Hybrid renormalization: matching and its renormalon ambiguity

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Topics

- Why hybrid renormalization? (review of Ji et al 2008.03886)
- Matching PDFs, GPDs (w/ CY Chou 2204.08343)
- Renormalon ambiguity (w/ WY Liu 2010.06623; YX Chen)

LaMET Factorization Theorem

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}, \frac{M^2}{P_z^2}\right) + \dots$$

- compensating the UV difference of quasi-PDF and PDF; renormalization scheme and scale dependent.
- PDF in MS-bar, quasi-PDF in lattice spacing--- lattice action dependent, slow convergence (linear divergence, Wilson line mass subtraction scheme, Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang;)

NPR

- Non-perturbative renormalization (NPR): quark bilinear operators multiplicatively renormalized, ratio scheme (same operator, different states, Radyushkin), RI/MOM (loop corrections removed at off shell momentum, Yong & Stewart; Constantinou et al), to continuum limit, no lattice discretization dependence (ChQCD (2012.05448): might not for RI/MOM)

Why Hybrid Renormalization?

- Ratio & RI/MOM schemes remove UV divergence, but their matching kernels have non-perturbative IR effects. Long (> Zs~0.3 fm) Wilson line op. using Wilson line mass subtraction scheme---Hybrid renormalization (X. Ji, Y. Liu, A. Schäfer, W. Wang, Y.B. Yang, J.H. Zhang, Y. Zhao, 2008.03886)

Our Contribution

- Hybrid-RI/MOM (q-PDF) to MS-bar (PDF) one loop matching kernel (isovector, unpolarized, helicity, and transversity PDFs and skewless GPDs) for any hadron. GPD to appear.
- Hybrid-Ratio as a special example (mu_R = 0, pz_R=0)
- Self-renormalization (SR) also a special case (Zs = 0), need modification at short distance

- Is the hybrid-ratio, perhaps with SR at long distance, the favorite scheme?

Scheme Conversions

- Multiplicative renormalization

$$\tilde{Q}^{B}_{\gamma^\mu}(z, P^z, \epsilon) = \frac{1}{2P^\mu}\langle P|\bar{\psi}(z)\gamma^\mu W(z,0)\psi(0)|P\rangle.$$

$$\tilde{Q}^{B}(z, P^z, \epsilon) = \tilde{Z}^{X}(z, P^z, \epsilon, \tilde{\mu})\,\tilde{Q}^{X}(z, P^z, \tilde{\mu}),$$

$$Z^{X}_{\overline{\rm MS}}(z, \tilde{\mu}, \tilde{\mu}') \equiv \frac{\tilde{Q}^{X}(z, P^z, \tilde{\mu})}{\tilde{Q}^{\overline{\rm MS}}(z, P^z, \tilde{\mu}')} = \frac{\tilde{Z}^{\overline{\rm MS}}(z, P^z, \epsilon, \tilde{\mu}')}{\tilde{Z}^{X}(z, P^z, \epsilon, \tilde{\mu})}$$

- Factorization proved in MS-bar; can be converted to other schemes

MS-bar to MS-bar matching

- Loose ends in Izubuchi, Ji, Jin, Stewart, Zhao:

Epsilon expansion and Fourier transform commute? Fermion number conservation and delta function at infinite x/y

$$\tilde{q}(x,\Lambda,P_z)=\int\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right)q(y,\mu)$$

$$-\int\frac{dzp^z}{2\pi}e^{ixzp^z}\ln(z^2\mu^2e^{\gamma_E})=-\left[\frac{d}{d\eta}\int\frac{dzp^z}{2\pi}e^{ixzp^z}\left(z^2\mu^2e^{\gamma_E}\right)^\eta\right]\Bigg|_{\eta=0}$$

$$-\int\frac{dzp^z}{2\pi}e^{ixzp^z}\ln(z^2\mu^2e^{\gamma_E})=-\int\frac{dzp^z}{2\pi}e^{ixzp^z}\ln(\frac{z^2\mu^2e^{\gamma_E}}{K^2})-\int\frac{dzp^z}{2\pi}e^{ixzp^z}\ln K^2$$

$$=\tilde{f}^C(x)-\ln K^2\left[\frac{1}{2}\left(\frac{1}{x^2}\delta^+\left(\frac{1}{x}\right)+\frac{1}{(-x)^2}\delta^+\left(-\frac{1}{x}\right)\right)\right]$$

Ratio to MS-bar matching

- Kernel has non-perturbative IR contribution

$$\tilde{Z}^{\overline{\rm MS}}(\tilde{\mu},\epsilon) = 1 + \frac{\alpha_s C_F}{2\pi}\frac{3}{2}\frac{1}{\epsilon_{UV}} + \mathcal{O}(\alpha_s^2).$$

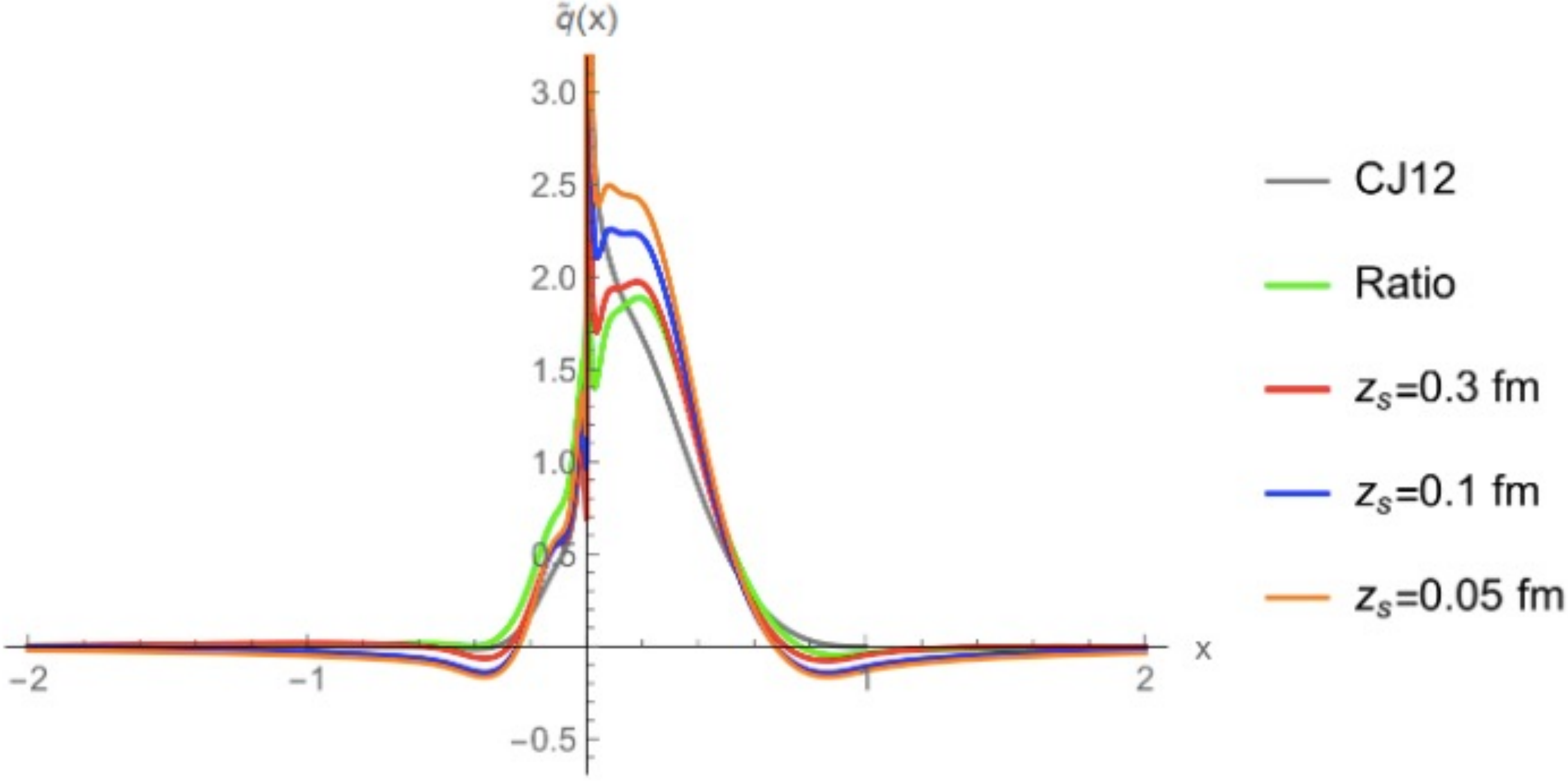
$$Z_{\overline{\rm MS}}^{\rm ratio}(z,\tilde{\mu}) = 1 - \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\frac{\tilde{\mu}^2 z^2}{4e^{-2\gamma_E}} + \frac{5}{2}\right) + \mathcal{O}(\alpha_s^2).$$

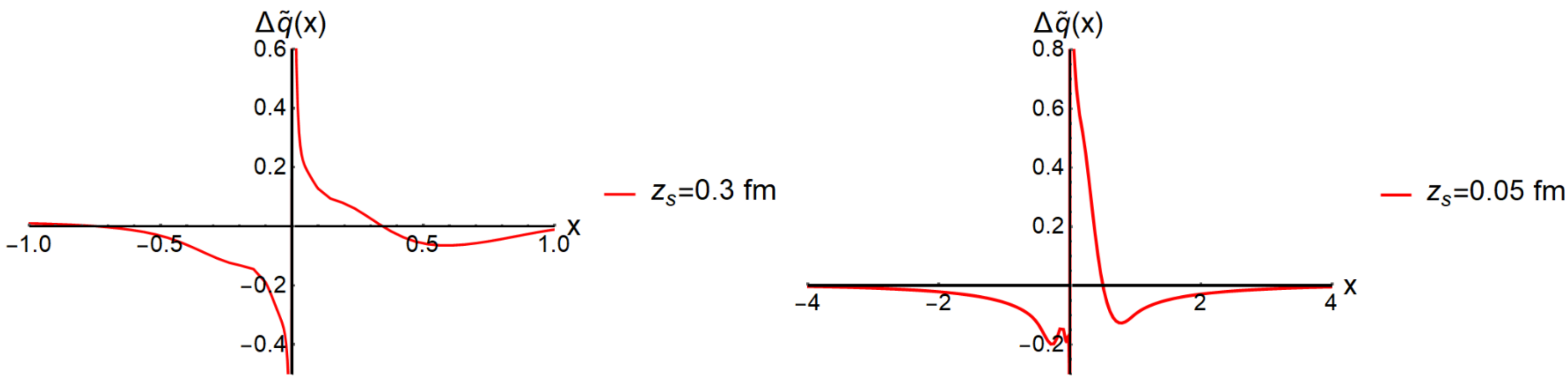
- Use Wilson line mass subtraction scheme for z > Zs, conversion factor is constant in z in dim reg

$$C^2 \exp(-\delta m|z|)$$

$$Z_{\overline{\rm MS}}^{\text{hybrid-X}}(z,z_s,\tilde{\mu},\tilde{\mu}') = Z_{\overline{\rm MS}}^{X}(z,\tilde{\mu},\tilde{\mu}')\,\theta(z_s-|z|) + Z_{\overline{\rm MS}}^{X}(z_s,\tilde{\mu},\tilde{\mu}')\,\theta(|z|-z_s),$$

Hybrid-Ratio to MS-bar Matching





$\Delta\tilde{q}(x)$
0.6
0.4
0.2
−0.2
−0.4
−1.0
−0.5
0.5
1.0
x
z_s=0.3 fm
$\Delta\tilde{q}(x)$
0.8
0.6
0.4
0.2
−0.2
−4
−2
2
4
x
z_s=0.05 fm

Renormalon in LaMET

$$\tilde{Q}(x, P_z, \mu') = \int_{-1}^{1} \frac{dy}{|y|} Z(\frac{x}{y}, yP_z, \mu', \mu) Q(y, \mu) + \mathcal{O}(\frac{1}{P_z^2})$$

In $\overline{\mathrm{MS}}$ and RI/MOM scheme, renormalon ambiguity arises. Braun, Vladimirov and Zhang (1810.00048): power correction $\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/x^2 P_z^2)$

Renormalon Ambiguity

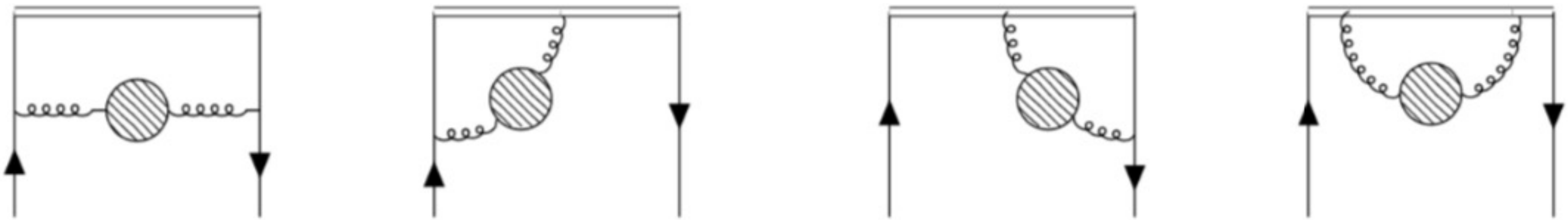
In an OPE,

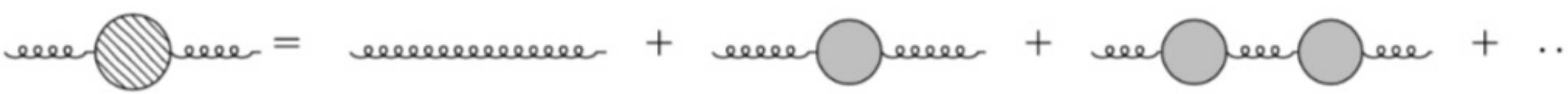
(1) use Borel transform to improve the convergence of the Wilson coefficients

(2) sum the series

(3) then perform inverse Borel transform. Poles in the integrant (renormalons) lead to ambiguity in the contour integrals which can be absorbed by the power corrections.

Studied by bubble chain diagrams

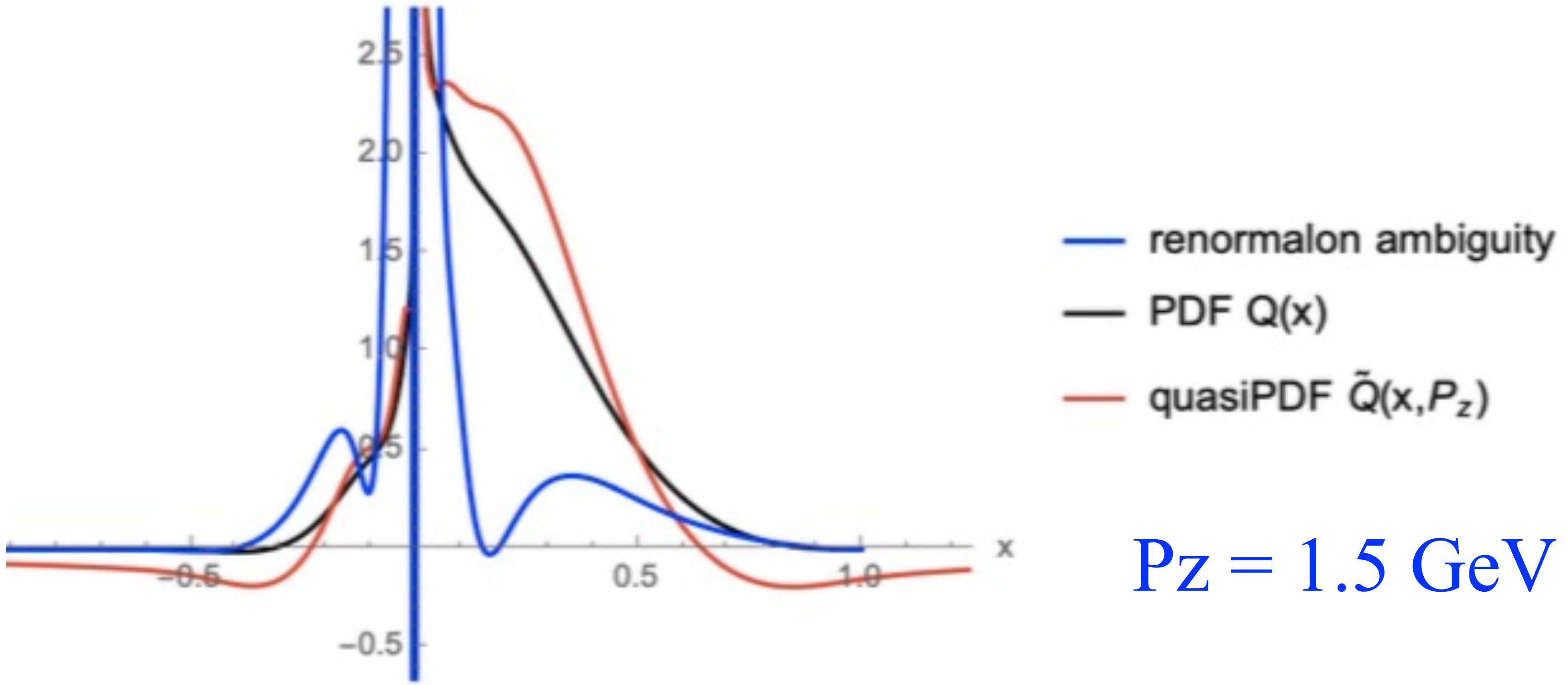




$\alpha_s n_f$ as $\mathcal{O}(1)$

$$\frac{1}{-k^2 - i\epsilon} \to \frac{(\Lambda_{\rm QCD}^2)^w}{(-k^2 - i\epsilon)^{1+w}}$$

Power Corrections suggested by Renormalon Ambiguity

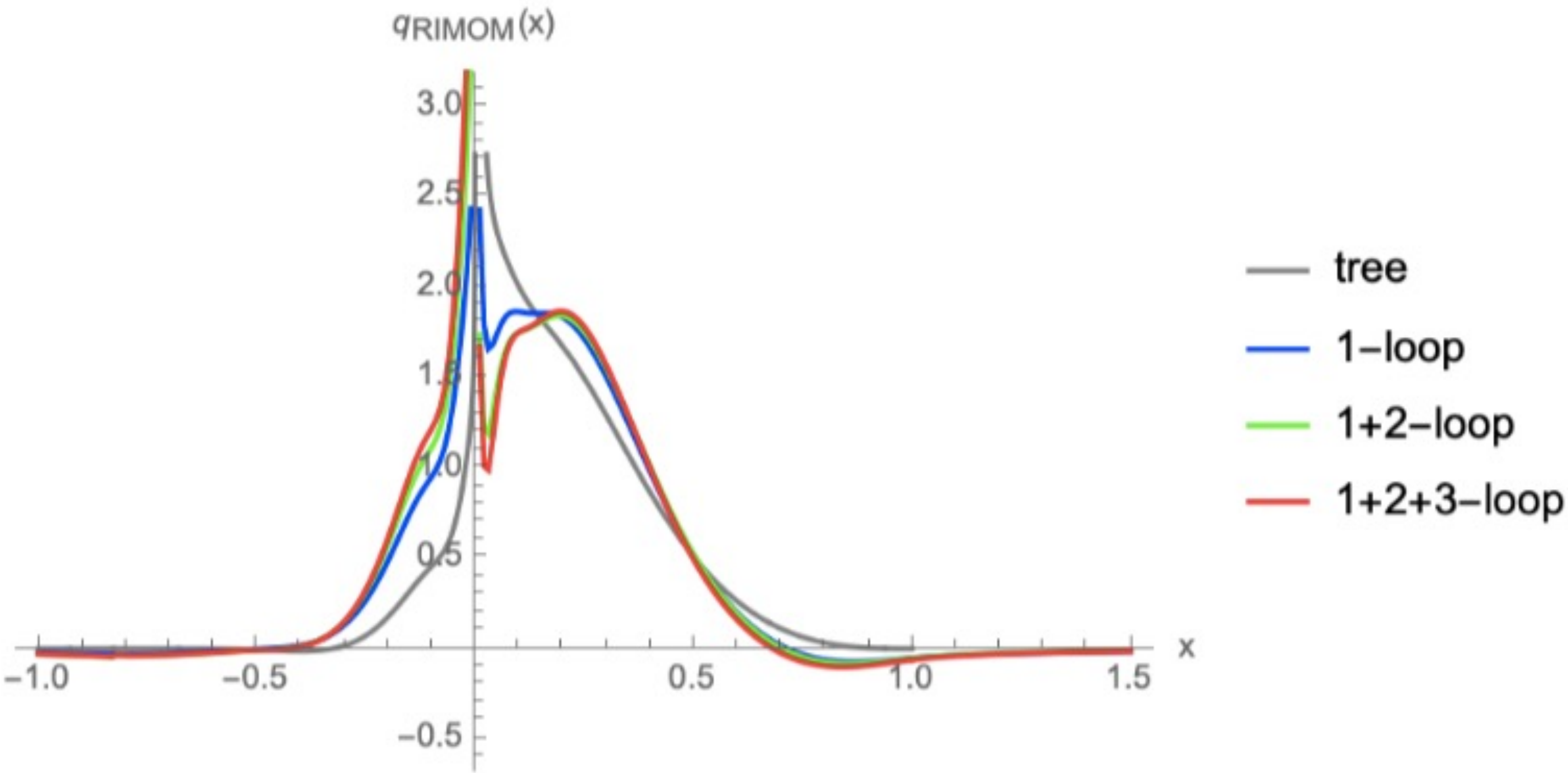


Pz = 1.5 GeV

$$\delta\tilde{Q}_{ren}(x,P_z) = \frac{\pi}{\beta_0} e^{5/3} C_F \frac{\Lambda^2_{\rm QCD}}{P_z^2} \int_{-1}^{1} \frac{dy}{|y|y^2} \left[\frac{\theta(1-\frac{x}{y})\theta(\frac{x}{y}) - \delta(1-\frac{x}{y})}{1-\frac{x}{y}} \right]_+ Q(y)$$

$$= \frac{\pi}{\beta_0} e^{5/3} C_F \frac{\Lambda^2_{\rm QCD}}{x^2 P_z^2} \left\{ \int_0^1 d\xi \frac{1}{1-\xi} [\xi Q(x/\xi) - Q(x)] + Q(x) - xQ'(x) \right\}$$

Bubble diagram contribution up to 3-loops (RI/MOM to MS-bar)



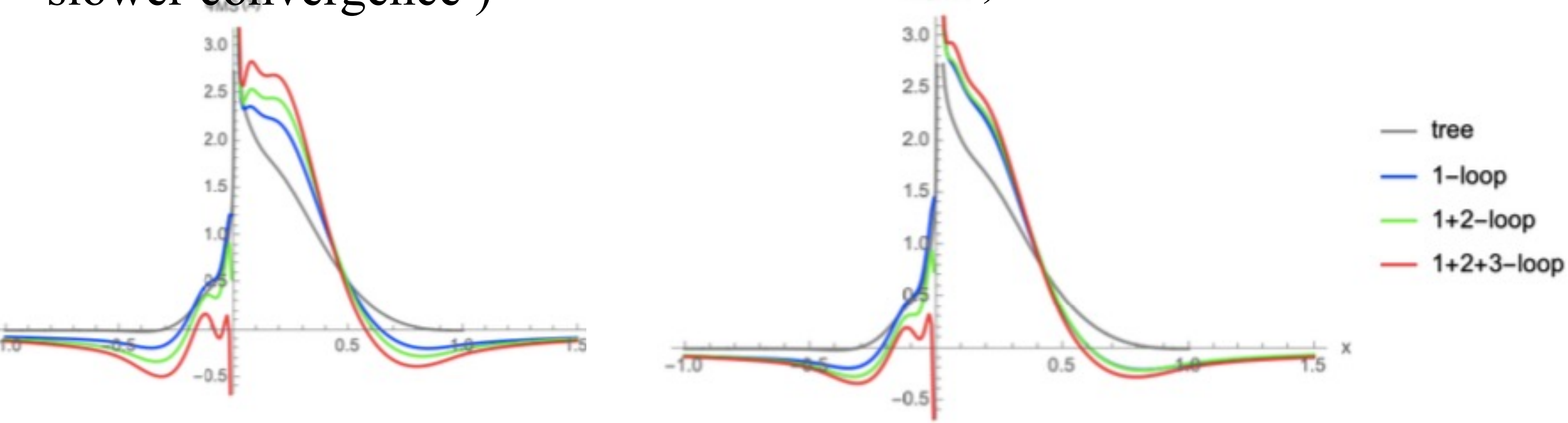
R-Scheme

(Hoang, Jain, Scimemi, Stewart, 0908.3189)

$$\tilde{Q}_R(x, P_z, P_z', \Lambda') = \frac{P_z^2 \tilde{Q}(x, P_z, \Lambda') - P_z'^2 \tilde{Q}(x, P_z', \Lambda')}{P_z^2 - P_z'^2}$$

(MS-bar to MS-bar slower convergence)

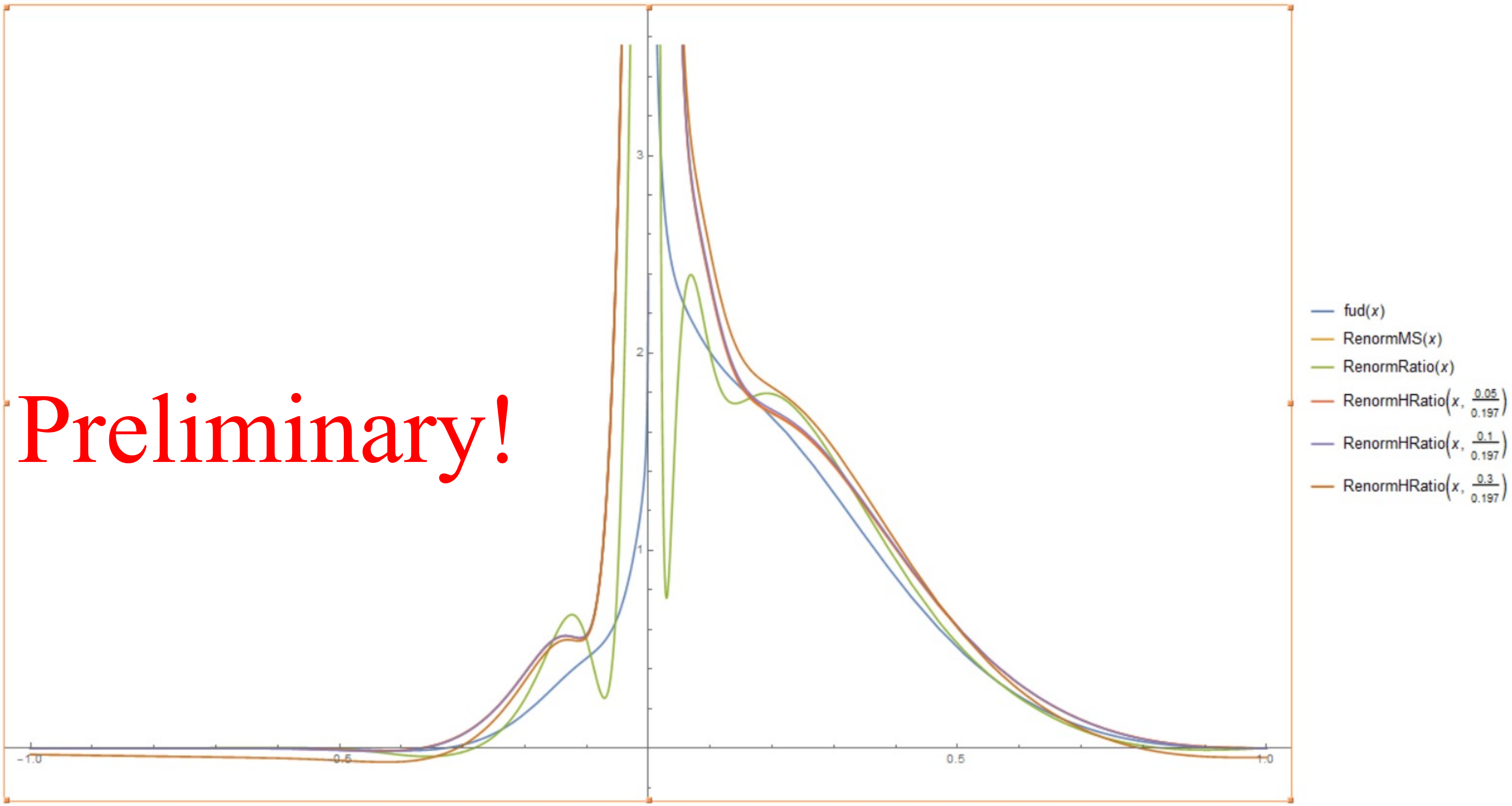
(faster convergence by adding the R-Scheme)



$P_z' = 3$ GeV, $\alpha_s = 0.283$, $P_z = 1.5$ GeV, and $\mu = 3$ GeV

Hybrid-ratio renormalon ambiguity

w/ YX Chen



- Hybrid-ratio closer to MS-bar than ratio
- Is 0.2 < x < 0.8 reliable?

Summary

- The hybrid-ratio scheme, perhaps with self-renormalization at long distance, seems to be the best scheme so far
- What is the reliable range of x? Trying the R-scheme?

Backup slides

Topics

- Renormalon (w/ WY Liu 2010.06623)
- Finite volume effect w/ ChPT (w/ WY Liu 2011.13536)
- Matching in hybrid renormalization (w/ CY Chou)

Factorization in LaMET

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right) + \ldots$$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P | \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) | P\rangle$$

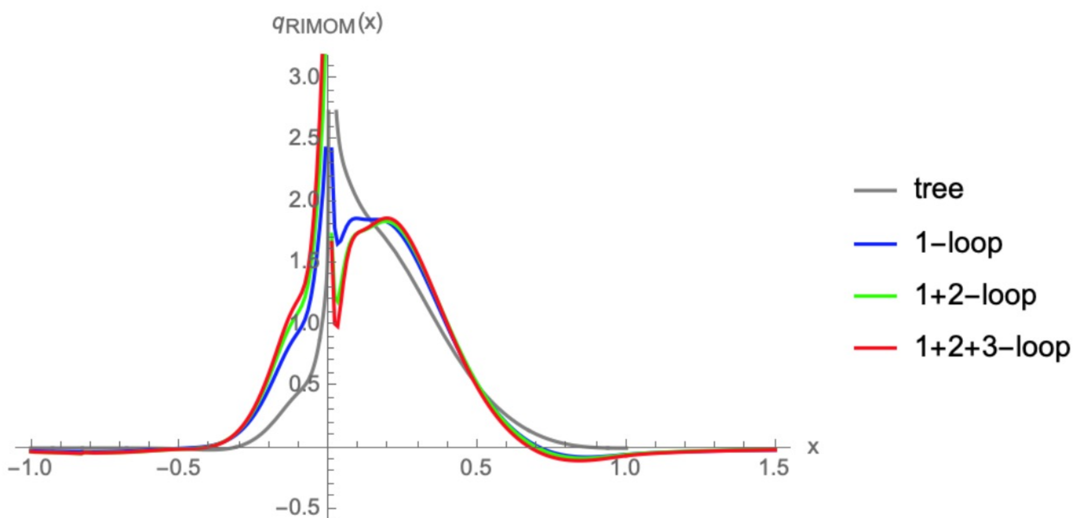
$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dz}{4\pi} e^{-izk} \times$$

$$\left\langle \vec{P} \right| \bar{\psi}(z)\gamma_z e^{ig\int_0^z A_z(z')dz'} \psi(0) \left| \vec{P} \right\rangle$$

Power Corrections

- $\mathcal{O}(M^2/(P^z)^2)$ corrections computed to all orders (JWC et al. 1603.06664)
- Renormalon effect: Braun, Vladimirov, Zhang (1810.00048) $\mathcal{O}(\Lambda^2_{\rm QCD}/\check{x}^2 P_z^2)$; But the slow convergence is not seen in bubble diagrams at 3-loops

(w/ Wei-Yang Liu 2010.06623)



ChPT for LaMET

$$\lambda_\mu \bar{\psi}(z)\Gamma^\mu W(z,0)\psi(0) \simeq \sum_{n=0}^{\infty} \frac{(iz)^n}{n!} \lambda_\mu \lambda_{\mu_1} \lambda_{\mu_2} \ldots \lambda_{\mu_n} \bar{\psi}\Gamma^\mu iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n}\psi,$$

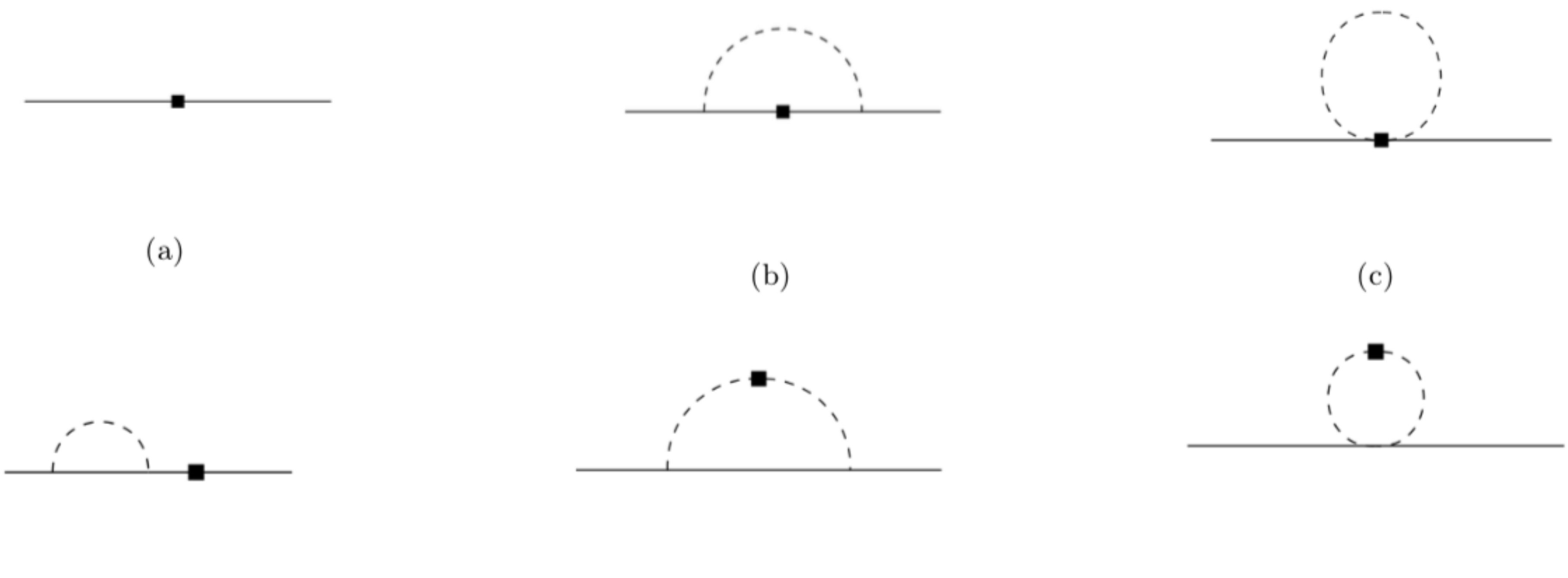
$$\mathcal{O}_q^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{(\mu} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n)}\psi,$$

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{tr}(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \eta\,\mathrm{tr}(\mathcal{M}\Sigma^\dagger + \mathcal{M}^\dagger \Sigma) + \overline{N} iv \cdot DN + 2g_A \overline{N} S \cdot AN + \ldots,$$

$$\Sigma = e^{\frac{i}{F_\pi}\Pi}, \quad \Pi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$\mathcal{M} = \mathrm{diag}(m_u, m_d) \qquad\qquad u^2 = \Sigma.$$

ChPT for LaMET

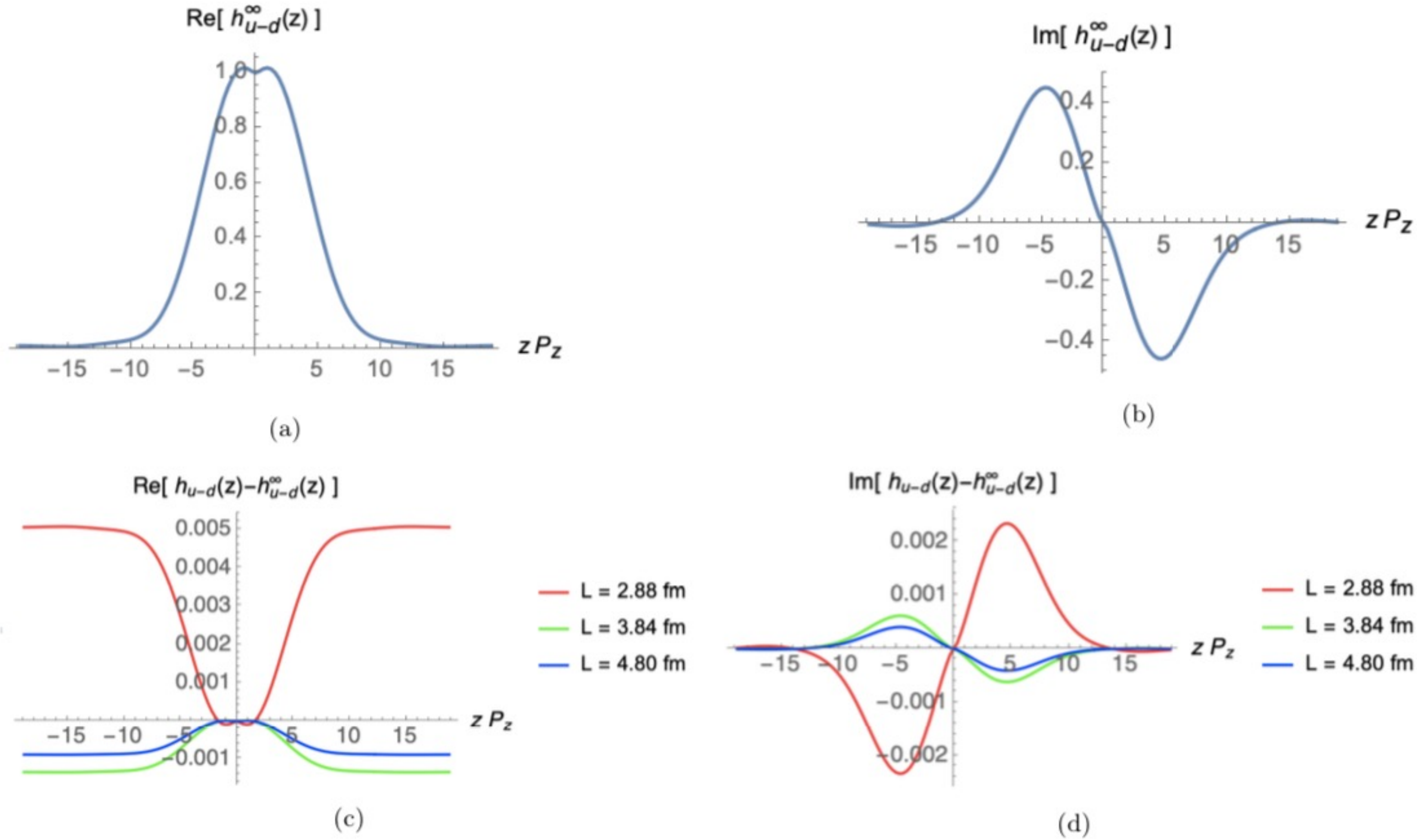


(a) (b) (c) (d) (e) (f)

$$\mathcal{O}_{u-d}^{\mu\mu_1\mu_2\ldots\mu_n} = c_1^{(n)}\bar{N}v^{(\mu}v^{\mu_1}\ldots v^{\mu_n)}(u\tau^3u^\dagger + u^\dagger\tau^3 u)N + \tilde{c}_1^{(n)}\bar{N}S^{(\mu}v^{\mu_1}\ldots v^{\mu_n)}(u\tau^3u^\dagger - u^\dagger\tau^3 u)N + \cdots$$

$$\mathcal{O}_{u-d,\pi}^{\mu} \simeq a^{(0)}F_\pi^2 \mathrm{tr}\left(\Sigma^\dagger\tau^3 i\partial^\mu\Sigma + \Sigma\tau^3 i\partial^\mu\Sigma^\dagger\right)$$

ChPT for LaMET



Finite volume effect less than 1% when $P_z/M \geq 1$ and $m_\pi L \geq 3$ consistent w/ Lin & Zhang (2019).

ChPT for LaMET

- Idea: Heavy Baryon ChPT can be used for a baryon with a large momentum, as long as its off-shellness in the loop is much smaller than the baryon mass.
- Equal time correlator is dominated by the symmetric traceless (twist-2) terms under OPE. Trace terms are suppressed by the baryon momentum.
- Matching of the twist-2 operators standard by now: JWC, Ji, **PLB**523 (2001) 107; **PRL** 87 (2001) 152002; **PRL** 88 (2002) 052003; JWC, Stewart, **PRL** 92 (2004) 202001; Arndt, Savage, **NPA**697 (2002) 429