, 1 - \alpha(t)\big)$$

found by Ademollo and Del Giudice and Landshoff and Polkinghorne in the pre-QCD era, extending the Veneziano duality model (1968)

- $\alpha(t)$ is the Regge trajectory of the VM which couples to the quark EM current in the hadron
- For $\tau = N$, the number of constituents in a Fock component, the FF is an N-1 product of poles

$$F_{\tau}(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{n=0}^2}\right)\left(1 + \frac{Q^2}{M_{n=1}^2}\right)\cdots\left(1 + \frac{Q^2}{M_{n=\tau-2}^2}\right)}$$

located at

$$-Q^{2} = M_{n}^{2} = \frac{1}{\alpha'} (n + 1 - \alpha(0))$$

It generates the radial excitation spectrum of the exchanged VM particles in the t-channel



Quark distributions

• Using integral representation of Beta function FF is expressed in a reparametrization invariant form

$$F(t)_{\tau} = \frac{1}{N_{\tau}} \int_0^1 dx \, w'(x) w(x)^{-\alpha(t)} \left[1 - w(x)\right]^{\tau-2}$$

with $w(0)=0, \quad w(1)=1, \quad w'(x)\geq 0$

• Flavor FF is given in terms of the valence GPD $H^q_\tau(x,\xi=0,t)$ at zero skewness

$$F_{\tau}^{q}(t) = \int_{0}^{1} dx H_{\tau}^{q}(x,t) = \int_{0}^{1} dx \, q_{\tau}(x) \exp[tf(x)]$$

with the profile function $f(\boldsymbol{x})$ and PDF $q(\boldsymbol{x})$ determined by $w(\boldsymbol{x})$

$$f(x) = \frac{1}{4\lambda} \log\left(\frac{1}{w(x)}\right)$$

$$q_{\tau}(x) = \frac{1}{N_{\tau}} [1 - w(x)]^{\tau - 2} w(x)^{-\alpha(0)} w'(x)$$

- Boundary conditions: At $x \to 0$, $w(x) \sim x$ from Regge behavior, $q(x) \sim x^{-\alpha(0)}$, and w'(1) = 0 to recover counting the rules at $x \to 1$, $q_{\tau}(x) \sim (1-x)^{2\tau-3}$ (inclusive-exclusive connection)
- If w(x) fixed by nucleon PDFs then pion PDF is a prediction. Example: $w(x) = x^{1-x}e^{-a(1-x)^2}$

Unpolarized GPDs and PDFs (HLFHS Collaboration, 2018)





• Transverse impact parameter quark distribution

$$u(x, \mathbf{a}_{\perp}) = \int \frac{d^2 \mathbf{q}_{\perp}}{(2\pi)^2} e^{-i\mathbf{a}_{\perp} \cdot \mathbf{q}_{\perp}} H^u(x, \mathbf{q}_{\perp}^2)$$



Polarized GPDs and PDFs (HLFHS Collaboration, 2019)

- Separation of chiralities in the AdS action allows computation of the matrix elements of the axial current including the correct normalization, once the coefficients c_{τ} are fixed for the vector current
- Helicity retention between quark and parent hadron at $x \to 1$: $\lim_{x \to 1} \frac{\Delta q(x)}{q(x)} = 1$
- No spin correlation with parent hadron at $x \to 0$: $\lim_{x\to 0} \frac{\Delta q(x)}{q(x)} = 0$



Intrinsic charm in the proton (2020)

- S. J. Brodsky, P. Hoyer, C. Peterson, N. Sakai (1980)
- Nonperturbative intrinsic heavy-quark content of hadron structure is now verified by LGTH
- EM form factors compjuted with three gauge ensembles (one at the physical pion mass)
- Nonperturbative intrinsic charm asymmetry $c(x) \overline{c}(x)$ determined from LQCD and LFHQCD analysis





Unpolarized gluon distribution (HLFHS Collaboration)

- Confinement in LFHQCD emerges from intrinsic gluon contribution: the graviton couples to the T^{++} component of the energy-momentum tensor which leads to the gluon distribution function g(x)
- Gravitational FF expressed in terms of Euler Beta function

 $A_{\tau}(t) \sim B\left(\tau - 1, 2 - \alpha(t)\right)$

 Pomeron-proton vertex described by Regge trajectory for a soft Donnachie-Landshoff Pomeron

 $\alpha(t) = \alpha_P(0) + \alpha'_P t$

interpreted in QCD as a bound state of two gluons with $\alpha'_P(0)\simeq 1.08~~{\rm and}~~\alpha'\simeq 0.25\,{\rm GeV}^{-2}$



- Gluon PDF normalization determined from the sum rule: $\sum_{q} \langle x \rangle_q + \sum_{\overline{q}} \langle x \rangle_{\overline{q}} + \langle x \rangle_g = 1$
- Basic parameters fixed in quark sector: No adjustable parameters for prediction of g(x)
- Red error band from initial evolution scale, uncertainty from sum rule not included

7 Outlook

- Classical equations of motion derived from the 5-dim theory have identical form of the semiclassical bound-state equations for massless constituents in LF quantization
- Implementation of superconformal algebra determines uniquely the form of the confining interaction for mesons, nucleons and tetraquarks: tetraquarks are as fundamental as mesons and baryons
- Approach incorporates basic properties which are not apparent from the QCD Lagrangian, such as the emergence of a mass scale and the connection between mesons and baryons
- Prediction of massless pion in chiral limit is a consequence of the superconformal algebra and not of the Goldstone mechanism: vacuum chosen *ab initio*
- Structural framework of LFHQCD also provides nontrivial connection between the structure of form factors and polarized and unpolarized quark distributions with pre-QCD nonperturbative results such as Regge theory and the Veneziano model