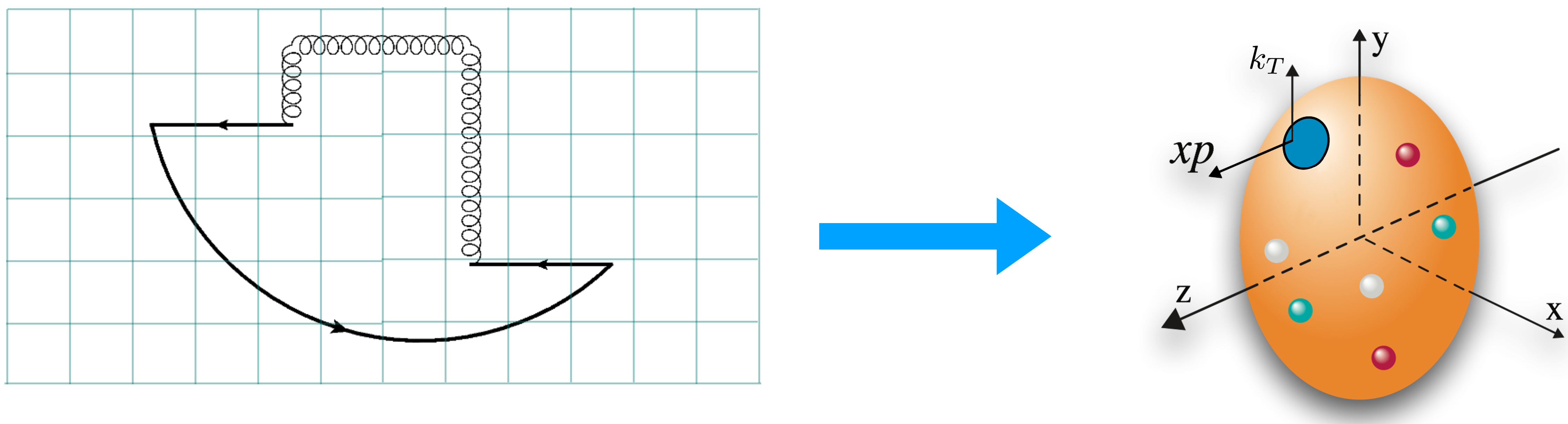
TMD evolution from lattice QCD

Michael Wagman



Shanahan, MW, Zhao, PRD 101 (2020)   Shanahan, MW, Zhao, PRD 102 (2020)   Shanahan, MW, Zhao, PRD 104 (2021)

+ work in progress with Artur Avkhadiev, Phiala Shanahan, and Yong Zhao

LaMET 2022

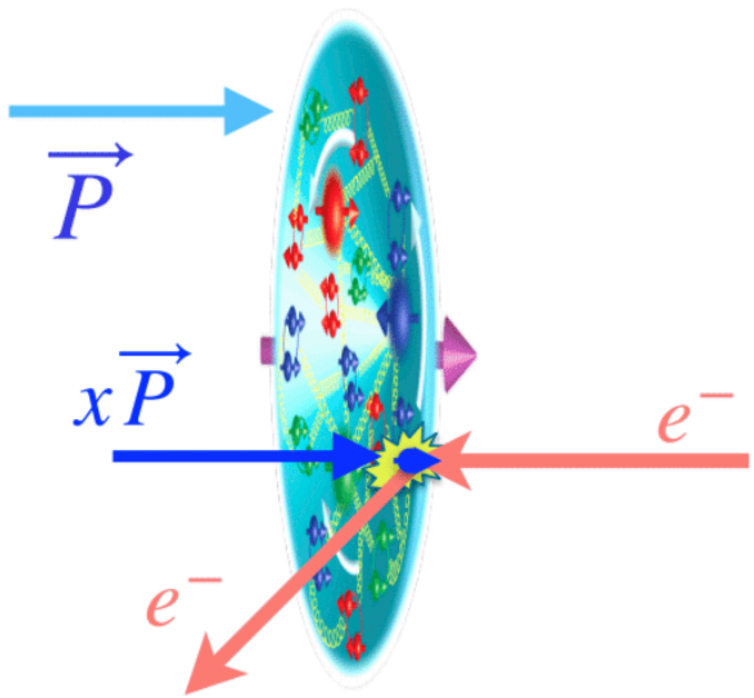
Argonne National Laboratory

Dec 1, 2022

Fermilab

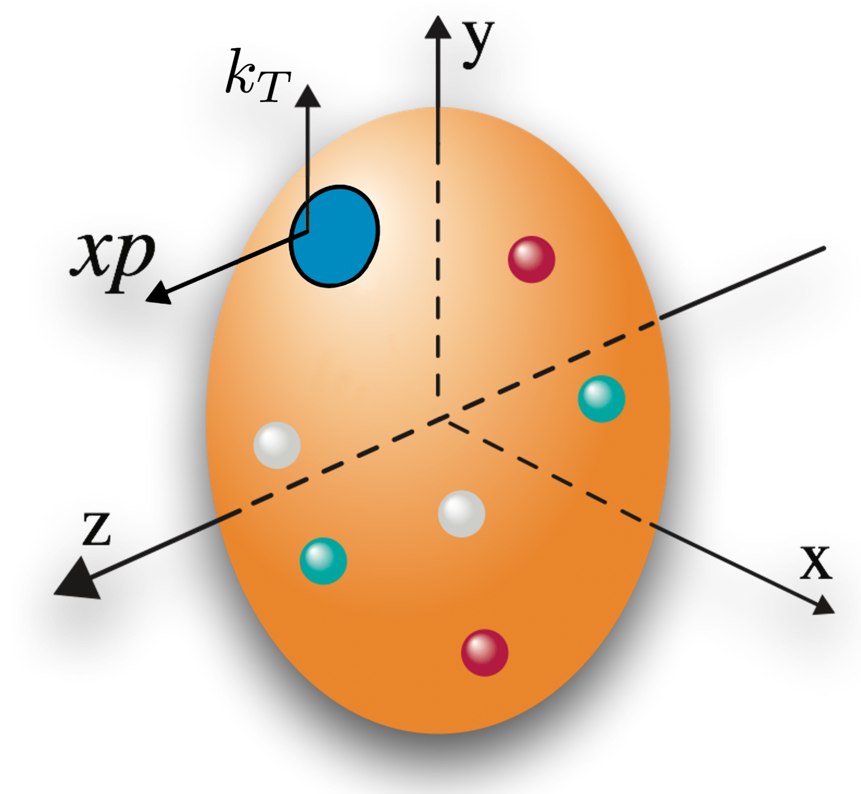
3D hadron structure

Our knowledge of proton structure has historically focused on collinear PDFs

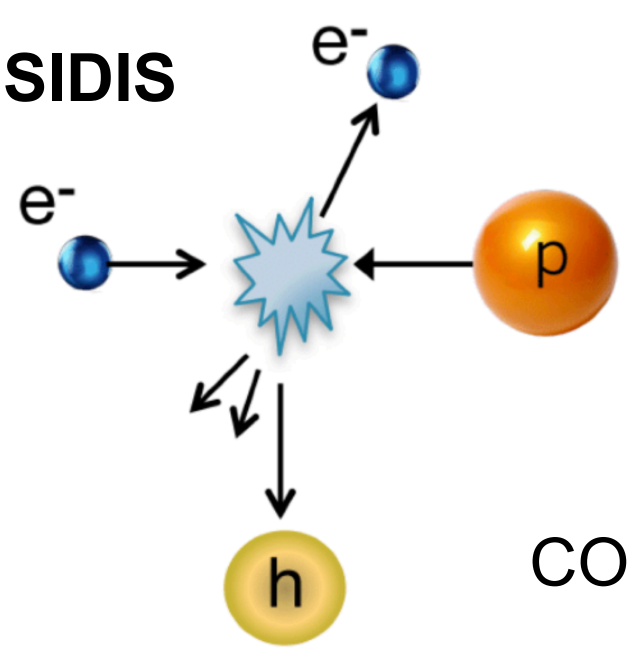


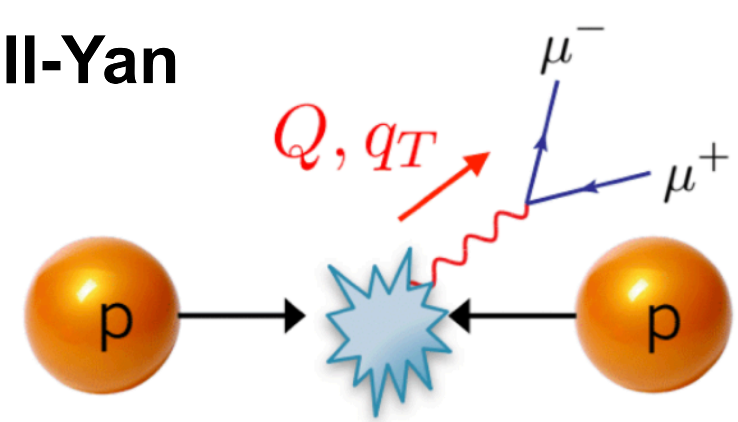
Hadrons further contain rich 3D structure encoded in TMDPDFs

$$f_i(x, \vec{b}_T) = \int d^2k_T \, e^{i\vec{k}_T \cdot \vec{b}_T} \, f_i(x, \vec{k}_T)$$



TMDPDFs are needed to describe cross-sections for SIDIS and the Drell-Yan process





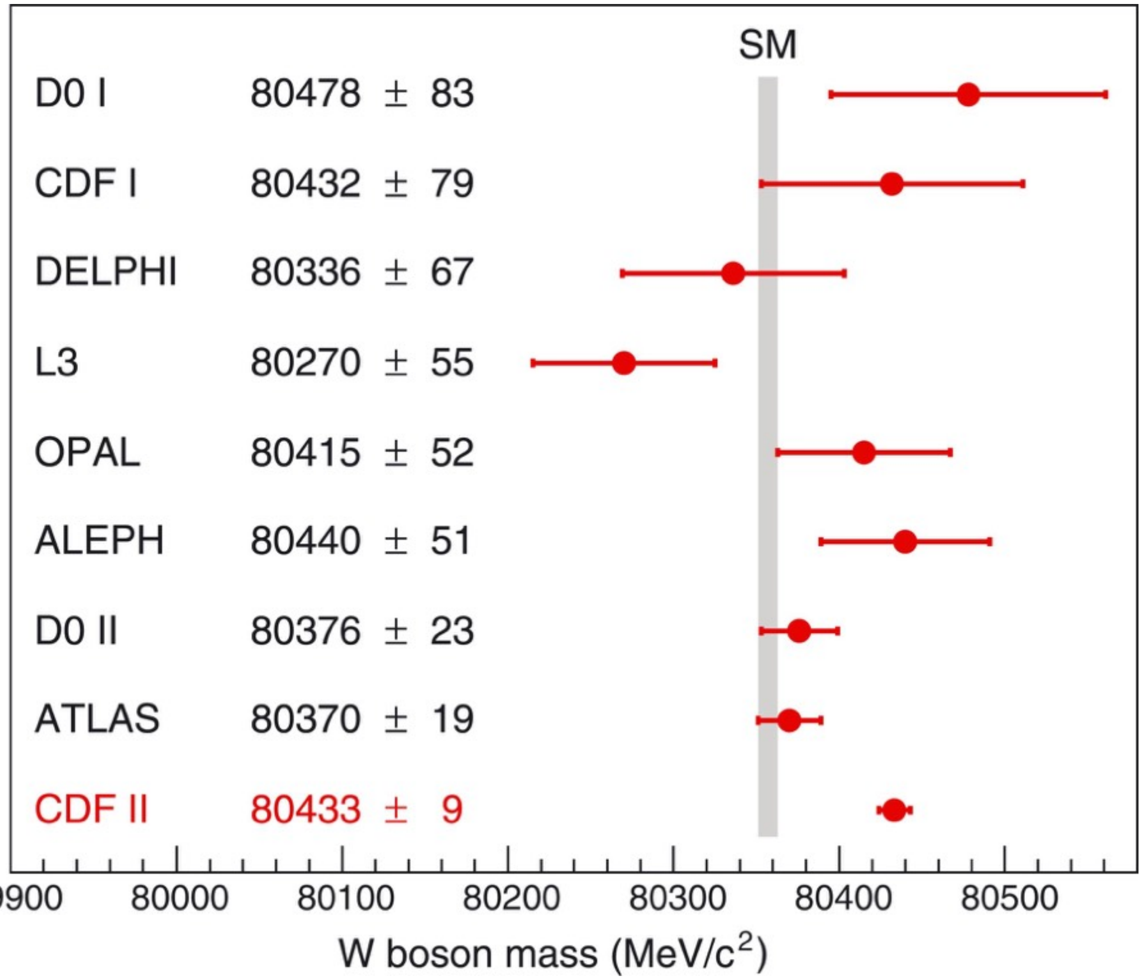
COMPASS, EIC, Fermilab, HERMES, JLab, LHC, RHIC, …

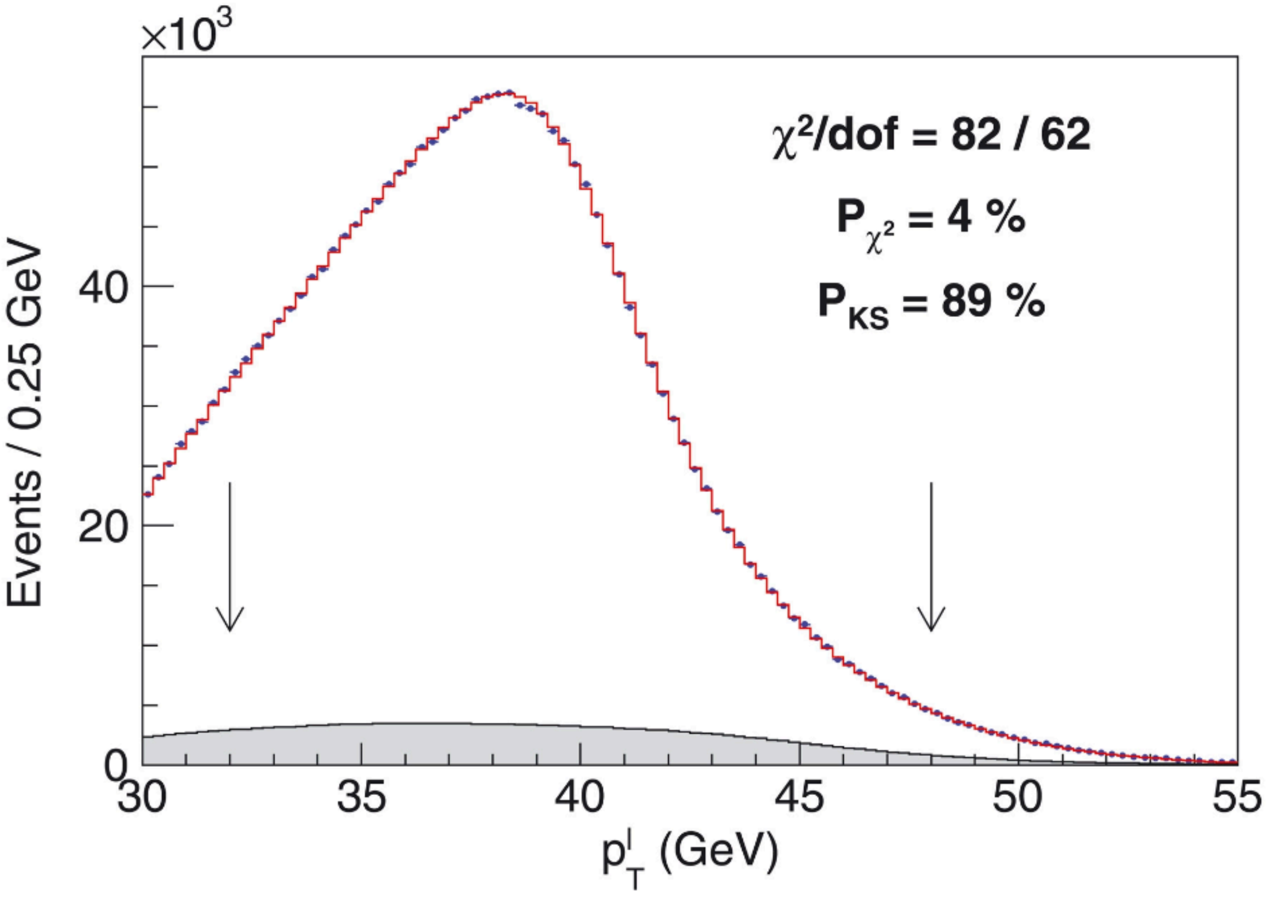
The W boson mass

Precise measurement of M_W from CDF disagrees at 7 sigma with M_W obtained from electroweak precision fits

New physics?

Robust understanding of all QCD theory uncertainties essential





Aaltonen et al [CDF], Science 376 (2022)

Measurement made by fitting shapes of transverse momentum distributions to theory predictions including resumed and nonperturbative QCD effects

Distribution shapes are insensitive to many aspects of TMDPDFs but are sensitive to flavor dependence and evolution effects

The Collins-Soper kernel

TMDPDFs depend on UV renormalization scale μ as well as a scale ζ associated with the renormalization of rapidity divergences

$$f_i(x, \vec{b}_T, \mu, \zeta) = f_i(x, \vec{b}_T, \mu_0, \zeta_0) \times \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^i(\mu', \zeta_0)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu, b_T) \ln \frac{\zeta}{\zeta_0}\right]$$

UV anomalous dimension

Collins-Soper kernel

(rapidity anomalous dimension)

Changing hard momentum scales requires evolving TMDPDFs in μ and ζ

Evolution in μ is perturbative as long as μ is large, but evolution in ζ is always nonperturbative for large b_T

CS kernel phenomenology

Fits to SIDIS and Drell-Yan data with multiple energy scales are sensitive to evolution effects and therefore the CS kernel

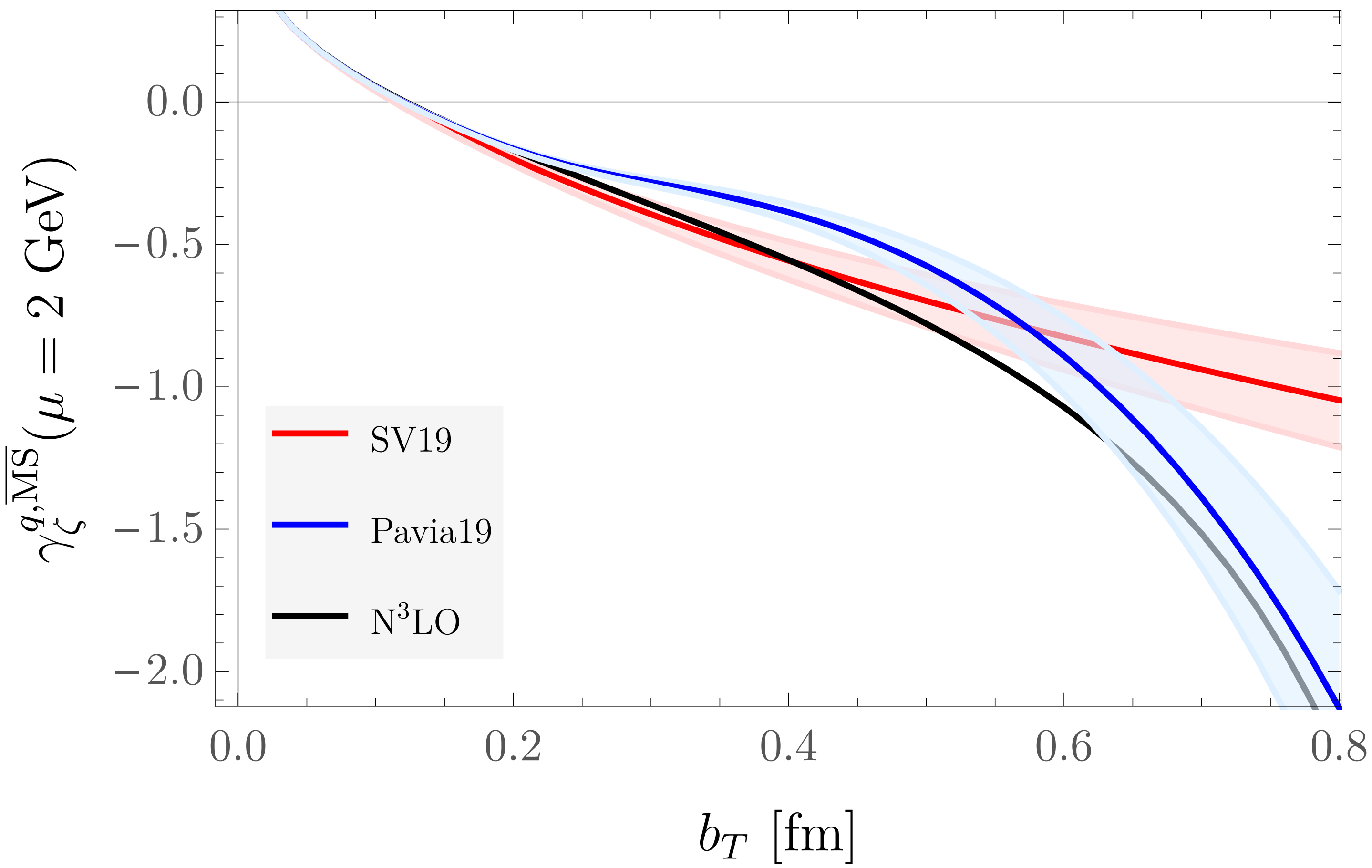
CS kernel can be extracted along with TMDPDF in global fits

SV19 - Scimemi and Vladimirov, JEHP 06 (2020)

(582 SIDIS + 457 DY data points)

Pavia19 - Bacchetta et al, JEHP 07 (2020)

(353 DY data points)



Modeling significant for

$$b_T \gtrsim 0.2\ \mathrm{fm}$$

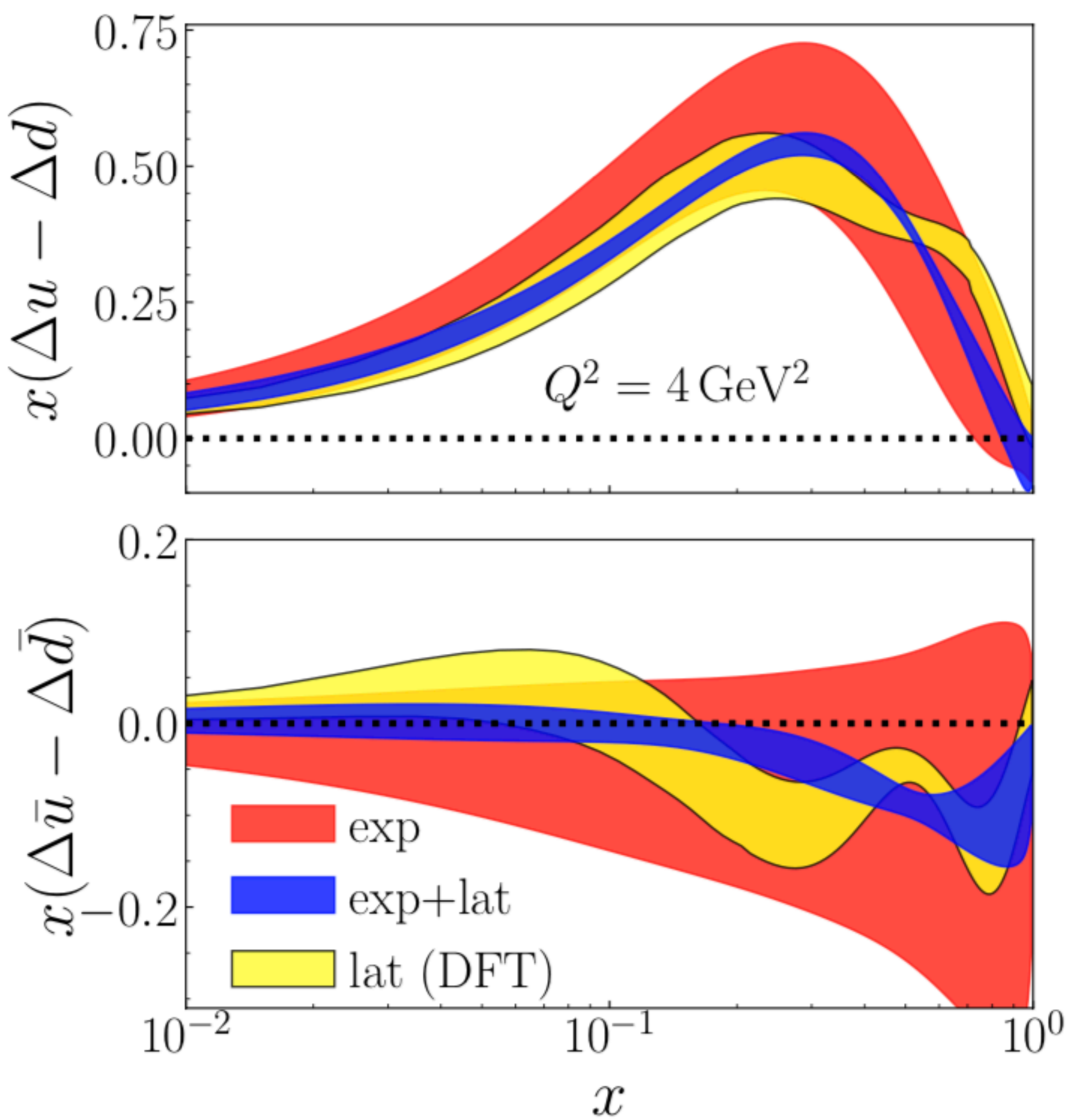
(nonperturbative region)

Quasi PDFs

Large momentum effective theory connects light-cone PDFs to Euclidean matrix elements that can be calculated using lattice QCD

Review: Ji et al, Rev. Mod. Phys. 93, 35005 (2021)

Quasi PDF: $$\widetilde{q}(x,P_z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi}\, e^{-ixzP_z} \, \langle h(P_z)|\overline{q}(z)\gamma_4 W(z,0) q(0)|h(P_Z)\rangle$$



Bringewatt et al [JAM], PRD 103 (2021)

For large P_z , quasi PDFs can be related to light-cone PDFs by perturbative matching coefficients

Several LQCD groups are performing increasingly refined quasi PDF calculations

See e.g. Snowmass white paper arXiv:2202.07193

For e.g. isovector polarized nucleon PDFs, LQCD results provide significant improvements to global fits

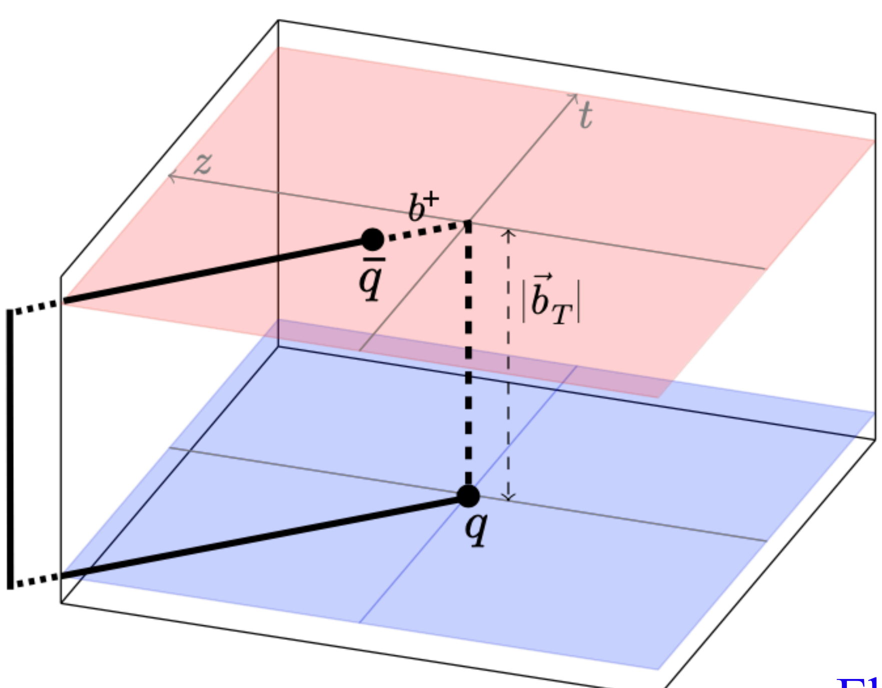
Quasi TMDPDFs

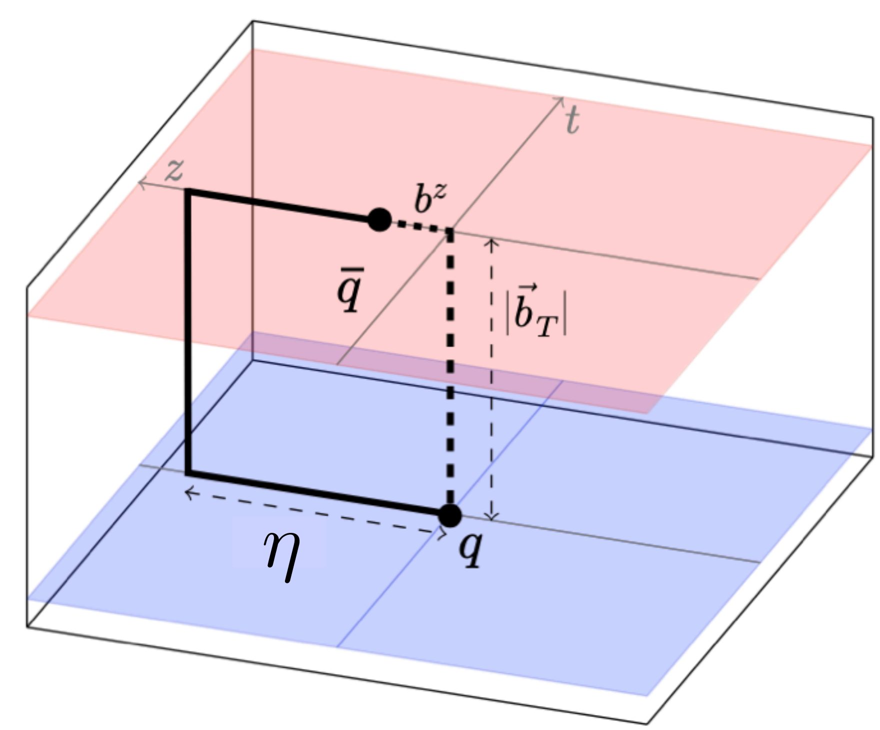
The construction of quasi TMDPDFs is more complicated Ji, PRL 110 (2013)

TMDPDF products appearing in e.g. Drell-Yan can be expressed as convolutions of "beam functions" and "soft functions"

Quasi beam functions can be constructed that are related to light-cone beam functions by a Lorentz boost

$$\widetilde{q}(x, b_T, P_z) = \lim_{\eta\to\infty} \int_{-\infty}^{\infty} \frac{dz}{4\pi}\, e^{-ixzP_z} \left\langle h(P_z)|\overline{q}(b_T)\gamma_4 W(b_T, \eta + b_T) W_T^\dagger(\eta + b_T, \eta) W_z^\dagger(\eta, 0) q(0)|h(P_Z)\right\rangle$$



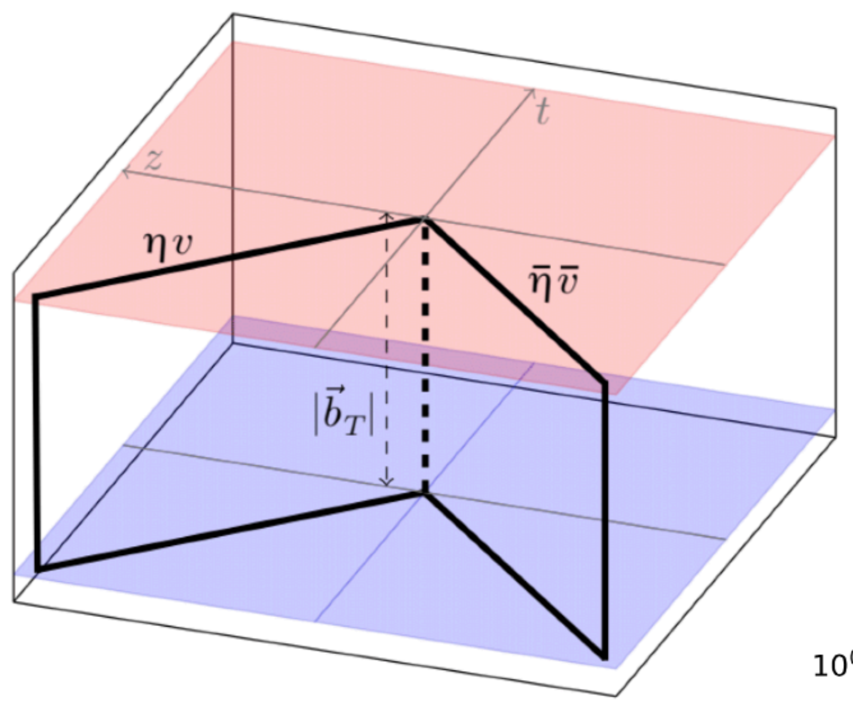


Ebert, Stewart, Zhao, JHEP 1909 (2019)

The soft function

TMDPDF products in Drell-Yan also involve a soft function that depends on the light-like momenta of both hadrons

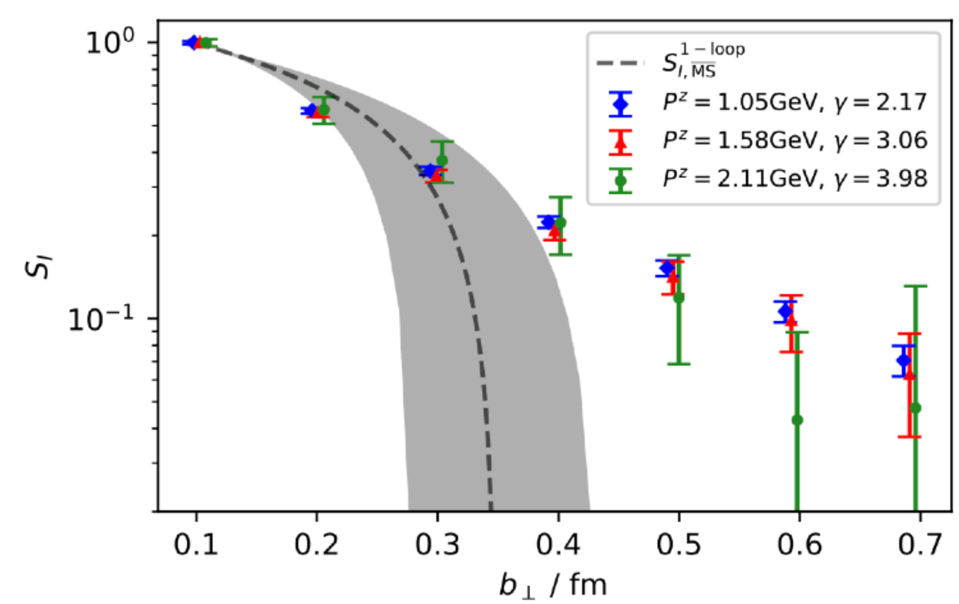
Soft function cannot be related to a matrix element of equal-time operator product by a Loretnz boost



Recent progress relates light-cone soft function to a large-momentum form factor that can be calculated with LQCD

Ji, Liu, and Liu, Nucl Phys B 955 (2020)

Proof of principle demonstration that LQCD can predict TMDPDFs



Zhang et al [LPC], PRL 125 (2020)

The CS kernel from LQCD

Ratios of TMDPDFs free from soft factors and can be calculated with LQCD

Musch et al, PRD 85 (2012) Engelhardt et al, PRD 93 (2016) Yoon et al, PRD 96 (2017)

CS kernel determination using quasi-TMDPDFs suggested

Ji, Sun, Xiong, Yuan PRD 91 (2015)

Formula relating CS kernel to quasi-TMDPDF ratios proposed and derived

Ebert, Stewart, Zhao, PRD 99 (2019)

$$\gamma_\zeta^{q,\overline{\mathrm{MS}}}(b_T,\mu) = 2\zeta \frac{d}{d\zeta} \ln f_q^{\overline{\mathrm{MS}}}(x, b_T, \mu, \zeta)$$

$$= \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xP_2^z) \int db^z e^{ib^z x P_1^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z, b_T, \eta, \mu, P_1^z)}{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xP_1^z) \int db^z e^{ib^z x P_2^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z, b_T, \eta, \mu, P_2^z)}$$

Euclidean quasi-beam function

Quasi/light-cone matching coefficient

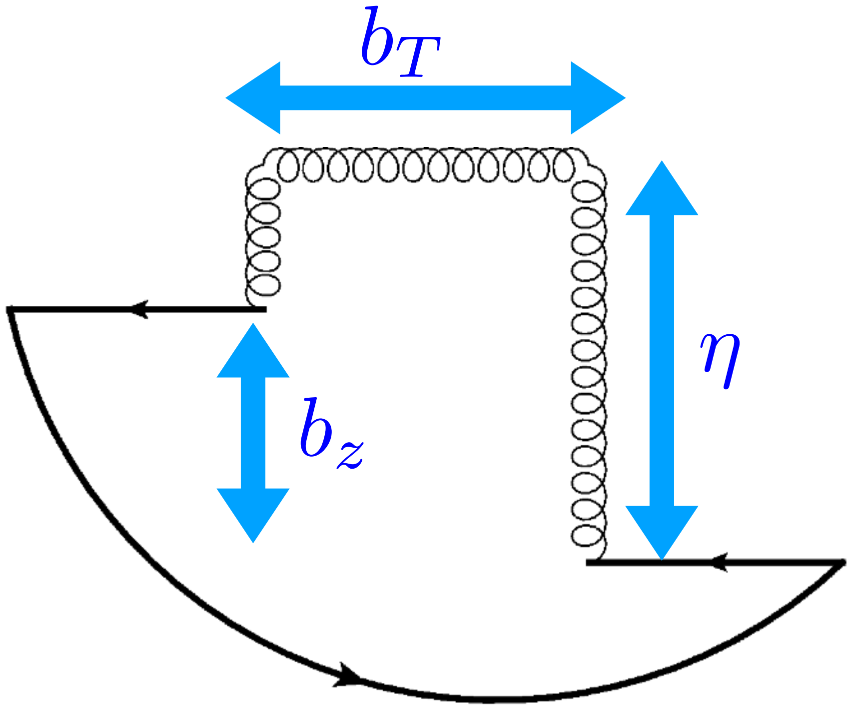
Quenched LQCD exploration

CS kernel property of QCD vacuum,
independent of hadronic state

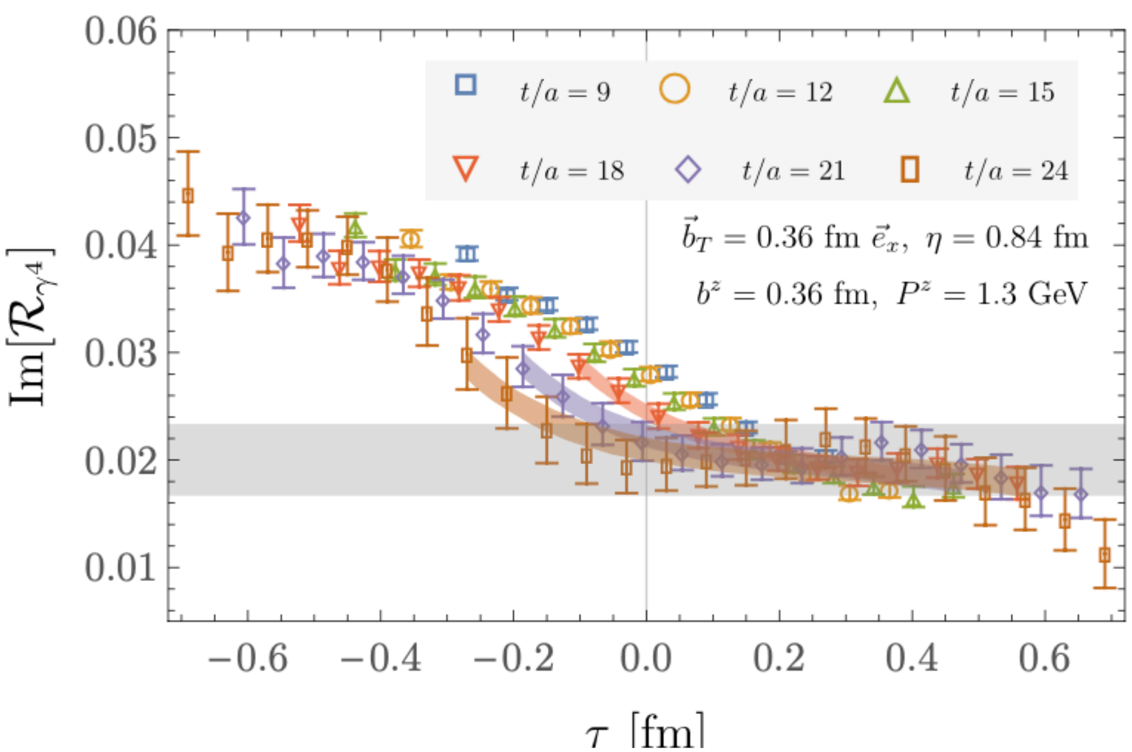
Calculate using pion state

In quenched ($N_f = 0$) QCD, exact results calculable using heavy quark probe

$$m_\pi \sim 1.2 \text{ GeV}$$



Allows high precision with only 400 quark propagator sources



Shanahan, MW, Zhao, PRD 102 (2020)

3 values of $\eta \in [0.6,\ 0.8]$ fm

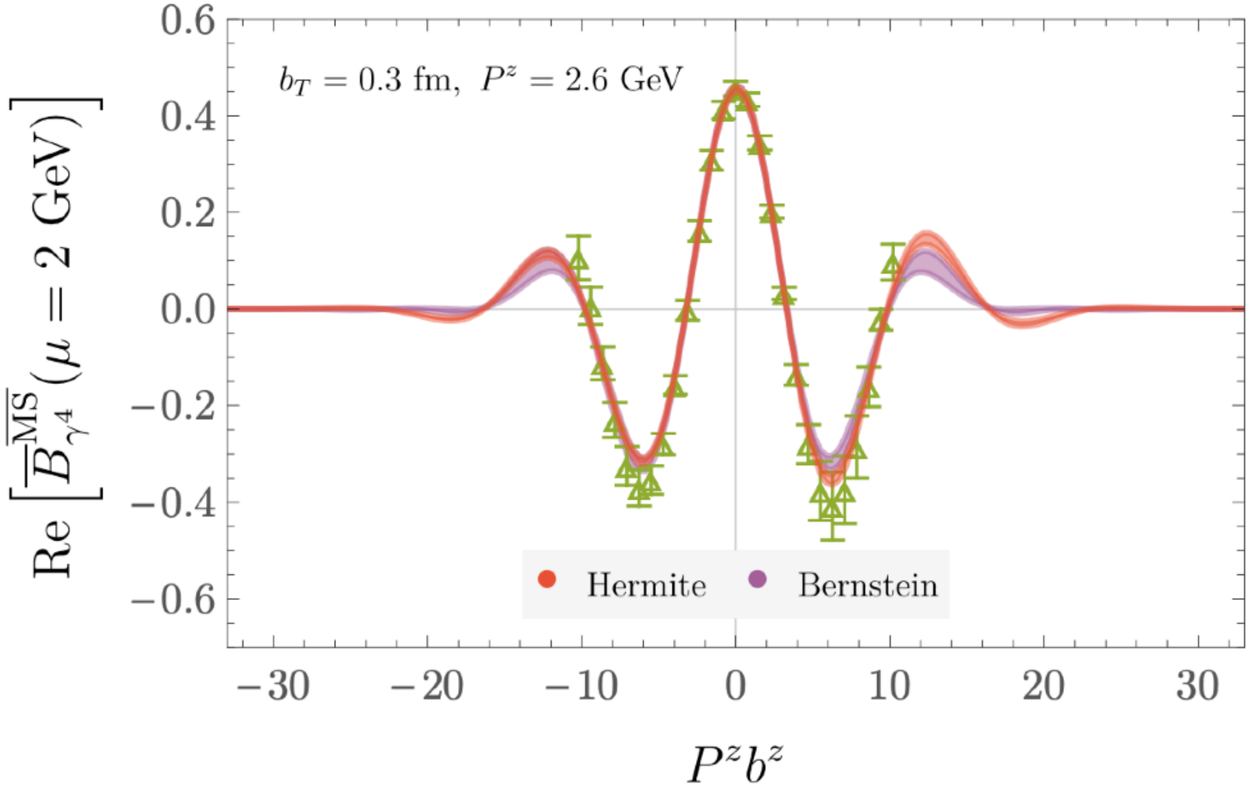
3 values of $P^z \in [1.3,\ 2.6]$ GeV

All 16 Dirac structures and
staple geometries b_T and b^z

35,660 bare matrix elements -
robust automated fitting essential

Fourier transform challenges

Fourier transform truncation effects: challenging systematic uncertainties to quantify

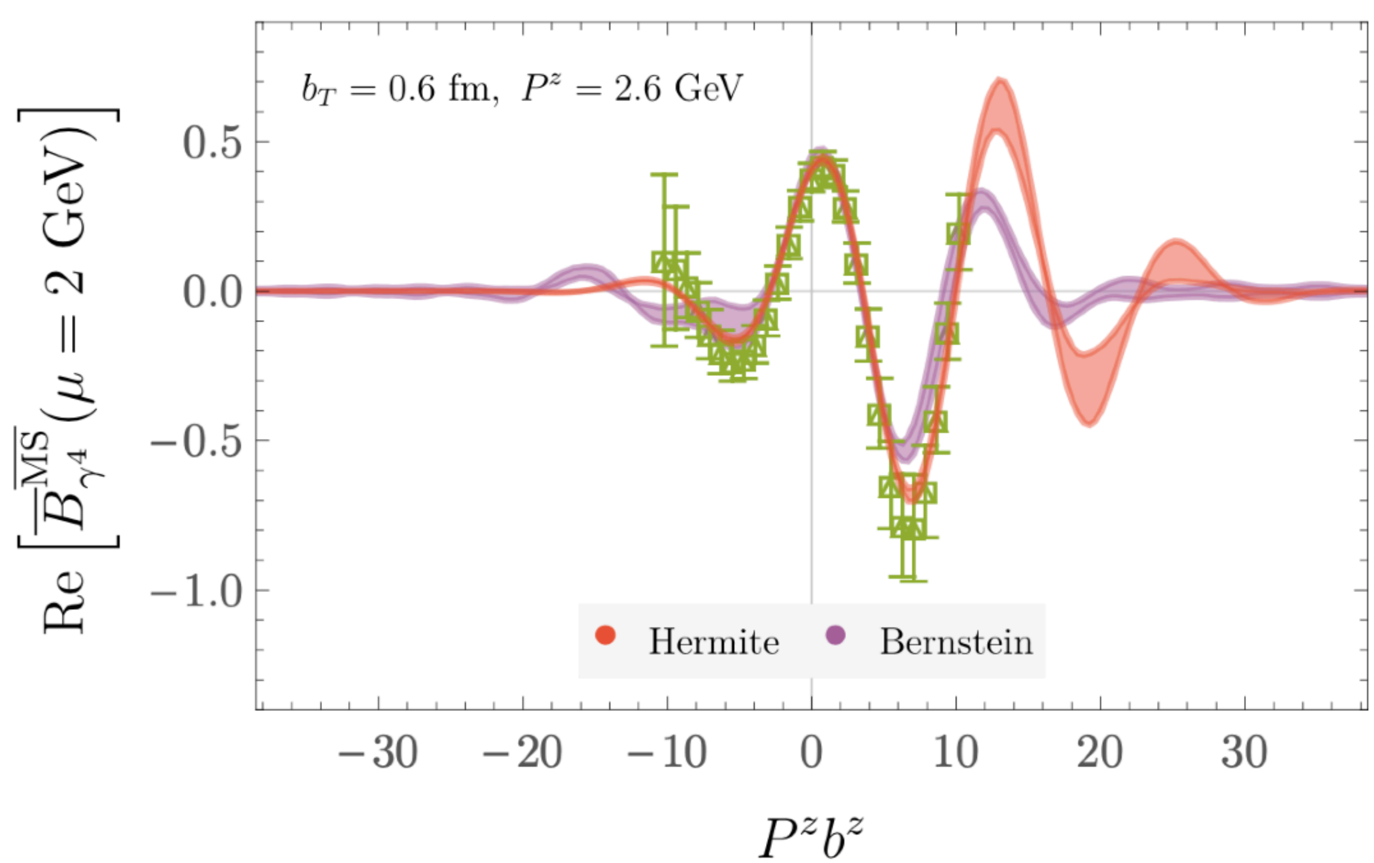


Shanahan, MW, Zhao, PRD 102 (2020)

Modeling of beam functions outside region with LQCD results necessary to cover region where Fourier transform integrand matters

Broadening of the (physical) beam function with increasing b_T leads to larger truncation effects

Unexpected asymmetry visible at large b_T

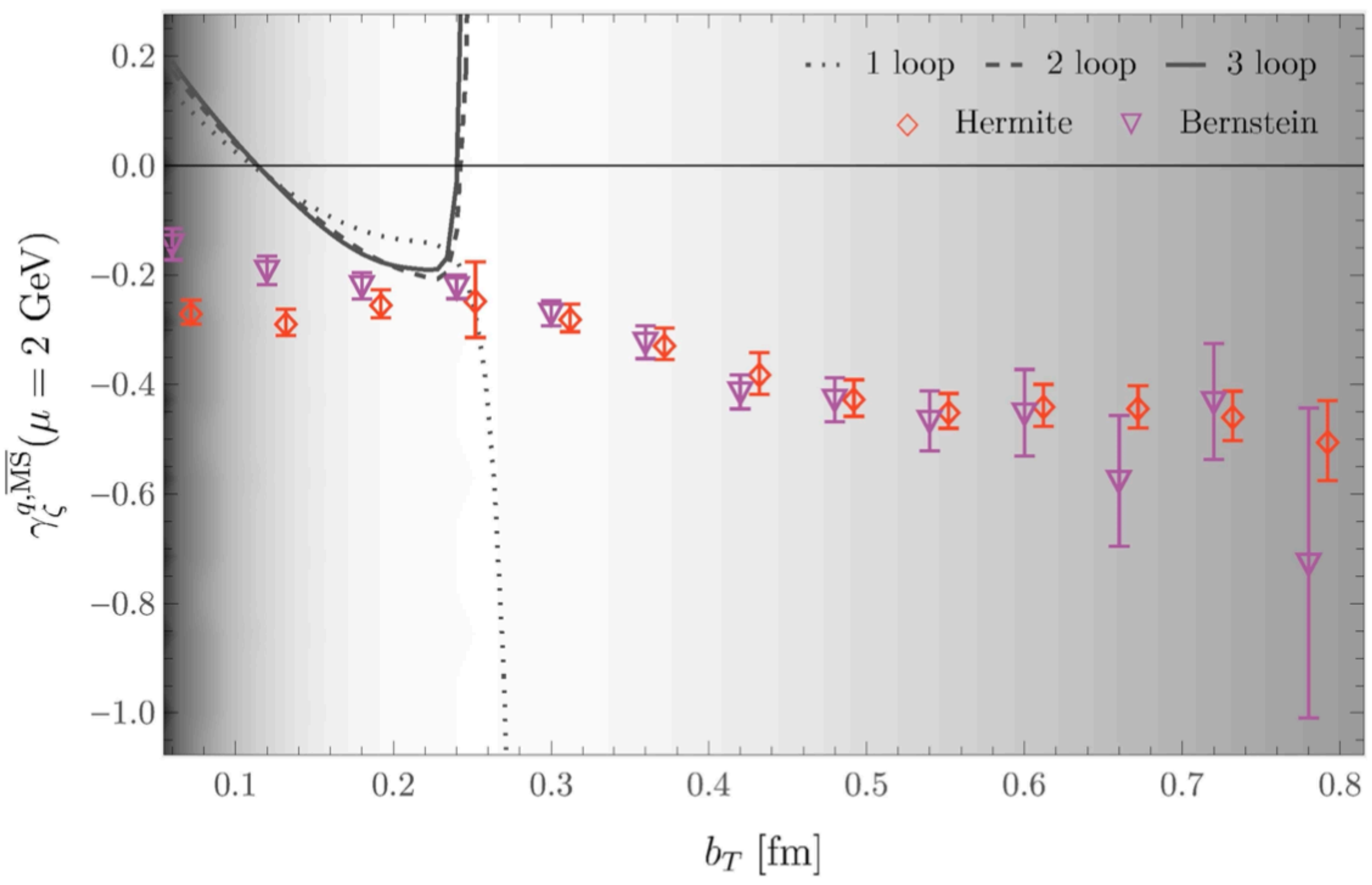


Renormalization issues?

Finite volume effects?

Briceño, Guerrero, Hansen, and Monahan, PRD 99 (2018)

Quenched LQCD results



Shanahan, MW, Zhao, PRD 102 (2020)

Fourier transform truncation effects challenging to quantify, two different models used to extrapolate beam functions outside range of data

CS kernel determined precisely for b_T extending into nonperturbative regime

$m_\pi = 1.2$ GeV

$L = 32a = 1.92$ fm

$P^z \in \{1.3,\ 1.9,\ 2.6\}$ GeV

$\eta \leq 0.8$ fm

Dynamical LQCD setup

Mixed action: $N_f = 2+1+1$ MILC ensembles with ~physical quark masses

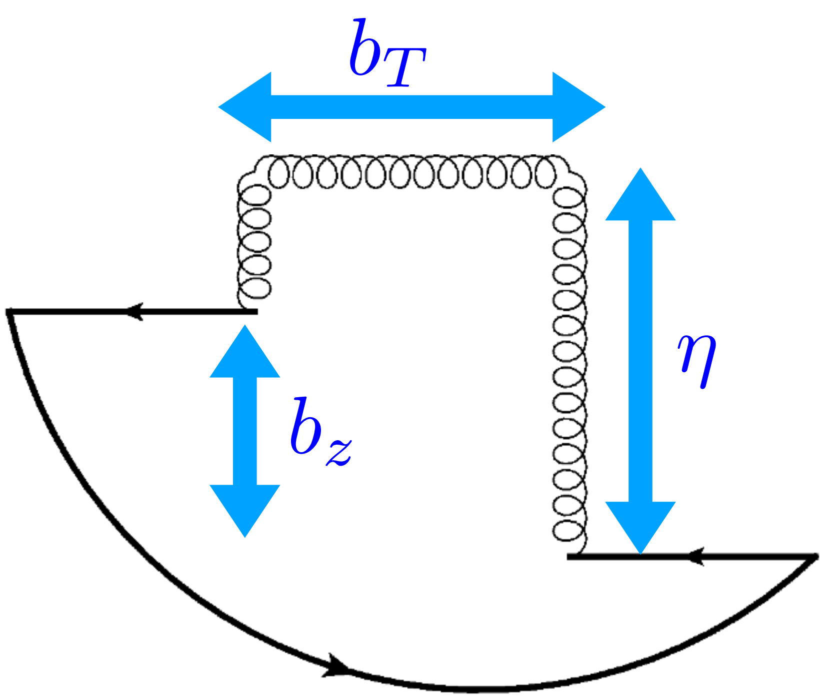
$a = 0.12\ \mathrm{fm} \qquad L = 48a = 5.6\ \mathrm{fm}$

Bazavov et al [MILC] PRD 87 (2013)

Wilson valence quarks with tree-level clover improvement,
Wilson flow $t = 1.0$ used as smearing in valence action

$m_\pi = 538(1)\ \mathrm{MeV}$

n_z	P^z [GeV]	η/a	n_{src}	n_{cfg}
3	0.65	{12,14}	4	96
3	0.65	23	16	100
5	1.1	{12,14}	4	449
7	1.5	{12,14}	16	596



Larger volumes enable larger staple extents than in quenched calculation

$$\eta \leq 1.7\ \mathrm{fm}$$

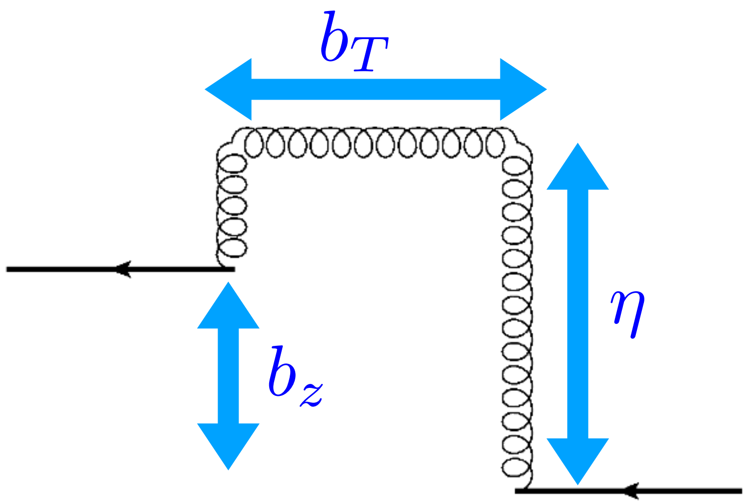
Shanahan, MW, Zhao, PRD 104 (2021)

$$\eta P^z_{\mathrm{max}} = 14.5 \quad \text{vs quenched} \quad \eta P^z_{\mathrm{max}} = 11.0$$

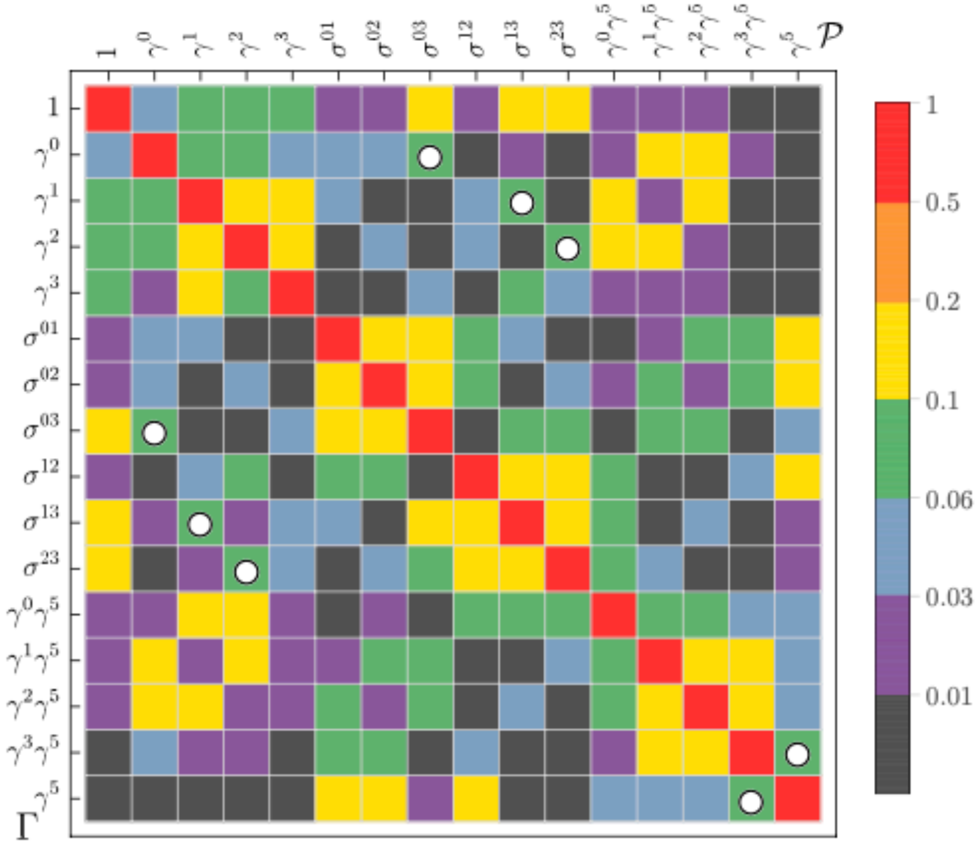
Renormalization and mixing

Nonlocal quark bilinear operators with stapled shaped Wilson lines renormalized using high-momentum quark vertex function (RI/MOM scheme)

$$\mathcal{O}^q_\Gamma(b^\mu,\eta) = \overline{q}(b^\mu)\frac{\Gamma}{2}W_{\hat{z}}(b^\mu;\eta-b^z)$$
$$\times W_T^\dagger(\eta\hat{z};b_T)W_{\hat{z}}^\dagger(0;\eta)q(0)$$



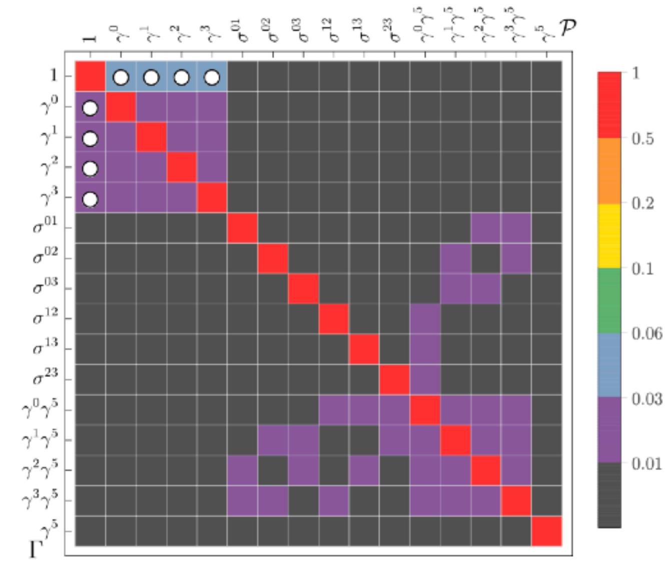
NLO (one-loop) matching used to convert to $\overline{\mathrm{MS}}$ scheme Ebert, Stewart, Zhao, JHEP 099 (2020)



Shanahan, MW, Zhao, PRD 101 (2020)

Nonperturbative operator mixing significant for operators with large staples

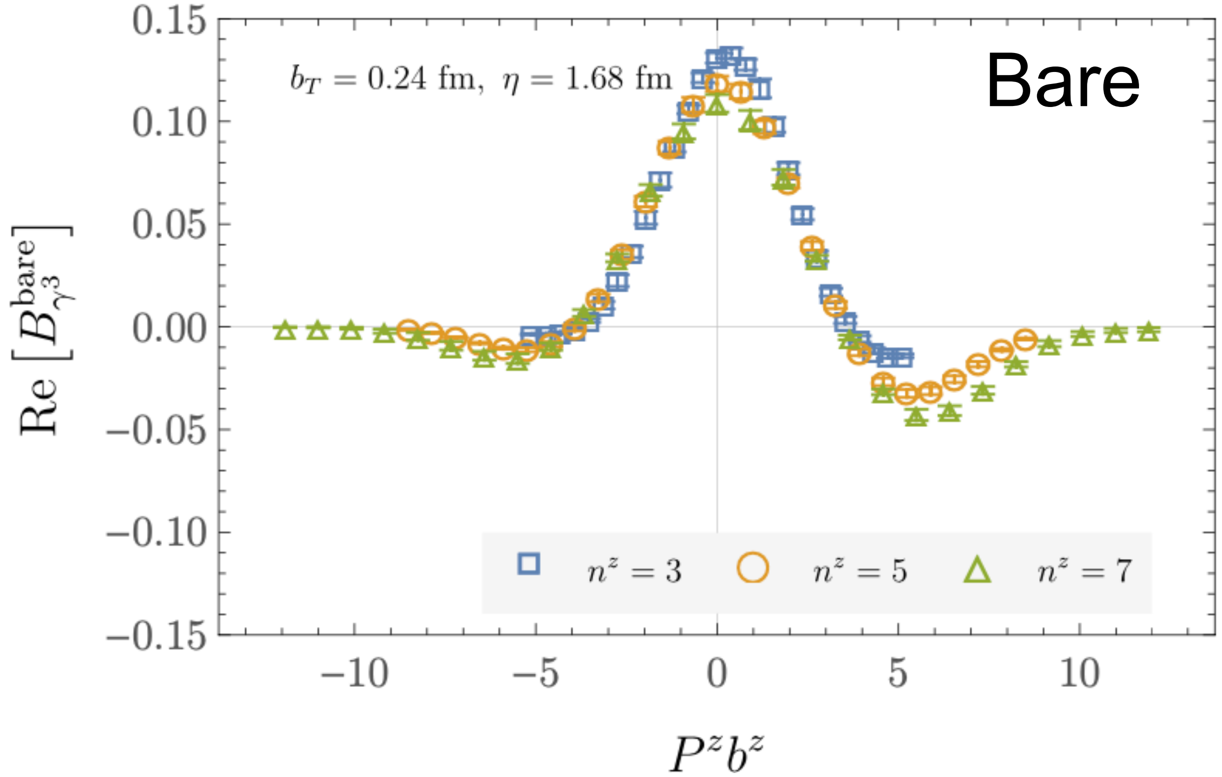
Much smaller mixing observed e.g. for local quark bilinears:

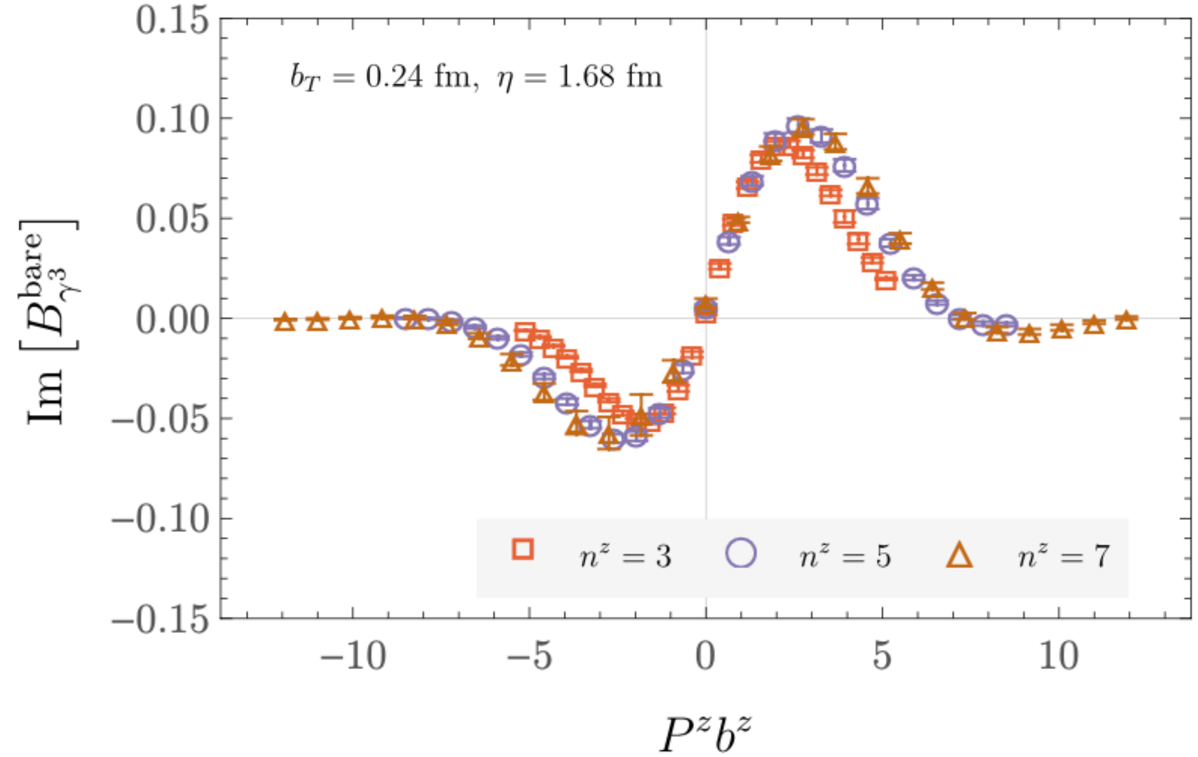


RI/MOM assumes nonlocal operators renormalize analogously to local operators

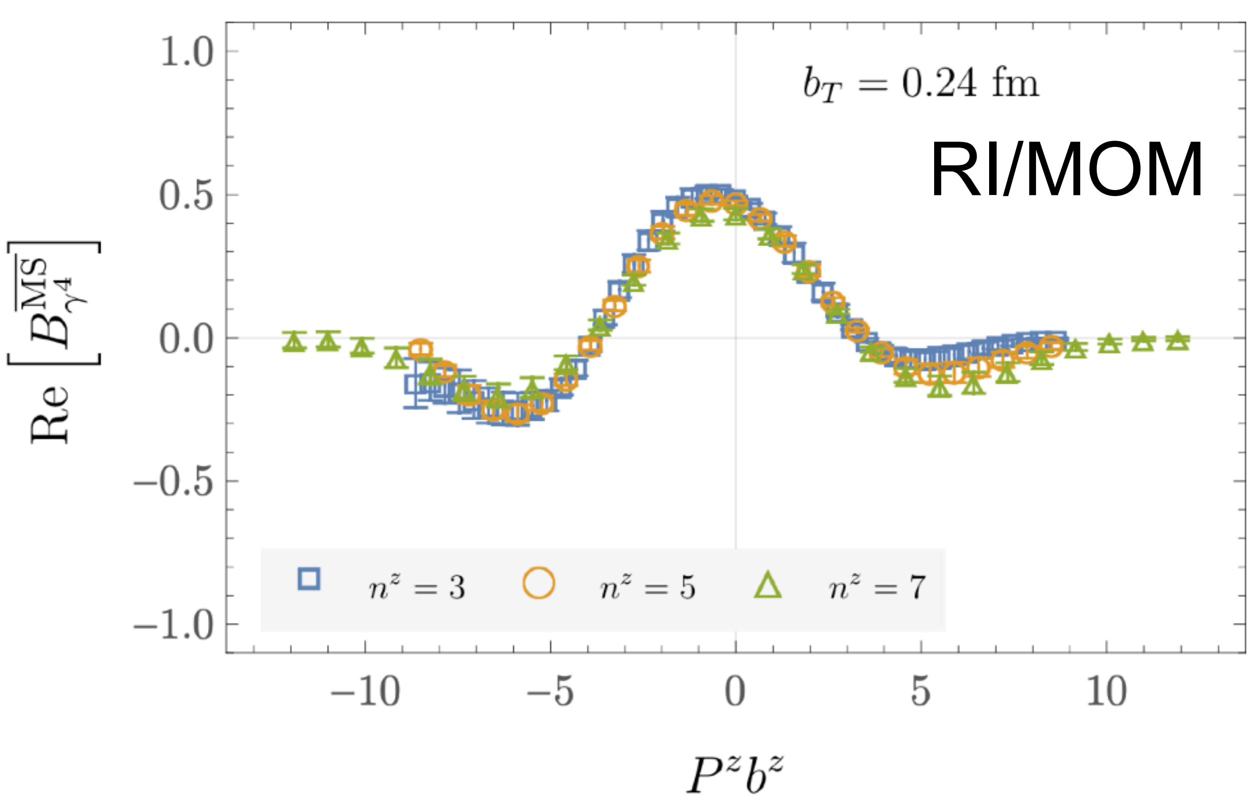
Problematic? Zhang et al [χQCD], PRD 104 (2021) Huo et al [LPC], Nucl. Phys. B 969 (2021)

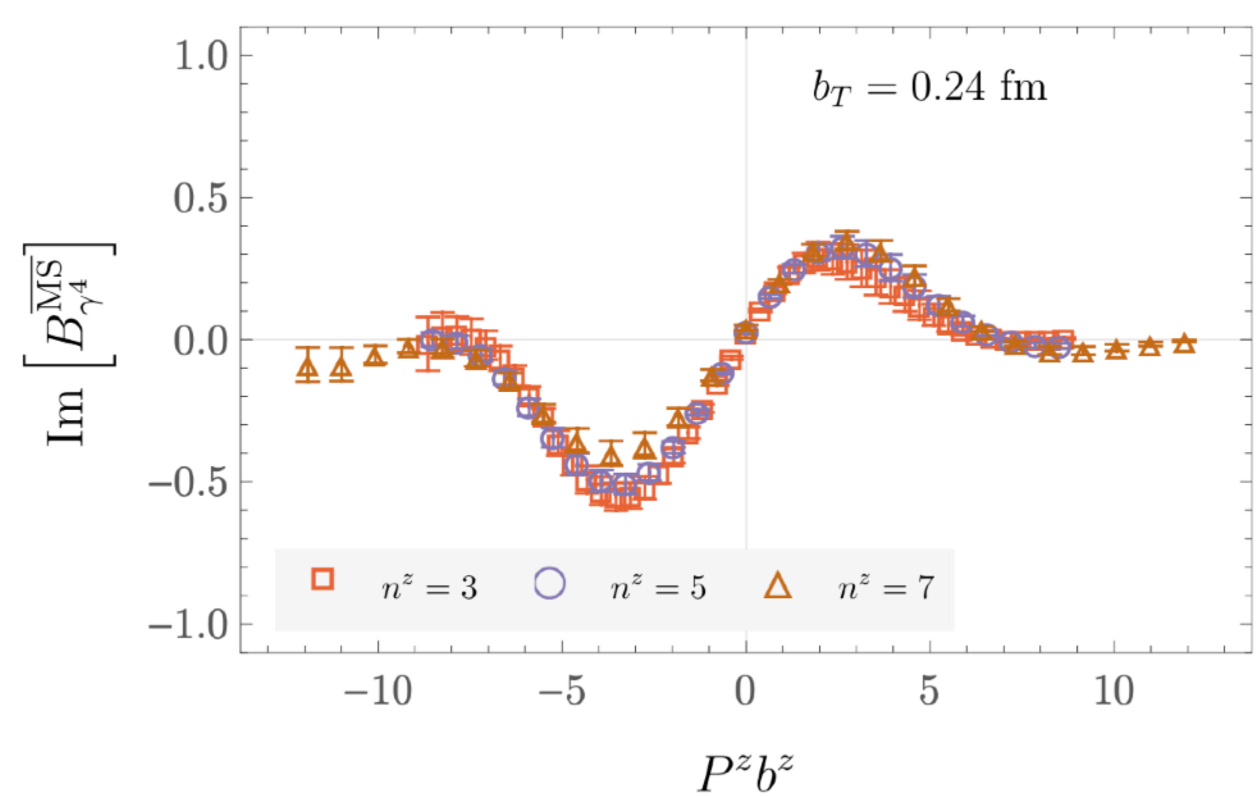
Trouble with RI/MOM





Asymmetry visible in beam functions for large volume after RI/MOM renormalization

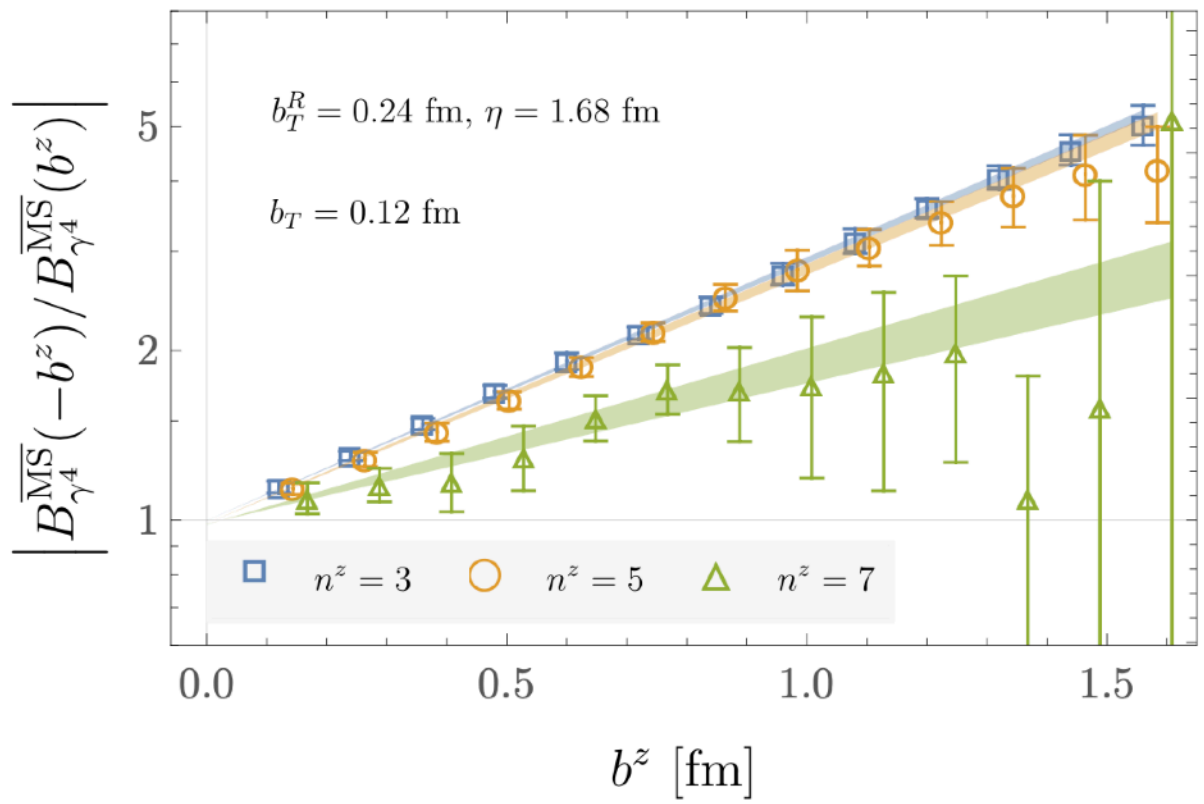




Shanahan, MW, Zhao, PRD 104 (2021)

Beam function asymmetry

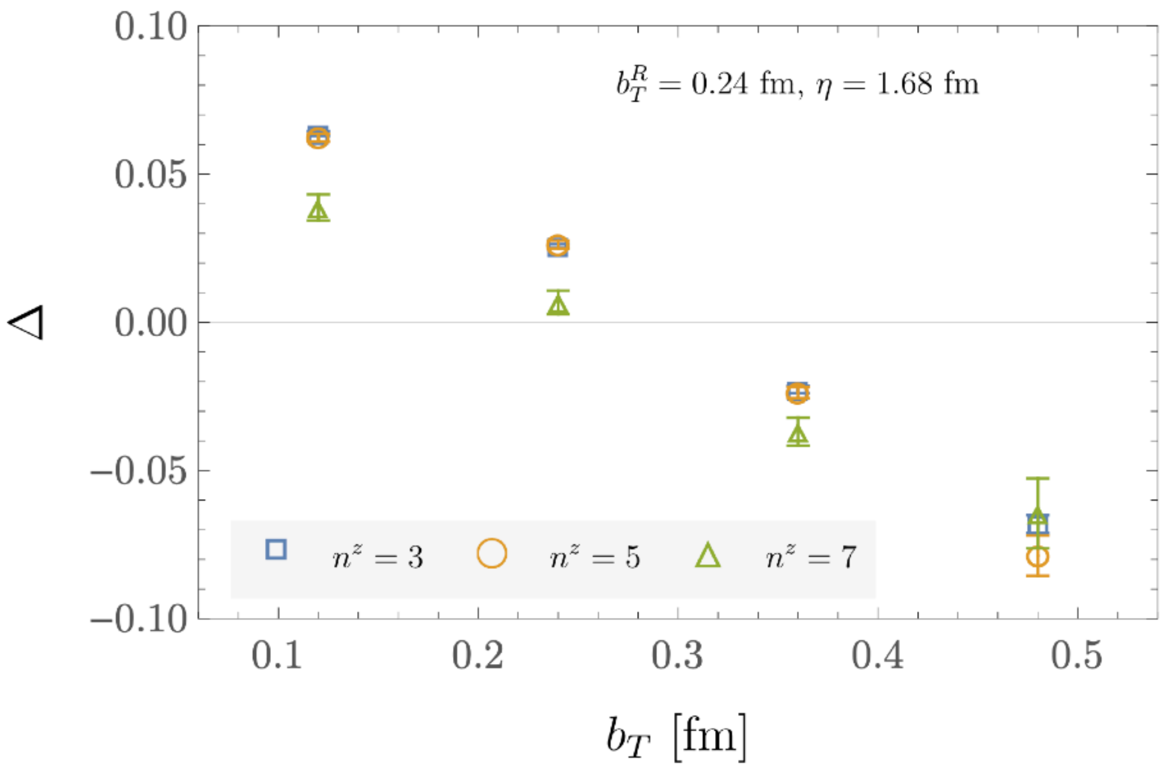
Asymmetry visible after RI/MOM renormalization could arise from state-dependence of static quark potential



Unexpected state dependence seen in previous calculations

Zhang et al [χQCD], PRD 104 (2021)

Huo et al [LPC], Nucl. Phys. B 969 (2021)



Shanahan, MW, Zhao, PRD 104 (2021)

Correction for difference in static quark potentials applied

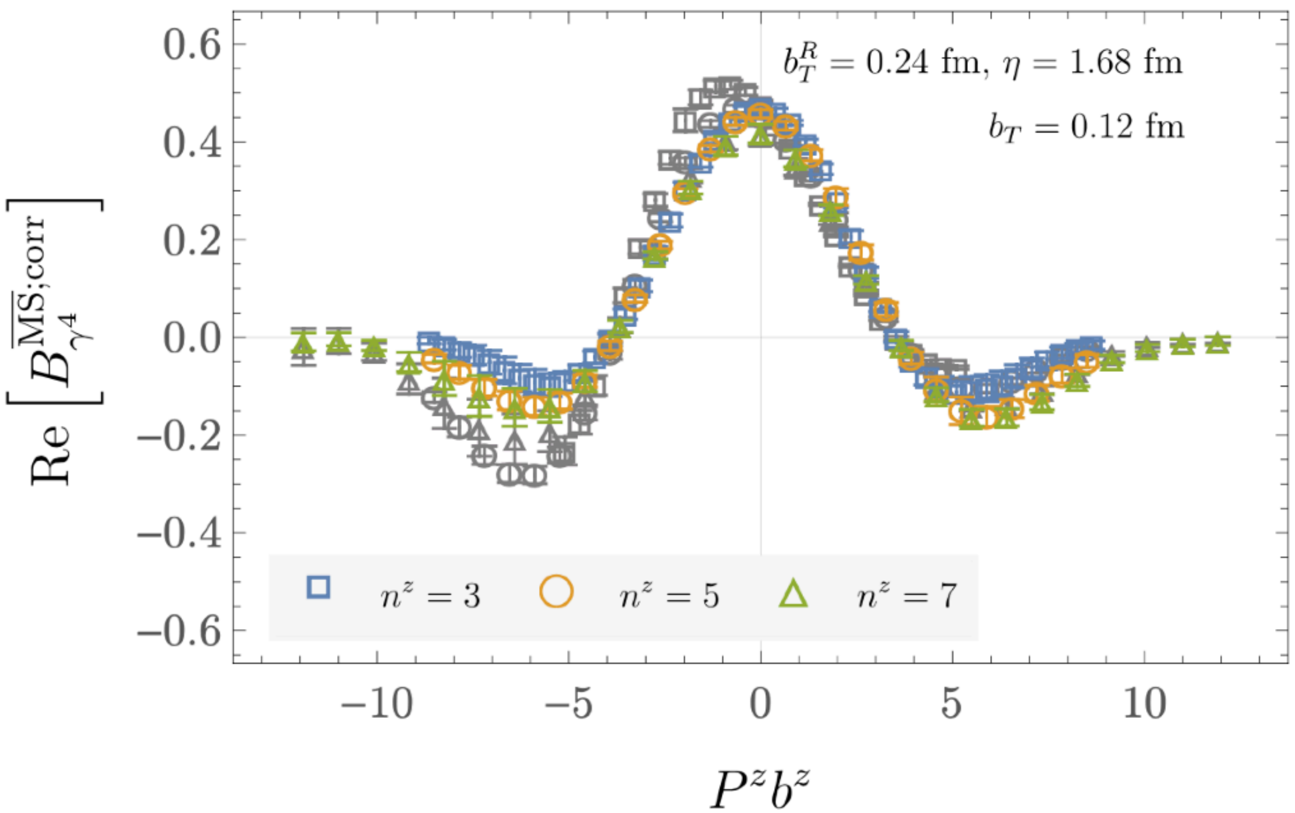
$$B_{\gamma_4}^{\overline{\mathrm{MS}};\mathrm{corr}}(b^z, b_T) = e^{\Delta(b_T)|b^z|} B_{\gamma_4}^{\overline{\mathrm{MS}}}(b^z, b_T)$$

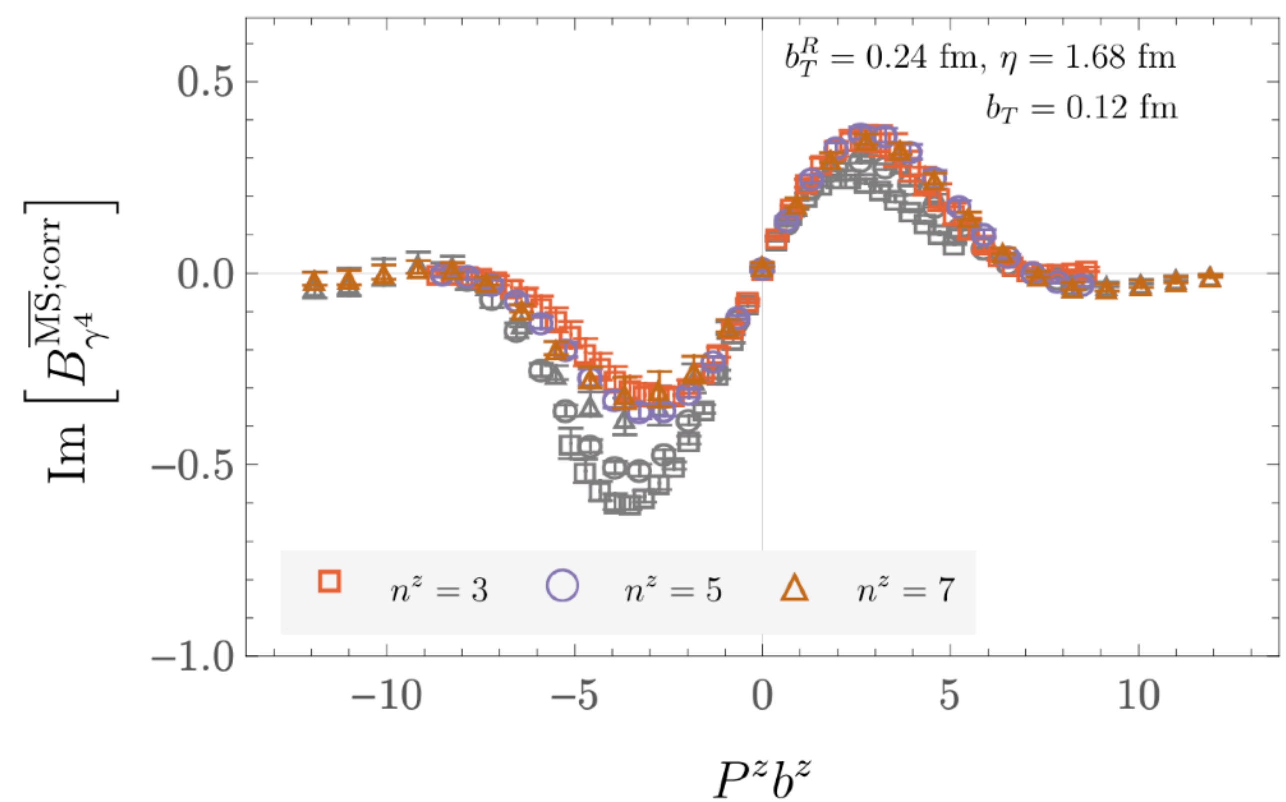
Roughly linear trend in b_T observed

$$\Delta(b_T) = V(b_T)_{\mathrm{quark}} - V(b_T)_{\mathrm{pion}} \sim \sigma\, b_T$$

Asymmetry correction

After correcting for state dependence of static quark potential, expected (anti)symmetrization of beam function emerges



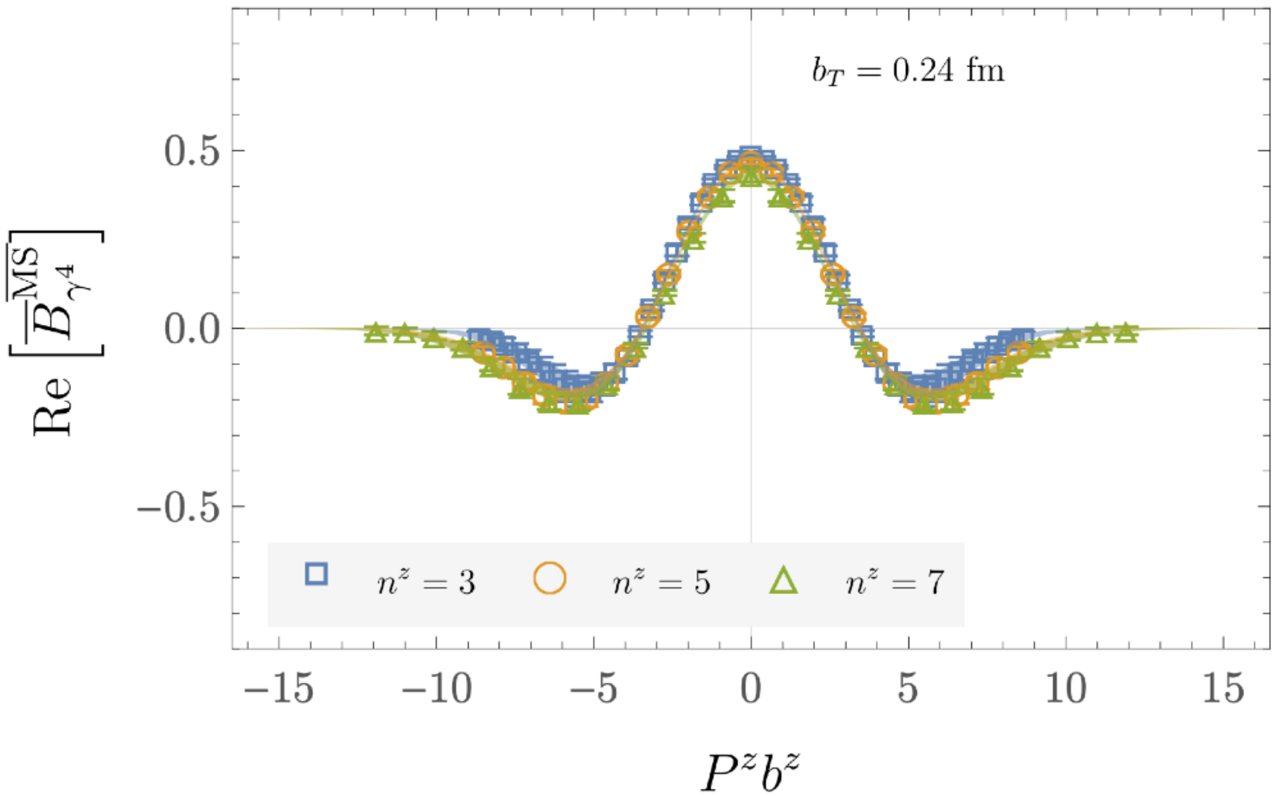


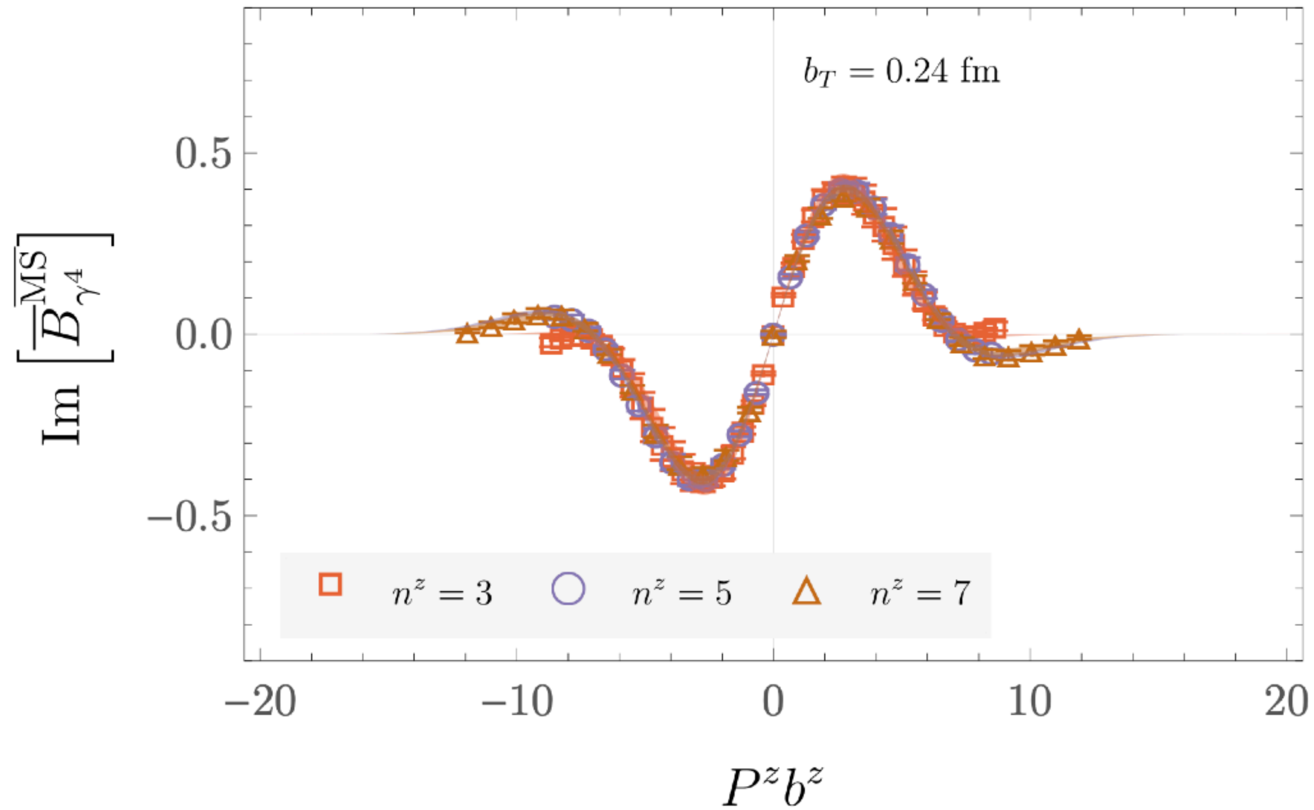
Extrapolation to large η (by a constant) and averaging over choice of b_T^R used in renormalization performed after correcting for asymmetry

Shanahan, MW, Zhao, PRD 104 (2021)

Large-distance extrapolation

Fits performed independently for each $b_T,\ P^z$ to analytic model in order to extrapolate to larger b^z



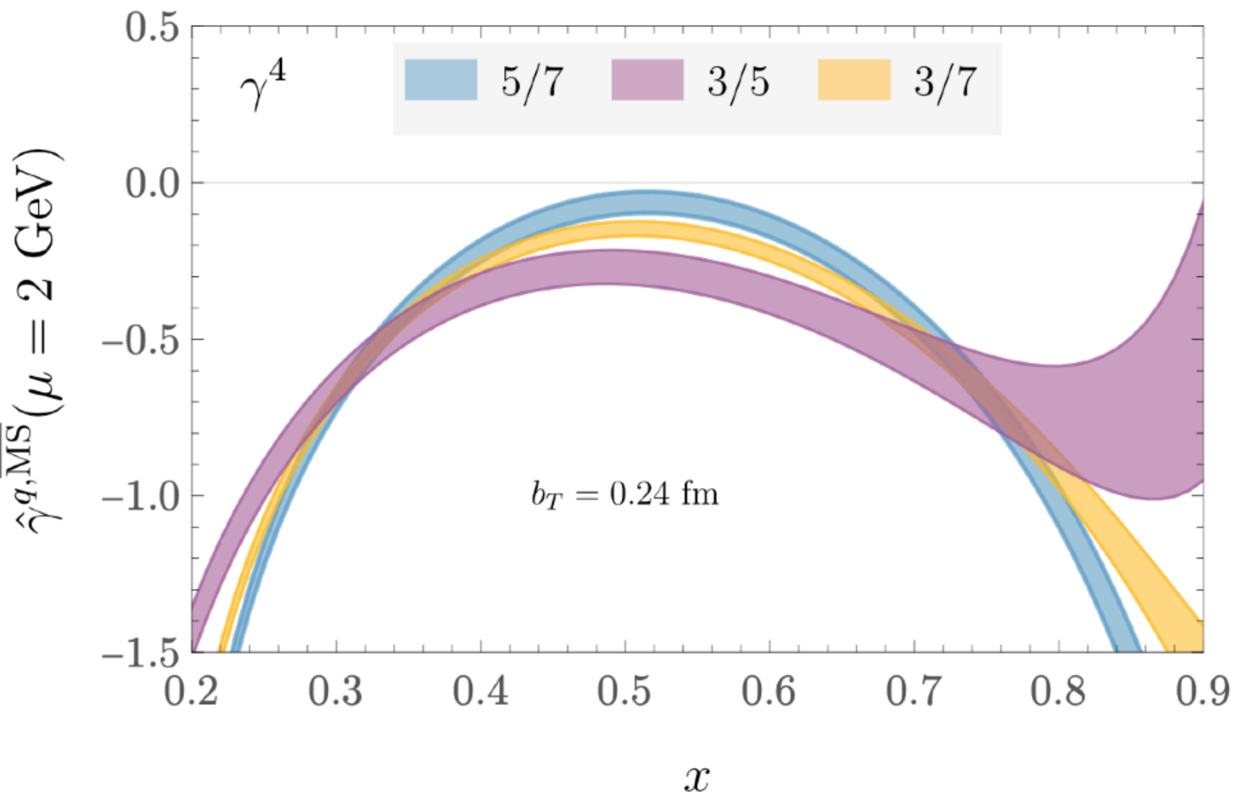


Untruncated Fourier transformations performed analytically after fitting

Improvement from quenched calculation: modeling in coordinate space instead of momentum space permits x dependent extraction of CS kernel and NLO quasi/light-cone matching

Shanahan, MW, Zhao, PRD 104 (2021)

CS kernel systematics



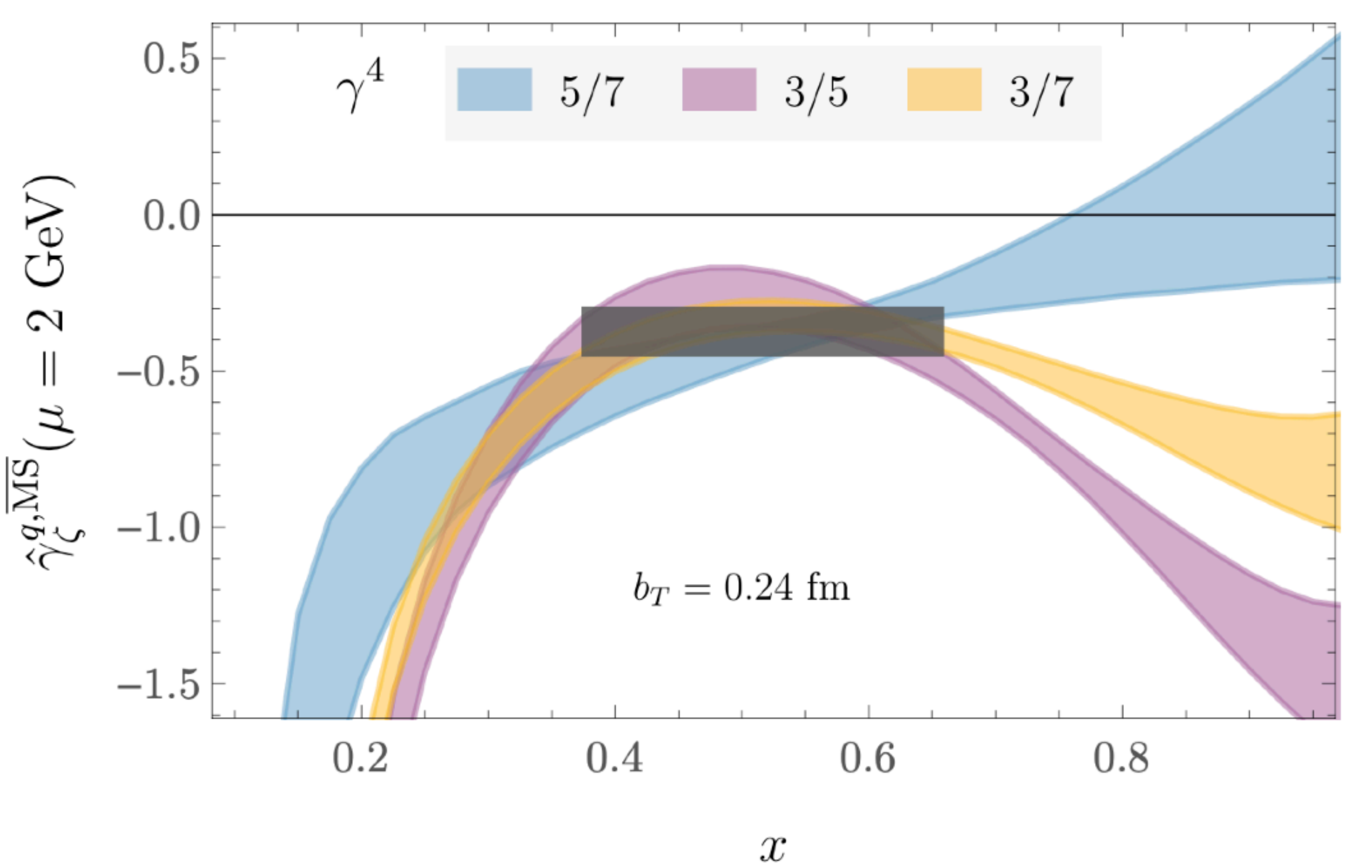
Discrete Fourier transform leads to significant x dependence of (asymptotically flat) CS kernel estimate

Differences between estimates with different momentum pairs visible

Fourier transforming the analytically extrapolated model leads to smaller (though still visible) x and P^z dependence

"Plateau region" identified by automated search for overlap between different P^z pairs

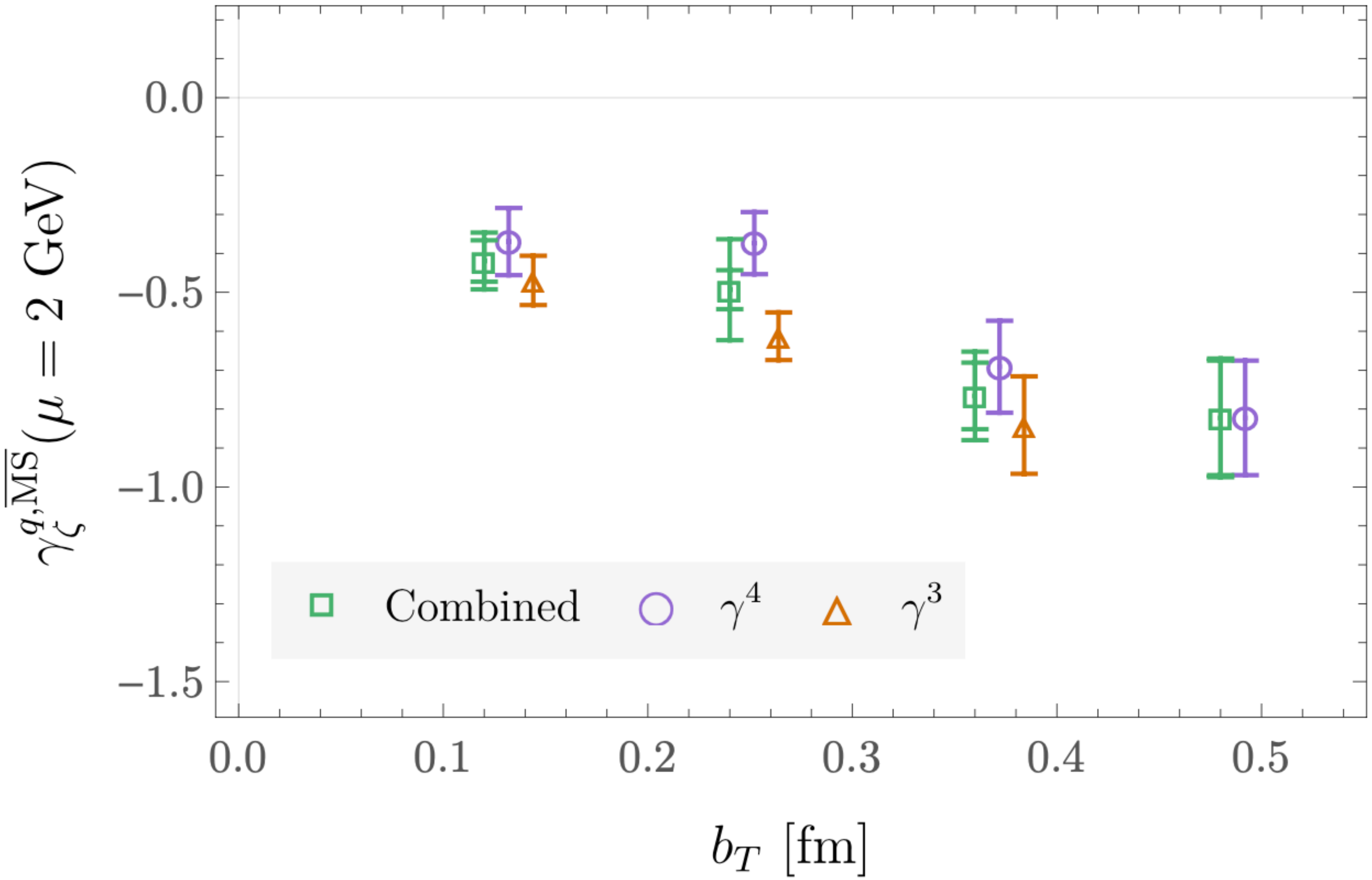
Fits of $1/P^z$ artifacts also attempted



Shanahan, MW, Zhao, PRD 104 (2021)

CS kernel results

Plateau regions of x-dependent CS kernel used to give final results and (bootstrap confidence interval) uncertainties

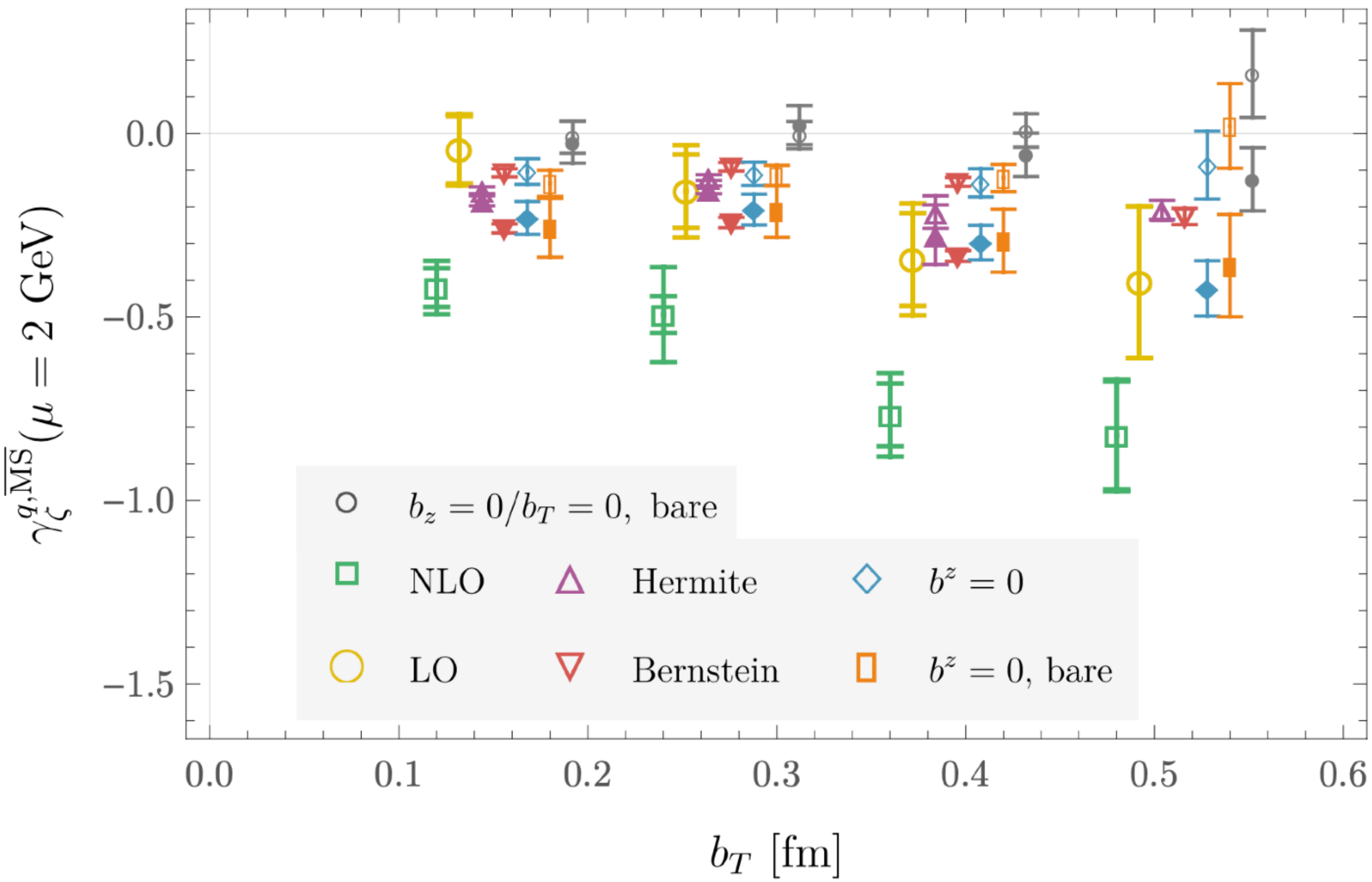


Results computed using both γ^4 and γ^3, difference used to estimate systematics due to finite P^z

Shanahan, MW, Zhao, PRD 104 (2021)

Comparing approximations

NLO matching leads to significant effects on CS kernel determination

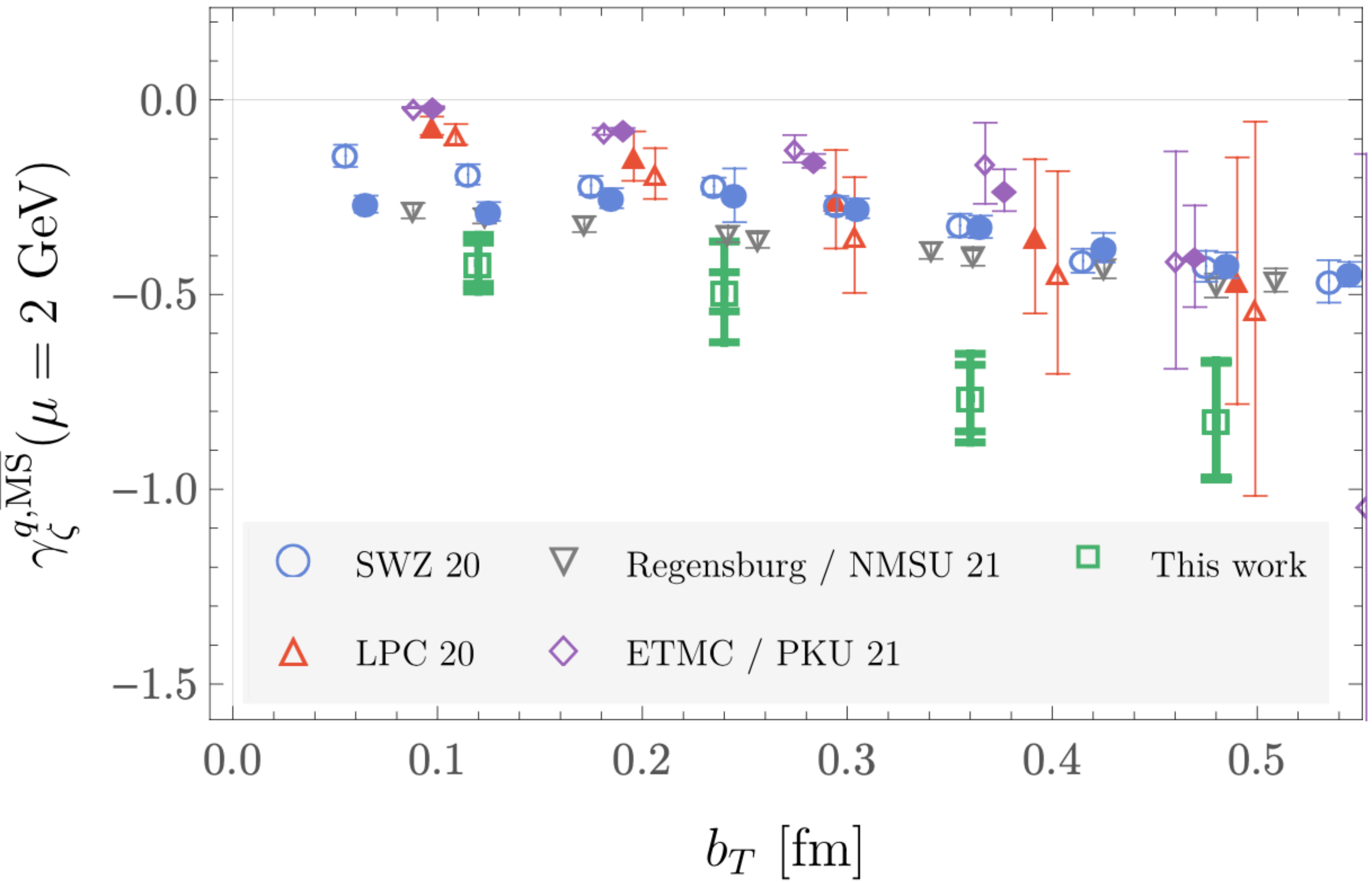


LO results using ratios of $b^z = 0$ beam functions or the momentum-space models used in quenched calculation are consistent with LO results using average over x dependence but give smaller uncertainty estimates

Shanahan, MW, Zhao, PRD 104 (2021)

Lattice comparison

Results are broadly consistent with other LQCD calculations (different actions and systematics)



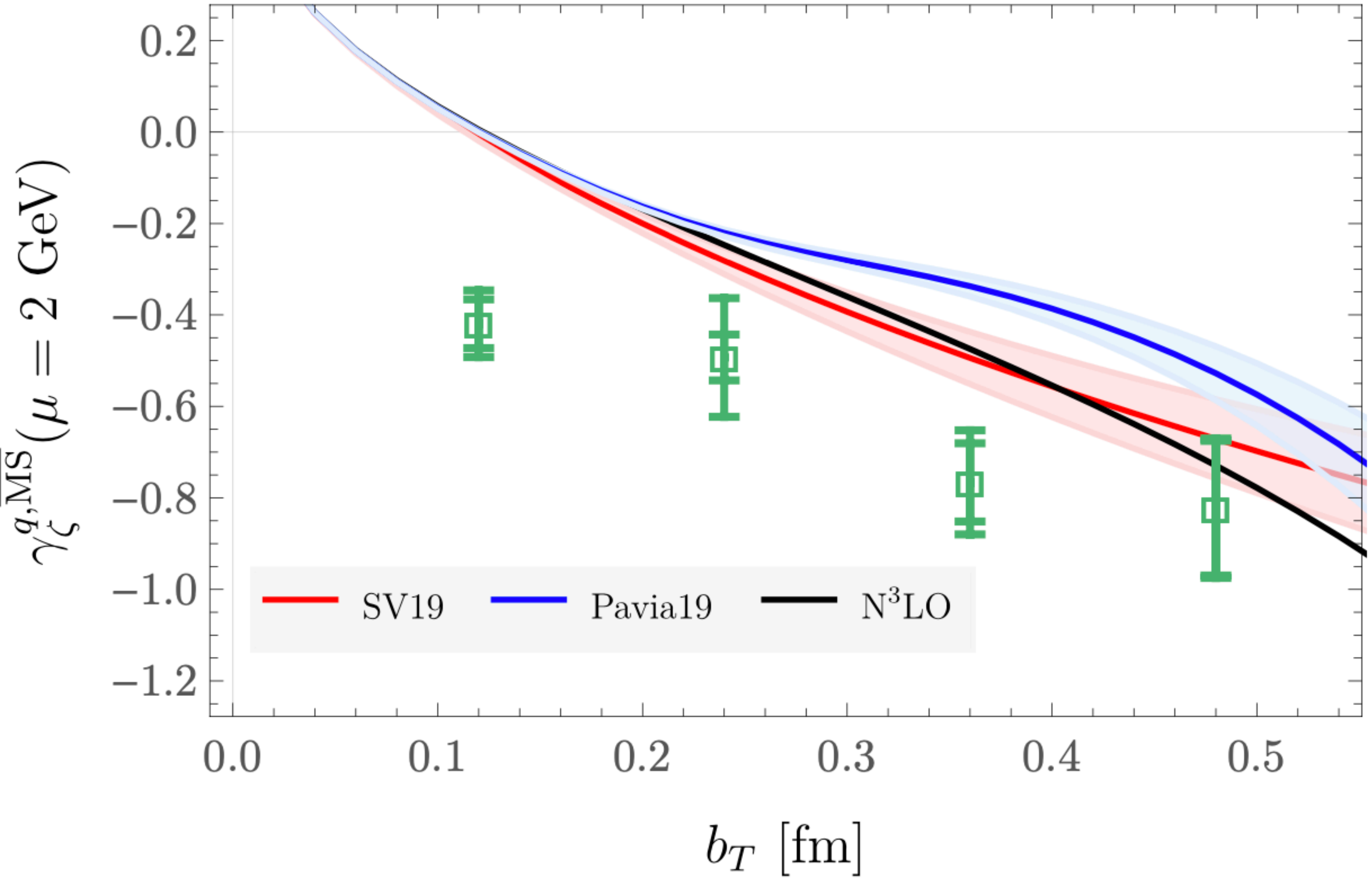
Differences with previous LO calculations (SWZ 20, LPC 20, ETMC / PKU 21) consistent with differences between Fourier transform schemes

Shanahan, MW, Zhao, PRD 104 (2021)

Phenomenological comparison

Results can be compared with phenomenology

Warning - no continuum extrapolation, unquantified systematics remain



Lattice artifacts at small b_T ? Underestimated Fourier transform systematics? Further studies needed!

Shanahan, MW, Zhao, PRD 104 (2021)

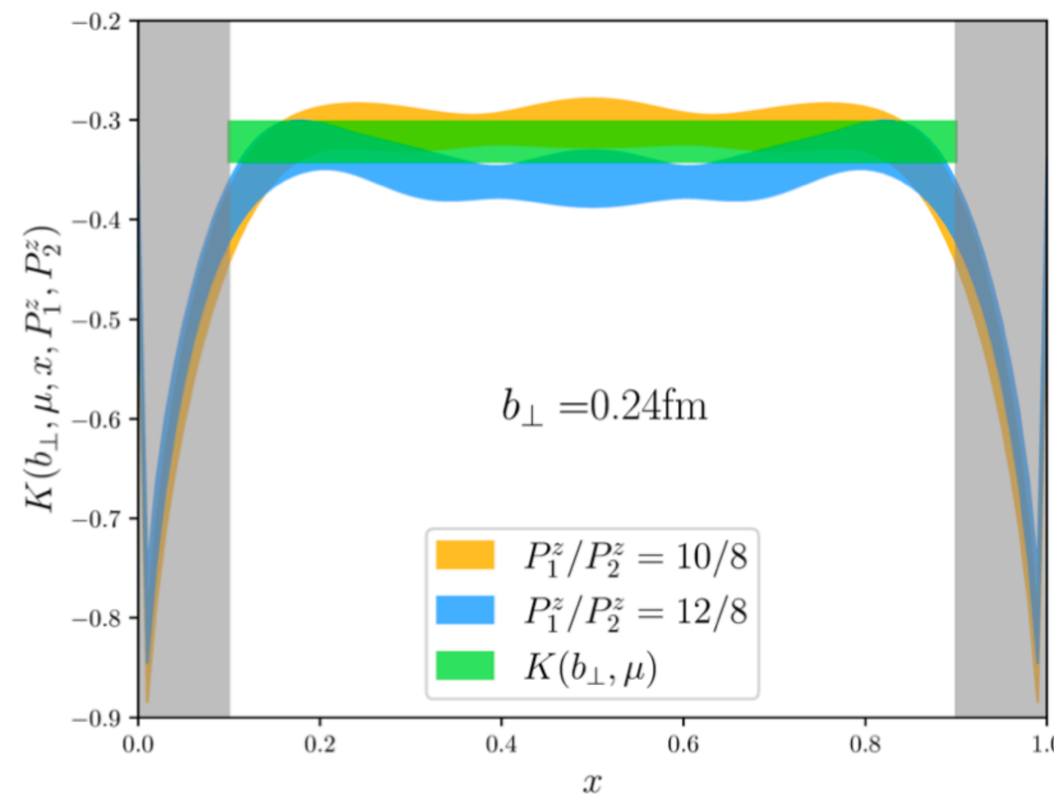
Towards fully controlled systematics

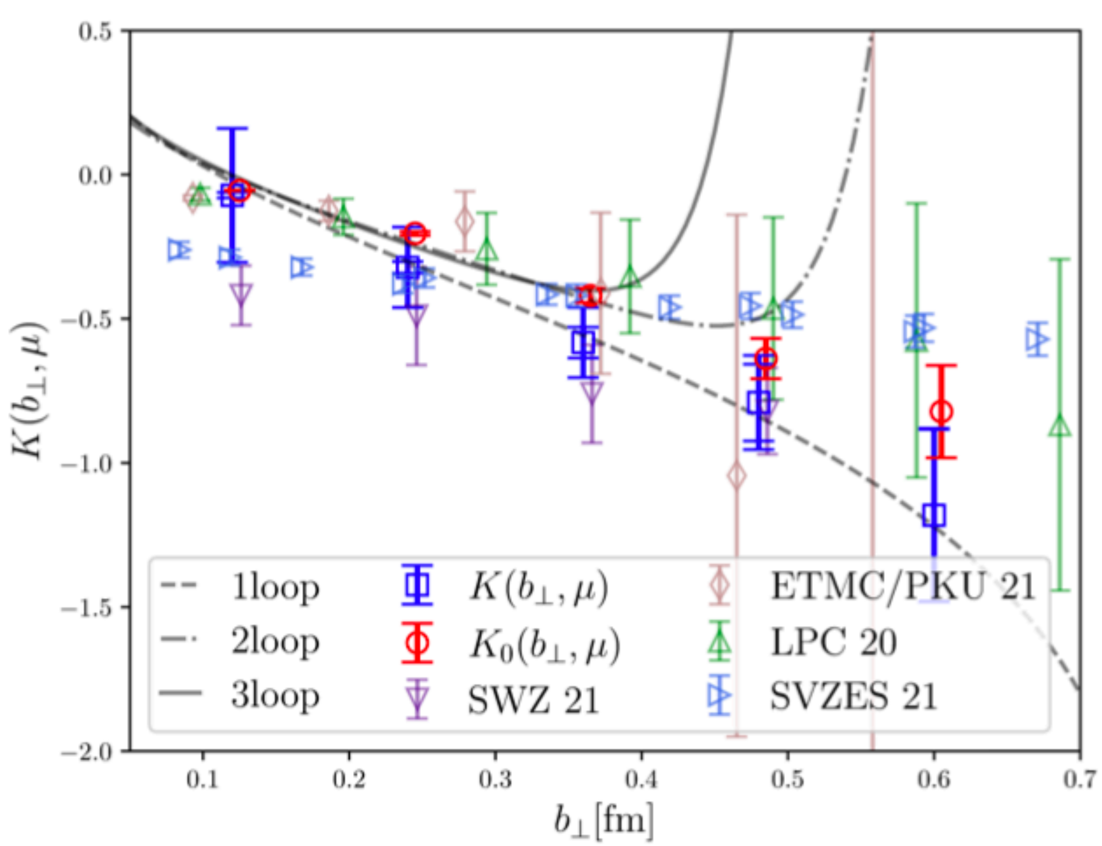
The CS kernel can also be extracted from ratios of TMD wavefunctions analogous to distribution amplitudes

$$\tilde{\psi}(b^z, b_T, \eta, P^z) \propto \langle 0|\mathcal{O}(b^z, b_T, \eta)|\pi(P^z)\rangle$$

Recent calculations by LPC using TMD wavefunctions achieve larger $b \cdot P$, smaller Fourier transform systematics

Chu et al [LPC], arXiv:2204.00200





Results consistent with previous determinations using TMDPDF beam functions

Chu et al [LPC], arXiv:2204.00200

TMD Wavefunctions

TMD WFs built from pion-to-vacuum amplitudes with staple-shaped operator sinks

$$B_q^{\pm}(x,\vec{b}_T,\epsilon,y_P-y_B) = \frac{1}{\bar{n}\cdot P}\int \frac{\mathrm{d}(P\cdot b)}{2\pi} e^{\mathrm{i}(1-x)P\cdot b}\left\langle 0\middle|\bar{d}(b^\mu)\frac{\not{\bar{n}}\gamma_5}{2}W_{\sqsubset}(b^\mu,0;n_B(y_B),\pm\infty)u(0)\middle|\pi^+(P)\right\rangle$$

UV renormalization factors and soft function needed to construct full TMD WF

$$\phi_q^{\pm}(x,\vec{b}_T,\mu,\zeta) = \lim_{\epsilon\to 0} Z_{\mathrm{uv}}^q(\mu,\zeta,\epsilon) \lim_{y_B\to-\infty} \frac{B_q^{\pm}(x,\vec{b}_T,\epsilon,y_n-y_B)}{\sqrt{S^q[b_\perp,\epsilon,\pm\infty n_A(2y_n),\pm\infty n_B(2y_B)]}}$$

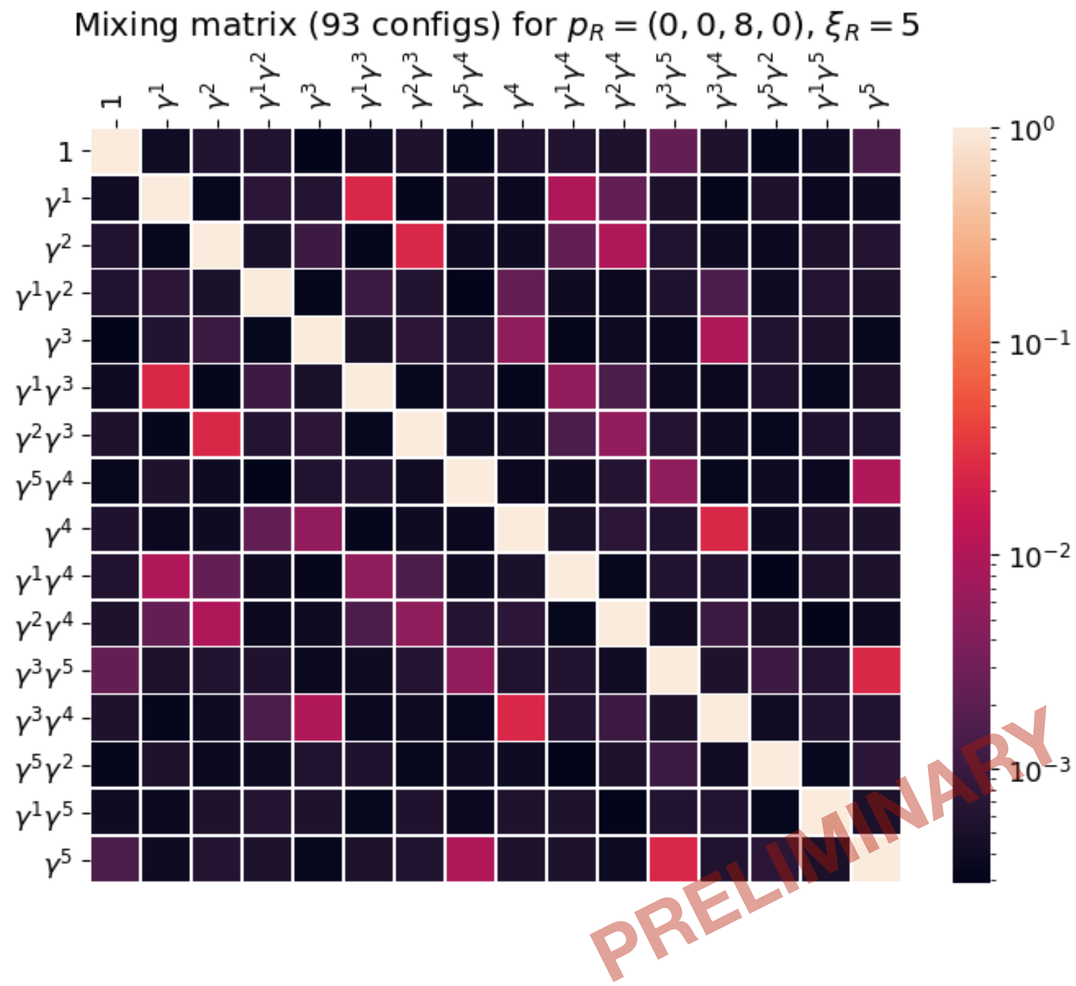
Ratios of TMD WFs can be used to determine CS kernel

$$\gamma_\zeta^q(b_T,\mu) = \frac{1}{\ln P_1^z/\ln P_2^z}\ln\frac{H^{\pm}(\mu,x,P_2^z)\tilde{\phi}_\pi^{\pm}(x,b_T,\mu,P_2^z)}{H^{\pm}(\mu,x,P_1^z)\tilde{\phi}_\pi^{\pm}(x,b_T,\mu,P_1^z)}$$

Zhang et al [LPC], PRL 125 (2020) Chu et al [LPC], arXiv:2204.00200

Renormalization factors cancel assuming no operator mixing

Mixing and RI-xMOM



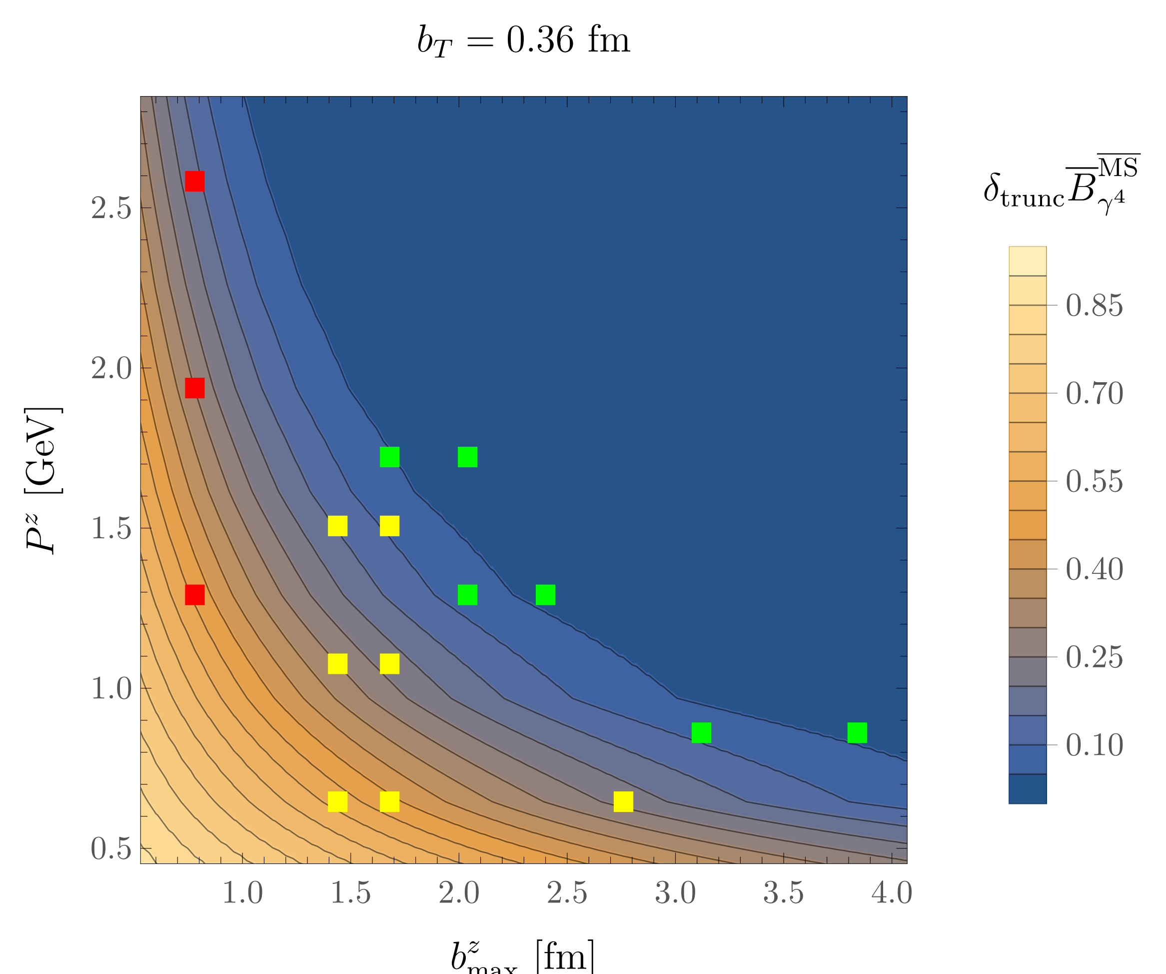
Operator mixing can be incorporated into the RI-xMOM scheme introduced in

Green et al, PRL 121 (2018)

RI-xMOM uses auxiliary fields to reduce nonlocal operator renormalization to local operator renormalization (in extended theory)

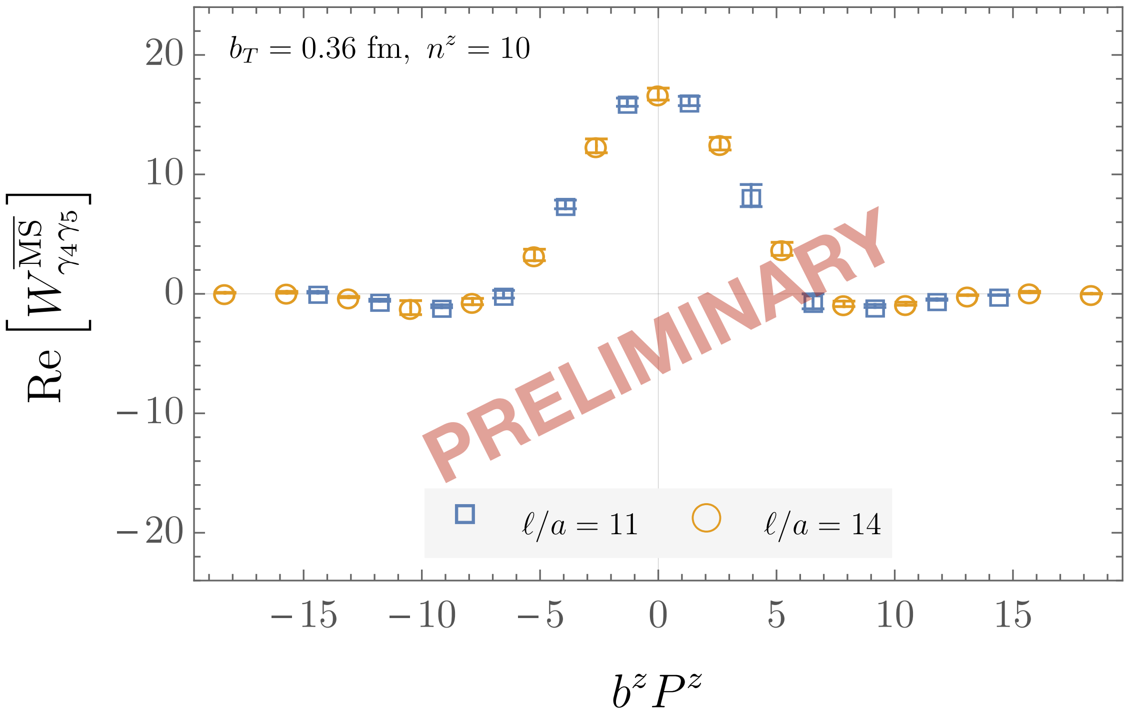
Expected mixing patterns seen in nonperturbative LQCD calculations

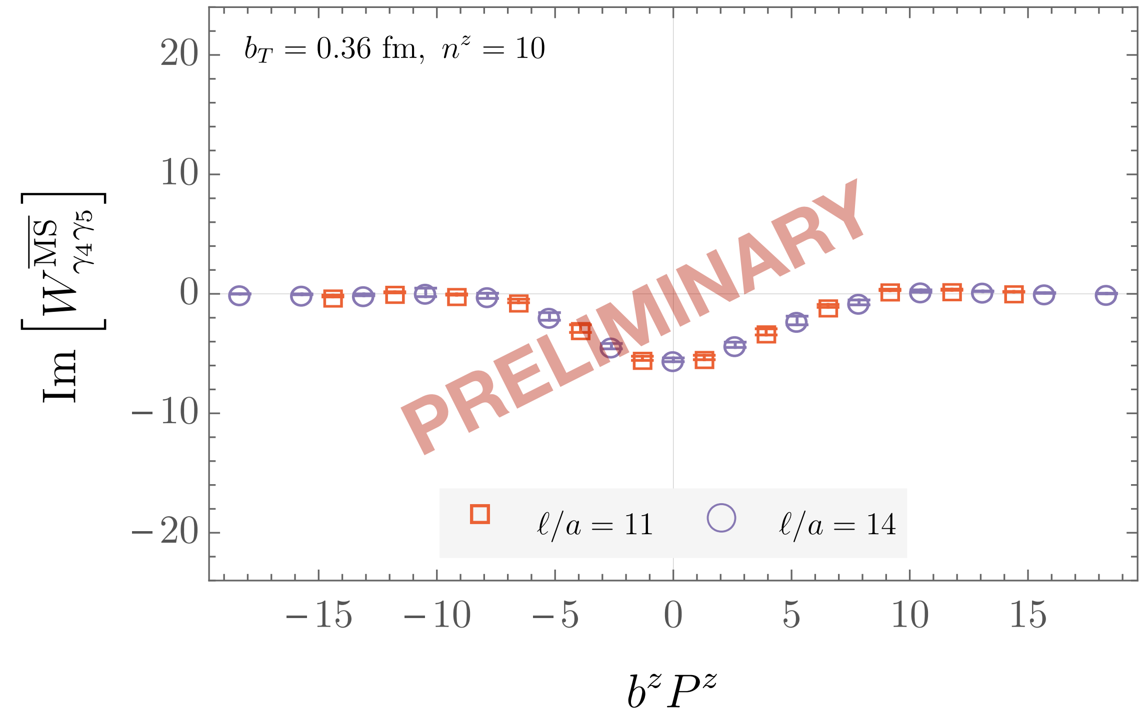
Calculations underway including larger boosts * separations to reduce Fourier transform systematics



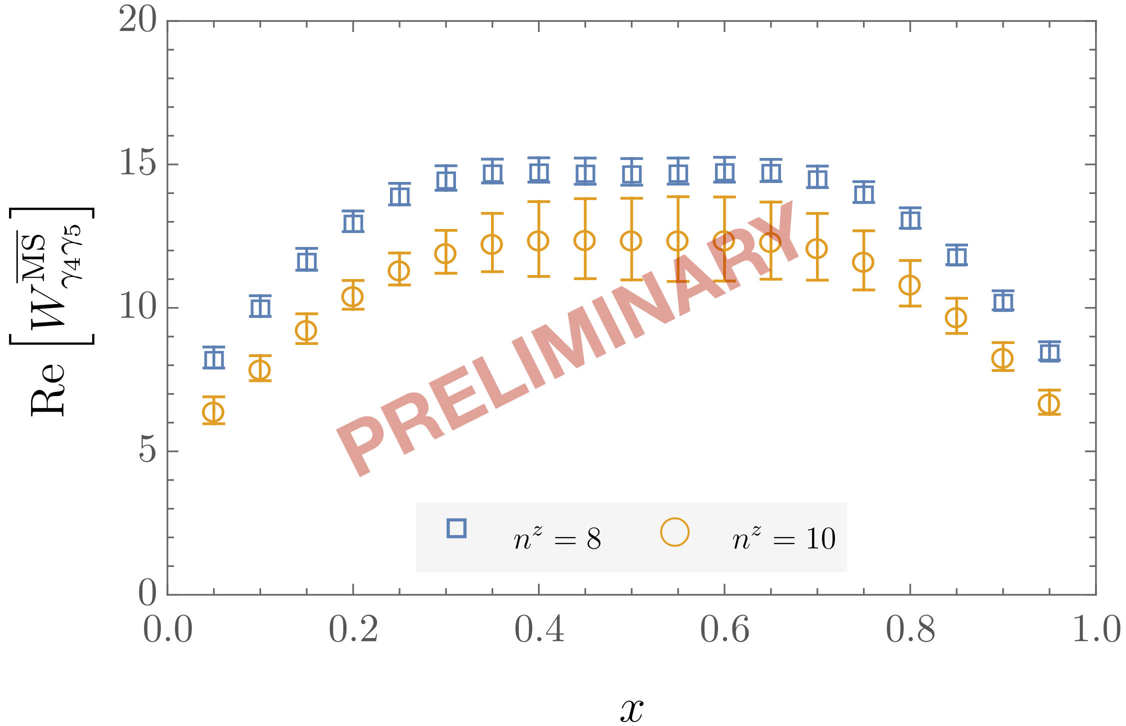
TMD Wavefunction results

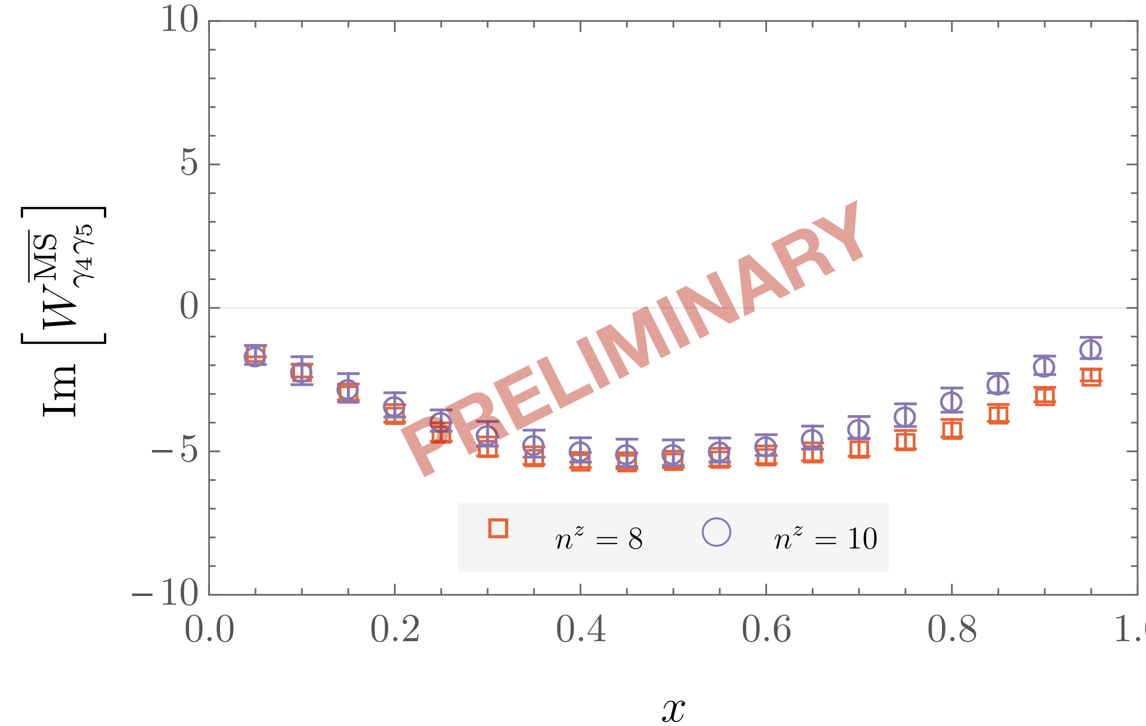
Signals obtained over large enough regions to control Fourier transform





Fourier transforms complex but show expected phase-independence of ratios

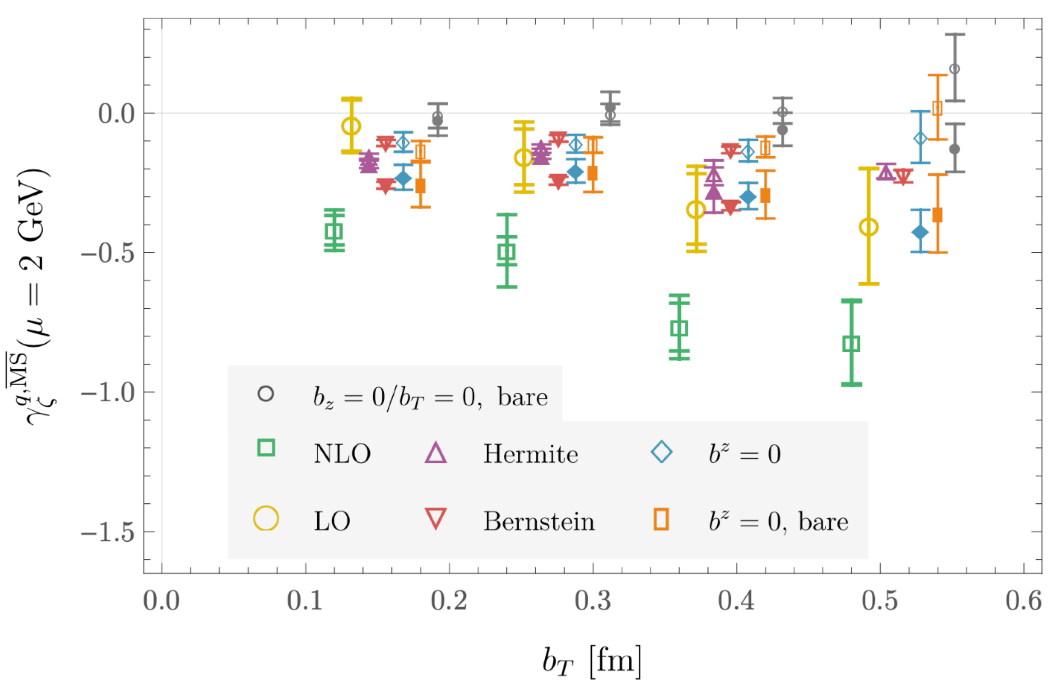




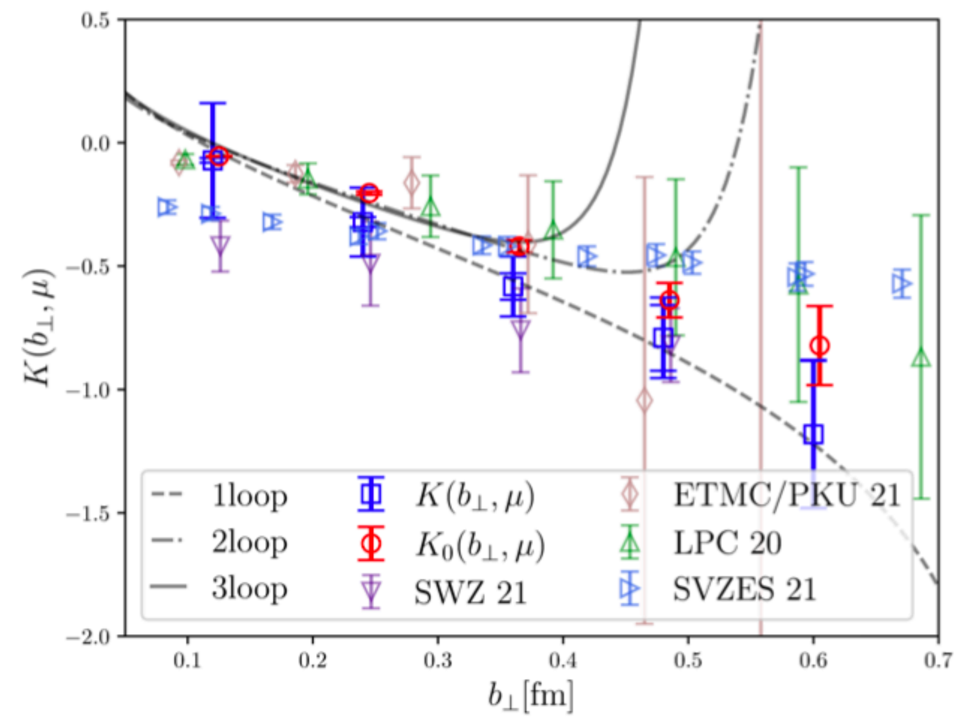
Outlook

Nonperturbative QCD input is required to determine the Collins-Soper kernel governing TMDPDF evolution and improve precision of SIDIS and Drell-Yan predictions / TMDPDF extractions

Recent LQCD results by several groups demonstrate increasing control over challenging systematics in CS kernel calculations



Shanahan, MW, Zhao, PRD 104 (2021)



Chu et al [LPC], arXiv:2204.00200

More sophisticated treatment of renormalization (“hybrid renormalization scheme”) and Fourier transform systematics needed

— Reliable LQCD determinations of the CS kernel (and full TMDPDFs) possible