The NNLO unpolarized isovector quark PDF of the nucleon at the physical point from lattice QCD

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Outline

- Motivations and formalism
- Lattice setup
- Analysis of two-point and three-point functions
- Model-independent extraction of lowest Mellin moments
- x-dependent PDF from model fits, deep neural network, and LaMET

All results are preliminary!

Motivations

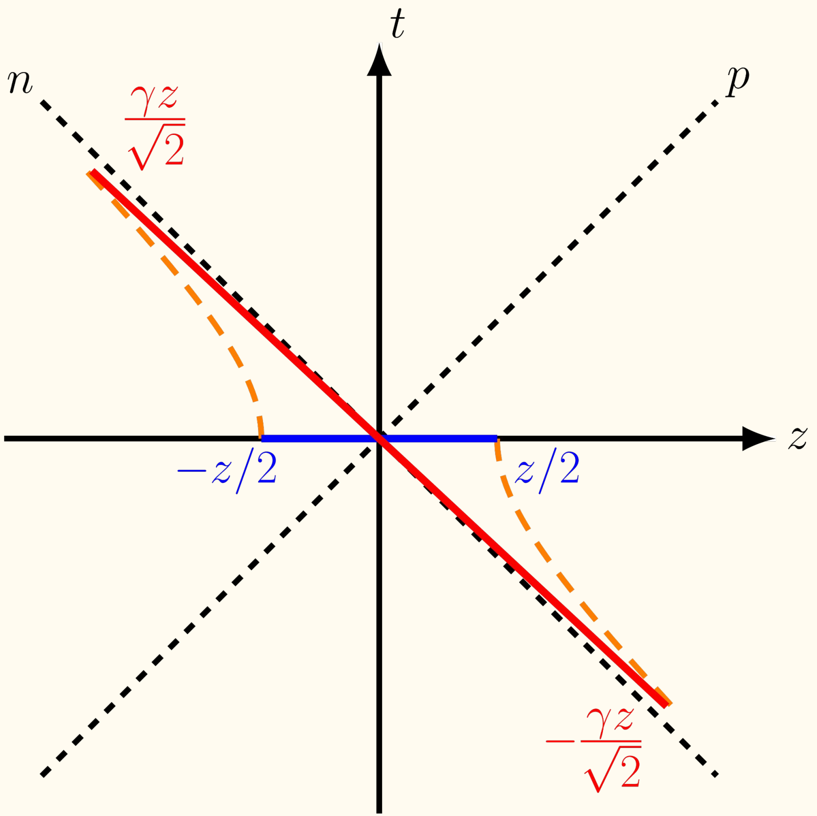
- Test methods against experimental data
- Extract nucleon PDFs using various methods
  - Leading-twist OPE
  - Model-dependent fits
  - Deep neural network
  - x-space matching with Hybrid renormalization
- When does short-distance factorization break down?
- Check perturbative uncertainty by including NNLO matching

Light-cone PDFs from lattice QCD

- Cannot calculate matrix elements separated along the light cone in lattice QCD
- Instead, calculate equal-time spatially-separated matrix element of highly-boosted hadron

$$h^B(z, P_z) = \langle N; P_z|\overline{\psi}(z)\Gamma W(z,0)\psi(0)|N; P_z\rangle$$

- Can be matched to light-cone PDF through Large-momentum Effective Theory or short-distance factorization



[X. Ji et al., Rev. Mod. Phys. **93**, 035005, arXiv: 2004.03543]

Theoretical Framework:
[V. Braun, D. Müller ‘07]
[X. Ji ’13]
[A. Radyushkin ’17]

Correlation functions

Use standard nucleon operator: $N_\alpha^{(s)}(x,t) = \varepsilon_{abc} u_{a\alpha}^{(s)}(x,t)(u_b^{(s)}(x,t)^T C\gamma_5 d_c^{(s)}(x,t))$

For two-point functions: $C^{\text{2pt}}(\vec{p}, t_{\text{sep}}; \vec{x}, t_0) = \sum_{\vec{y}} e^{-i\vec{p}\cdot(\vec{y}-\vec{x})} \mathcal{P}^{\text{2pt}}_{\alpha\beta} \langle N_\alpha(\vec{y}, t_{\text{sep}}+t_0) \overline{N}_\beta(\vec{x}, t_0) \rangle$

And three-point functions:

$$C^{\text{3pt}}(\vec{p}_f, \vec{q}, t_{\text{sep}}, t_{\text{ins}}; \vec{x}, t_0) = \sum_{\vec{y},\vec{z}} e^{-i\vec{p}_f\cdot(\vec{y}-\vec{x})} e^{-i\vec{q}\cdot(\vec{x}-\vec{z})} \mathcal{P}^{\text{3pt}}_{\alpha\beta} \langle N_\alpha(\vec{y}, t_{\text{sep}} + t_0) \mathcal{O}^\Gamma(\vec{z}, \hat{\mathcal{L}}, t_{\text{ins}} + t_0) \overline{N}_\beta(\vec{x}, t_0) \rangle$$

$$\mathcal{O}^\Gamma(\vec{z}, \hat{\mathcal{L}}, t_{\text{ins}}+t_0) = \overline{q}(\vec{z}, t_{\text{ins}}+t_0)\Gamma\tau_3 W(\vec{z}, t_{\text{ins}}+t_0; \vec{z}+\hat{\mathcal{L}}, t_{\text{ins}}+t_0) q(\vec{z}, +\hat{\mathcal{L}}, t_{\text{ins}}+t_0)$$

For unpolarized distribution: $\mathcal{P}^{\text{2pt}} = \mathcal{P}^{\text{3pt}} = \frac{1}{2}(1+\gamma_t)$, $\Gamma = \gamma_t, \gamma_z$ ← **No mixing**

Smeared-smeared (SS) and smeared-point (SP) two-point correlators

Only smeared-smeared three-point correlators

Calculation setup

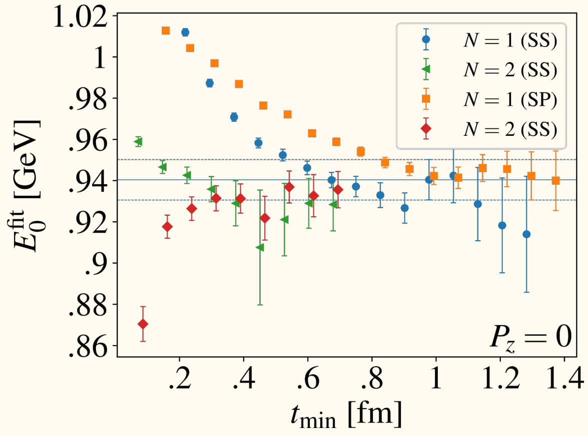
- Mixed fermion action
  - Sea quark action: $N_f = 2+1$ HISQ with physical quark masses, $L^3 \times T = 64^3 \times 64$, $a = 0.076$ fm
  - Valence quark action: $N_f = 2 + 1$ Wilson-Clover with physical quark masses, 1-HYP smeared gauge links
- Calculations done with Qlua, which utilizes the multigrid solver in QUDA
- Use momentum smearing for quarks to achieve better overlap with boosted hadrons
- Included four momentum projections to $P_x^{(f)}$ at the sink for three-point functions

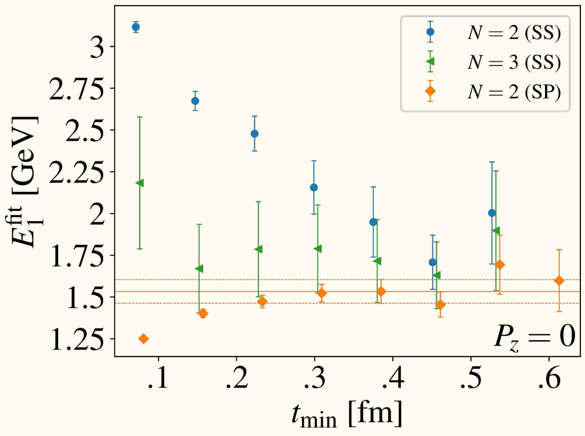
$P_x^{(f)}$	k_x	$t_{\rm sep}$	$N_{\rm samp}$
0	0	6	16
0	0	8,10	32
0	0	12	64
1	0	6,8,10,12	32
4	2	6	32
4	2	8,10,12	128
6	3	6	20
6	3	8	100
6	3	10,12	140

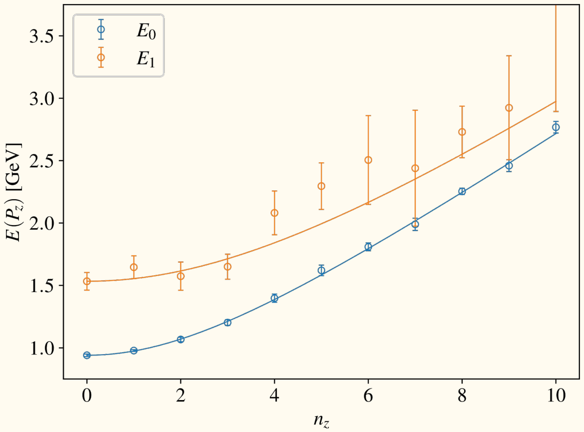
Analysis of two-point functions

- Fit two-point functions to $C_N^{2\mathrm{pt}}(\vec{p}, t_{\mathrm{sep}}) = C_0 e^{-E_0 t_{\mathrm{sep}}} \left[1 + \sum_{i=1}^{N-1} R_i \prod_{j=1}^{i} e^{-\Delta_{j,j-1} t_{\mathrm{sep}}}\right]$

- Use SP and SS correlators to help control excited states
- All energies below largest energy are priored
- Three states required to fit full t_{sep}







Three-point function analysis

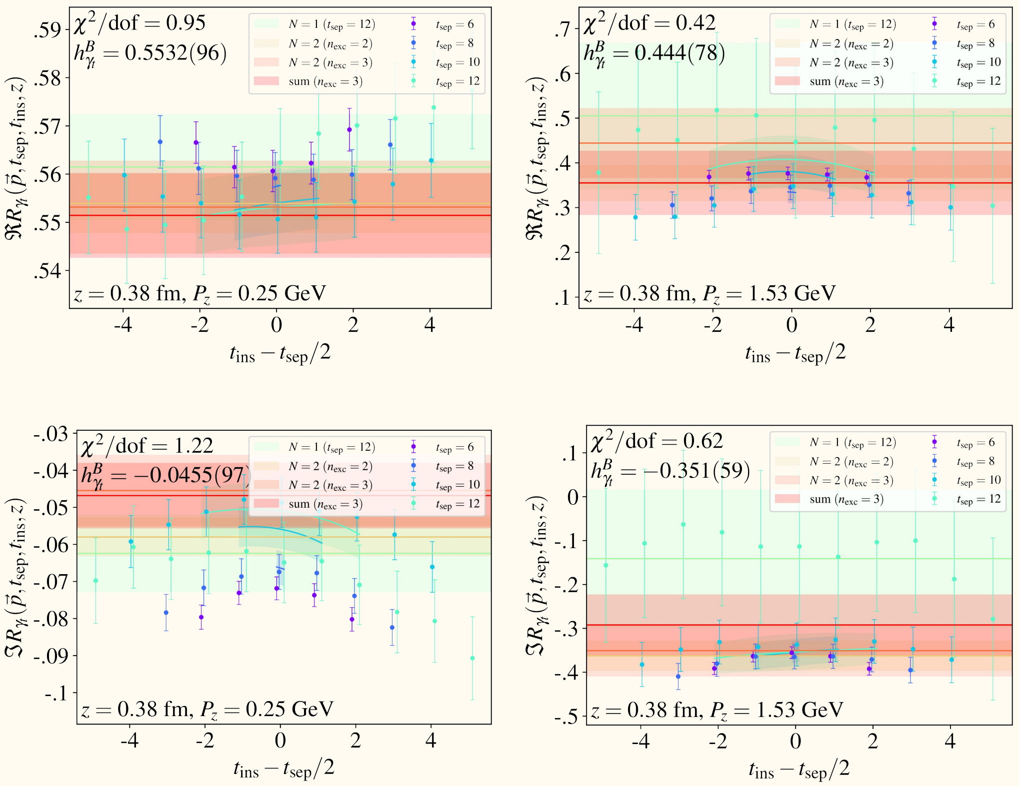
- Fit ratio of three-point to two-point data

$$R(\vec{p}_f, t_{\text{ins}}, t_{\text{sep}}) = \frac{C^{3\text{pt}}(\vec{p}_f, \vec{q}=0, t_{\text{ins}}, t_{\text{sep}})}{C^{2\text{pt}}(\vec{p}_f, t_{\text{sep}})}$$

- Two-state fits to three-point ratio priored with ‘effective’ energy gap and amplitudes from two-state fits to two-point SS correlators

- Reasonable agreement between two-state and other fit strategies, like summation fits

$$R_{\text{sum}}(\vec{p}_f, t_{\text{sep}}) = \sum_{t_{\text{ins}}=n_{\text{exc}}a}^{t_{\text{sep}}-n_{\text{exc}}a} R(\vec{p}_f, t_{\text{ins}}, t_{\text{sep}})$$



Ratio-scheme renormalization

- The operator $\mathcal{O}_\Gamma(z)$ is multiplicatively renormalizable

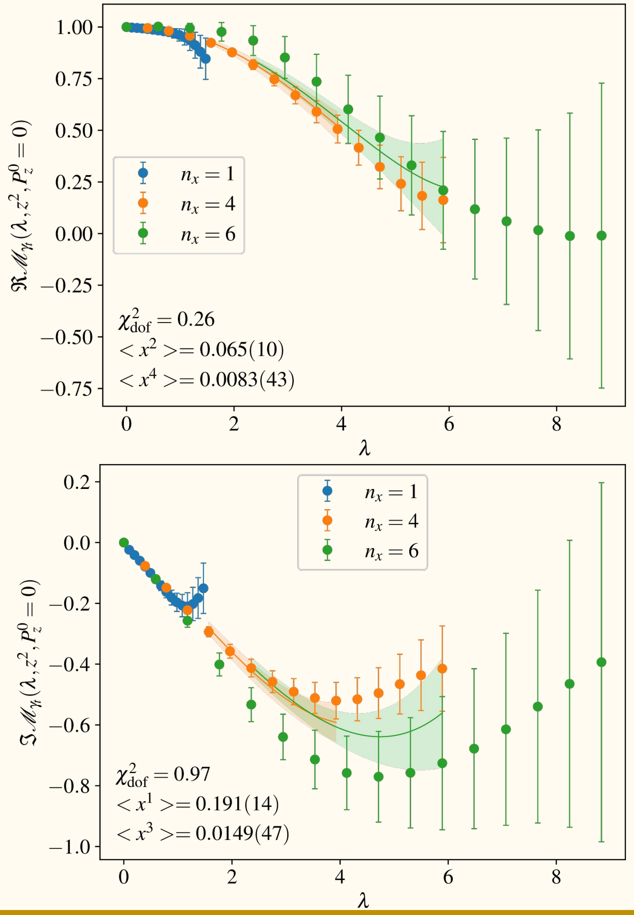
$$h^B_\Gamma(z, P_z, a) = e^{-\delta m(a)|z|} Z_O(a) h^R_\Gamma(z, P_z, \mu)$$

- Can form renormalization-group invariants with the double ratio ($z = 0$ for exact normalization)

$$\mathcal{M}(\lambda, z^2; P_z^0, a) = \frac{h^B(z, P_z, a)}{h^B(z, P_z^0, a)} \Big/ \frac{h^B(0, P_z, a)}{h^B(0, P_z^0, a)} \quad , \lambda \equiv z P_z$$

- Consider $\mathcal{M}(\lambda, z^2; P_z^0 = 0, a)$, referred to as the reduced Ioffe Time Distribution (rITD)

- rITD can be perturbatively matched to light-cone ITD $Q(\lambda, \mu^2)$

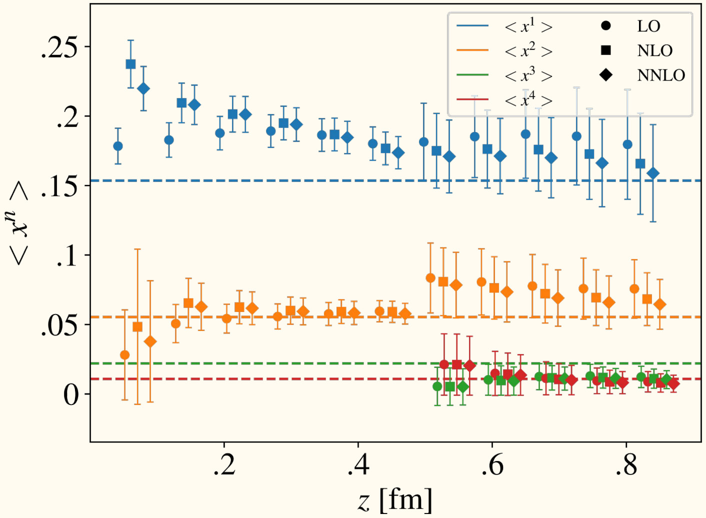


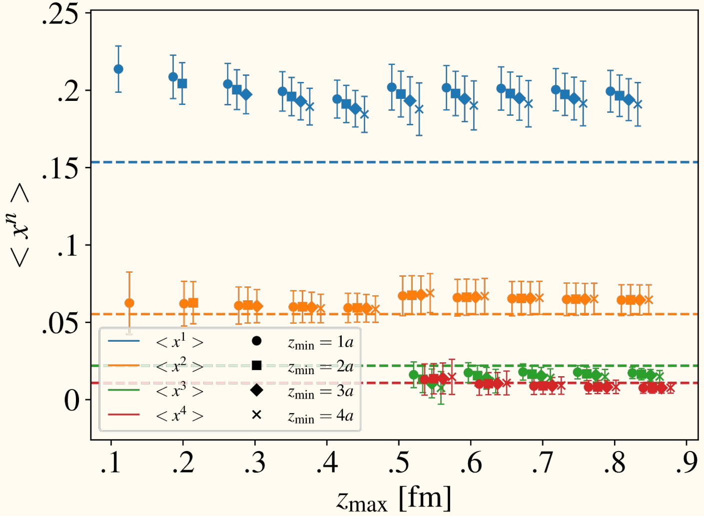
Lowest moments from leading-twist OPE

The lowest few moments can be extracted from the rITD by fits to

$$\mathcal{M}(\lambda, z^2; \lambda^0 \equiv zP_z^0) = \frac{\sum_{n=0} c_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(\Lambda_{QCD}^2 z^2)}{\sum_{n=0} c_n(\mu^2 z^2) \frac{(-i\lambda^0)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(\Lambda_{QCD}^2 z^2)}$$

where $c_n(\mu^2 z^2) \equiv C_n(\mu^2 z^2)/C_0(\mu^2 z^2)$, and $C_n(\mu^2 z^2)$ are Wilson coefficients, which have been computed up to next-to-next-leading-order (NNLO)





Model dependent fits

- Model the PDF $q_{\text{model}}(x)$ and evaluate the moments $< x^n >_{\text{model}} = \int_0^1 \mathrm{d}x\, x^n q_{\text{model}}(x)$
- Substitute $< x^n >_{\text{model}}$ into leading-twist OPE to obtain $\mathcal{M}_{\text{model}}(\lambda, z^2; \lambda^0)$
- Fit by minimizing, $$\chi^2 = \sum_{P_z > P_z^0}^{P_z^{\max}} \sum_{z_{\min}}^{z_{\max}} \frac{(\mathcal{M}(\lambda, z^2; \lambda^0) - \mathcal{M}_{\text{model}}(\lambda, z^2; \lambda^0))^2}{\sigma^2(z, P_z, P_z^0)}$$
- Real and Imaginary part of rITD related to

$$q^-(x) \equiv q^u(x) - q^d(x) - (q^{\bar{u}}(x) - q^{\bar{d}}(x))\,, \quad q^+(x) \equiv q^u(x) - q^d(x) + (q^{\bar{u}}(x) - q^{\bar{d}}(x))\,, \qquad x \in [0,1]$$

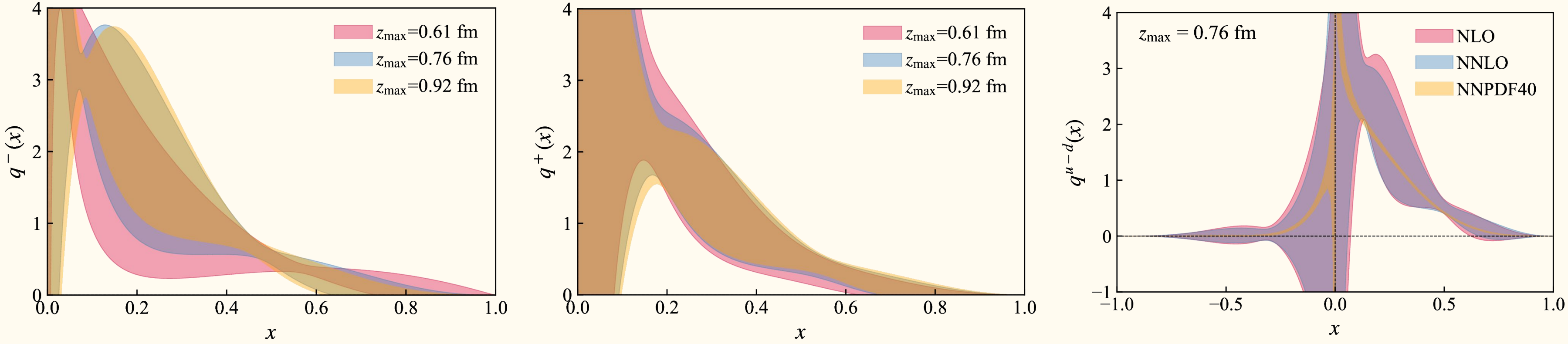
respectively, and can be expressed via a simple model

$$q^-(x; \alpha, \beta) = \frac{\Gamma(2+\alpha+\beta)}{\Gamma(1+\alpha)\Gamma(2+\beta)} x^\alpha (1-x)^\beta\,, \qquad q^+(x; \alpha, \beta, A) = A x^\alpha (1-x)^\beta$$

Isovector PDF from model fits

- Include all P_z and $z \in [2a, z_{\max}]$ for fit
- Use $q^f(-x) = -q^{\bar{f}}(x)$ to form isovector PDF from

$$q^{u-d}(x) = \begin{cases} \frac{q^-(x)+q^+(x)}{2}, & x > 0 \\ \frac{q^-(-x)-q^+(-x)}{2}, & x < 0 \end{cases}$$



Deep Neural Network

- Use leading-twist factorization formula

$$h^R(z, P_z, \mu) = \int_{-1}^{1} d\alpha\, \mathcal{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy\, e^{-iy\alpha\lambda} q(y, \mu)$$

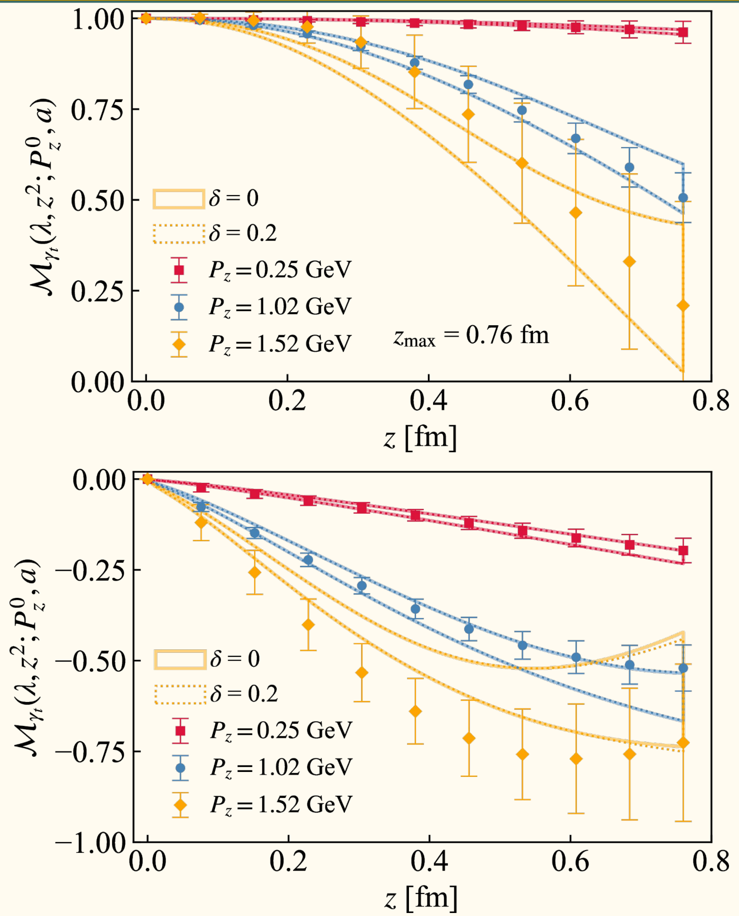
- Consider modifications to model fits with a function represented by a deep neural network

$$q^-(x; \alpha, \beta, \theta) \equiv A^- x^\alpha (1-x)^\beta [1+\delta \sin(f^-_{\mathrm{DNN}}(x, \boldsymbol{\theta}))],$$

$$q^+(x; \alpha, \beta, A, \theta) \equiv A^+ x^\alpha (1-x)^\beta [1+\delta \sin(f^+_{\mathrm{DNN}}(x, \boldsymbol{\theta}))].$$

- Minimize loss function

$$J(\boldsymbol{\theta}) \equiv \frac{\eta}{2} \boldsymbol{\theta} \cdot \boldsymbol{\theta} + \frac{1}{2} \chi^2(\boldsymbol{\theta}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \ldots)$$



Hybrid renormalization

- Need full range of z for Fourier transform
  - Must be careful about renormalization at large z

- Determine δm from static q-$\bar{\text{q}}$ potential, and change scheme to $\overline{\text{MS}}$ with

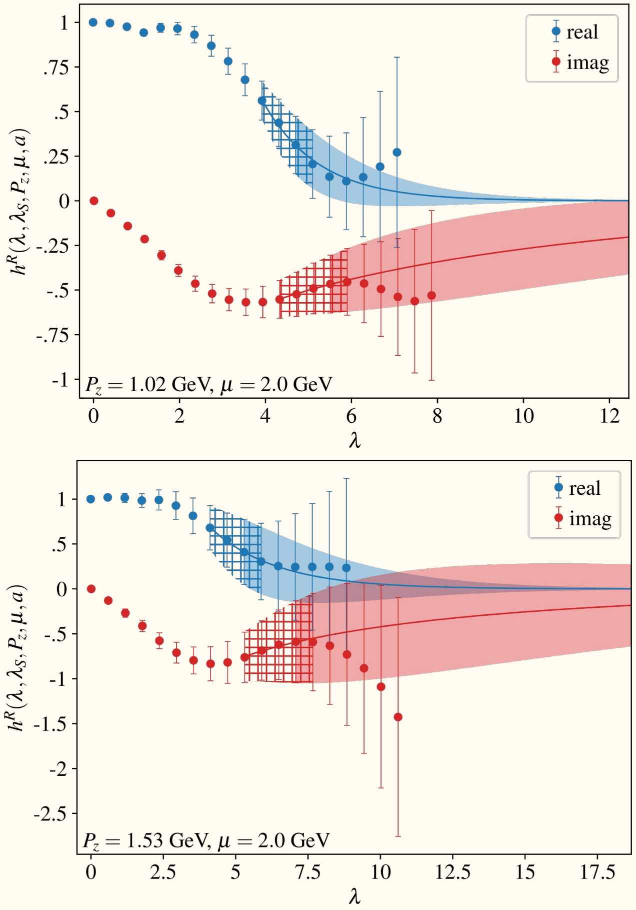
$$e^{\delta m(z-z_0)}\frac{h^B(z,0,a)}{h^B(z_0,0,a)} = e^{-\overline{m}_0(z-z_0)}\frac{C_0(\mu^2 z^2) - \Lambda z^2}{C_0(\mu^2 z_0^2) - \Lambda z_0^2}$$

- Ratio scheme at short distances and include Wilson-line mass correction at large distances

$$\tilde{h}^R(z, z_S, P_z, \mu) = \begin{cases} N\frac{h^B(z,P_z,a)}{h^B(z,0,a)}\frac{C_0(z^2\mu^2)+\Lambda z^2}{C_0(z^2\mu^2)}, & z \le z_S \\ N\frac{h^B(z,P_z,a)}{h^B(z_S,0,a)}\frac{C_0(z_S^2\mu^2)+\Lambda z_S^2}{C_0(z_S^2\mu^2)}e^{\delta m'(z-z_S)}, & z > z_S. \end{cases}$$

- Extrapolate with exponential decay model

$$\frac{Ae^{-m_{\text{eff}}\lambda/P_z}}{|\lambda|^d}$$



$\boldsymbol{x}$-space matching

- Obtain quasi-PDF from Fourier transform

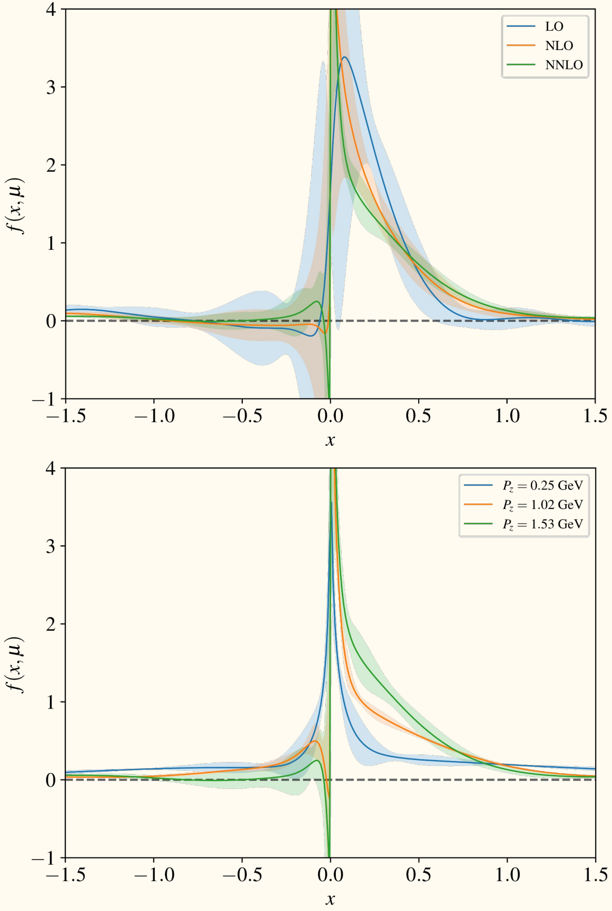
$$\tilde{q}^v(x, \lambda_S, P_z, \mu) = \int \frac{dz}{2\pi} e^{ixP_z z}\, \tilde{h}^R(z, z_S, P_z, \mu)$$

- Match quasi-PDF to PDF

$$q^v(x, \mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP_z}, |y|\lambda_S\right) \tilde{q}^v(y, \lambda_S, P_z, \mu)$$

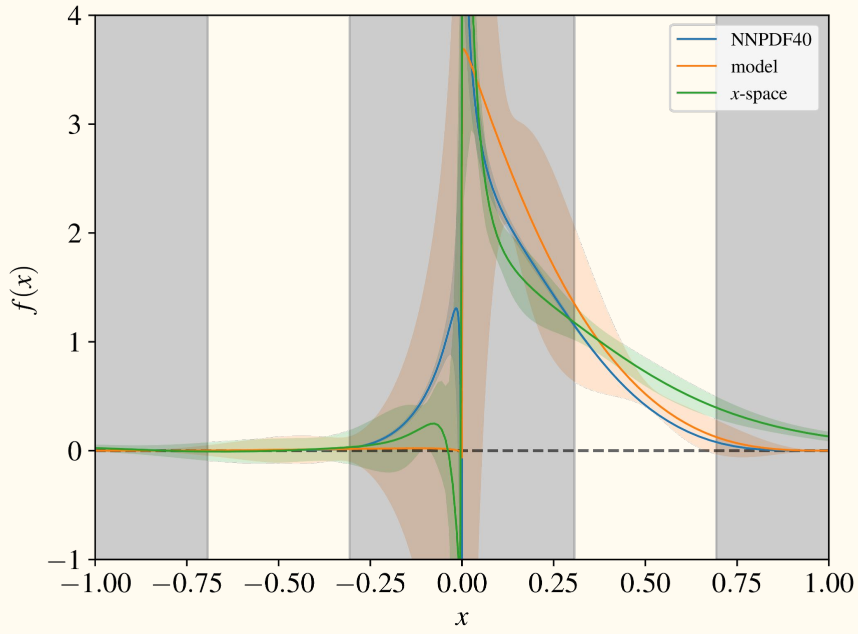
$$+\mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xP_z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{((1-x)P_z)^2}\right)$$

- Need to estimate valid range of $\boldsymbol{x}$
    - Use DGLAP evolution to remove $\ln \frac{\mu^2}{4x^2P_z^2}$, which breaks down at $2xP_z \sim \Lambda_{\mathrm{QCD}}$



Conclusions and Outlooks

- Conclusions
  - Excited-state contamination at the physical point can be controlled
  - Leading-twist OPE can describe the proton ratio data for $z \sim 0.8$ fm
    - First four moments extracted
    - $\langle x \rangle$ is above result from NNPDF40
  - Some tension between various extraction methods
  - Perturbative convergence appears to have been achieved



- Future work/Outlooks
  - More statistics and source-sink separations would be helpful
  - Larger P_z would also be useful if possible
  - Helicity and transversity distributions