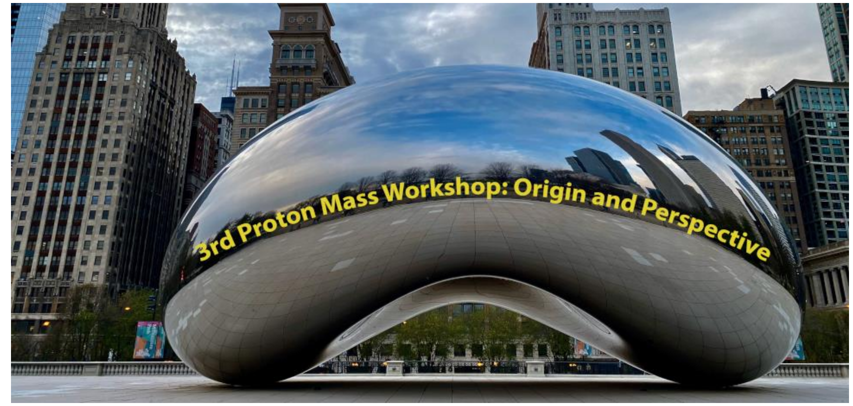

The physical meaning of the various mass decompositions

Cédric Lorcé

CPHT Centre de Physique Théorique

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What is mass?

Formal definition $$p^\mu p_\mu = M^2$$ A global Lorentz-invariant quantity characterizing a physical system

⚠ Not additive! $$p^\mu = p_q^\mu + p_g^\mu \quad \Rightarrow \quad M^2 = p_q^2 + p_g^2 + 2\, p_q \cdot p_g$$

Physical interpretation $$p^\mu u_\mu = M$$ Proper inertia (i.e. energy) of a system

↑ **CM four-velocity** $u^\mu = p^\mu / M$

Additive $$p^\mu = p_q^\mu + p_g^\mu \quad \Rightarrow \quad M = p_q \cdot u + p_g \cdot u$$

$$= \quad p_q^0 \quad + \quad p_g^0$$ **CM frame**

A mass decomposition is a proper energy sum rule

Link with the energy-momentum tensor (EMT)

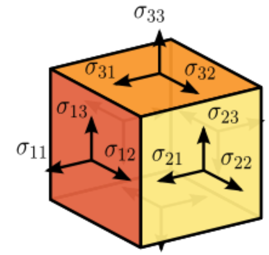
Energy density — Momentum density

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Shear stress

Normal stress (pressure)

Energy flux — Momentum flux



Four-momentum operator $$P^\mu_a = \int \mathrm{d}^3r\, T^{0\mu}_a(r) \qquad a = q, g$$

Expectation value $$p^\mu_a \equiv \langle P^\mu_a \rangle = \frac{\langle p|P^\mu_a|p\rangle}{\langle p|p\rangle} = \underbrace{\frac{\int \mathrm{d}^3r}{(2\pi)^3\delta^{(3)}(0)}}_{=1} \frac{\langle p|T^{0\mu}_a(0)|p\rangle}{2p^0}$$

Poincaré constraints (spin-0 or spin-1/2 targets)

$$\langle p|T_a^{\mu\nu}(0)|p\rangle = 2p^\mu p^\nu A_a(0) + 2M^2 g^{\mu\nu}\bar{C}_a(0)$$

$$\Rightarrow \quad \langle P_a^\mu\rangle = p^\mu A_a(0) + \frac{M^2}{p^0}\,g^{0\mu}\bar{C}_a(0)$$

Not a four-vector !
(unless state is massless)

Light-front version

$$\langle P_{a,\mathrm{LF}}^\mu\rangle = p^\mu A_a(0) + \frac{M^2}{p^+}\,g^{+\mu}\bar{C}_a(0) \quad \Rightarrow \quad \langle P_{a,\mathrm{LF}}^+\rangle = \underbrace{A_a(0)}_{=\langle x\rangle_a}\,p^+$$

Four-momentum sum rules

$$p^\mu = \langle P_q^\mu\rangle + \langle P_g^\mu\rangle \quad \Rightarrow \quad \boxed{\begin{cases} A_q(0) + A_g(0) = 1 \\ \bar{C}_q(0) + \bar{C}_g(0) = 0 \end{cases}}$$

[Ji, PRD58 (1998)]

From the experimental side, all we need is:

1. to measure *all* the coefficients and
2. to check the four-momentum sum rules

Mass decomposition

Once a decomposition of the EMT into quark and gluon contributions is determined, a well-defined mass decomposition follows automatically

$$\boxed{M = \langle P_q \cdot u \rangle + \langle P_g \cdot u \rangle}$$

$$\langle P_a \cdot u \rangle = \left[A_a(0) + \bar{C}_a(0) \right] M$$

Each term represents in a covariant way an energy contribution in the CM frame

[C.L., EPJC78 (2018)]

Renormalized QCD operators

$$T^{\mu\nu} = T^{\mu\nu}_q + T^{\mu\nu}_g$$

$$T^{\mu\nu}_q = \overline{\psi} \gamma^\mu \tfrac{i}{2} \overleftrightarrow{D}^\nu \psi$$

$$T^{\mu\nu}_g = -G^{\mu\lambda} G^\nu{}_\lambda + \tfrac{1}{4}\, g^{\mu\nu}\, G^2$$

Refinements of the mass decomposition

In CM frame

[C.L., EPJC78 (2018)]
[Rodini, Metz, Pasquini, JHEP09 (2020)]
[Metz, Pasquini, Rodini, PRD102 (2020)]

$$M = \underbrace{\left[\langle \int d^3r\, \overline{\psi}\gamma^0 iD^0\psi\rangle - \langle \int d^3r\, \overline{\psi}m\psi\rangle\right]}_{\text{Quark kinetic and potential energy}} + \underbrace{\langle \int d^3r\, \overline{\psi}m\psi\rangle}_{\text{Quark rest mass energy}} + \underbrace{\langle \int d^3r\, \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)\rangle}_{\text{Gluon total energy}}$$

See talks by Metz and Rodini

Spatial distribution in CM (or Breit) frame

$$M = \int d^3r\, T^{00}(r) = \sum_a \int d^3r\, T^{00}_a(r)$$

$$T^{00}(r) = M \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ A(t) + \frac{t}{4M^2}\left[B(t) - 4C(t)\right] \right\}$$

See talks by Polyakov and Schweitzer

[Donoghue, Holstein, Gambrecht, Konstandin, PLB529 (2002)]
[Polyakov, PLB555 (2003)]
[Polyakov, Schweitzer, IJMPA33 (2018)]

$$T^{00}_a(r) = M \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2}\left[B_a(t) - 4C_a(t)\right] \right\}$$

[C.L., Moutarde, Trawinski, EPJC79 (2019)]

Trace decomposition

Quantum corrections break conformal symmetry

$$T^{\mu}{}_{\mu} = \underbrace{\frac{\beta(g)}{2g}\,G^2}_{\text{Trace anomaly}} + (1+\gamma_m)\,\overline{\psi}m\psi$$

Trace anomaly

Quark mass matrix

[Crewther, PRL28 (1972)]
[Chanowitz, Ellis, PRD7 (1972)]
[Adler, Collins, Duncan, PRD15 (1977)]
[Collins, Duncan, Joglekar, PRD16 (1977)]
[Nielsen, NPB120 (1977)]

Textbook approach $\langle p|T^{\mu\nu}(0)|p\rangle = 2p^\mu p^\nu$

[Shifman, Vainshtein, Zakharov, PLB78 (1978)]
[Donoghue, Golowich, Holstein, CMPPNPC2 (1992)]
[Kharzeev, PISPF130 (1996)]
[Roberts, FBS58 (2017)]

$$\langle p|\frac{\beta(g)}{2g}\,G^2|p\rangle + \langle p|(1+\gamma_m)\,\overline{\psi}m\psi|p\rangle = 2M^2$$

$\mu = 2\,\mathrm{GeV}$

~ 90% (to be measured)

~ 10% (measurement to be improved)

Caveats:

1) Depends on state normalization (not an expectation value!)
2) Decomposition of squared mass and not mass
3) No explanation for the evaluation of EMT at a single spacetime point
4) Relation to mass only at the level of matrix element of total trace (meaning of individual terms?)

Trace decomposition

Expectation value (independent of state normalization)

[C.L., EPJC78 (2018)]
[Krein, Thomas, Tsushima, PPNP100 (2018)]

$$\underbrace{\frac{\langle p|\frac{\beta(g)}{2g}\,G^2|p\rangle}{2p^0}}_{=\langle\int \mathrm{d}^3r\,(T^\mu{}_\mu)_a\rangle} + \underbrace{\frac{\langle p|(1+\gamma_m)\,\overline{\psi}m\psi|p\rangle}{2p^0}}_{=\langle\int \mathrm{d}^3r\,(T^\mu{}_\mu)_m\rangle} = \frac{2M^2}{2p^0}$$

$$(T^\mu{}_\mu)_a = \frac{\beta(g)}{2g}\,G^2$$
$$(T^\mu{}_\mu)_m = (1+\gamma_m)\,\overline{\psi}m\psi$$

Operator mixing

[Hatta, Rajan, Tanaka, JHEP12 (2018)]
[Tanaka, JHEP01 (2019)]

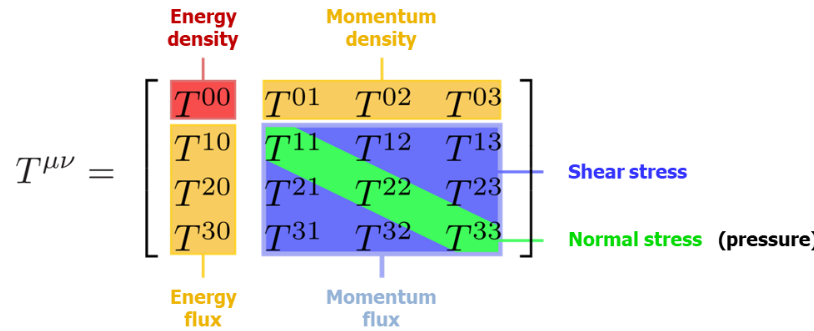
$$g_{\mu\nu}T_q^{\mu\nu} = c_a\,(T^\mu{}_\mu)_a + c_m\,(T^\mu{}_\mu)_m$$
$$g_{\mu\nu}T_g^{\mu\nu} = (1-c_a)\,(T^\mu{}_\mu)_a + (1-c_m)\,(T^\mu{}_\mu)_m$$

$c_a,\ c_m$ are scheme and scale-dependent calculable coefficients

See talks by Hatta, Metz and Rodini

Quark-gluon separation and trace operation do not commute

Trace decomposition



Physical interpretation in CM frame

$$\underbrace{\langle \textstyle\int \mathrm{d}^3r\, T^\mu{}_\mu \rangle}_{=M} = \underbrace{\langle \textstyle\int \mathrm{d}^3r\, T^{00} \rangle}_{=M} - \sum_i \underbrace{\langle \textstyle\int \mathrm{d}^3r\, T^{ii} \rangle}_{=0}$$

[von Laue, AP340 (1911)]
[Landau, Lifshitz, CTF (1962)]

Stable system → Total pressure-volume work must vanish!

$$\underbrace{\langle \textstyle\int \mathrm{d}^3r\, T^\mu_{a\mu} \rangle}_{\text{Can be negative!}} = \underbrace{\langle \textstyle\int \mathrm{d}^3r\, T^{00}_a \rangle}_{=\langle P^0_a \rangle} - \sum_i \underbrace{\langle \textstyle\int \mathrm{d}^3r\, T^{ii}_a \rangle}_{\neq 0} \qquad a = q, g$$

[C.L., EPJC78 (2018)]

Trace decomposition combines mass decomposition with mechanical equilibrium

Ji's decomposition

Step 1 $$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu}$$

$$\bar{T}^{\mu\nu} = T^{\mu\nu} - \tfrac{1}{4}\, g^{\mu\nu} T^{\alpha}{}_{\alpha}$$
$$\hat{T}^{\mu\nu} = \tfrac{1}{4}\, g^{\mu\nu} T^{\alpha}{}_{\alpha}$$

[Ji, PRL74 (1995)]
[Ji, PRD52 (1995)]

Step 2 $$\bar{T}^{\mu\nu} = \bar{T}^{\mu\nu}_q + \bar{T}^{\mu\nu}_g$$

<u>NB:</u> quark and gluon contributions also traceless

$$\hat{T}^{\mu\nu} = (\hat{T}^{\mu\nu})_m + (\hat{T}^{\mu\nu})_a$$ or $$\hat{T}^{\mu\nu}_q + \hat{T}^{\mu\nu}_g$$

⚠ Operator mixing

Physical interpretation in CM frame

$$T^{00}_a = \underbrace{\bar{T}^{00}_a}_{= \frac{3}{4} T^{00}_a + \frac{1}{4} \sum_i T^{ii}_a} + \underbrace{\hat{T}^{00}_a}_{= \frac{1}{4} T^{00}_a - \frac{1}{4} \sum_i T^{ii}_a} \qquad a = q, g$$

[C.L., EPJC78 (2018)]

Ji's decomposition is a sum of terms representing physical quantities of <u>different</u> nature

Ji's decomposition

Step I $T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu}$

$$\bar{T}^{\mu\nu} = T^{\mu\nu} - \frac{1}{4}\, g^{\mu\nu} T^{\alpha}{}_{\alpha}$$

$$\hat{T}^{\mu\nu} = \frac{1}{4}\, g^{\mu\nu} T^{\alpha}{}_{\alpha}$$

$\bar{T}^{\mu\nu}, \hat{T}^{\mu\nu}$ belong to different Lorentz representations ➡ separately renormalized

$$T^{00} = \psi^\dagger i\vec{D}\cdot\vec{\alpha}\psi + \overline{\psi} m\psi + \frac{1}{2}(\vec{E}^2 + \vec{B}^2) + \frac{1}{4}\,\gamma_m\,\overline{\psi}m\psi + \frac{1}{4}\,\frac{\beta(g)}{2g}\,G^2$$

[Ji, PRL74 (1995)]
[Ji, PRD52 (1995)]
[Ji, Liu, arXiv:2101.04483]

… but the total EMT is conserved and should not contain the anomalous terms

See talks by Metz and Rodini

[Hatta, Rajan, Tanaka, JHEP12 (2018)]
[Tanaka, JHEP01 (2019)]
[Rodini, Metz, Pasquini, JHEP09 (2020)]
[Metz, Pasquini, Rodini, PRD102 (2020)]

➡ This undermines the simple interpretation attributed to the individual terms in Ji's decomposition

$$M = M_q + M_g + M_m + M_a$$

E.g. $M_g \neq \langle \int \mathrm{d}^3 r\, \frac{1}{2}(\vec{E}^2 + \vec{B}^2)\rangle$

To sum up

Fundamental sum rules

Mass (i.e. CM energy)

$$M = \left[A_q(0) + \bar{C}_q(0)\right] M + \left[A_g(0) + \bar{C}_g(0)\right] M$$

Momentum

$$\vec{p} = A_q(0)\,\vec{p} + A_g(0)\,\vec{p}$$

Mechanical equilibrium

$$0 = \left[-\bar{C}_q(0)\right] M + \left[-\bar{C}_g(0)\right] M$$

in CM frame

Trace decomposition (i.e. energy – 3 x pressure-volume work)

$$\frac{M^2}{p^0} = \left[A_q(0) + 4\bar{C}_q(0)\right] \frac{M^2}{p^0} + \left[A_g(0) + 4\bar{C}_g(0)\right] \frac{M^2}{p^0}$$

To sum up

Ji's decomposition (obscure mix of energy and pressure-volume work in CM frame)

$$M = M_q + M_g + M_m + M_a$$

$$M_q = \tfrac{3}{4}\left(a - \tfrac{b}{1+\gamma_m}\right) M \quad \neq \langle \int \mathrm{d}^3r\, \psi^\dagger i\vec{D}\cdot\vec{\alpha}\psi\rangle \quad ⚠$$

$$M_g = \tfrac{3}{4}\,(1-a)\,M \quad \neq \langle \int \mathrm{d}^3r\, \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)\rangle \quad ⚠$$

$$M_m = \frac{4+\gamma_m}{4(1+\gamma_m)}\, b\, M \quad = \langle \int \mathrm{d}^3r\, \left(1 + \tfrac{1}{4}\gamma_m\right)\overline{\psi} m\psi\rangle$$

$$M_a = \tfrac{1}{4}\,(1-b)\,M \quad = \langle \int \mathrm{d}^3r\, \tfrac{1}{4}\,\tfrac{\beta(g)}{2g}\, G^2\rangle$$

NB: one has $M_q - \frac{3\gamma_m}{4+\gamma_m} M_m + M_g - 3M_a = 0$ as a result of mechanical equilibrium!

Relations between Ji's parameters and gravitational FFs

$$A_q(0) = a \qquad A_q(0) + 4\bar{C}_q(0) = c_a(1-b) + c_m b$$

$$A_g(0) = 1 - a \qquad A_g(0) + 4\bar{C}_g(0) = (1-c_a)(1-b) + (1-c_m)b$$

My opinion is that the interpretation associated with the terms in Ji's decomposition is questionable

What remains unclear to me

One has to be very careful because renormalization, quark-gluon separation and trace operation do not commute!

Renormalization should preserve the EOM $(\overline{\psi} i \not{D} \psi)_R = (\overline{\psi} m \psi)_R$

Standard manipulation $(\overline{\psi}\gamma^0 iD^0\psi)_R = (\psi^\dagger i\vec{D}\cdot\vec{\alpha}\psi)_R + (\overline{\psi}m\psi)_R$

[Ji, PRL74 (1995)]
[Ji, PRD52 (1995)]
[Rodini, Metz, Pasquini, JHEP09 (2020)]
[Metz, Pasquini, Rodini, PRD102 (2020)]

… but $g_{\mu\nu}(\overline{\psi}\gamma^\mu iD^\nu\psi)_R = c_a\,(1+\gamma_m^R)(\overline{\psi}m\psi)_R + c_m\,\frac{\beta_R(g_R)}{2g_R}\,(G^2)_R$

➡ It seems that the anomalous contributions come from the spatial part of the trace

Seems at odds with Lorentz covariance!?!