MAKING SENSE OF THE NAMBU-JONA-LASINIO MODEL VIA SCALE INVARIANCE

Philip D. Mannheim

University of Connecticut

Presentation at Third Proton Mass Workshop: Origin and Perspective

January 2021

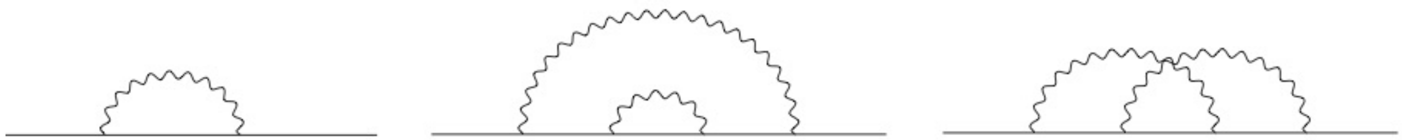
P. D. Mannheim, Living Without Supersymmetry – the Conformal Alternative and a Dynamical Higgs Boson, J. Phys. G 44, 115003 (2017). (arXiv:1506.01399 [hep-ph])

P. D. Mannheim, Mass Generation, the Cosmological Constant Problem, Conformal Symmetry, and the Higgs Boson, Prog. Part. Nucl. Phys. 94, 125 (2017). (arXiv:1610:08907 [hep-ph])

The status of the chiral-invariant Nambu-Jona-Lasinio (NJL) four-fermi model is quite equivocal. It serves as the paradigm for dynamical symmetry breaking and yet it is not renormalizable. So one looks to obtain dynamical symmetry breaking in a gauge theory instead. An early attempt was Maskawa and Nakajima (1974). They studied the quenched (i.e. bare) photon, planar graph approximation to an Abelian gauge theory,

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^{\mu}(i\partial_{\mu} - e_0 A_{\mu})\psi - m_0\bar{\psi}\psi. \quad (1)$$

They kept the first two graphs and their iterations but not the non-planar third:



With $S^{-1}(p) = \not{p} - B(p^2)$ they solved the fermion propagator Schwinger-Dyson equation and obtained

$$B(p^2) = m_0 + \frac{3\alpha}{4\pi}\left[\int_0^{p^2} dq^2 \frac{q^2 B(q^2)}{p^2[q^2+B^2(q^2)]} + \int_{p^2}^{\infty} dq^2 \frac{B(q^2)}{[q^2+B^2(q^2)]}\right], \quad B(p^2) = m\left(\frac{-p^2}{m^2}\right)^{\gamma_\theta/2}, \quad \gamma_\theta + 1 = \pm\left(1-\frac{3\alpha}{\pi}\right)^{1/2}, \quad m_0 = \frac{3\alpha m}{2\pi\gamma_\theta}\frac{\Lambda^{\gamma_\theta}}{m^{\gamma_\theta}}. \quad (2)$$

Solutions to this equation depend on whether $\alpha = e_0^2/4\pi$ is less than or greater than $\pi/3$. For $\alpha > \pi/3$ we get $m_0 = 0$ identically and have dynamical symmetry breaking and a dynamical Goldstone boson. However, for $\alpha < \pi/3$ it initially again appears that the bare mass is zero. However this time m_0 only vanishes in the limit of infinite cutoff. As noted by Baker and Johnson (1971) at the same time the multiplicative renormalization constant $Z_\theta^{-1/2}$ that renormalizes $\bar{\psi}\psi$ diverges as $\Lambda^{-\gamma_\theta}$, so that $m_0\bar{\psi}\psi$ is non-zero, and the chiral symmetry **is broken in the Lagrangian**. Despite the fact that the Schwinger-Dyson equation now becomes homogeneous and despite the fact that one is looking at its non-trivial self-consistent solution, there is then no Goldstone boson and this is known as the Baker-Johnson evasion of the Goldstone theorem. Conventional wisdom: **One can get dynamical symmetry breaking in a gauge theory if the coupling is big enough. But BCS is a counterexample.**

But for large α (viz. $> \pi/3$) the third graph cannot be ignored. So is it valid to claim that there is a phase transition? Johnson, Baker, Wiley (1961) found all-order quenched planar plus non-planar graph solution scales for any value of α. So phase transition at $\alpha = \pi/3$ is just an artifact of using a perturbative result outside of its domain of validity. Even if dress photon propagator find same result if fixed point with $\beta(\alpha) = 0$. γ_θ is the anomalous dimension of $\theta = \bar{\psi}\psi$. At $\alpha = \pi/3$, $\gamma_\theta = -1$, we have $d[\bar{\psi}\psi] = 3 + \gamma_\theta = 2$. Is dressed four-Fermi vertex then renormalizable? Suggested in Mannheim (1975), proven to all orders in Mannheim (2017). But not point-coupled Nambu-Jona-Lasinio model.

NJL Model is a Mean-Field plus Residual Interaction Theory. Introduce mass term with m as a trial parameter and note $m^2/2g$ term.

$$I_{\rm NJL} = \int d^4x \left[i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \frac{g}{2}[\bar{\psi}\psi]^2 - \frac{g}{2}[\bar{\psi}i\gamma_5\psi]^2\right] = \int d^4x \left[i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi + \frac{m^2}{2g}\right] + \int d^4x\left[-\frac{g}{2}\left(\bar{\psi}\psi - \frac{m}{g}\right)^2 - \frac{g}{2}\left(\bar{\psi}i\gamma_5\psi\right)^2\right] \quad (3)$$

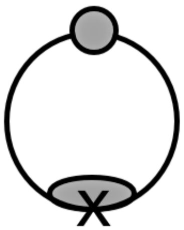
$$\langle\Omega_{\rm m}|\left[\bar{\psi}\psi - \frac{m}{g}\right]^2|\Omega_{\rm m}\rangle = \langle\Omega_{\rm m}|\left[\bar{\psi}\psi - \frac{m}{g}\right]|\Omega_{\rm m}\rangle^2 = 0, \quad \langle\Omega_{\rm m}|\bar{\psi}\psi|\Omega_{\rm m}\rangle = -i\int\frac{d^4k}{(2\pi)^4}{\rm Tr}\left[\frac{1}{\not{p} - m + i\epsilon}\right] = \frac{m}{g}, \quad -\frac{M\Lambda^2}{4\pi^2} + \frac{M^3}{4\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) = \frac{M}{g}. \quad (4)$$

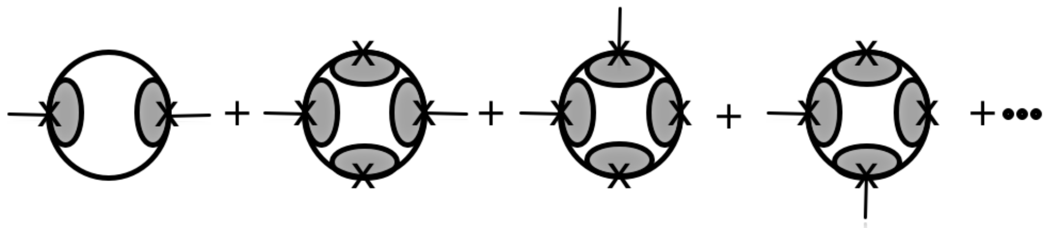
Dilemma: Gauge theory renormalizable, no dynamical symmetry breaking, NJL not renormalizable but has dynamical symmetry breaking.

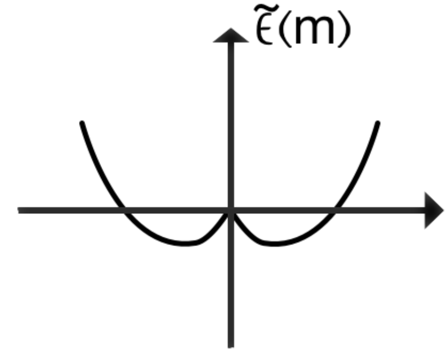
Solution: They cure each other.

$$\mathcal{L}_{\rm QED-FF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^{\mu}(i\partial_{\mu} - eA_{\mu})\psi - m\bar{\psi}\psi + \frac{m^2}{2g} - \frac{g}{2}\left(\bar{\psi}\psi - \frac{m}{g}\right)^2 - \frac{g}{2}\left(\bar{\psi}i\gamma_5\psi\right)^2 \tag{5}$$

$$\tilde{S}^{-1}(p) = \not{p} - m\left(\frac{-p^2 - i\epsilon}{\mu^2}\right)^{-1/2} + i\epsilon, \quad \tilde{\Gamma}_{\rm S}(p,p,0) = \left(\frac{-p^2 - i\epsilon}{\mu^2}\right)^{-1/2} \tag{6}$$







$$\langle \Omega_m|\bar{\psi}\psi|\Omega_m\rangle = -\frac{m\mu^2}{4\pi^2}{\rm ln}\left(\frac{\Lambda^2}{m\mu}\right) = \frac{m}{g}, \quad -\frac{\mu^2}{4\pi^2}{\rm ln}\left(\frac{\Lambda^2}{M\mu}\right) = \frac{1}{g}, \quad M = \frac{\Lambda^2}{\mu}\exp\left(\frac{4\pi^2}{\mu^2 g}\right). \tag{7}$$

Gap equation gives $-g \sim 1/{\rm ln}\Lambda^2$. Thus g is negative, i.e. attractive, and becomes very small as $\Lambda \to \infty$, with BCS-type essential singularity in gap equation at $g = 0$. Hence dynamical symmetry breaking with weak coupling.

$$\epsilon(m) = \frac{i}{2}\int\frac{d^4p}{(2\pi)^4}{\rm Tr\ ln}\left[1 - \frac{m^2}{p^2 + i\epsilon}\left(\frac{-p^2 - i\epsilon}{\mu^2}\right)^{-1}\right] = -\frac{m^2\mu^2}{8\pi^2}\left[{\rm ln}\left(\frac{\Lambda^2}{m\mu}\right) + \frac{1}{2}\right], \quad \tilde{\epsilon}(m) = \epsilon(m) - \frac{m^2}{2g} = \frac{m^2\mu^2}{16\pi^2}\left[{\rm ln}\left(\frac{m^2}{M^2}\right) - 1\right], \tag{8}$$

$\epsilon(m)$ is only log divergent. Due to presence of $m^2/2g$ term, $\tilde{\epsilon}(m)$ is **completely finite**. Dynamically induce double-well potential with no fundamental $-\mu^2\phi^2$ term (Mannheim 1975). We thus see the power of dynamical symmetry breaking. It reduces divergences. Moreover, since $m^2/2g$ is a cosmological term, dynamical symmetry breaking has a control over the cosmological constant problem that an elementary Higgs field potential does not. When coupled to conformal gravity, the cosmological constant problem is completely solved (Mannheim 2017).

Higgs-Like Lagrangian (Mannheim 1978)

$$\mathcal{L}_{\rm EFF} = -\frac{m^2(x)\mu^2}{16\pi^2}\left[{\rm ln}\left(\frac{m^2(x)}{M^2}\right) - 1\right] + \frac{3\mu}{256\pi m(x)}\partial_{\mu}m(x)\partial^{\mu}m(x) + ... \tag{9}$$

From residual interaction with $T = g + g\Pi g + .. = g/(g - \Pi)$ get Goldstone and Higgs bound states (Mannheim 2017) with completly finite

$$T_{\rm P}(q^2) = \frac{128\pi M}{7\mu q^2} = \frac{57.446M}{\mu q^2}, \quad q^2({\rm Goldstone}) = 0, \quad q^2({\rm Higgs}) = (2.189 - 0.051{\rm i}){\rm M}^2. \tag{10}$$

Dynamical Higgs boson solves hierachy problem. Dynamical Higgs mass is close to dynamical fermion mass, but above threshold with narrow width. In a double-well elementary Higgs field theory Higgs mass is real. Width can be used to distinguish an elementary Higgs from a dynamical one.