Flavor Non-Singlet Structure of the Nucleon

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Outline

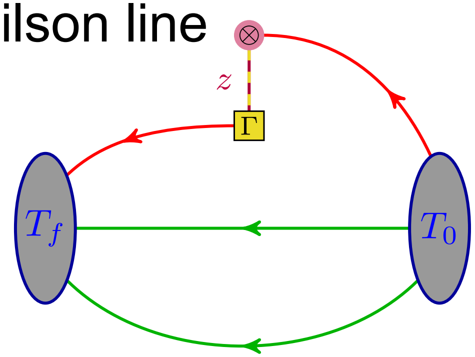
- Brief reminder - pseudo-PDF formulation
- 2 flavor studies
- Understanding systematic effects
  - Distillation + momentum smearing to reach high momenta
- Isovector PDF
- Transversity
- Helicity
- Summary

Pseudo-PDFs

Lattice "building blocks" that of quasi-PDF approach.

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Wilson line



A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

$$M^\alpha(p,z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $\qquad$ $z = (0, z_-, 0_T)$

$$\Downarrow$$

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

$$\mathfrak{M} = \frac{\mathscr{M}(\nu, z^2)}{\mathscr{M}(0,z^2)} \equiv \left(\frac{\mathscr{M}(\nu, z^2)}{\mathscr{M}(\nu,0)}\right) / \left(\frac{\mathscr{M}(0,z^2)}{\mathscr{M}(0,0)}\right)$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu,\mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

$\Downarrow$

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

Need data for all ν, *or additional physics input*

Inverse problem

Ioffe-Time Distribution to PDF

J.Karpie, K.Orginos, A.Radyushkin, S.Zafeiropoulos, Phys.Rev.D 96 (2017)

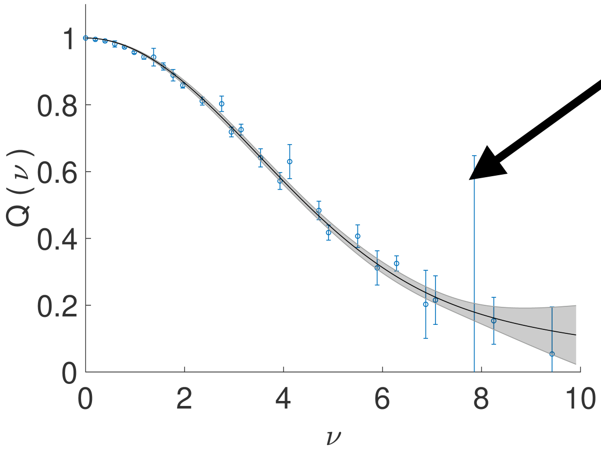
B.Joo *et al.*, HEP 12 (2019) 081, J.Karpie et al., *Phys.Rev.Lett.* 125 (2020) 23, 232003

To extract PDF requires additional information - *use a phenomenologically motivated parametrization*

$$f(x) = x^a(1-x)^b P(x)$$

MSTW, CJ

$$P(x) = \frac{1 + c\sqrt{x} + dx}{B(a+a, b+1) + cB(a+1.5, b+1) + dB(a+2, b+1)}$$



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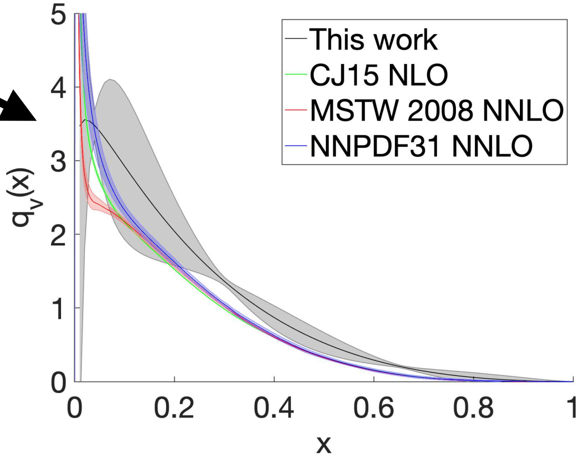
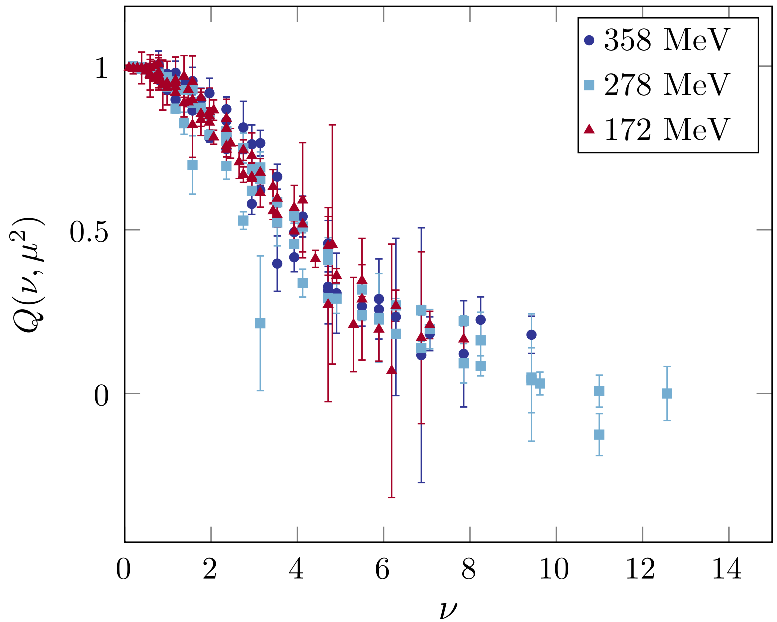
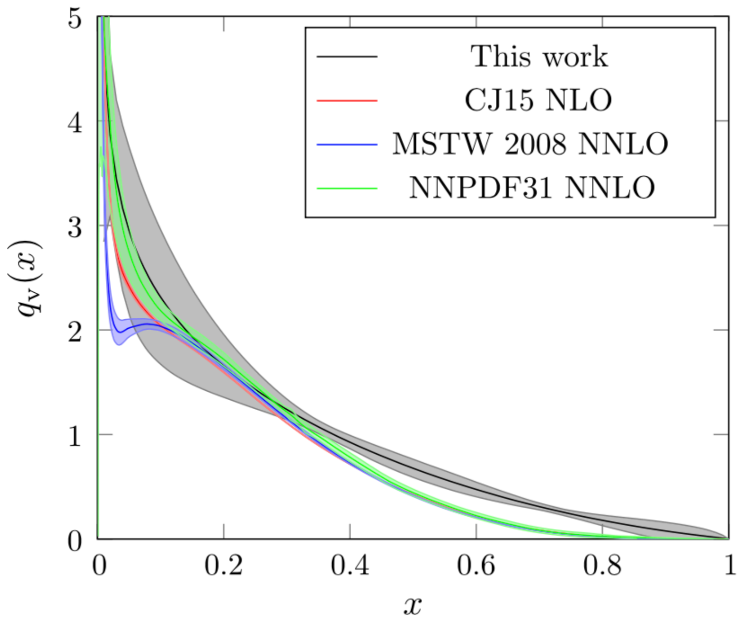


PDFs at Physical Quark Masses

ID	a(fm)	M_π(MeV)	β	c_{SW}	am_l	am_s	$L^3 \times T$	N_{cfg}
$a094m360$	0.094(1)	358(3)	6.3	1.20536588	-0.2350	-0.2050	$32^3 \times 64$	417
$a094m280$	0.094(1)	278(3)	6.3	1.20536588	-0.2390	-0.2050	$32^3 \times 64$	500
$a091m170$	0.091(1)	172(6)	6.3	1.20536588	-0.2416	-0.2050	$64^3 \times 128$	175





B.Joo *et al.*,Phys.Rev.Lett. 125 (2020) 23, 232003

Physical pion

$$q_v(x,\mu^2,m_\pi) = q_v(x,\mu^2,m_0) + a\Delta m_\pi + b\Delta m_\pi^2$$

Pseudo-PDF in Precision Era

FLAG 2019 Flavour Lattice Averaging Group

What do we need?

J.Karpie,K.Orginos,A.Radyushkin,S.Zafeiropoulos,
arXiv:2105.13313

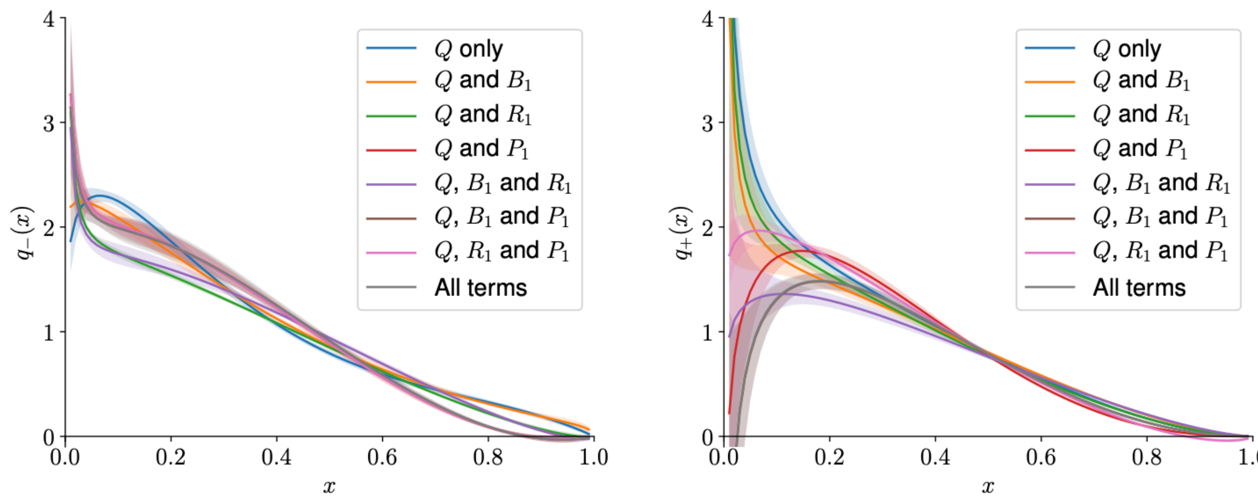
ID	a(fm)	M_π(MeV)	β	$c_{\rm SW}$	κ	$L^3 \times T$	$N_{\rm cfg}$
$\widetilde{A}5$	0.0749(8)	446(1)	5.2	2.01715	0.13585	$32^3 \times 64$	1904
E5	0.0652(6)	440(5)	5.3	1.90952	0.13625	$32^3 \times 64$	999
N5	0.0483(4)	443(4)	5.5	1.75150	0.13660	$48^3 \times 96$	477

$$\mathfrak{M}(p, z, a) = \mathfrak{M}_{\rm cont}(\nu, z^2) + \sum \left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\rm QCD})^n R_n(\nu)\,.$$

Lattice spacing

$$\mathfrak{M}_{\rm cont}(\nu, z^2) = \mathfrak{M}_{\rm lt}(\nu, z^2) + \sum_{n=1} (z^2\Lambda_{\rm QCD}^2)^n B_n(\nu)\,.$$

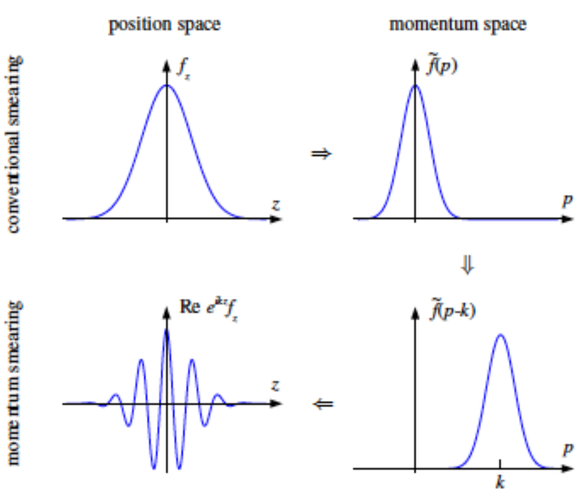
Higher twist



Challenges of Higher Momenta

Achieving high momenta in a lattice calculation presents several challenges

- Discretization errors
- "Compression" of energy spectrum as spatial momentum increased
- Reduced symmetries for states in motion - parities are mixed, helicity defines the basis
- Poor overlaps of e.g. Jacobi smearing on states in motion - poor signal-to-noise ratio.



Neat solution **Boosted interpolating operators**

Bali *et al.*, Phys. Rev. D 93, 094515 (2016)

Now essentially ubiquitous

Can we combine momentum smearing with distillation to address some of the other issues?

N.B Bali et al does indeed suggest application to distillation.

Look at

- *Nucleon energies and dispersion relation*
- *Nucleon charges*

Distillation

Low-rank approximation to (typically) Jacobi-smearing kernel

$$-\nabla^2(t)\xi^{(k)}(t) = \lambda^{(k)}(t)\xi^{(k)}(t)$$

M. Peardon *et al.*, PRD80,054506 (2009)

Rank →

$$\Box(\vec{x},\vec{y};t)_{ab} = \sum_{k=1}^{R_{\mathcal{D}}} \xi_a^{(k)}(\vec{x},t)\,\xi_b^{(k)\dagger}(\vec{y},t)\,,$$

Components of distillation:

$$\tau_{\alpha\beta}^{(l,k)}(t',t) = \xi^{(l)\dagger}(t')\,M_{\alpha\beta}^{-1}(t',t)\,\xi^{(k)}(t)$$

Perambulators → quark propagation

$$\Phi_{\alpha\beta\gamma}^{(i,j,k)}(t) = \epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a\left(\mathcal{D}_2\xi^{(j)}\right)^b\left(\mathcal{D}_3\xi^{(k)}\right)^c(t)\,S_{\alpha\beta\gamma}$$

Elementals → (baryon) operators

Projection to irrep

$$C_{rs}(t) = \sum_{\vec{x},\vec{y}}\langle 0 \mid \mathcal{O}_r(t,\vec{x})\mathcal{O}_s^\dagger(0,\vec{y}) \mid 0\rangle \equiv \mathrm{Tr}\left[\Phi_r(t)\otimes\tau(t,0)\tau(t,0)\tau(t,0)\otimes\Phi_s(0)\right]$$

Matrix of correlators

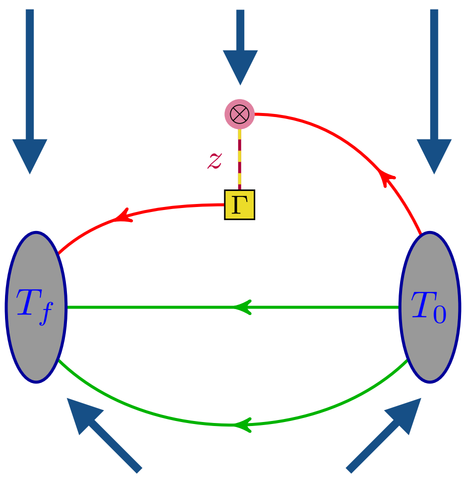
Extension to 3pt functions straightforward

Distillation and Hadron Structure

To control systematic uncertainties, need precise computations over a wide range of momentum.

- Use a low-mode projector to capture states of interest "distillation" M.Peardon *et al* (Hadspec), Phys.Rev.D 80 (2009) 054506

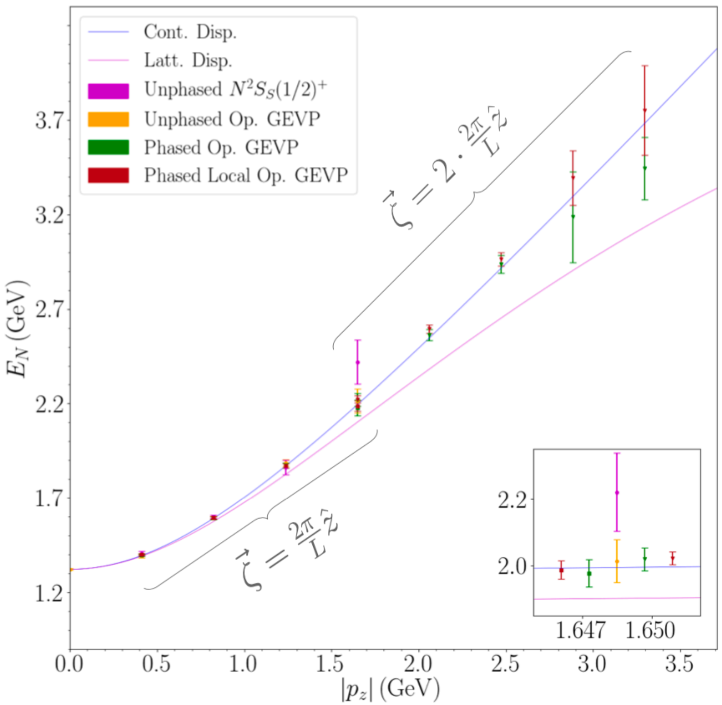
Momentum projection



Variational basis

\+ momentum smearing

G.Bali *et al*, Phys.Rev.D 93 (2016) 9, 094515



C.Egerer *et al* (Hadstruc), Phys. Rev. D 103, 034502 (2021)

Isovector PDF using Distillation

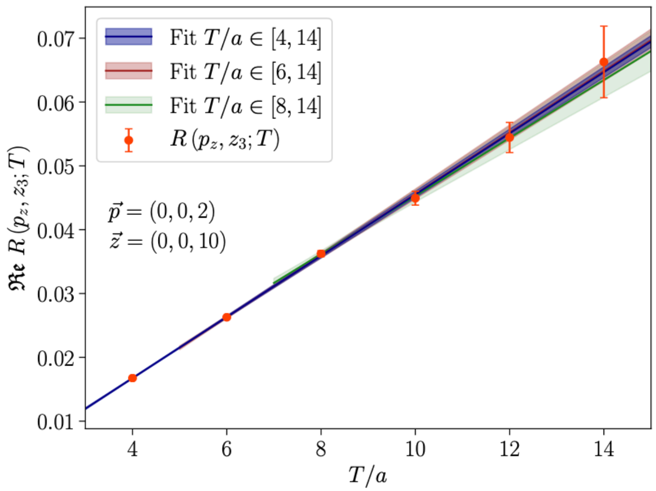
C.Egerer *et al.* (hadstruc), JHEP 11 (2021) 148

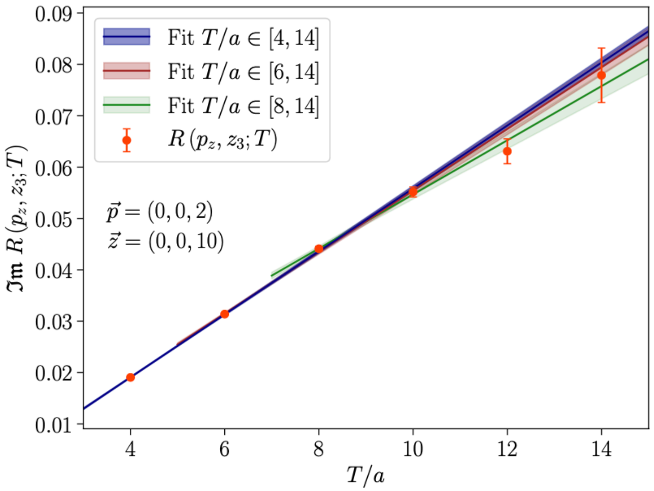
Numerics

ID	a_s (fm)	m_π (MeV)	$L_s^3 \times N_t$	N_{cfg}	N_{srcs}	$R_{\mathcal{D}}$
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	4	64

Used throughout rest of this talk

Matrix elements extracted using summation method - *reduced excited-state contributions*





Expand the x-dependence in terms of (shifted) Jacobi Polynomials

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \mathfrak{Re}\int_0^1 dx\, \mathcal{K}_{\rm v}\left(x\nu, z^2\mu^2\right) x^\alpha (1-x)^\beta\, \Omega_n^{(\alpha,\beta)}(x)$$

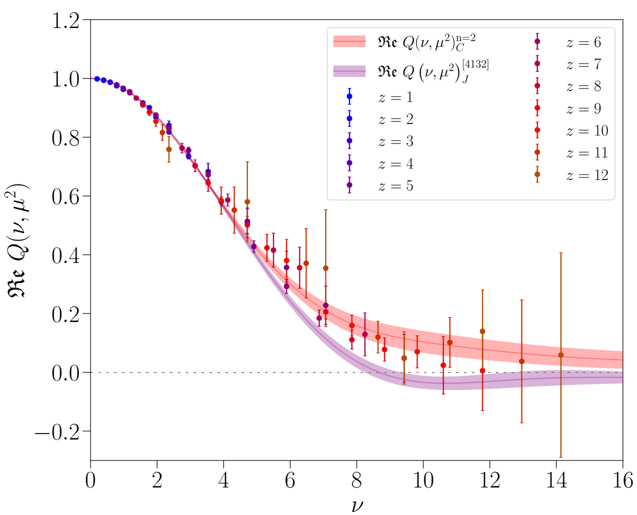
Matching kernel

J.Karpie,K.Orginos,A.Radyushkin,S.Zafeiropoulos, arXiv:2105.13313

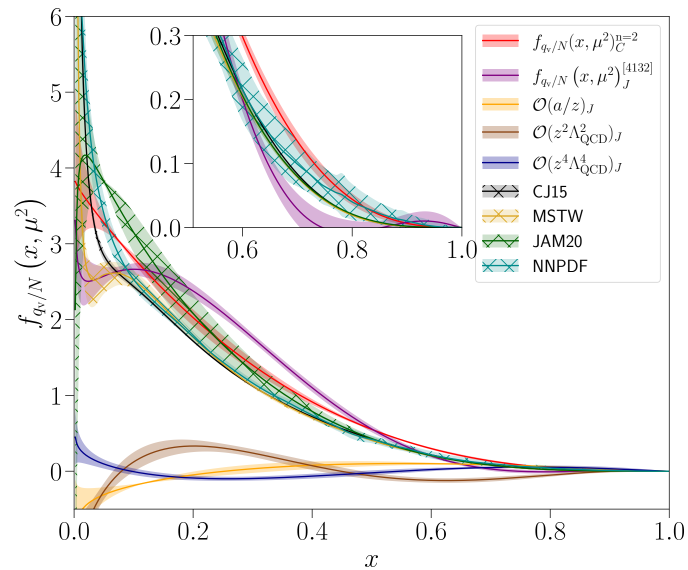
$$\mathfrak{Re}\,\mathfrak{M}_{\rm fit}\left(\nu, z^2\right) = \sum_{n=0}^{\infty}\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{{\rm v},n}^{lt\,(\alpha,\beta)} + \left(\frac{a}{z}\right)\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{az\,(\alpha,\beta)} + z^2\Lambda_{\rm QCD}^2\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{t4\,(\alpha,\beta)}$$

Discretization

Higher twist

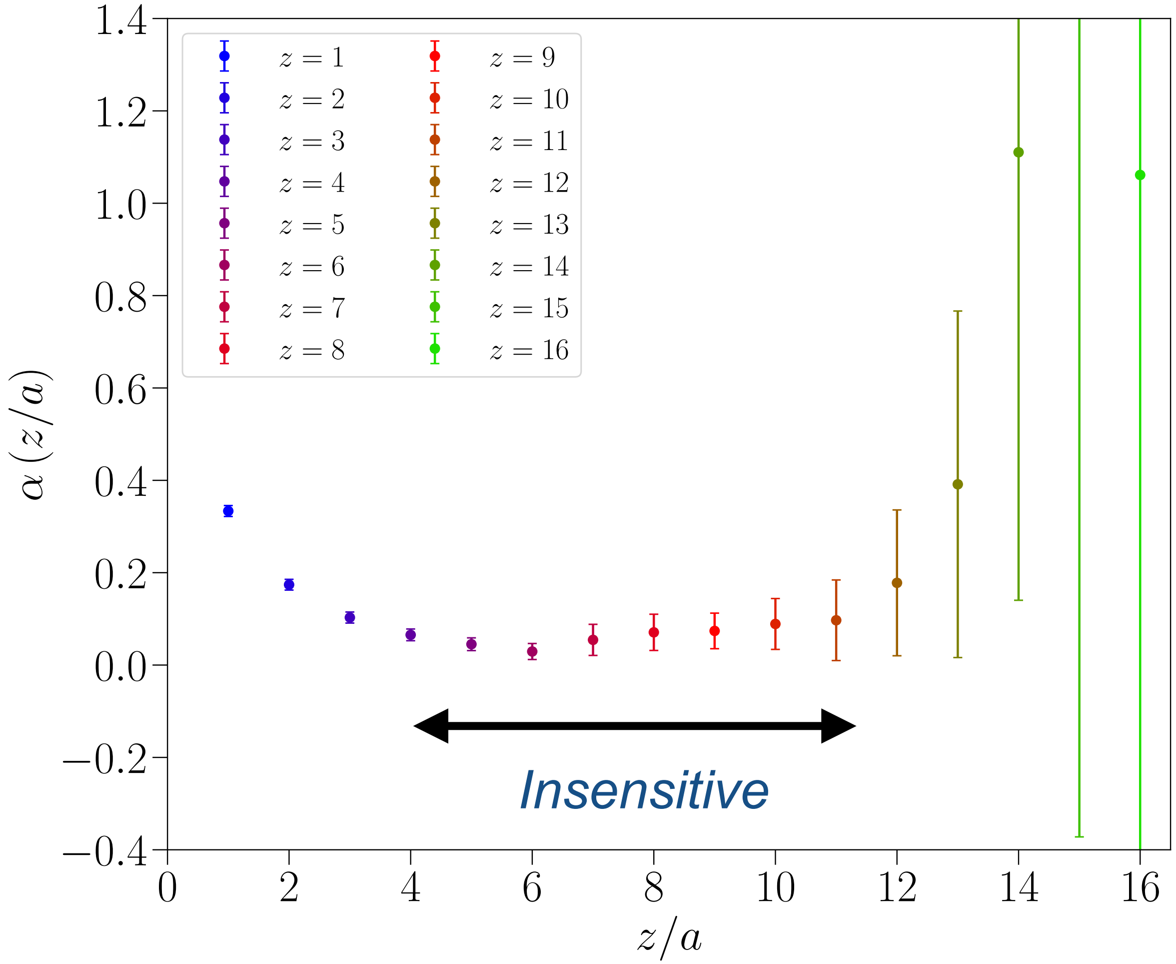


$m_\pi \simeq 358$ MeV



DGLAP Evolution

- Look at two-parameter, matched fit



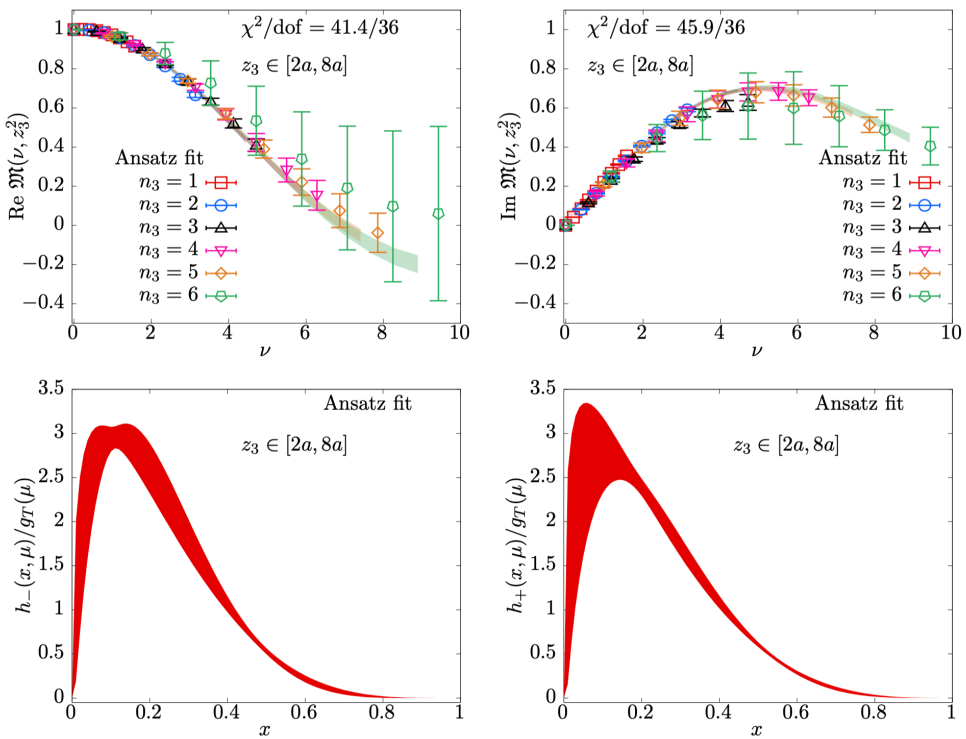
Transversity

Phys.Rev.D 105 (2022) 3, 034507, Hadstruc Collaboration, (C.Egerer et al).

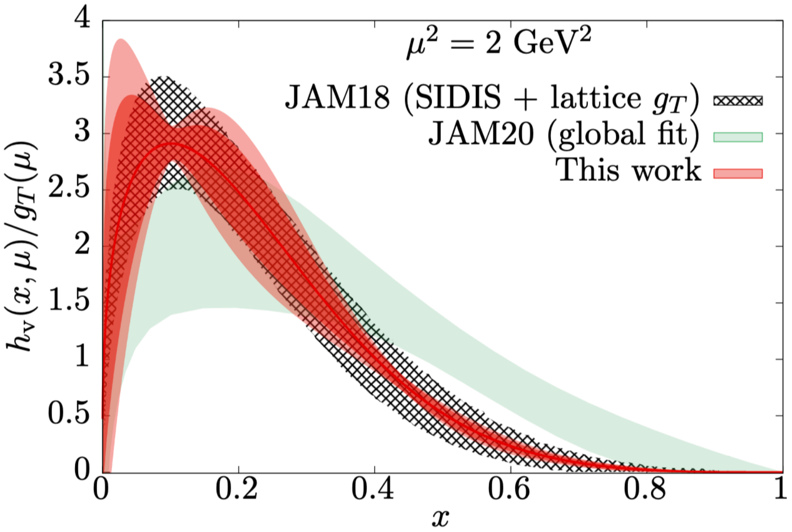
$$2P^+ S^{\rho_\perp} \mathcal{I}(P^+ z^-, \mu) = \langle P, S^{\rho_\perp} | \bar{\psi}(z^-) \gamma^+ \gamma^{\rho_\perp} \gamma_5 W_+(z^-, 0) \psi(0) | P, S^{\rho_\perp} \rangle$$

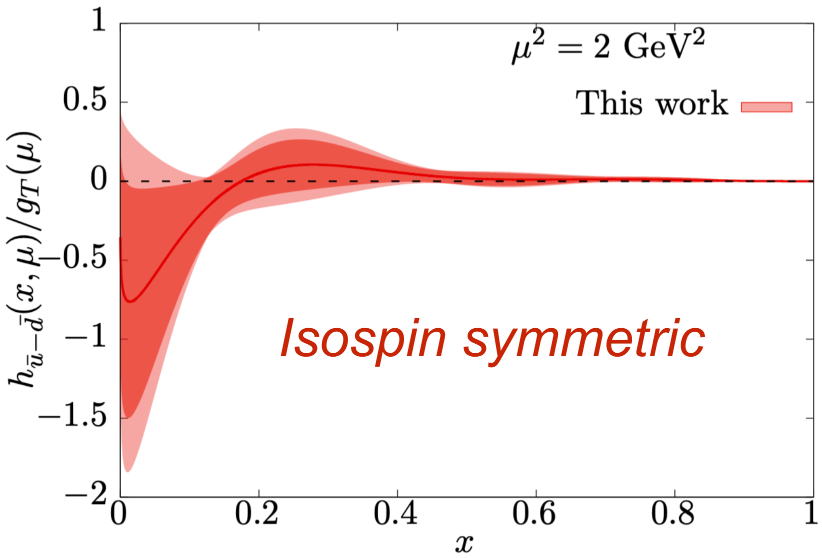
$$h(x, \mu) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} e^{-ix\nu} \mathcal{I}(\nu, \mu)$$

In contrast to unpolarized PDF, there is no conserved current - so express in terms of the (renormalized) tensor charge.



Transversity Distribution





Helicity Distribution

arXiv:2211.04434, HadStruc Collaboration, R.Edwards et al., Colin Egerer

Formalism

$$M^{\mu 5}(p,z) = \langle N(p,S)\overline{\psi}(z)\gamma^{\mu}\gamma^{5}W^{(f)}(z,0)\psi(0)\rangle N(p,S)$$

Lorentz invariance

$$M^{\mu 5}(p,z) = -2m_N S^{\mu}\mathcal{M}(\nu,z^2) - 2im_N p^{\mu}(z\cdot S)\mathcal{N}(\nu,z^2) + 2m_N^3 z^{\mu}(z\cdot S)\mathcal{R}(\nu,z^2)$$

Spin polarization

As before, we exploit Lorentz invariance and choose matrix element that can be calculated on a Euclidean lattice

$$M^{35}(p,z_3) = -2m_N S^3[p_z\hat{z}]\{\mathcal{M}(\nu,z_3^2) - ip_z z_3\mathcal{N}(\nu,z_3^2)\} - 2m_N^3 z_3^2 S^3[p_z\hat{z}]\mathcal{R}(\nu,z_3^2)$$

$$M^{35}(p,z_3) = -2m_N S^3[p_z\hat{z}]\{\mathcal{Y}(\nu,z_3^2) + m_N^2 z_3^2\mathcal{R}(\nu,z_3^2)\}$$

$$\widetilde{\mathcal{Y}}(\nu,z_3^2)$$

Reduced distribution: $\mathfrak{Y}(\nu,z_3^2) = \left(\frac{\widetilde{\mathcal{Y}}(\nu,z_3^2)}{\widetilde{\mathcal{Y}}(0,z_3^2)|_{p_z=0}}\right) \Big/ \left(\frac{\widetilde{\mathcal{Y}}(\nu,0)|_{z_3=0}}{\widetilde{\mathcal{Y}}(0,0)|_{p_z=0,z_3=0}}\right)$

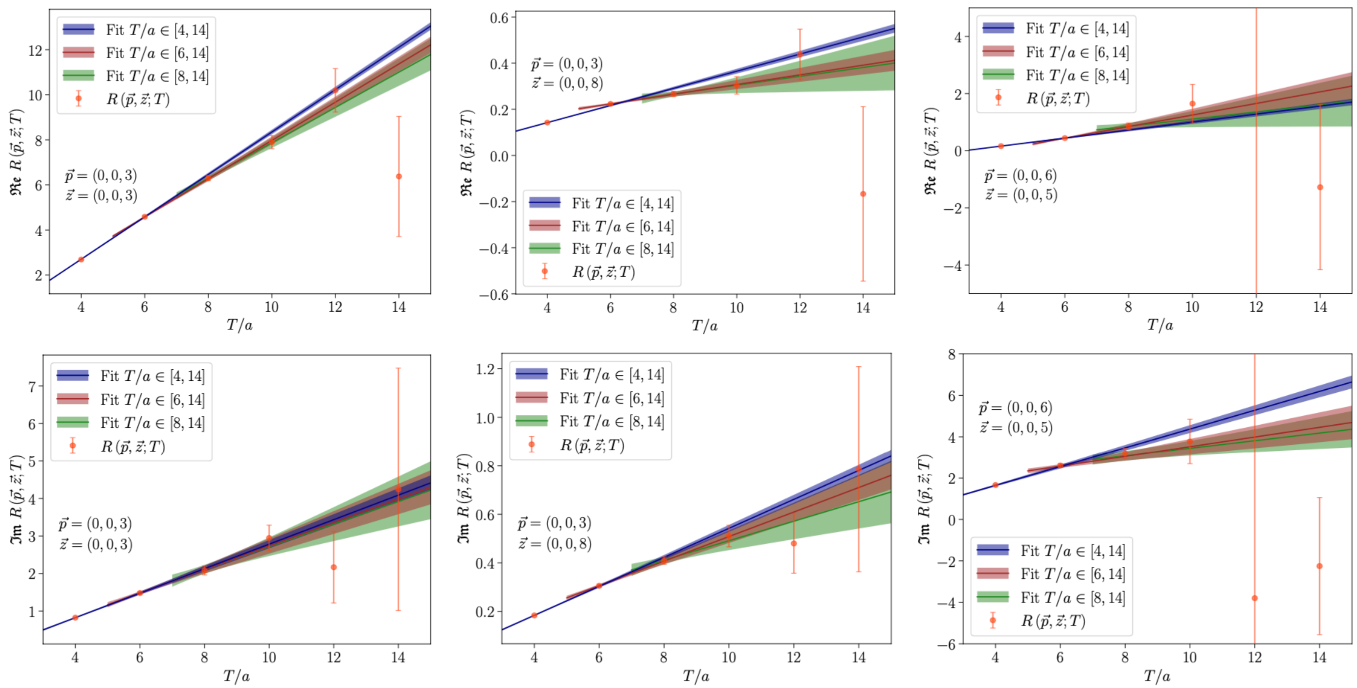
Numerical Method

$$\mathfrak{Y}\left(\nu, z_3^2\right) = \frac{1}{g_A(\mu^2)} \int_0^1 \mathrm{d}u\, \mathcal{C}\left(u, z_3^2\mu^2, \alpha_s\left(\mu^2\right)\right) \mathcal{I}\left(u\nu, \mu^2\right) + \mathcal{O}\left(z_3^2 \Lambda_{\mathrm{QCD}}^2\right)$$

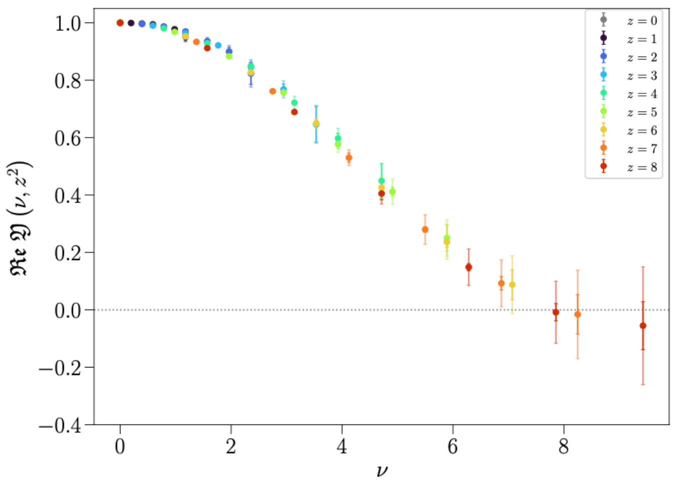
Not conserved current - normalize to g_A

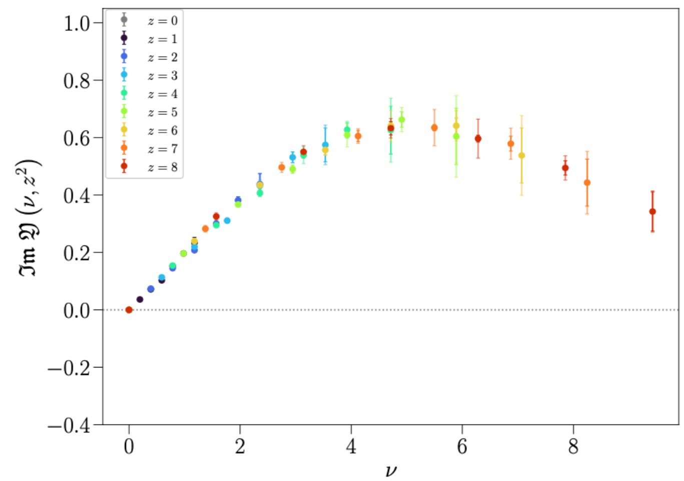
where $\mathcal{I}\left(\nu, \mu^2\right) = \int_{-1}^1 \mathrm{d}x\, e^{i\nu x} g_{q/N}\left(x, \mu^2\right)$

We use summation method to extract matrix elements:



Reduced ITD



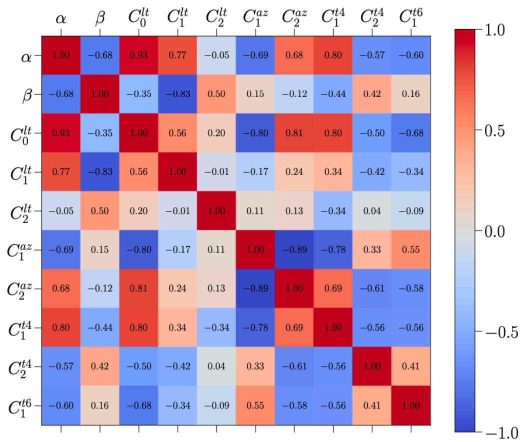


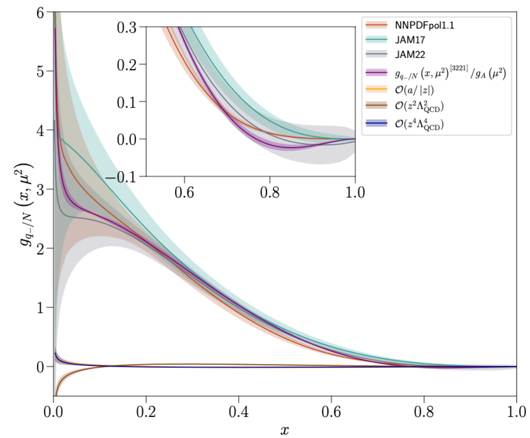
Express as parametrization over Jacobi polynomials:

$$\begin{aligned}
\mathfrak{Re}\,\mathfrak{M}_{\text{fit}}\left(\nu, z_3^2\right) = &\sum_{n=0}^{N_{lt}} \sigma_n^{(\alpha,\beta)}\left(\nu, z_3^2\mu^2\right) C_{-,n}^{lt\ (\alpha,\beta)} + \frac{a}{|z_3|} \sum_{n=1}^{N_{az}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{-,n}^{az\ (\alpha,\beta)} \\
&+ z_3^2 \Lambda_{\text{QCD}}^2 \sum_{n=1}^{N_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{-,n}^{t4\ (\alpha,\beta)} + z_3^4 \Lambda_{\text{QCD}}^4 \sum_{n=1}^{N_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{-,n}^{t6\ (\alpha,\beta)} \\
\mathfrak{Im}\,\mathfrak{M}_{\text{fit}}\left(\nu, z_3^2\right) = &\sum_{n=0}^{N_{lt}} \eta_n^{(\alpha,\beta)}\left(\nu, z_3^2\mu^2\right) C_{+,n}^{lt\ (\alpha,\beta)} + \frac{a}{|z_3|} \sum_{k=0}^{N_{az}} \eta_{0,n}^{(\alpha,\beta)}(\nu)\, C_{+,n}^{az\ (\alpha,\beta)} \\
&+ z_3^2 \Lambda_{\text{QCD}}^2 \sum_{n=0}^{N_{t4}} \eta_{0,n}^{(\alpha,\beta)}(\nu)\, C_{+,n}^{t4\ (\alpha,\beta)} + z_3^4 \Lambda_{\text{QCD}}^4 \sum_{n=0}^{N_{t6}} \eta_{0,n}^{(\alpha,\beta)}(\nu)\, C_{+,n}^{t6\ (\alpha,\beta)},
\end{aligned}$$

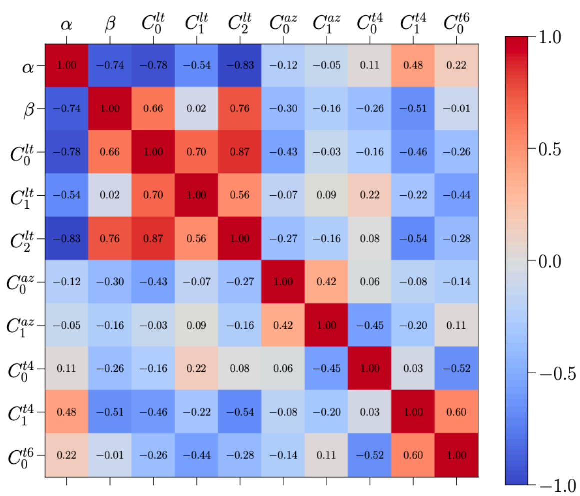
Results

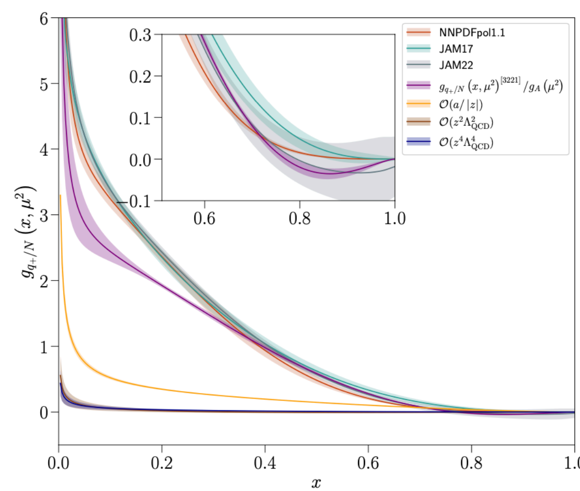
Valence quark helicity distribution, together with contamination terms



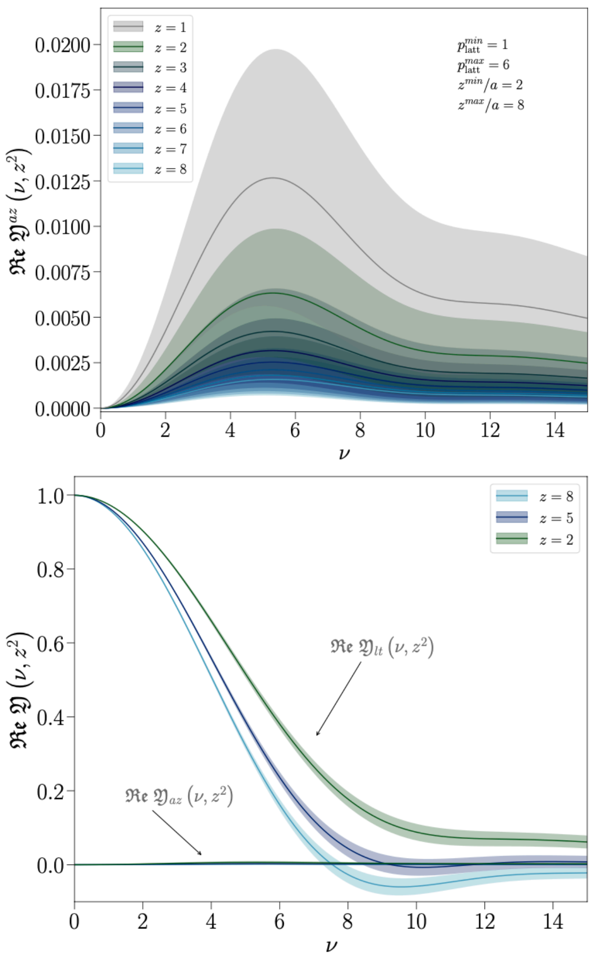


CP-odd helicity distribution, together with contamination terms

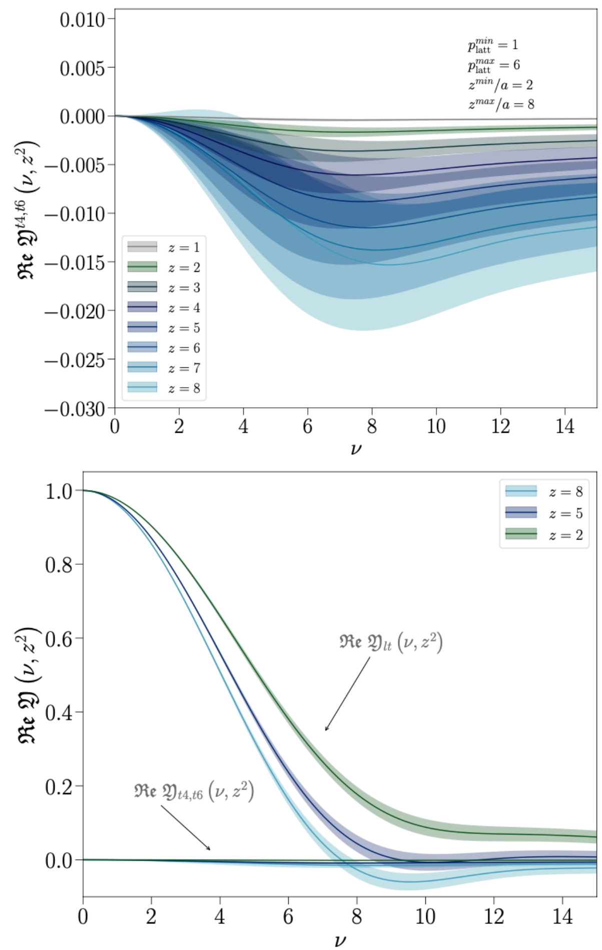




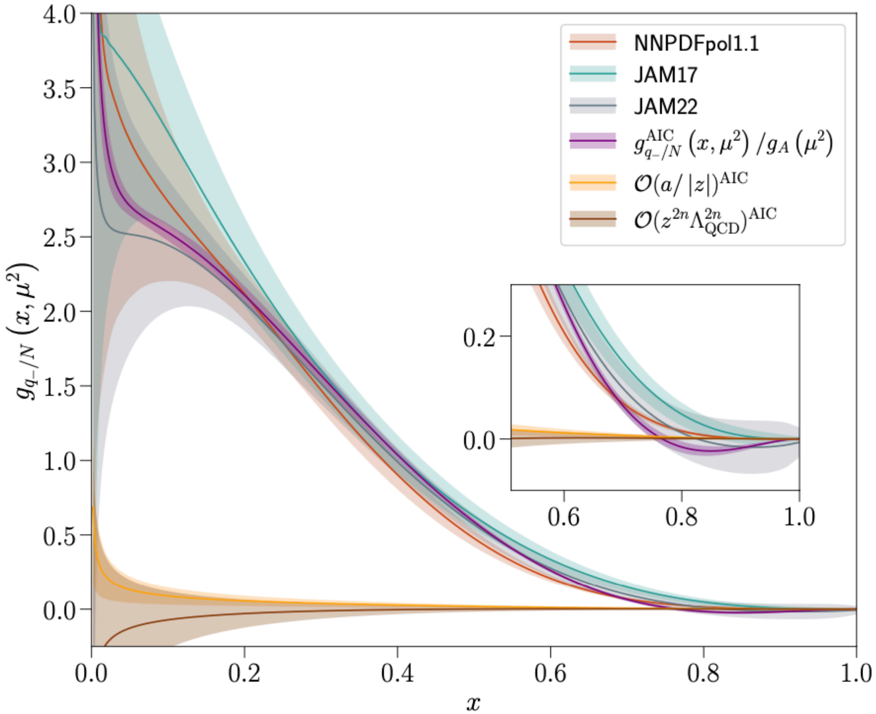
Discretization

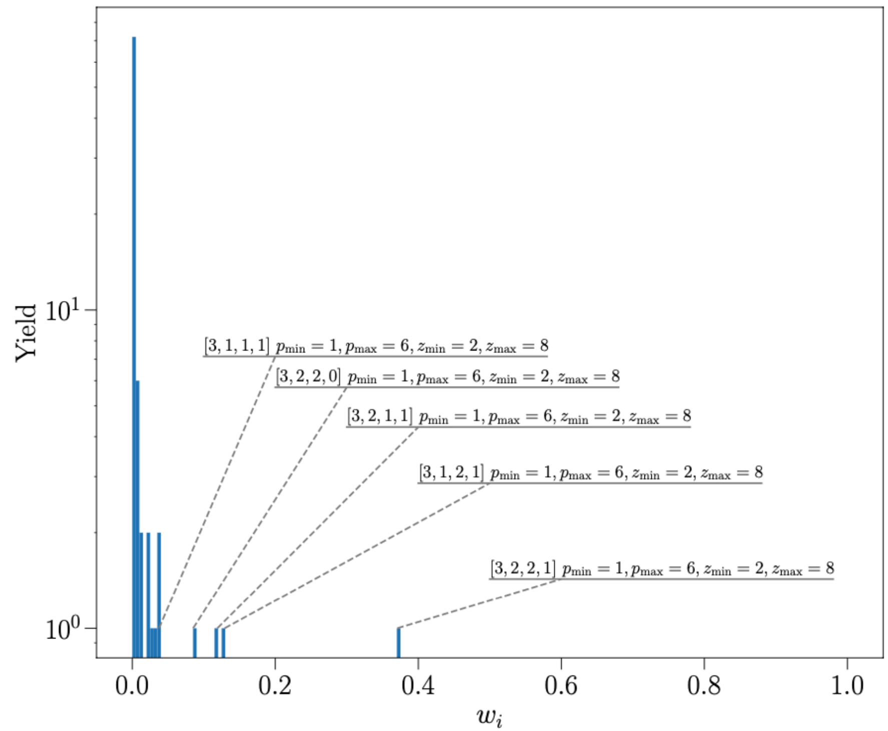


Higher Twist



AICc Prescription





Summary

- Focus on understanding systematic contributions in pseudo-PDF framework
- Distillation + boosting enables both far increased reach in momentum, and improved sampling of lattice
  - *Essential in calculations of gluon contributions*
- Are able to isolate leading twist from higher-twist and discretization contamination
- Framework admits calculation of GPDs and meson structure using many of the same components - Calculations in Progress!