CALCULATION OF PION DISTRIBUTION AMPLITUDE II

Large-log resummation and complementarity.

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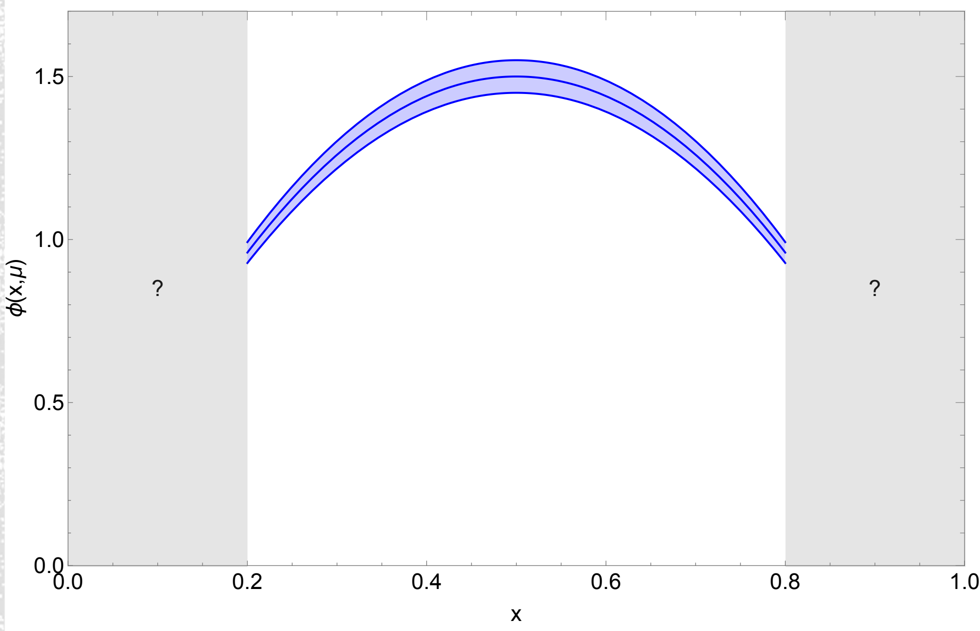
December 1st 2022
14:50-15:15

OUTLINE

- Light-cone matching introduces large-logarithms.
- Corrections to $\phi(x,\mu)$ are $\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2P_z^2},\frac{\Lambda_{QCD}^2}{(1-x)^2P_z^2}\right)$. How to extend the x-range.
- Results with resummation, endpoints and LRR (see Rui Zhang's talk) are shown.



LARGE-λ

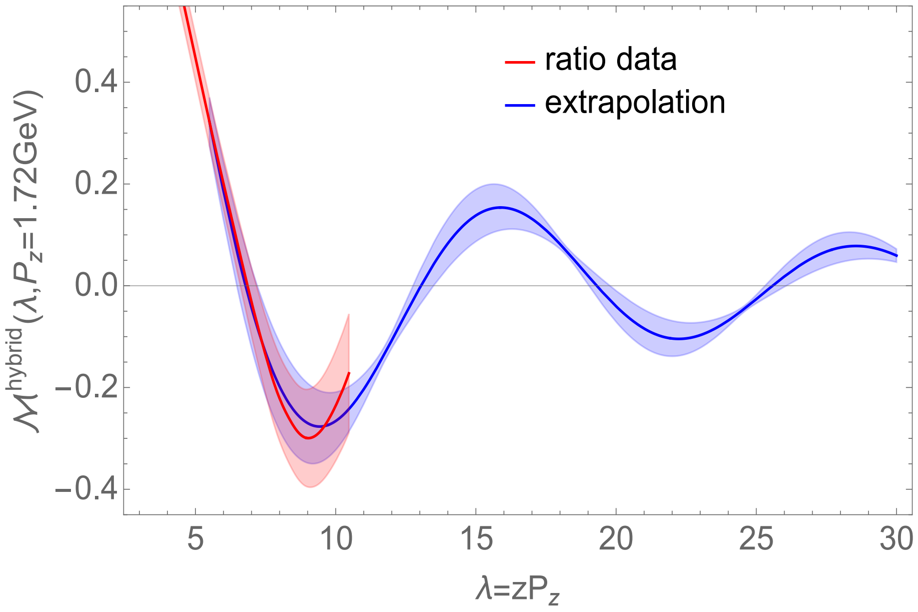
- Extrapolate quasi-DA to infinite distance to perform a Fourier transform.
- Longtail model

$$\tilde{H}^R(\lambda, P_z) = \left(e^{i\lambda/2}\frac{c_1}{(i\lambda)^a} + e^{-i\lambda/2}\frac{c_1}{(-i\lambda)^a}\right)e^{-|\lambda|/\lambda_0}$$

$\sim x^{a-1}$ in momentum space: Regge behavior.

Correlation length $\lambda_0 \to \infty$ as $P_z \to \infty$

- The longtail affects endpoint region.



LIGHTCONE MATCHING. Z-SPACE

- Short distance factorization (SDF).

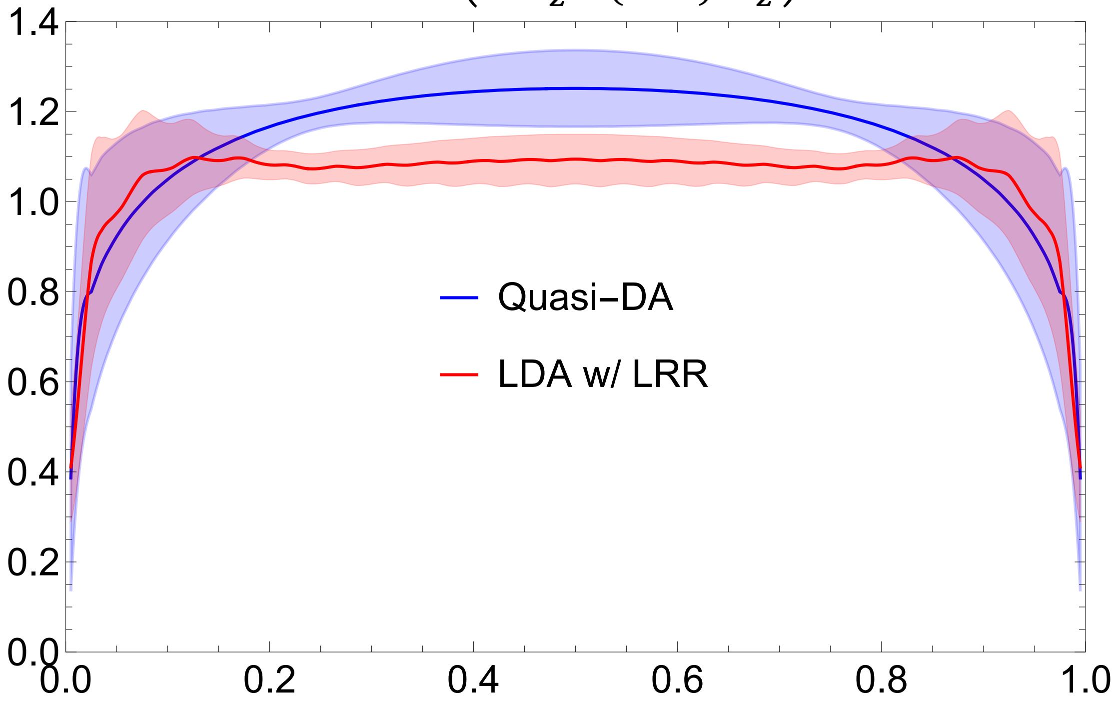
$$\widetilde{H}^R(\lambda, P_z) = \int_0^1 d\nu \mathcal{Z}(\nu, z^2, \mu^2, \lambda) H(\nu\lambda, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

- $\mathcal{Z}(\nu, z^2, \mu^2, \lambda) \sim \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)$.
- Renormalized qDA, $\widetilde{H}^R(\lambda, P_z)$, is scale independent.
- SDF valid only when $z \ll \Lambda_{QCD}^{-1}$.

LIGHTCONE MATCHING. X-SPACE

- Match in momentum space instead.

$$\tilde{\phi}(x,P_z) = \int_0^1 dy\mathcal{C}\,(x,y,\mu,P_z)\phi(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2P_z^2},\frac{\Lambda_{QCD}^2}{(1-x)^2P_z^2}\right) = \mathcal{C}(x,y,\mu,P_z)\otimes\phi(y,\mu)$$



STRATEGY

- Can evolve DA to different scales with ERBL kernel:

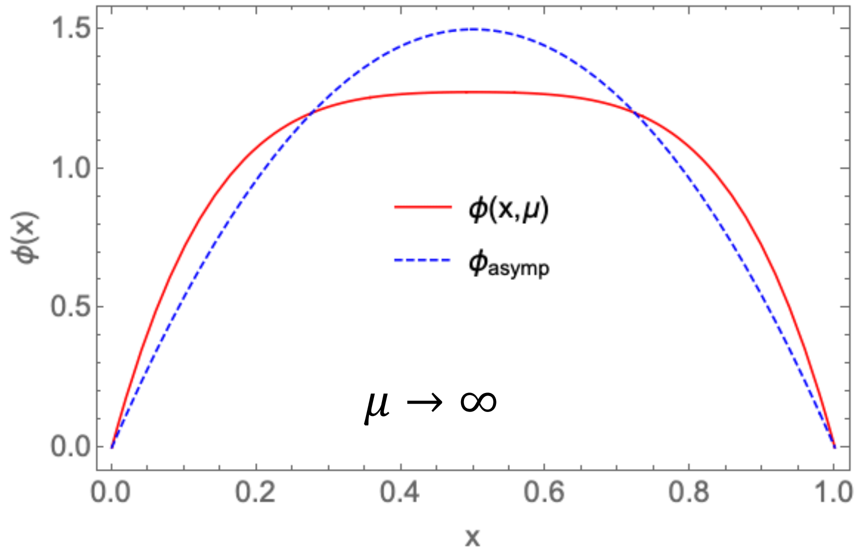
$$\frac{d\phi(x,\mu)}{d\ln(\mu^2)} = \int_0^1 dy V^{(0)}(x,y)\phi(y,\mu) + \mathcal{O}(\alpha_s^2)$$

$$V^{(0)}(x,y) = \frac{C_F\alpha_S}{2\pi}\left(\frac{x}{y}\frac{1-x+y}{y-x}\theta(y-x) + \begin{bmatrix} x \to 1-x \\ y \to 1-y \end{bmatrix}\right)_+$$

- Mellin moments $\xi = x-(1-x) = 2x-1$ also evolve

$$\frac{d\langle\xi^n\rangle}{d\ln(\mu^2)} = \sum_m \gamma_{nm}\langle\xi^m\rangle$$

- γ_{nm} is a triangular matrix.

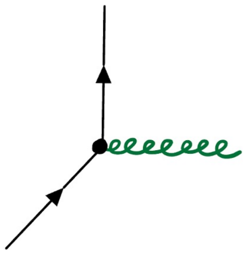


Phys.Lett.B 87 (1979), 359-365.
Phys.Lett.B 94 (1980), 245-250

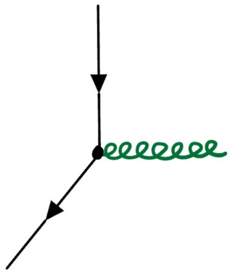
RENORMALIZATION GROUP RESUMMATION (RGR)

- Two logarithms appear

$$\mathcal{Z}(\nu, z^2, \mu^2, \lambda) \sim \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) \xrightarrow{\text{Double F.T.}} \mathcal{C}(\mathrm{x}, \mathrm{y}, \mu, P_z) \sim \begin{cases} \ln\left(\frac{\mu^2}{4x^2 P_z^2}\right) & x < y \\ \ln\left(\frac{\mu^2}{4(1-x)^2 P_z^2}\right) & y < x \end{cases}$$



$x < y$



$y < x$

- Cannot eliminate both logs with a single μ value.

STRATEGY

- First order matching:

$$\tilde{\phi}(x) = \phi(x,\mu) + \mathcal{C}^{(1)}(x,y,\mu,P_z) \otimes \phi(x,\mu) + \mathcal{O}(\alpha_s^2)$$

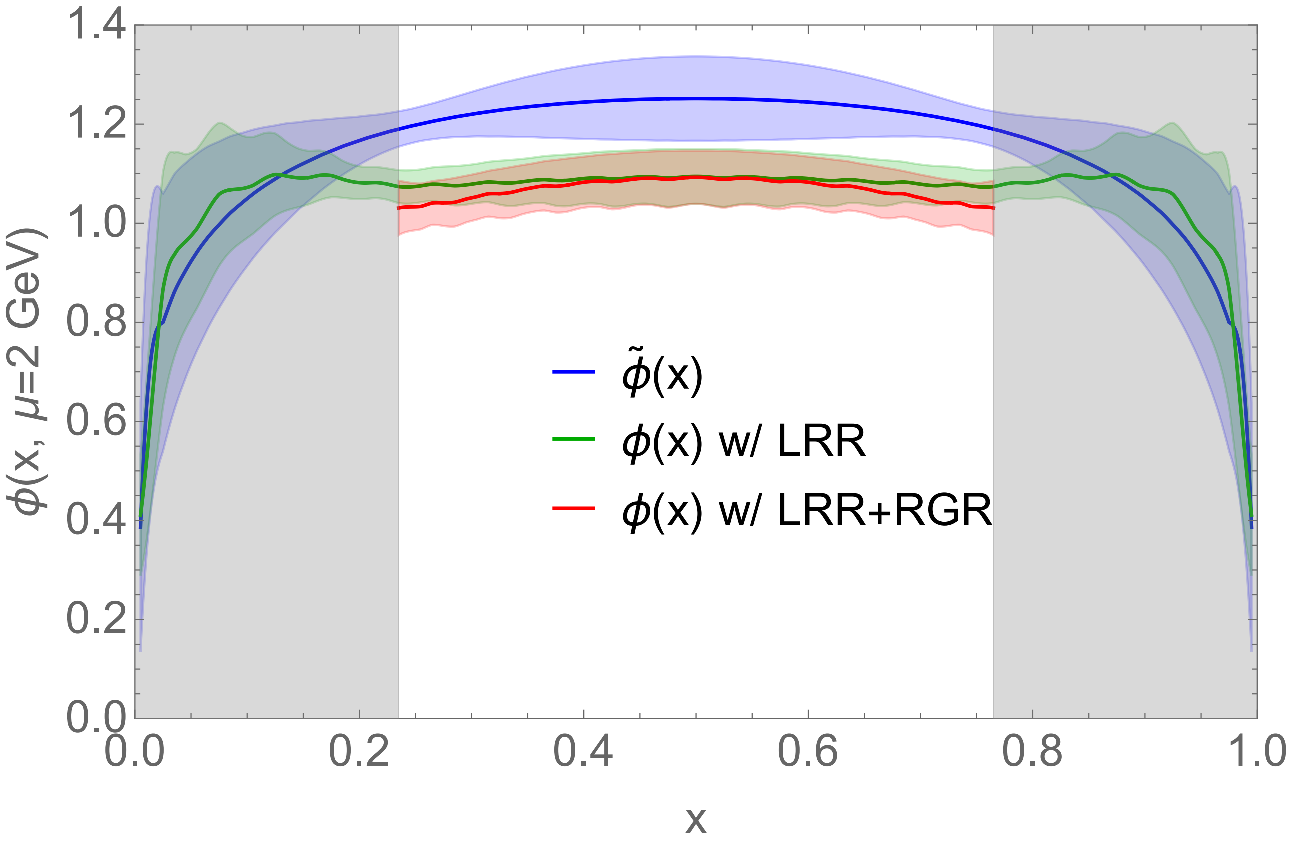
- Split $\mathcal{C}$ into two parts

$$\tilde{\phi}(x) = \boxed{w_L(x)\phi(x,\mu_1) + \int_x^1 dy\, \mathcal{C}^{(1)}(x,y,\mu_1,P_z)\phi(y,\mu_1)}$$

$$+ \boxed{w_R(x)\phi(x,\mu_2) + \int_0^x dy\, \mathcal{C}^{(1)}(x,y,\mu_2,P_z)\phi(y,\mu_2)} + \mathcal{O}(\alpha_s^2)$$

- ▢ are each scale independent.
- $\mu_1 = 2xP_z$ and $\mu_2 = 2(1-x)P_z$.

RESULTS

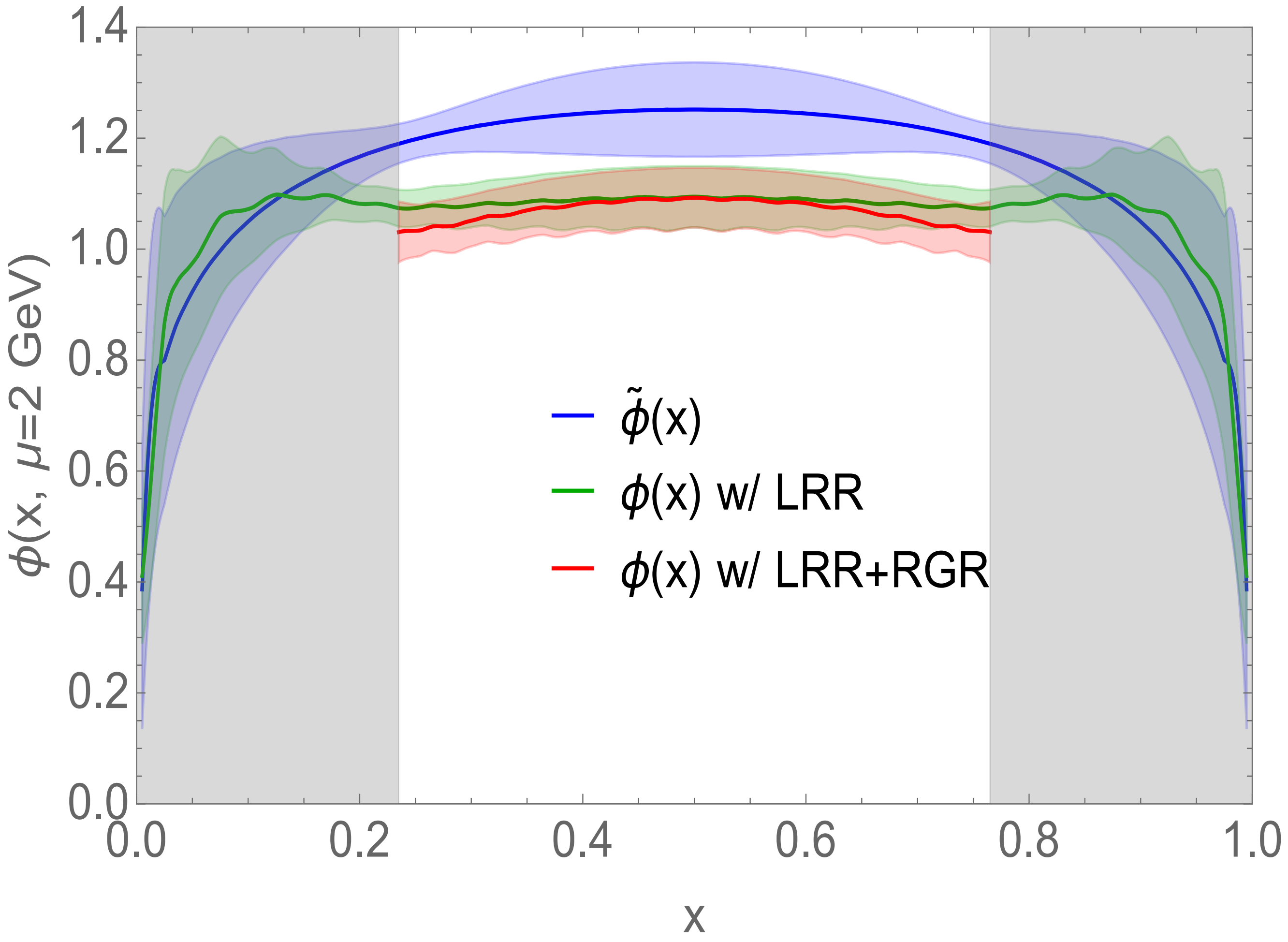


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\phi(x, \mu=2$ GeV$)$
0.0
0.2
0.4
0.6
0.8
1.0
x
$\tilde{\phi}(x)$
$\phi(x)$ w/ LRR
$\phi(x)$ w/ LRR+RGR

SMALL- AND LARGE-X

$$\tilde{\phi}(x, P_z) = \int_0^1 dy\,\mathcal{C}\,(x, y, \mu, P_z)\phi(y, \mu)$$

$$+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$



- ▢ becomes ~1 at endpoints: $x_{min} \sim \frac{\Lambda_{QCD}}{P_z}$ and $x_{max} = 1 - x_{min}$.
- We fill these gaps using the Operator Product Expansion (OPE).

X. Ji. arXiv:2209.09332v1

OPERATOR PRODUCT EXPANSION (OPE)

- At short distances

$$\widetilde{H}^R(\lambda, P_z) = \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{-i\lambda}{2}\right)^n \sum_{m=0}^{n} C_{nm}(z,\mu)\langle\xi^n\rangle + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

Renormalized qDA ✅ — Wilson coefficients ✅ — Mellin moments ?

$$\langle\xi^n\rangle(\mu) = \int_0^1 dx\phi(x,\mu)(2x-1)^n$$

COMPLEMENTARITY

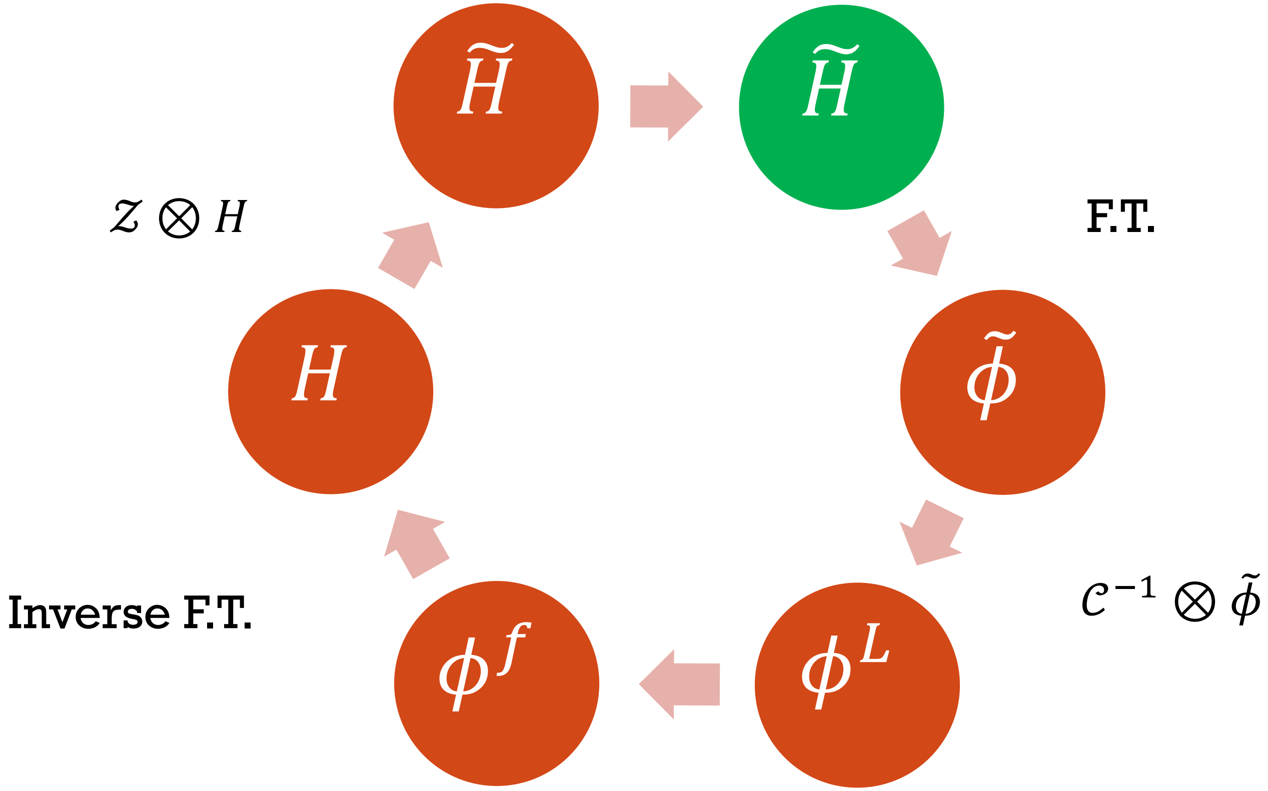
- LaMET gives $\phi^L(x,\mu)$ for $x \in [x_{min}, x_{max}]$.
- Model the endpoints:

$$\phi(x,\mu) = \begin{cases} \phi^L(x_{min},\mu)\left(\frac{x}{x_{min}}\right)^m & x \leq x_{min} \\ \phi^L(x,\mu) & x_{min} \leq x \leq x_{min} \\ \phi^L(x_{max},\mu)\left(\frac{1-x}{1-x_{max}}\right)^m & x \geq x_{max} \end{cases}$$

ENDPOINT MODELING

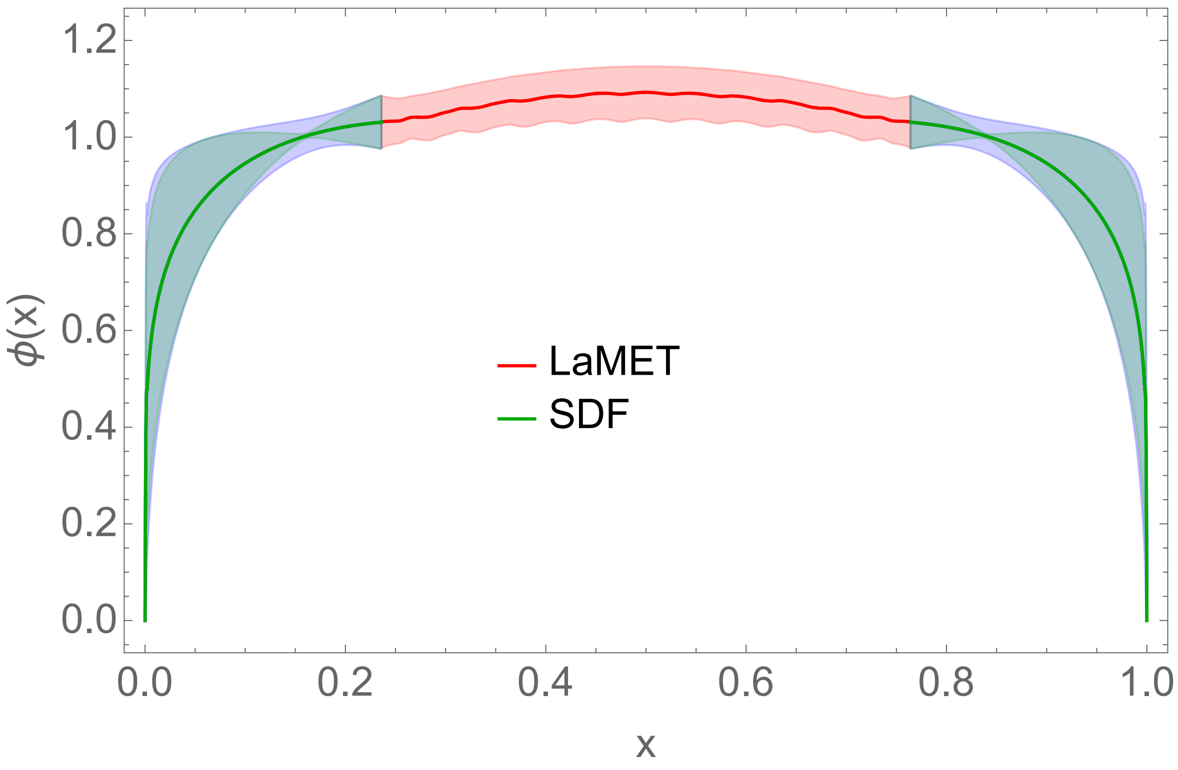
$$\phi(x,\mu) = \begin{cases} \phi^L(x_{min},\mu)\left(\frac{x}{x_{min}}\right)^m & x \le x_{min} \\ \phi^L(x,\mu) & x_{min} \le x \le x_{min} \\ \phi^L(x_{max},\mu)\left(\frac{1-x}{1-x_{max}}\right)^m & x \ge x_{max} \end{cases}$$

Tune the value of m to complete the circuit.

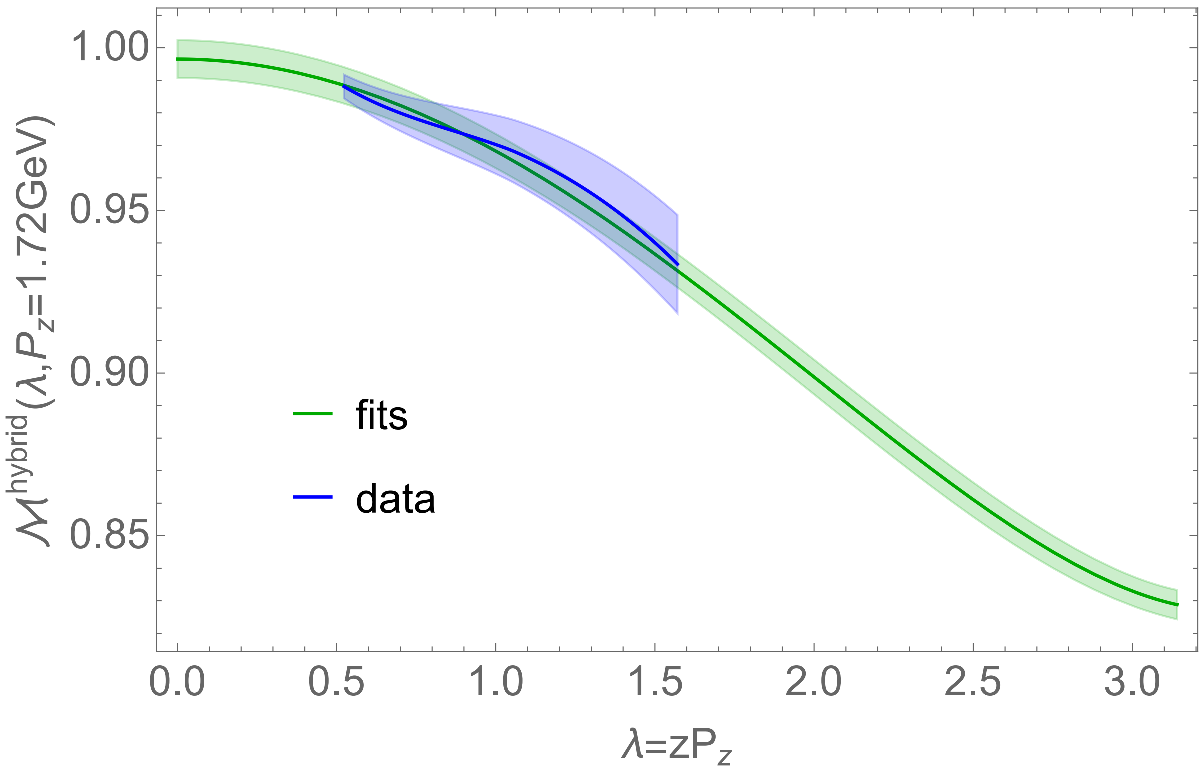


Model endpoints.

RESULTS



Momentum space



Coordinate space. (Valid for short distances)

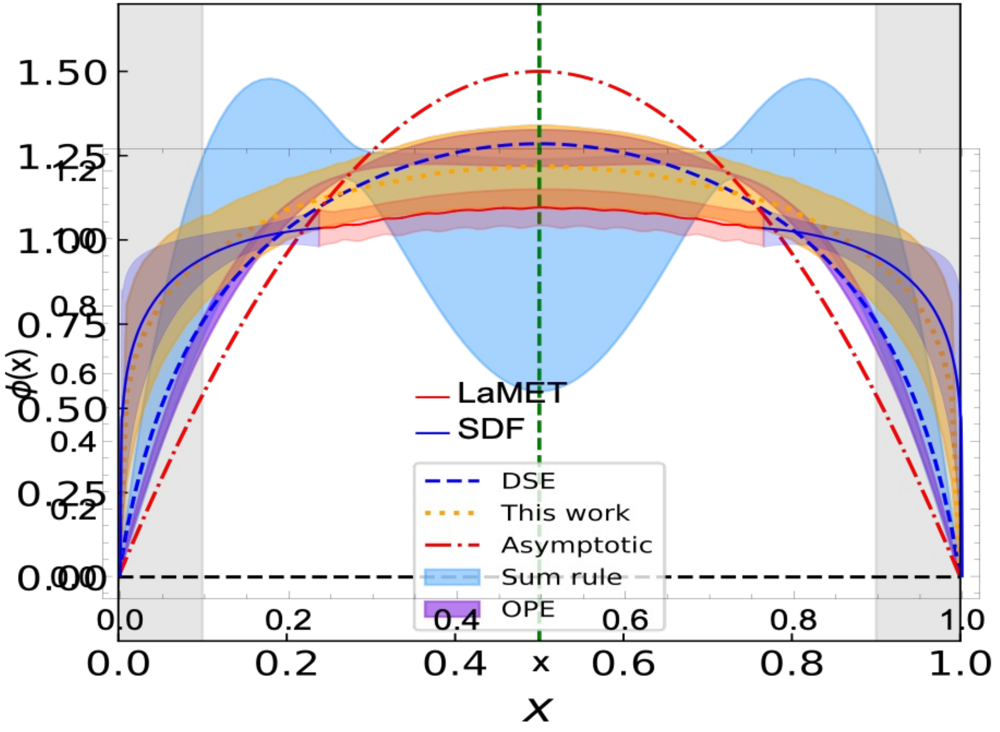
RESULTS AND CONCLUSIONS

- Mellin moments:

$\langle \xi^0 \rangle = 0.996(6)$ $\langle \xi^2 \rangle = 0.302(23)$

$\langle \xi^2 \rangle_{OPE} = 0.298(39)$

- Resummed logarithms using renormalization group.
- Extended the x-range using the OPE.
- Endpoint modeling and OPE give consistent results.



Hua et. al. arXiv:2201.09173