

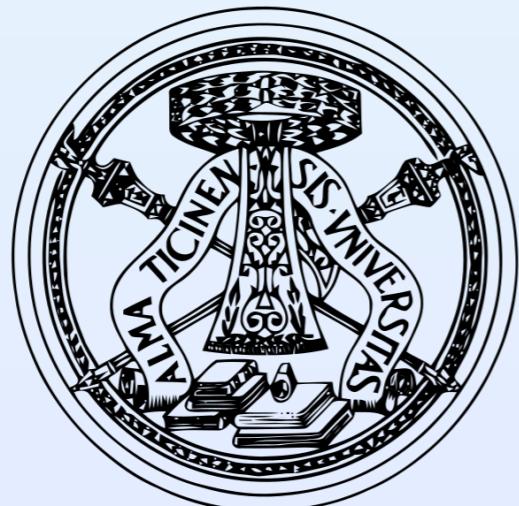
Energy-Momentum Tensor form factors for QED, a one-loop example

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Based on

Rodini et al., JHEP 09 (2020) 067

Metz et al., Phys. Rev. D102 (2020) 114042



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Outline

Renormalization procedure for multiple local operators

Scheme dependence and scheme choice

QED form factors (also off-forward matrix elements, a quick peek)

Definition of the Energy-Momentum Tensor

Lagrangian renormalization is understood

$$T^{\mu\nu} = \mathcal{O}_1 + \frac{\mathcal{O}_2}{4} + \mathcal{O}_3$$

Hatta et al., JHEP 12 (2018) 008

Tanaka, JHEP 01 (2019) 120

$$\mathcal{O}_1 = -F^{\mu\alpha} F^\nu_\alpha$$

$$\mathcal{O}_2 = g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

$$\mathcal{O}_3 = \frac{i}{4} \bar{\psi} \gamma^{\{\mu} \overset{\leftrightarrow}{D}^{\nu\}} \psi$$

$$\mathcal{O}_4 = g^{\mu\nu} m \bar{\psi} \psi$$

Renormalize fields, coupling and masses is not enough.

We must also renormalize the operators:

$$\mathcal{O}_{1,R} = Z_T \mathcal{O}_1 + Z_M \mathcal{O}_2 + Z_L \mathcal{O}_3 + Z_S \mathcal{O}_4$$

$$\mathcal{O}_{2,R} = Z_F \mathcal{O}_2 + Z_C \mathcal{O}_4$$

$$\mathcal{O}_{3,R} = Z_\psi \mathcal{O}_3 + Z_K \mathcal{O}_4 + Z_Q \mathcal{O}_1 + Z_B \mathcal{O}_2$$

$$\mathcal{O}_{4,R} = \mathcal{O}_4$$

The total EMT is not affected by the additional renormalization

$$T_R^{\mu\nu} = T^{\mu\nu}$$

$$T^\mu_\mu = (T_R)^\mu_\mu = (T^\mu_\mu)_R = (1 + \gamma_m)(m\bar{\psi}\psi)_R + \frac{\beta}{2e}(F^{\alpha\beta}F_{\alpha\beta})_R$$

However, in general, trace and renormalization do not commute

$$\text{Tr}[O_R^{\mu\nu}] \neq (\text{Tr}[O^{\mu\nu}])_R$$

In particular $T_{e,R}^{\mu\nu} = \mathcal{O}_{3,R}$ $T_{\gamma,R}^{\mu\nu} = \mathcal{O}_{1,R} + \frac{\mathcal{O}_{2,R}}{4}$

$$(T_{e,R})^\mu_\mu = (1 + y)(m\bar{\psi}\psi)_R + x(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$(T_{\gamma,R})^\mu_\mu = (\gamma_m - y)(m\bar{\psi}\psi)_R + \left(\frac{\beta}{2e} - x\right)(F^{\alpha\beta}F_{\alpha\beta})_R$$

How to fix the counterterms (1)

$Z_{F,C}$

Are known from

R. Tarrach, Nucl. Phys. B 196 (1982) 45

$Z_{L,T,Q,\psi}$

**Are given by the evolution equations
for the second moment of the
flavor-singlet unpolarized parton distributions.**

Tanaka, JHEP 01 (2019) 120

$$\text{AMF}(\text{R}[e]) = Z_1 \text{AMF}(e) + Z_2 \text{AMF}(\gamma),$$

**Average Moment Fractions (AMF)
from PDFs**

$$\text{AMF}(\text{R}[\gamma]) = Z_3 \text{AMF}(\gamma) + Z_4 \text{AMF}(e)$$

$$\text{R}[\text{AMF}(e)] = Z_\psi \text{AMF}(e) + Z_Q \text{AMF}(\gamma),$$

**Average Moment Fractions (AMF)
from EMT (++ components)**

$$\text{R}[\text{AMF}(\gamma)] = Z_T \text{AMF}(\gamma) + Z_L \text{AMF}(e)$$

$$Z_\psi^{[\epsilon]} = Z_1^{[\epsilon]} \quad Z_Q^{[\epsilon]} = Z_2^{[\epsilon]} \\ Z_T^{[\epsilon]} = Z_3^{[\epsilon]} \quad Z_L^{[\epsilon]} = Z_4^{[\epsilon]}$$

**The counterterms have
the same divergent part**

How to fix the counterterms (2)

The other counterterms are not independent!

$$Z_T + Z_Q = 1$$

$$Z_L + Z_\psi = 1$$

$$Z_M + Z_B + \frac{Z_F}{4} = \frac{1}{4}$$

$$Z_S + Z_K + \frac{Z_C}{4} = 0$$

$$Z_M = \frac{Z_T}{d} - \frac{Z_F}{d} \left(1 - \frac{\beta}{2g} + x \right)$$

$$Z_S = -\frac{Z_L}{d} - \frac{Z_C}{d} \left(1 - \frac{\beta}{2g} + x \right) - \frac{y - \gamma_m}{d}$$

$$Z_B = \frac{Z_Q}{d} + \frac{x}{d} Z_F$$

$$Z_K = -\frac{Z_\psi}{d} + \frac{x}{d} Z_C + \frac{1+y}{d}$$

$$\tilde{\mathcal{O}}_{1,R} = \mathcal{O}_{1,R} + \frac{1}{4} \left(1 - \frac{\beta}{2e} + x \right) \mathcal{O}_{2,R} + \frac{y - \gamma_m}{4} \mathcal{O}_{4,R},$$

$$\tilde{\mathcal{O}}_{3,R} = \mathcal{O}_{3,R} - \frac{x}{4} \mathcal{O}_{2,R} - \frac{1+y}{4} \mathcal{O}_{4,R}$$

Tanaka, JHEP 01 (2019) 120

Hatta et al., JHEP 12 (2018) 008

Fixing x and y it corresponds to choose a scheme

Diagonal schemes:

D1: $x = 0$ $y = \gamma_m$

Rodini et al., JHEP 09 (2020) 067

$$(T_{e,R})^\mu_\mu = (1+y) (m\bar{\psi}\psi)_R + x (F^{\alpha\beta} F_{\alpha\beta})_R$$

$$(T_{\gamma,R})^\mu_\mu = (\gamma_m - y) (m\bar{\psi}\psi)_R + \left(\frac{\beta}{2e} - x \right) (F^{\alpha\beta} F_{\alpha\beta})_R$$

D2: $x = 0$ $y = 0$

Metz et al., Phys. Rev. D102 (2020) 114042

MS-like schemes:

Tanaka, JHEP 01 (2019) 120

Hatta et al., JHEP 12 (2018) 008

Alternative construction of MSbar scheme

Metz et al., Phys. Rev. D102 (2020) 114042

MS-like schemes

Tanaka, JHEP 01 (2019) 120

Impose vanishing finite contributions to the derived counterterms

$$Z_X = \delta_{X,T} + \delta_{X,\psi} + \delta_{X,F} + \frac{a_X}{\epsilon} + \frac{b_X}{\epsilon^2} + \frac{c_X}{\epsilon^3} + \dots$$

From the definition of $Z_{M,S}$

$$\frac{1}{32} \left[(8 + 4a_T + 2b_T + c_T + \dots) - \left(1 + x - \frac{\beta}{2e}\right) (8 + 4a_F + 2b_F + c_F + \dots) \right] = 0$$
$$\frac{1}{32} \left[-(4a_L + 2b_L + c_L + \dots) - \left(1 + x - \frac{\beta}{2e}\right) (4a_C + 2b_C + c_C + \dots) + 8(\gamma_m - y) \right] = 0$$

Variant of MSbar

Metz et al., Phys. Rev. D102 (2020) 114042

MSbar counterterms with finite contributions, which are uniquely determined by the divergent part

$$Z|_{\overline{\text{MS}}} = (1, 0) + \alpha \frac{\bar{a}_1}{\epsilon} S_\epsilon + \alpha^2 \left(\frac{\bar{b}_2}{\epsilon^2} + \frac{\bar{b}_1}{\epsilon} \right) S_\epsilon^2 + \alpha^3 \left(\frac{\bar{c}_3}{\epsilon^3} + \frac{\bar{c}_2}{\epsilon^2} + \frac{\bar{c}_1}{\epsilon} \right) S_\epsilon^3$$

$$S_\epsilon|_{\overline{\text{MS}}_1} = \frac{(4\pi)^\epsilon}{\Gamma(1 - \epsilon)}$$

$$Z_B = \frac{Z_Q}{d} + \frac{x}{d} Z_F$$

$$S_\epsilon|_{\overline{\text{MS}}_2} = (4\pi e^{-\gamma_E})^\epsilon$$

$$x = x_1 \alpha + x_2 \alpha^2 + \dots$$

What we find (in MSbar 1)

$$O(\alpha_s) : \frac{1}{8} \left(\bar{a}_{1,Q} + 2\bar{a}_{1,Q} (\log(4\pi) - \gamma_E) + 2x_1 \right)$$

What we want

$$O(\alpha_s) : \frac{1}{4} \bar{a}_{1,Q} (\log(4\pi) - \gamma_E)$$

EMT matrix elements

Ji, Phys. Rev. Lett. 78 (1997) 610

$$\langle \mathcal{O} \rangle = \langle P | \mathcal{O} | P \rangle$$

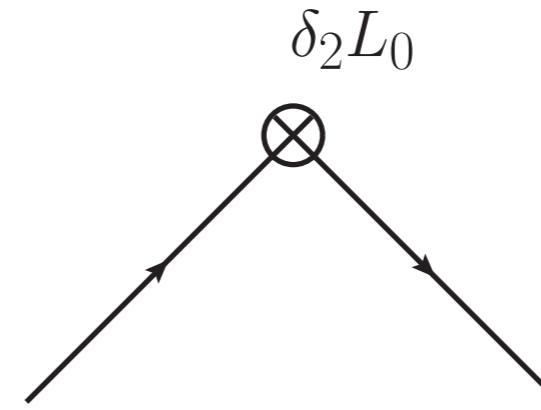
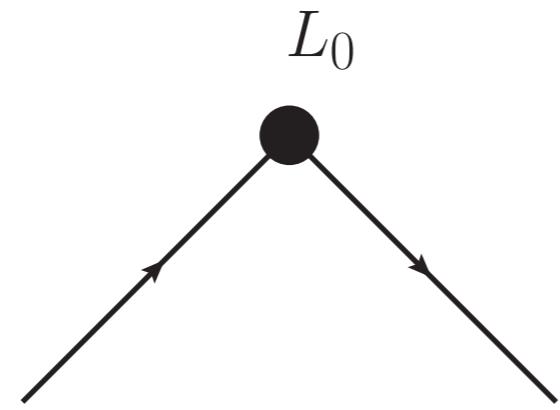
$$\begin{aligned} \langle e(p'), s' | T_{i,R}^{\mu\nu} | e(p), s \rangle &= \left\langle e\left(P + \frac{\Delta}{2}\right), s' \middle| T_{i,R}^{\mu\nu} \middle| e\left(P - \frac{\Delta}{2}\right), s \right\rangle \\ &= \bar{u}' \left(A_i(\Delta^2) \frac{P^\mu P^\nu}{m} + J_i(\Delta^2) \frac{i P^{\{\mu} \sigma^{\nu\}} \rho \Delta_\rho}{2m} \right. \\ &\quad \left. + D_i(\Delta^2) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} + m \bar{C}_i(\Delta^2) g^{\mu\nu} + C_i(\Delta^2) P^{[\mu} \gamma^{\nu]} \right) u \end{aligned}$$

Two form factors for the forward limit

$$\langle T_{i,R}^{\mu\nu} \rangle = 2P^\mu P^\nu A_i(0) + 2M^2 g^{\mu\nu} \bar{C}_i(0) \quad i = e, \gamma$$

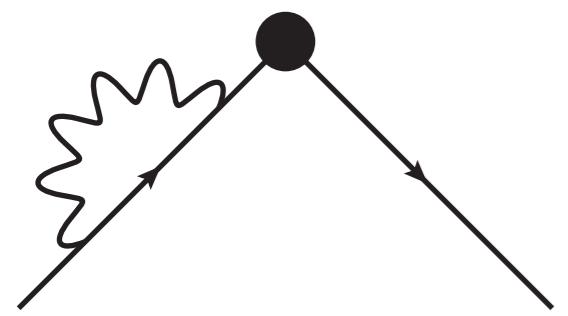
$$A_e(0) + A_\gamma(0) = 1 \quad \bar{C}_e(0) + \bar{C}_\gamma(0) = 0$$

How to obtain the form factors (QED)

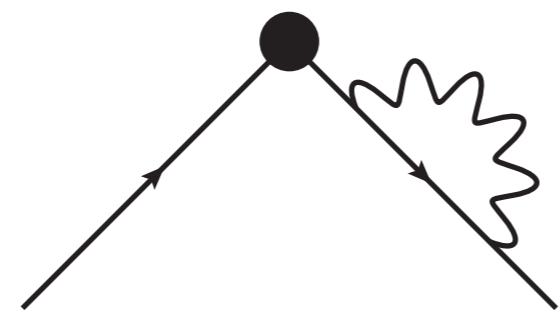


L_{tot}

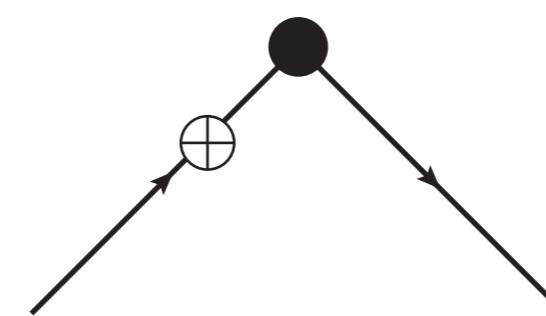
L_1



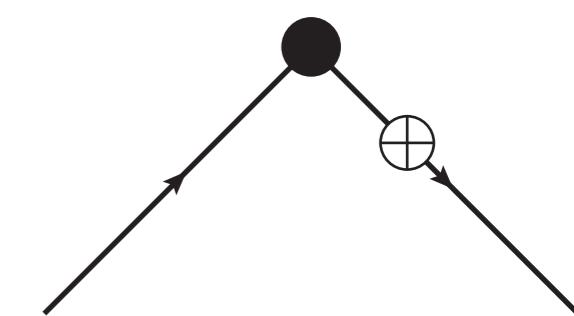
L_2



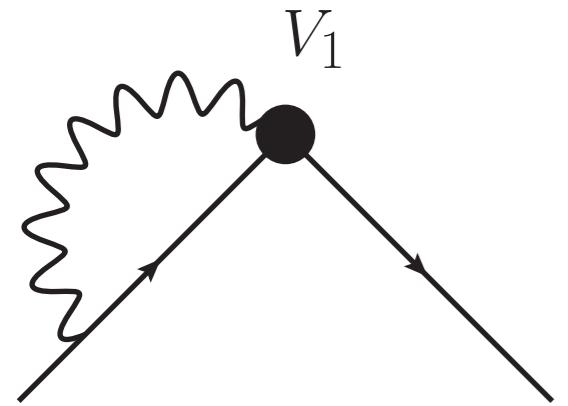
$L_1^{c.t.}$



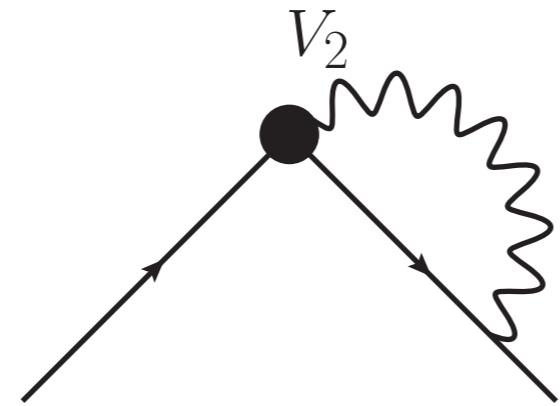
$L_2^{c.t.}$



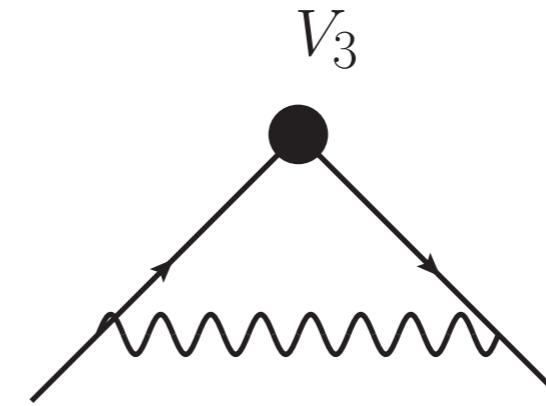
V_1



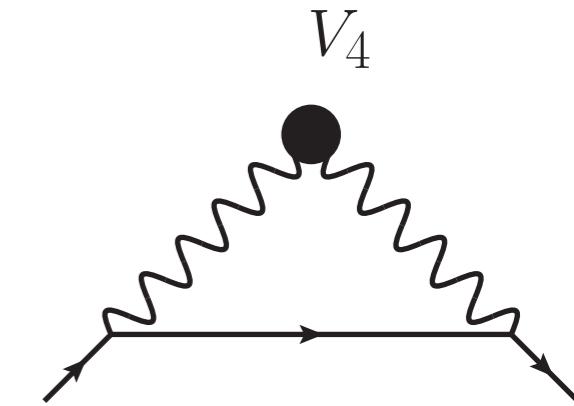
V_2



V_3



V_4



QED

$$Z_T + Z_Q = 1$$

$$Z_L + Z_\psi = 1$$

$$Z_M + Z_B + \frac{Z_F}{4} = \frac{1}{4}$$

$$Z_S + Z_K + \frac{Z_C}{4} = 0$$

$$x = 0 \quad y = \begin{cases} \frac{\alpha}{3\pi} & \overline{\text{MS}} \\ \frac{3\alpha}{2\pi} & \text{D}_1 \\ 0 & \text{D}_2 \end{cases}$$

$$Z_F = 1 + \frac{\beta}{e} \Delta_{\text{UV}} \quad Z_C = 2\gamma_m \Delta_{\text{UV}}$$

$$Z_T = 1 \quad Z_Q = 0 \quad Z_\psi = 1 + \frac{2\alpha}{3\pi} \Delta_{\text{UV}}$$

$$Z_L = -\frac{2\alpha}{3\pi} \Delta_{\text{UV}} \quad Z_M = -\frac{\alpha}{12\pi} \Delta_{\text{UV}} \quad Z_B = 0$$

$$Z_S = -\frac{7\alpha}{12\pi} \Delta_{\text{UV}} \begin{cases} +0 & \overline{\text{MS}} \\ -\frac{7\alpha}{24\pi} & \text{D}_1 \\ +\frac{\alpha}{12\pi} & \text{D}_2 \end{cases}$$

$$Z_K = -\frac{\alpha}{6\pi} \Delta_{\text{UV}} \begin{cases} +0 & \overline{\text{MS}} \\ +\frac{7\alpha}{24\pi} & \text{D}_1 \\ -\frac{\alpha}{12\pi} & \text{D}_2 \end{cases}$$

$$\Delta_{\text{UV}} = \frac{1}{\varepsilon} + \log(4\pi) - \gamma_E$$

How to obtain the form factors: QED (1/2)

1-loop calculation!

$$\mathcal{L} = \log\left(\frac{\mu^2}{m^2}\right)$$

$$L_{tot,R} (\Delta = 0) = 2P^\mu P^\nu \left(1 + \frac{\alpha}{\pi\varepsilon_I} - \frac{\alpha}{\pi} - \frac{\alpha\mathcal{L}}{4\pi} \right)$$

$$(V_1 + V_2)_R (\Delta = 0) = 2P^\mu P^\nu \left(-\frac{\alpha\mathcal{L}}{2\pi} - \frac{3\alpha}{2\pi} \right) - 2m^2 g^{\mu\nu} \times \begin{cases} \left(\frac{\alpha\mathcal{L}}{4\pi} + \frac{\alpha}{4\pi} \right) & \overline{\text{MS}} \\ \left(\frac{\alpha\mathcal{L}}{4\pi} - \frac{\alpha}{24\pi} \right) & D_1 \\ \left(\frac{\alpha\mathcal{L}}{4\pi} + \frac{\alpha}{3\pi} \right) & D_2 \end{cases}$$

$$V_{3,R} (\Delta = 0) = 2P^\mu P^\nu \left(-\frac{\alpha}{\pi\varepsilon_I} + \frac{14\alpha}{9\pi} + \frac{\alpha\mathcal{L}}{12\pi} \right) + 2m^2 g^{\mu\nu} \left(\frac{5\alpha\mathcal{L}}{12} + \frac{7\alpha}{36\pi} \right)$$

$$V_{4,R} (\Delta = 0) = 2P^\mu P^\nu \left(\frac{2\alpha\mathcal{L}}{3\pi} + \frac{17\alpha}{18\pi} \right) + 2m^2 g^{\mu\nu} \times \begin{cases} \left(-\frac{\alpha\mathcal{L}}{6\pi} + \frac{\alpha}{18\pi} \right) & \overline{\text{MS}} \\ \left(-\frac{\alpha\mathcal{L}}{6\pi} - \frac{17\alpha}{72\pi} \right) & D_1 \\ \left(-\frac{\alpha\mathcal{L}}{6\pi} + \frac{5\alpha}{36\pi} \right) & D_2 \end{cases}$$

How to obtain the form factors: QED (2/2)

$$\mathcal{L} = \log \left(\frac{\mu^2}{m^2} \right)$$

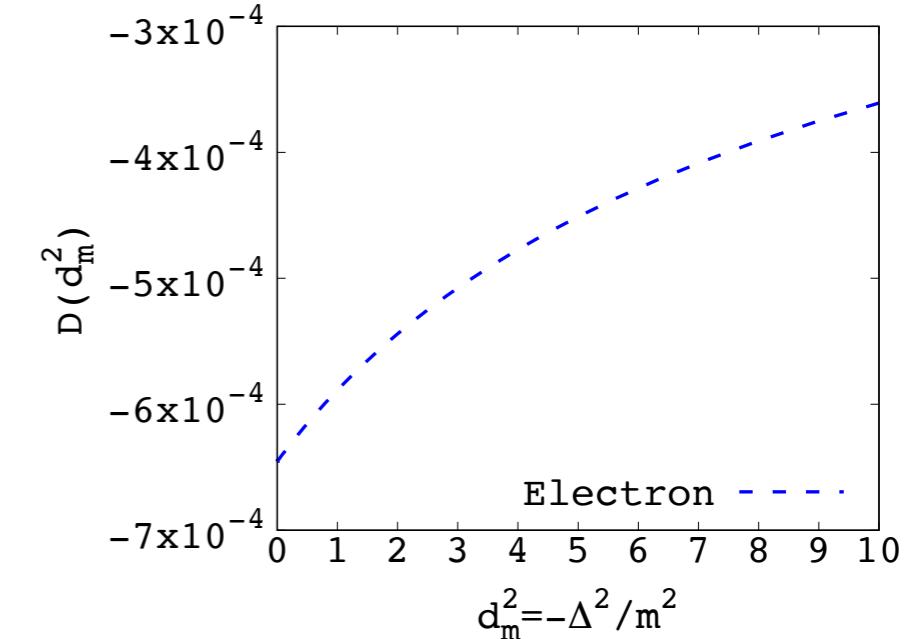
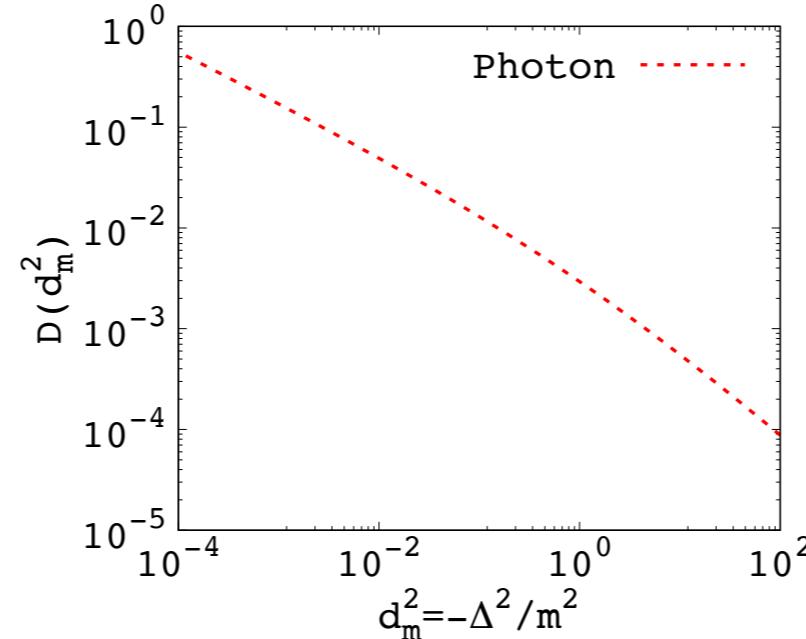
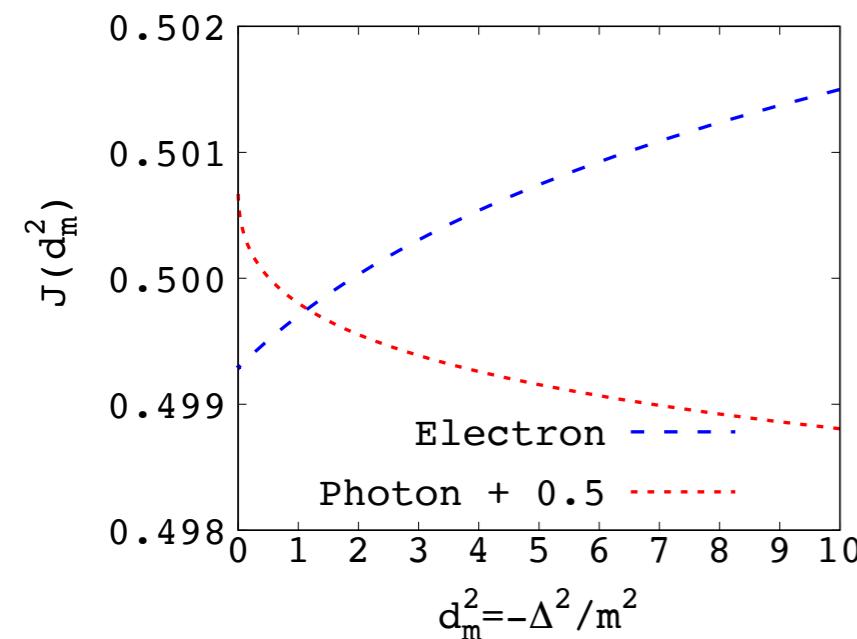
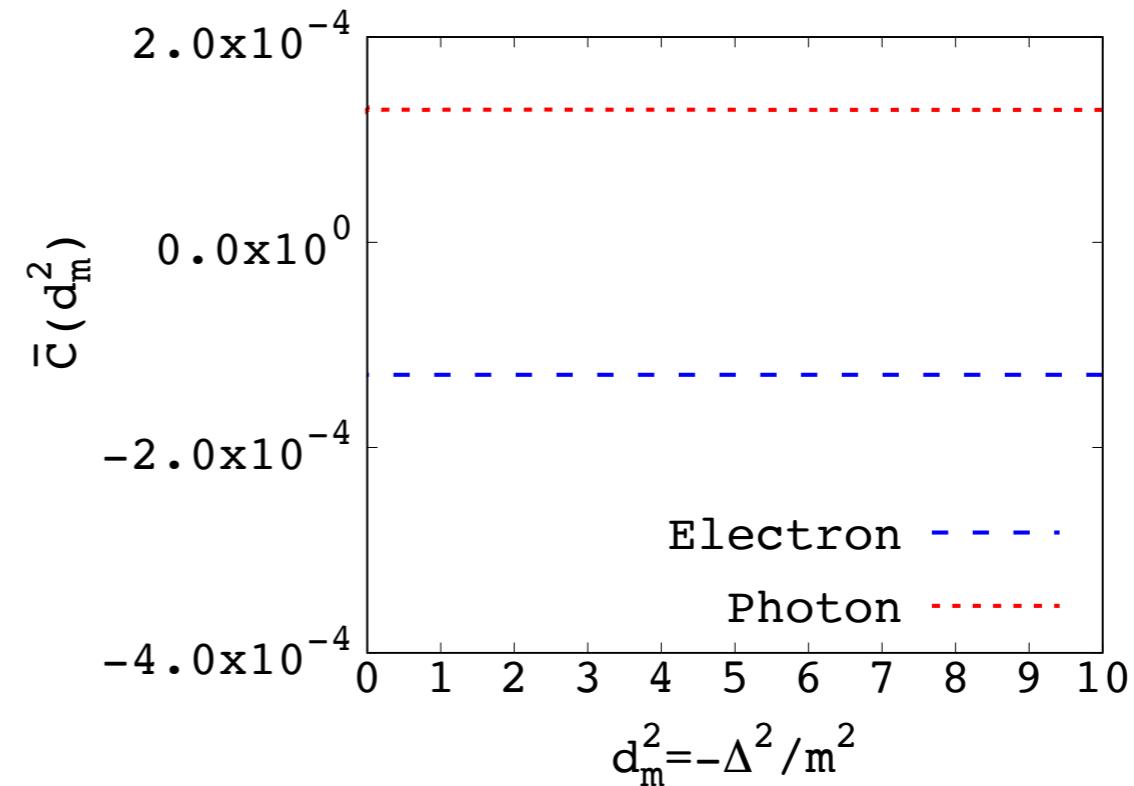
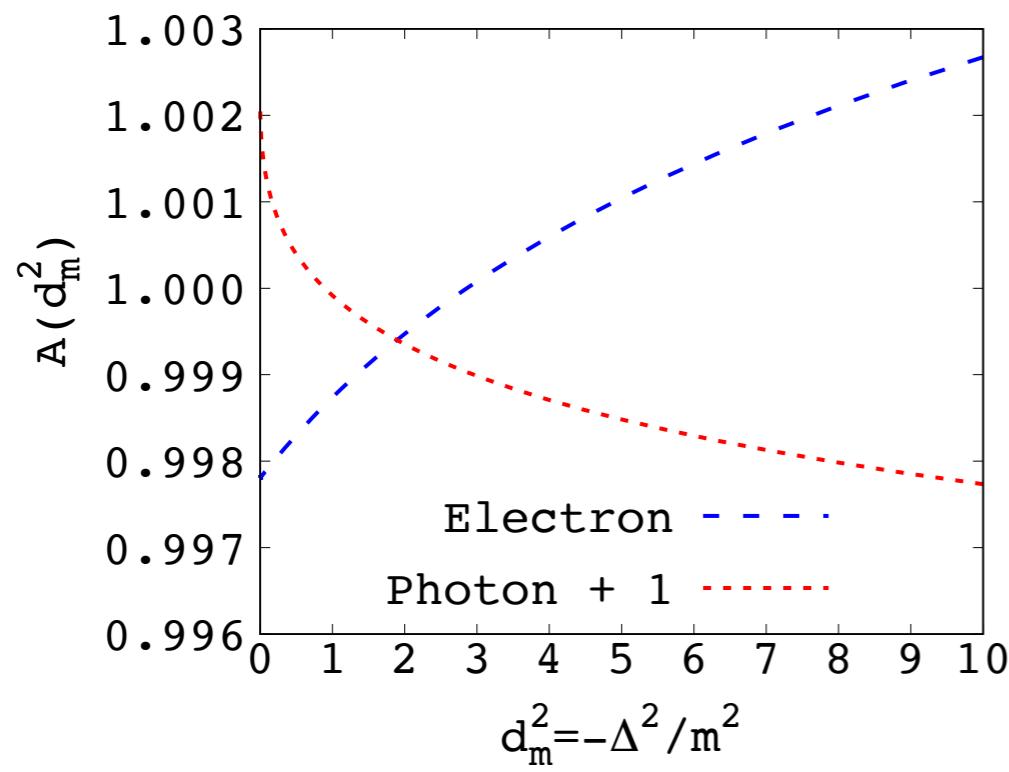
$$A_e^R(0) = 1 - \frac{2\alpha\mathcal{L}}{3\pi} - \frac{17}{18}\frac{\alpha}{\pi}$$

$$A_\gamma^R(0) = \frac{2\alpha\mathcal{L}}{3\pi} + \frac{17}{18}\frac{\alpha}{\pi}$$

$$\bar{C}_e^R(0) = \frac{\alpha\mathcal{L}}{6\pi} \begin{cases} -\frac{\alpha}{18\pi} & \overline{\text{MS}} \\ +\frac{17\alpha}{72\pi} & D_1 \\ -\frac{5\alpha}{36\pi} & D_2 \end{cases}$$

$$\bar{C}_\gamma^R(0) = -\frac{\alpha\mathcal{L}}{6\pi} \begin{cases} +\frac{\alpha}{18\pi} & \overline{\text{MS}} \\ -\frac{17}{72}\frac{\alpha}{\pi} & D_1 \\ +\frac{5\alpha}{36\pi} & D_2 \end{cases}$$

Off-forward form factors: a quick peek



Summary

Renormalization of multiple local operators



Renormalization of the form factors (scheme dependence)



1-loop QED calculation as simple example

Conclusions

**The EMT form factors are the fundamental building blocks
for all the mass decompositions.**

**Scheme dependence prevents a unique interpretation
of the terms of the decompositions**