COLLEGE OF COMPUTER, MATHEMATICAL, & NATURAL SCIENCES

CALCULATION OF PION DISTRIBUTION AMPLITUDE I: *RENORMALIZATION AND POWER ACCURACY*

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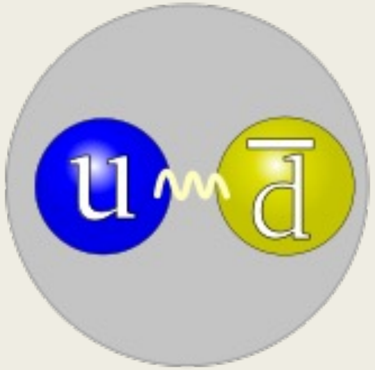
In collaboration with Xiangdong Ji, Huey-Wen Lin, Jack Holligan, and Yushan Su

Outline

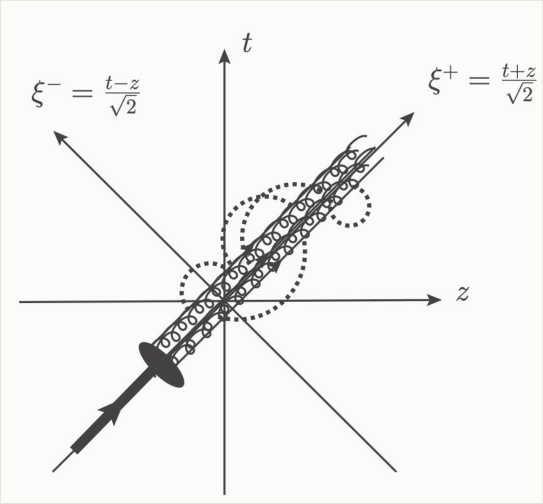
- Introduction to Pion DA
- Renormalization of DA
- Power Accuracy in LaMET
- Application to DA Analysis

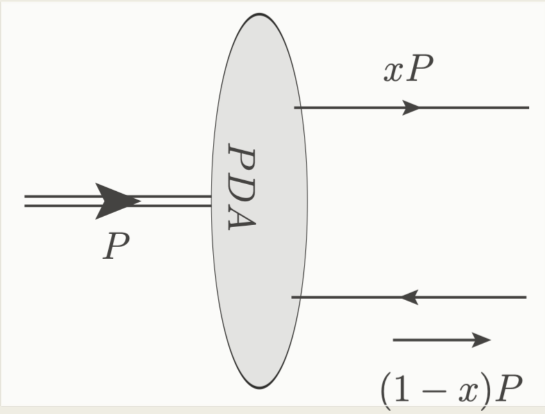
Pion Distribution Amplitude (DA)



Pion lightfront DA $\phi(x)$: probability amplitude of pion in the bound state's minimal Fock component $|q\bar{q}\rangle$

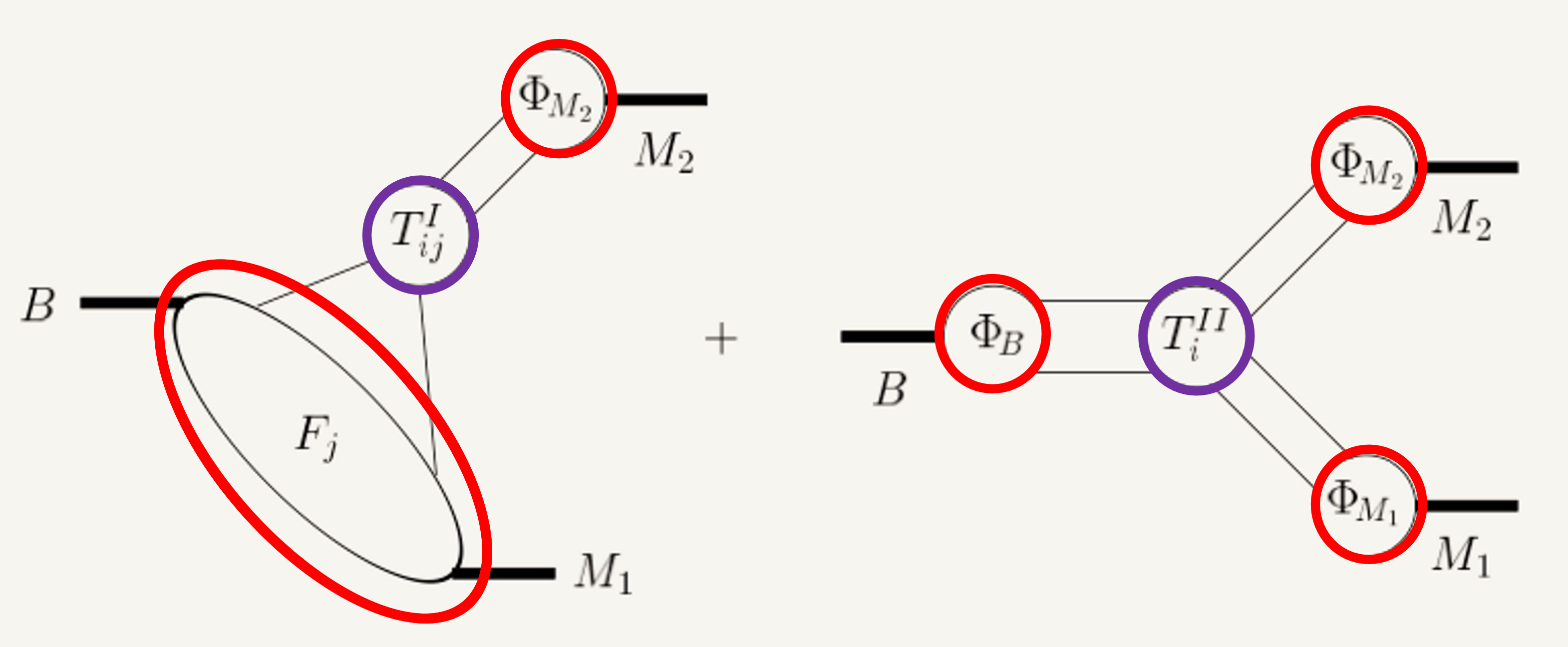
$$\phi(x,\mu) = \frac{1}{if_\pi}\int \frac{d\xi^-}{2\pi} e^{i\left(\frac{1}{2}-x\right)\xi^- p^+} \langle 0|\bar{q}\left(\frac{\xi^-}{2}\right)\gamma^-\gamma_5 U\left(\frac{\xi^-}{2},-\frac{\xi^-}{2}\right) q\left(-\frac{\xi^-}{2}\right)|\pi(p)\rangle$$

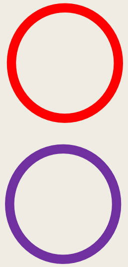




Factorization of hard exclusive process

DA as important input to hard exclusive process at $Q^2 \gg \Lambda_{\mathrm{QCD}}^2$: Beneke, et al. NPB(2001)

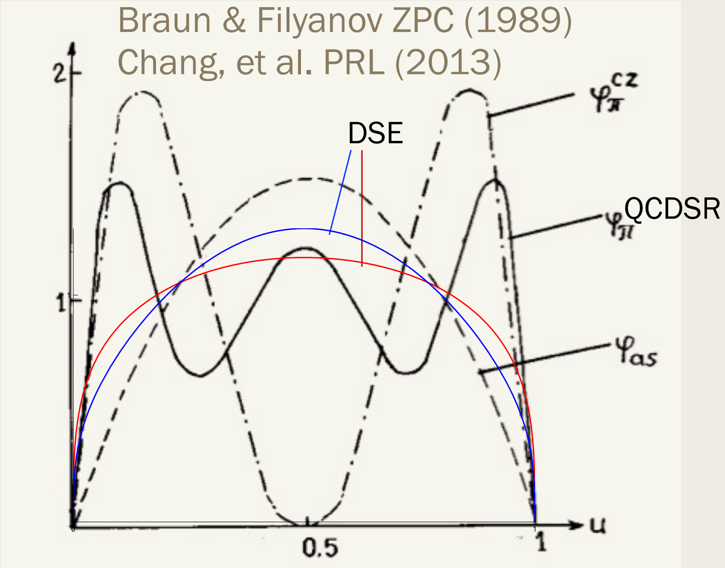




$$\langle \pi K | Q_i | B \rangle = F_0^{B\to\pi}\, T^{\mathrm{I}}_{K,i} * f_K \Phi_K + F_0^{B\to K}\, T^{\mathrm{I}}_{\pi,i} * f_\pi \Phi_\pi + T^{\mathrm{II}}_i * f_B \Phi_B * f_K \Phi_K * f_\pi \Phi_\pi \, ,$$

Previous DA Calculations

- Theoretical calculations
  1. QCD sum rule Chernyak, et al., NPB (1982)
  2. Dyson-Schwinger Equation



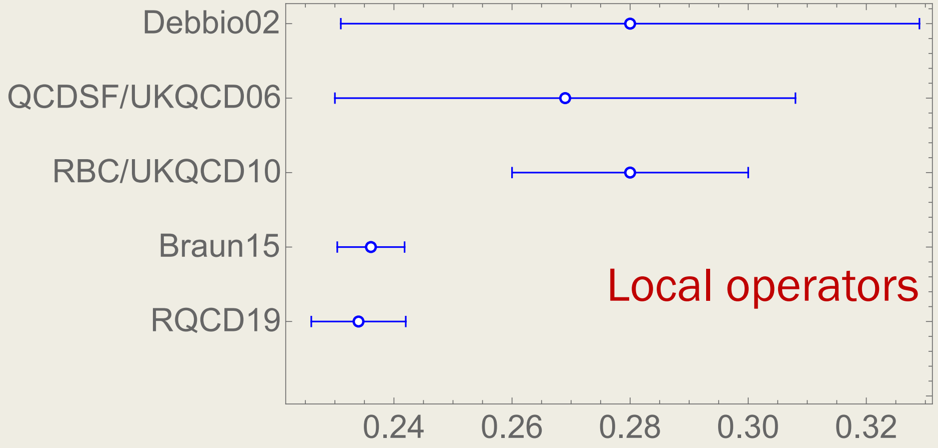
Model-dependent

- Moment calculations on Lattice

$$\langle \xi^n \rangle = \int_0^1 dx\, \phi(x)(2x-1)^n$$

HOPE PRD (2022)
RQCD PRD (2019)
Braun, et al., EPJC (2007)

  1. Current-current correlators
  2. Local twist-2 operators

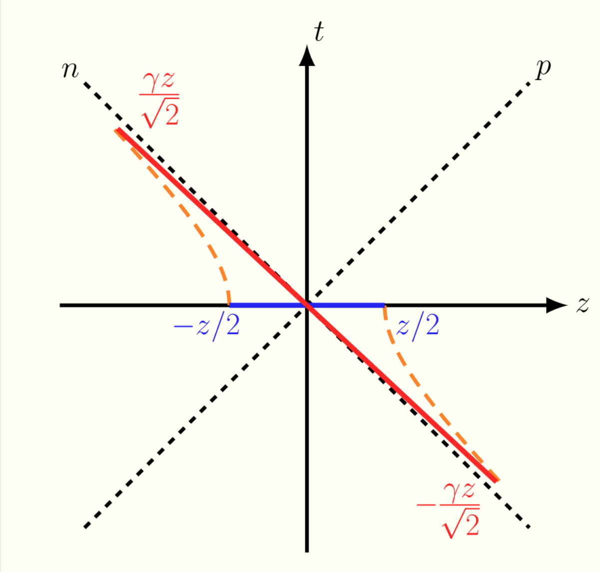


Pros: Precise lowest moment

Cons: Unable to extract x-dependence

X-dependence calculation

Ji, et al., RMP (2021)



- Large Momentum Effective Theory
- quasi-DA: Same IR behavior/ different UV behavior

$$\tilde{\phi}(x, P_z) = \frac{1}{i f_\pi} \int \frac{dz}{2\pi} e^{i\left(\frac{1}{2}-x\right)zP_z} \langle 0|\bar{q}(z)\gamma_z\gamma_5 U(z,-z) q(-z)|\pi(P_z)\rangle$$

- Approach $P \to \infty$ limit through large P_z expansion
- Matching to lightcone distribution

$$\tilde{\phi}(x, P_z) = \int_0^1 dy\, C(x, y, \mu, P_z)\phi(y, \mu) + O\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

Xiong, et al., PRD (2014)
Ma, et al., PRD (2018)
Izubuchi, et al., PRD (2018)
Liu, et al., PRD, (2019)
Ji, et al., RMP (2021)

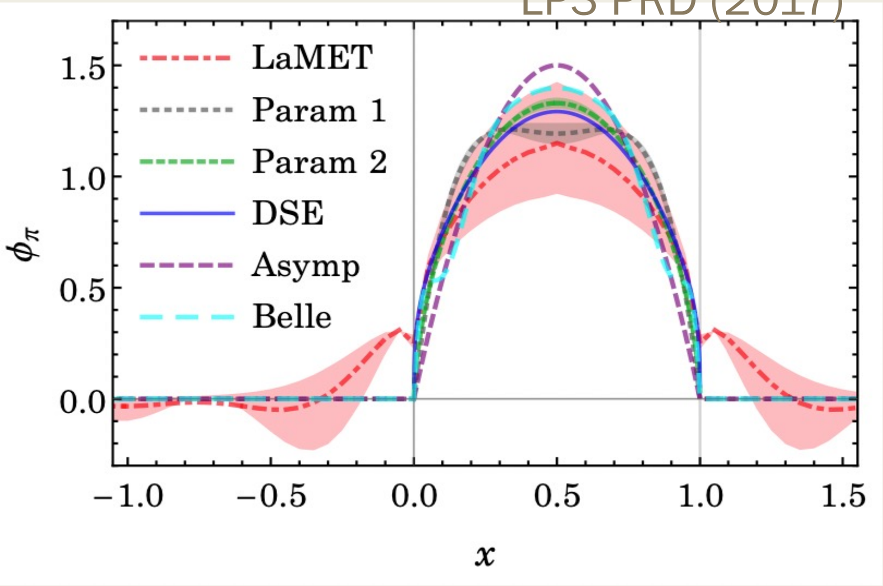
Pros: Direct x-dependence calculation, works well in mid-x region

Cons: Large P_z expansion breaks down near endpoints

More complicated renormalization

Progress in x-dependence calculations

LP3 PRD (2017)



- Lattice efforts
  - *Boost to large momentum*

Progress in x-dependence calculations

- Lattice efforts
  - *Boost to large momentum*
  - *Continuum extrapolation*

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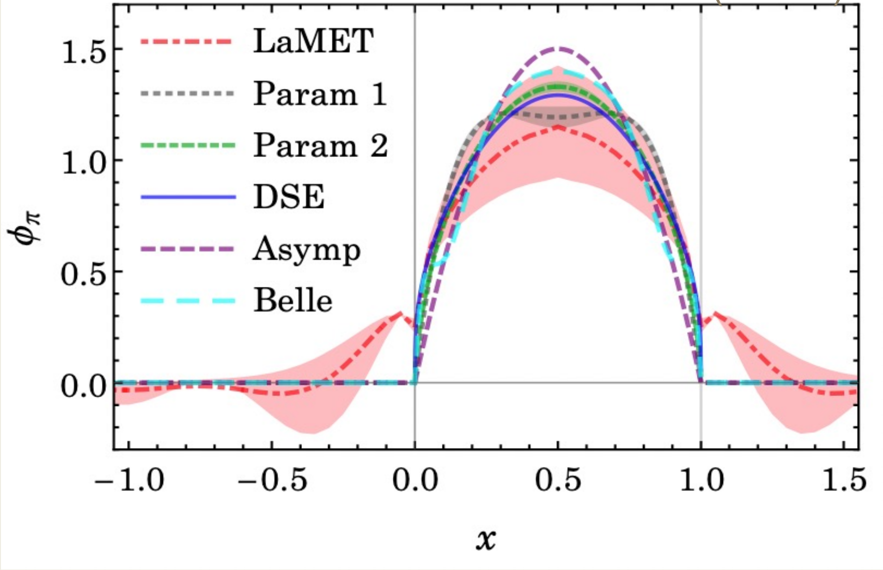

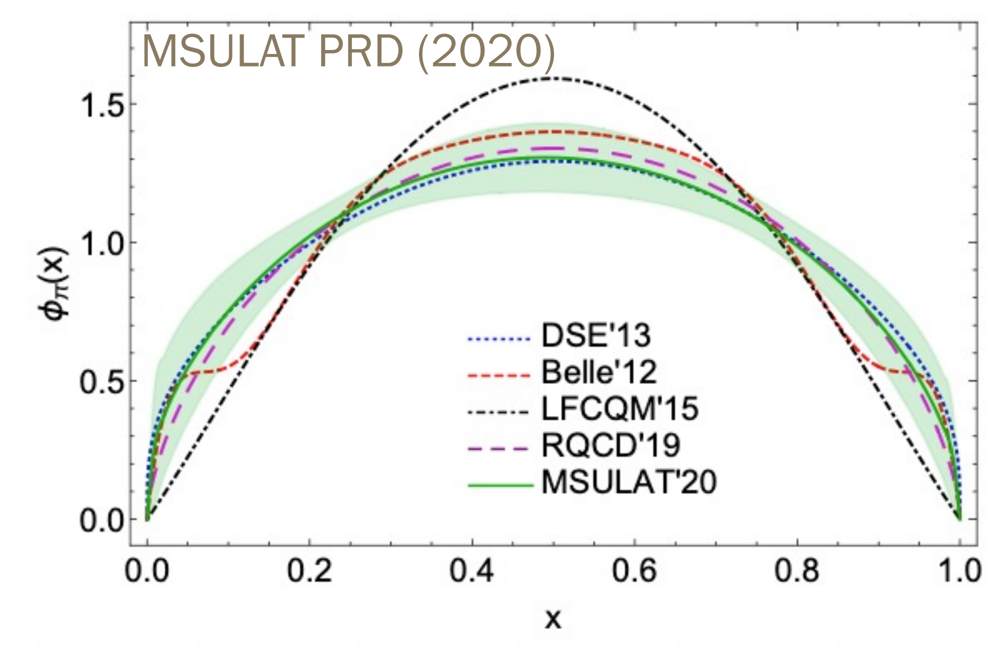

Progress in x-dependence calculations

- Lattice efforts
  - *Boost to large momentum*
  - *Continuum extrapolation*
  - *Physical pion mass*

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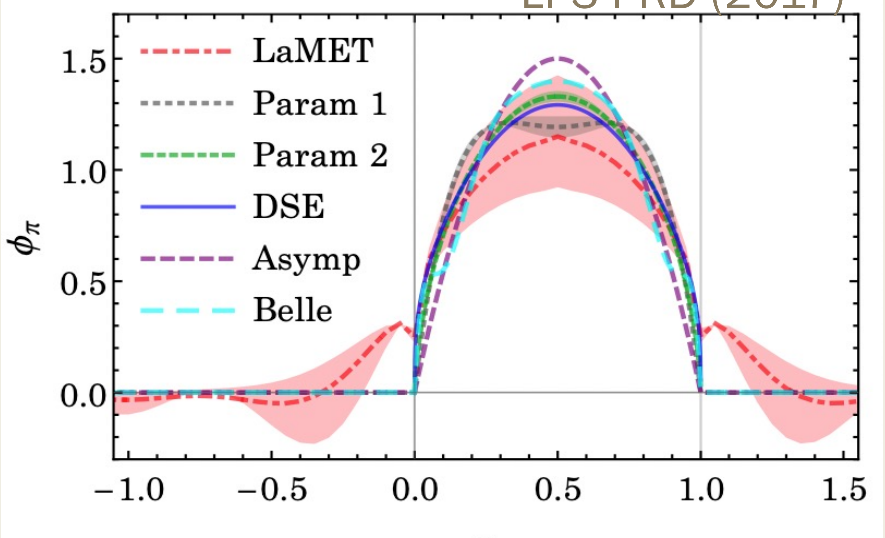
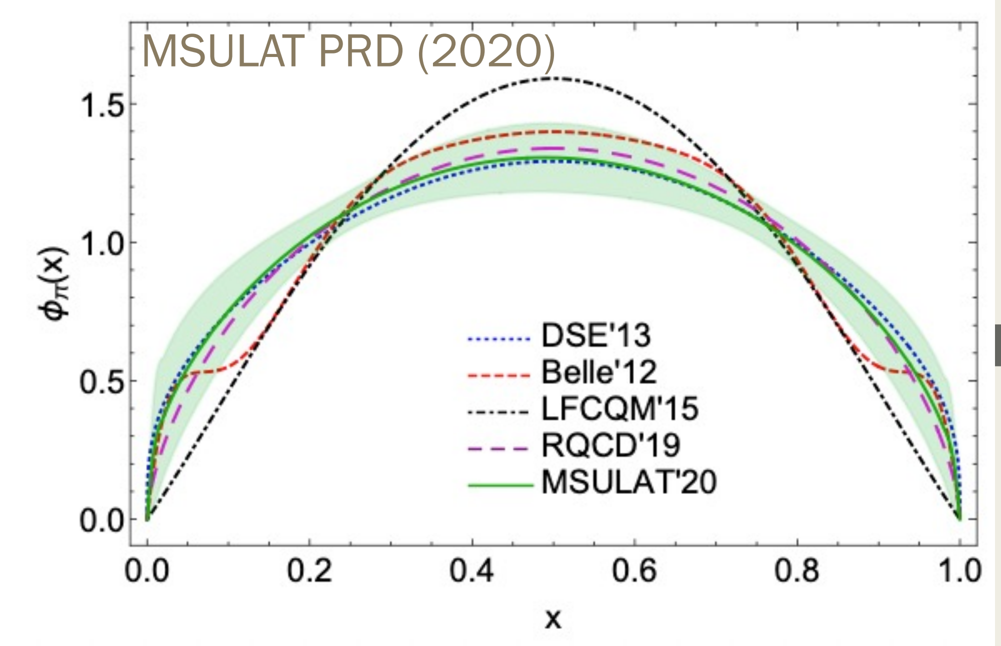
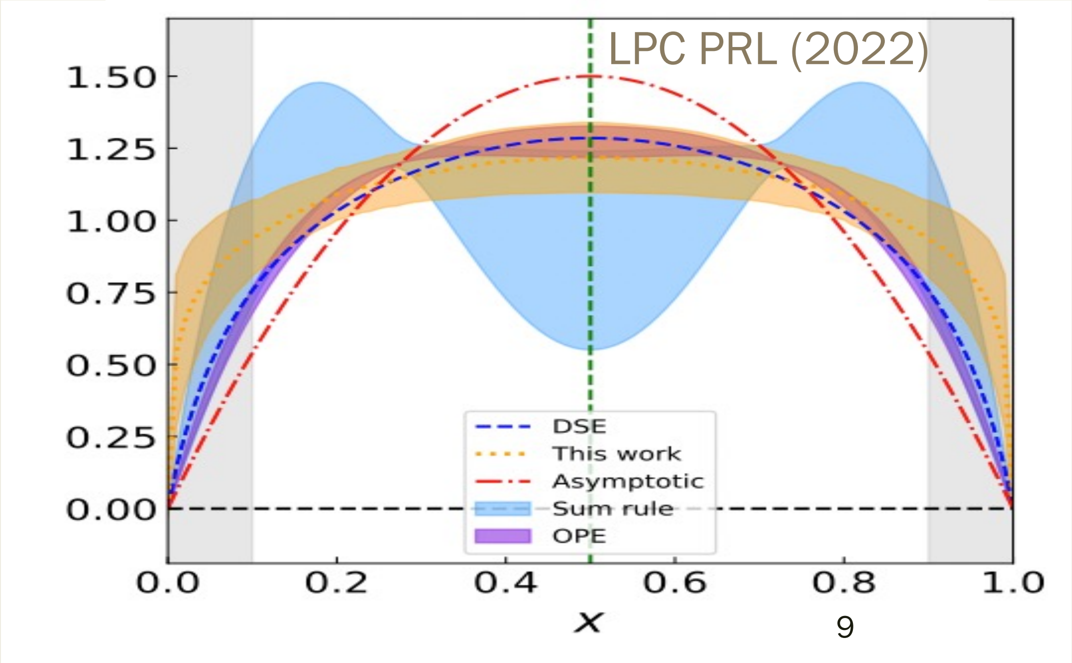


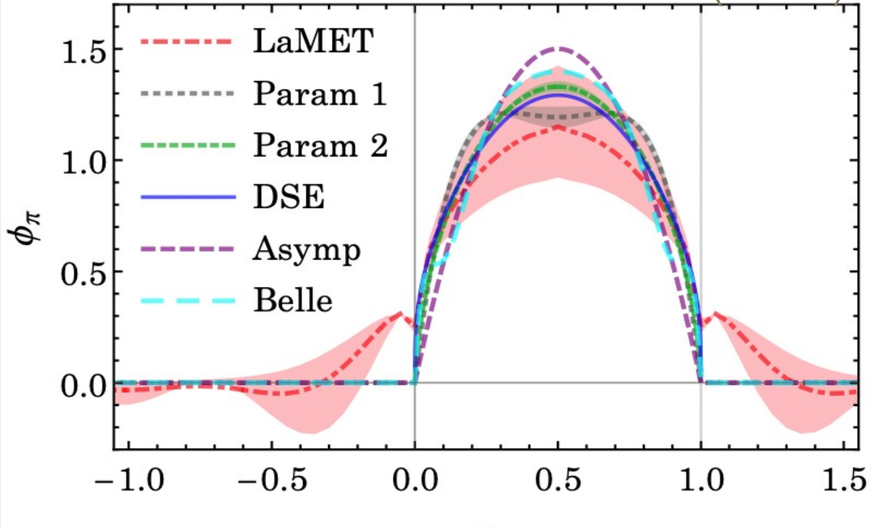


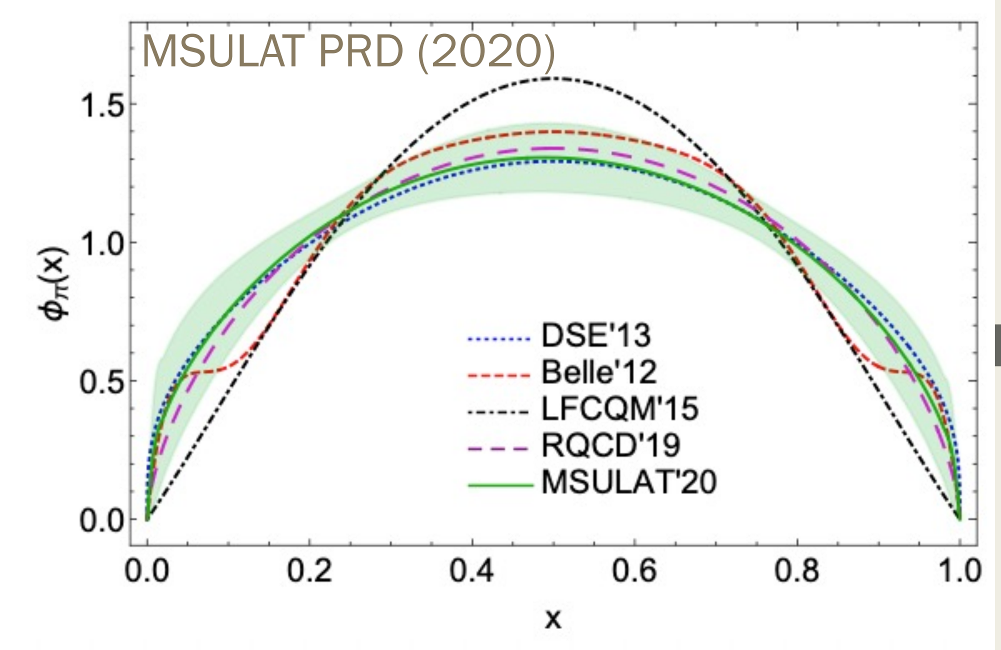


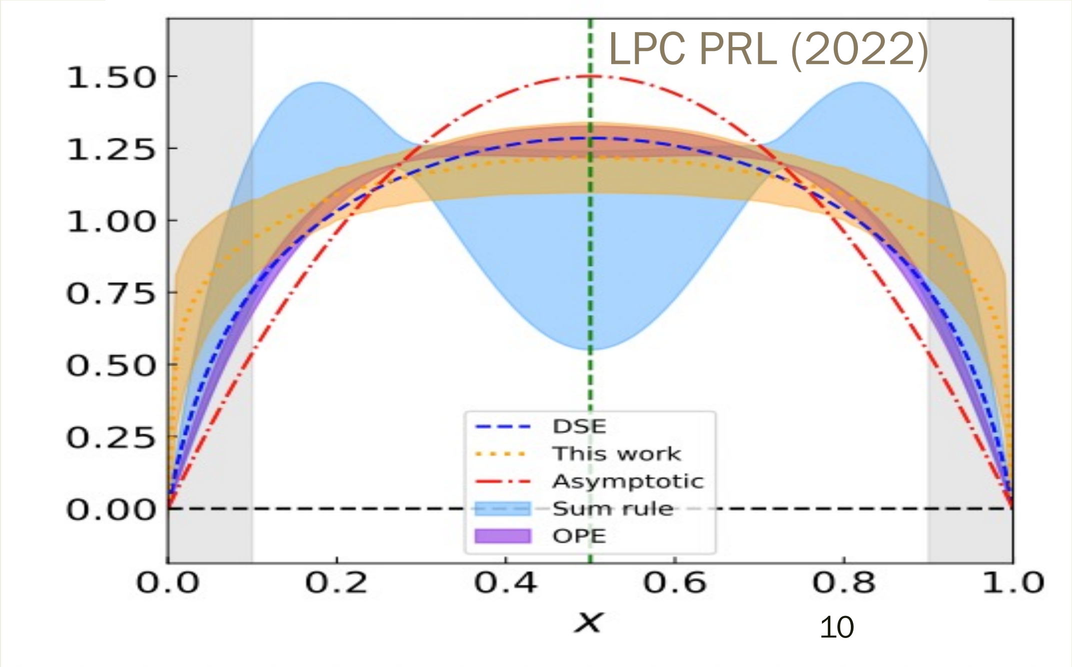
Progress in x-dependence calculations

- Lattice efforts
  - *Boost to large momentum*
  - *Continuum extrapolation*
  - *Physical pion mass*
- Theory: renormalization scheme

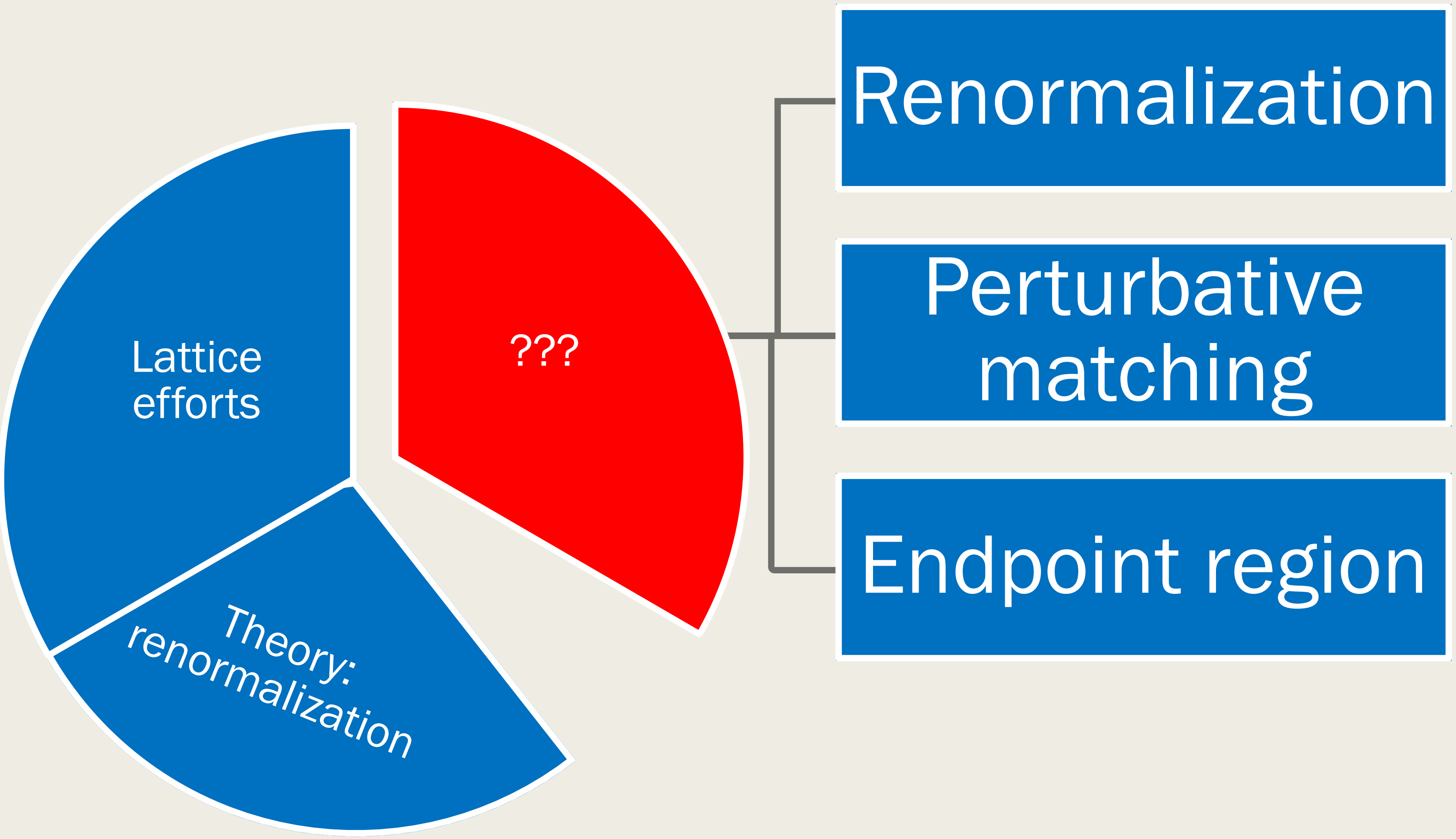
LP3 PRD (2017)







What is still missing?



IR renormalon

IR renormalon
Large log resummation

Constraints from SDF

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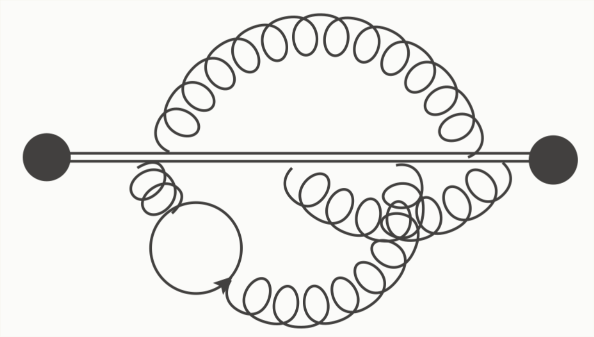
Why is renormalization necessary



- Non-local operator: $\hat{O}(z) = \bar{q}(z)W(z,0)\Gamma q(0)$
- Wilson line self energy: $\delta m \sim \frac{1}{a}$, linear divergence!
- Multiplicative renormalization

$$\hat{O}^R(z,a) = \hat{O}^{bare}(z,a)/Z^R(z,a)$$

$a \to 0$ 😀 $a \to 0$ 😰 $\sim e^{\delta m\, z}$

Renormalization with lattice data

Use lattice data with the same divergence

- Ratio scheme:

Radyushkin, PRD (2017)

$$Z^R(z,a) = \langle P_z = 0 | \hat{O}(z) | 0 \rangle$$

The lattice correlator vanishes for $P_z = 0$, not applicable

- RI/MOM scheme:

Martinelli, et al. NPB (1995)
Zhang, et al. PRD (2020)

$$Z^R(z,a) = \frac{\langle q | \hat{O}(z) | q \rangle}{\langle q | \hat{O}(z) | q \rangle_{\text{tree}}}$$

Extra non-perturbative effects at large z.

The scheme conversion to $\overline{\text{MS}}$ scheme at large z is not perturbative.

Improvements from perturbation theory

- Self renormalization:
  - Fit the a dependence from $P_z = 0$ lattice data
  - Match the lattice data to perturbative results in $\overline{\text{MS}}$ scheme

LPC, NPB (2021)

Hybrid scheme framework:

Short distance $|z| < z_s : Z^R(z, a)$ from another scheme

Long distance $|z| > z_s : Z^R(z_s, a)e^{\delta m(|z|-z_s)}$

Ji, et al., NPB (2021)
LPC, PRL (2022)

The scheme conversion to $\overline{\text{MS}}$ scheme is perturbative in all regions.

How to determine δm?

- Perturbation theory/Fit lattice data (from self-renormalization)?

Outline

- Introduction to Pion DA
- Renormalization
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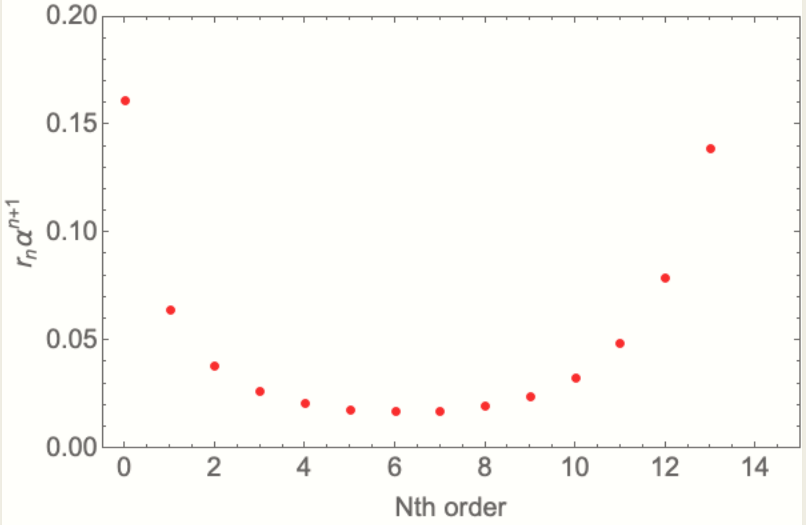
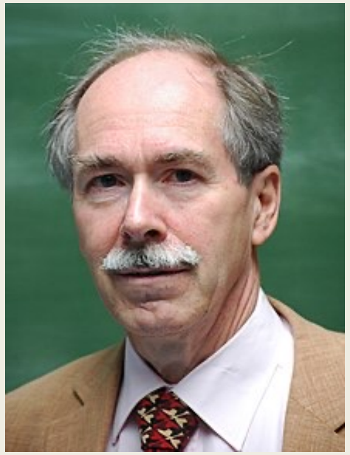
Renormalon in perturbation series

- $\delta m = \frac{1}{a}\sum \alpha_s^{n+1}(a) r_n$
- At higher orders:
  - $r_n \sim n!$
  - Divergent for any α_s
  - No well-defined sum

Data from Bali, et al. PRD(2013)



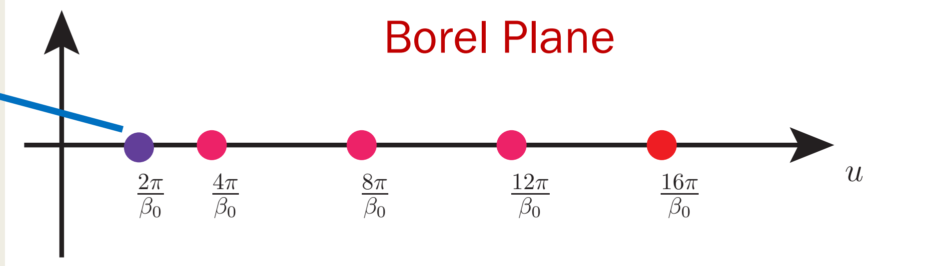


Gerard 't Hooft
1999 Nobel Prize

n! growth comes from IR renormalon

Need a regularization!

Ambiguity $\Delta(\delta m) \sim O(\Lambda_{\rm QCD})$



The matching $C(x, y, \mu, P_z)$ also contains renormalons

Braun, et al. PRD (2019)

Ambiguity in extracting δm from data

- Following the self-renormalization, we can parametrize the linear divergence and logarithmic divergence in a for $C_0(z,a)$

LPC, NPB (2021)

$$C_0(z,a) = \exp[g(z)] \exp\left[\frac{kz}{a \ln a\Lambda}\right] \exp\left[\frac{3C_F}{\beta_0} \ln\left(\ln \frac{1}{a\Lambda}\right) + \ln(1 + \frac{d}{\ln a\Lambda})\right]$$

Physical z dependence

- When fitting to lattice data, k and Λ are correlated and uncertain

$$\Delta(\delta m) = \Delta\left(\frac{k}{a \ln a\Lambda}\right) \sim O(\Lambda_{\mathrm{QCD}})$$

This uncertainty comes from the renormalon ambiguity

$$\Delta(g(z)) \sim O(z\Lambda_{\mathrm{QCD}})$$

The physical z dependence is dependent on our choice of k and Λ

Power Accuracy

- Leading renormalon ambiguity results in $\mathrm{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right)$ correction in the LaMET matching

$$\tilde{\phi}(x,P_z)=\int_0^1 dy\, C(x,y,\mu,P_z)\phi(y,\mu)+\mathrm{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right)+\mathrm{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2P_z^2},\frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2P_z^2}\right)$$

- $P_z\to\infty$: $O\left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right)$ not important
- $P_z\sim\mathrm{GeV}$: $O\left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right)$ dominate the power correction

Need an approach to eliminate $\mathrm{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right)$ in the matching: Power Accuracy!

How to Achieve Power Accuracy?

- LaMET Matching:

$$\tilde{\phi}(x, P_z) = \int_0^1 dy\, C(x, y, \mu, P_z)\phi(y, \mu) \qquad + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\right)$$

How to Achieve Power Accuracy?

- LaMET Matching:

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Leading Renormalon Resummation

Bali, et al. PRD(2013)

How to Achieve Power Accuracy?

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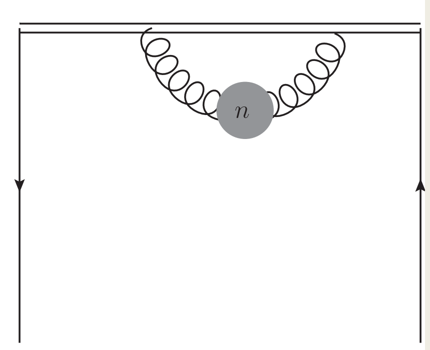
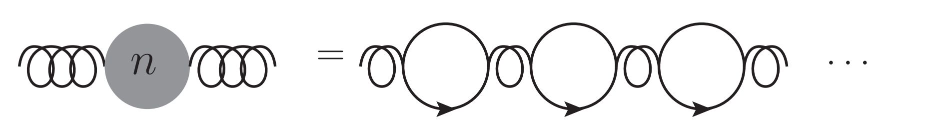
Leading Renormalon Resummation $= \sum_n$

Large β_0 approximation



Braun, et al. PRD (2019)



Why large-β_0 approximation?

1. Pert theory very difficult at high order
2. The approximation is good in δm Bali, et al. PRD(2013)
3. Independent of Dirac structure and hadron momentum

How to Achieve Power Accuracy?

- LaMET Matching:

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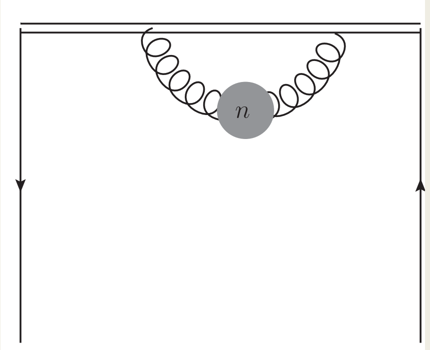
Renormalization with $e^{\delta m\cdot z+m_0\cdot z}$

Leading Renormalon Resummation $= \sum_n$

Large β_0 approximation



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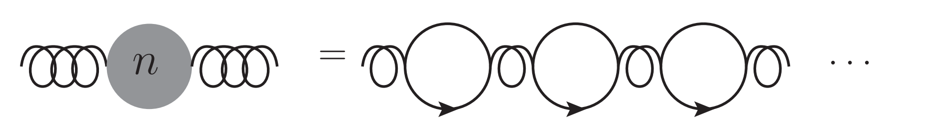

How to Achieve Power Accuracy?

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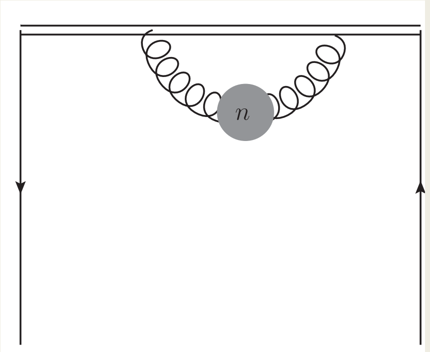
Renormalization with $e^{\delta m \cdot z + m_0 \cdot z}$

Leading Renormalon Resummation $= \sum_n$

Large β_0 approximation



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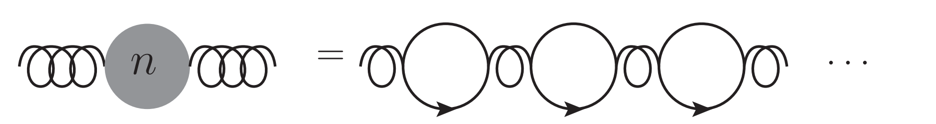

How to calculate m_0

- Work with $P_z = 0$ lattice data

LPC, NPB (2021)

Perturbative calculation

$$e^{m_0 \cdot z} e^{\delta m \cdot z} \langle P_z = 0 | \bar{q}(z) \gamma_t W(z,0) q(0) | P_z = 0 \rangle = C_0^{\overline{\mathrm{MS}}}(z,a)$$

Lattice data

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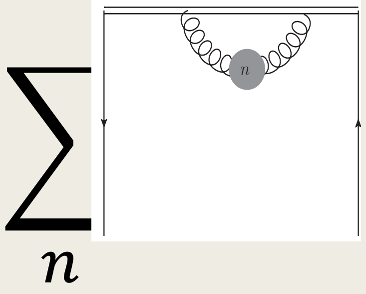

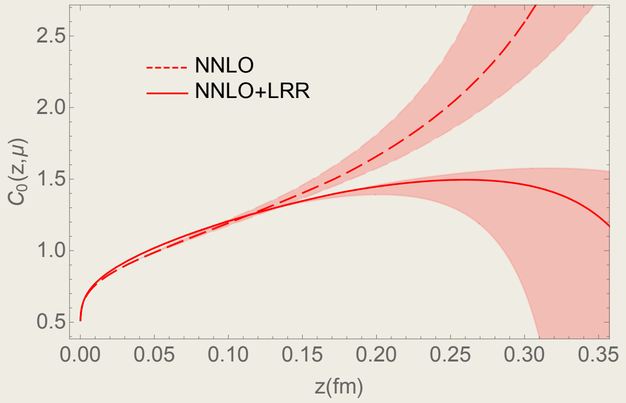

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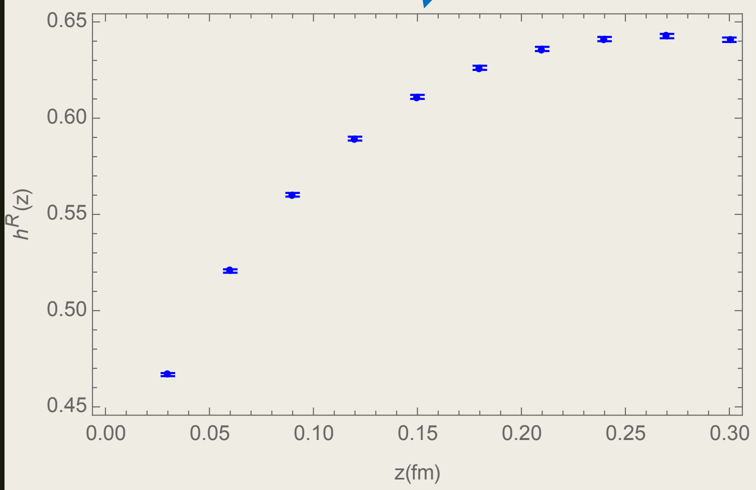

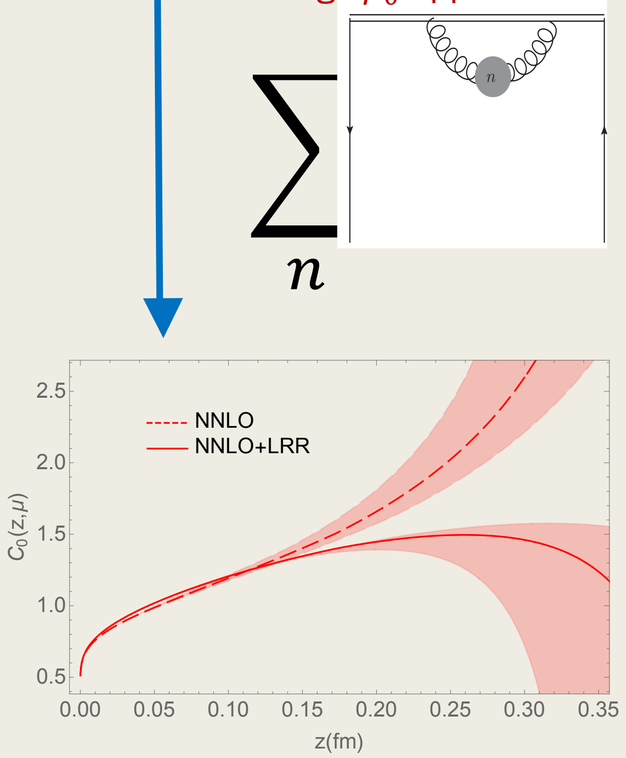

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Lattice data

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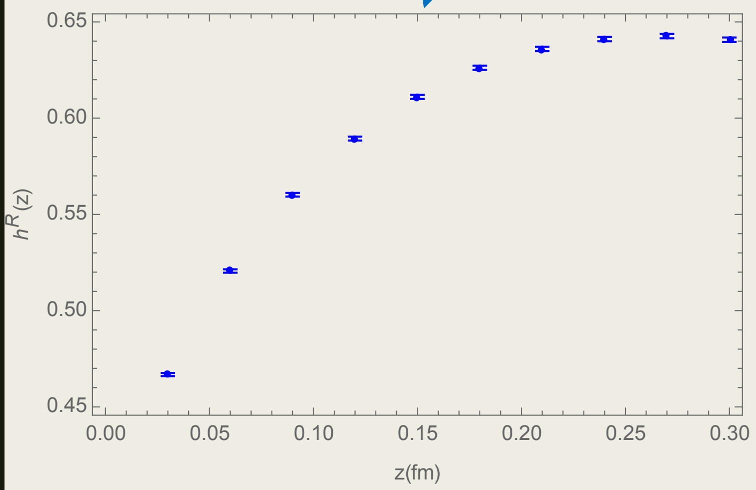

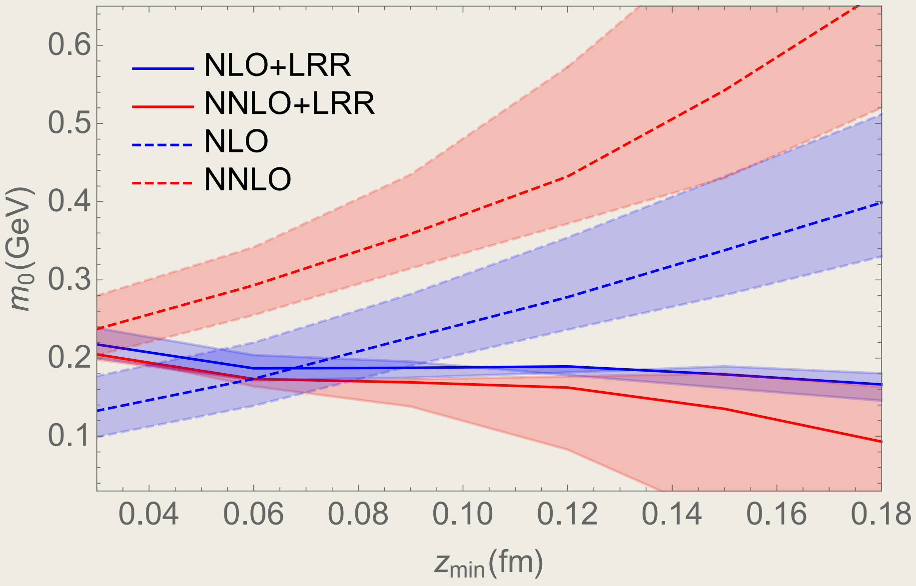

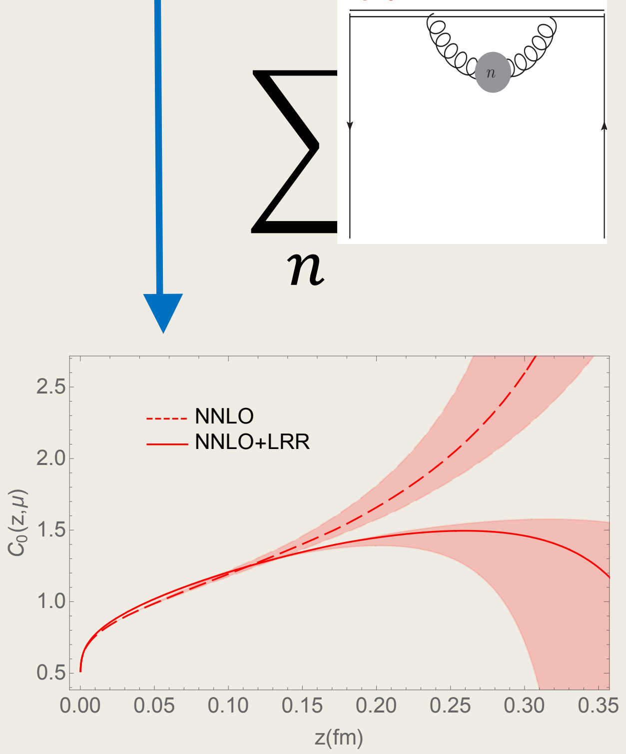

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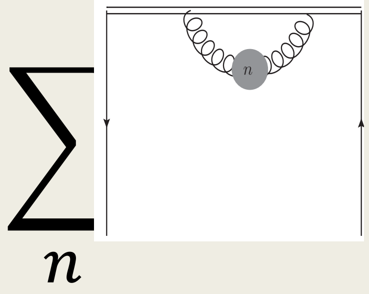

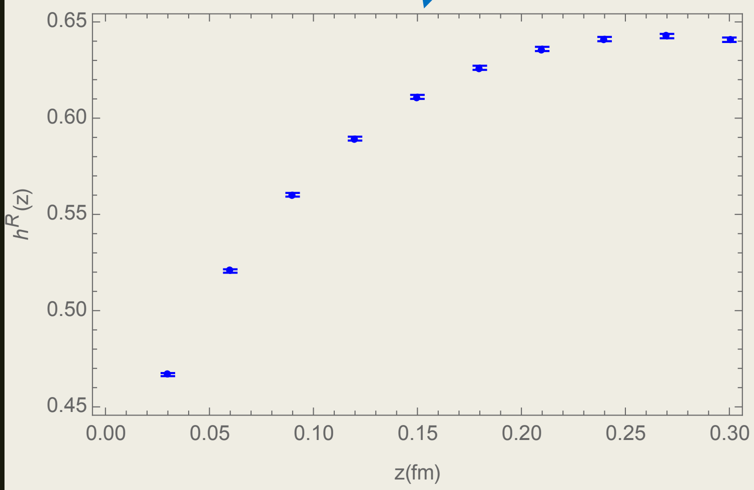

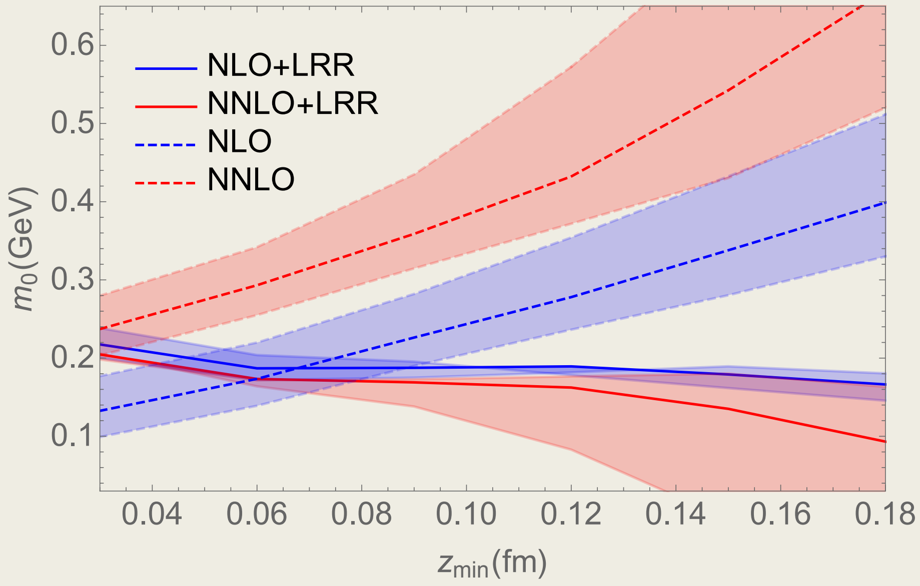

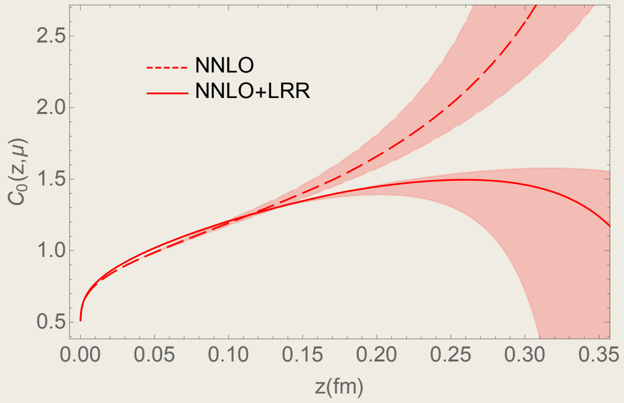

Such an extraction is independent of Dirac structure and external state's momentum

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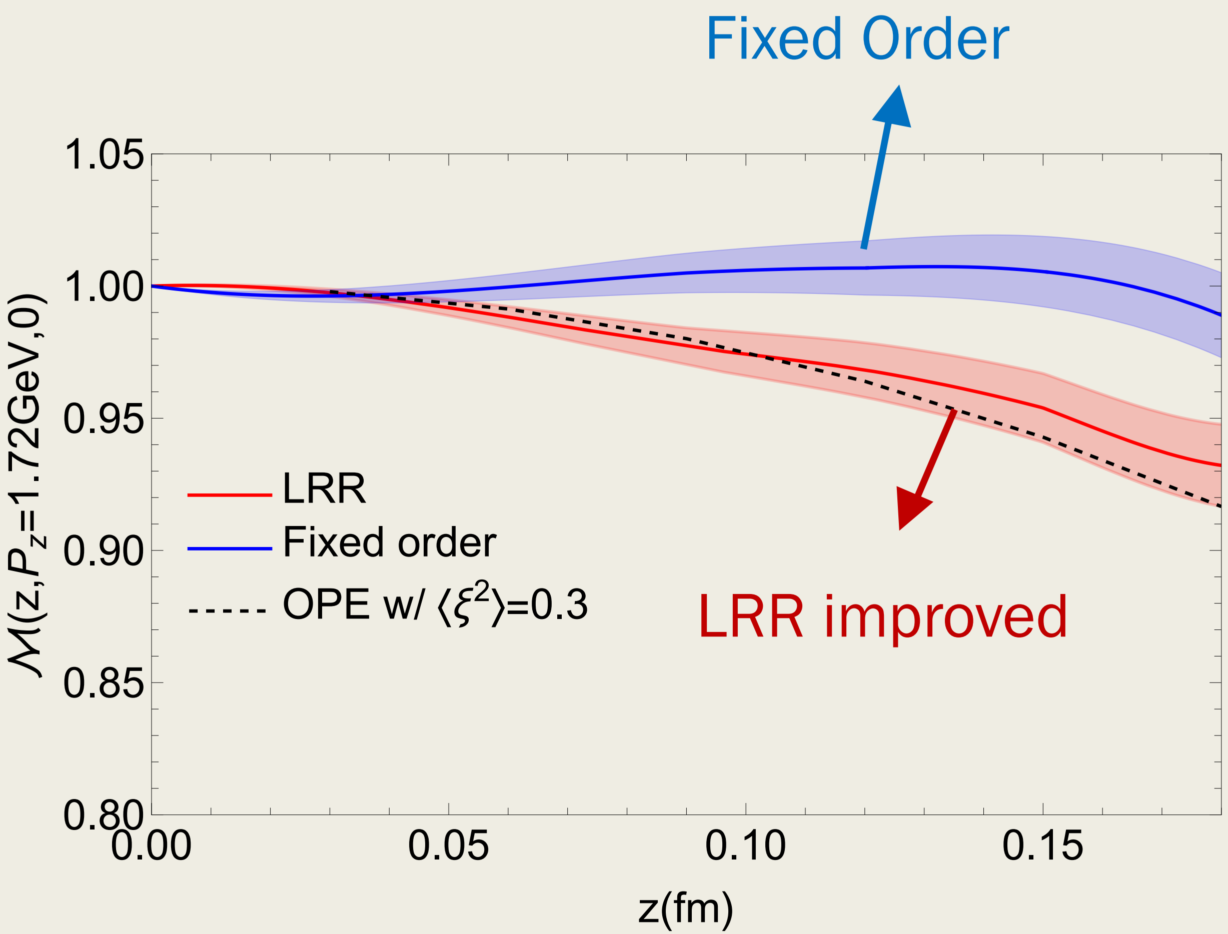
- Application to DA Analysis

LRR-improved renormalization



- Fixed order renormalization suggests a hump at small z
- LRR improved renormalization suggests a decaying distribution

Can be tested in short distance OPE

Testing renormalization through moments extraction

OPE ($z \ll \Lambda_{\text{QCD}}^{-1}$):

$$\tilde{h}^R(z, P_z, \mu) = \sum_{n=0} \frac{\left(-\frac{izP_z}{2}\right)^n}{n!} \sum_{m=0}^{n} C_{nm}(z^2\mu^2)\langle\xi^m\rangle + O\left(z^2\Lambda_{\text{QCD}}^2\right)$$

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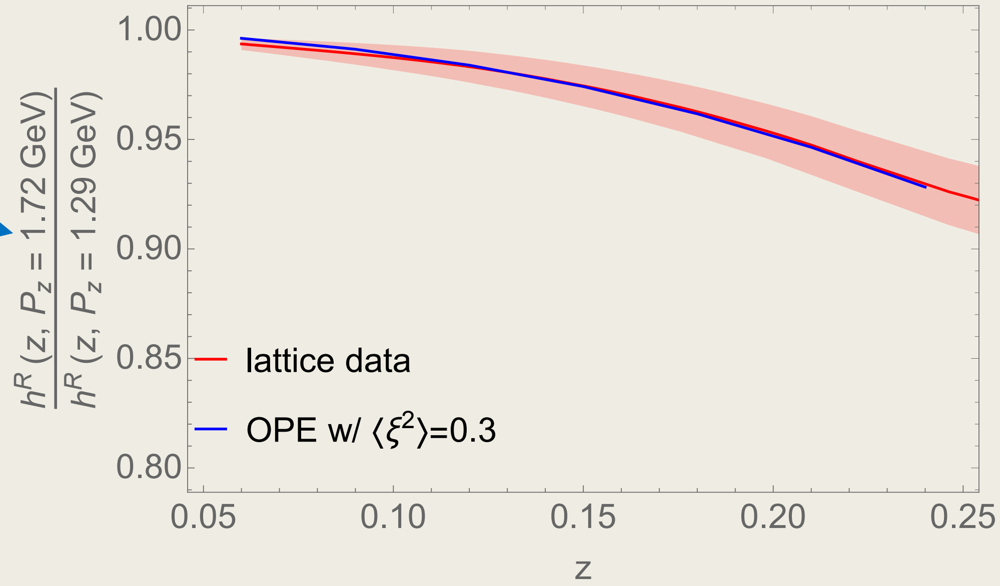
Method 1:

Gao, et al., PRD (2022)

- Ratio: $\frac{\tilde{h}^B(z,P_1)}{\tilde{h}^B(z,P_2)}$
- Renormalization independent
- $\langle\xi^2\rangle = 0.298(39)$



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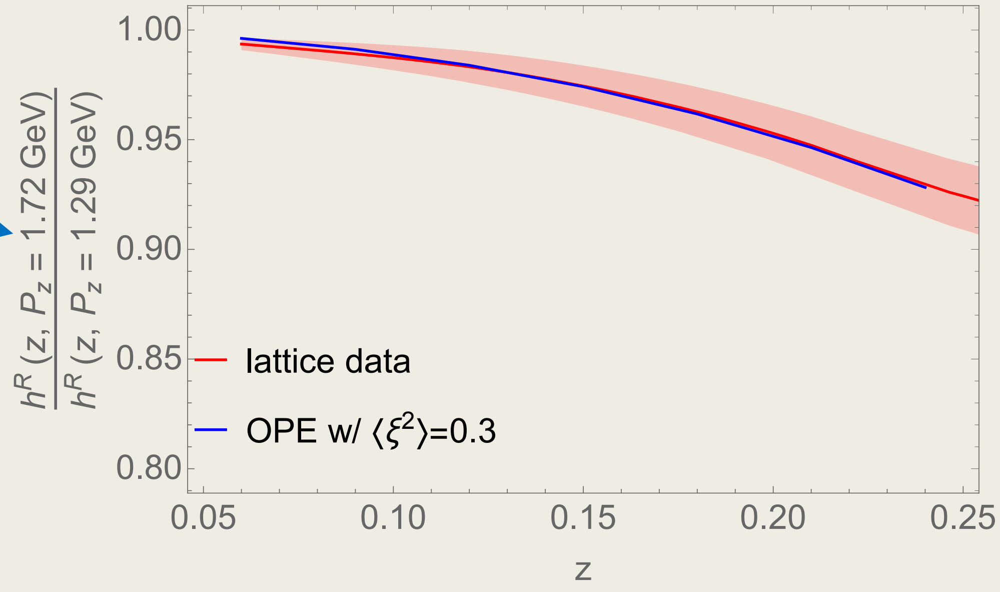

Method 2:

- Renormalized ME: $\tilde{h}^R(z, P_z, \mu)$
- Fixed-order result suggests $\langle\xi^2\rangle < 0$
- LRR result suggests $\langle\xi^2\rangle \approx 0.3$

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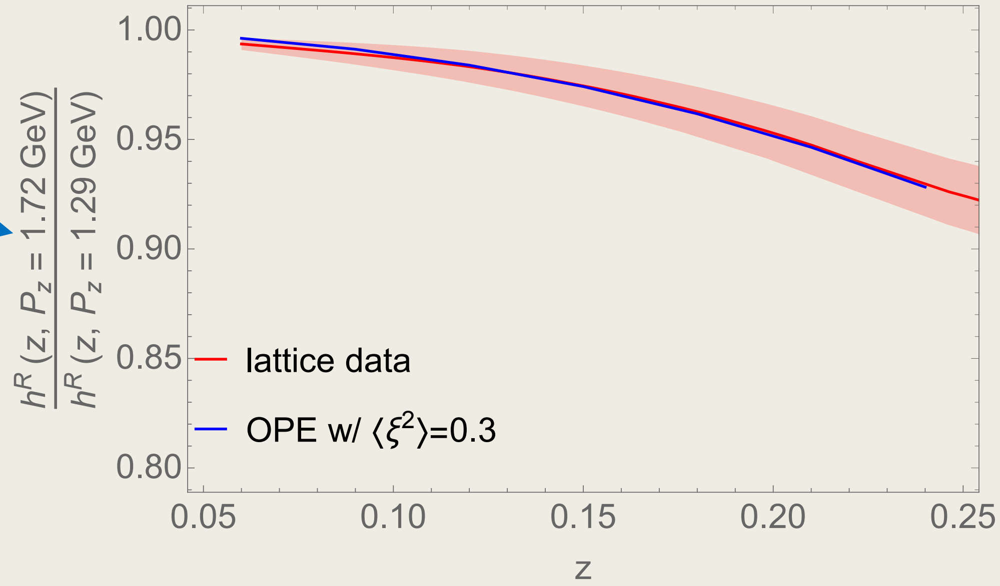

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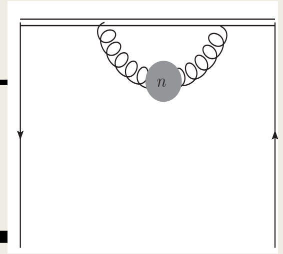
Consistency suggests the LRR-renormalization is correct!

How LRR improves matching?

The diagrams are the same for different momenta/Dirac structure:

Large β_0 approximation

$\sum_n$ 

$$\Delta Q(z, P_z, \mu) = \left(C_0^{\mathrm{LRR}}(z,\mu) - C_0(z,\mu)\right) e^{-iyzP_z}$$

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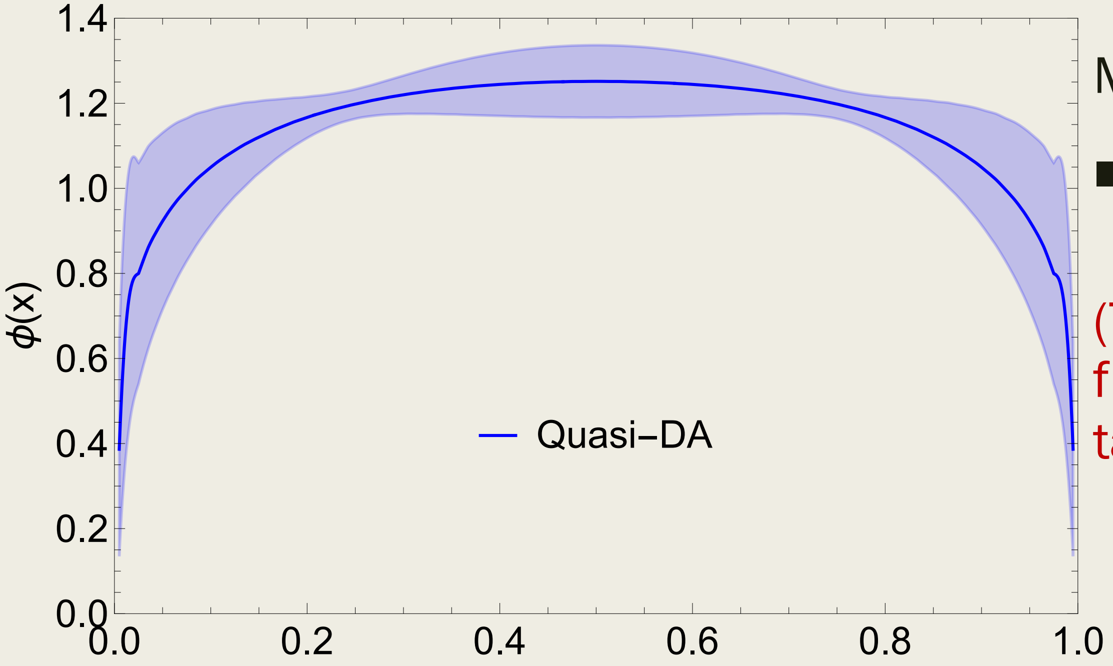

Matching with LRR (the same quasi-DA):

- LRR introduces correction to the matching, more important near endpoints

(This is just the effect of LRR matching, not the final result, which will be presented in Jack's talk.)

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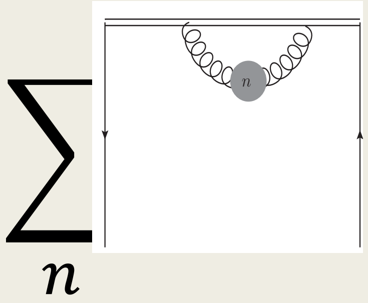

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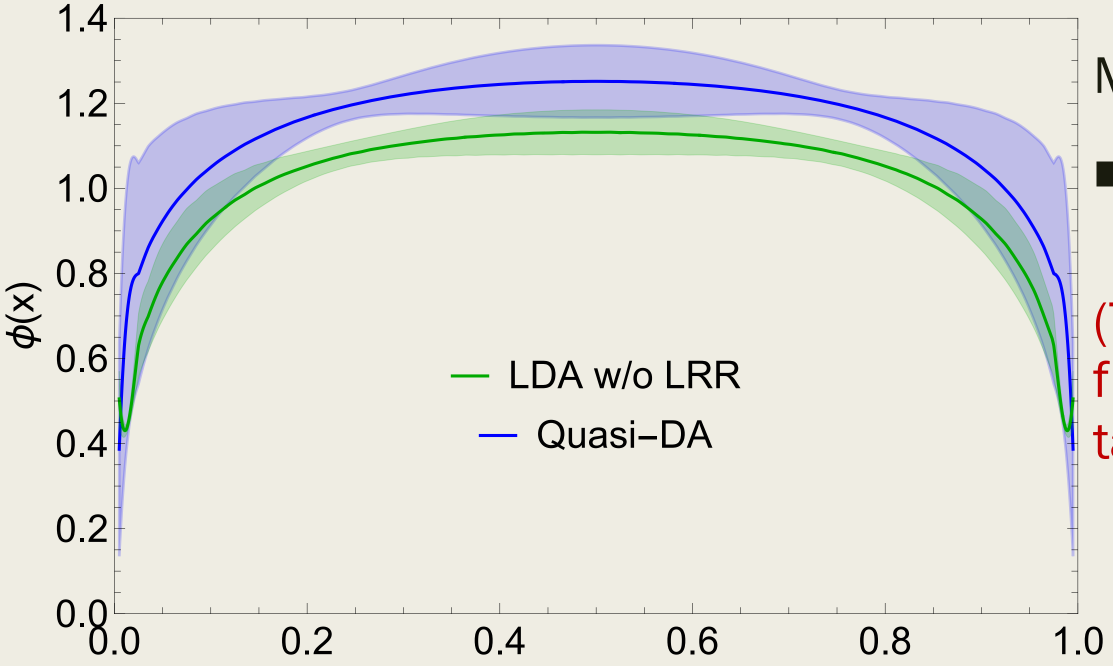

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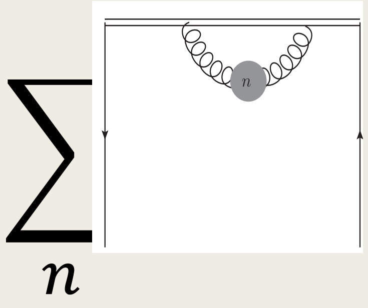

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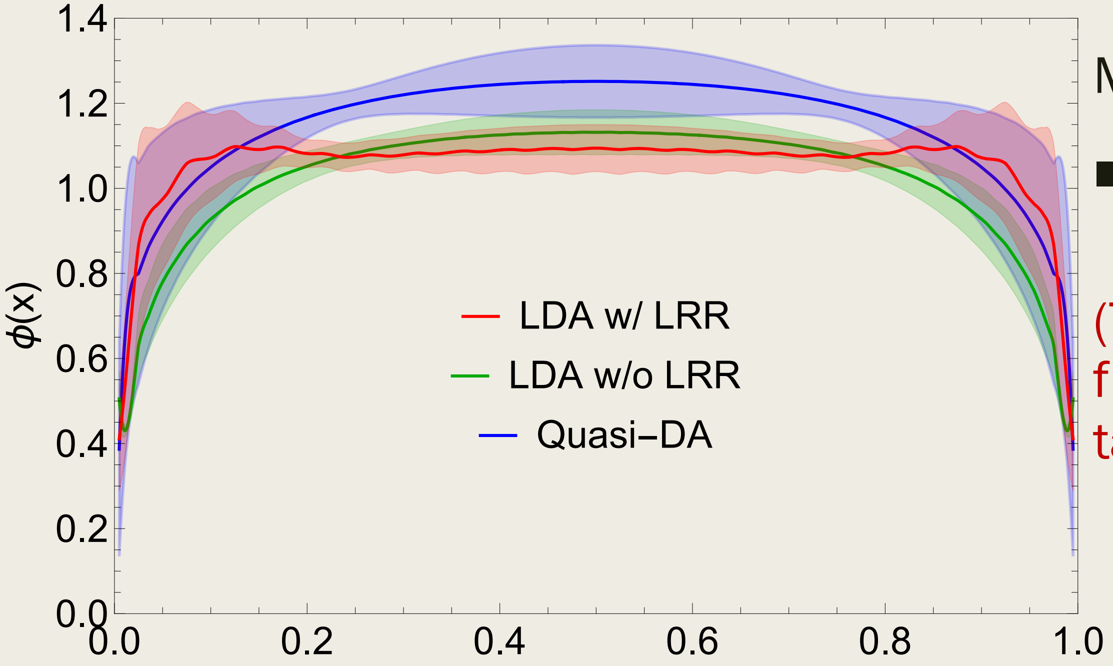

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Conclusion

- Renormalons exist in the renormalization of the LaMET operator, and the perturbative matching kernel
- The leading renormalon results in an $O\left(\frac{\Lambda_{QCD}}{xP_z}\right)$ correction to the LaMET factorization
- We perform a leading renormalon resummation in the large-β_0 limit and use a term $m_0^{\rm eff}$ in renormalization to eliminate the correction
- The LRR-improved renormalization improves the lattice calculation of short distance correlations, consistent with OPE
- A corresponding modification is made in the matching kernel

THANK YOU!