Nuclear Science Computing Center at CCNU

Lattice QCD prediction of pion & kaon form factor at large momentum transfer Q^2

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Outline

- Motivation
- Lattice Setup
- Compute form factor on the lattice
- Results & Summarize

Motivation

☞ Form factor: the Fourier transform of electromagnetic current distribution in space

- The hadron structure
- The interplay between the emergent hadron mass (EHM) & the Higgs-mass mechanism
- FF + PDF -> GPD, three-dimensional image of the hadron
- Pion & Kaon

☞ Approaches:

- Experiment: JLab, EIC, EicC …
- Effective theories: QCD sum rules, DSE …
- **Lattice QCD: from first principle, no model dependence**

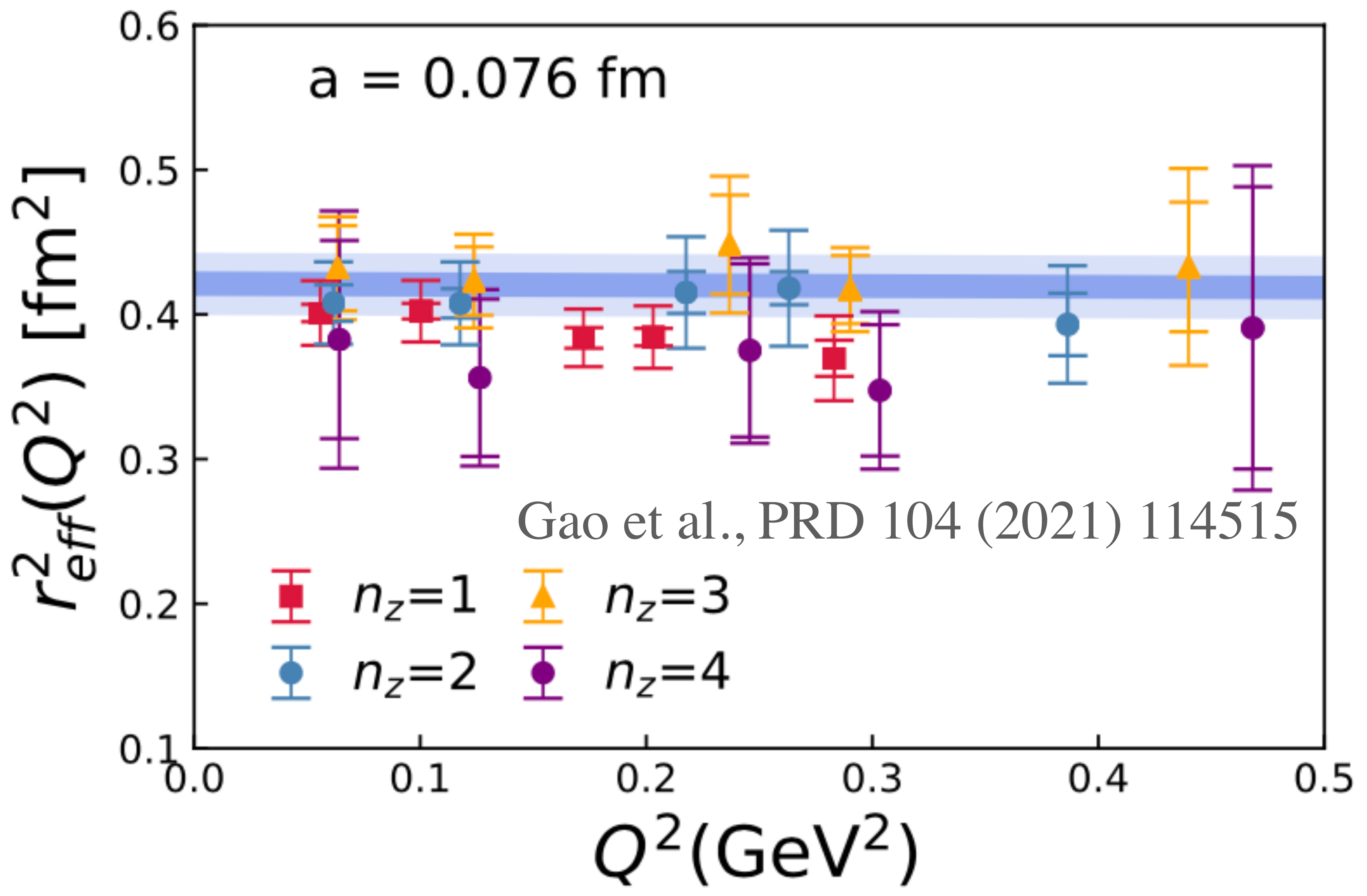
Motivation

Small Q^2 limit: hadronic picture

- Direct: Length of Wilson line $z = 0$, no matching
- **V**ector **M**eson **D**ominance -> Charge radius

$$r^2_{eff}(Q^2) = \frac{6(1/F_\pi(Q^2) - 1)}{Q^2}.$$

$\langle r^2_\pi \rangle = 0.42(2)$ fm^2, $\langle r^2_\pi \rangle_{PDG} = 0.434(5)$ fm^2

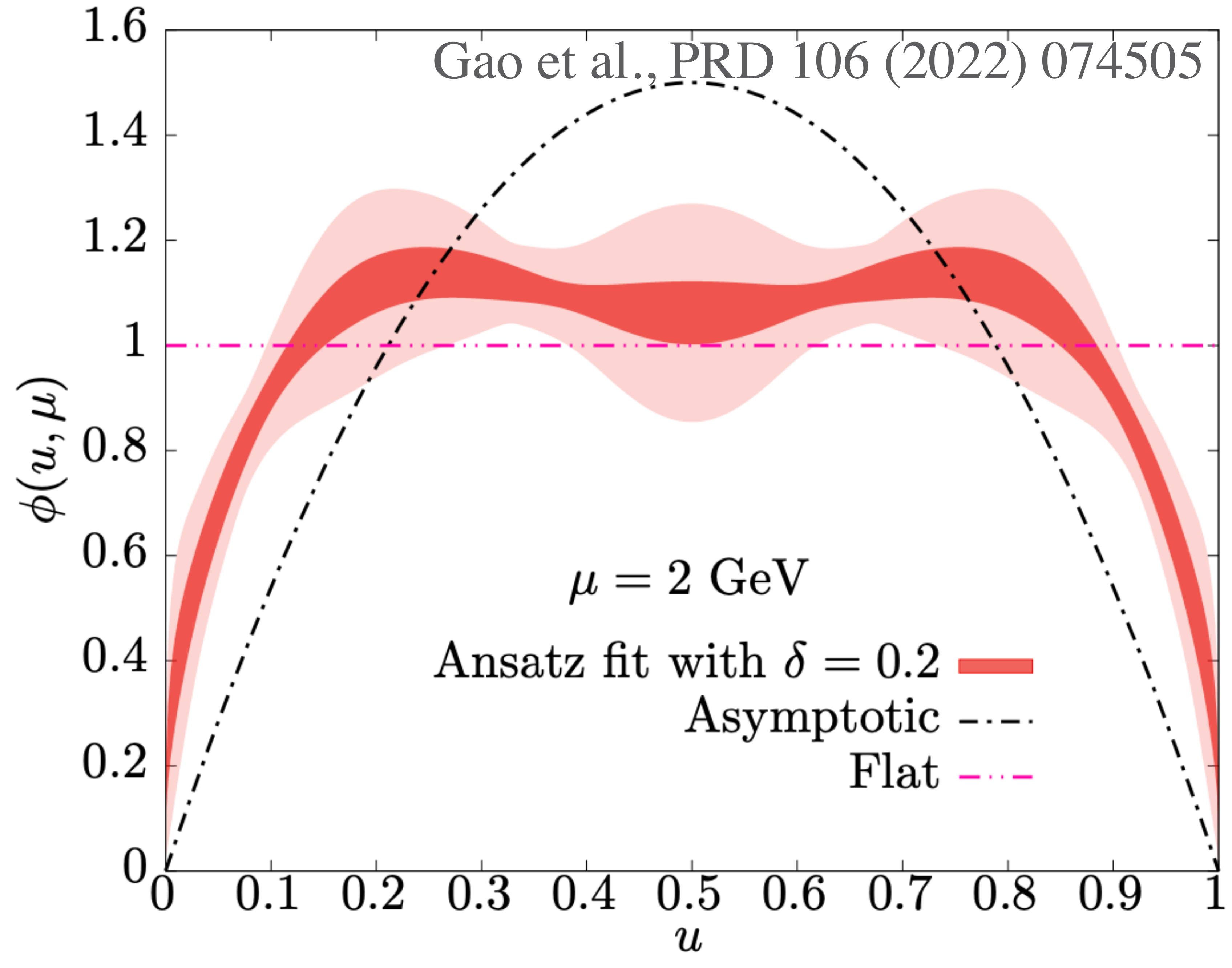


Large Q^2 limit: partonic picture

- Indirect: **D**istribution **A**mplitude, need matching,
- **L**arge-**M**omentum **E**ffective **T**heory

Hard-process kernel

$$F(Q^2) = \int_0^1 \int_0^1 \mathrm{d}x\mathrm{d}y \phi^*(v, \mu_F^2) T_F(u, v, Q^2, \mu_R^2, \mu_F^2) \phi(u, \mu_F^2),$$



Motivation

Small Q^2 limit: hadronic picture

- Direct: Length of Wilson line $z=0$, no matching
- **V**ector **M**eson **D**ominance -> Charge radius

$$r^2_{eff}(Q^2) = \frac{6(1/F_\pi(Q^2)-1)}{Q^2}.$$

$\langle r_\pi^2 \rangle = 0.42(2)$ fm^2, $\langle r_\pi^2 \rangle_{PDG} = 0.434(5)$ fm^2

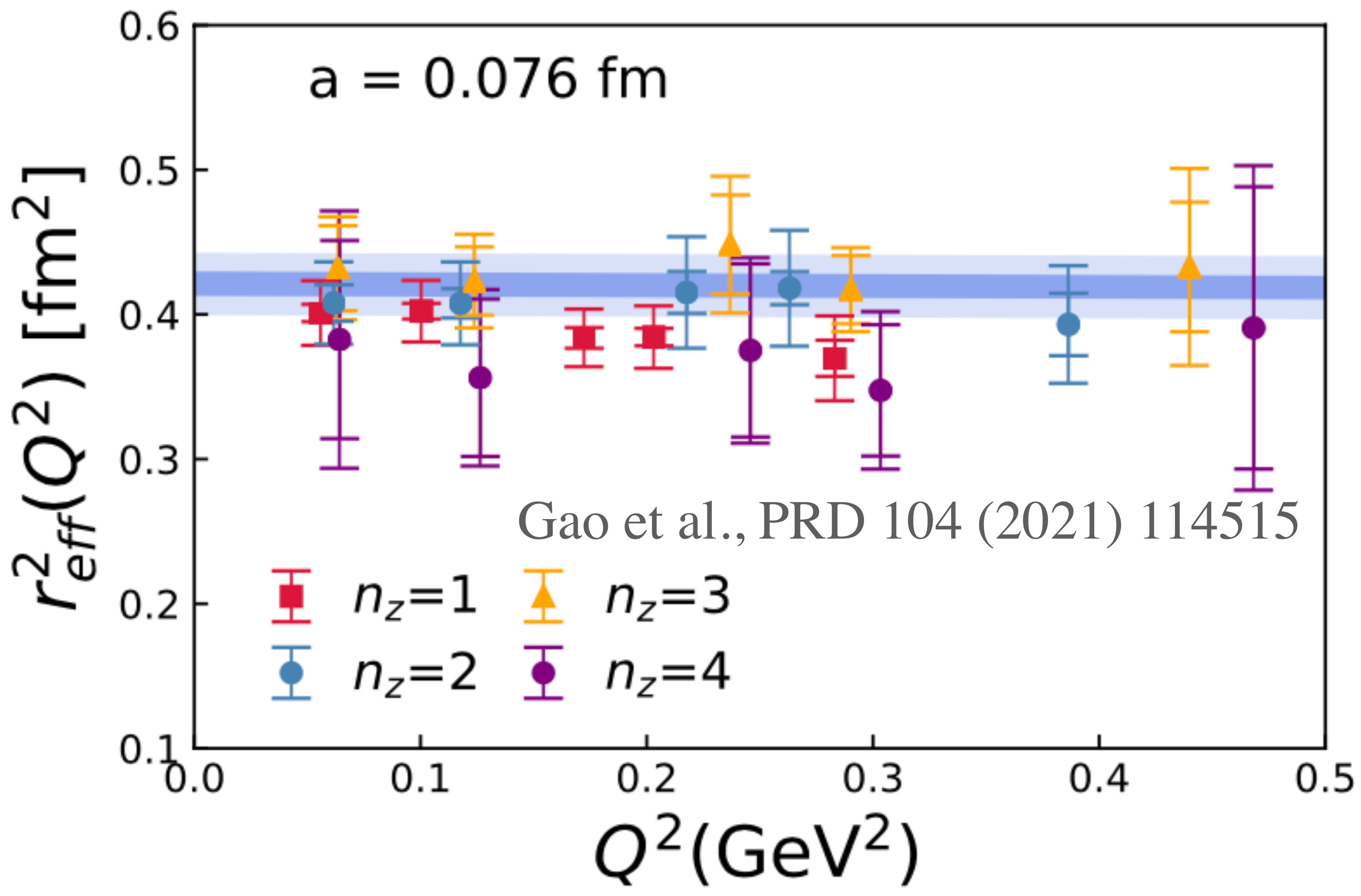
How about the intermediate range?

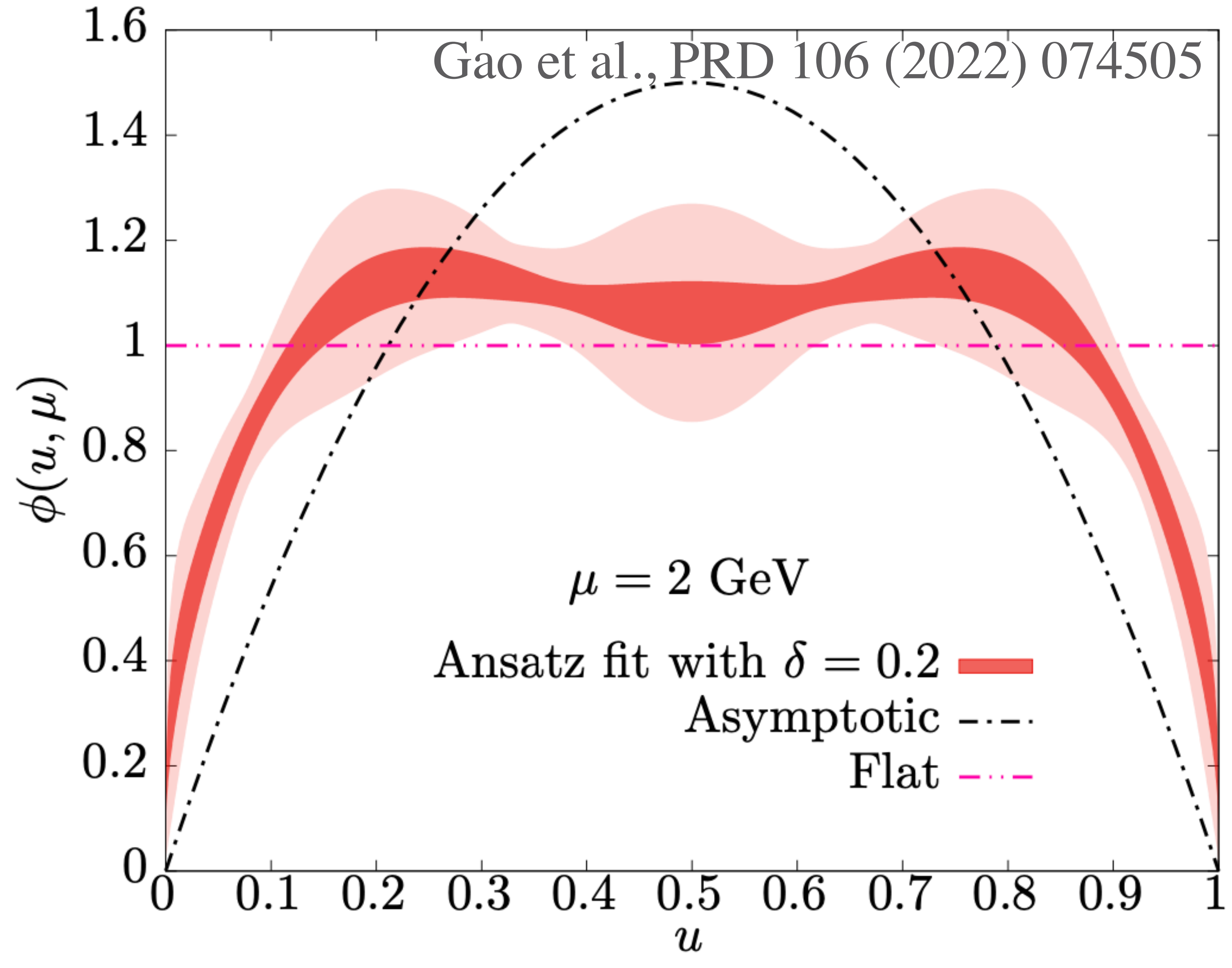
Large Q^2 limit: partonic picture

- Indirect: **D**istribution **A**mplitude, need matching,
- **La**rge-**M**omentum **E**ffective **T**heory

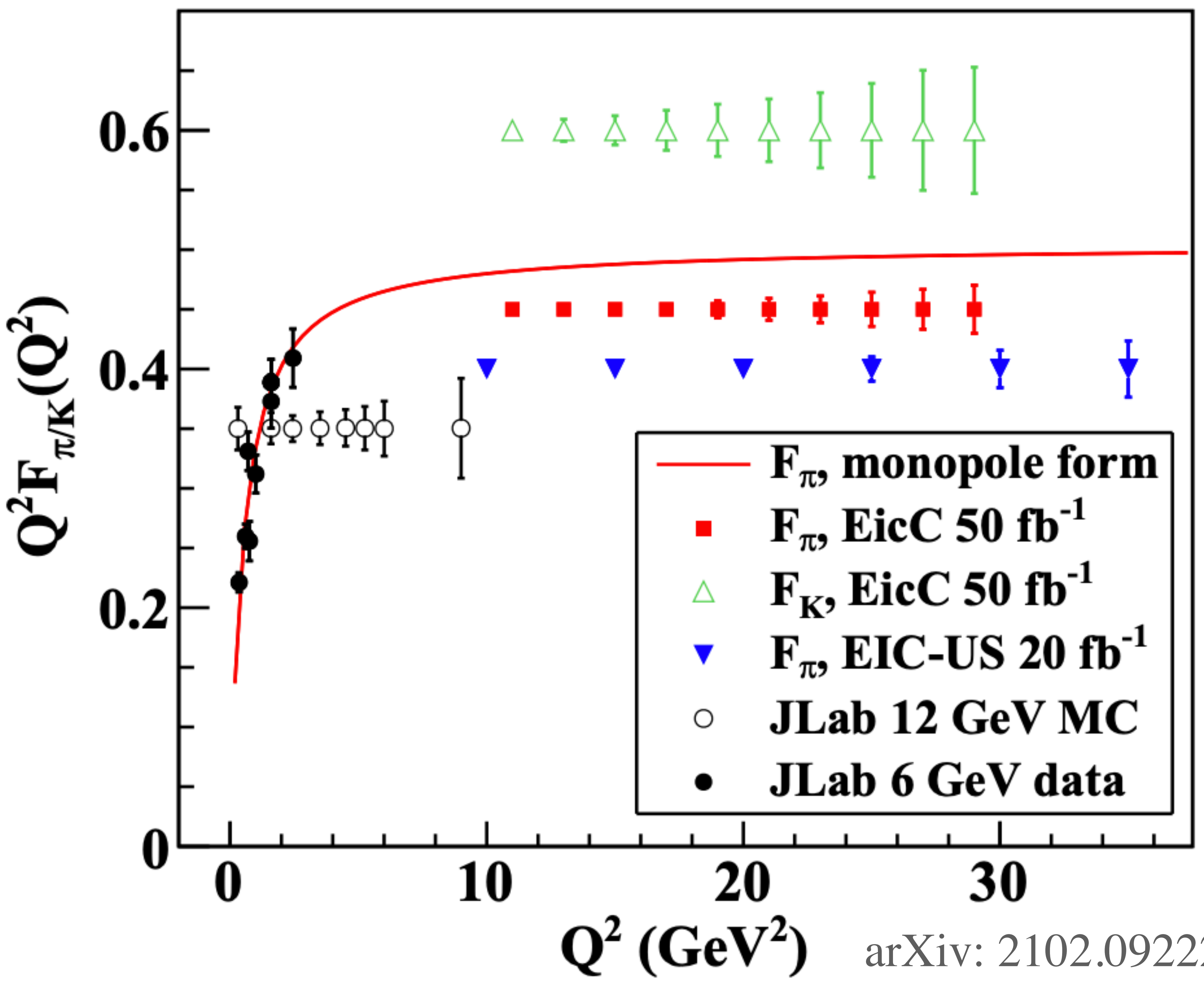
Hard-process kernel

$$F(Q^2) = \int_0^1 \int_0^1 \mathrm{d}x\mathrm{d}y \phi^*(v, \mu_F^2) \underline{T_F(u, v, Q^2, \mu_R^2, \mu_F^2)} \phi(u, \mu_F^2),$$





Motivation



arXiv: 2102.09222

☞ Approaches:

EPJA 48 (2012) 187 EPJA 52 (2016) 268 arXiv: 2102.09222

- Experiment: JLab, EIC, EicC …

Gao et al., PRD 96 (2017) 034024

- Effective theories: QCD sum rules, DSE …

- **Lattice QCD: from first principle**

ETMC, PRD 105 (2022) 054502

❍ **State-of-the-art: $Q^2 \leq$ 2.5, 3 GeV^2**

❍ **This work: Q^2 up to 10, 28 GeV^2**

Lattice setup:

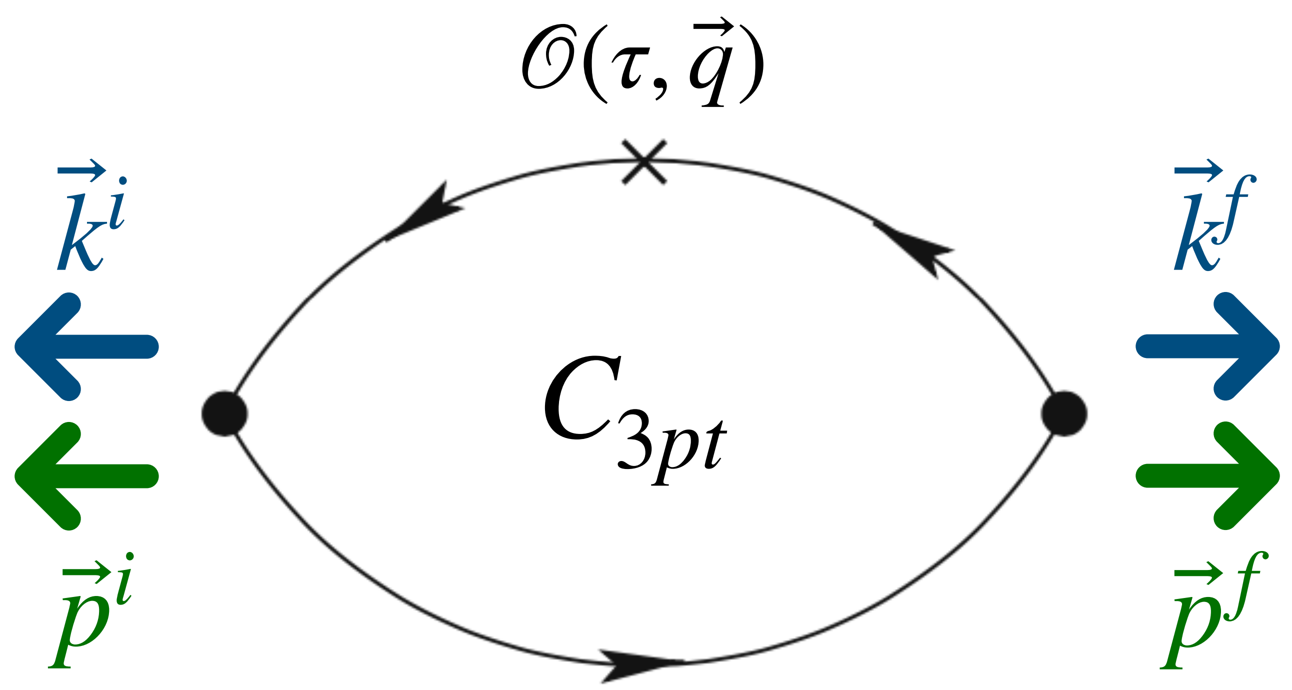
- Lattice size: $N_s^3 \times N_t = 64^3 \times 64$
- lattice spacing: $a = 0.076$ fm for Pion, $a = 0.076, 0.04$ fm for Kaon
- Sea quark: Highly Improved Staggered Quark (HISQ) action

  Valence quark: Wilson-Clover action

  $\Rightarrow m_{\pi^+} = 140/134$ MeV, $m_{K^+} = 497$ MeV for $a = 0.076/0.04$ fm, **at the physical point**
- Multiple time separations: $t_s = \{6,8,10,12\}, \{8,10,12,14,16\}$ for $a = 0.076/0.04$ fm
- Keep the directions of **quark boost momentum** $\vec{k}$ & **quark momentum** $\vec{p}$ consistent

  to improve the signal

  $\Rightarrow$ The largest Q^2 up to **10** (Pion) and **28 GeV^2** (Kaon)



Compute form factor on the Lattice

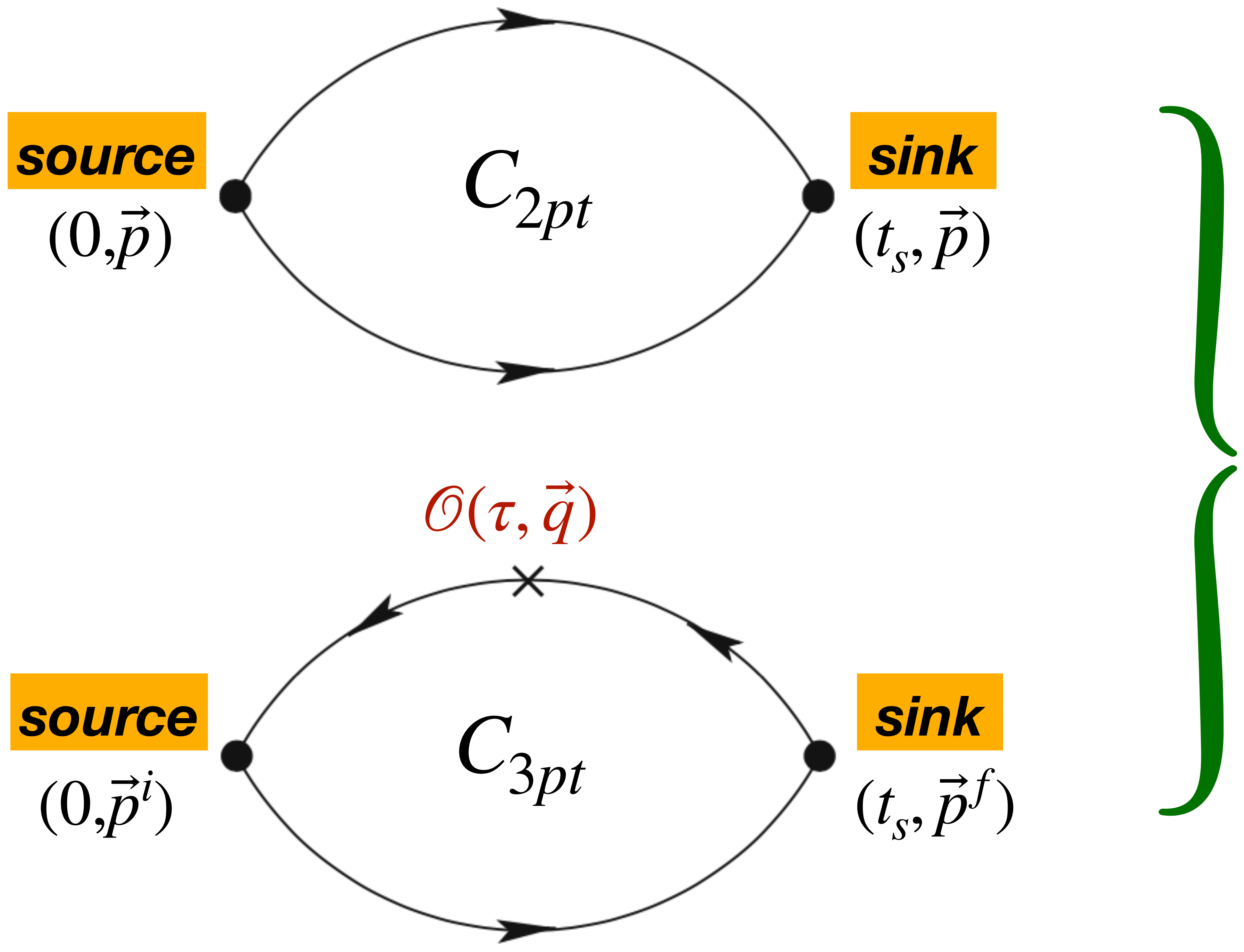
Construct a hadron state: $C_{2pt}(t,\vec{p}) = \langle H(t_s,\vec{p})H^{\dagger}(0,\vec{p})\rangle$

Insert an electromagnetic current
$\mathcal{O}(\tau,\vec{q})$ to probe the hadron

$\vec{p}^f = \vec{p}^i + \vec{q}$

$$C_{3pt}(\tau,t_s;\vec{p}^i,\vec{p}^f) = \langle H(t_s,\vec{p}^f)\hat{\mathcal{O}}_{\Gamma}(\tau,\vec{q})H^{\dagger}(0,\vec{p}^i)\rangle$$

$\Gamma : \hat{1}, \gamma^{\mu}, \sigma^{\mu\nu}$



Extract the bare form factor:

$$F^B = \langle E_0,\vec{p}^f | \hat{\mathcal{O}}_{\gamma^{\mu}}(\tau,\vec{q}) | E_0,\vec{p}^i\rangle$$

from ~ C_{3pt} / C_{2pt}

Renormalization

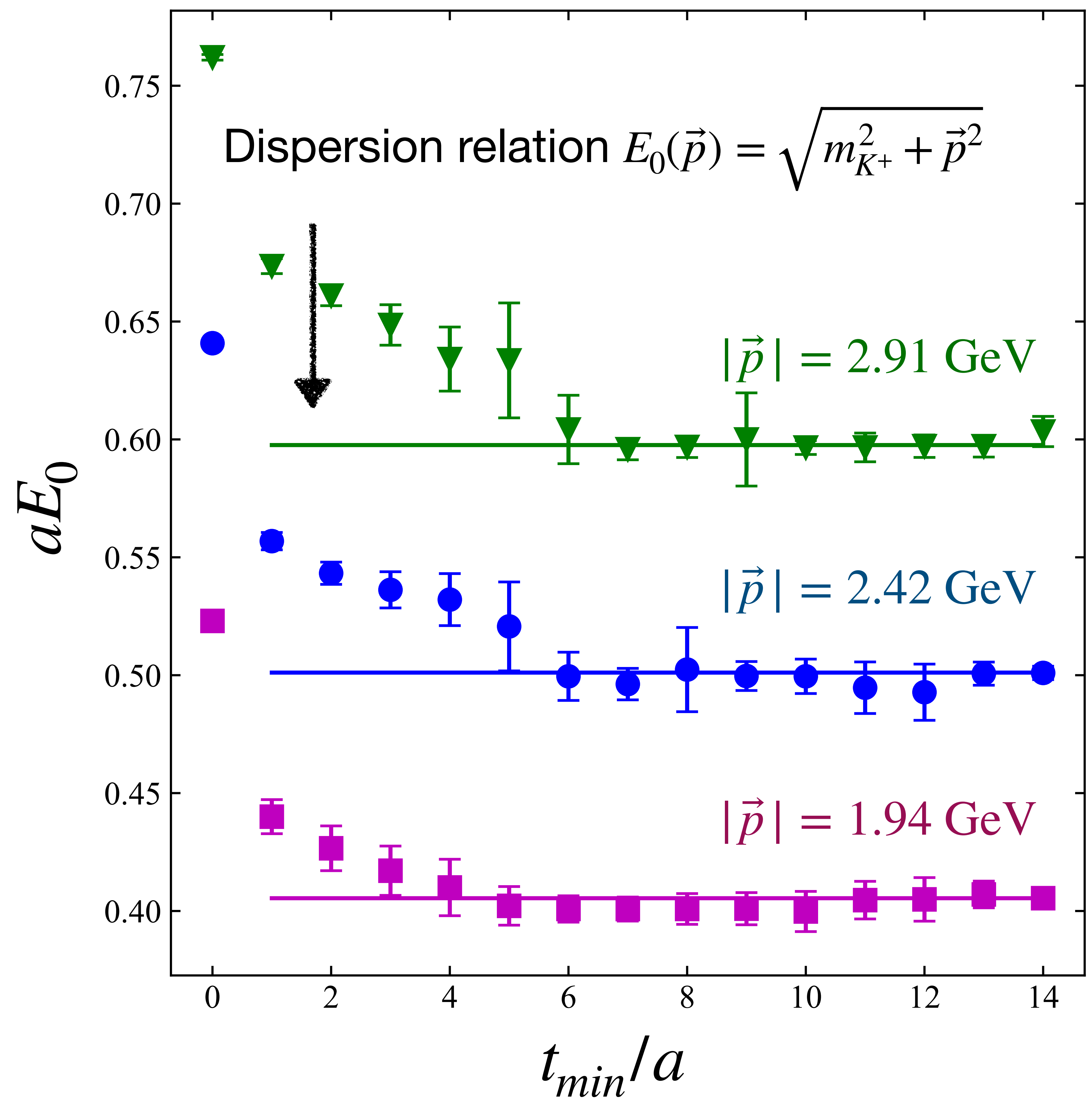
Form factor: $F(Q^2) = F^B \times Z_V^{-1}$
with $Q^2 = -q^2$

1. Construct the hadron state C_{2pt}

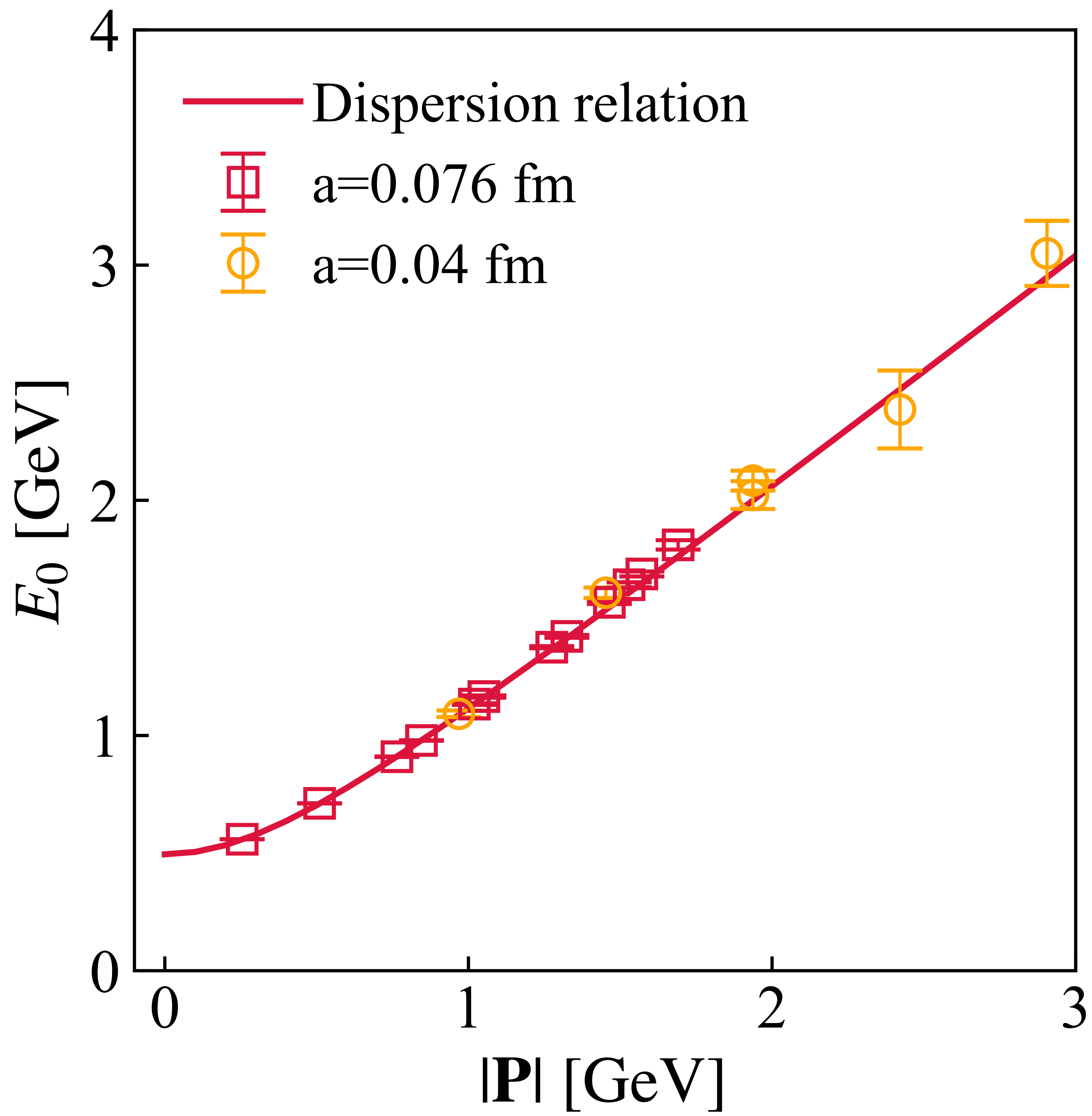
Take kaon data as an example

$$C_{2pt}(t_s) = \sum_{k=0}^{N_{state}-1} A_k \left[e^{-E_k t_s} + e^{-E_k(aN_t - t_s)}\right]$$

Fit with $N_{state} = 2$ → $E_0, E_1;\ A_0, A_1$



Fit with different range of $t_s/a \in [t_{min}/a, N_t/2]$



All results of E_0

2. Extract the bare form factor

Take kaon data as an example

$$C_{3pt}(\vec{p}^f, t_s; \vec{q}, \tau; \vec{p}^i, 0) = \sum_{n,k=0}^{N_{state}-1} \underbrace{\overbrace{\left\langle \Omega \middle| H(\vec{p}^f) \middle| n \right\rangle}^{A_n} \overbrace{\left\langle k \middle| H^\dagger(\vec{p}^i) \middle| \Omega \right\rangle}^{A_k} e^{-(E_k^{\vec{p}^i} - E_n^{\vec{p}^f})\tau} e^{-E_n^{\vec{p}^f} t_s}} \times \left\langle n \middle| \hat{\mathcal{O}}_{\gamma^\mu}(\tau) \middle| k \right\rangle$$

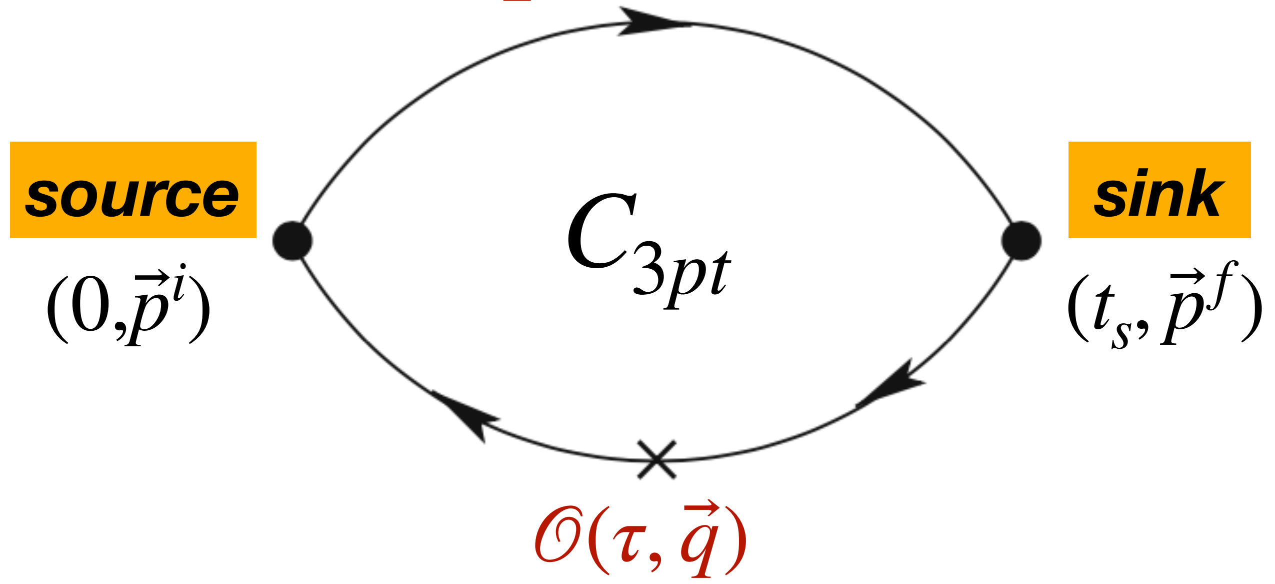
Can be suppressed by C_{2pt}

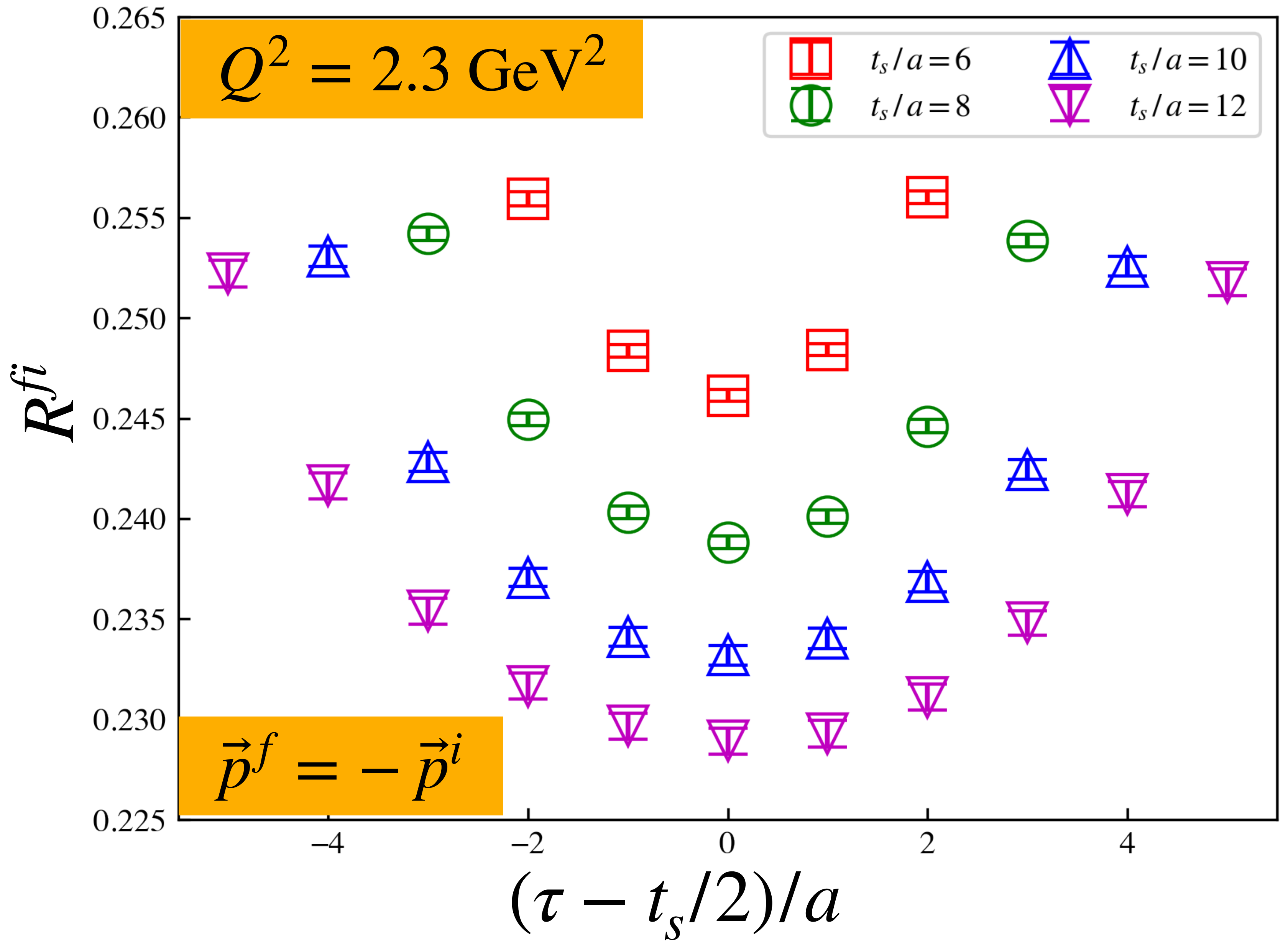
$n = k = 0$ → F^B

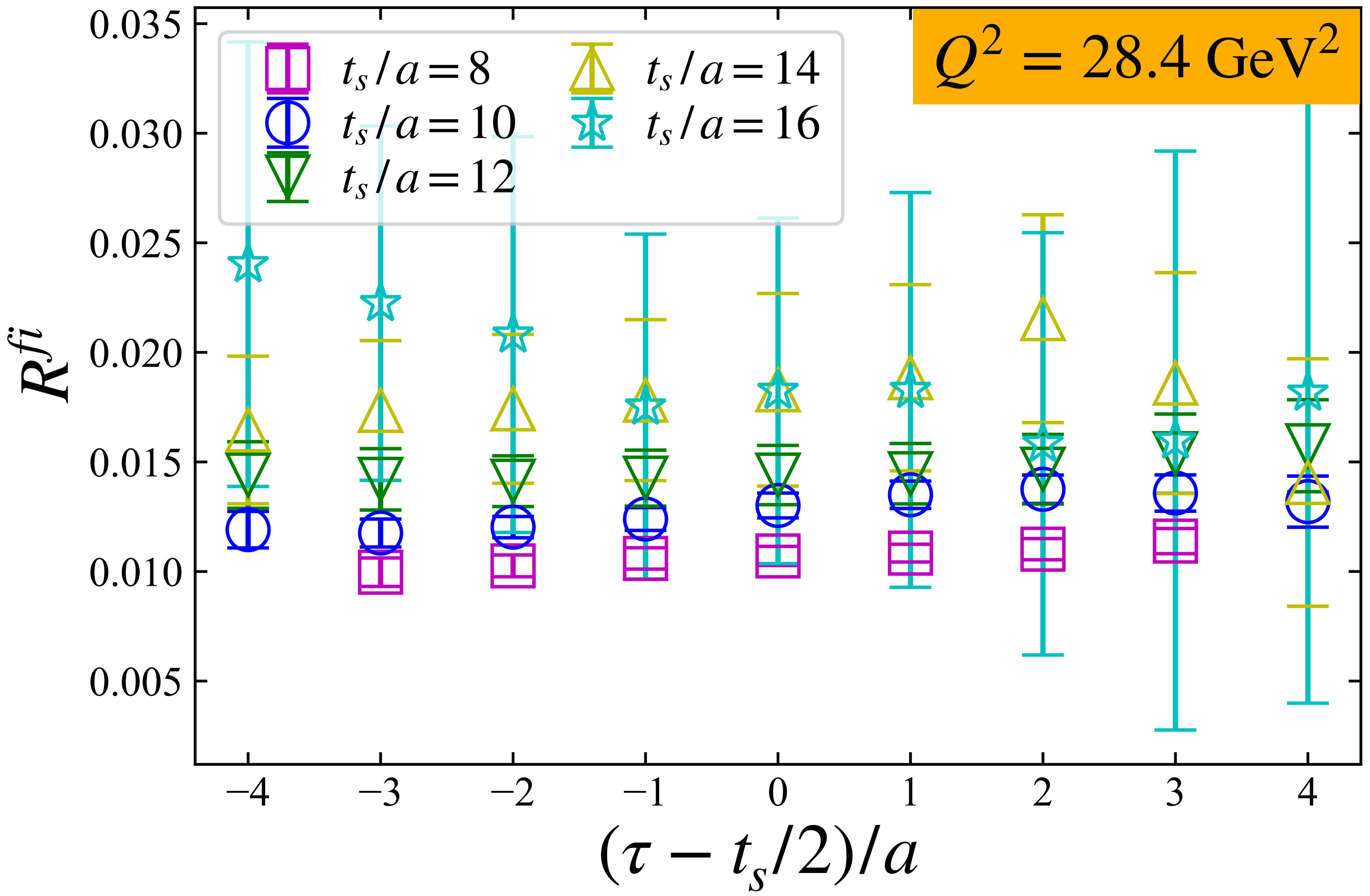
- Take a special case $\vec{p}^f = -\vec{p}^i$ as an example

Gao et al., PRD 104 (2021) 11

$$R^{fi}(\vec{p}^f, t_s; \vec{q}, \tau; \vec{p}^i, 0) \equiv \frac{C_{3pt}(\tau, t_s; \vec{p}^i, \vec{p}^f)}{C_{2pt}(t_s, \vec{p}^f)}, \quad F^B = \lim_{\tau,\, (t_s-\tau),\, t_s \to \infty} R^{fi}$$





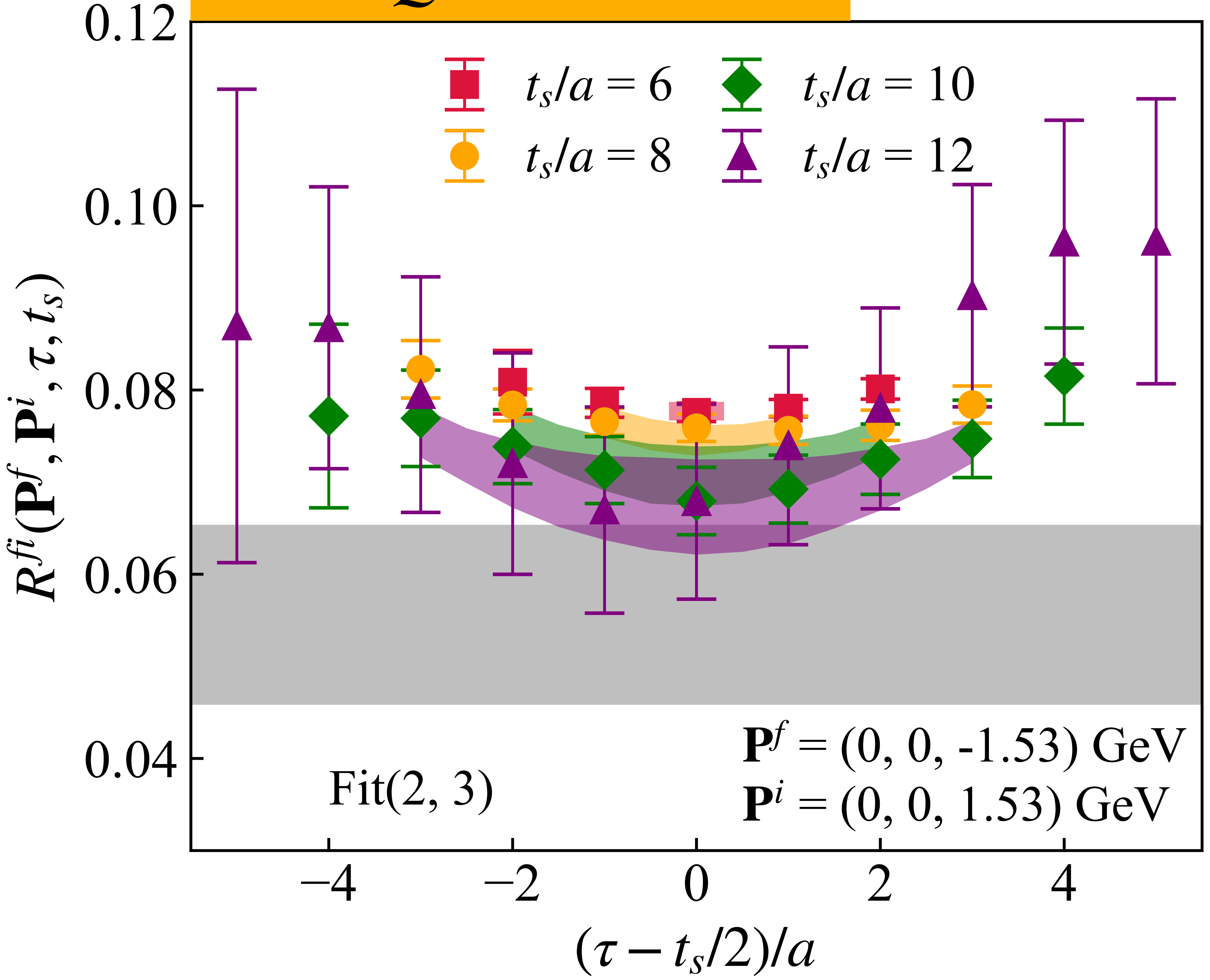


2. Extract the bare form factor

$$N_{state}=2:\ R^{fi}(\tau,t_s)=\left(\underbrace{\mathcal{O}_{00}}_{F^B}+\frac{A_1}{A_0}\mathcal{O}_{11}e^{-t_s\Delta E}+\sqrt{\frac{A_1}{A_0}}\mathcal{O}_{01}e^{-\tau\Delta E}+\sqrt{\frac{A_1}{A_0}}\mathcal{O}_{10}e^{-(t_s-\tau)\Delta E}\right)\Big/\left(1+\frac{A_1}{A_0}e^{-t_s\Delta E}\right),\ \Delta E=E_1-E_0$$

- Use the values of energy E_n and amplitude A_n extracted from C_{2pt}
- Perform a 4-parameter fit to the ratio R^{fi} to extract F^B

Pion: $Q^2 = 9.4\ \mathrm{GeV}^2$



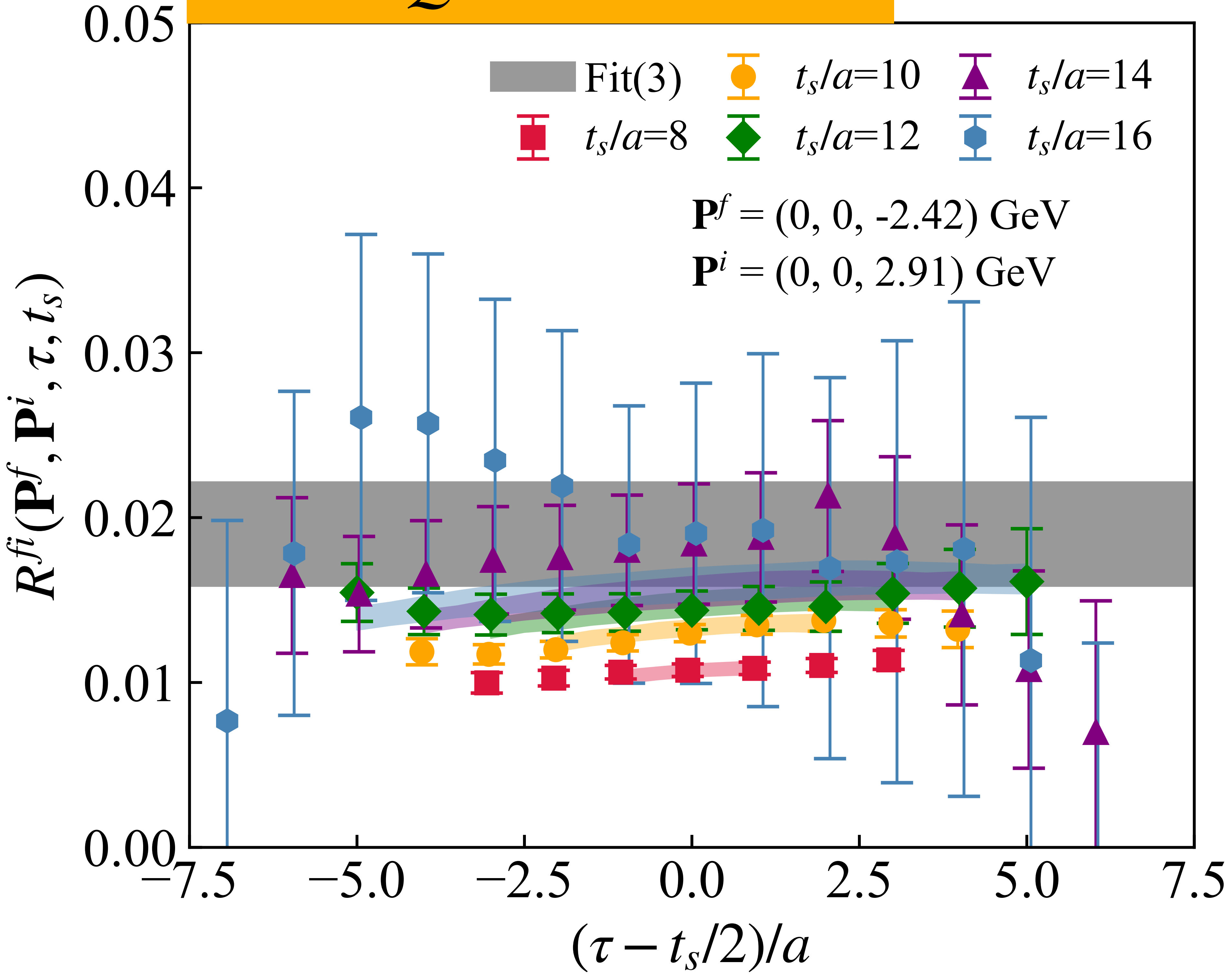
Extrapolation

↓

Grey band

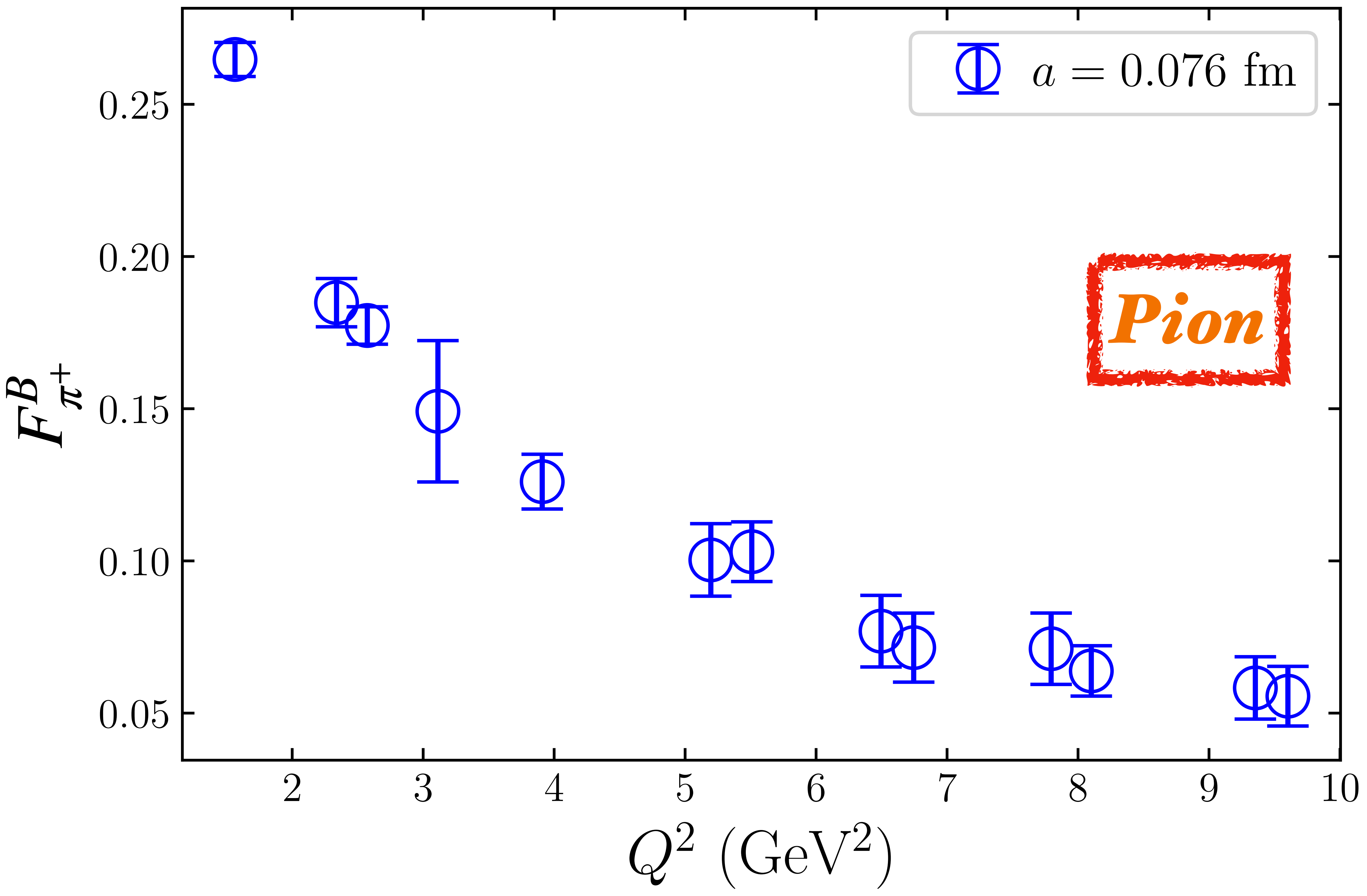
$$F^B=\lim_{\tau,\,(t_s-\tau),\,t_s\to\infty}R^{fi}$$

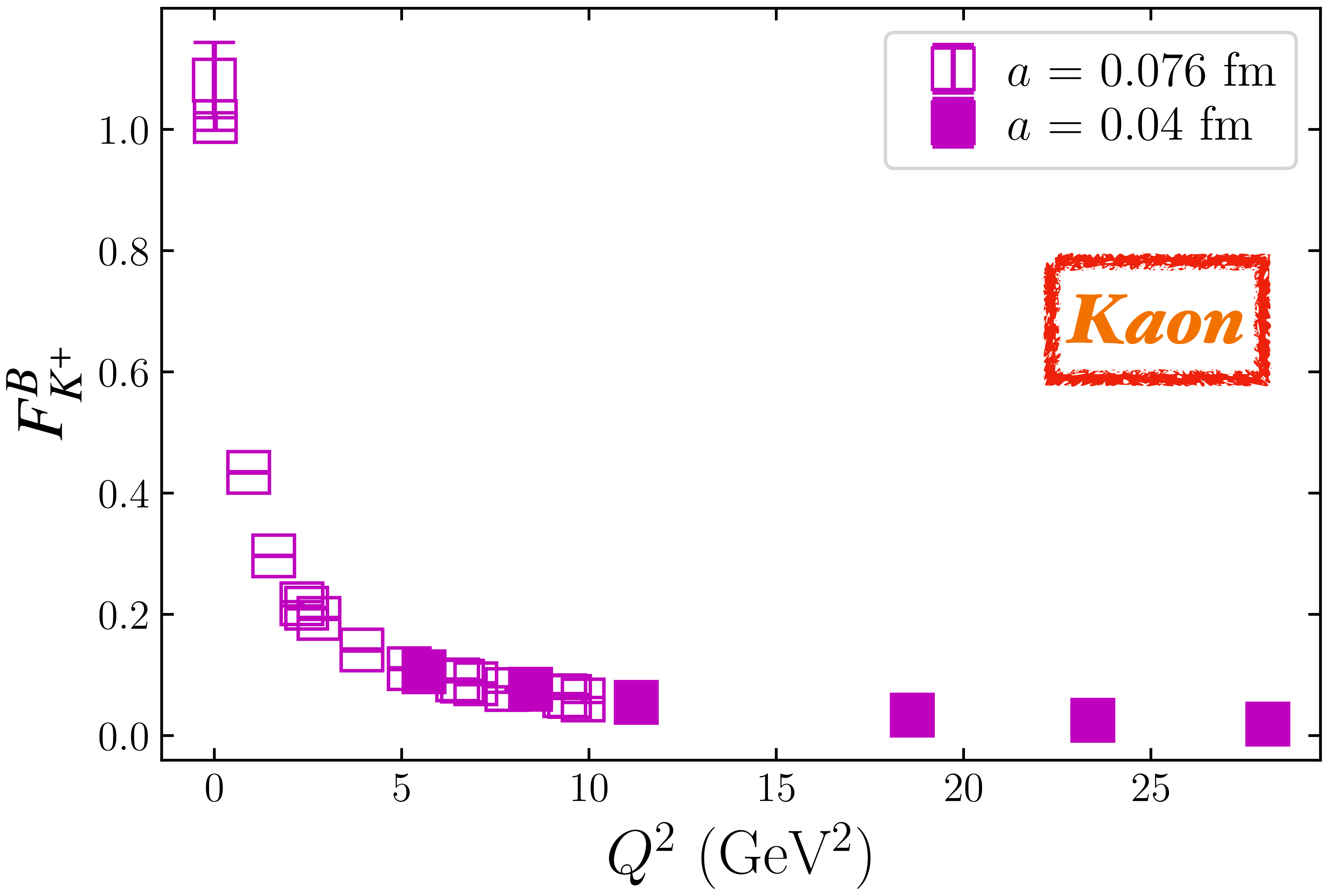
Kaon: $Q^2 = 28.4\ \mathrm{GeV}^2$



3. Renormalization

All results of bare form factor





- Renormalization: $F = F^B \times Z_V^{-1}$

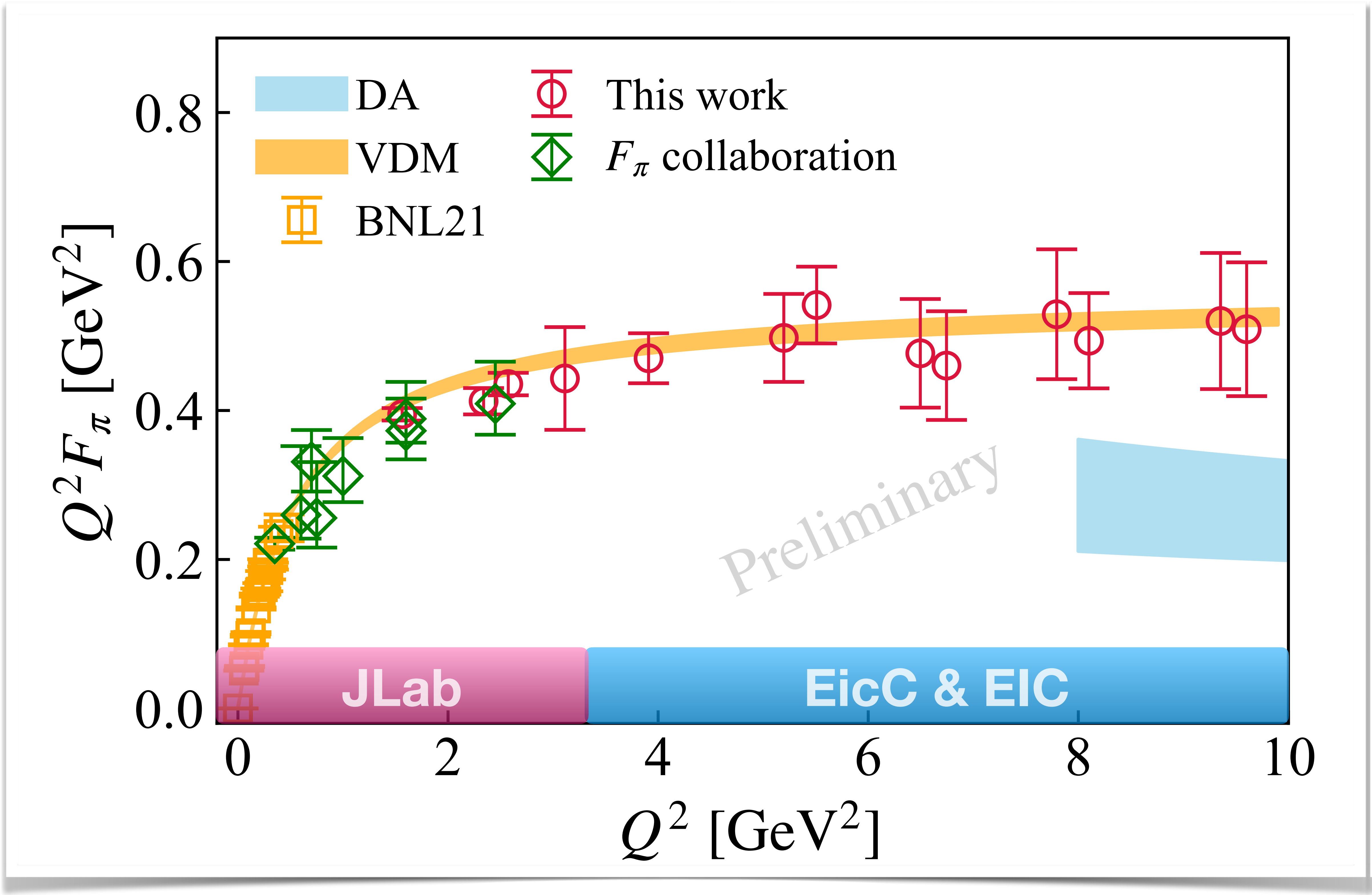
Gao et al., PRD 104 (2021) 114515

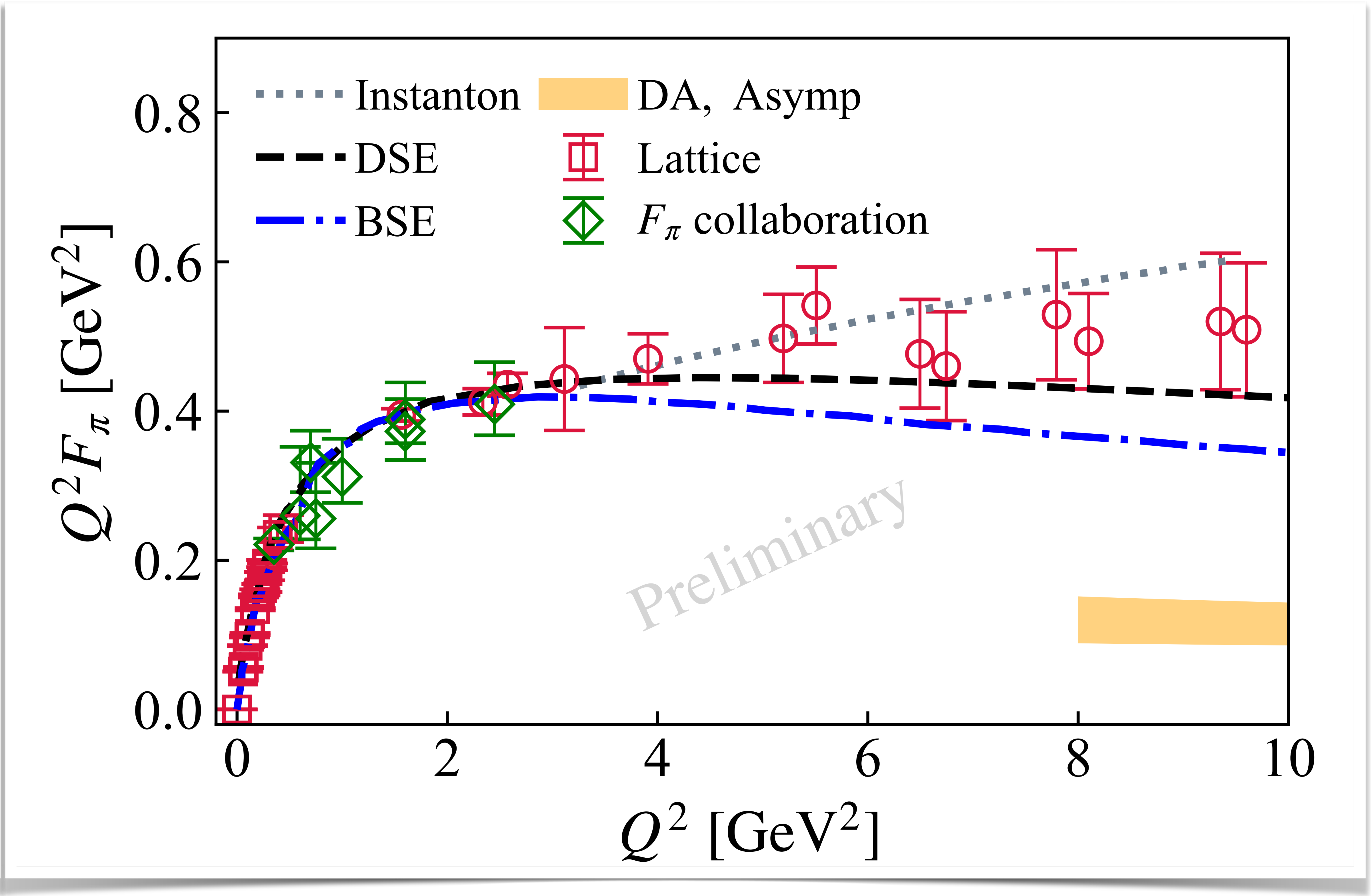
$Z_V = \left\langle 0; \vec{p} \,|\, \hat{\mathcal{O}} \,|\, \vec{p}; 0 \right\rangle = 1.048, 1.024$ for $a = 0.076, 0.04$ fm, extracted in our previous work of pion

Results

Pion

- The lattice results overlap with the experimental extraction
- Up to $Q^2 \sim 10\ \mathrm{GeV}^2$, no trend towards the partonic picture





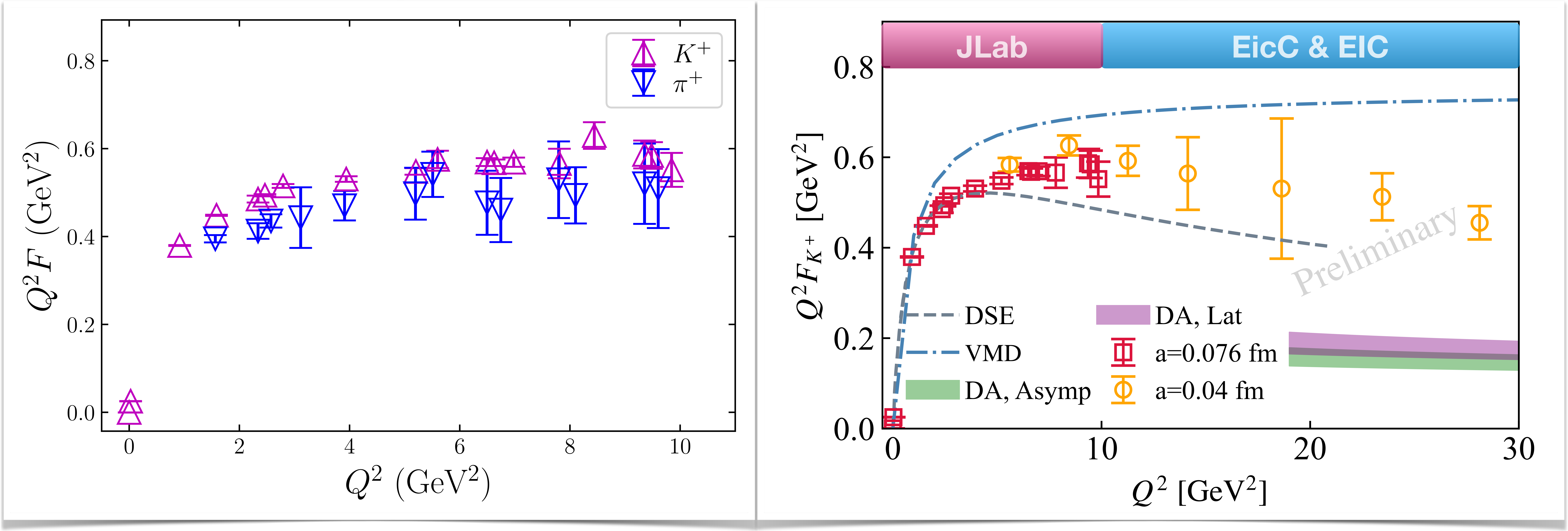
DA: Gao et al., arXiv:2206.04084
VDM: $Q^2/(1 + Q^2\langle r_\pi^2\rangle/6)$
BNL21: Gao et al., PRD 104 (2021) 114515
F^π collaboration: Huber et al., PRC 78 (2008) 045203

Instanton: Shuryak et al., PRD 103 (2021) 054028
DSE: Gao et al., PRD 96 (2017) 034024
BSE: Ydrefors et al., PLB 820 (2021) 136494
DA, Asymp: $\phi(x) = 6x(1-x)$

Results

Kaon

- Up to $Q^2 \sim 10\ \mathrm{GeV}^2$, no apparent flavor dependence
- Up to $Q^2 \sim 30\ \mathrm{GeV}^2$, doesn't reach the partonic picture



VDM: $Q^2/(1+Q^2\langle r_K^2\rangle/6)$

DA, Asymp: $\phi(x) = 6x(1-x)$

DSE: Gao et al., PRD 96 (2017) 034024

DA, Lat: Bali et al., JHEP 08 (2019) 065

Summary

- We calculate the pion and kaon electromagnetic form factor at the physical point from the first principle using Lattice QCD
- Our results contain a wide range of Q^2 , up to 10 and 28 GeV^2 for pion and kaon, which could cover the full range of JLab12 and go into the EicC and EIC region.
- Pion: Up to $Q^2 \sim 10\,\mathrm{GeV}^2$, no trend towards the partonic picture

  Kaon: Up to $Q^2 \sim 10\,\mathrm{GeV}^2$, no apparent flavor dependence

  Even up to $Q^2 \sim 30\,\mathrm{GeV}^2$, doesn't reach the partonic picture

Thanks

Backup

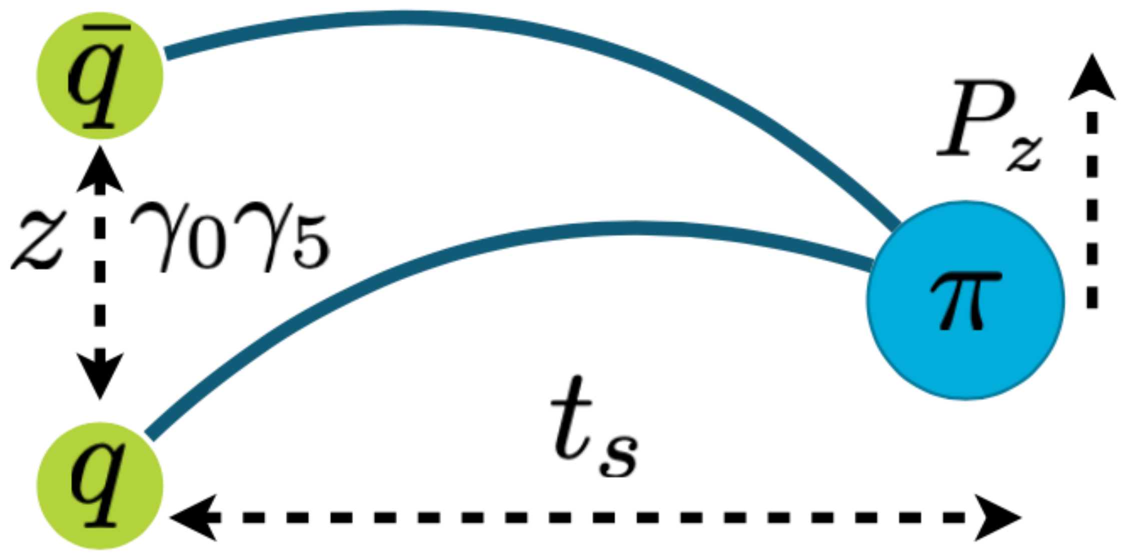
Pion Distribution Amplitude

Gao et al., PRD 106 (2022) 074505

$$C_{\pi\pi}(t_s, P_3) = \langle \pi(\mathbf{P}, t_s)\pi^\dagger(\mathbf{x}_0, 0)\rangle \,,$$

$$C_{\pi\tilde{O}_3}(t_s; z_3, P_3) = \left\langle \tilde{O}_3^B(z_3; \mathbf{P}, t_s)\pi^\dagger(\mathbf{x}_0, 0)\right\rangle \,,$$

$$R(t_s) = \frac{-iC_{\pi\tilde{O}_3}(t_s; P_3, z_3)}{C_{\pi\pi}(t_s; P_3)} \,,$$



Extrapolation

Bare quasi-DA matrix element F^B

Renormalization

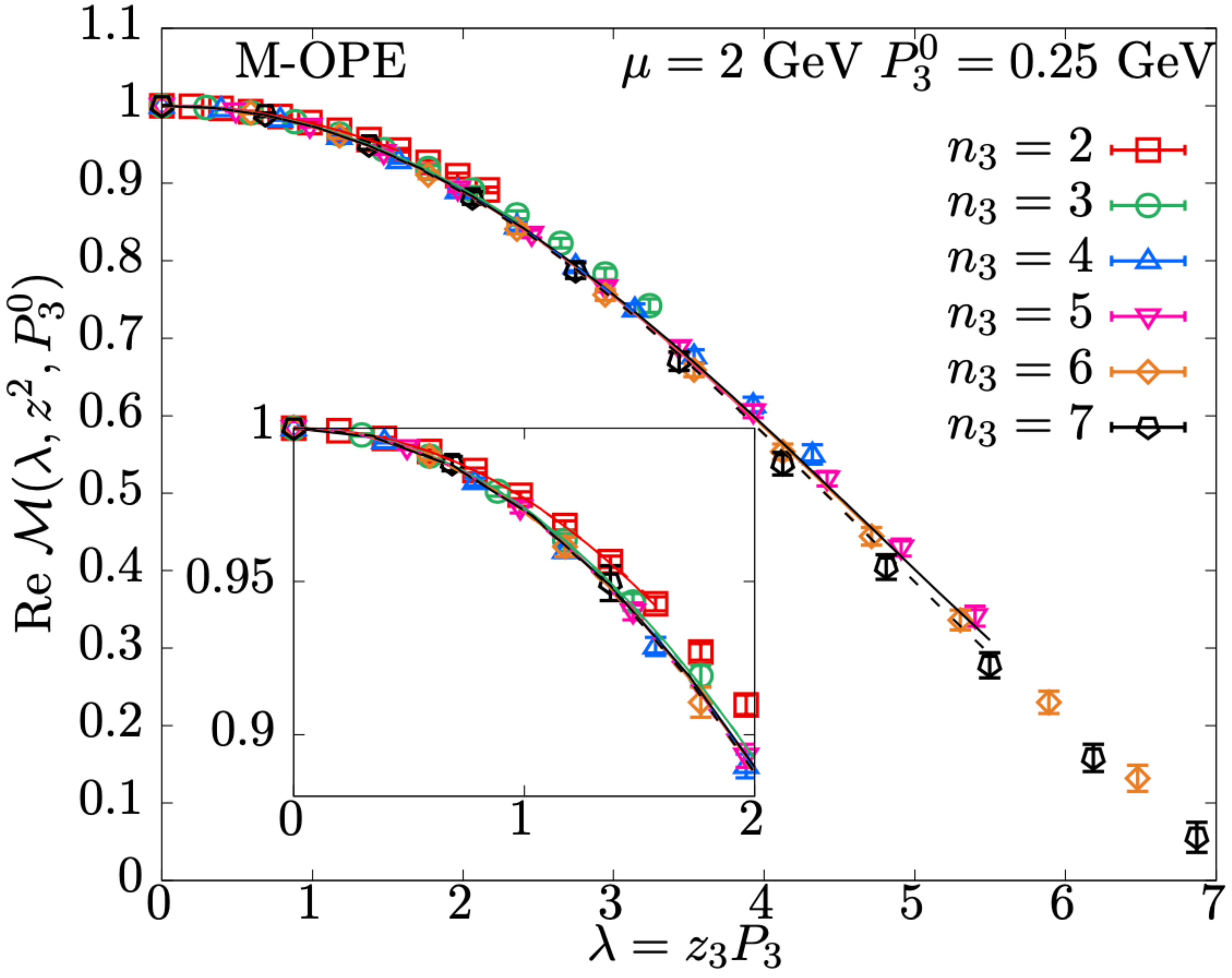
Renormalization **G**roup **I**nvariant ratios $\mathcal{M}(\lambda, z^2, P^0) \equiv \dfrac{F^B(\lambda, z_3^2)}{F^B(\lambda_0, z_3^2)}$

Pion Distribution Amplitude

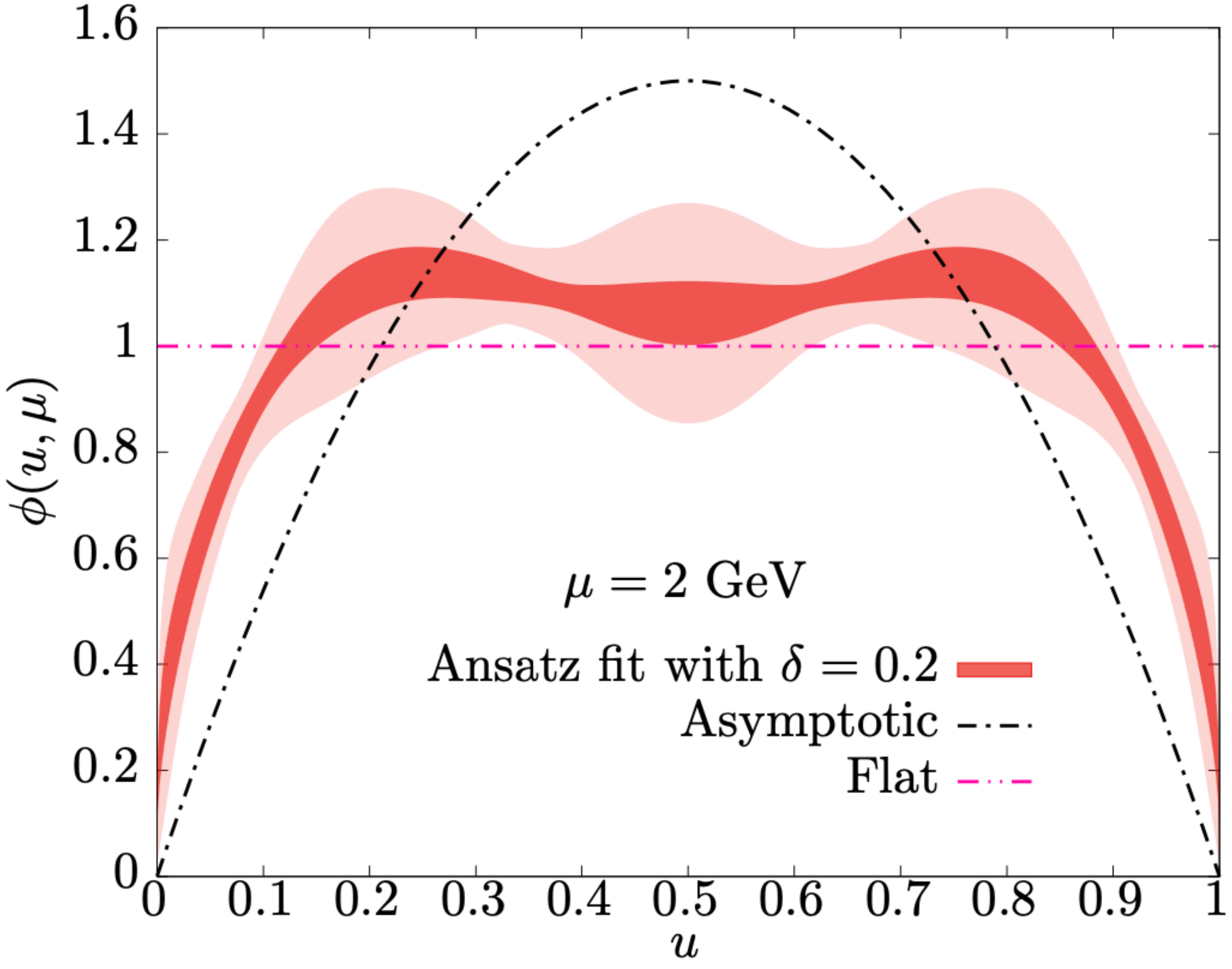
Gao et al., PRD 106 (2022) 074505

Renormalized ratio

$$\mathcal{M}\left(zP_z, z^2, \mu\right) = \sum_n C_n\left(z^2\mu^2\right)\langle x^n\rangle(\mu)\frac{(-izP_z)^n}{n!}$$



$$\phi(u) = \mathcal{N}u^{\alpha}(1-u)^{\alpha}\sum_{n=0}^{N_G+1} s_n C_{2n}^{\frac{1}{2}+\alpha}(1-2u),$$



Hard-process kernel

$$F(Q^2) = \int_0^1\int_0^1 \mathrm{d}x\mathrm{d}y\,\phi^*(v,\mu_F^2)T_F(u,v,Q^2,\mu_R^2,\mu_F^2)\phi(u,\mu_F^2),$$