Large-Momentum Effective Theory vs. Short-Distance OPE: Contrast and Complementarity

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Outline

- Physics of LaMET matching
- Precision control
- LaMET vs. short-distance OPE: contrast and complementarity
- Outlook

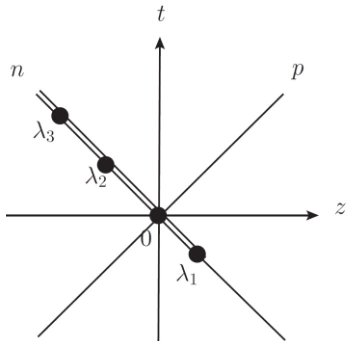
Physics of LaMET matching

Partons as static correlations in IMF

- Parton observables usually formulated as light-front correlators of fields

$$\hat{O} = \phi_1(\lambda_1 \mathrm{n}) W \phi_2(\lambda_2 n) \ldots W \phi_k(\lambda_k n)$$

ϕ_i: quark/gluon fields, W: Wilson link



- They are in fact matrix elements of equal time correlators

$$\hat{O} = \phi_1(\lambda_1 \mathrm{z}) W \phi_2(\lambda_2 z) \ldots W \phi_k(\lambda_k z)$$

in the hadron's infinite momentum frame (IMF) $P^z = \infty$

- However, taking the IMF limit in QFT is non-trivial!

Example: PDF

- PDFs have their origin in Mom. Dis. in a moving hadron $n\left(\vec{k}, P^z\right)$

  fundamental property of a quantum (many-body) system,

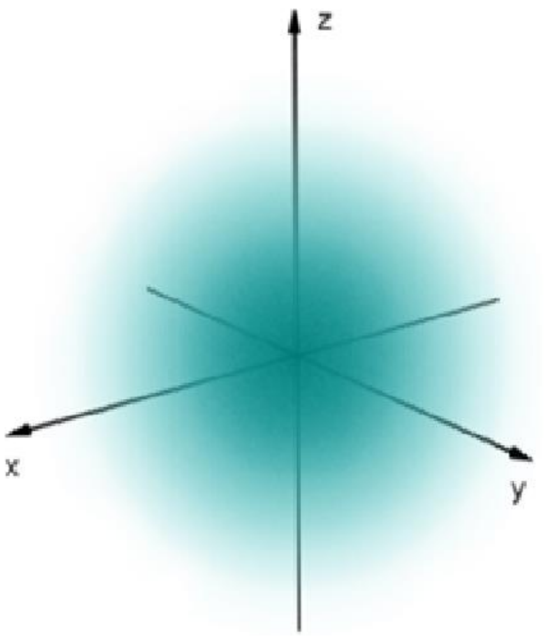
$$n\left(\vec{k}\right) = \left|\psi\left(\vec{k}\right)\right|^2 \sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3\vec{r}$$

- Static correlation functions can be calculated on lattice QCD

  With lattice spacing a

  imposes a UV cutoff $\Lambda_{UV} \sim 1/a$

$$n\left(\vec{k}, P^z, a\right)$$



IMF limit

- Longitudinal mom. dis. is

$$n(k^z, P^z, a) = \int d^2\vec{k}_\perp\, n(k^z, \vec{k}_\perp, P^z, a)$$

- When P^z is large, one must first take the continuum limit with

$$P^z \ll \Lambda_{UV} \sim 1/a, \quad \rightarrow \quad n\left(\vec{k}, P^z\right)$$

- Followed infinite momentum limit to get PDF

$$n(k^z, P^z) \rightarrow_{P^z \rightarrow \infty} f(x)\,? \quad \text{with } x = \frac{k^z}{P^z},$$

Does the limit exist?

Large momentum expansion

- When it does, $n(k^z, P^z)$ has a Taylor expansion around $P^z = \infty$,

$$n(k^z, P^z) = f(x) + f_4(x)(M/P^z)^2 + \ldots$$

A precise statement about large-P symmetry!

- One can get the PDFs from Mom. Dis. at large but finite P^z so long as M/P^z is small.

t' Hooft model

- 1+1D QCD with $N_c = \infty$

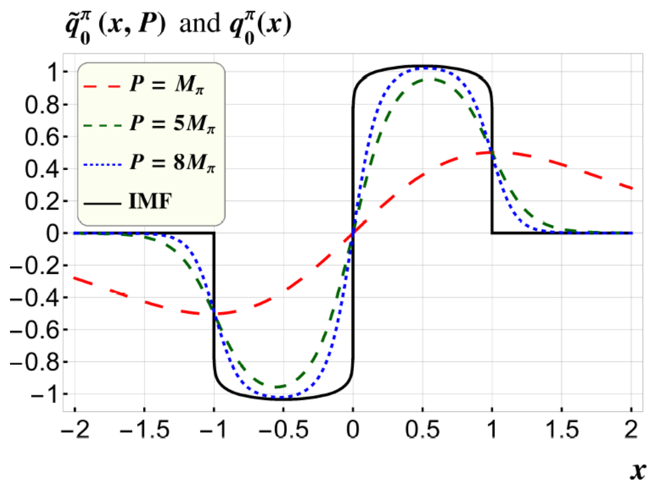
  Can be solved exactly at any finite P^z.

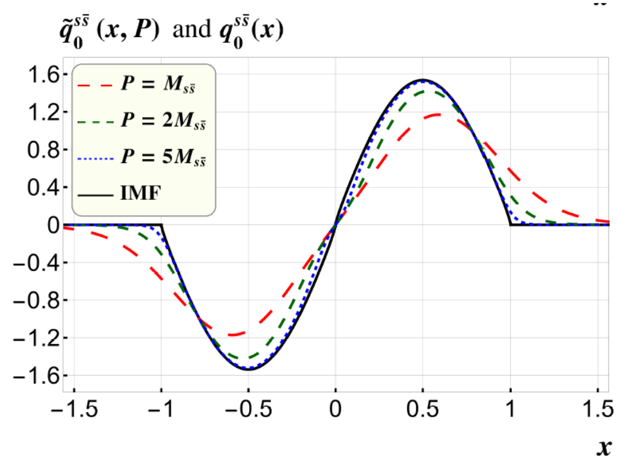
- Mom dis. Calculated at various mom:

  $p_\pi^z = m_\pi, 5m_\pi, 8m_\pi \ldots$

  $p_\phi^z = m_\phi, 2m_\phi, 5m_\phi \ldots$

- PDF obtained from the smooth limit of $p^z \to \infty$





Infinite momentum limit in 3+1 QCD

- A simple Feynman integral

$$\int^{\Lambda_{UV}} d^4k \frac{1}{(P+k)^2 k^2}$$

- Integral is UV divergent, Λ_{UV} shall be larger than any physics scales. The result depends on lnP.
- Naïve infinite momentum limit does not exist!
- However, parton physics is obtained by taking a different limit $P^z \to \infty$ first under the integral sign, followed by $\Lambda_{UV} \to \infty$ (Weinberg 1966, Dirac's LFQ 1949)

Matching relation between two limits

Instead of the simple Taylor expansion,

$$n(k^z, P^z) = f(x) + f_4(x)(M/P^z)^2 + \ldots$$

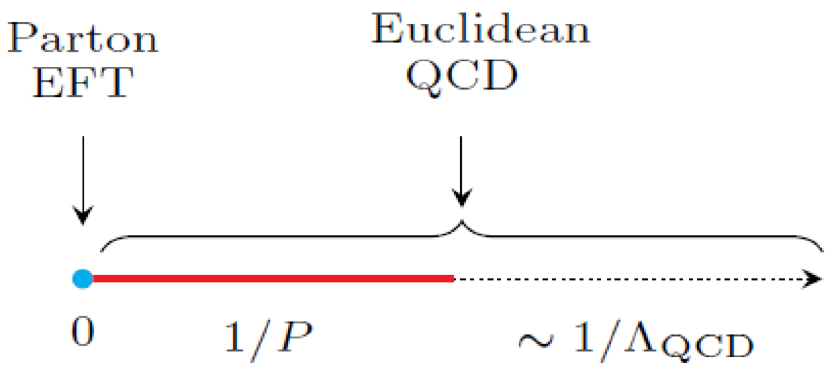
We have the relation between mom dis. in full QCD and PDFs in IMF disregarding UV div (Ji, 2013)

$$n(y, P^z) = \int Z(y/x, xP^z/\mu) f(x, \mu) dx + \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\Big),$$

All order in pert. QCD Ma and Qiu (2018), Izubuchi et al. (2018)

Generalization 1: Universality

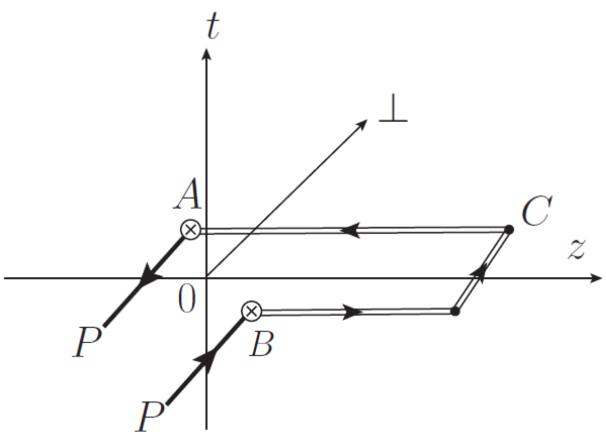
- The most natural quantities starting large momentum expansion are the corresponding finite P physical quantities (quasi-PDFs).
- One can use infinite number of Euclidean observables to achieve the same parton physics, such as current correlators, etc.

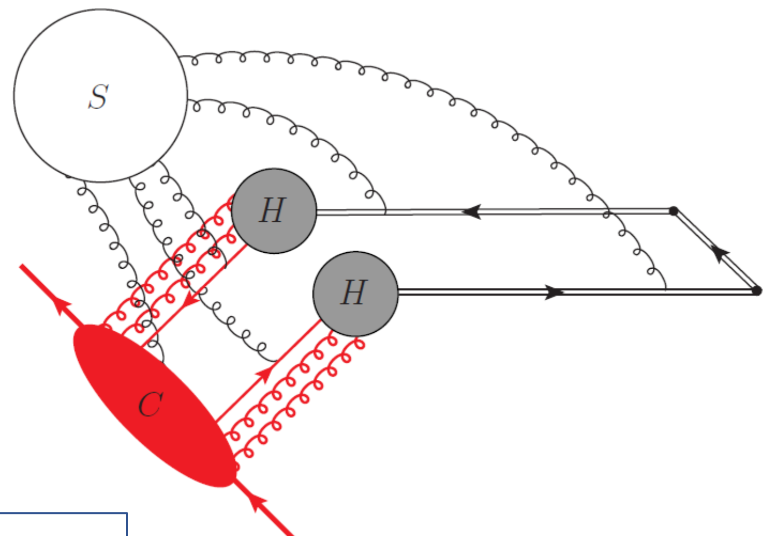


Generalization 2: TMDs, high twists, etc

- Large-momentum expansion can be naturally applied to TMDs.
  - TMD PDFs
  - TMD Wave Functions
- Soft functions
- Higher twists for parton correlations
- Other light-ray observables? Jet functions?

TMDPDF Matching (Ji, Liu, Liu, 2020, Ebert et al 2022)





$$\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)}$$
$$= H\left(\frac{\zeta_z}{\mu^2}\right) e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})} f^{\rm TMD}(x, b_\perp, \mu, \zeta) + \ldots$$

$$\mu \frac{d}{d\mu} \ln H\left(\frac{\zeta_z}{\mu^2}\right) = \Gamma_{\rm cusp} \ln \frac{\zeta_z}{\mu^2} + \gamma_C$$

Generalization 3: Light-Front Quantization (LFQ)

- Light-front quantized theory is formal (undefined!) and cannot be solved without regularizing light-cone singularities.
- If the regularization breaks Lorentz symmetry (almost all regulators in the LFQ literature do), theory ends up non-renormalizable.
- LFQ can be defined through large-momentum effective theories, including wave functions.

(X. Ji & Y. Liu, 2022 & to be published)

Precision Control

Power counting

- In the large-momentum expansion, small parameters are

$$\epsilon_i = \left(\frac{\Lambda_{QCD}}{k_i}\right)$$

where k is ANY physical momentum scale.

- In PDF calculation, k can be
  - Active quark/gluon, $k^z = xP^z$
  - Spectator, $k^z = (1-x)P^z$
- Thus, LaMET approach cannot calculate small and large-x partons unless P^z is very large, such that

$$xP^z, (1-x)P^z \gg \Lambda_{QCD}$$

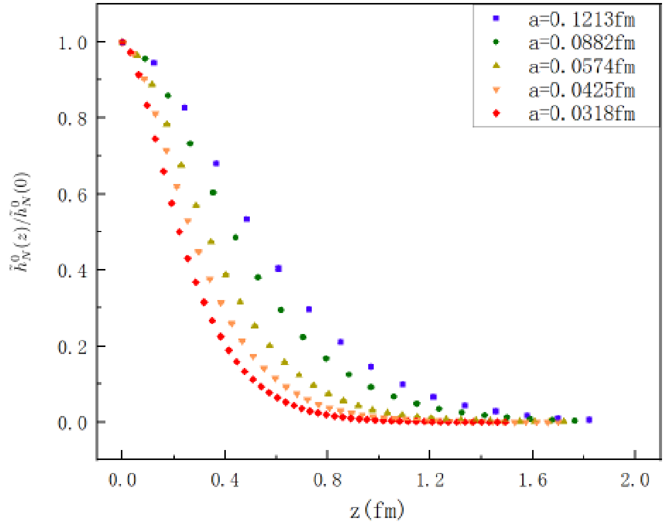
Linear divergence and continuum limit

- The quasi-PDF operator has linear Wilson line, which generate power law divergence (mass renorm.)
- These divergences must be subtracted carefully to take the continuum limit.
- Hybrid renormalization scheme (LPC)
- Renormalon ambiguity

  in mass subtraction

$$O_\Gamma(z)_R = Z_O^{-1} e^{\delta \bar{m} z} O_\Gamma(z),$$

$$\delta \bar{m} = m_{-1}(a)/a - m_0 \ ,$$



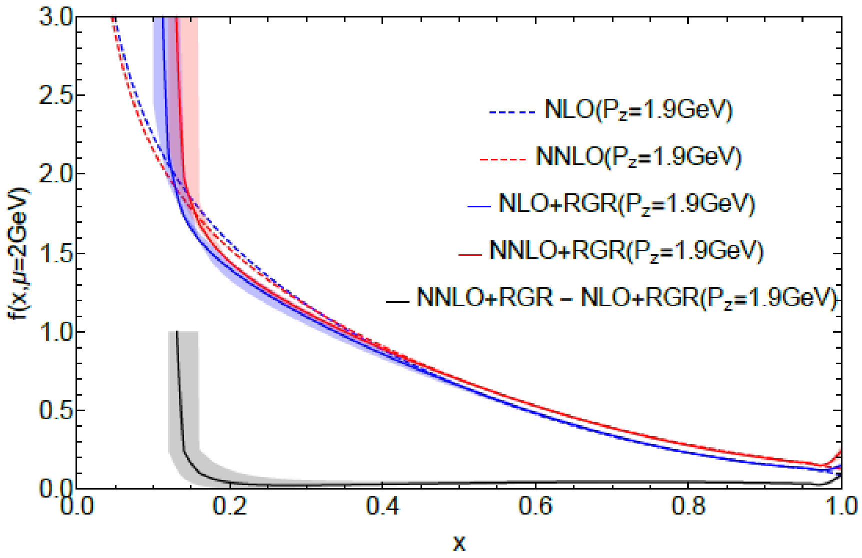
One-loop matching

$$\tilde{f}(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{|x|P_z}\right) f(y, \mu) + \mathcal{O}\left[\frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{\rm QCD}^2}{(1-x)^2 P_z^2}\right]$$

$$C^{(1)}\left(\xi, \frac{\mu}{|x|P_z}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left(\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} + 1 - \frac{3}{2(1-\xi)}\right)_{+(1)}^{[1,\infty]} & \xi > 1 \\ \left(\frac{1+\xi^2}{1-\xi} \left[-\ln \frac{\mu^2}{4x^2 P_z^2} + \ln(\frac{1-\xi}{\xi}) - 1\right] + 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[0,1]} & 0 < \xi < 1 \\ \left(-\frac{1+\xi^2}{1-\xi} \ln \frac{-\xi}{1-\xi} - 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]} & \xi < 0 \end{cases}$$

RG resummation (Y. Su et al, e-Print: 2209.01236)

- There is a large scale-gap between μ and 2xP, when x is small, or when P is large.



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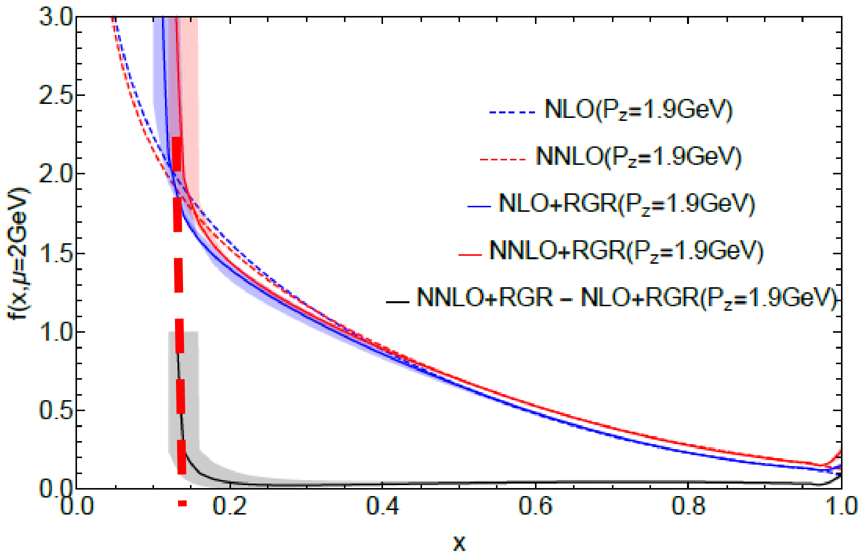
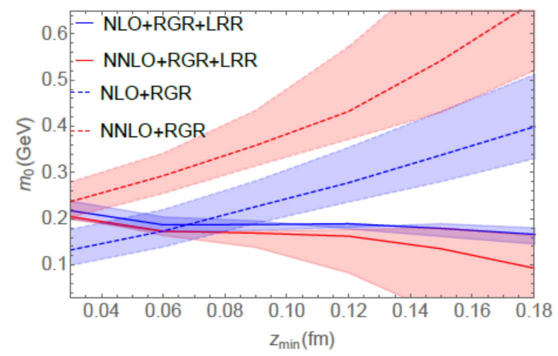
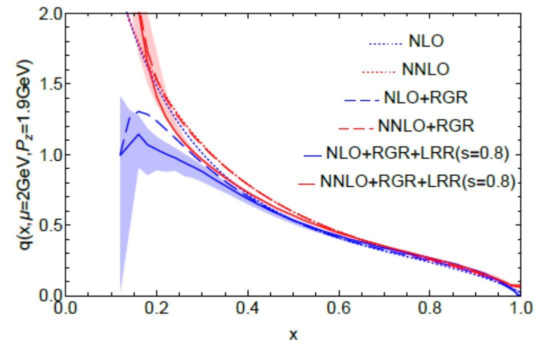


- Consistent with power counting.

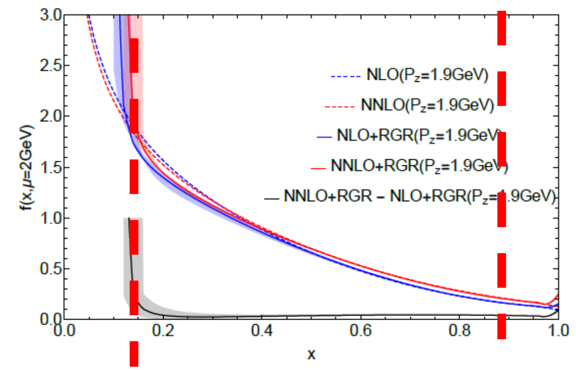
IR Renormalons (R Zhang et al)

- Mass renormalization contains IR renormalons
- The matching coefficient has also similar renormalon ambiguity.
- Both renormalon effects cancel.
- To obtain matching to the leading power accuracy, one must determine m_0 in consistency with the leading renormalon estimation.





Threshold resummation



- When $x \to 1$, the hadron remnant moment $(1-x)P^z$ becomes soft.
- This is now an incomplete cancellation of IR divergences between real and virtual contributions.

$$C^{(1)}\left(\xi, \frac{\mu}{|x|P_z}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1 - \frac{3}{2(1-\xi)}\right)_{+(1)}^{[1,\infty]} & \xi > 1 \\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{4x^2P_z^2} + \ln(\frac{1-\xi}{\xi}) - 1\right] + 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[0,1]} & 0 < \xi < 1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi} - 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]} & \xi < 0 \end{cases}$$

- Large logs of type $\left[\frac{\ln(1-x)}{1-x}\right]_+$ shall be resumed, can be done in momentum space as in DIS

LaMET vs. Short distance OPE contrast and complementarity

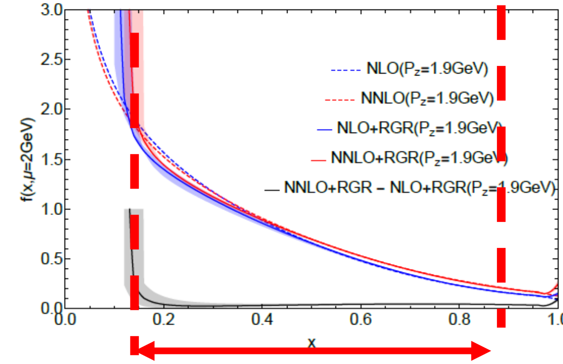
Beyond moments for collinear PDFs: OPE

- There has been continuous efforts in going beyond calculating *individual* PDF moments:
  - Hadron Tensor (Liu & Dong, 1994…)
  - OPE without OPE & Compton Tensor (Aglietti, Martinelli, 1998…Chambers et al, 2018)
  - Heavy-quark OPE (HOPE) (Detmold, Lin, 2006…)
  - Coordinate-space OPE (Braun & Muller,2008…)
  - Pseudo PDFs (Radyushkin, 2017…)
  - Good lattice cross section (Qiu & Ma, 2018…)
  - …

Operator product expansion (OPE):

first few moments /
a range of coordinate-space twist-2 correlator

Differences at finite P



- LaMET yields directly x-dependent parton physics at any $x \in [x_{min}, x_{max}]$ → local info on partons
- Short-distance OPE yields a twist-2 spatial correlation in a segment $\lambda \in [0, \lambda_{max}]$ → global info on partons

$$\lambda_{max} = zP^z$$

$\sim$ 0.2 fm x3 GeV

~ 3

The results cannot be converted into x-dependence without additional assumption

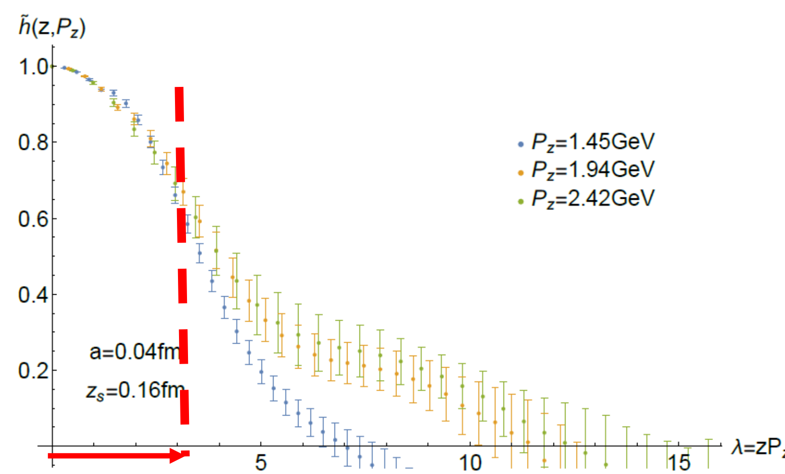


FIG. 3. Renormalized lattice matrix elements in the hybrid scheme (colorful points). $a = 0.04$ fm and $z_s = 0.16$ fm.

Fitting PDFs in short-distance OPE
(“Inverse Problem”)

- Given a finite range of coordinate space correlation, one can fit
  - Moments of PDF
  - Fits parametrization of PDF/structure functions, $f(x) = x^{\alpha}(1-x)^{\beta}\tilde{f}(x)$ as in global analysis

  Approximately equivalent
- See relevant talks

Complementarity (Ji, e-Print: 2209.09332)
)

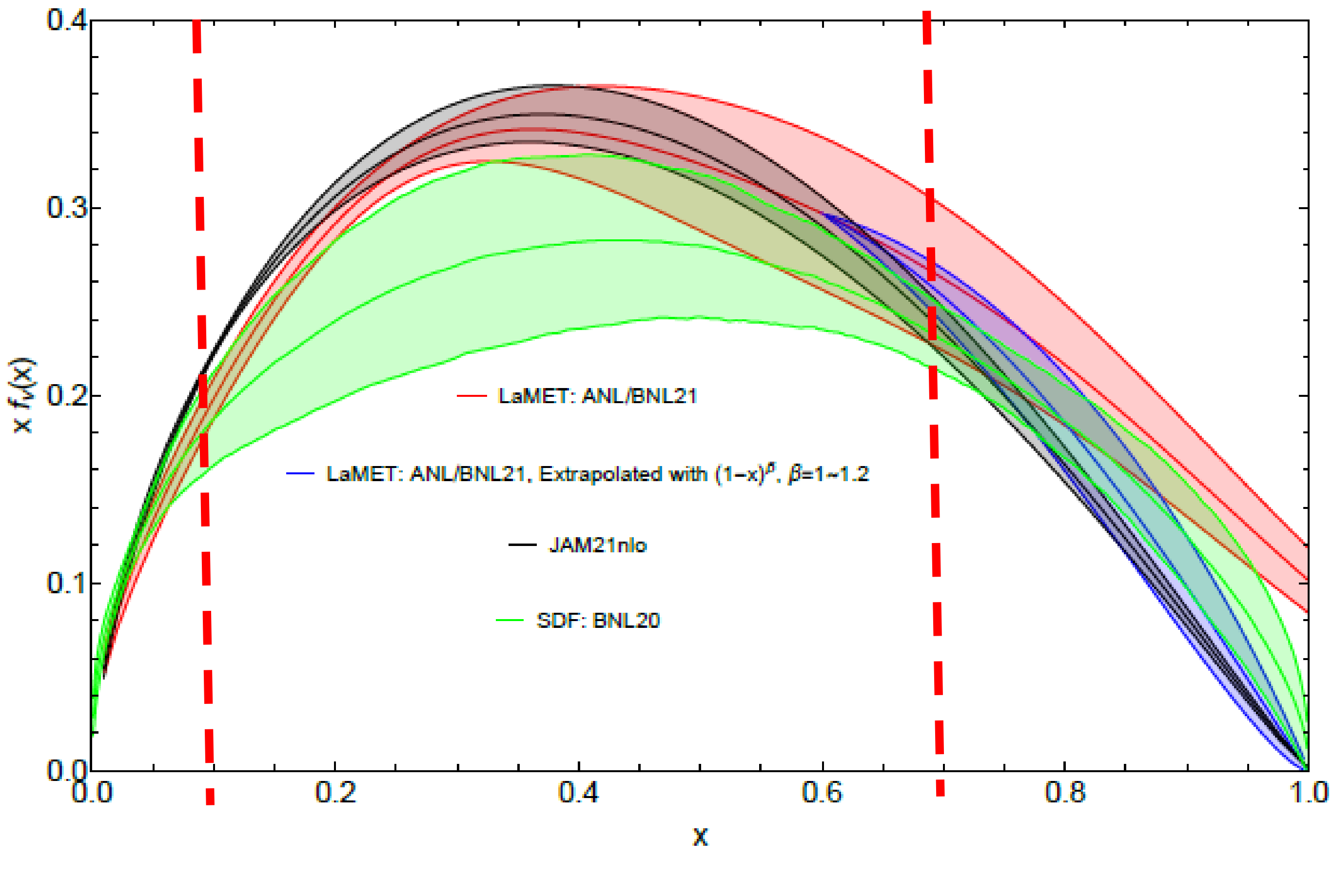
- LaMET generates a prediction for PDFs at a finite range $[x_{min}, x_{max}]$
- However, it cannot say anything in the end-point regions $[0, x_{min}]$ & $[x_{max}, 1]$
- Assuming phenomenological forms in these regions

$f(x) = Ax^{\alpha}$ for small x~0

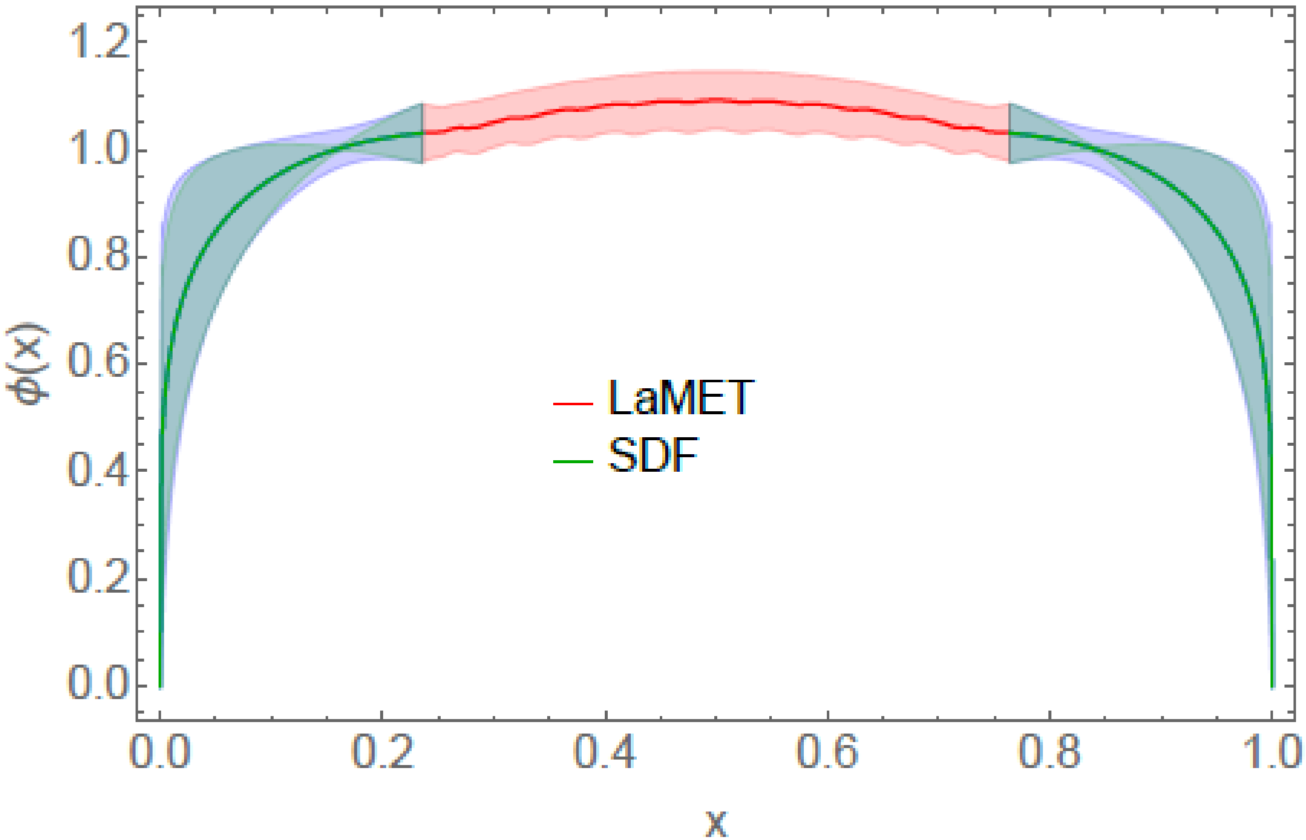
$f(x) = B(1-x)^{\beta}$ for x~1

One can fit these parameters to the global parton properties: moments or short distance correlations in OPE.

An example, pion PDF (Y. Zhao et al)



Pion Distribution Amplitude (R Zhang et al)



Summary

- LaMET aims to calculate parton physics at intermediate x region without doing global fitting.
- Short-distance OPE allows to calculate coordinate space correlations in a finite range (global parton properties). With an assumption on the functional form of PDFs at end points, one can use to this to constrain the local parton densities.
- LaMET augmented with SD-OPE can be used to constrain the partons in the entire x-range.