北京航空航天大學
BEIHANG UNIVERSITY

Unpolarized Transverse-Momentum-Dependent Parton Distributions from Lattice QCD: I

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December, 02 @ LaMET22

LPC Lattice Parton Collaboration

Based on *hep-lat/2211.02340*

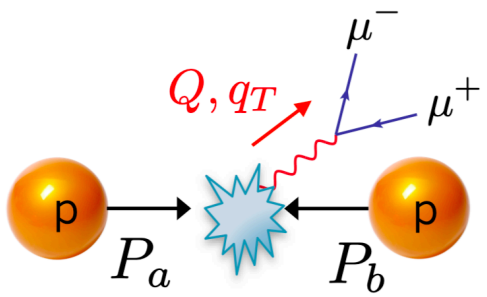
OUTLOOK

- **Motivation**
  - **What's and why TMDs?**
  - **Progress in the studies of TMDPDFs**
- **TMDs from lattice QCD**
  - **LaMET formalism**
  - **Extract TMDPDFs from LaMET**
  - **Preliminary studies: intrinsic and rapidity dependent soft function**
- **Simulating quasi TMDPDFs on a Euclidean lattice**

What's and why TMDs?

TMD processes:

Drell-Yan

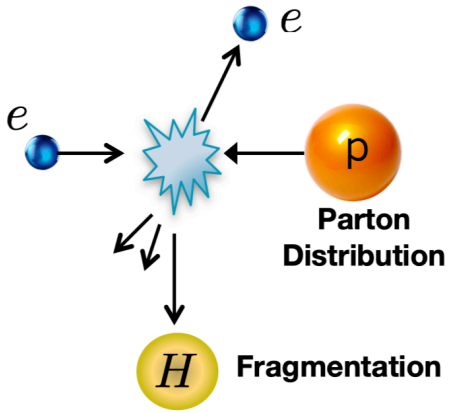


$$q_T \ll Q$$

LHC, FermiLab, RHIC, ...

$$\sigma \sim f_{q/P}(x, k_T)\, f_{q/P}(x, k_T)$$

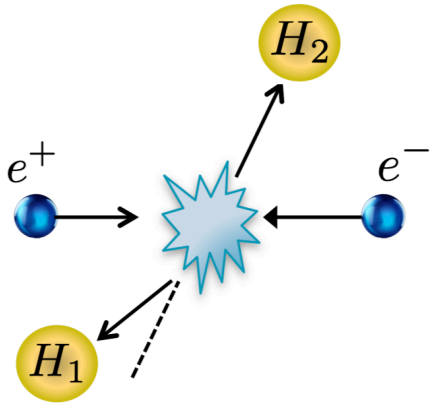
Semi-Inclusive DIS



HERMES, COMPASS, JLab, EIC, ...

$$\sigma \sim f_{q/P}(x, k_T)\, D_{h/q}(x, k_T)$$

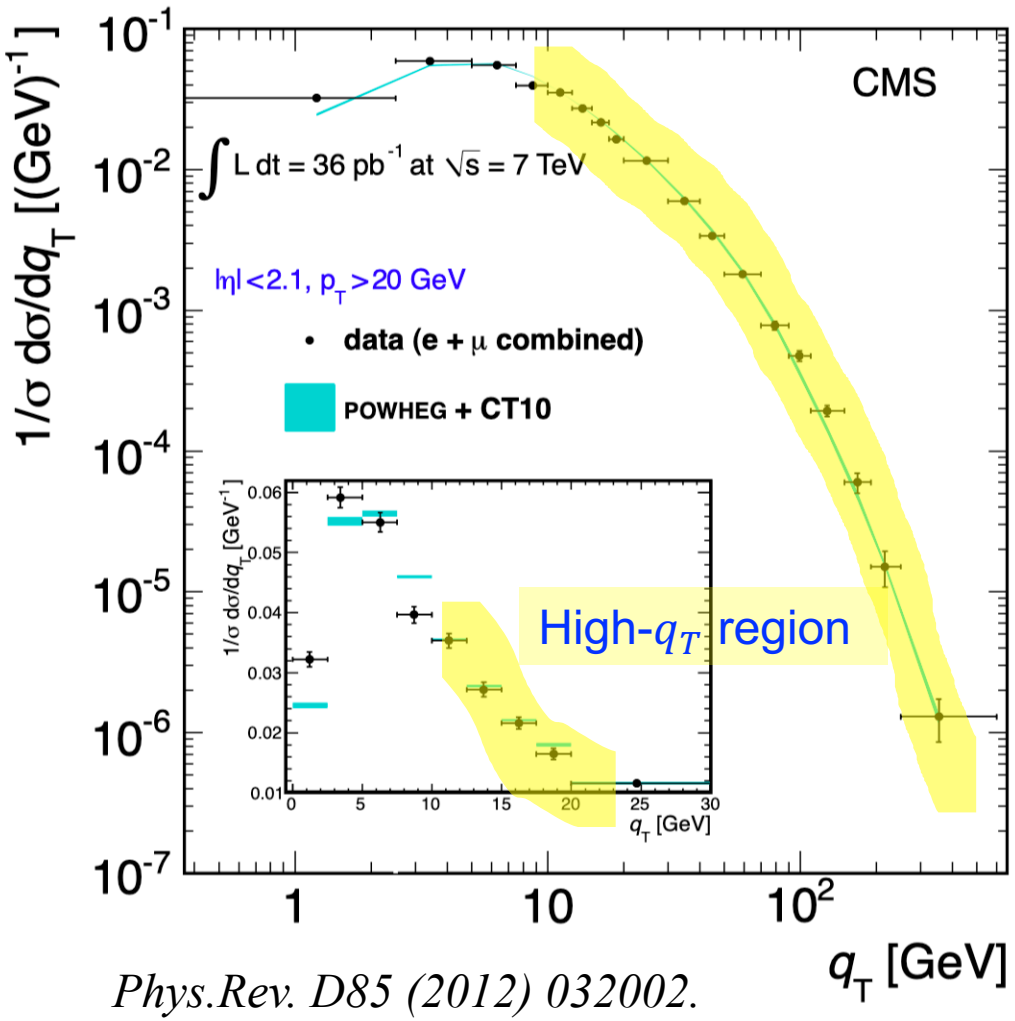
Dihadron in e^+e^-



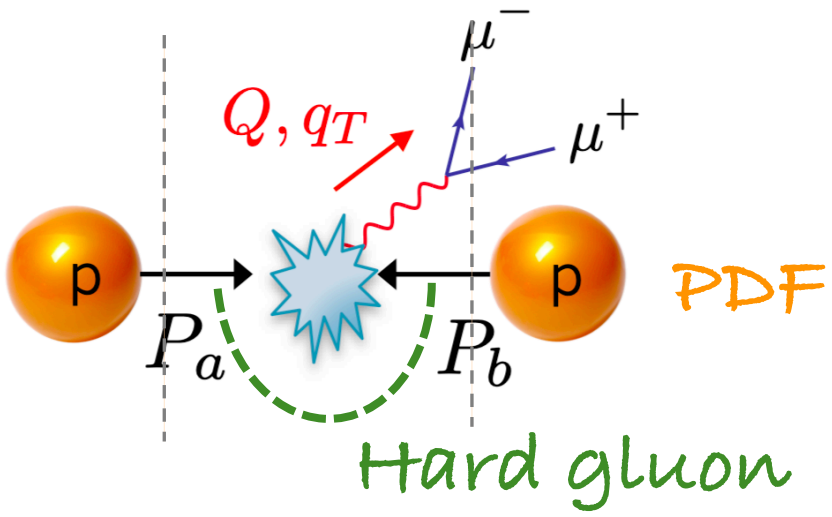
BESIII, Babar, Belle, ...

$$\sigma \sim D_{h_1/q}(x, k_T)\, D_{h_2/q}(x, k_T)$$

Z-production q_T spectrum at LHC



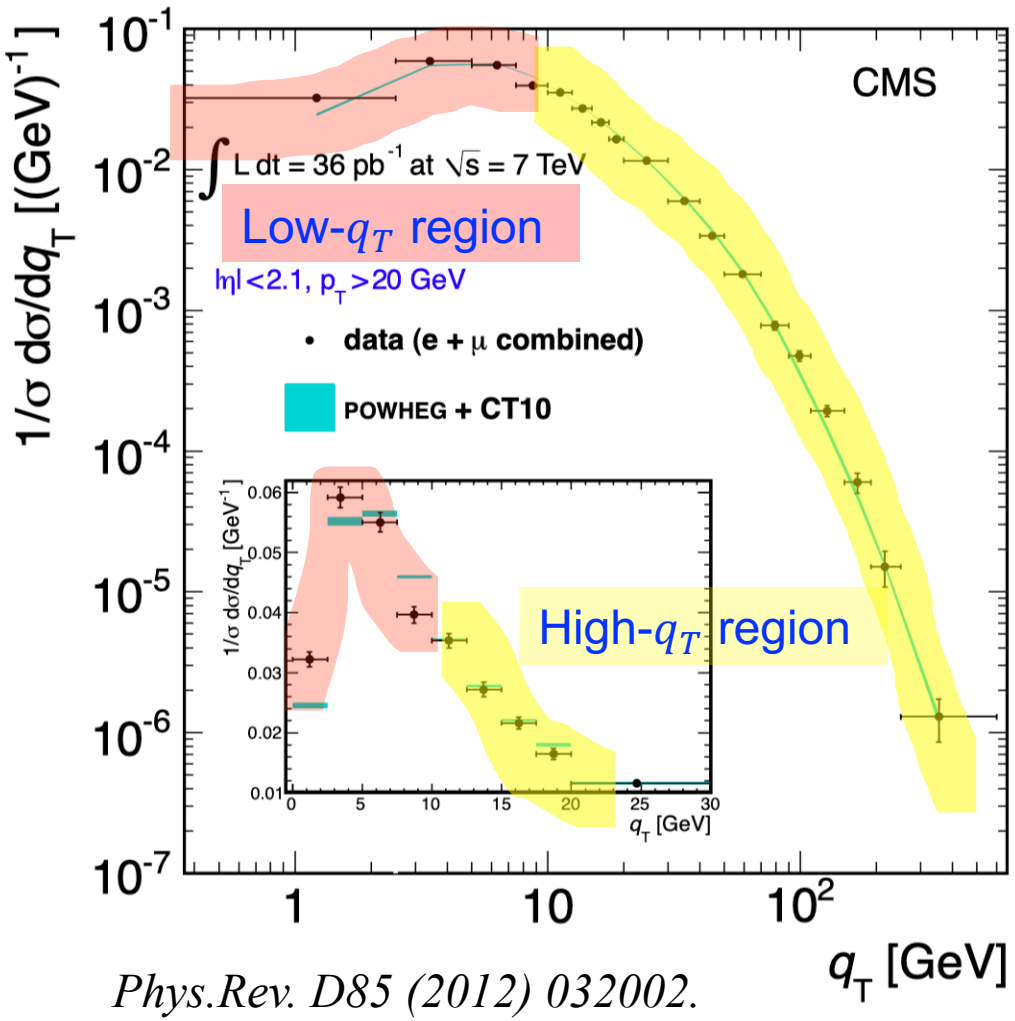
Phys.Rev. D85 (2012) 032002.



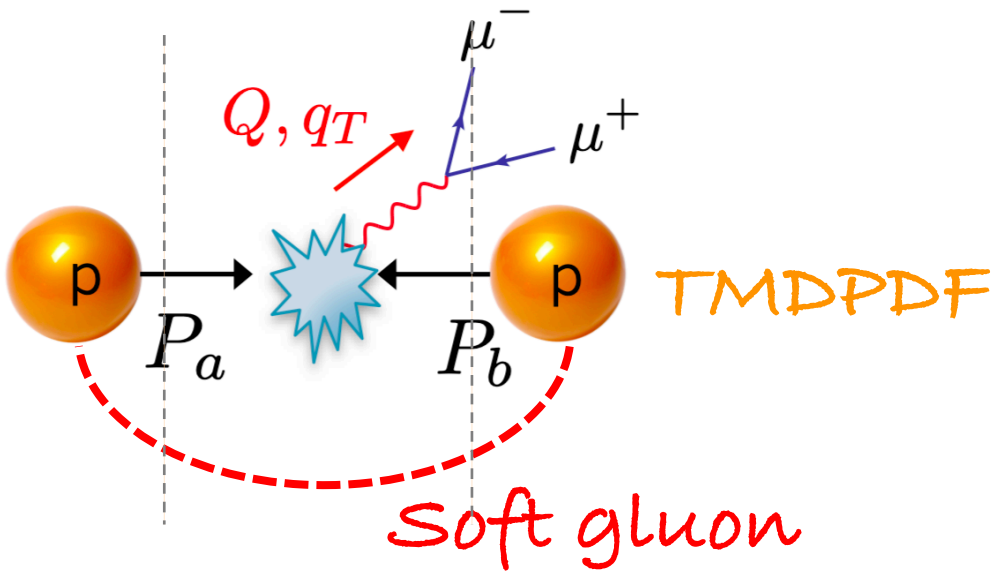
- **High-q_T region:**

  Collinear factorization => PDF

Z-production q_T spectrum at LHC



Phys.Rev. D85 (2012) 032002.



- **High-q_T region:**

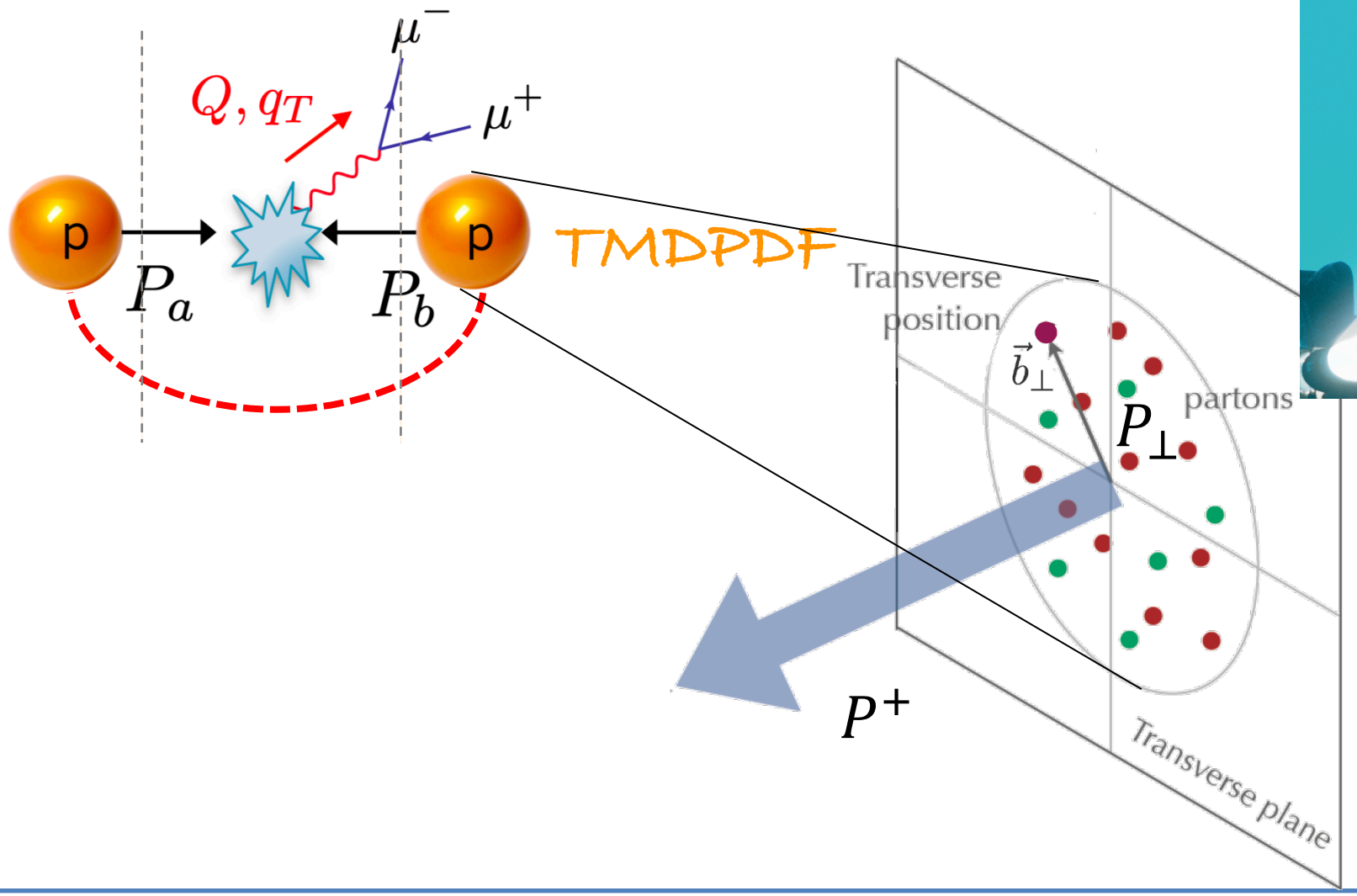
  Collinear factorization => PDF

- **Low-q_T region:**

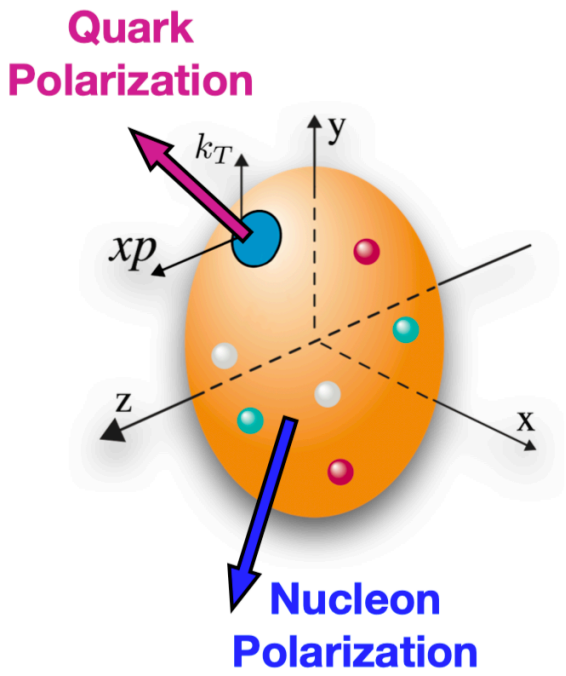
  TMD factorization

  => Generalize to TMDPDFs

TMDPDFs: 3D tomography of the nucleon



TMDPDFs: 3D tomography of the nucleon



Leading Quark TMDPDFs

Nucleon Spin Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
L		g_{1L} = Helicity	$h_{1L}^\perp$ = Worm-gear
T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity; $h_{1T}^\perp$ = Pretzelosity

TMD Handbook, TMD Collaboration, to appear soon

Progress in the study of TMDs

- **Theoretical analysis**
  - **TMD factorization, evolution and resummation:**

    Collins, Foundations of perturbative QCD; ……

- **Phenomenological parametrizations and extractions**
  - **Unpolarized:**

    Bacchetta, JHEP06 (2017); Scimem, JHEP06 (2020); Bury, 2201.07114; Bacchetta, 2206.07598; ……

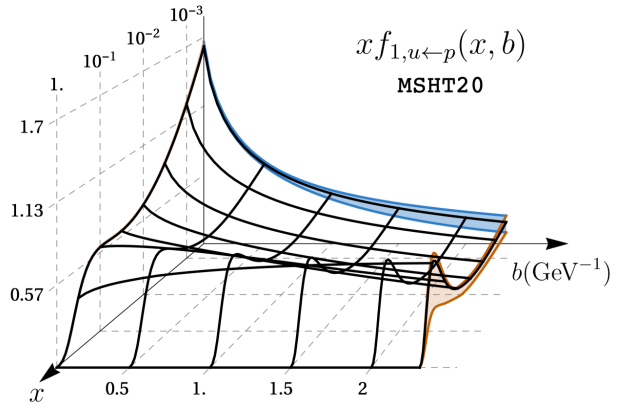
  - **Sivers:**

    Bury, PRL126 (2021), JHEP05 (2021) ; ……

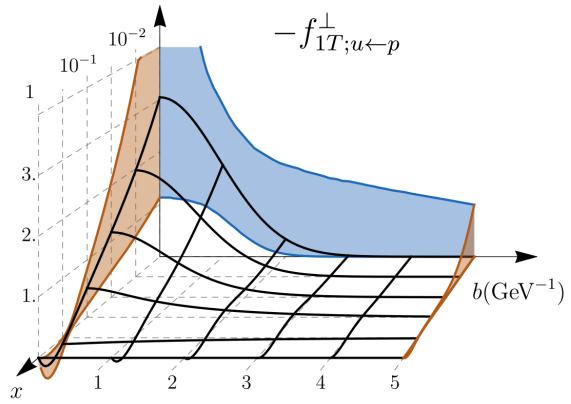
  - **Boer-Mulders:**

    Zhang, PRD77 (2008), Lu, PRD81 (2010) ; ……

  - **Others: worm-gear, gluon TMDs, ……**



u-quark unpolarized TMDPDF, 2201.07114



u-quark Sivers function, PRL126 (2021)

➢ Lattice calculations

- **Lorentz-invariant approach:** ratios of Mellin moments

  Hagler, EPL88(2009); Musch, PRD85(2012); Yoon, 1601.05717, PRD96(2017); ……

- **LaMET formalism:**

  ✓ **Preparation I:** theoretical framework of calculating TMDs and related soft function, Collins-Soper kernel, beam function, ……

  Ji, RMP93(2021), NPB955(2020), PLB811(2020); Ebert, JHEP04(2022); Deng, JHEP09(2022)……

  ✓ **Preparation II:** lattice calculation of intrinsic soft function, Collins-Soper kernel, beam function, ...

  LPC, PRL125(2020); Li, PRL128(2022); LPC, PRD106(2022); Shanahan, PRD104(2021); Schlemmer, JHEP08(2021); ……

  see Min-Huan Chu's talk

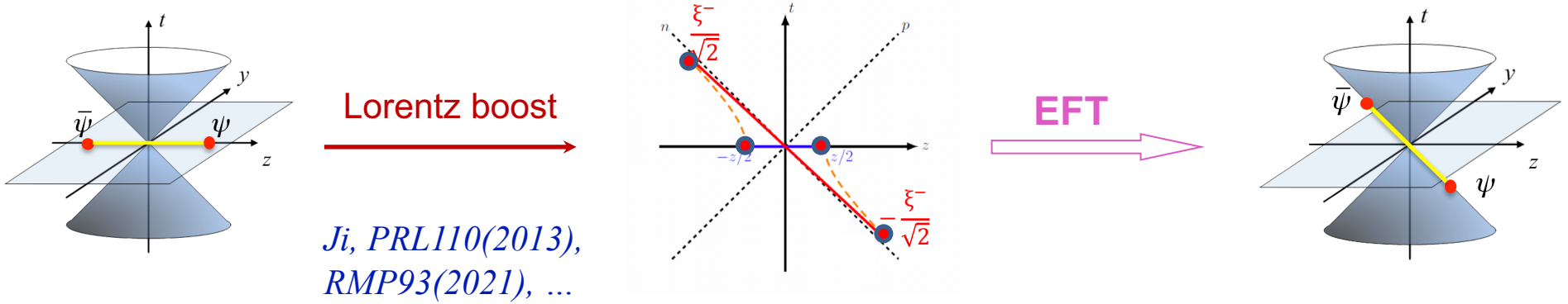
  ✓ **Preparation III:** Renormalization, resummation

  LPC, PRL129(2022); 2209.01236……

  see Kuan Zhang and Yu-Shan'Su's talk

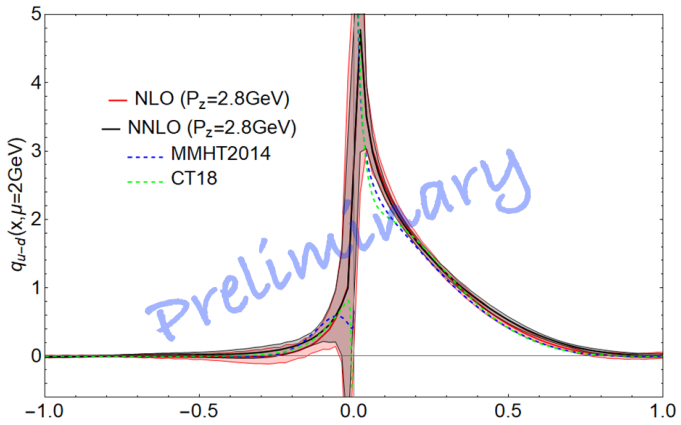
Extracting TMDs in LaMET formalism

- **Large-momentum effective theory: connecting Euclidean lattice and physical observables**

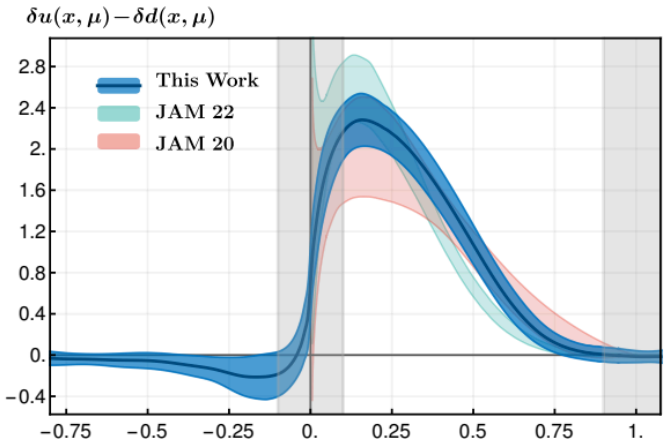


Ji, PRL110(2013), RMP93(2021), …

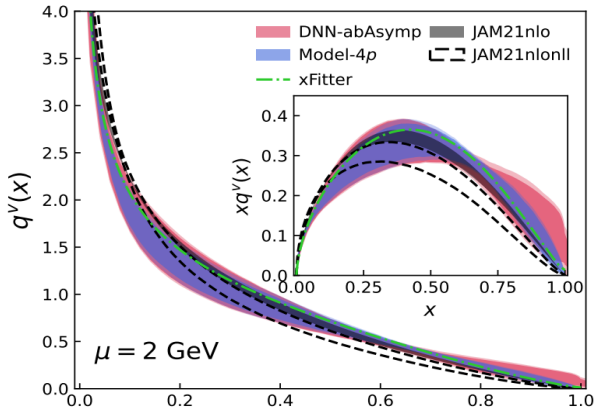
- **Achieved great success in the studies of PDF:**



Proton unpolarized PDF, in preparation

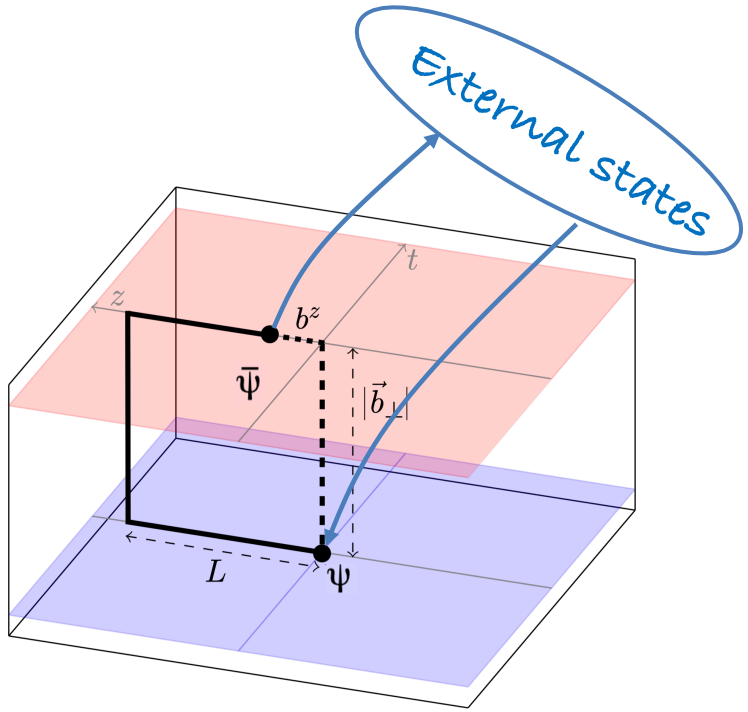


Proton transversity PDF, 2208.08008



Pion valance PDF, 2208.02297

- **Matching from quasi TMDs to TMDs**



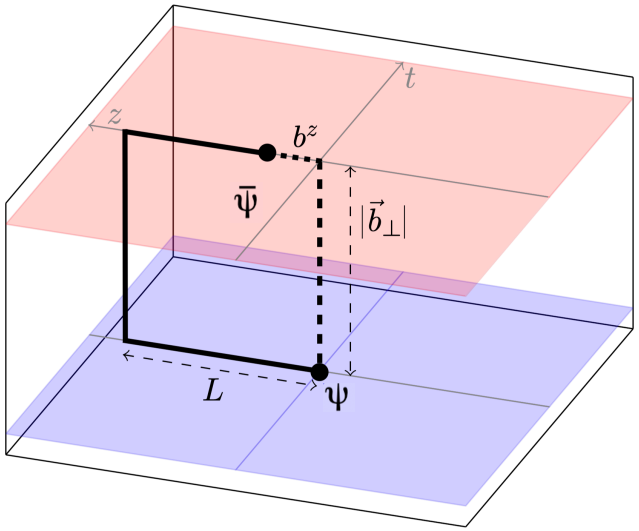
Equal-time correlators
with staple-shaped Wilson link,
directly calculable on lattice

- Hadronic matrix element reduced from equal-time correlators:

$$\tilde{h}^0_\Gamma\left(z, b_\perp, P^z\right) = \lim_{L\to\infty} \left\langle P^z \middle| \bar{\psi}(b_\perp \hat{n}_\perp)\Gamma \right.$$
$$\times U_{\sqsubset}\left(b_\perp\hat{n}_\perp \leftarrow b_\perp\hat{n}_\perp + L\hat{n}_z; b_\perp\hat{n}_\perp + L\hat{n}_z \leftarrow L\hat{n}_z; L\hat{n}_z \leftarrow z\hat{n}_z\right)$$
$$\left. \times\ \psi(z\hat{n}_z) \middle| P^z \right\rangle$$

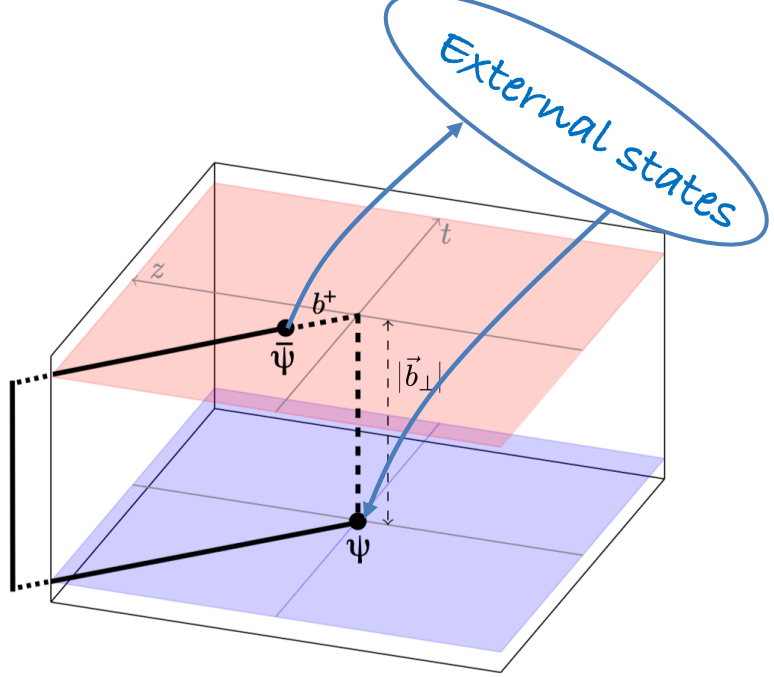
- Subtracted quasi TMDPDFs:

$$\tilde{f}_\Gamma\left(x, b_\perp, P^z, \mu\right) \equiv \lim_{\substack{a\to 0 \\ L\to\infty}} \int \frac{dz}{2\pi} e^{-iz(xP^z)} \frac{\tilde{h}^0_\Gamma\left(z, b_\perp, P^z, a, L\right)}{\sqrt{Z_E\left(2L+z, b_\perp, a\right)} Z_O\left(1/a, \mu, \Gamma\right)}$$



Lorentz boost

$L \to \infty$



Equal-time correlators,
directly calculable on lattice

Space-like correlators,
NO effective method for directly calculation

Connected at large-momentum limit

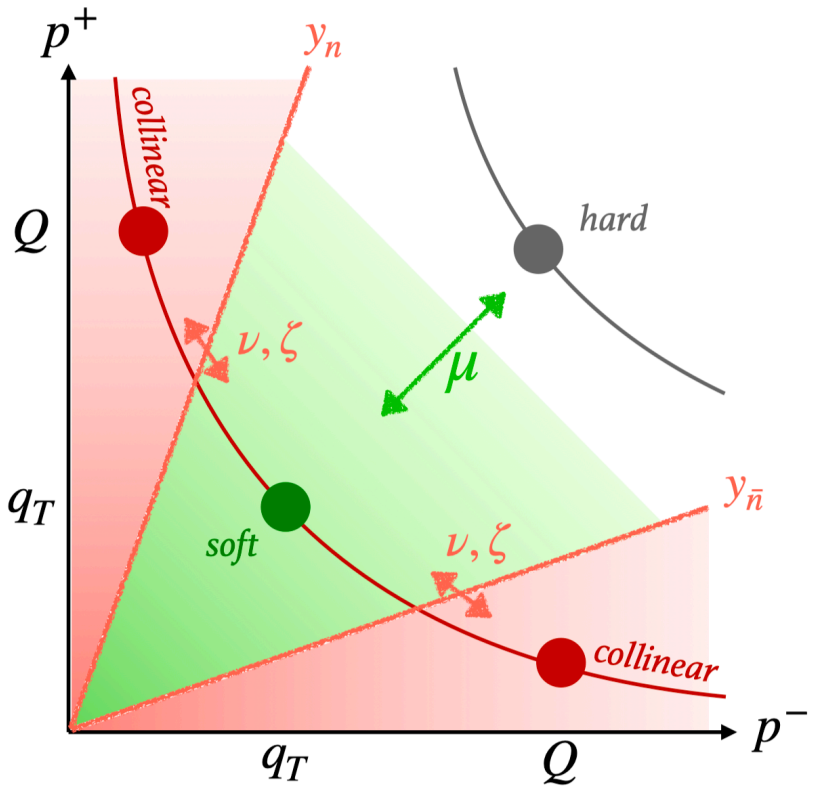
Ji, PLB811(2020); Ebert, JHEP04(2022)

Matching kernel

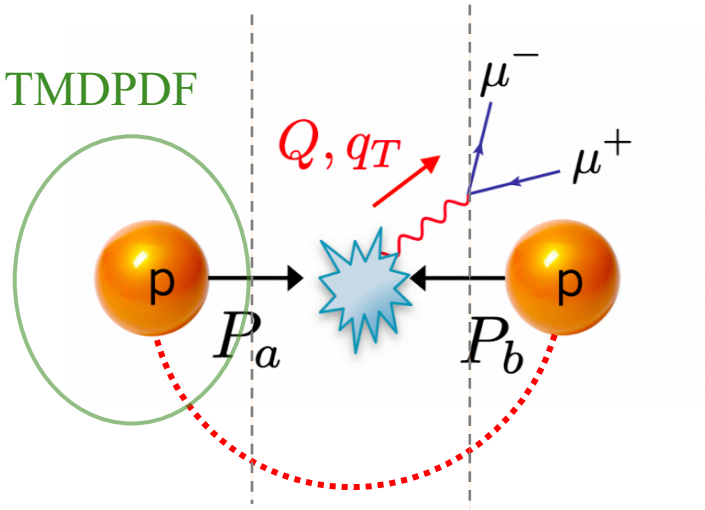
$$\tilde{f}_\Gamma\left(x, b_\perp, \zeta_z, \mu\right) \sqrt{S_I\left(b_\perp, \mu\right)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} f\left(x, b_\perp, \mu, \zeta\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

Quasi TMDPDF | Intrinsic soft function | Collins-Soper kernel | Light-cone TMDPDF

Evolution of TMDPDF



- **μ renormalization group evolution: perturbative**
- **Rapidity evolution: nonperturbative**



Soft gluon emission => **soft function**

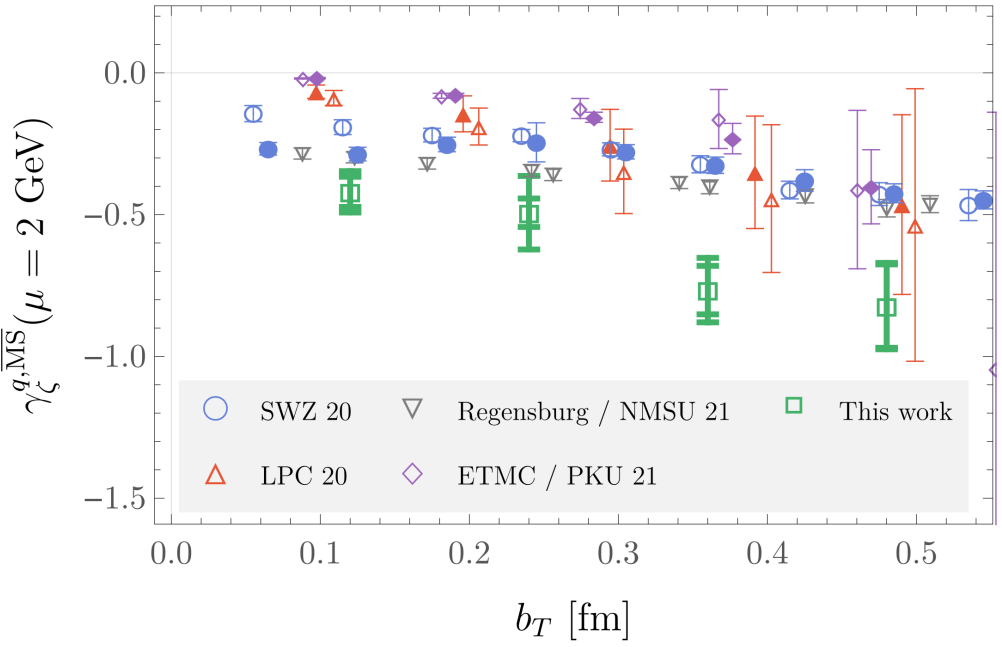
Rapidity evolution: soft function

- **Rapidity-dependent part: Collins-Soper kernel**

From quasi beam function:

Shanahan, PRD104(2021), PRD102(2020);

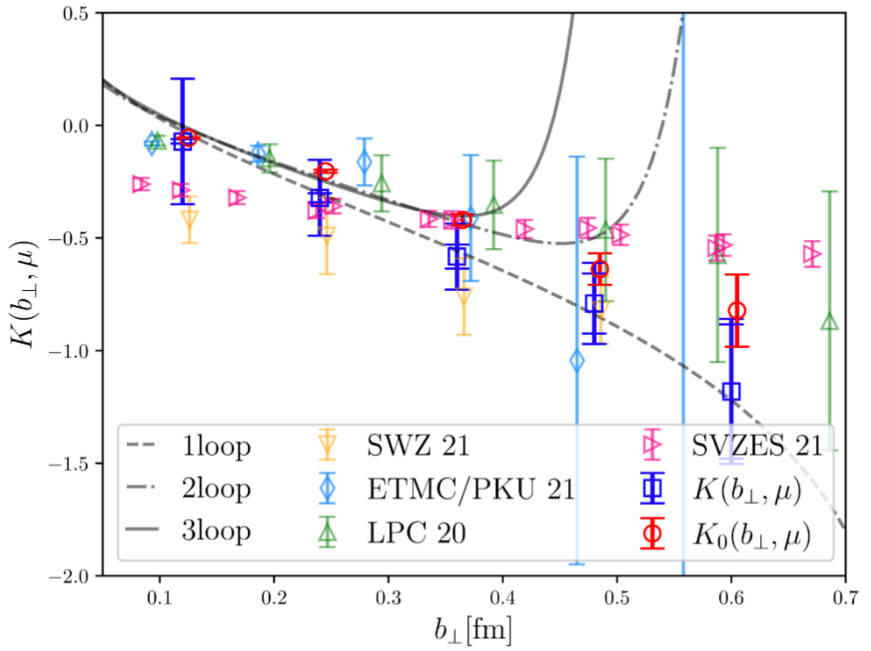
Schlemmer, JHEP08(2021); ……



From quasi TMDWF:

Chu, PRD106(2022); Zhang, PRL125(2020);

Li, PRL128(2022); ……

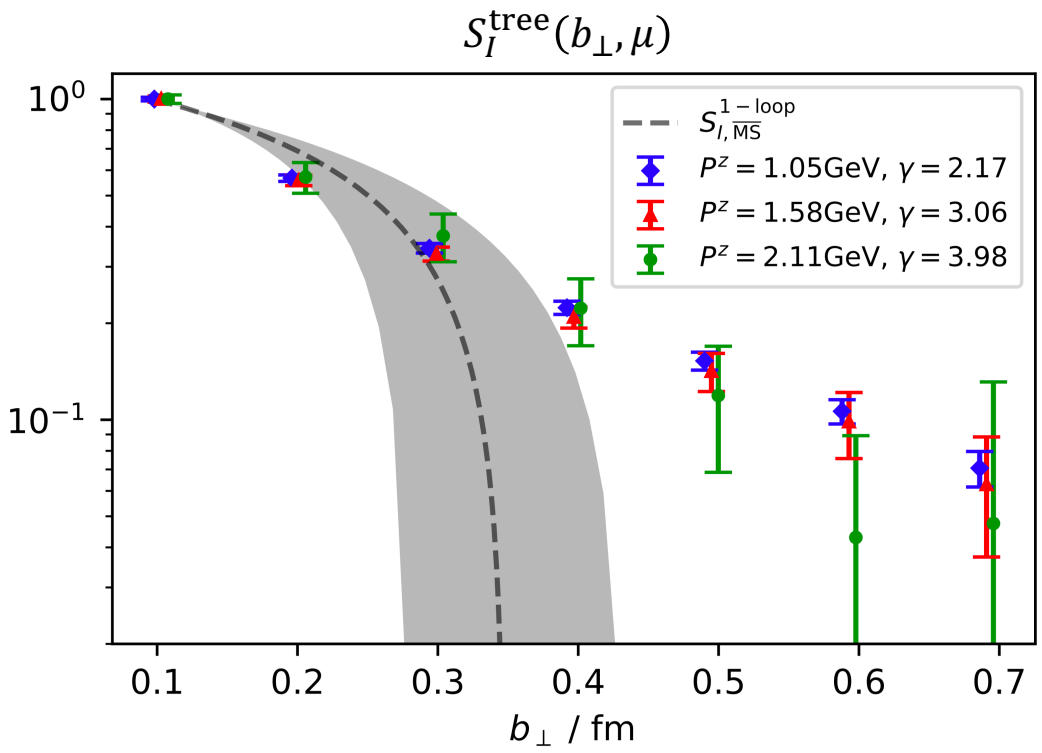


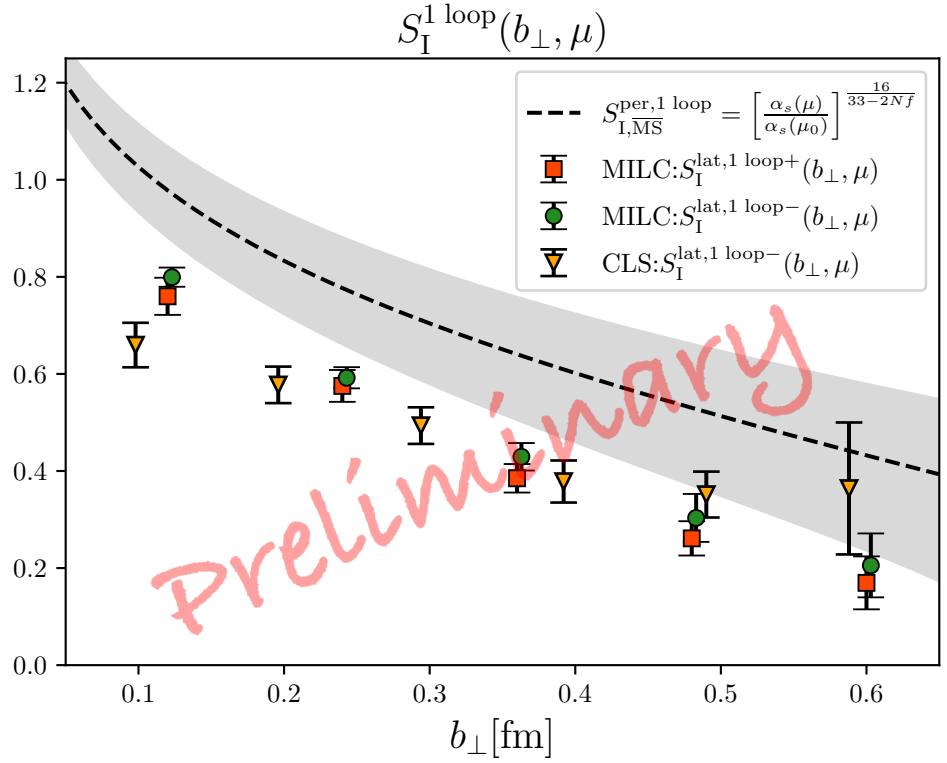
Rapidity evolution: soft function

- **Rapidity-independent part: intrinsic/reduced soft function**

  From quasi TMDWF:

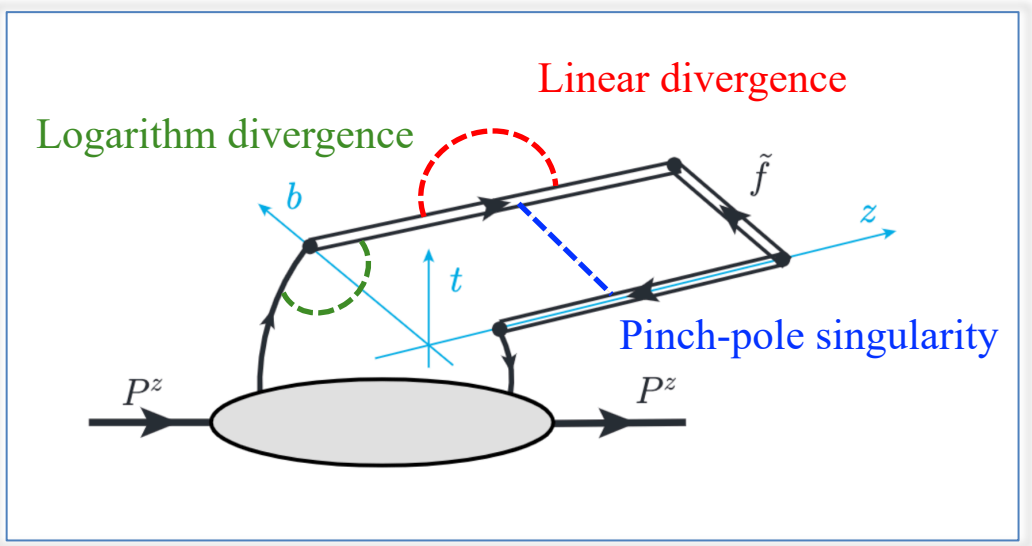
  Ji, NPB955(2020); Zhang, PRL125(2020); Li, PRL128(2022); Chu, in preparation……



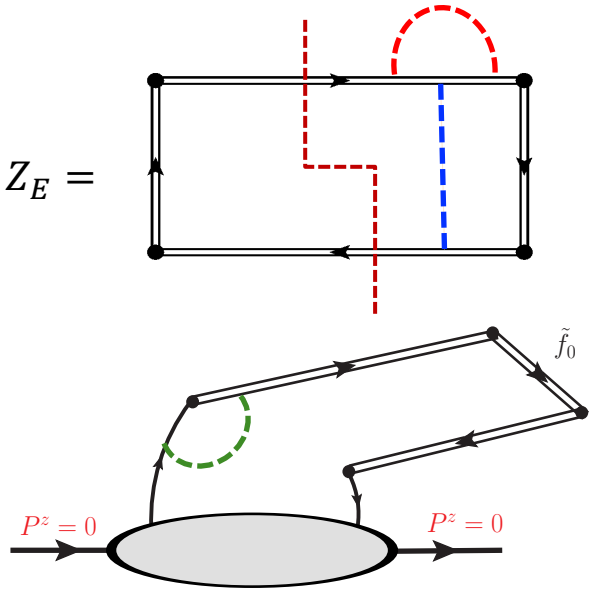


Simulating quasi TMDPDF on a Euclidean lattice

1. Bare quasi TMDPDF matrix element



2. Renormalization



Wilson loop:

Linear divergence

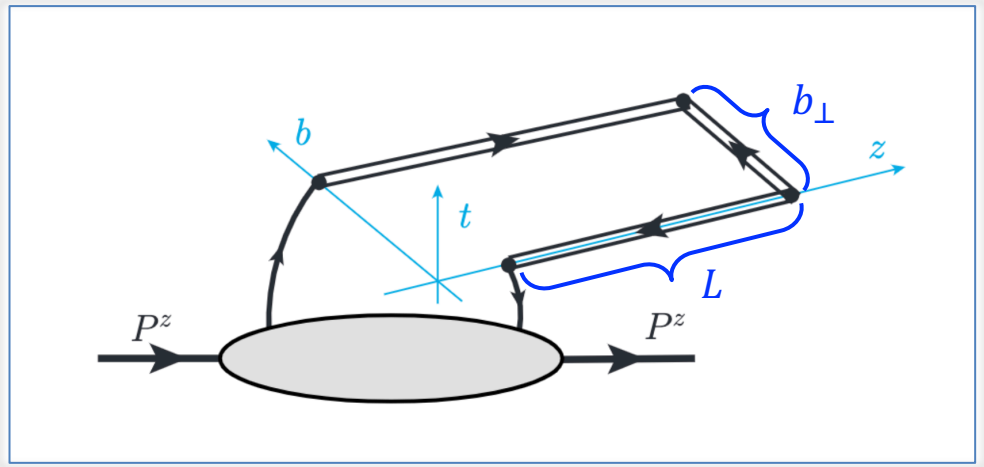
Pinch-pole singularity

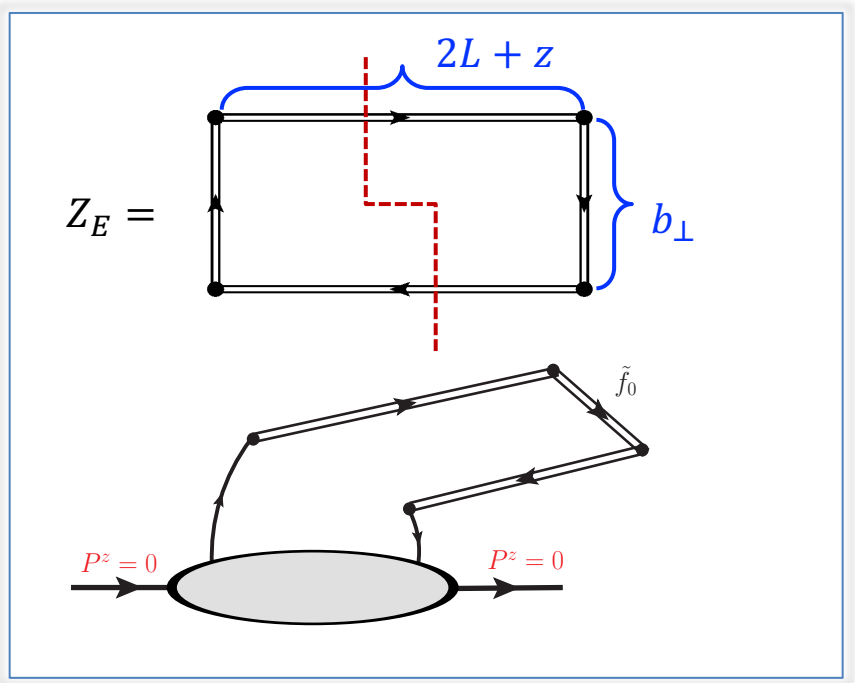
Zero-momentum matrix element:

Logarithm divergence

Ji, PRL120(2018), NPB964(2021), PLB257(1991); Zhang, PRD95(2017), NPB939(2019); Ishikawa, PRD96(2017); Green, PRL121(2018); Huo, NPB969(2021); Chen, NPB915(2017); Musch, PRD83(2011); ……

Simulating quasi TMDPDF on a Euclidean lattice





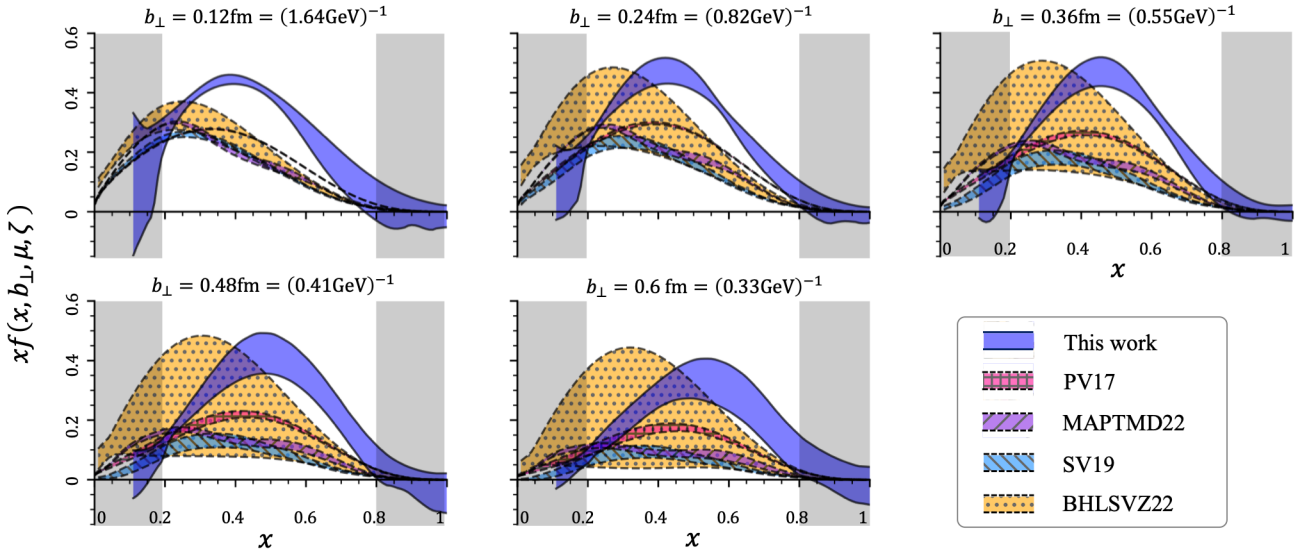
Lattice setup:

- MILC configuration: $48^3 \times 64$, $a = 0.12\text{fm}$;
- Pion mass: $\mathrm{m}_\pi^{\text{sea}} = 130\text{MeV}$, $m_\pi^{\text{val}} = \{310, 220\}\text{MeV}$ $\Rightarrow$ extrapolate to physical mass
- Large momentum: $P^z = \{1.72,\ 2.15,\ 2.58\}\text{GeV}$ $\Rightarrow$ extrapolate to infinity
- Saturated length of Wilson link $L = 0.72\text{fm}$;
- $z_{\max} = 1.44\text{fm}$, $b_{\perp\max} = 0.6\text{fm}$ $\Rightarrow$ momentum distribution.

Simulating quasi TMDPDF on a Euclidean lattice

Next, stage to Jin-Chen He:

- Lattice calculation of quasi TMDPDF;
- λ-extrapolation and FT;
- Matching to light-cone TMDPDF;
- Comparison and discussion;
- Uncertainties estimation;



Thanks for your attention!