Inelastic Reactions on a Quantum Computer

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Realistic many-nucleon calculations rely on stochastic methods. These methods use the imaginary time. Scattering is a real-time process.

Physics problem: Calculate inelastic processes $a(b,\gamma)c$, $a(b,c)d$, ...

The issue: *In nature, entanglement is a friend and $a(b,\gamma)c \leftrightarrow c(\gamma,b)a$ is negligible. Moreover, the photon has many allowed modes.*

- **Generic algorithm** : Transition $\langle E_f|O_T|E_i\rangle$ initiated by coupling to an ancilla "photon" qubit through an X- or Y-gate. Photon counting register $|0\rangle \leftrightarrow |1\rangle$ signals transition.
- The photon energy can be tuned to drive the system to a resonance like NMR, and achieve 100% transition probability.

1: Encode the system and add an extra "photon" qubit.

2: Prepare the initial state as $|1\rangle \otimes |E_i\rangle$.

3: Evolve initial state with $e^{-iH_T t}$.

4: Measure the photon qubit.

$$H_T = \mathbb{1} \otimes H + \frac{\omega}{2}(\mathbb{1} - Z) \otimes \mathbb{1} + c_0\,(\phi^\dagger + \phi) \otimes O_T$$

"Photon" creation and annihilation

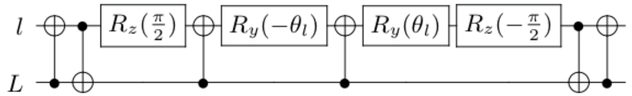
Physics near a resonance is two-level:

$$H_T^{\rm eff} = \begin{pmatrix} E_f & \omega_1/2 \\ \omega_1/2 & E_i + \omega \end{pmatrix}, \quad \text{with} \quad \omega_1 = 2c_0\langle E_f|O_T|E_i\rangle$$

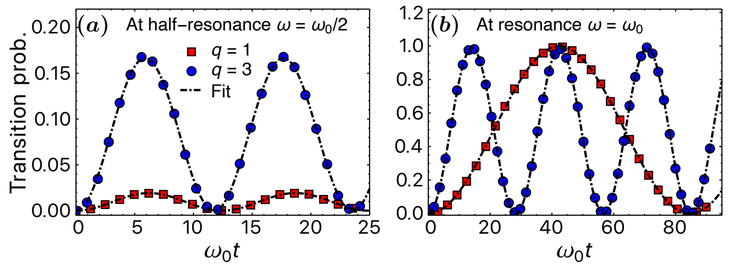
Transition probability: $$P(t) = \frac{\omega_1^2}{(E_f - E_i - \omega)^2 + \omega_1^2}\sin^2\frac{\sqrt{(E_f - E_i - \omega)^2 + \omega_1^2}\,t}{2}$$

Example 1: E1 dipole transition in single particle SHO levels

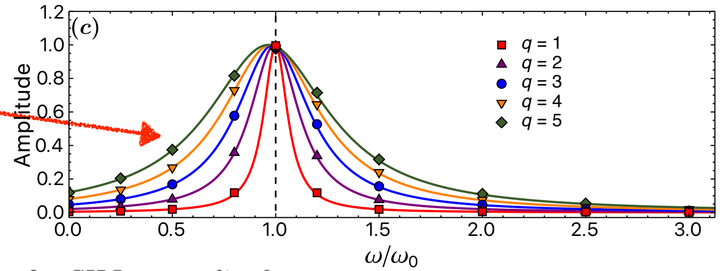
Interaction motivated by minimal coupling $$H_I = qe\sigma^2 X \sum_l l\hat{\psi}_l^\dagger\hat{\psi}_l$$



Results from Qulacs simulator (21+1 qubits)



Fitted probability amplitude at half-resonance frequency to the two-level Lorentzian form

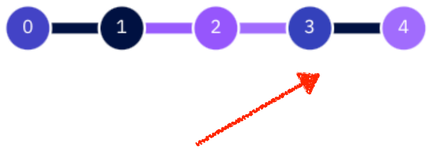


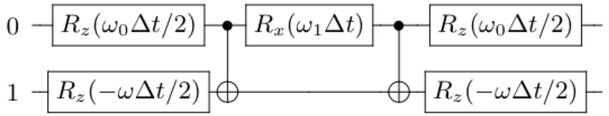
Recover known continuum, infinite volume results

TABLE I. Fit parameter $\Delta\bar{E}$ and $\bar{\omega}_1$ for SHO normalized to expected oscillator frequency $\omega_0 = 8\,\text{MeV}$ and $\omega_1^{(\text{th})}$, respectively. The first set of $(\Delta\bar{E}, \bar{\omega}_1)$ is from fits at $\omega = \omega_0/2$ and the second set with the superscript R is at resonance $\omega = \omega_0$.

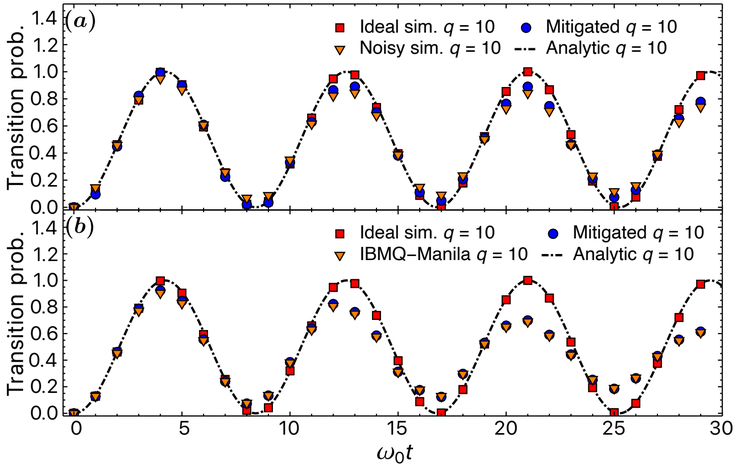
q	$\Delta\bar{E}$	$\bar{\omega}_1$	$\Delta\bar{E}^{(R)}$	$\bar{\omega}_1^{(R)}$
1	1.000(6)	0.936(2)	0.996(8)	0.9911(6)
2	0.994(5)	0.967(4)	0.988(11)	0.9866(10)
3	0.987(3)	0.972(3)	0.976(8)	0.9834(9)
4	0.977(2)	0.966(2)	0.961(10)	0.9793(13)
5	0.965(2)	0.958(2)	0.940(10)	0.9735(16)

Example 2: Two-level systems on IBMQ (ibmq_manila)





Qubits 3 and 4 were used for 1024 measurements



Algorithmic Robustness

Locations of peaks and valleys sufficient for transition element

$$\omega_1^{(\mathrm{meas.})} \approx 1.03 \pm 0.03\, \omega_1^{(\mathrm{th})}$$

Details in arXiv: 2209.09962 [nucl-th]

Why does it work? Tuning ω near the resonance, the H_T^{eff} eigenstates $|0\rangle_x$, $|1\rangle_x$ are orthogonal to the initial state $|1\rangle$. Time evolution then efficiently mixes $|0\rangle = \dfrac{|0\rangle_x + |1\rangle_x}{2}$ and $|1\rangle = \dfrac{|0\rangle_x - |1\rangle_x}{2}$

Collaborators

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Some Conclusions

1. Quantum computing might provide an advantage in real-time calculation of electro-weak processes. Efforts in this direction is important.
2. A generic algorithm for inelastic processes, final/excited state preparation.
3. Work is in progress to calculate $p(n,\gamma)d$, building from lower to higher dimensions.
4. Green's function approach to calculate triton binding energy.
5. Initial state preparation.
6. Common tools for the community.
7. Training of graduate students in the technology of the future. Diversity in funding.

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