Nucleon Transversity PDF From Lattice QCD

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Lattice Parton Collabration

LaMET2022

Outline

LPC, arXiv:2208.08008 [hep-lat]

- Definition and application of transversity PDF
- LaMET frame work
- Lattice calculation of quark transversity PDF inside proton
- Summary

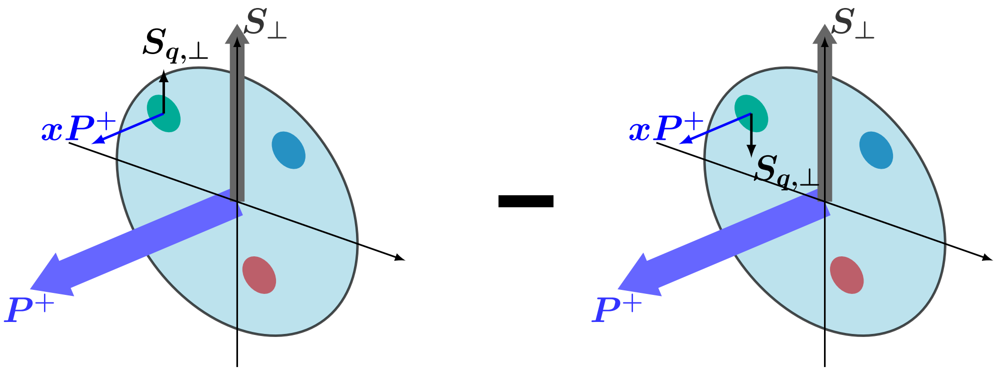
Definition of transversity PDF

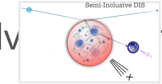
- Light-cone correlation function

$$\delta q_T(x,\mu) = \int \frac{d\xi^-}{2\pi P^+} e^{-ixP^+\xi^-} \left\langle P, S_\perp \left| \overline{\psi}\left(\frac{\xi^-}{2}\right) i\gamma^+\gamma^5\gamma^\perp \mathcal{W}\left[\frac{\xi^-}{2}, -\frac{\xi^-}{2}\right] \psi\left(-\frac{\xi^-}{2}\right) \right| P, S_\perp \right\rangle$$

- $S_\perp$: Spin of the nucleon (transversely polarized)
- Light cone coordinate: $\xi^\pm = \frac{\xi^0 \pm \xi^3}{\sqrt{2}}$
- Gauge link: $\mathcal{W}[\xi_1^-, \xi_2^-] = \mathcal{P}\left\{\exp\left[ig_s \int_{\xi_1^-}^{\xi_2^-} d\eta A^{+,a}(\eta) \mathrm{t}^a\right]\right\}$
  - Ensures the gauge invariant of the non-local correlation function
  - Origin: gluon exchnage between active parton and spectators

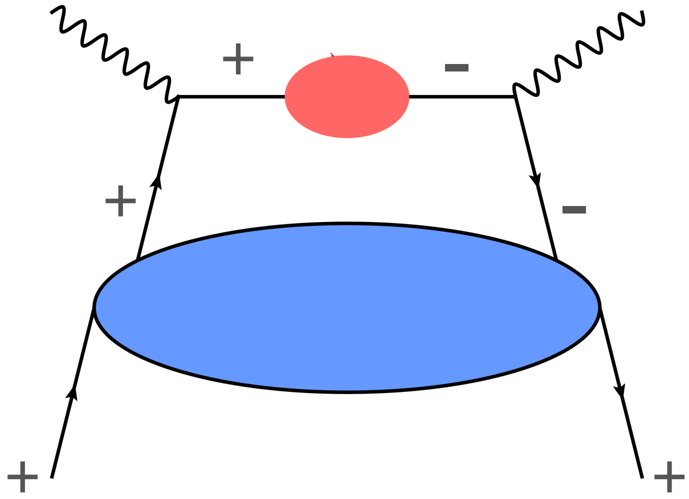
Probability Interpretation:



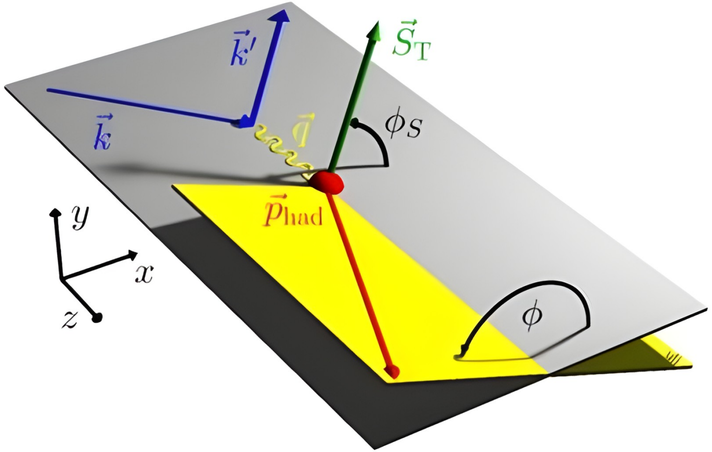
- Probablity distribution of finding a parton with spin parallel/anti-parallel to nucleon spin
- **δq_T is chiral odd, must be combined with a chiral odd hard process to be measured**
  - SIDIS process
  - Usually involv  nonperturbative quantities, e.g. Chiral odd fragmentation function
  - Less precisely determined experimentally

Measuring Transversity PDF,
— A chiral-odd distributon

- $\gamma^+\gamma^5\gamma^\perp$ ⇨ chiral odd, flips quark helicty
- Must combined with another chiral-odd process
  - Can't be partontic process ⇨
    QCD, QED preserve quark helicity
  - Could be nonperturbative process, e.g. fragmentation
    - e.g. $l\ N^\uparrow \to l'\ h\ X$,... ⇨ Collins mechanisum



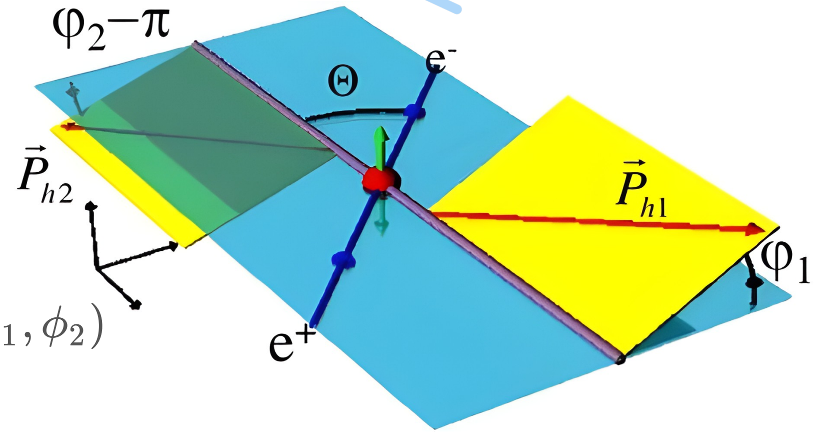
Collins Mechanism in Semi-Inclusive Deep Inelastic Scattering



$$l\, N^{\uparrow} \to l'\, h\, X \Rightarrow \boldsymbol{\delta q(x)} \otimes \boxed{D_h(z)} \sim \sin, \cos(\phi, \phi_S)$$

Universal chrial-odd fragmentation function

Very challenging to measure



$$e^-\, e^+ \to h_1\, h_2 \Rightarrow D_{h_1}(z_1) \otimes D_{h_2}(z_2) \sim \sin, \cos(\phi_1, \phi_2)$$

Calculate nonlocal light-cone correlation

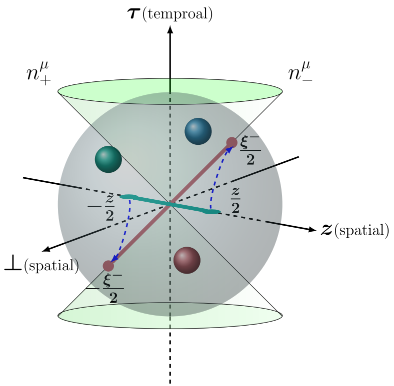
- Requires Euclidean spacetime to have probablity interpretation ⇨ MC sampling
- Light-Cone coordianates $\xi^{\pm} = \frac{\xi^0 \pm \xi^3}{\sqrt{2}}$ becomes complex after Wick rotation $\xi^{\pm} = \frac{i\xi^4 \pm \xi^3}{\sqrt{2}}$
- Mellin moments ⇦⇨ Matrix elements of local operators

$$\int dx\, x^{n-1} \delta q_T(x,\mu) \propto \langle P, S_\perp | \boxed{\overline{\psi}(0) \overleftrightarrow{D}^{\{+} \cdots i\gamma^+\gamma^5\gamma^\perp \cdots \overleftrightarrow{D}^{+\}} \psi(0)} | P, S_\perp \rangle$$

- 😄 local operator, no imaginary time dependence
- 😟
  - high order derivatives requires fine lattice spacing
  - operator mixing among higher momentens

LaMET Frame Work

- **1. Calculate Euclidean quasi quantities** $\langle P|\tilde{O}_E|P\rangle$



- The IMF quantities are large momentum limit of quasi quantities

$$\langle P_\infty|\mathcal{O}|P_\infty\rangle = \lim_{P\to\infty}\langle P|\tilde{O}_E|P\rangle$$

- e.g. quasi transveristy PDF

$$\delta\tilde{q}_T(x, P^z, \mu) = \int \frac{dz}{2\pi P^z} e^{ixP^z z} \left\langle P^z, S_\perp \left| \overline{\psi}\left(\frac{z}{2}\right) i\gamma^t\gamma^5\gamma^\perp \mathcal{W}\left[\frac{z}{2}, -\frac{z}{2}\right] \psi\left(-\frac{z}{2}\right) \right| P^z, S_\perp \right\rangle$$

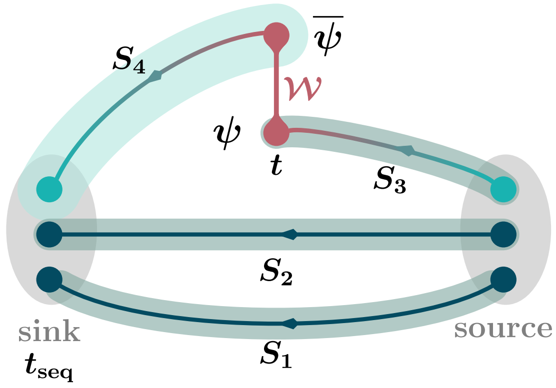
$\tilde{h}(z, P_z)$

  - : pure spatial correlation, can be calculated directly on lattice
  - $\delta\tilde{q}_T(x, P_z, \mu)$ has the same IR behavior of the light cone quantity $\delta q_T(x, \mu)$

X. Ji et al, Large-momentum effective theory, Rev. Mod. Phys. 93, 035005

Sequential propagator

1. Setup source
2. 1st inversion
3. Combine $S_{1,2}$ ⇨ seq. src
4. 2nd inv. with seq.src ⇨ S_4
5. Construct gauge link W
6. Assemble $S_{3,4}$, W and projectors



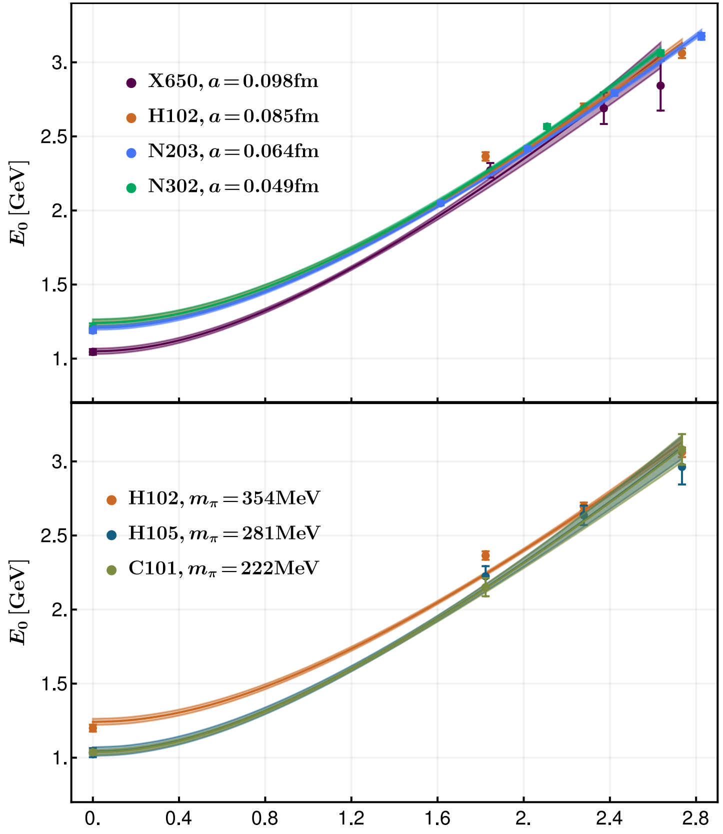
Access high momentum nucleon state

⇨ momentum smeared source

- Momentum smeared source

$$\psi(x) \Rightarrow \frac{1}{1+6\alpha}\left[\psi(x)+\alpha\sum_{j} U_j(x)e^{ik\hat{j}}\psi(x+\hat{j}a)\right]$$

> G. S. Bali, B. Lang, B. U. Musch and A. Schäfer, PRD 93, 094515 (2016)



Supercomputers

SuperMUC@LRZ

HPC@Kunshan

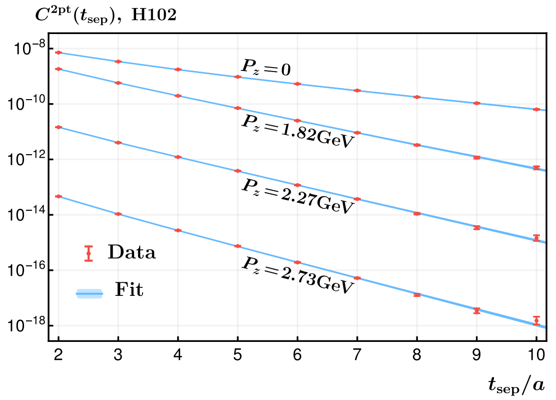
HPC@CSU

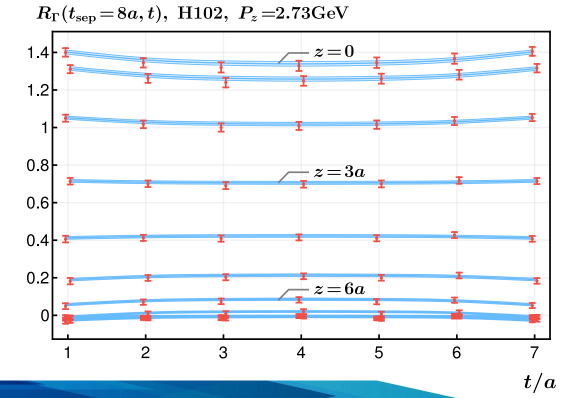
Extract ground state matrix element $\widetilde{h}(z, P_z)$

- Two states analysis (0: ground state, 1: 1st excited state)
  - $C^{2\text{pt}}(P_z, t_{\text{sep}}) = \underbrace{\boxed{|\mathcal{A}_0|^2 e^{-E_0 t_{\text{sep}}}}}_{\text{ground}} + |\mathcal{A}_1|^2 e^{-E_1 t_{\text{sep}}}$
  - $C_\Gamma^{3\text{pt}}(z, P_z, t, t_{\text{sep}}) = \underbrace{\boxed{|\mathcal{A}_0|^2 \langle 0|O_\Gamma|0\rangle e^{-E_0 t_{\text{sep}}}}}_{\text{ground}} + \mathcal{A}_1 \mathcal{A}_0^* \langle 1|O_\Gamma|0\rangle e^{-E_1(t_{\text{sep}}-t)} e^{-E_0 t}$
    $+\mathcal{A}_0 \mathcal{A}_1^* \langle 0|O_\Gamma|1\rangle e^{-E_0(t_{\text{sep}}-t)} e^{-E_1 t} + |\mathcal{A}_1|^2 \langle 1|O_\Gamma|1\rangle e^{-E_1 t_{\text{sep}}} + \cdots,$
- Combined fit ($0 < t < t_{\text{sep}}$)
  - $C^{2\text{pt}}(P_z, t_{\text{sep}}) = \underbrace{\boxed{|\mathcal{A}_0|^2 e^{-E_0 t_{\text{sep}}}}}_{\text{ground}} + |\mathcal{A}_1|^2 e^{-E_1 t_{\text{sep}}} + \cdots$
  - $R_\Gamma \equiv \dfrac{C_\Gamma^{3\text{pt}}(P_z, z, t_{\text{sep}}, t)}{C^{2\text{pt}}(P_z, t_{\text{sep}})} = \langle 0|O_\Gamma|0\rangle + (t, t_{\text{sep}} \text{ dependent terms})$

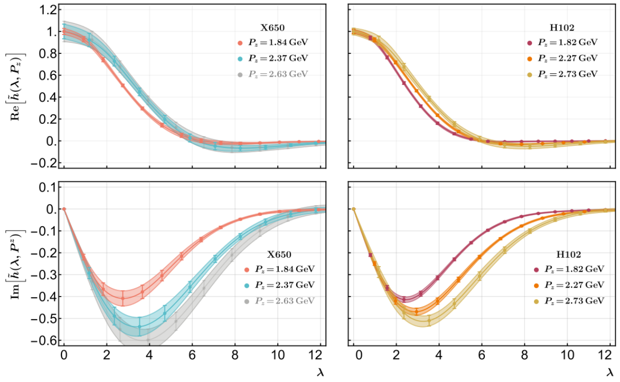
Ground state matrix element $\widetilde{h}(z, P_z)$

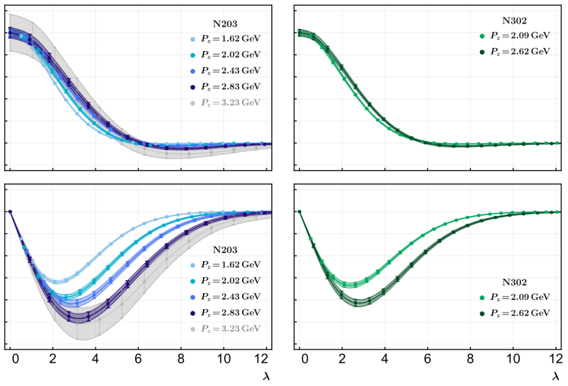
- Two-states fit





- Bare matrix element $\tilde{h}(z, P_z)$





2. Properly Renormalize $\tilde{h}(z, P_z)$ ⇨ $\tilde{h}_R(z, P_z)$

- Quasi lightfront correlator suffers from lineat UV divergence (but do not depend on external state)
- Orginate from gauge link self-energy ~ $e^{-|z|/a}$ ⇨ 😕 significantly dpressed the signal
- **The linear divergence must be removed before extracting light-cone correlations**
- The linear divergence is independent of hadron state
  - Use quark state ⇨ RI/MOM
  - Use hadron state at rest ⇨ Ratio scheme
- 😕 Bring undisired IR effect at long distance thus invalidates the LaMET factorization
- Hybrid renormalization scheme - so far the only reliable scheme won't introduce extra IR effects

> X. Ji *et al.*, NPB 964, 115311 (2021)   Y.K Huo *et al.* arXiv:2103.02965
> Jiunn-Wei Chen's takl today (session II)

Hybrid renormalization scheme

$$\delta\tilde{q}_T(x, P^z, \mu) = \int \frac{dz}{2\pi P^z} e^{ixP^z z} \boxed{\left\langle P^z, S_\perp \left| \overline{\psi}\left(\frac{z}{2}\right) i\gamma^t\gamma^5\gamma^\perp \mathcal{W}\left[\frac{z}{2}, -\frac{z}{2}\right] \psi\left(-\frac{z}{2}\right) \right| P^z, S_\perp \right\rangle}$$

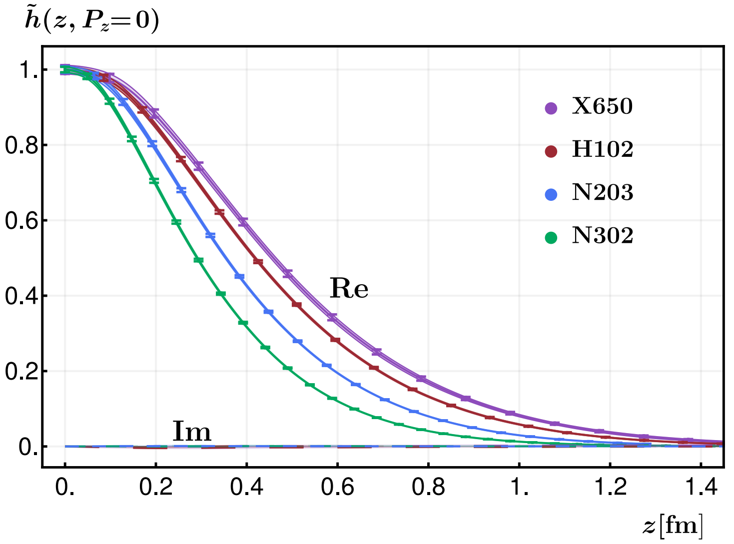
→ $\tilde{\boldsymbol{h}}(\boldsymbol{z}, \boldsymbol{a}, \boldsymbol{P_z})$

- $|z| < z_s$ ⇨
  - Linar divergence is indepdent of external state
    ⇨ $h(z, a, P_z)$ and $h(z, a, P_z = 0)$ have same lin. div.
  - Ratio renormalization scheme

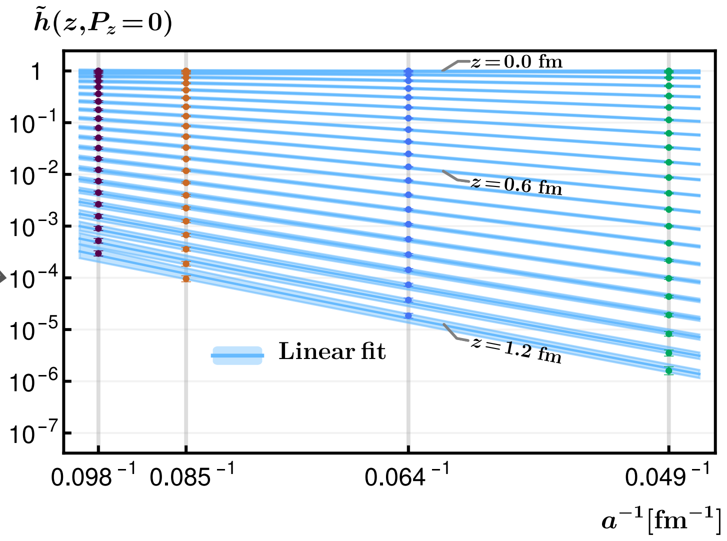
$$\tilde{h}_R(z, a) = \frac{\tilde{h}(z, a, P_z)}{\tilde{h}(z, a, P_z = 0)}$$

Hybrid renormalization scheme

- $|z| > z_s$ ⇨ Extract renormalization from $\tilde{h}(z, a, P_z = 0)$



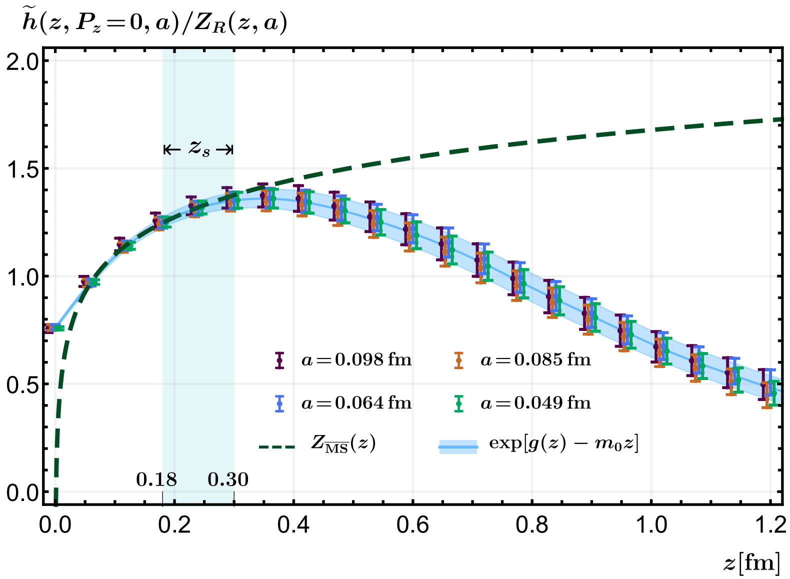
⇨ln⇨



- Detailed hybrid renormalization procedure see

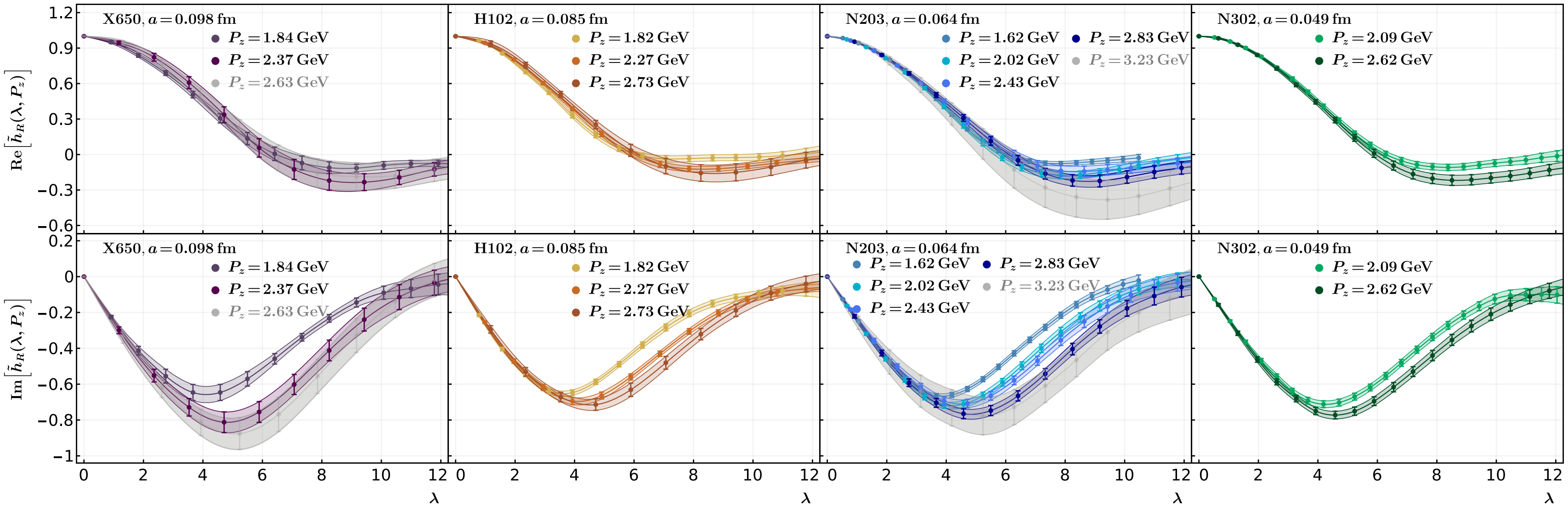
> X. Ji, Y. Liu, A. Schäfer, W. Wang, Y.-B. Yang, J.-H.Zhang, and Y. Zhao, Nucl. Phys. B 964, 115311 (2021)

- Renormalized matrix element ($P_z = 0$)



  - Mild lattice spacing dependence
- Renormalized matrix element should not depends on choice of z_s
  - As long as z_s in perturbative region (matches with $\overline{\rm MS}$ shceme pQCD) : 0.18~0.3fm
  - Freedom in chosing z_s ⇨ systematic error

Renormalized matrix elements



- 😳 Error grows in large $\lambda = zP_z$ region ($|\lambda| > \lambda_L$)
- 😳 Brutal truncation leads to unphysical oscillation

Large $\lambda = zP_z$ Extrapolation

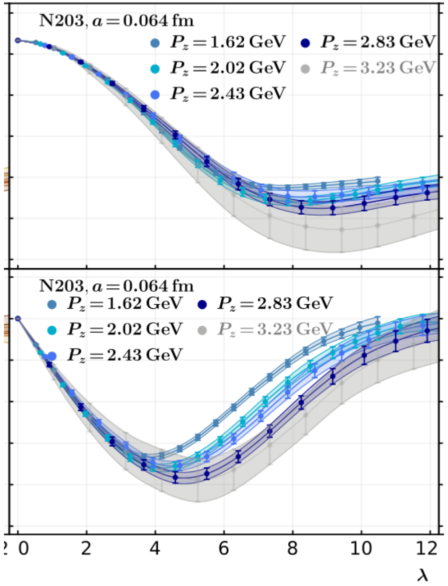
- Extrapolate $|\lambda| \geq \lambda_L$ with a physics based form

$$\tilde{H}_R(\lambda, P_z) = \boxed{\left[\frac{c_1}{(i\lambda)^a} + e^{-i\lambda}\frac{c_2}{(-i\lambda)^b}\right]}_{\text{algbr.}} \times \boxed{e^{-\lambda/\lambda_0}}_{\text{exp.}}$$

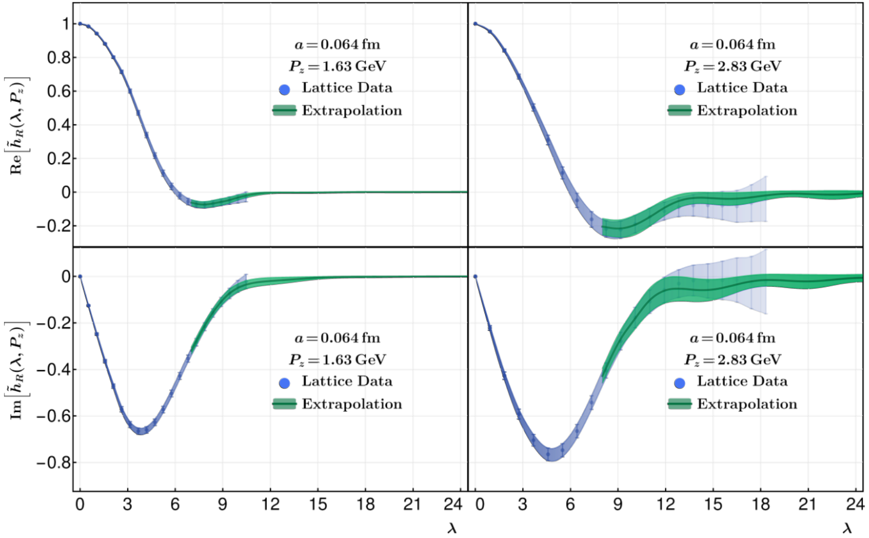
  - $\boxed{}_{\text{algbr}}$ ⇨ Regge behavior $x^a(1-x)^b$
  - $\boxed{}_{\text{exp}}$ ⇨ correlation function has a finite correlation length (λ_0)
- **The details of extroplation affect small/large x region**
- **LaMET is not reliable in small/large x region**

Large λ extrapolation: an example

- Before extrapolation



- After extrapolation



- Error in $|\lambda| > \lambda_L$ region under control
- Freedom of choicing λ_L ⇨ systematic error

3.

Fourier transform

- $$\delta\tilde{q}(x,P_z) = \int_{|\lambda|\le\lambda_L} d\lambda\, e^{i\lambda x}\,\tilde{h}_R(\lambda,P_z) + \int_{|\lambda|\ge\lambda_L} d\lambda\, e^{i\lambda x} \boxed{\tilde{H}_R(\lambda,P_z)}_{\text{(extrplt.)}}$$

quasi transversity

Perturbative Matching (Factorization)

higher twist corrections

$$\delta\tilde{q}(x,P_z) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} \boxed{C\left(\frac{x}{y},P_z,\mu\right)}_{\text{(pQCD)}} \times \boldsymbol{\delta q(y,\mu)} + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{(xP_z)^2},\frac{\Lambda^2_{\text{QCD}}}{((1-x)P_z)^2}\right)$$

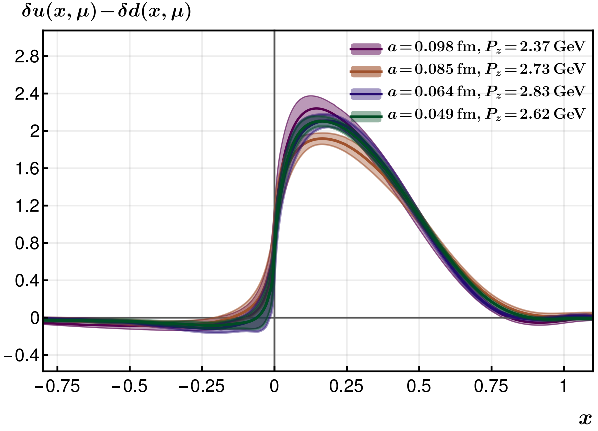
-

matching factor

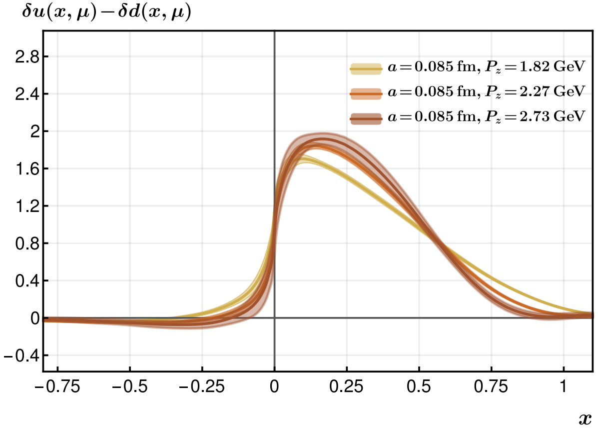
light-cone transversity

4. Continum, Physical point, Infinite momentum Extrapolation

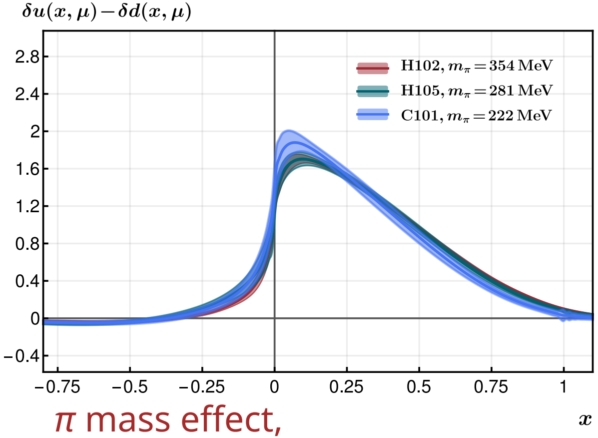
- $a \to 0$



- $P_z \to \infty$



- $m_\pi \to 135\mathrm{MeV}$



π mass effect,
J-W.Chen, X.Ji PLB5 23:107-110, 2001

Combined extrapolation form

discretization error

$$\delta q(x,\mu;a,P_z,m_\pi)=\left[\boldsymbol{\delta q^*(x,\mu)}+a^2 f(x)+a^2 P_z^2 h(x)+\frac{g(x,a)}{P_z^2}\right]\left[\frac{1-g' m_\pi^2 \ln m_\pi^2+m_\pi^2 k(x)}{1-g' m_\pi^2 \ln m_\pi^2}\right]$$

higher twist correction

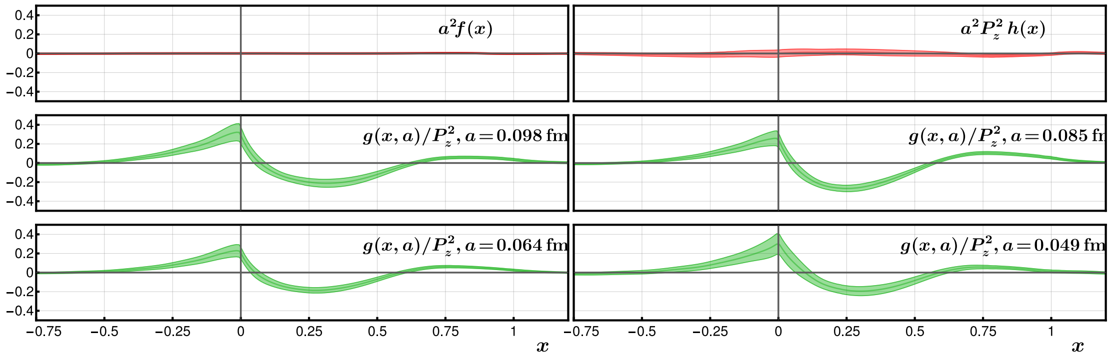
The desired transversity PDF ⇨ $\boldsymbol{\delta q(x,\mu)\equiv\delta q^*(x,\mu)\left[\frac{1-g' m_{\pi,\mathrm{phy}}^2 \ln m_{\pi,\mathrm{phy}}^2+m_{\pi,\mathrm{phy}}^2 k(x)}{1-g' m_{\pi,\mathrm{phy}}^2 \ln m_{\pi,\mathrm{phy}}^2}\right]}$

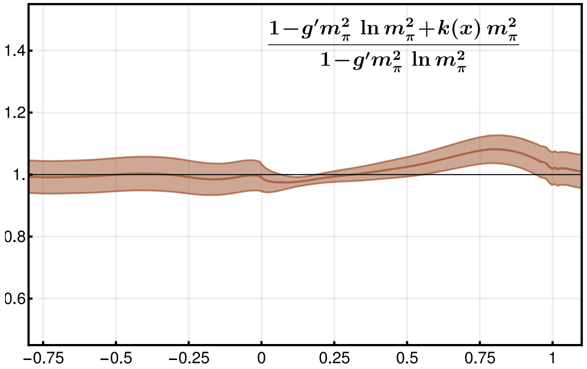
4. Continum, Physical point, Infinite momentum Extrapolation

- Combined extrapolation form

$$\delta q(x,\mu;a,P_z,m_\pi)=\left[a^2 f(x)+a^2P_z^2 h(x)+\frac{g(x,a)}{P_z^2}+\delta q^*(x,\mu)\right]\left[\frac{1-g' m_\pi^2 \ln m_\pi^2+m_\pi^2 k(x)}{1-g' m_\pi^2 \ln m_\pi^2}\right]$$

discretization error — higher twist correction — π mass effect

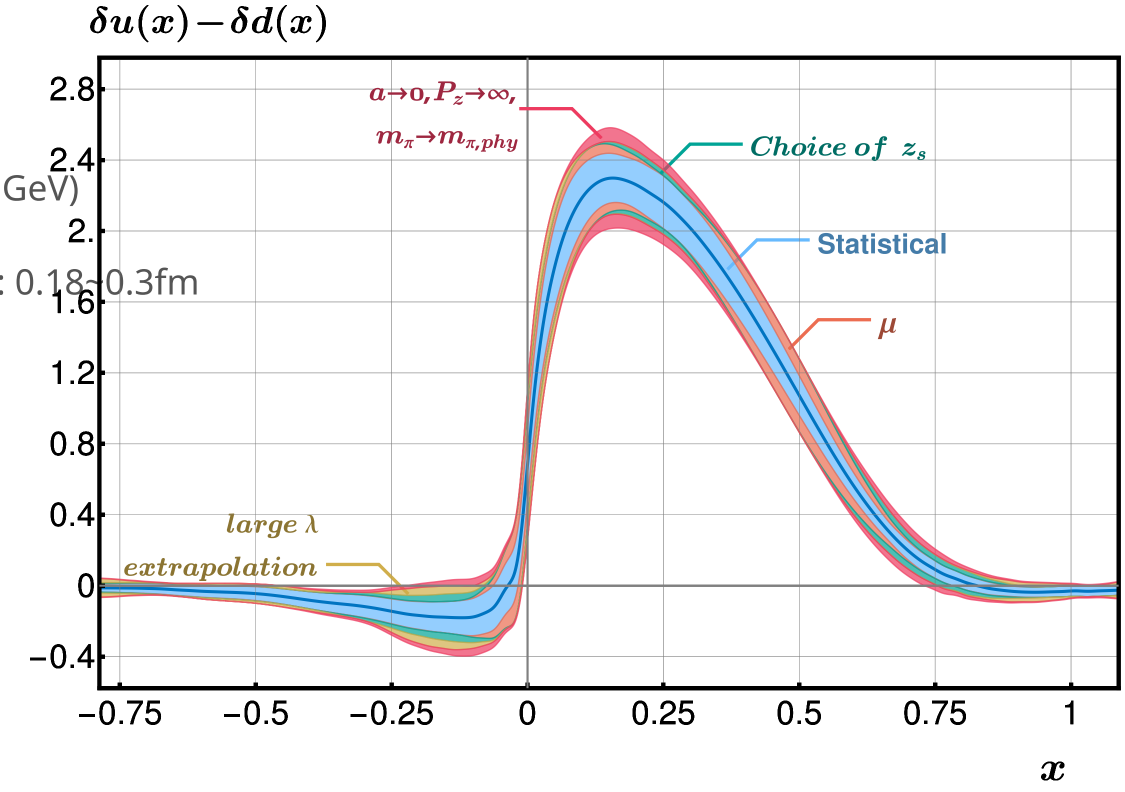




- **The largest correction is higher twist correction**
- **It is crucial to have large momentum on lattice**

5. Sources of uncertainties

- Statistical uncertaintity
- Renormalization scale μ (2~3GeV)
- z_s in Hybrid renormalization: 0.18~0.3fm
- Large λ extrapolation
- Combined $a \to 0$, $P_z \to \infty$, $m_\pi \to m_{\pi,phy}$ extrapolation



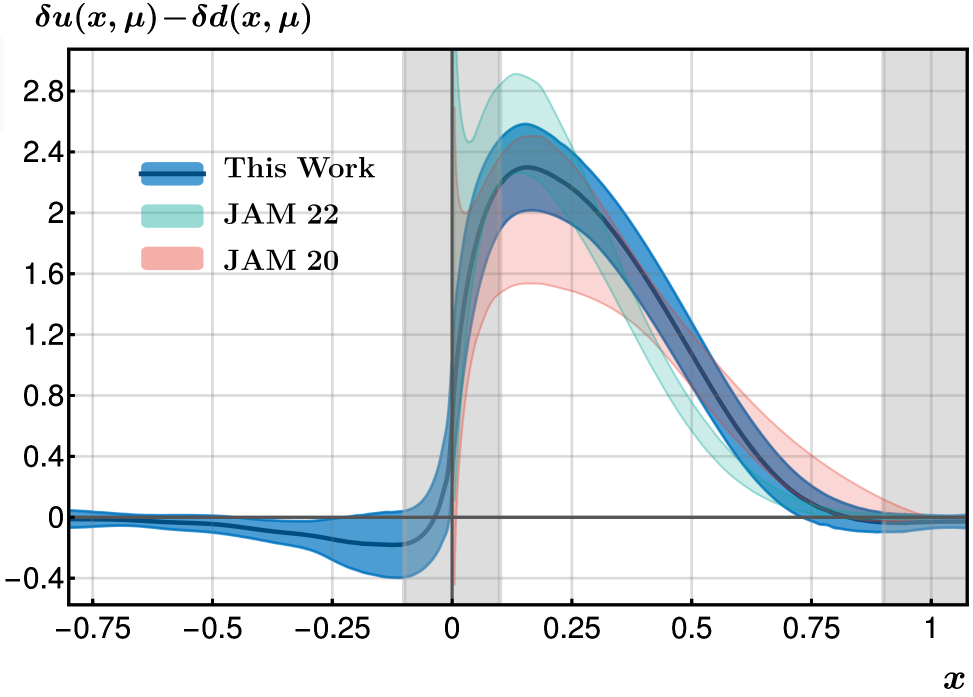
Light-cone transversity PDF at continuum, infinite momentum and physical pion mass limit

- Gray-shaded bands: $|x| < 0.1$ and $|1-x| < 0.1$ ⇨ LaMET is unreliable
  - Estimated by requireing $\frac{\Lambda_{\text{QCD}}}{xP_z}, \frac{\Lambda_{\text{QCD}}}{((1-x)P_z)} \sim 1$

    Renormalization Group Rsummation (RGR)

    Y.Su, *et al.*, arXiv:2209.01236
  - Yushan Su's talk today (session II)
- Global fit: JAM Collabration
  - 2020 ⇨ SIDIS data
  - 2022 ⇨ 2020+new data+lattice g_T
- $x < 0$ region consistent with zero

  ⇨ no flavor asymmetry in sea quark



Summary

- A state-of-the-art lattice calculation of nucleon quark transversity distribution

- Under the framework of LaMET, hybrid renromalized with self renormalization

- Extrapolated to continuum, physical pion mass, infinite momentum limit

- Provide guidence information for measurements at EIC, JLab 12GeV

Thanks for your attention !

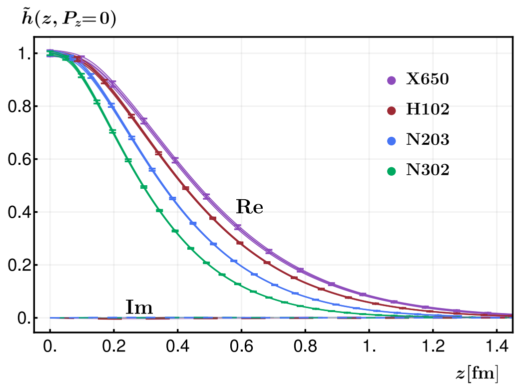
Backup slides

CLS ensembles

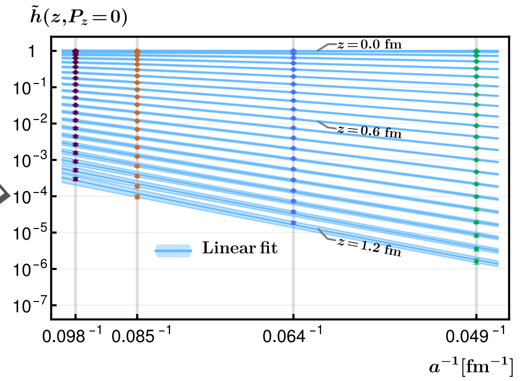
Ensemble	a(fm)	$L^3 \times T$	m_π(MeV)	$m_\pi L$	N_{conf}
X650	0.098	$48^3 \times 48$	338	8.1	1000
H102	0.085	$32^3 \times 96$	354	4.9	500
H105	0.085	$32^3 \times 96$	281	3.9	500
C101	0.085	$48^3 \times 96$	222	4.6	500
N203	0.064	$48^3 \times 128$	348	5.4	500
N302	0.049	$48^3 \times 128$	348	4.2	500

Hybrid renormalization scheme

- $|z| > z_s$ ⇨ Extract renormalization from $\tilde{h}(z, a, P_z = 0)$



⇨ln⇨



- Fit with pQCD functional form

$$\ln \tilde{h}(z, P_z=0) = \frac{kz}{a\ln(a\Lambda_{\rm QCD})} + \boxed{g(z)} + f(z)a^2 + \frac{3C_F}{11-2N_f/3}\ln\frac{\ln 1/a\Lambda_{\rm QCD}}{\ln \mu/\Lambda_{\rm QCD}} + \ln\left[1+\frac{d}{\ln(a\Lambda_{QCD})}\right]$$

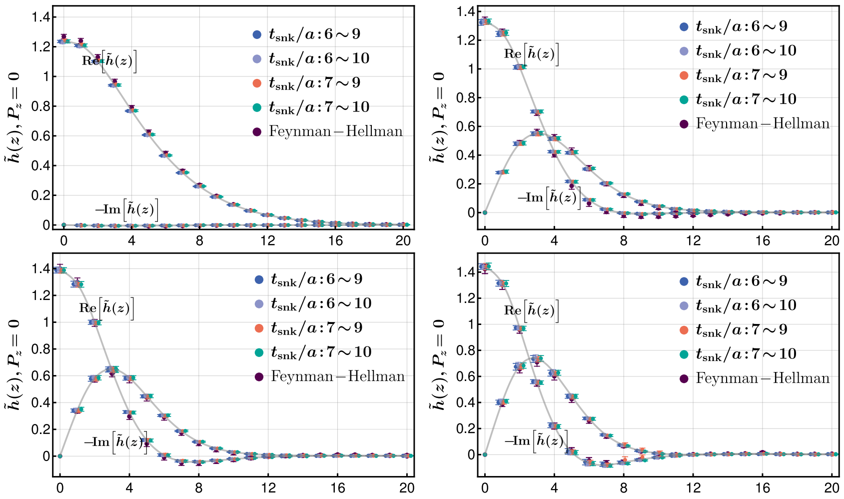
- Fit $g(z) - \ln Z_{\overline{\rm MS}} = m_0 z + b$
- m_0: renormalon ambiguity effects, $f(z)a^2$: (partially) discretization effect
- $\tilde{h}_R(z) = \exp[g(z) - m_0 z]$: **matrix element encodes hadron's intrinsic structure**

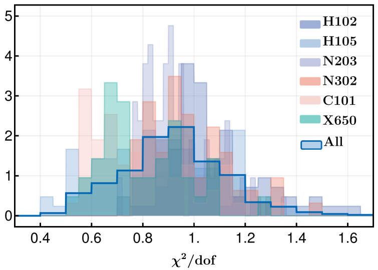
Quality of two-stae fit

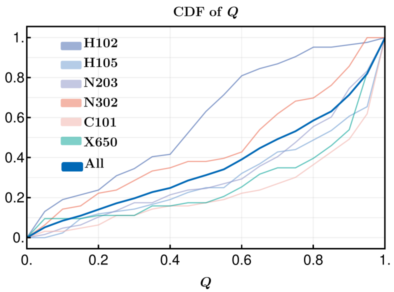
- Different choices of t_{snk}
- Feynman-Hellmann method

> J.C.He *et al.*.., PRC 105, 065203 (2022)

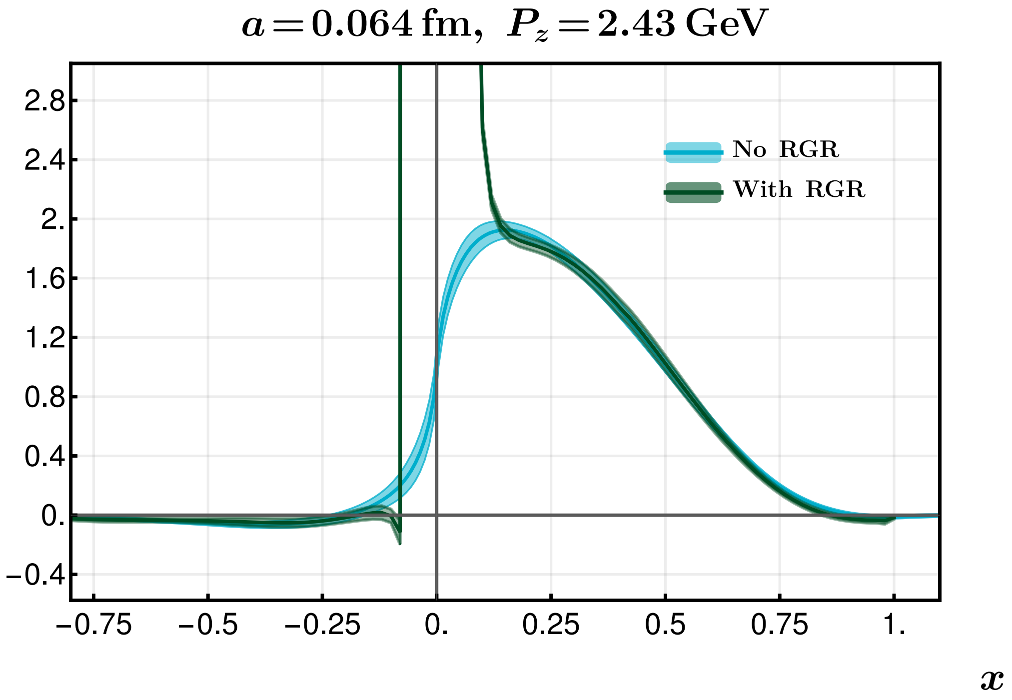
- Distribution of $\chi^2/\mathrm{d.o.f}$
- CDF of Q-value of fitting





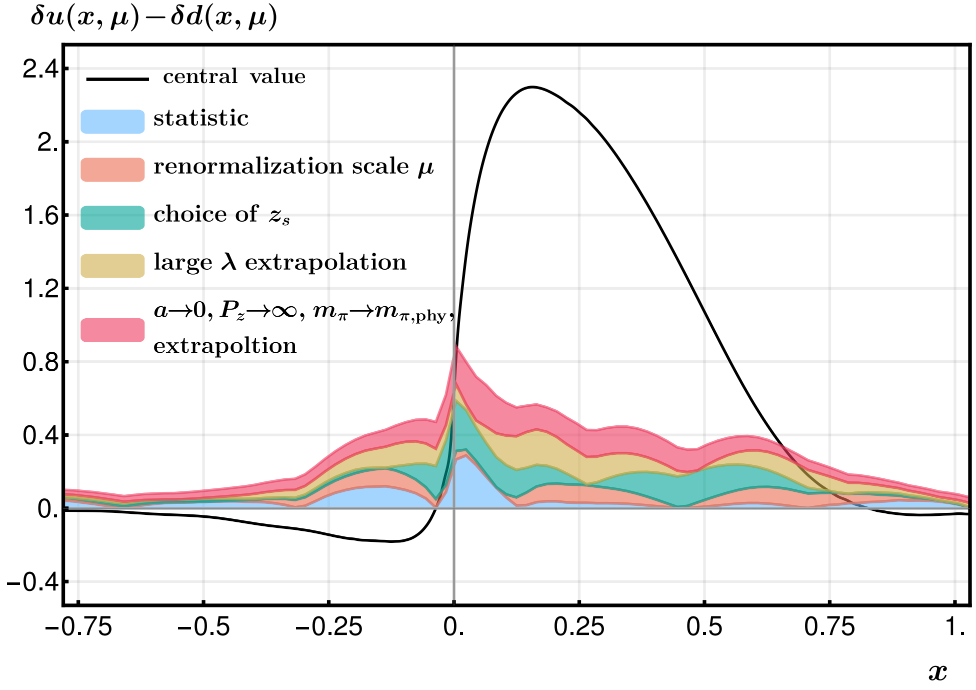


RGR vs no RGR


$a = 0.064\,\mathrm{fm},\ \ P_z = 2.43\,\mathrm{GeV}$
No RGR
With RGR
2.8
2.4
2.
1.6
1.2
0.8
0.4
0.
−0.4
−0.75
−0.5
−0.25
0.
0.25
0.5
0.75
1.
x

Sources of uncertanties

- Width of colored band indicates the size of uncertainty



Combined extrapolation in momentum space v.s. in coordianate space

- Combined extrapolation form

$$\delta q(x,\mu;a,P_z,m_\pi)=\left[\boldsymbol{\delta q^*(x,\mu)}+\boxed{a^2 f(x)+a^2 P_z^2 h(x)}+\boxed{\frac{g(x,a)}{P_z^2}}\right]\left[\frac{1-g' m_\pi^2 \ln m_\pi^2 + m}{1-g' m_\pi^2 \ln m}\right.$$

- $a \to 0$, $P_z \to \infty$ should be done in momentum space
  - Leading P_z dependence is removed by matching in momentum space
  - $a \to 0$ and $P_z \to \infty$ can not be done seperately (due to lattice ensembles)
- $m_\pi \to 135\mathrm{MeV}$ extrapolation can be done in both momentum and coordinate space
  - The difference is minor