Unpolarized TMDPDFs of the Nucleon from LQCD Pt.2

LaMET 2022

Collaborators: Qi-An Zhang, Wei Wang, et al.

arXiv:2211.02340

Unpolarized Transverse-Momentum-Dependent Parton Distributions of the Nucleon from Lattice QCD
(Lattice Parton Collaboration (LPC))

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University of Maryland, College Park

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- Precondition
- Methodology
- Result
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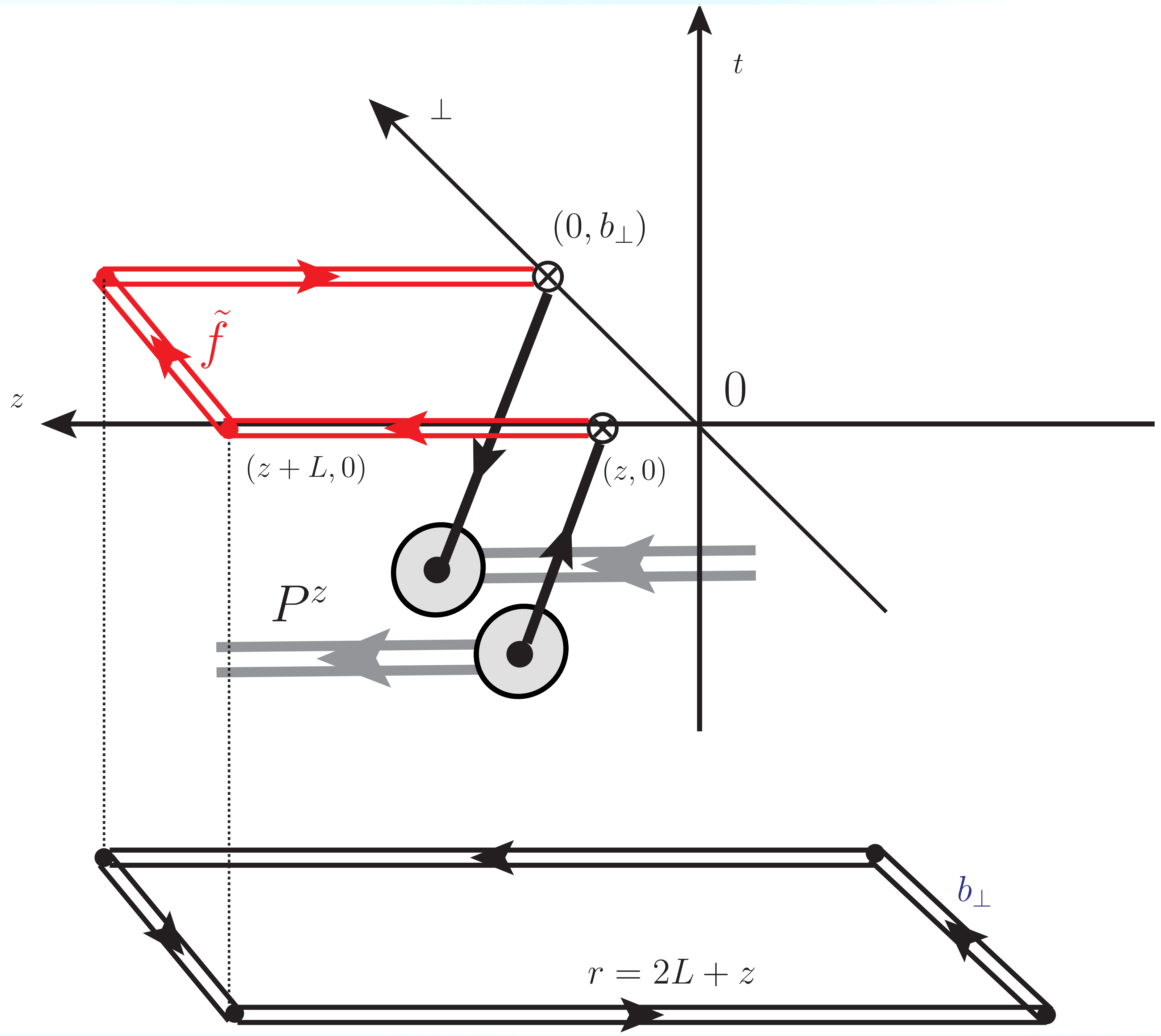
Precondition

Lattice Setup

- 2+1+1 flavors of HISQ action by MILC (a=0.12 fm)
- Valance pion mass: 310 MeV, 220 MeV
- Gamma structure: γ^t and γ^z
- Hadron momentum: 1.72 GeV, 2.15 GeV, 2.58 GeV



$$\tilde{h}^0_\Gamma\left(z, b_\perp, P^z, a, L\right) = \langle P^z | \tilde{O}^0_{\Gamma,\sqsubset}(z, b_\perp, P^z; L) | P^z \rangle$$

$$\tilde{O}^0_{\Gamma,\sqsubset}(z, b_\perp, L) \equiv \bar{\psi}(b_\perp \hat{n}_\perp)\Gamma U_{\sqsubset,L}\left(b_\perp \hat{n}_\perp, z\hat{n}_z\right)\psi(z\hat{n}_z)$$

$$U_{\sqsubset,L}\left(b_\perp \hat{n}_\perp, z\hat{n}_z\right) \equiv U_z^\dagger\left((L+z)\hat{n}_z + b_\perp\hat{n}_\perp, b_\perp\hat{n}_\perp\right) \times U_\perp\left((L+z)\hat{n}_z + b\hat{n}_\perp, (L+z)\hat{n}_z\right) U_z\left((L+z)\hat{n}_z, z\hat{n}_z\right)$$

Precondition

Ready for the calculation of TMDPDFs

Quasi distribution:
Equal time;
Directly calculable on the lattice

Light-cone distribution:
Separated on the time axis;
Cannot be calculated on the lattice

$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} \times f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

Intrinsic soft function:
Latest LPC work, to be published
Q. A. Zhang, et al. Phys. Rev. Lett. 125 (2020)

Hard kernel: 1 loop with RGR
X. Ji, et al. Phys. Lett. B 811 (2020)
Y. Su, et al. arXiv:2209.01236

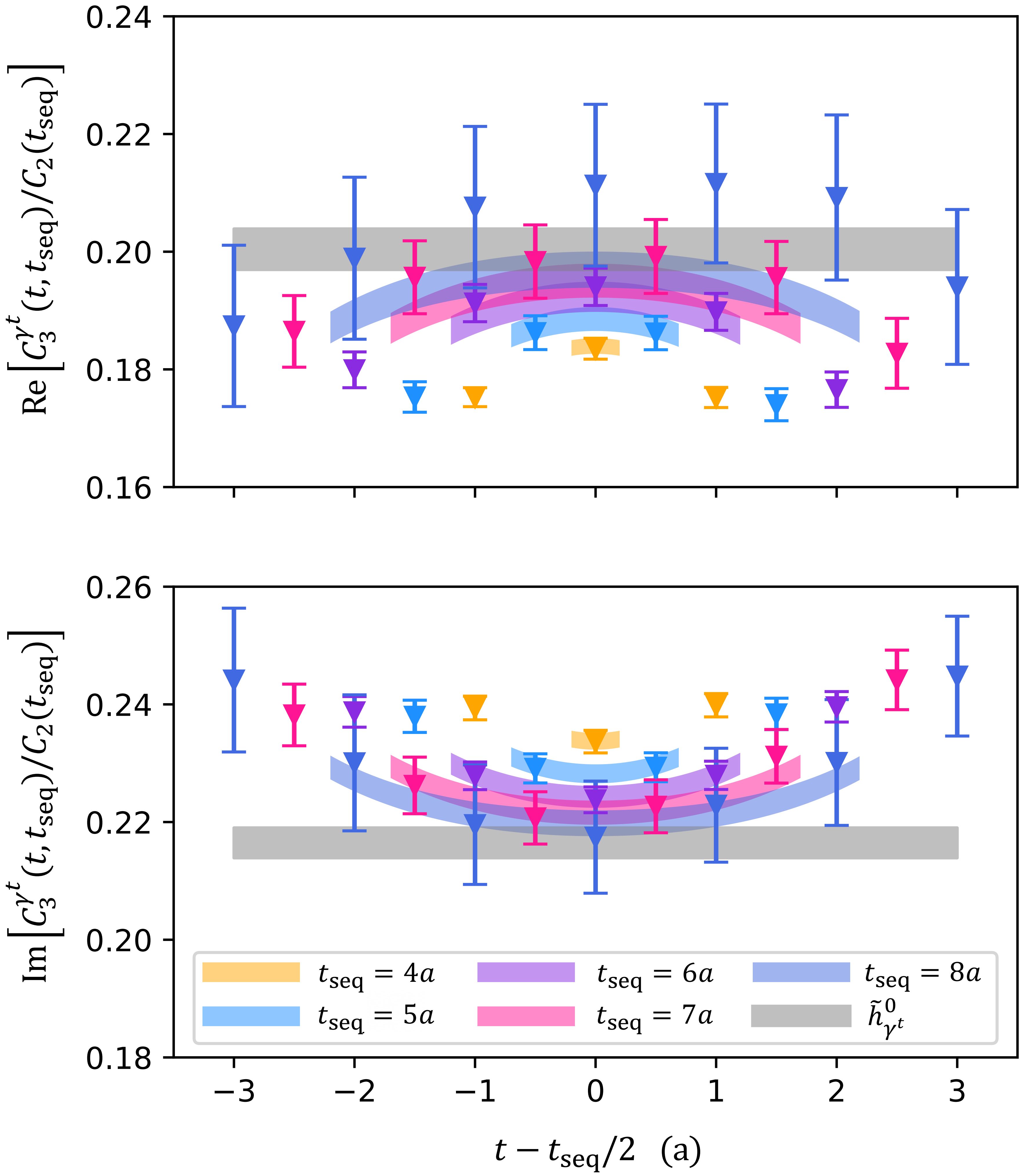
Collins-Soper kernel:
M. Chu, et al. Phys. Rev. D 106 (2022)

Methodology

To get quasi distribution: ground state fit

$$C_2(t_{\text{seq}}) = 1 + c_1 e^{-\Delta E t_{\text{seq}}}$$

$$\frac{C_3^{\Gamma}\left(t, t_{\text{seq}}\right)}{C_2(t_{\text{seq}})} = \frac{\boxed{\tilde{h}_{\Gamma}^{0}} + c_2\left(e^{-\Delta E t} + e^{-\Delta E (t_{\text{seq}} - t)}\right) + c_3 e^{-\Delta E t_{\text{seq}}}}{1 + c_1 e^{-\Delta E t_{\text{seq}}}}$$



Methodology

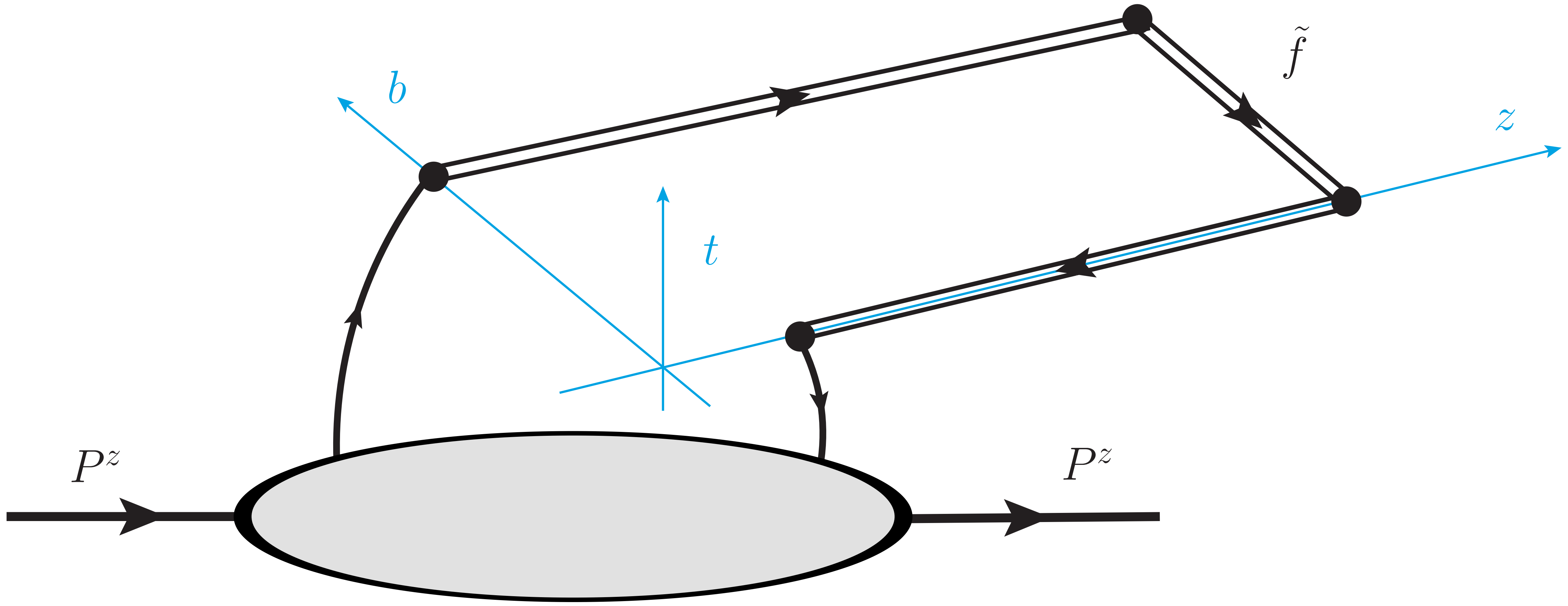
To get quasi distribution: renormalization

Divergence from Wilson link:

- **Linear divergence**
- **Pinch-pole singularity**

Divergence from quark-gauge link vertices:

- **Logarithmic divergence**



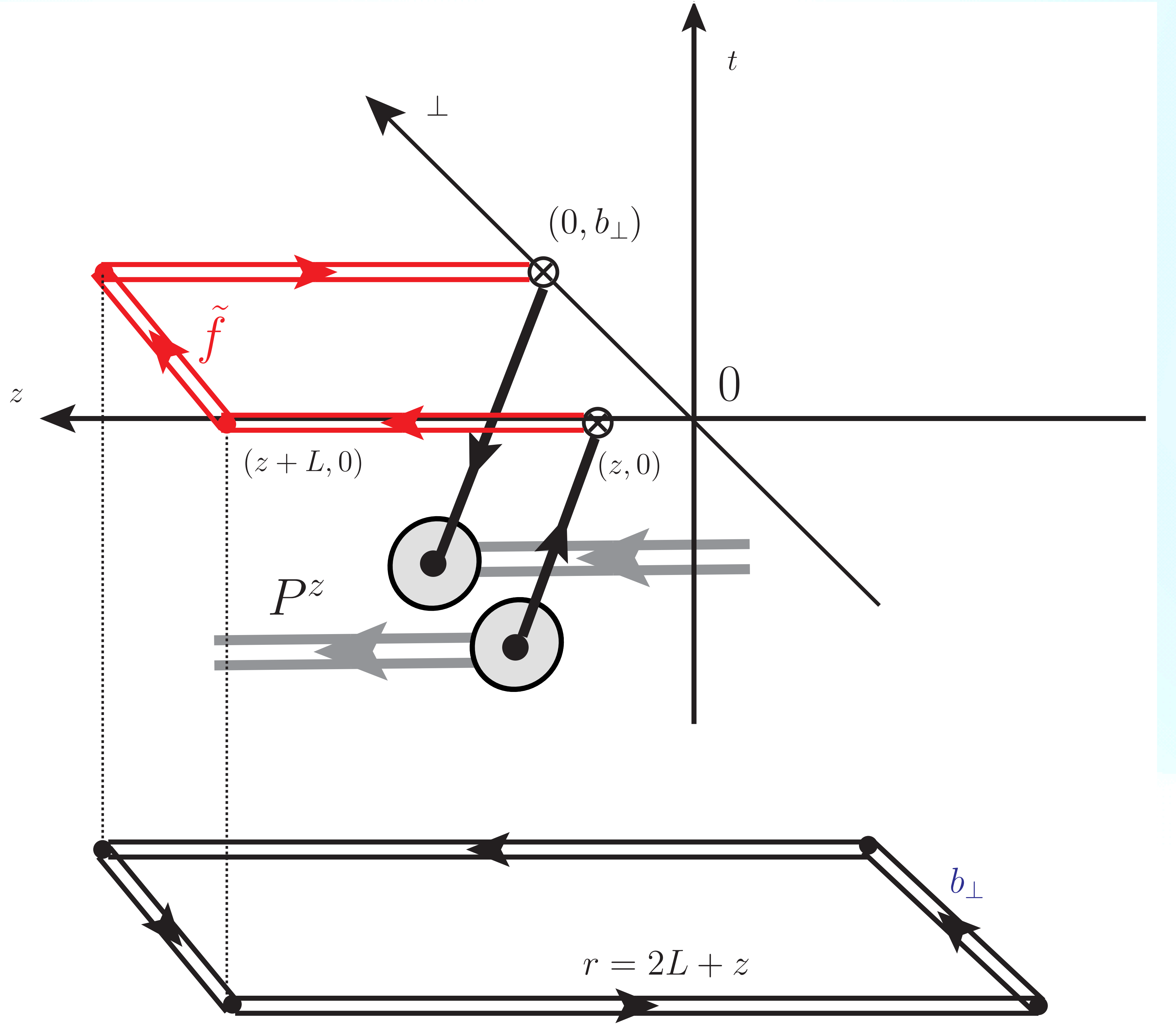
Methodology

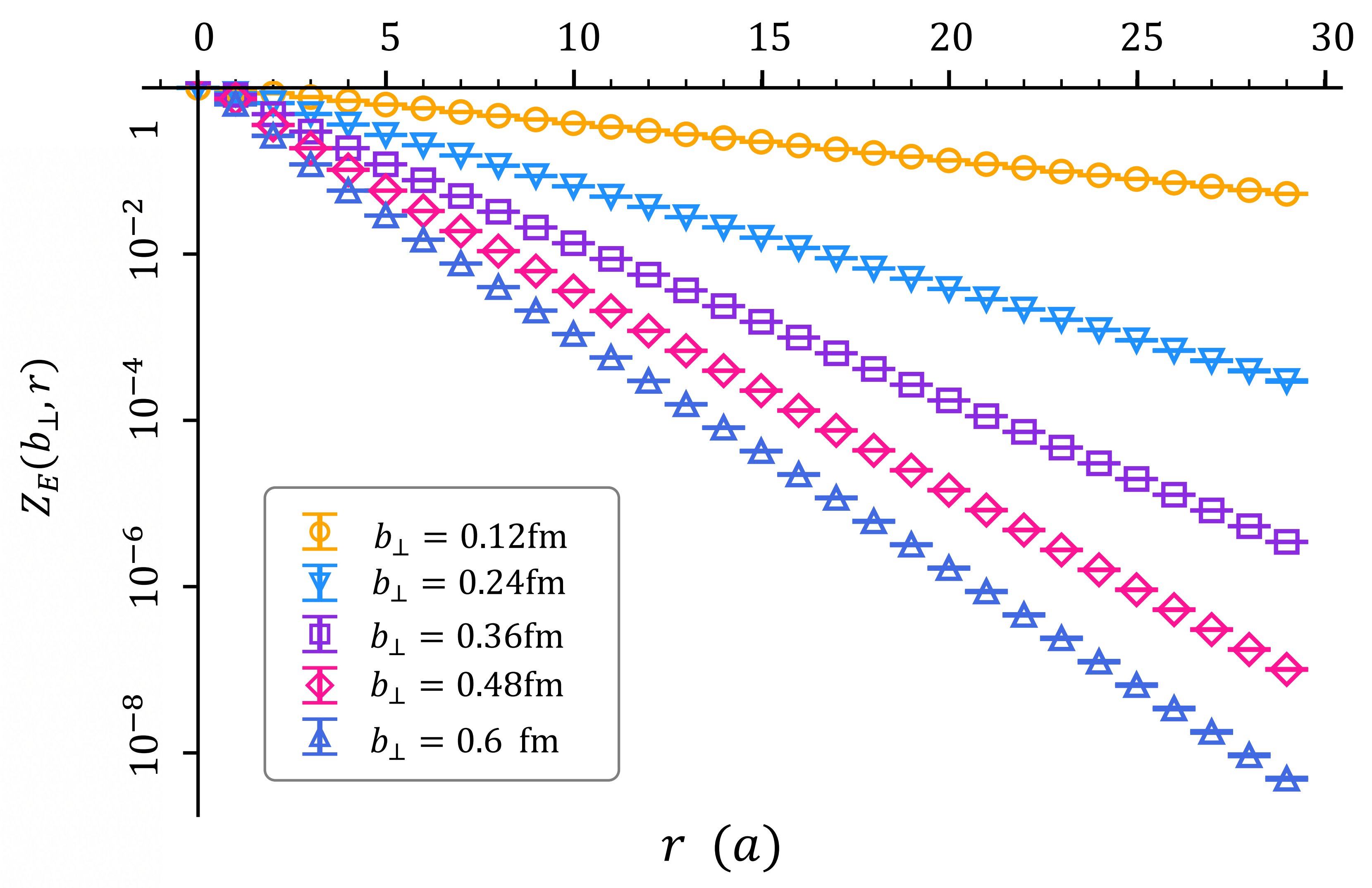
To get quasi distribution: renormalization

Divergence from Wilson link

$$\tilde{h}_\Gamma\left(z, b_\perp, P^z, a, \mu\right) = \lim_{L\to\infty} \frac{\tilde{h}^0_\Gamma\left(z, b_\perp, P^z, a, L\right)}{\sqrt{Z_E\left(2L+z, b_\perp, a\right)}\, Z_O(1/a, \mu, \Gamma)}$$

K. Zhang, et al. Phys. Rev. Lett. 129 (2022)





Methodology

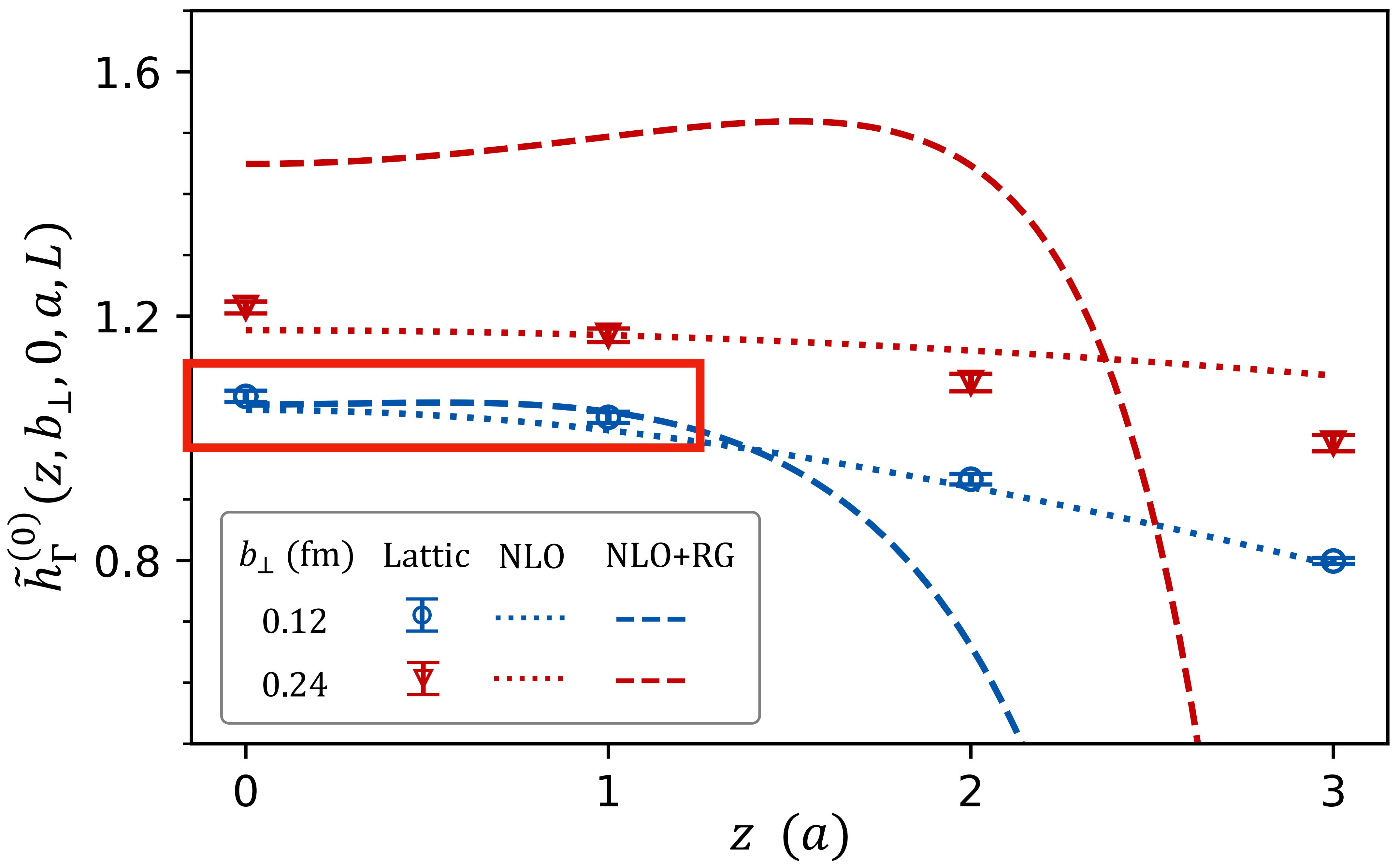
To get quasi distribution: renormalization

$$\tilde{h}_\Gamma\left(z, b_\perp, P^z, a, \mu\right) = \lim_{L\to\infty} \frac{\tilde{h}^0_\Gamma\left(z, b_\perp, P^z, a, L\right)}{\sqrt{Z_E\left(2L+z, b_\perp, a\right)}\, Z_O(1/a, \mu, \Gamma)}$$

<u>Divergence from quark-gauge link vertices</u>

$$\tilde{h}^{\overline{\rm MS}}_\Gamma\left(z, b_\perp, \mu\right) = \tilde{h}_\Gamma\left(z, b_\perp, 0, a, \mu\right)$$

$$Z_O(1/a, \mu, \Gamma) = \lim_{L\to\infty} \frac{\tilde{h}^0_\Gamma\left(z, b_\perp, 0, a, L\right)}{\sqrt{Z_E\left(2L+z, b_\perp, a\right)}\, \tilde{h}^{\overline{\rm MS}}_\Gamma\left(z, b_\perp, \mu\right)}$$



K. Zhang, et al. Phys. Rev. Lett. 129 (2022)

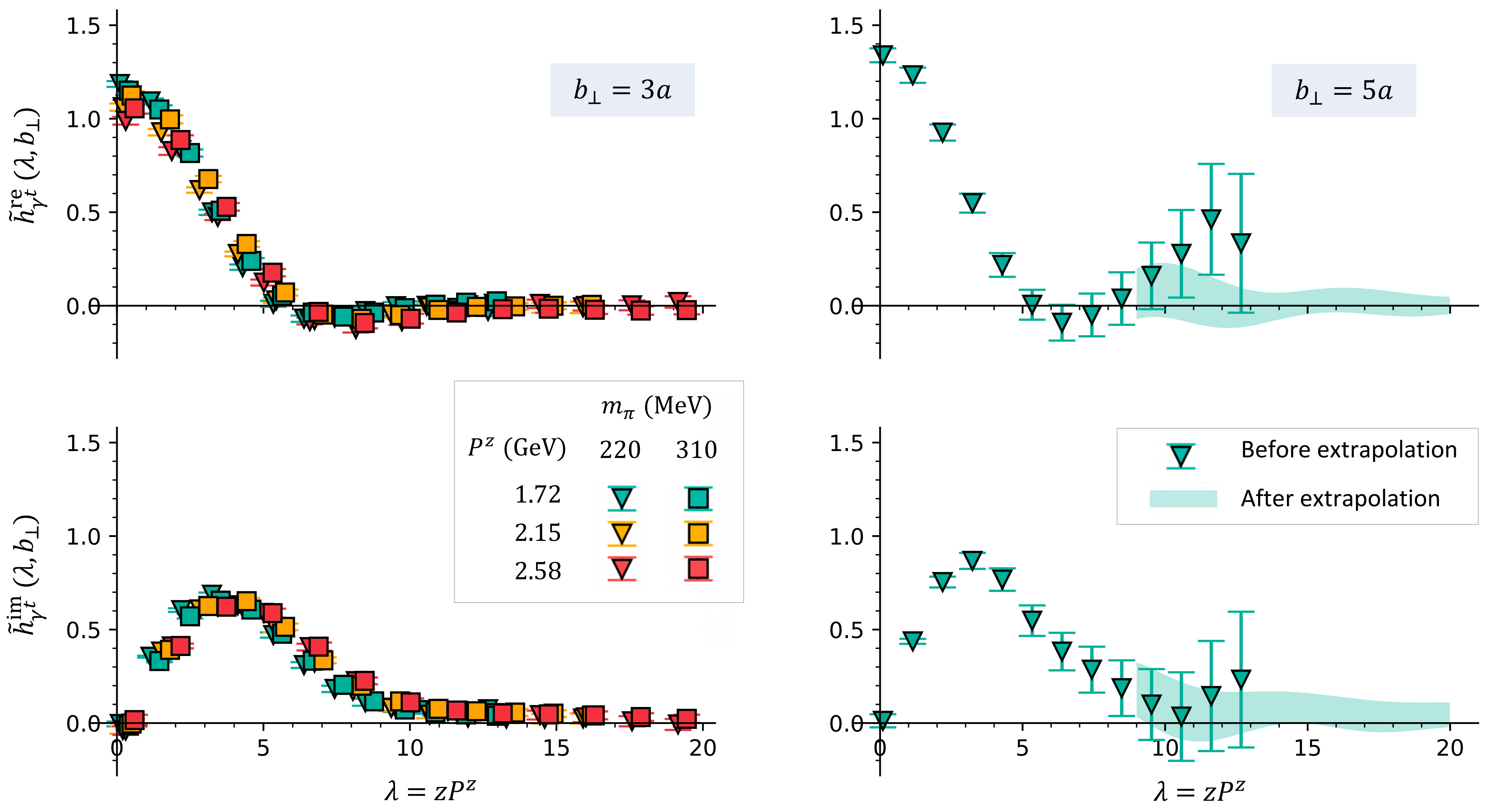
Y. Su, et al. arXiv:2209.01236

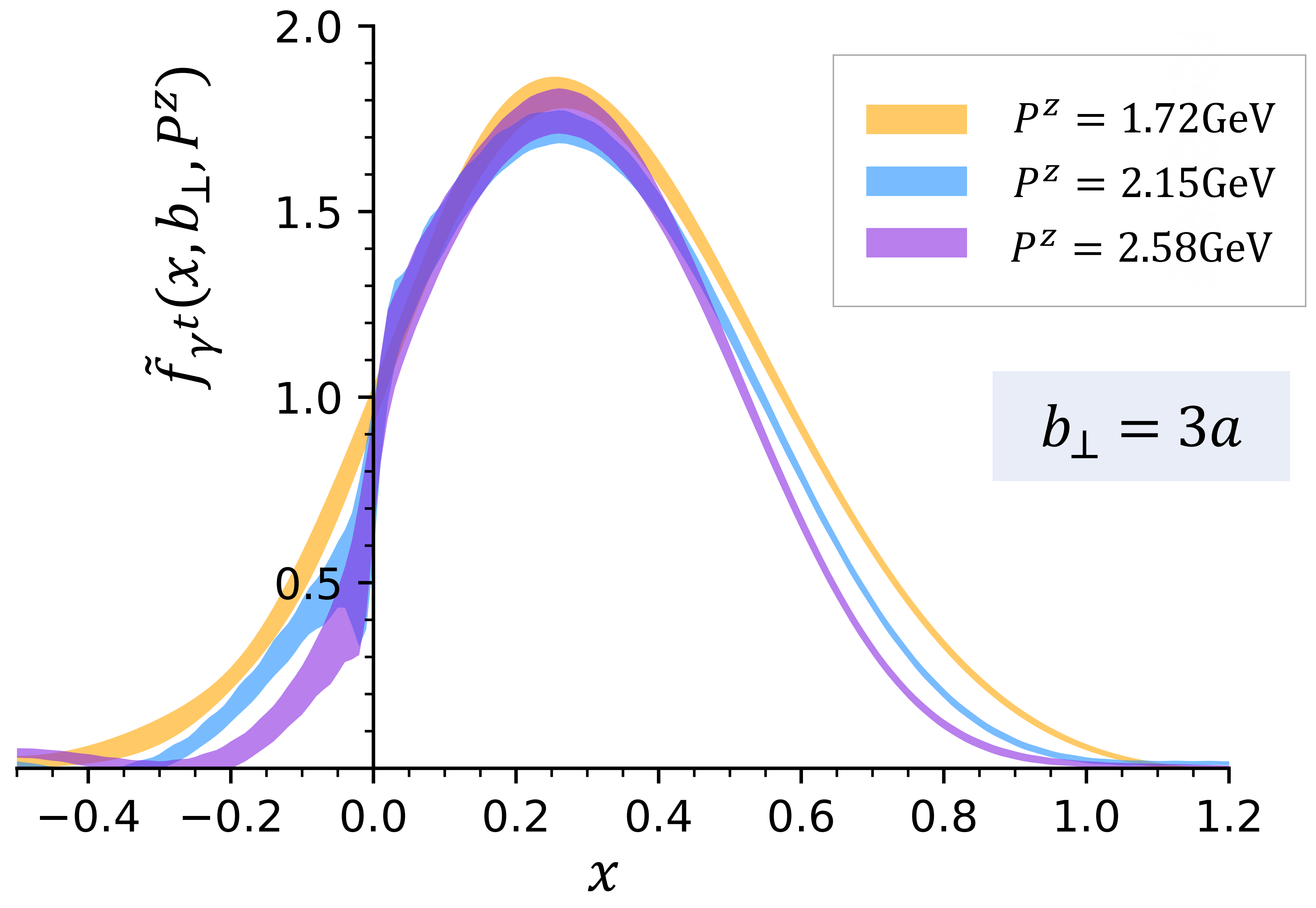
Methodology

To get quasi distribution: extrapolation

$$\tilde{h}_{\Gamma,\mathrm{extra}}(\lambda) = \left[\frac{m_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{m_2}{(i\lambda)^{n_2}}\right] e^{-\lambda/\lambda_0}$$

X. Ji, et al. Nucl. Phys. B 964, 115311 (2021)

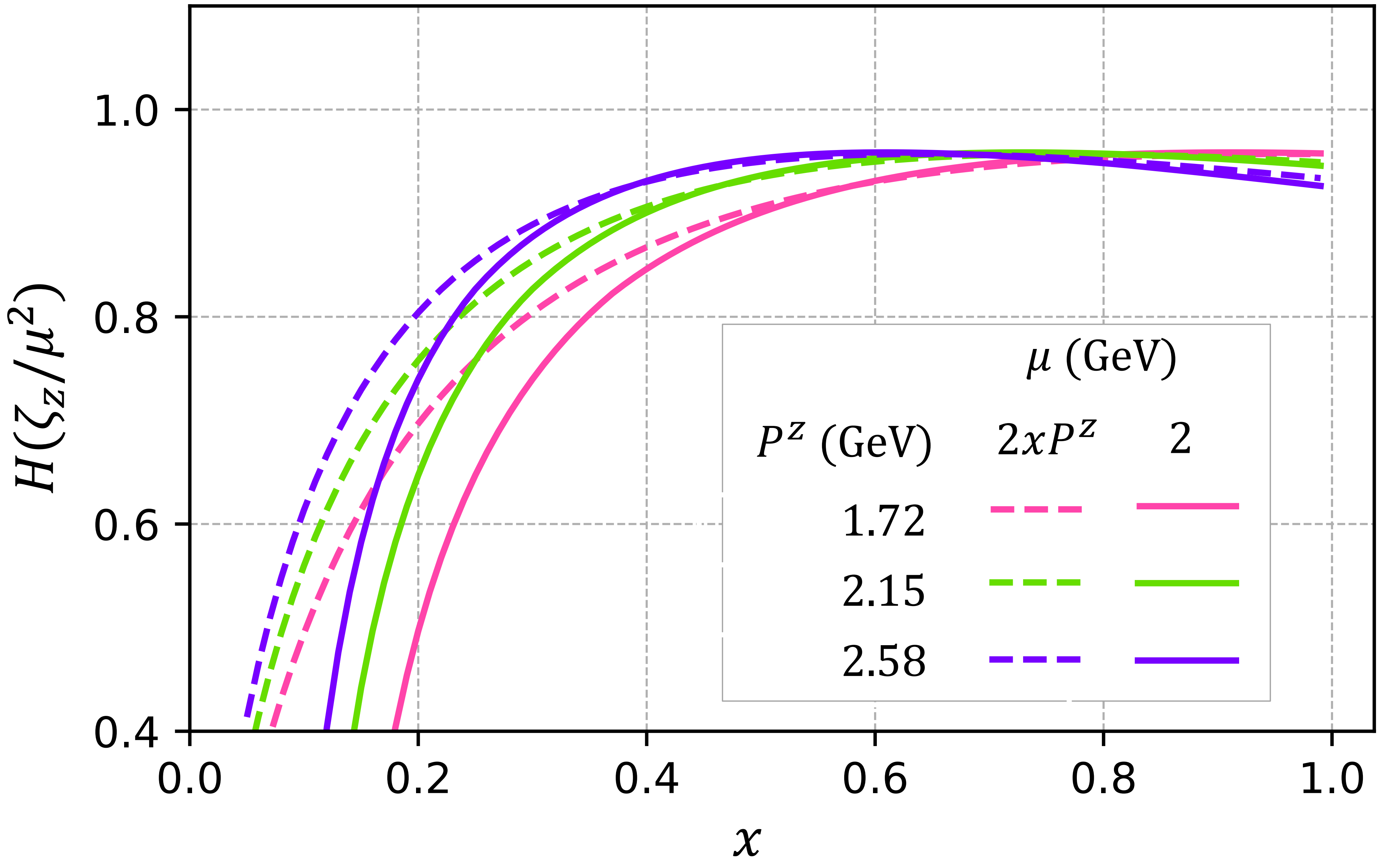




Methodology

To get light-cone distribution

$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} \times f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$



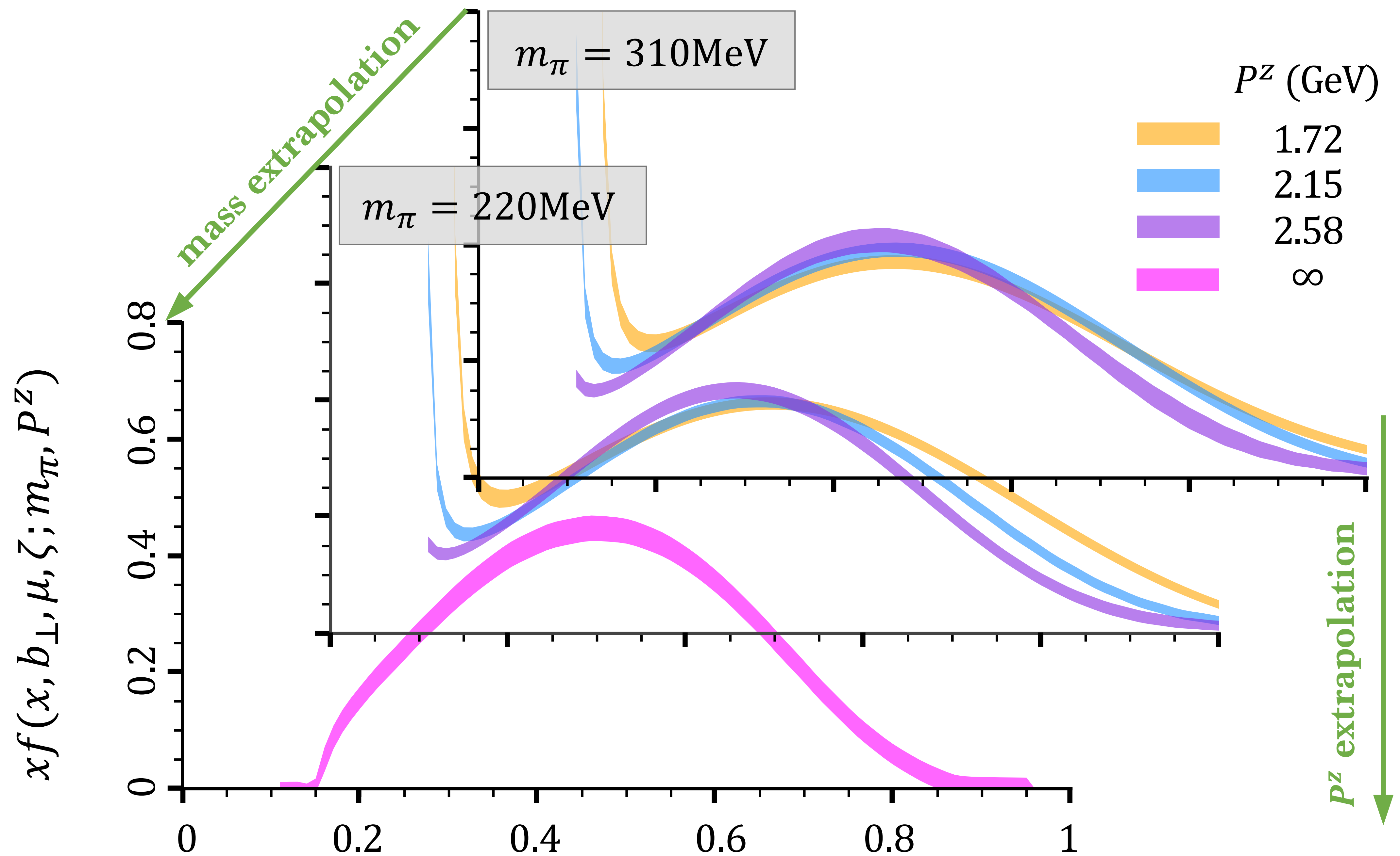
$$\mu^2 \frac{d}{d\mu^2} \ln H\left(\frac{\zeta_z}{\mu^2}\right) = \frac{1}{2}\Gamma_{\mathrm{cusp}}(\alpha_s) \ln \frac{\zeta_z}{\mu^2} + \frac{\gamma_C(\alpha_s)}{2}$$

X. Ji, et al. Phys. Lett. B 811 (2020)
Y. Su, et al. arXiv:2209.01236

Methodology

To get light-cone distribution

$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} \times f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$



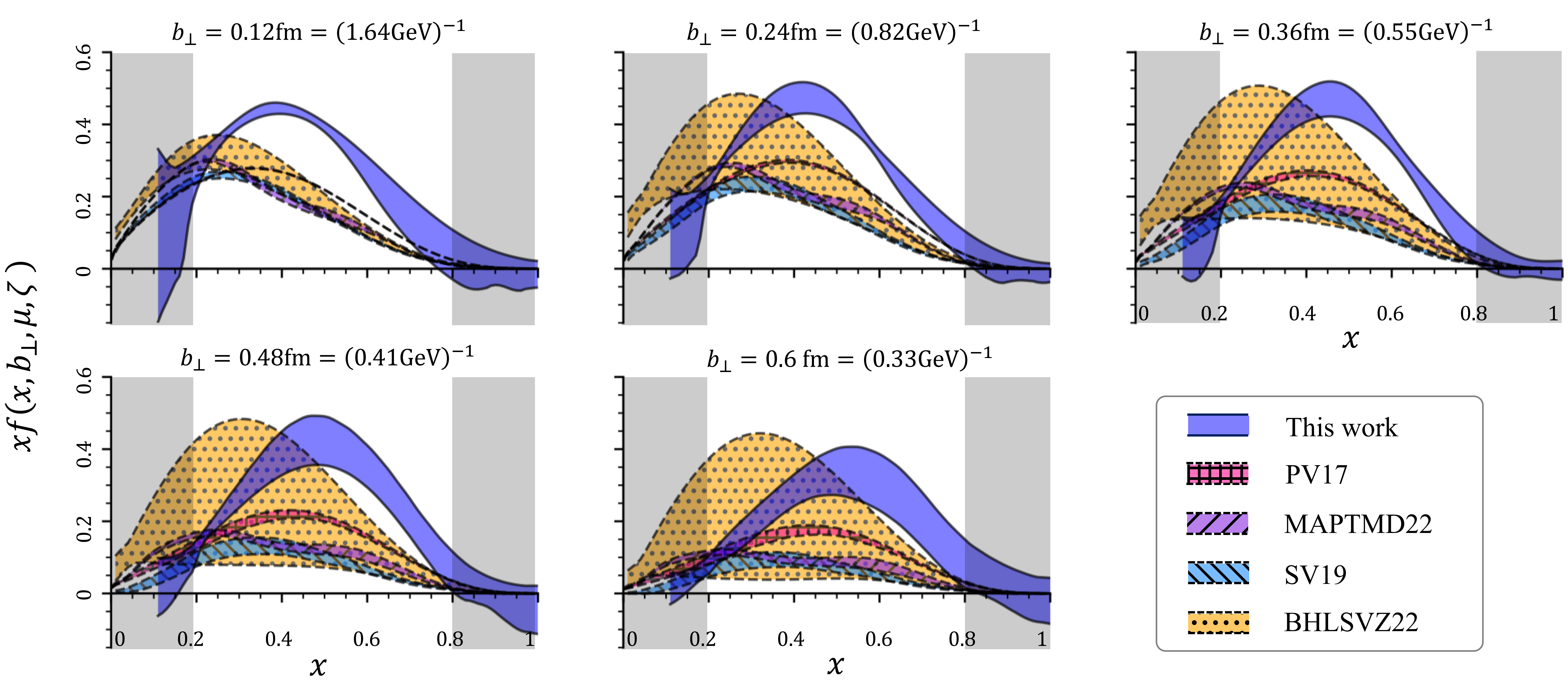
$b_\perp = 0.36$ fm, $\Gamma = \gamma^t$

$$f_\Gamma(x, b_\perp, \mu, \zeta; m_\pi, P^z) = f_\Gamma(x, b_\perp, \mu, \zeta)\Big|_{\substack{m_\pi \to m_{\pi,\rm phy} \\ P^z \to \infty}} \times \left[1 + d_0\left(m_\pi^2 - m_{\pi,\rm phy}^2\right) + \frac{d_1}{(P^z)^2}\right]$$

H. W. Lin, et al., arXiv:2011.14971 [hep-lat]

Result

Compared with phenomenological results



A. Bacchetta, et al. JHEP 10, 127 (2022)

M. Bury, et al. JHEP 10, 118 (2022)

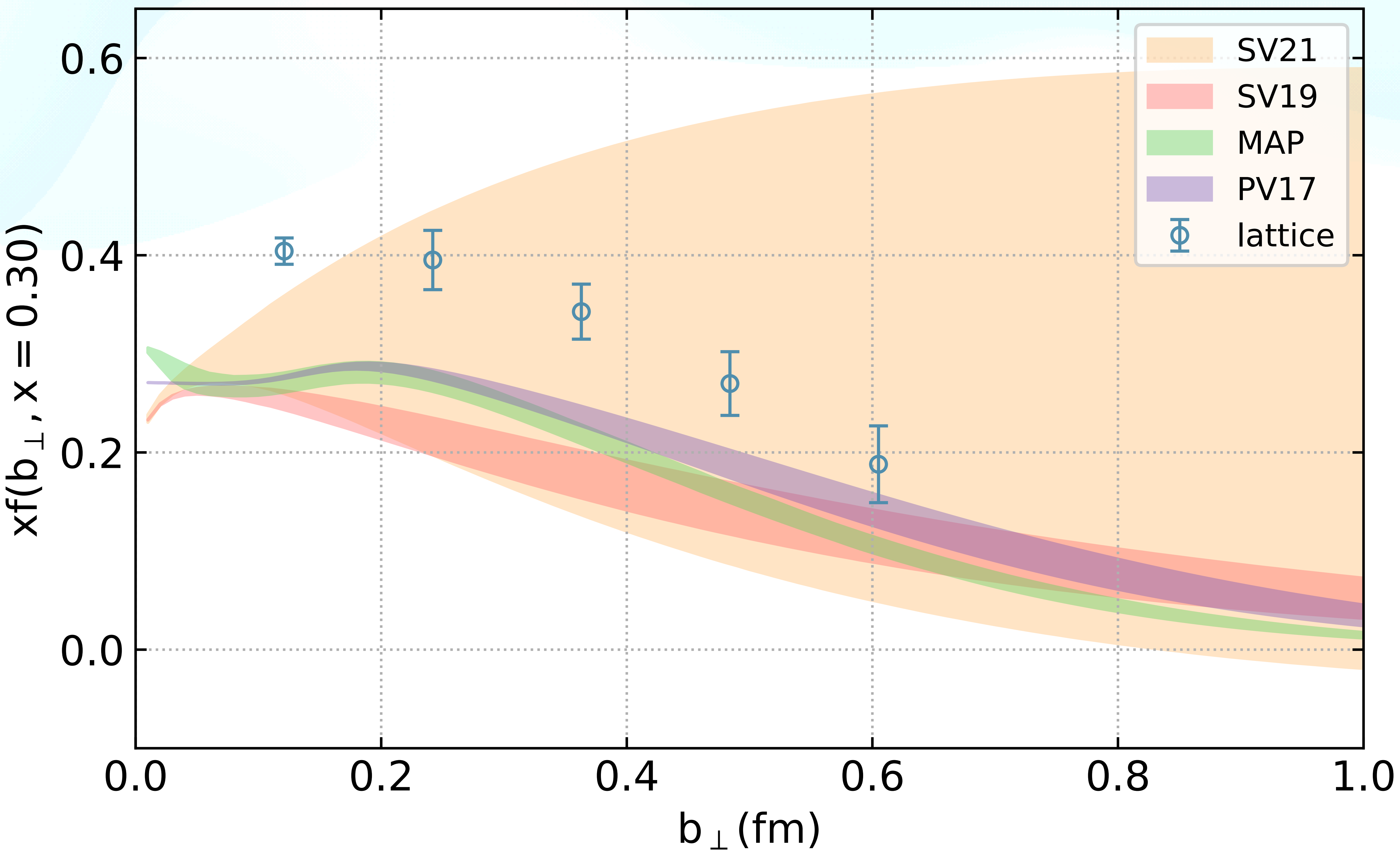
I. Scimemi, et al. JHEP 06, 137 (2020)

A. Bacchetta, et al. JHEP 06, 081 (2017)

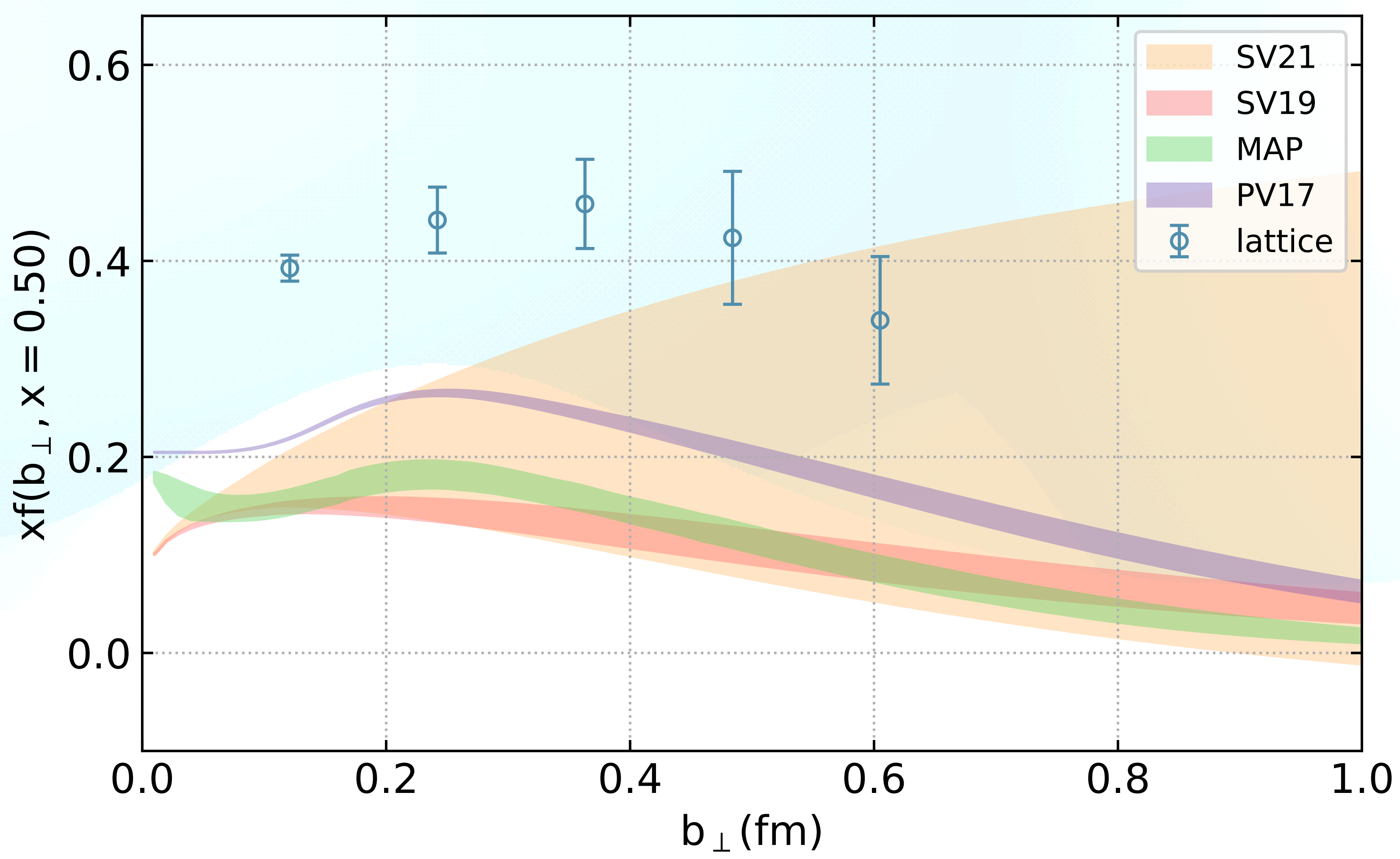
Result

Compared with phenomenological results

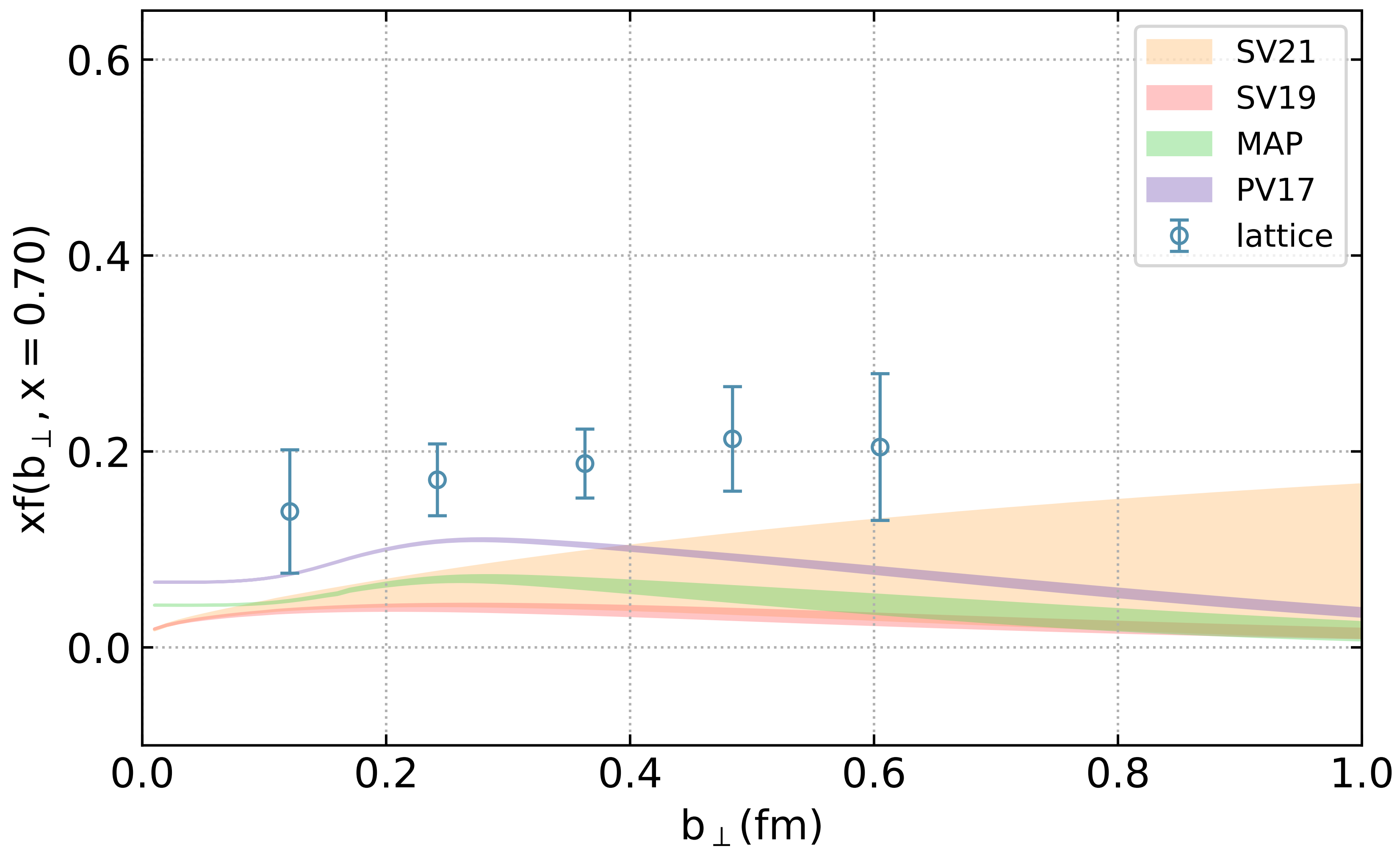
x=0.3



x=0.5



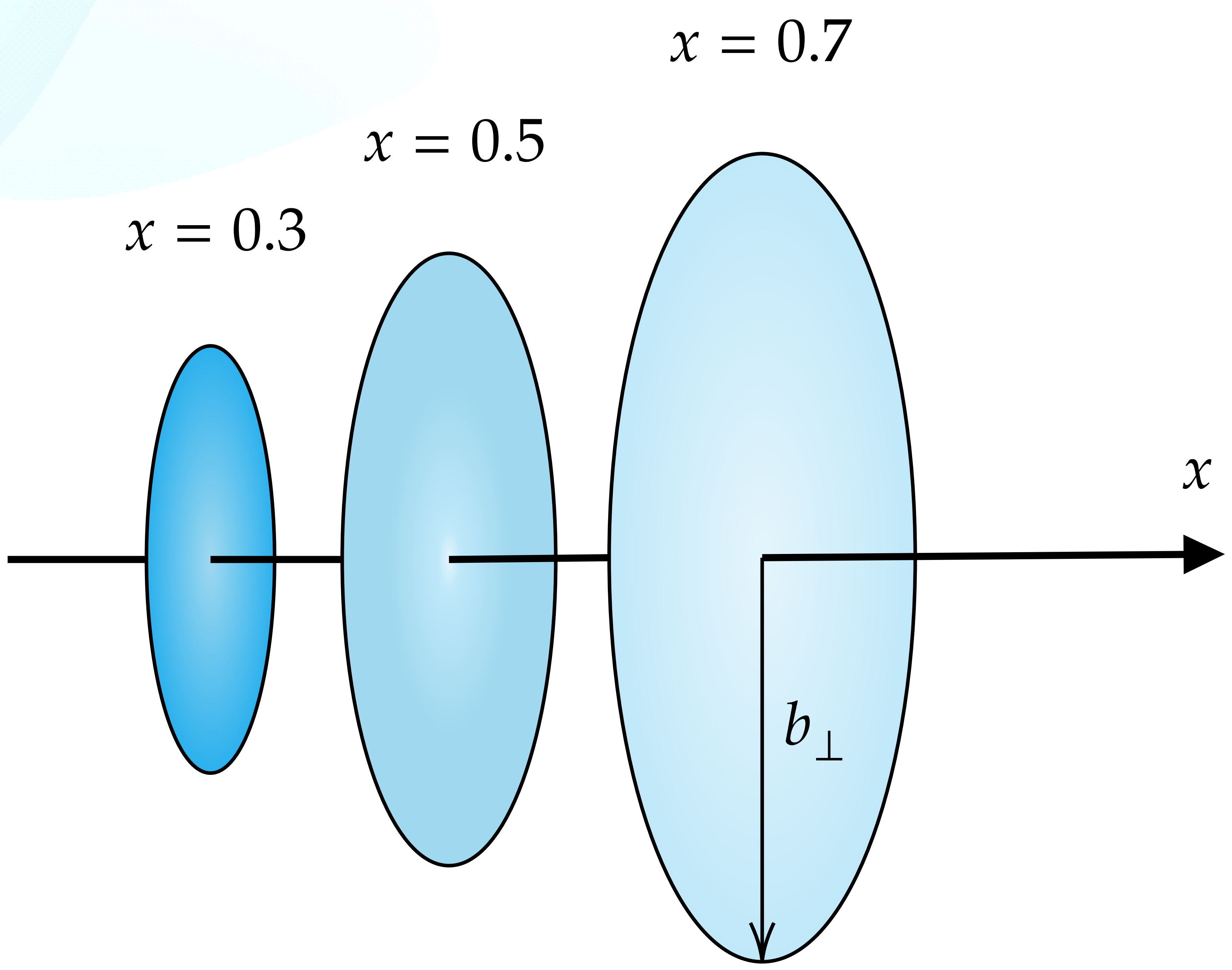
x=0.7



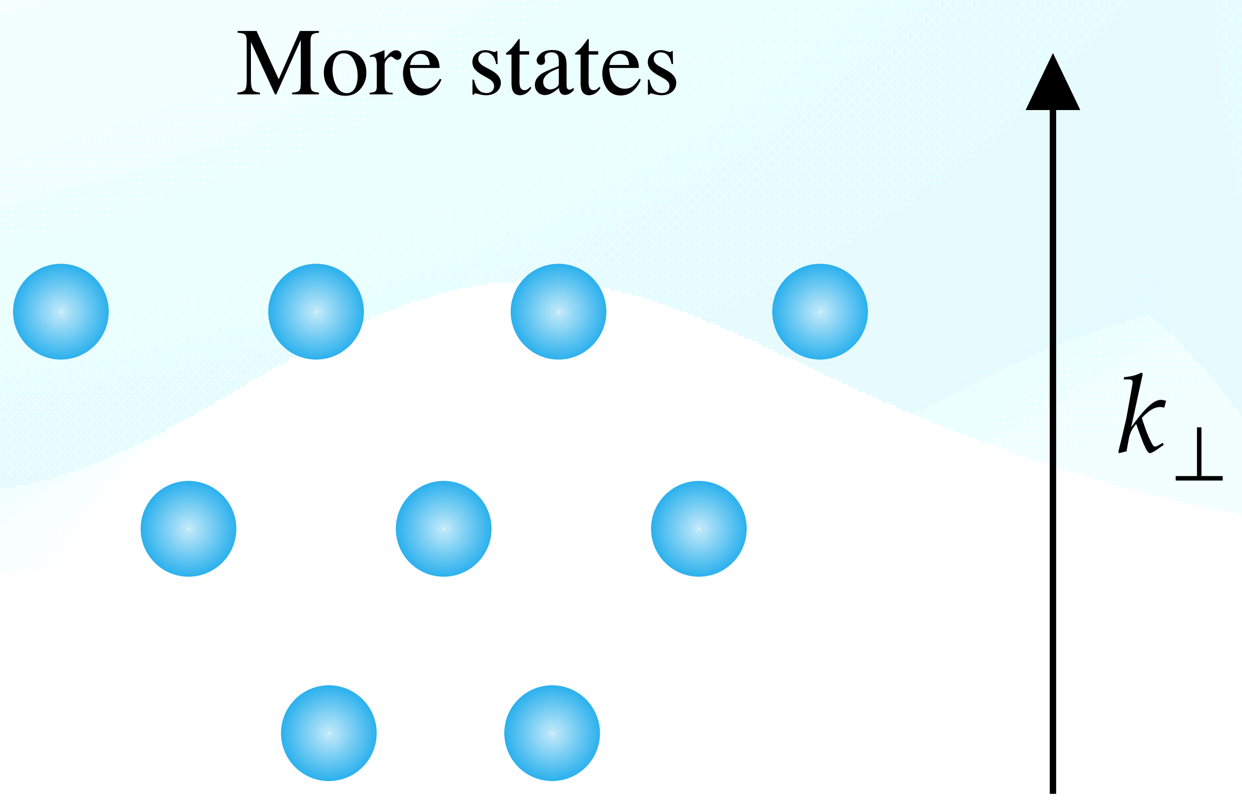
Result

TMDPDFs in transverse direction

Our results: large x — long correlation length



For small x



Wide distribution in $k_\perp$

FT

Narrow distribution in $b_\perp$

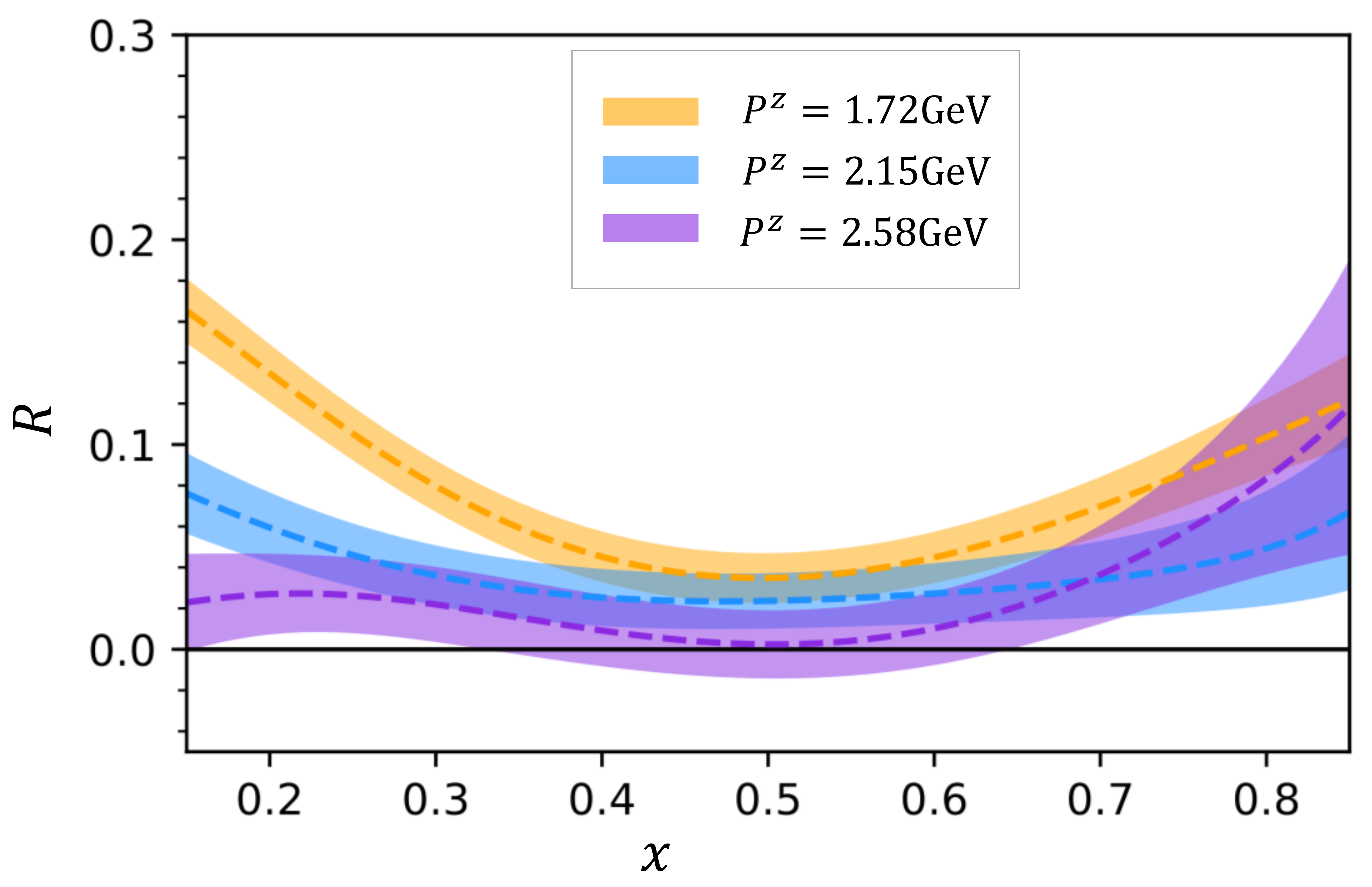
Result

Error estimation

$$f_{\gamma^t} \to f_{\mathrm{LP}} + \frac{m^2}{(P^z)^2} f_{\mathrm{NLP}} + \cdots$$

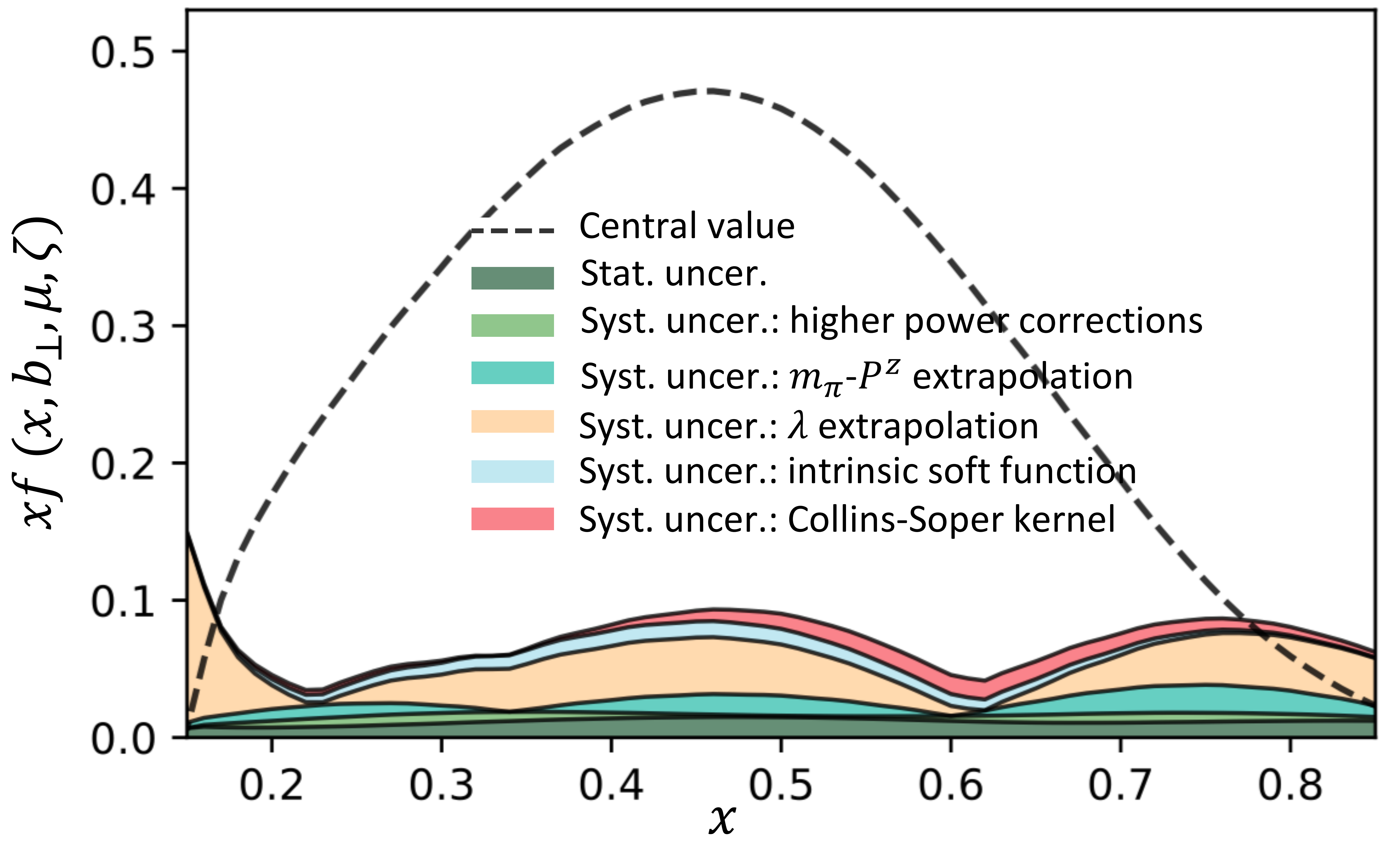
$$f_{\gamma^z} \to f_{\mathrm{LP}} - \frac{m^2}{(P^z)^2} f_{\mathrm{NLP}} + \cdots$$

$$R \equiv \frac{f_{\gamma^t} - f_{\gamma^z}}{f_{\gamma^t} + f_{\gamma^z}}$$



Result

Error estimation



Higher power correction: $f_{\gamma^t} - f_{\gamma^z}$

Combined extrapolation: reference point

Coordinate extrapolation: different start points

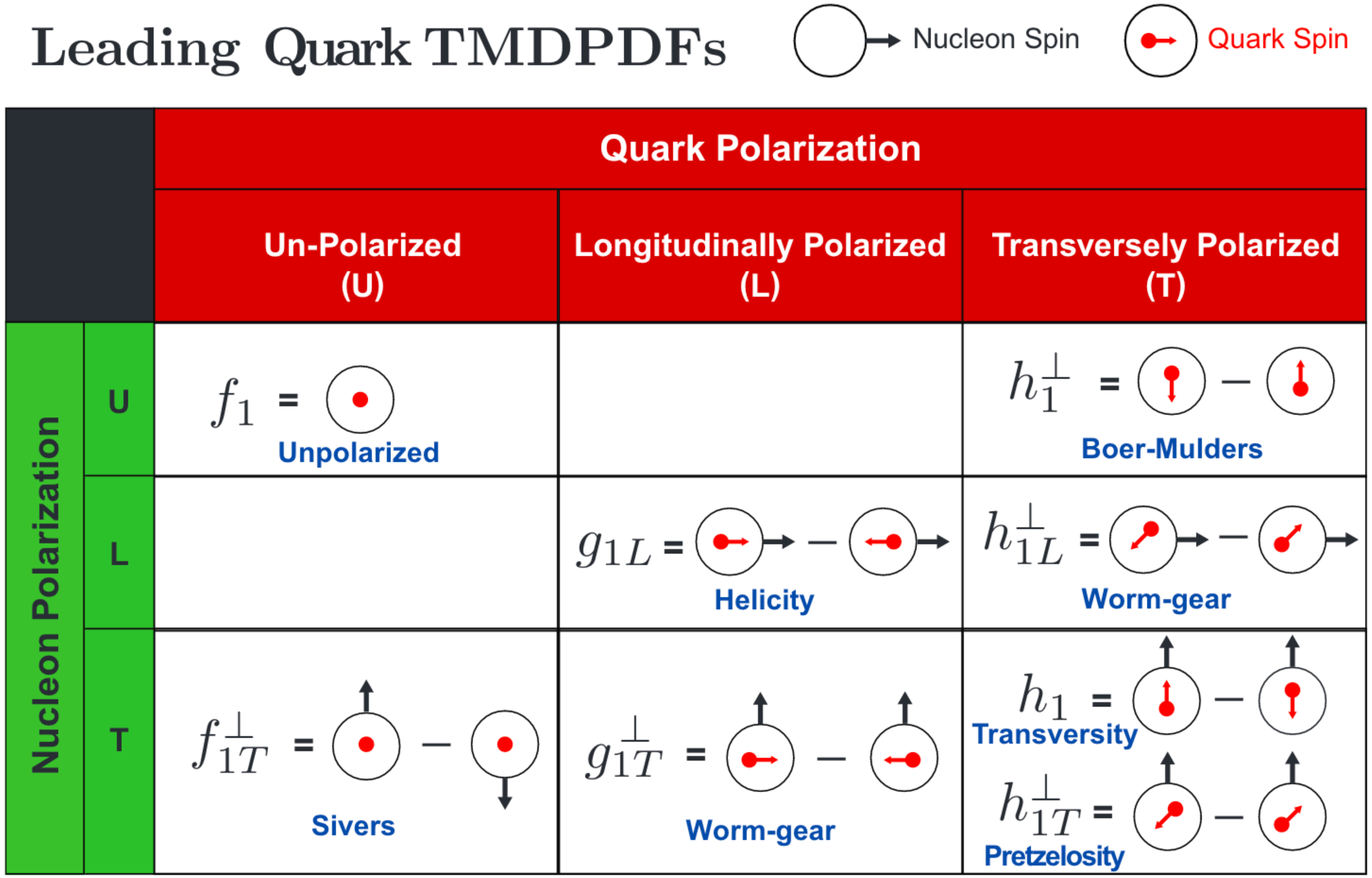
Prospect

What can we do next?

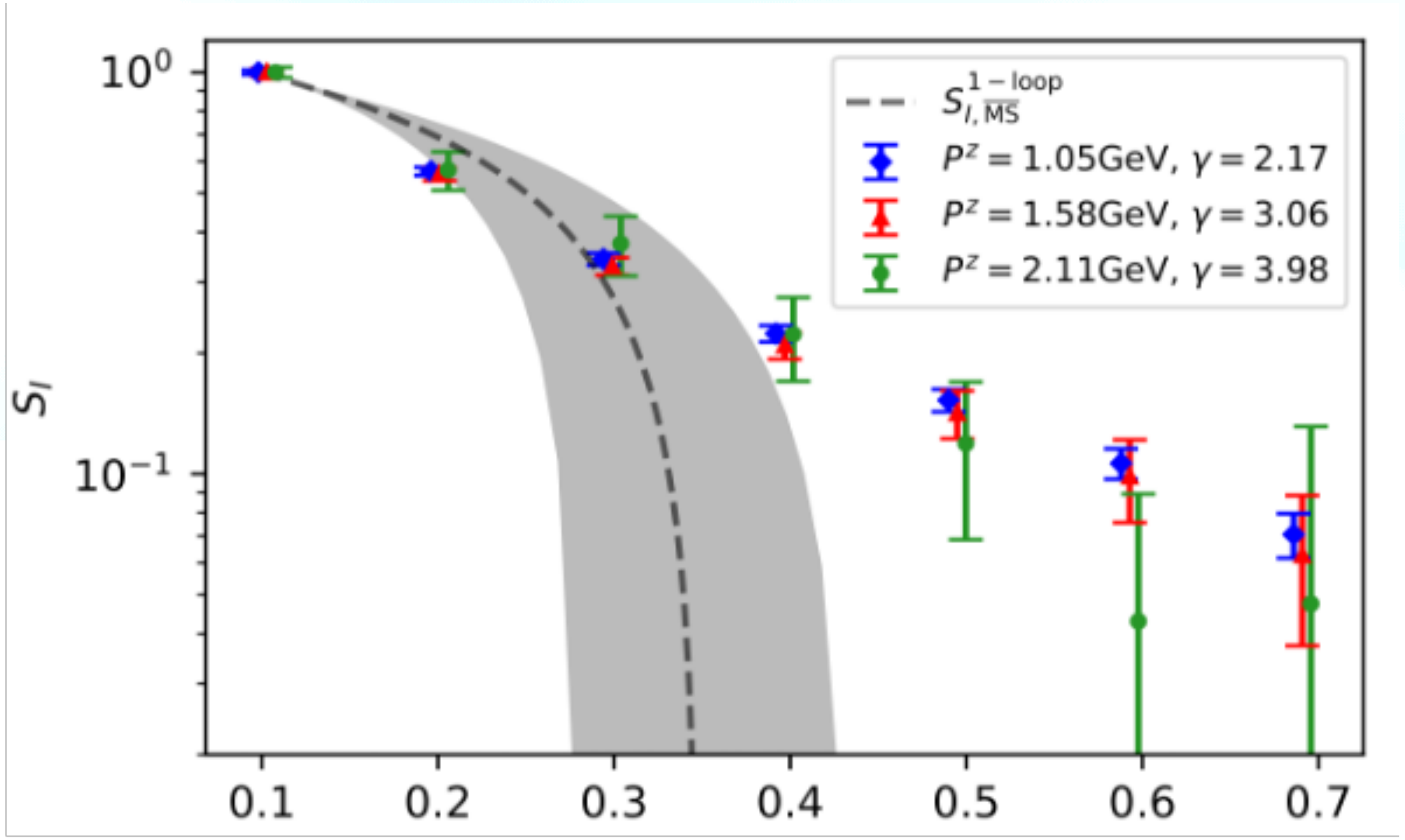
Directions	Status	Prospect
Lattice spacing	a=0.12 fm	Continuum limit
Hadron momentum	Pz=2.58 GeV	Larger gamma factor
Transverse behavior	b=0.6 fm	Better description / FT Confinement
More		Threshold resummation Operator mixing

Prospect

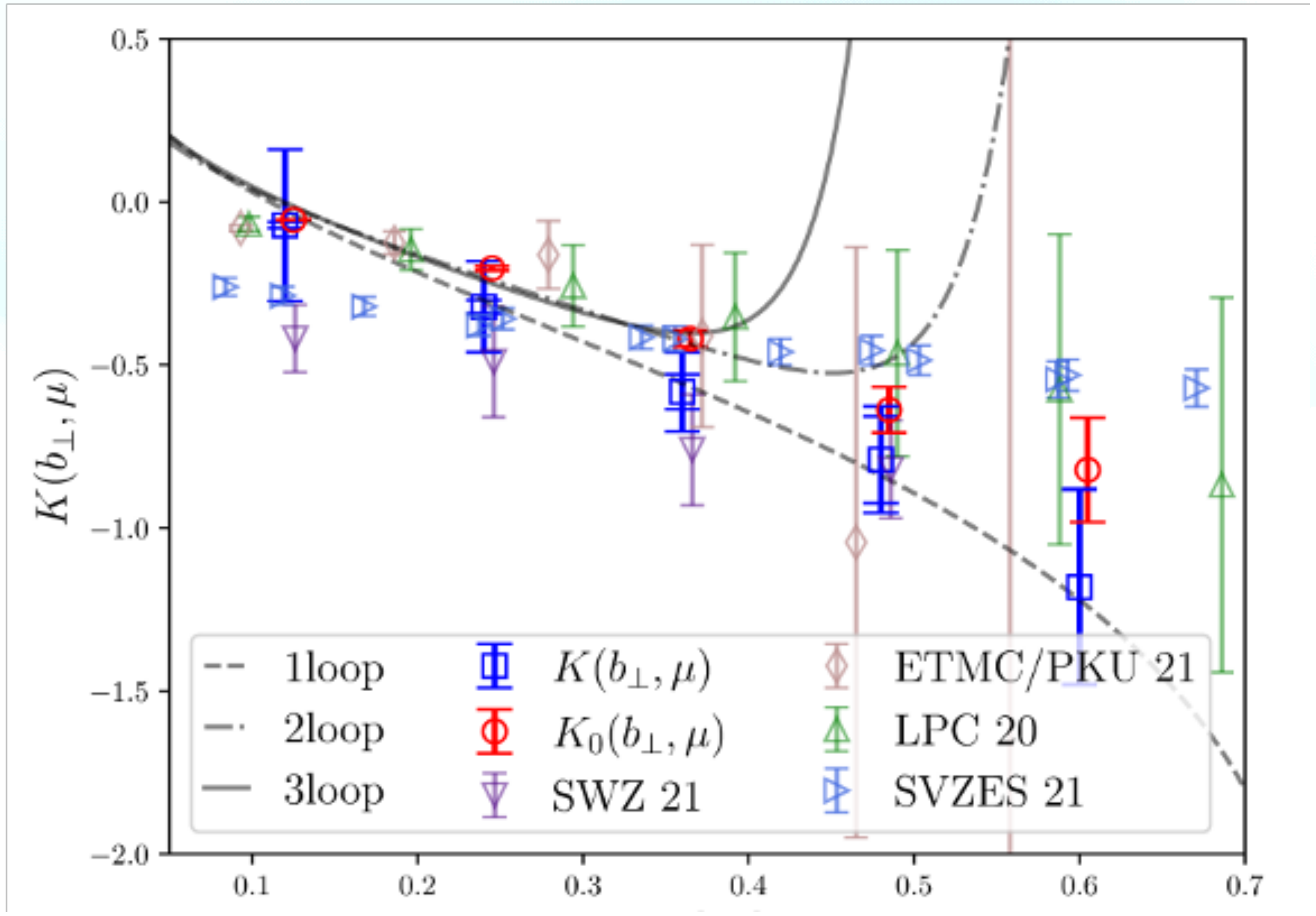
What we can do next?



Back up



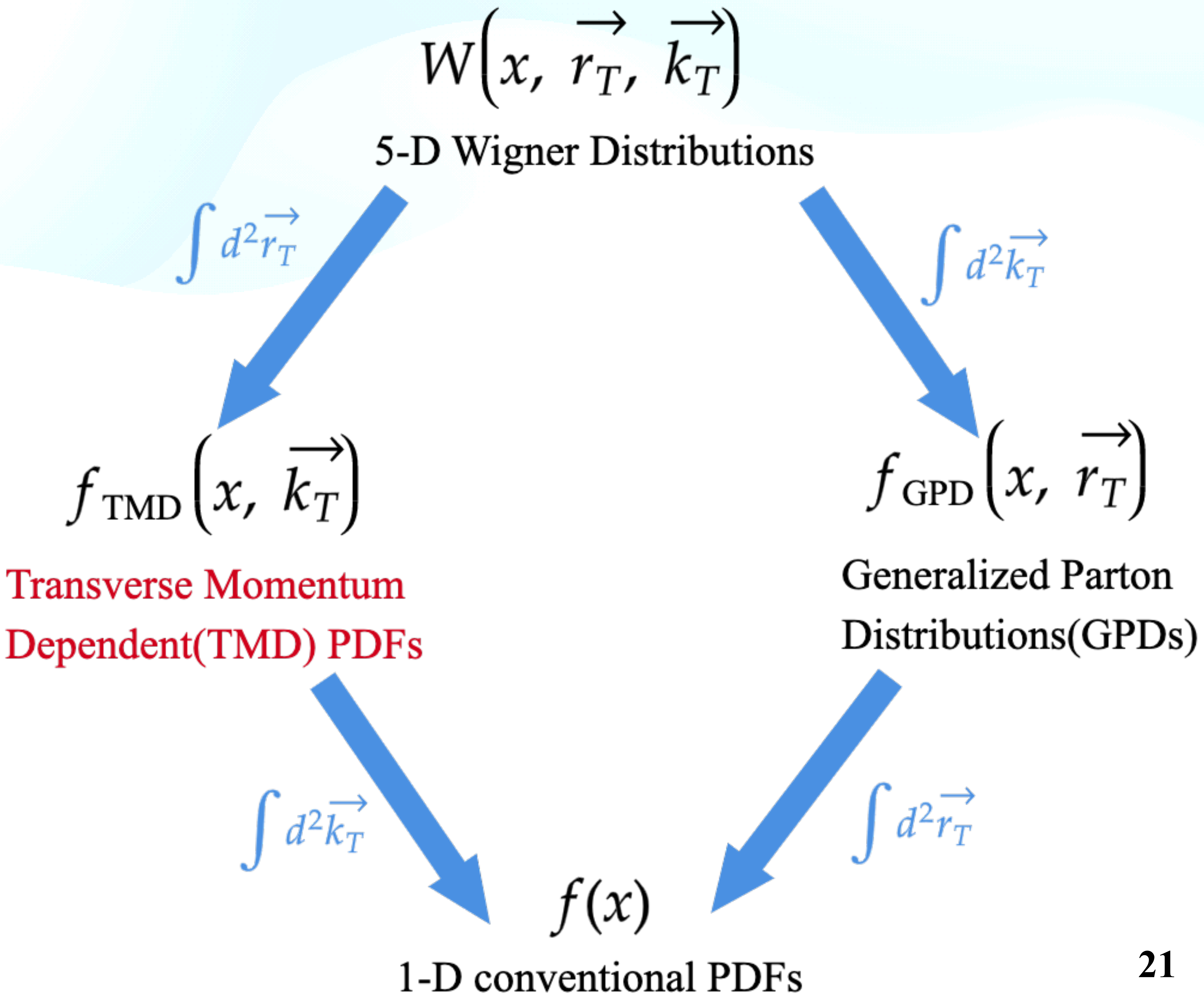
First lattice result of the soft function, PRL125(2020)



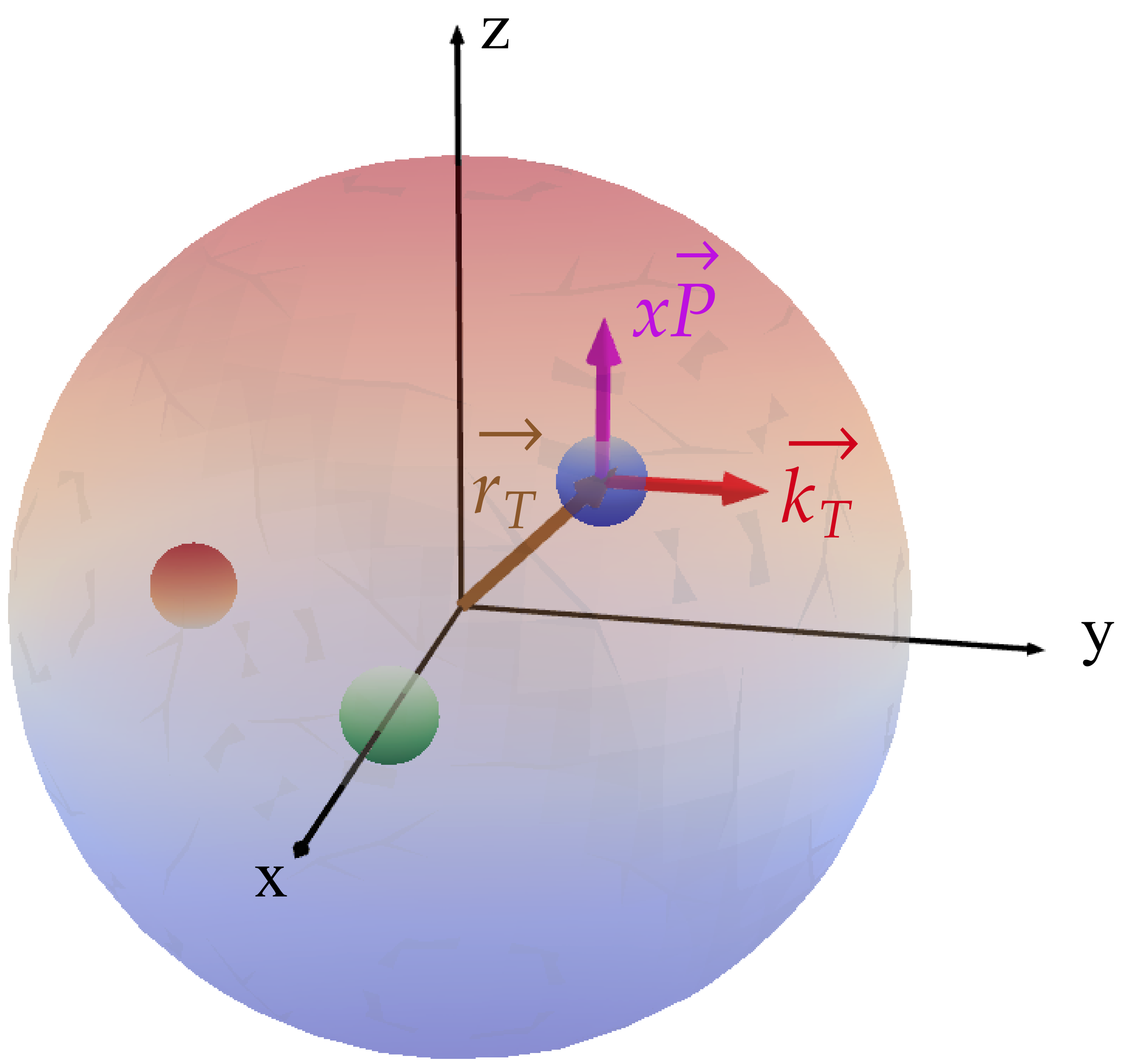
Lattice calculations of Collins-Soper kernel, PRD106(2022)

Background

What are TMDPDFs?



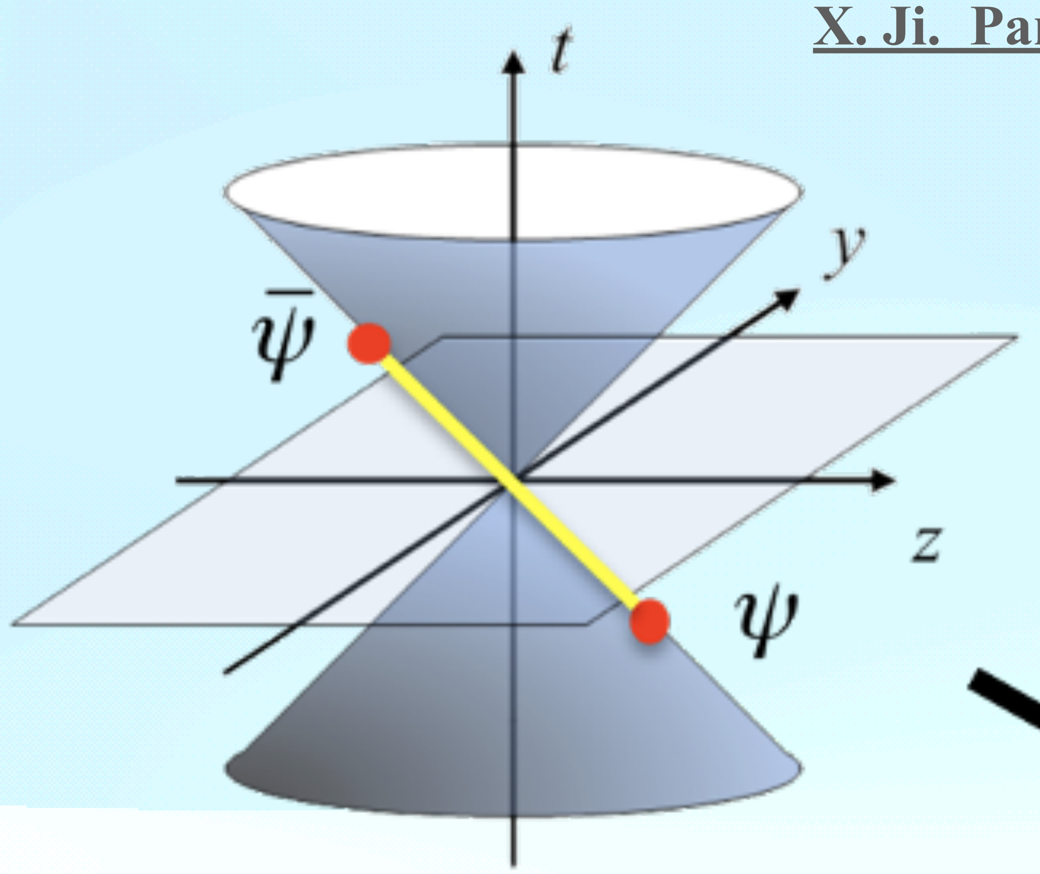
$x\overrightarrow{P}$: **longitudinal momentum of the parton**
$\overrightarrow{k_T}$: **transverse momentum of the parton**
$\overrightarrow{r_T}$: **average position corresponding to the center of the nucleon**

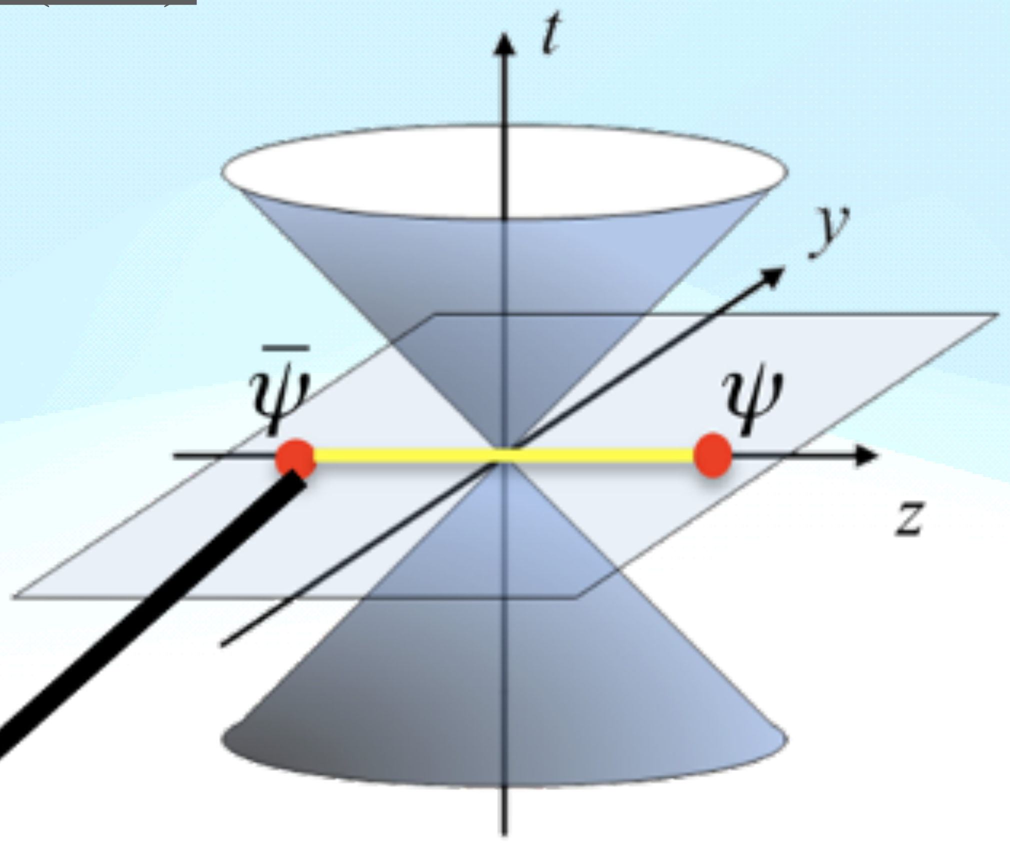


Methodology

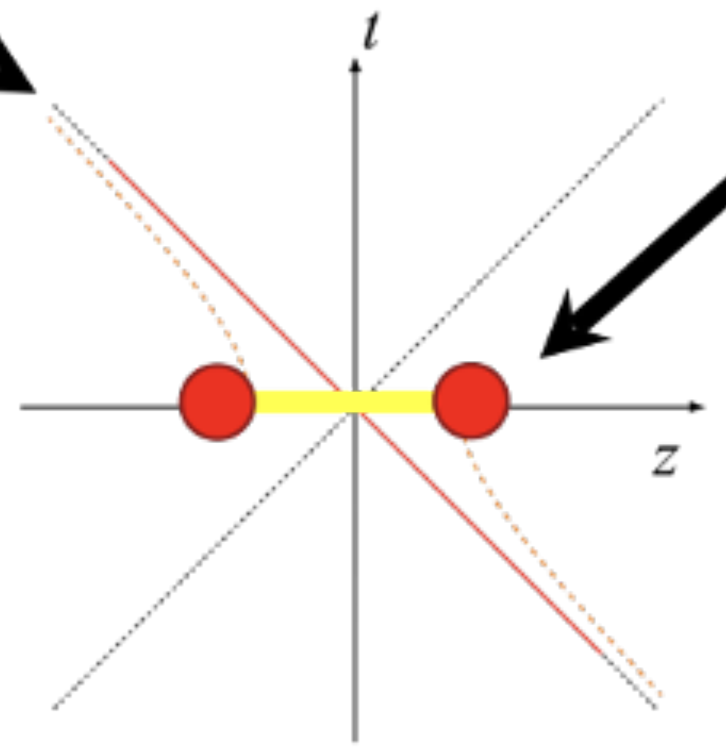
LaMET: First-principles calculation of TMDPDFs is feasible

X. Ji. Parton Physics on a Euclidean Lattice, Phys.Rev.Lett. 110, 262002 (2013).



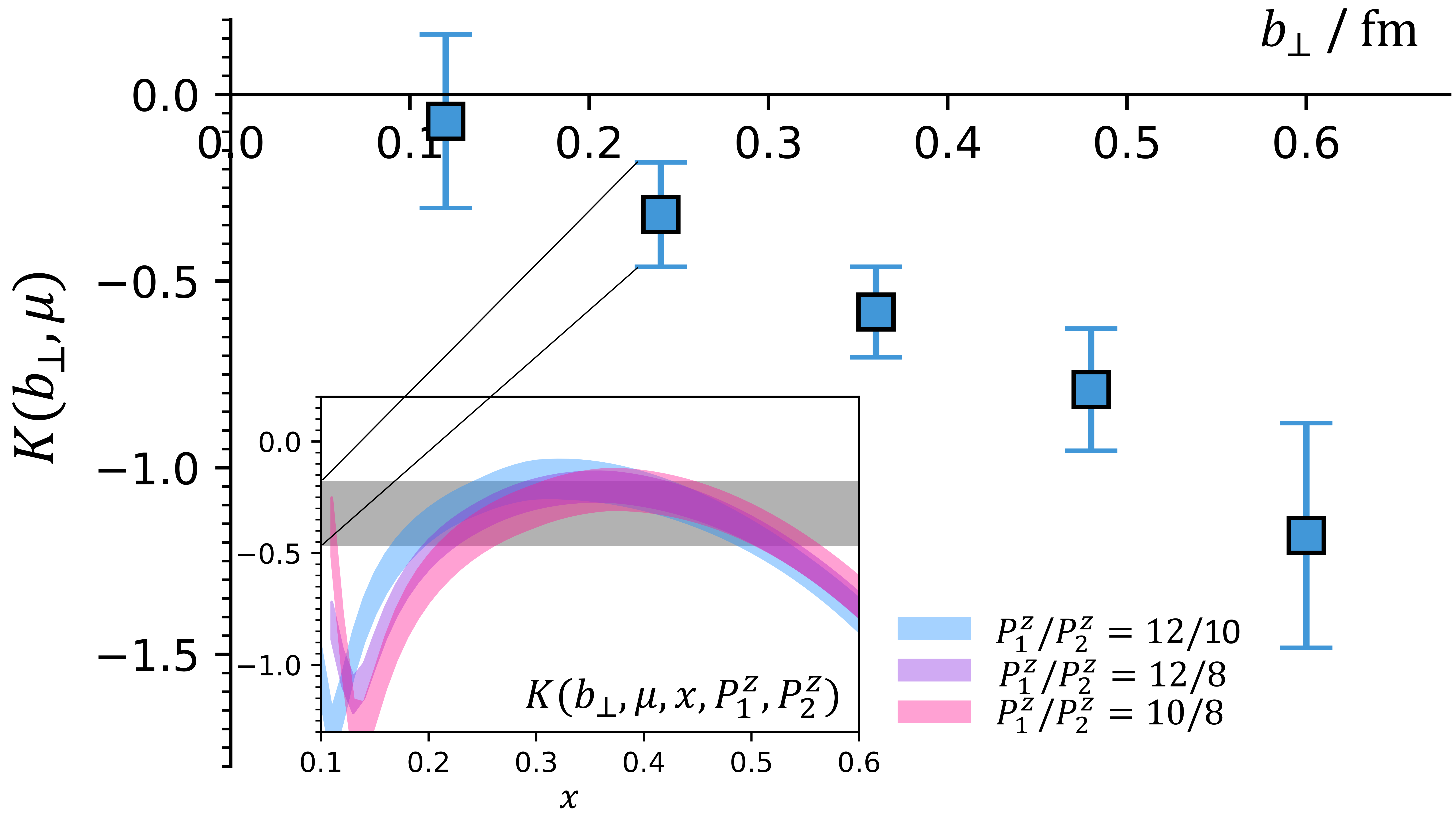


Light-cone distribution:
Separated on the time axis;
Cannot be calculated on the lattice



Quasi distribution:
Equal time;
Directly calculable on the lattice

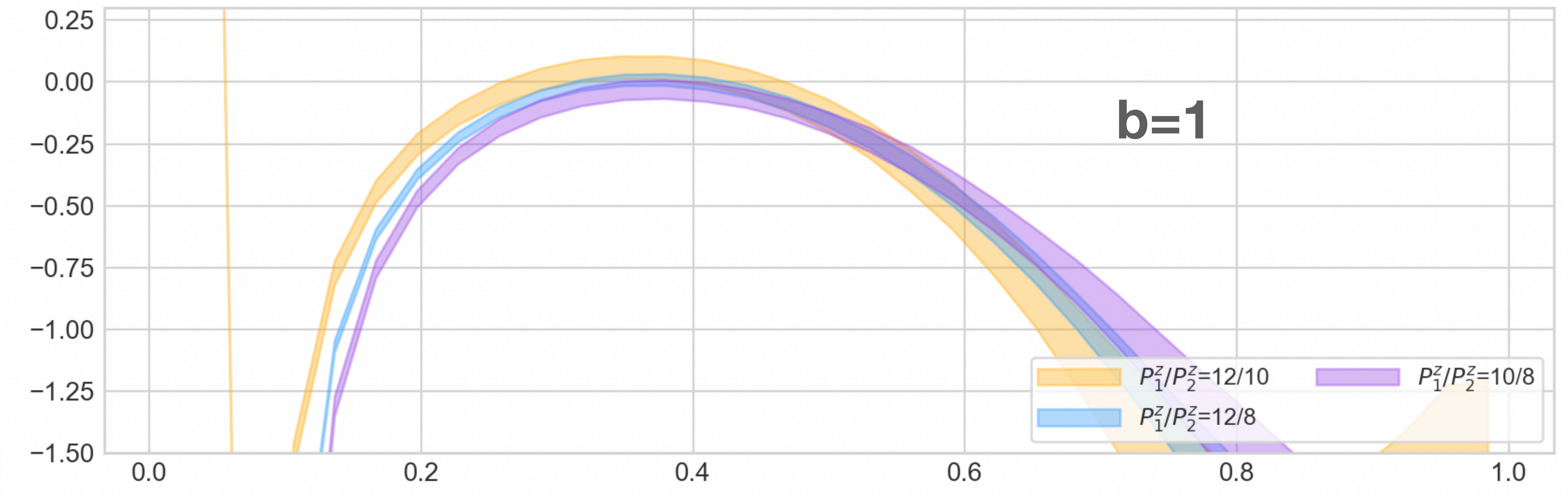
$$\tilde{f}_\Gamma\left(x, b_\perp, \zeta_z, \mu\right)\sqrt{S_I\left(b_\perp, \mu\right)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right)K\left(b_\perp,\mu\right)} \times f\left(x, b_\perp, \mu, \zeta\right) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2\zeta_z}\right)$$



$b_\perp$ / fm
0.0
−0.5
−1.0
−1.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
$K(b_\perp,\mu)$
$K(b_\perp,\mu,x,P_1^z,P_2^z)$
x
$P_1^z/P_2^z = 12/10$
$P_1^z/P_2^z = 12/8$
$P_1^z/P_2^z = 10/8$

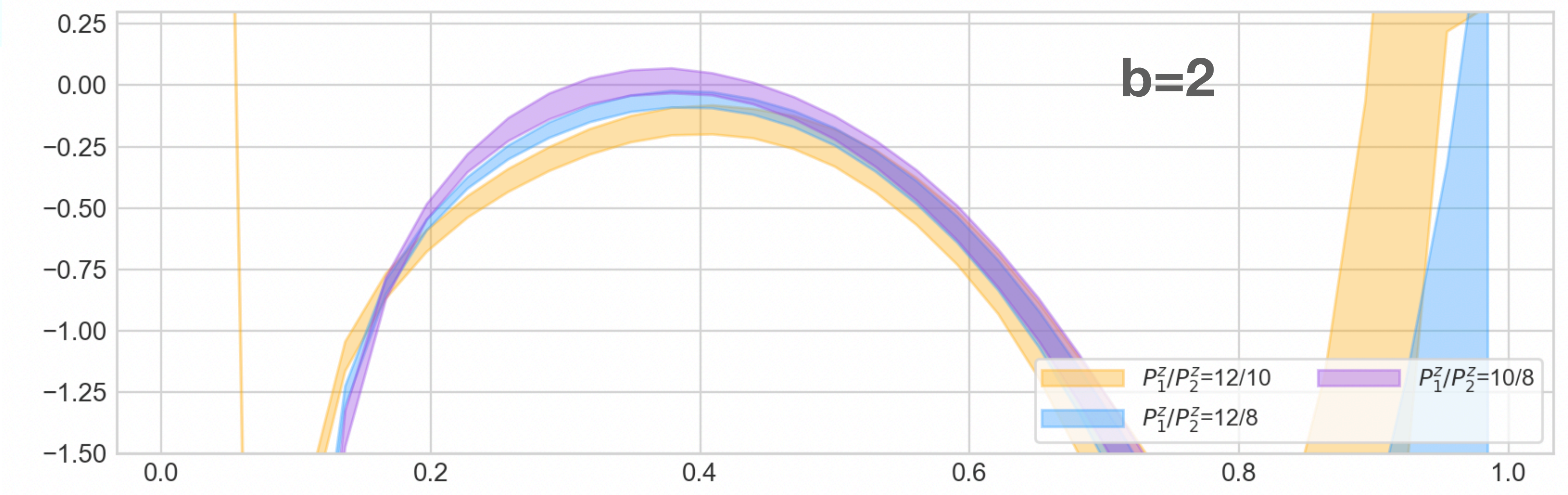
CS kernel

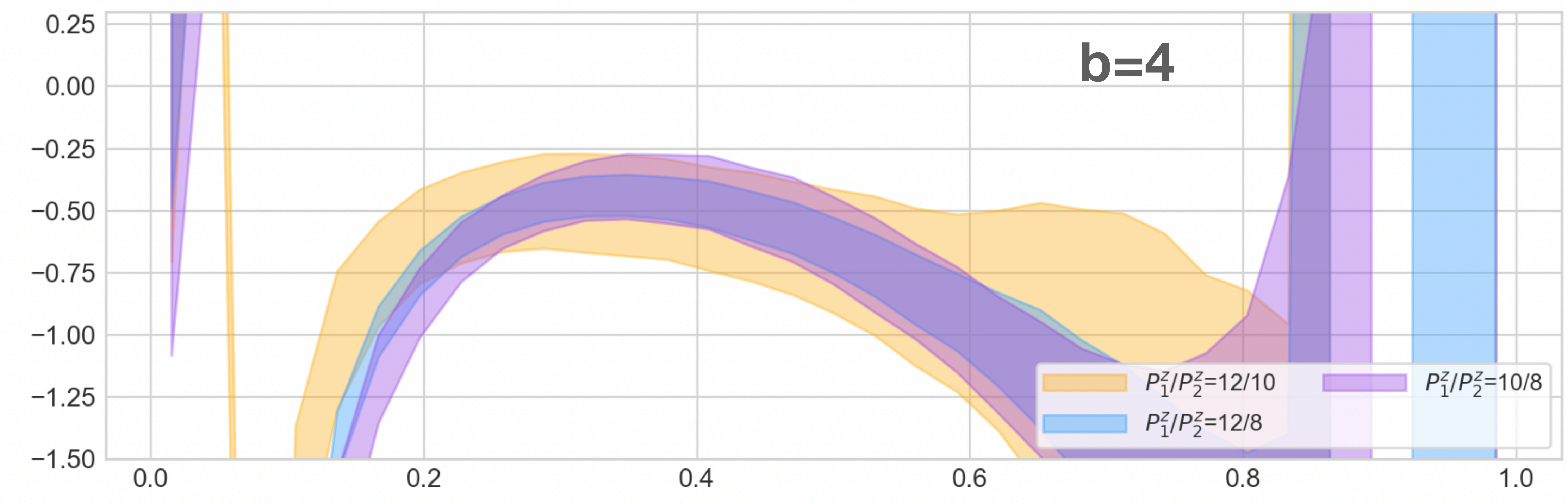
$$K\left(b_{\perp},\mu\right)=\frac{1}{\ln(P_1^z/P_2^z)}\ln\frac{H^{\pm}(xP_2^z,\mu)\tilde{\Psi}^{\pm}(x,b_{\perp},\mu,P_1^z)}{H^{\pm}(xP_1^z,\mu)\tilde{\Psi}^{\pm}(x,b_{\perp},\mu,P_2^z)}.$$

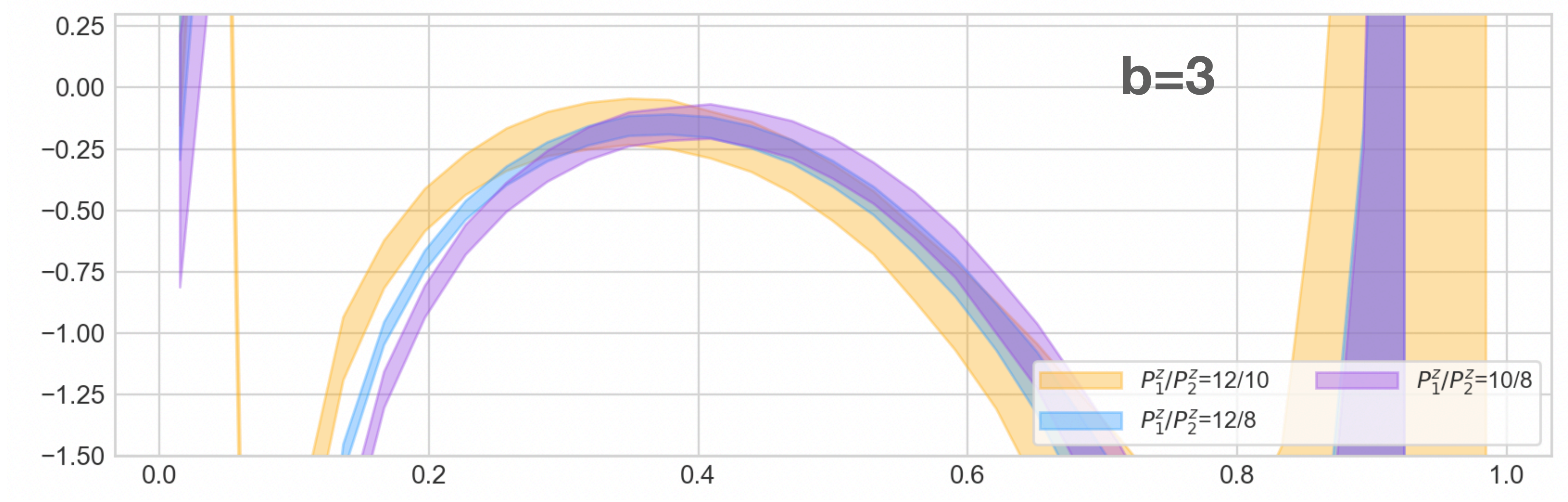


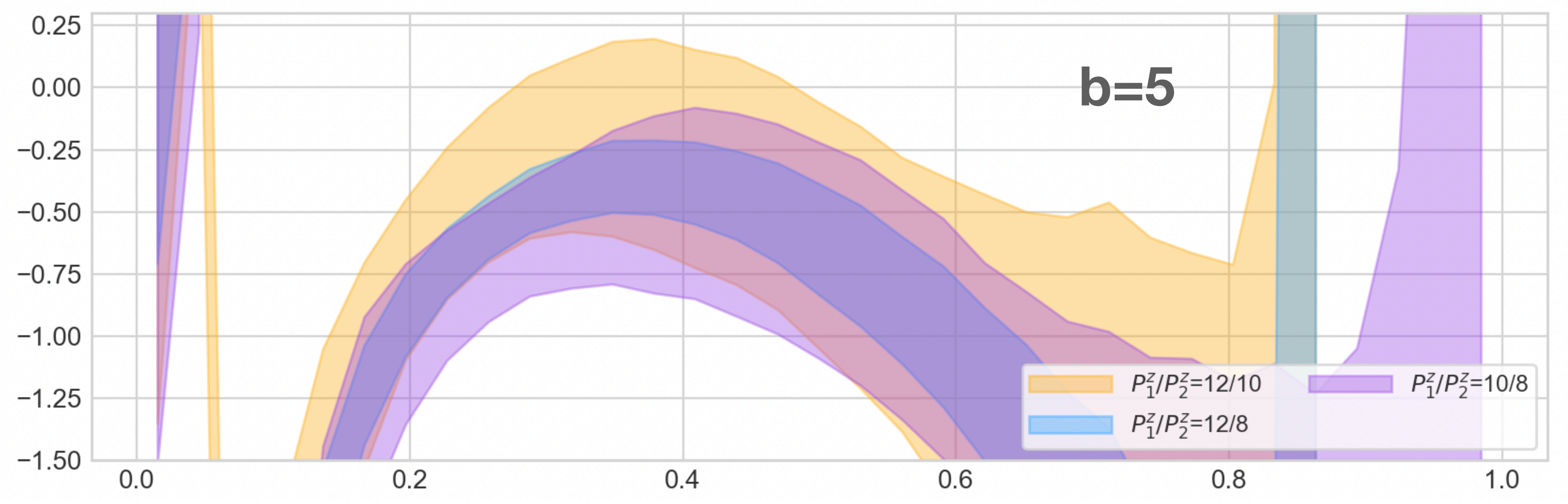
NLO

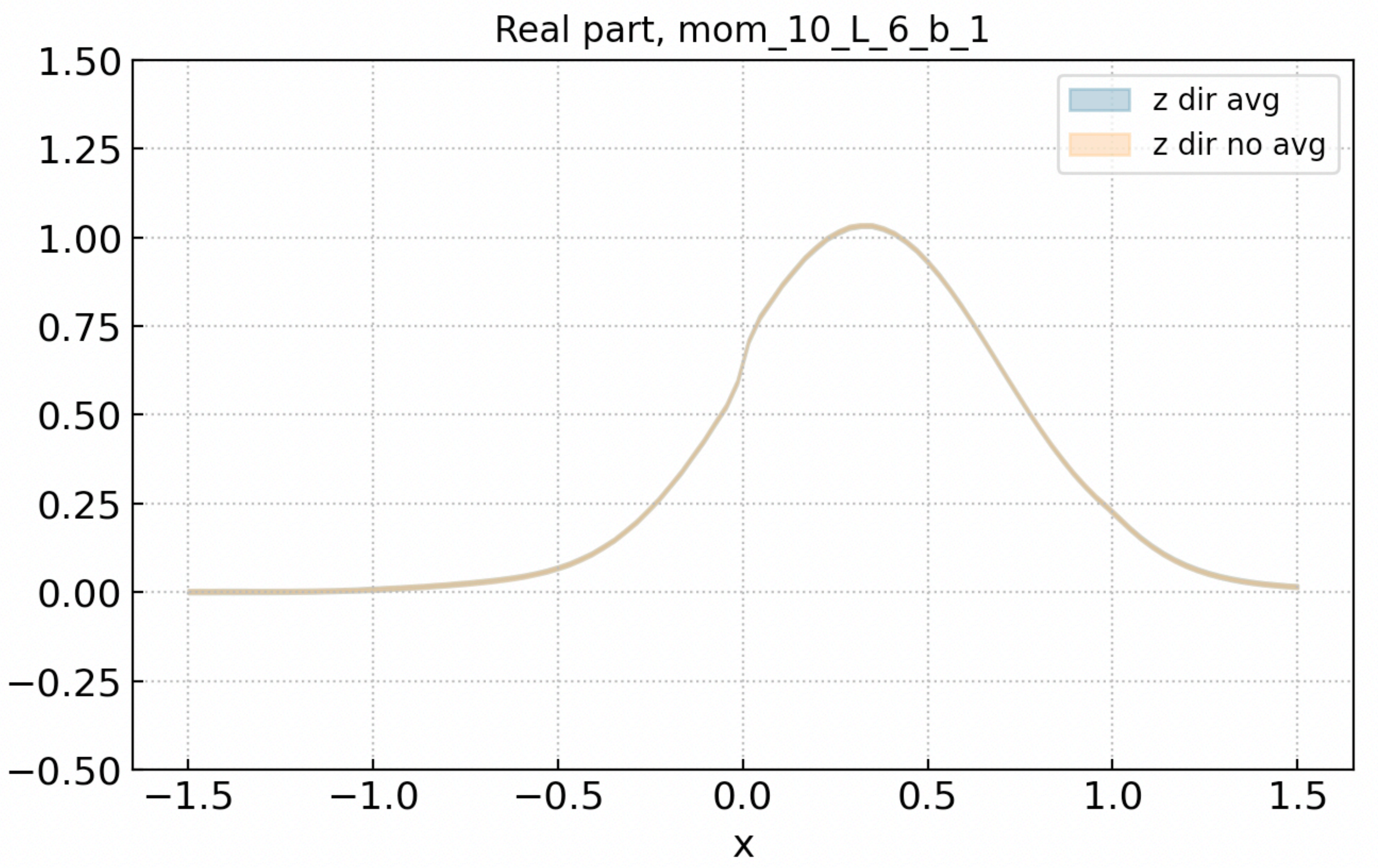
$$\begin{aligned}C_{\rm ns}^{\rm TMD}(\mu,xP^z)&\equiv C_{\rm ns}^{\rm TMD}[x,\mu,P^z,\tilde{\zeta}(x,P^z)=(2xP^z)^2]\\&=1+\frac{\alpha_sC_F}{4\pi}\left(-\ln^2\frac{(2xP^z)^2}{\mu^2}+2\ln\frac{(2xP^z)^2}{\mu^2}-4+\frac{\pi^2}{6}\right)+\mathcal{O}(\alpha_s^2)\,.\end{aligned}$$





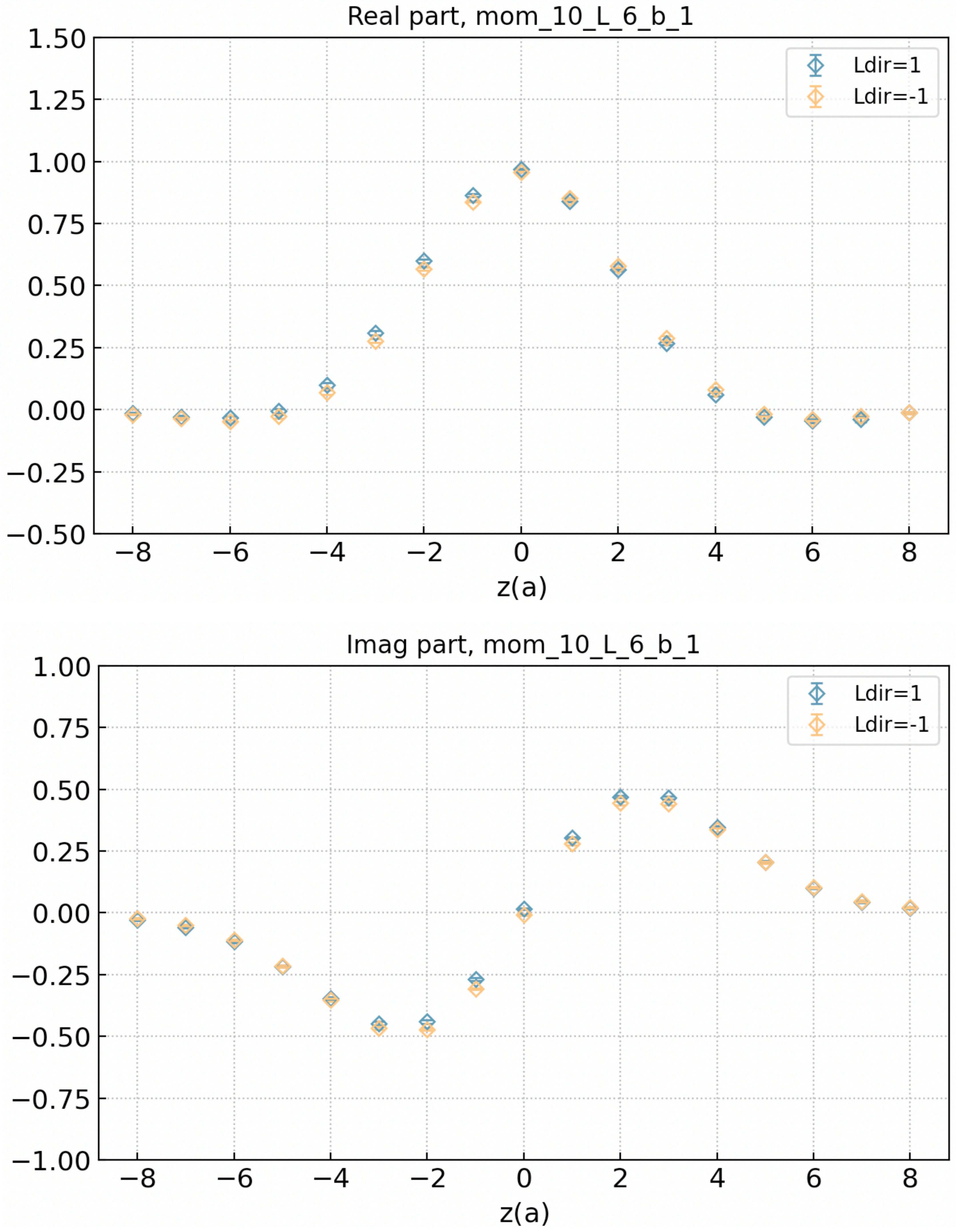




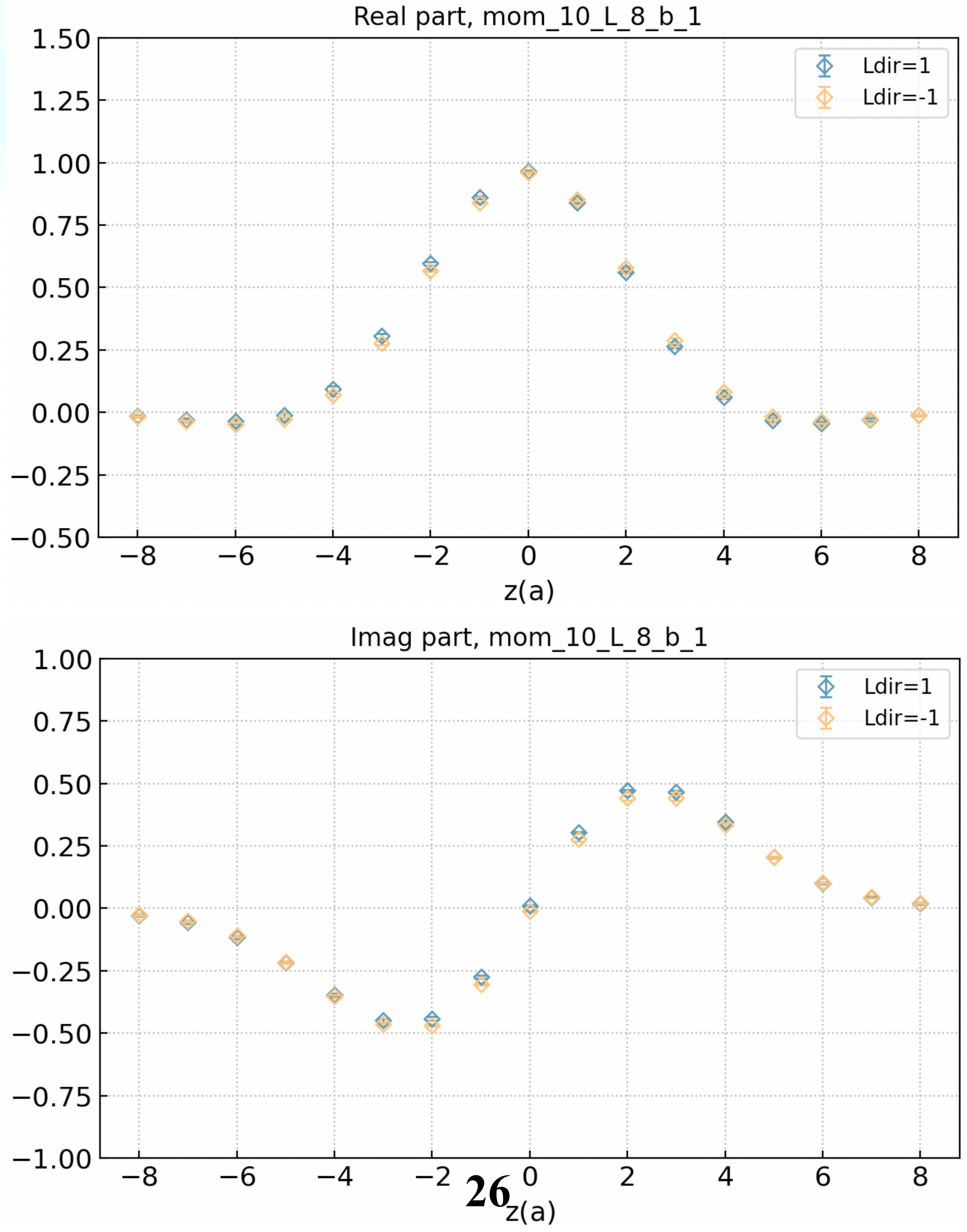


Real part, mom_10_L_6_b_1
z dir avg
z dir no avg
1.50
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
x

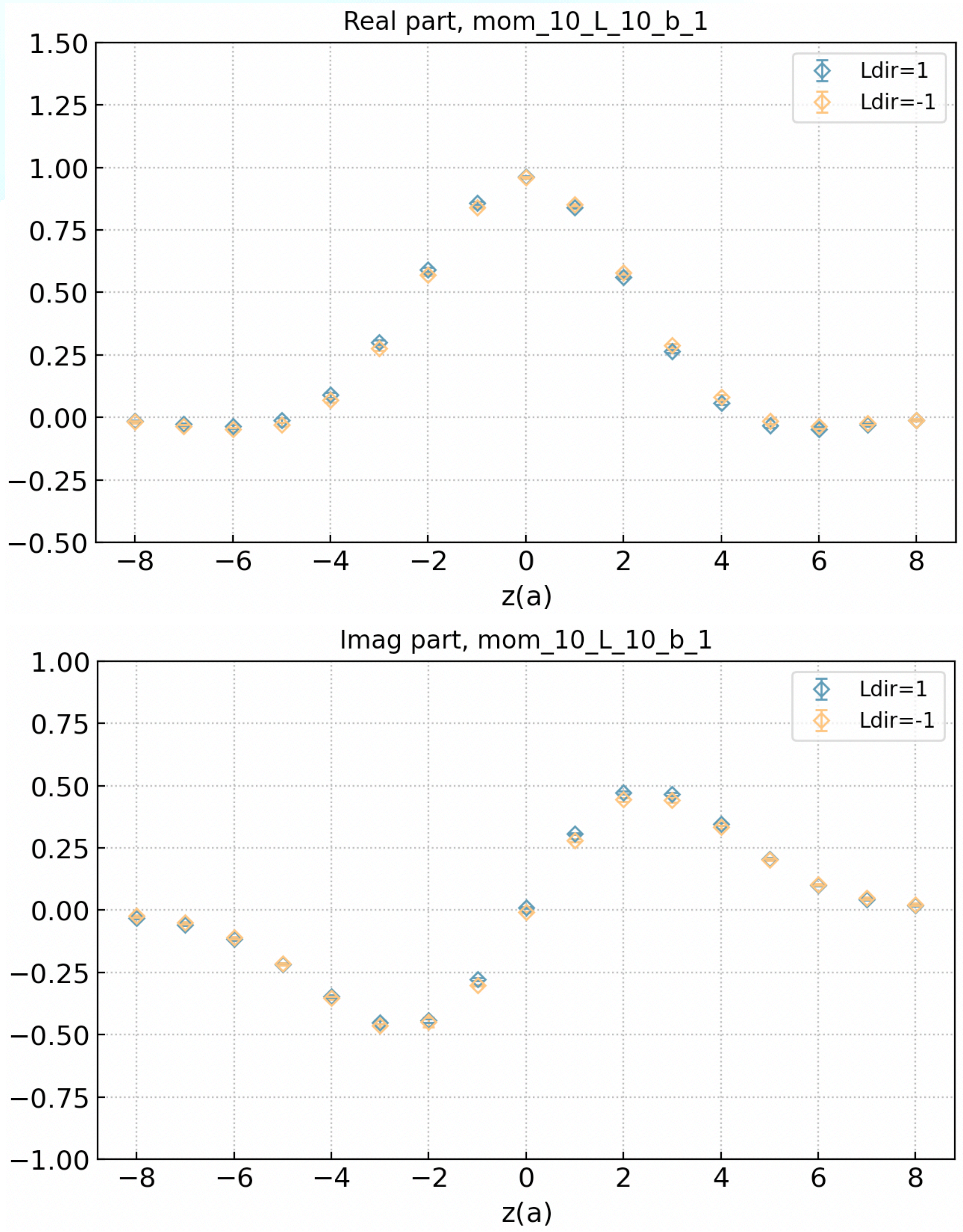
L=6


Real part, mom_10_L_6_b_1
Ldir=1
Ldir=-1
z(a)
Imag part, mom_10_L_6_b_1
Ldir=1
Ldir=-1
z(a)

L=8


Real part, mom_10_L_8_b_1
Ldir=1
Ldir=-1
z(a)
Imag part, mom_10_L_8_b_1
Ldir=1
Ldir=-1
z(a)

L=10

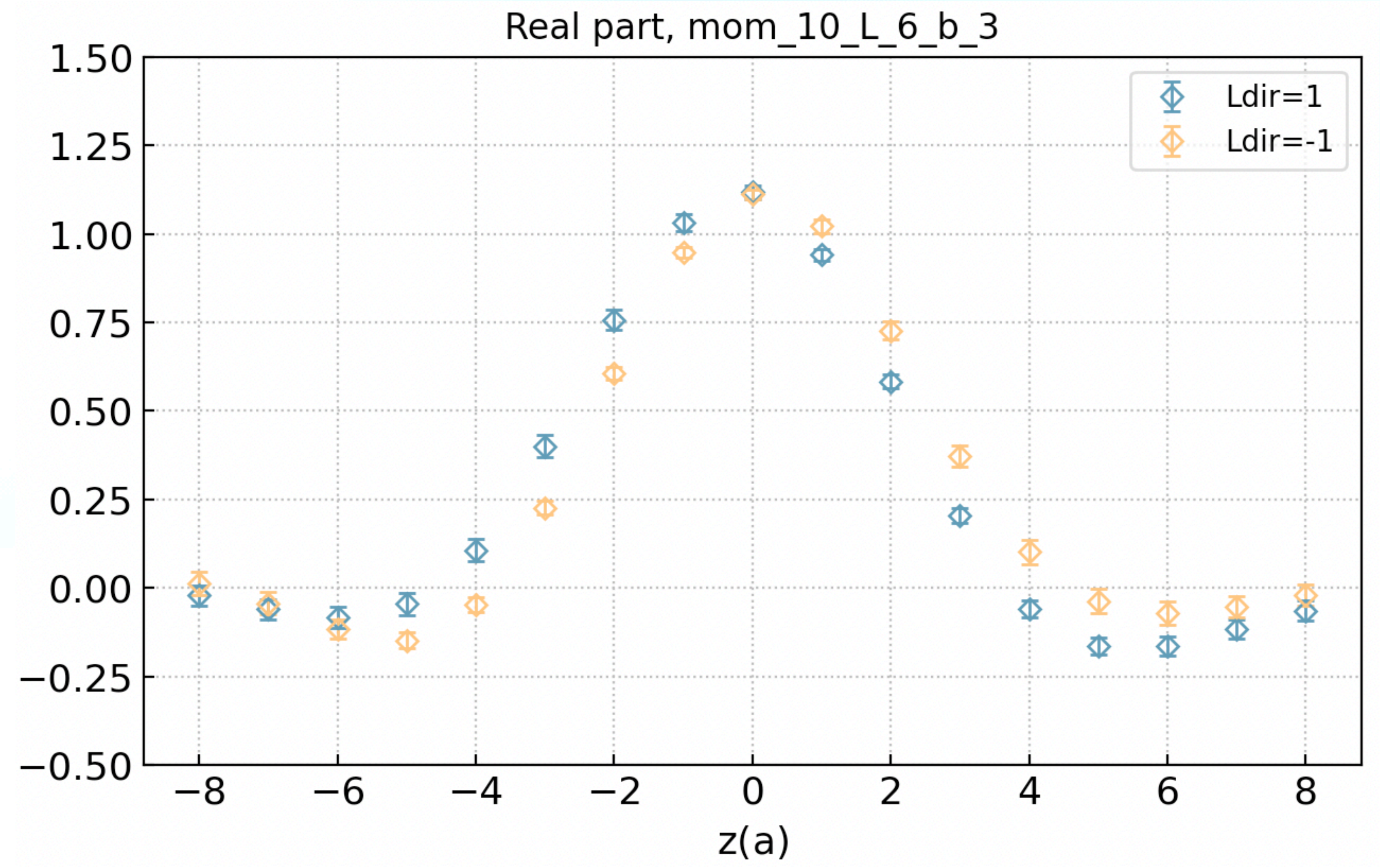

Real part, mom_10_L_10_b_1
Ldir=1
Ldir=-1
z(a)
Imag part, mom_10_L_10_b_1
Ldir=1
Ldir=-1
z(a)

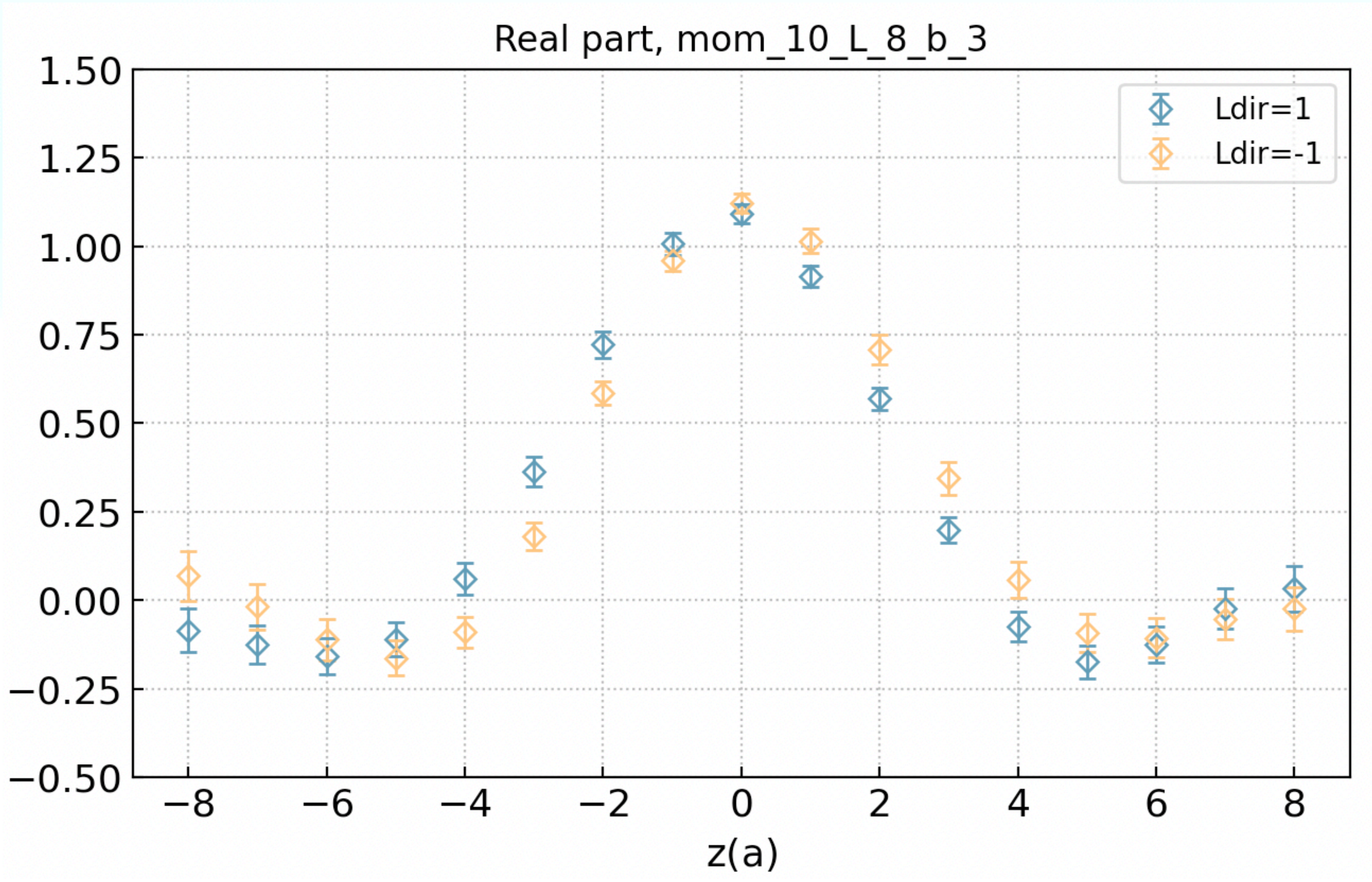
L dependence check, N configs = 998, mom10, b3, Z_O=1.05, tseq=[4,9]

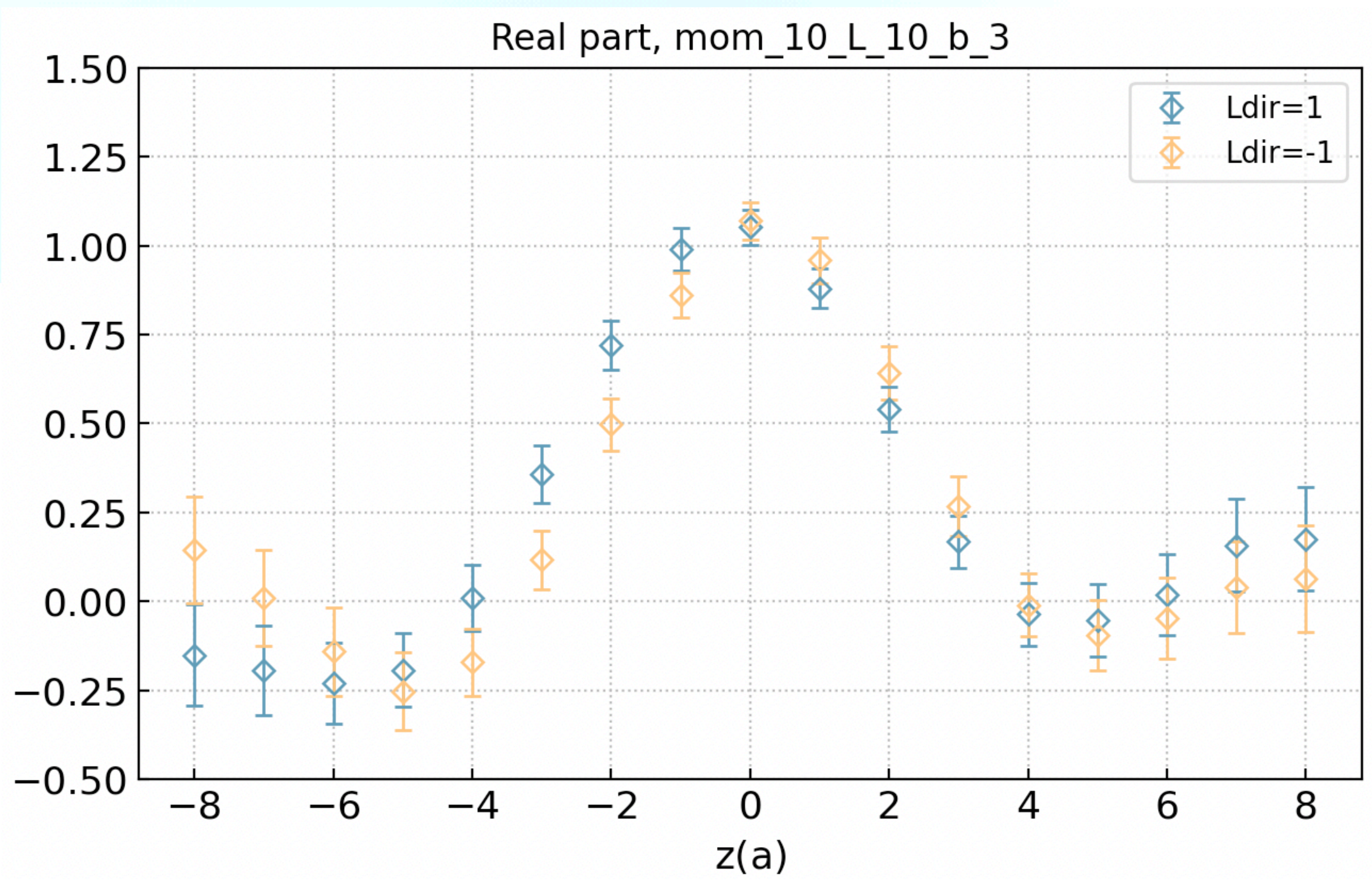
L=6

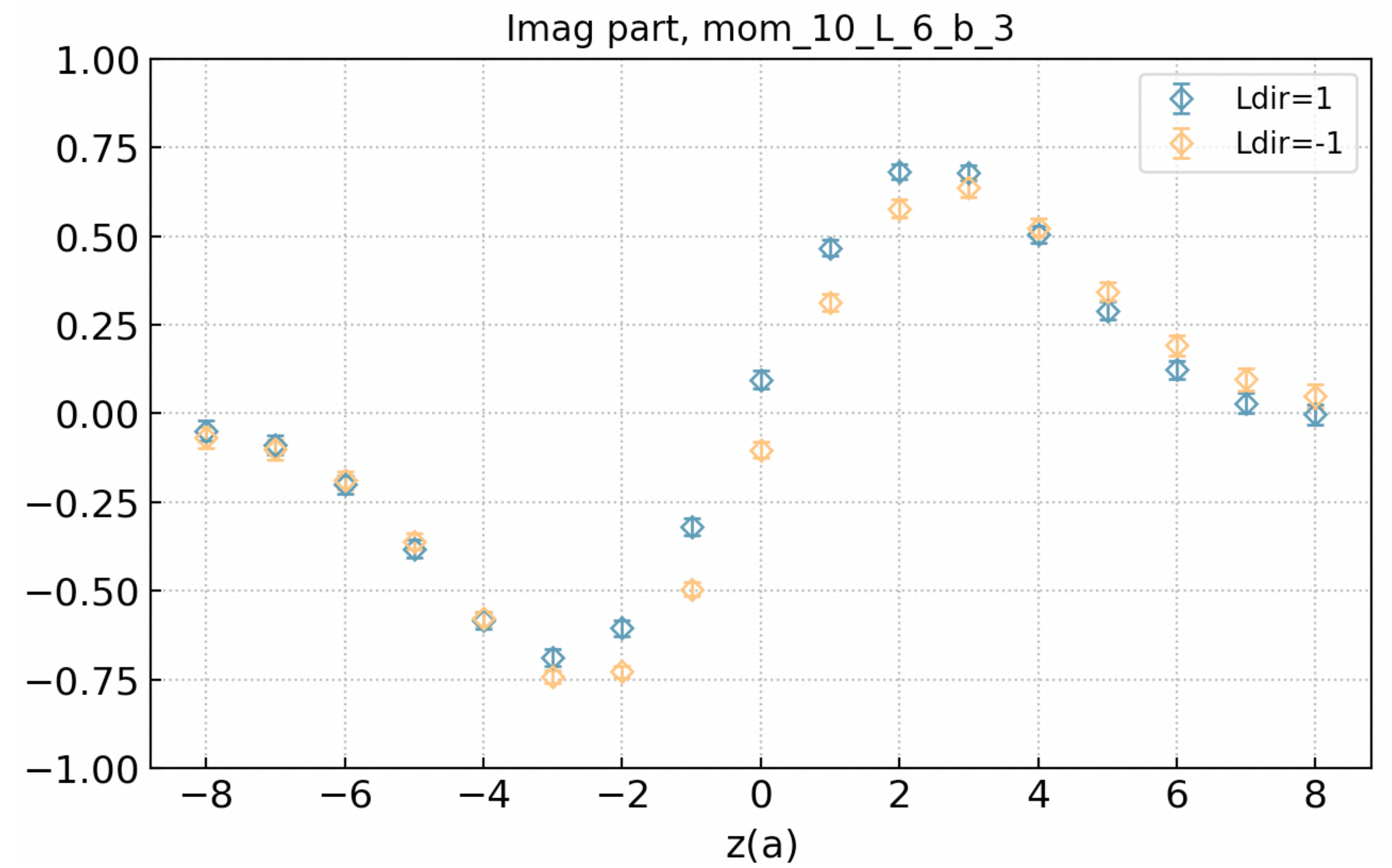
L=8

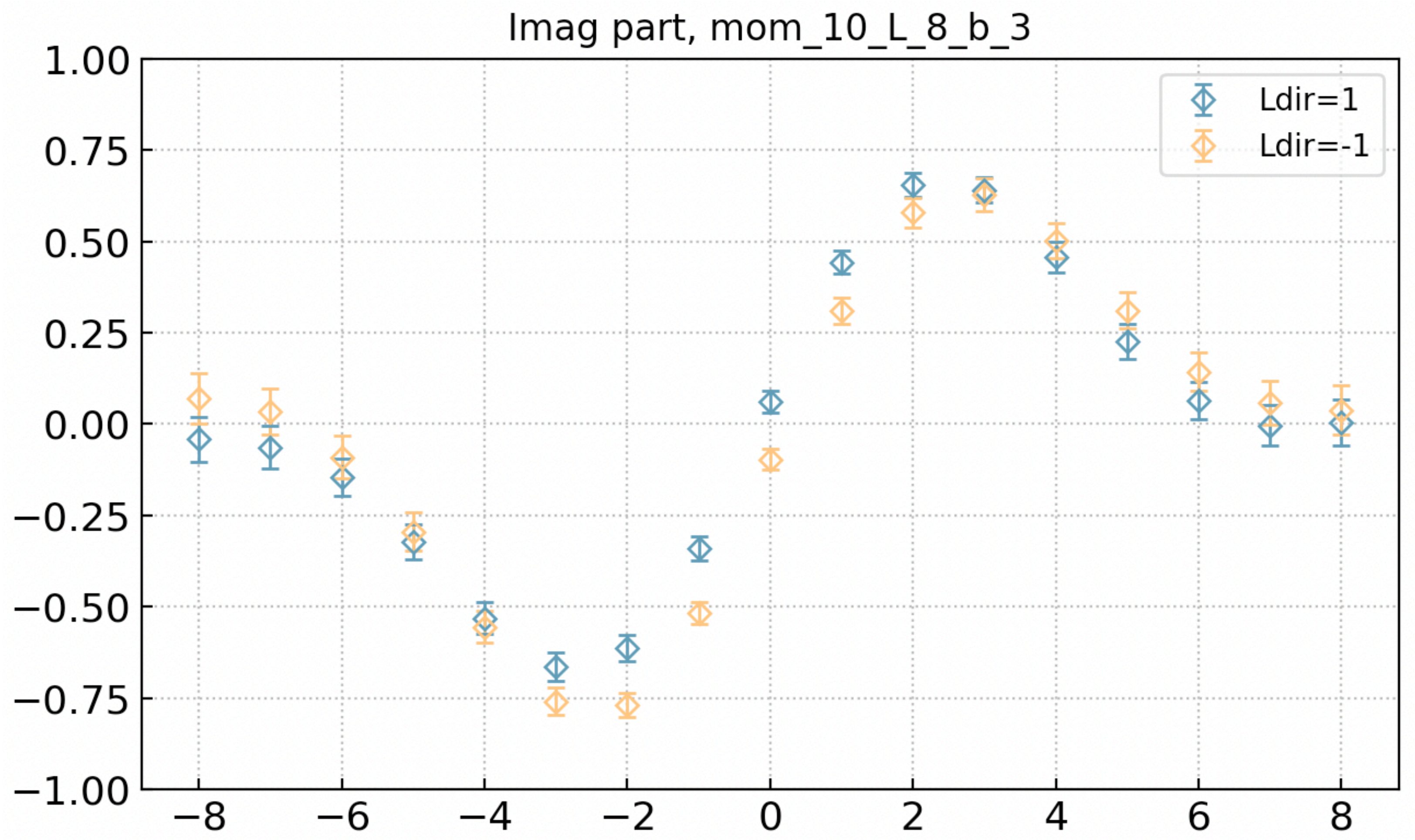
L=10

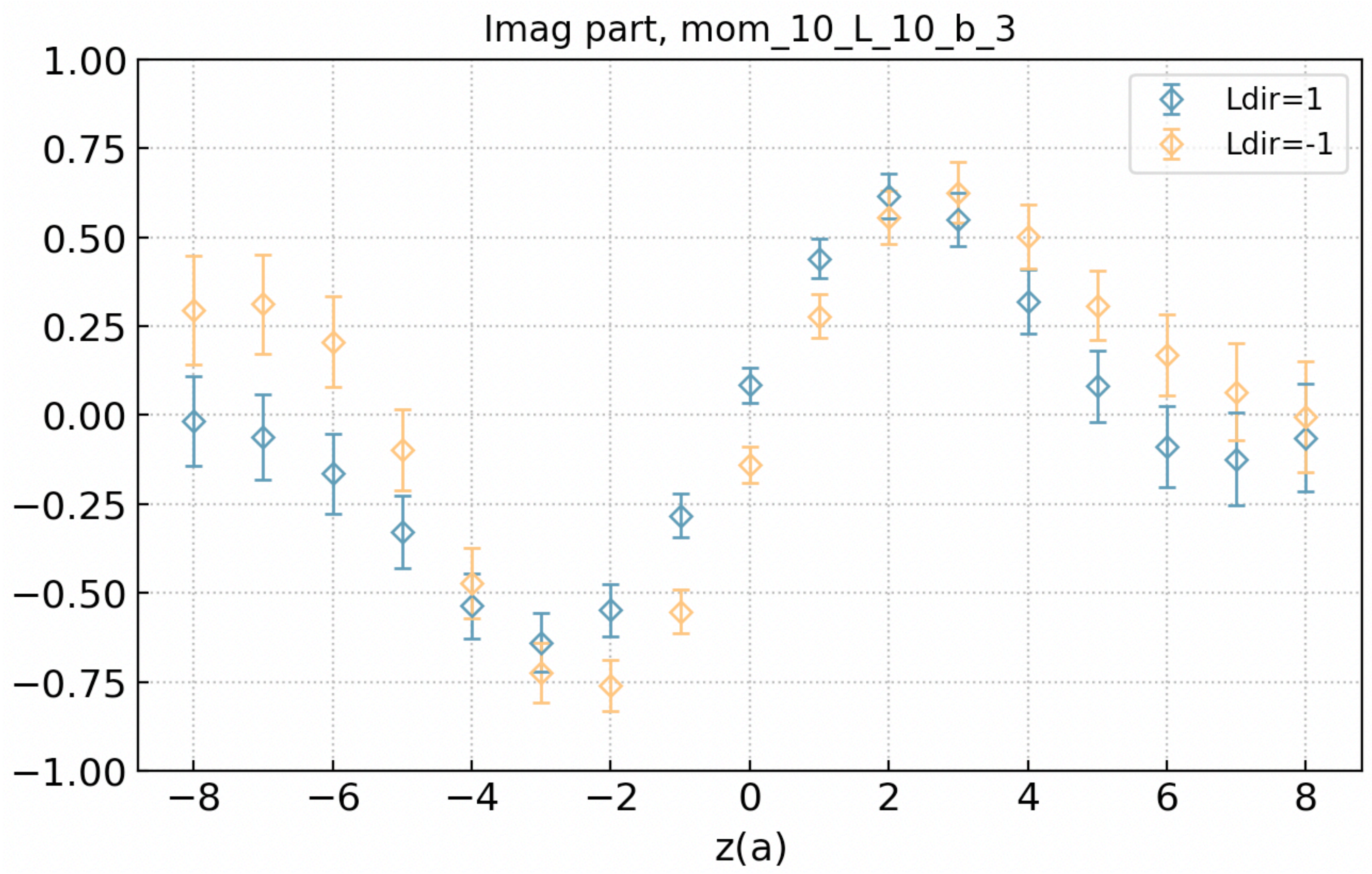








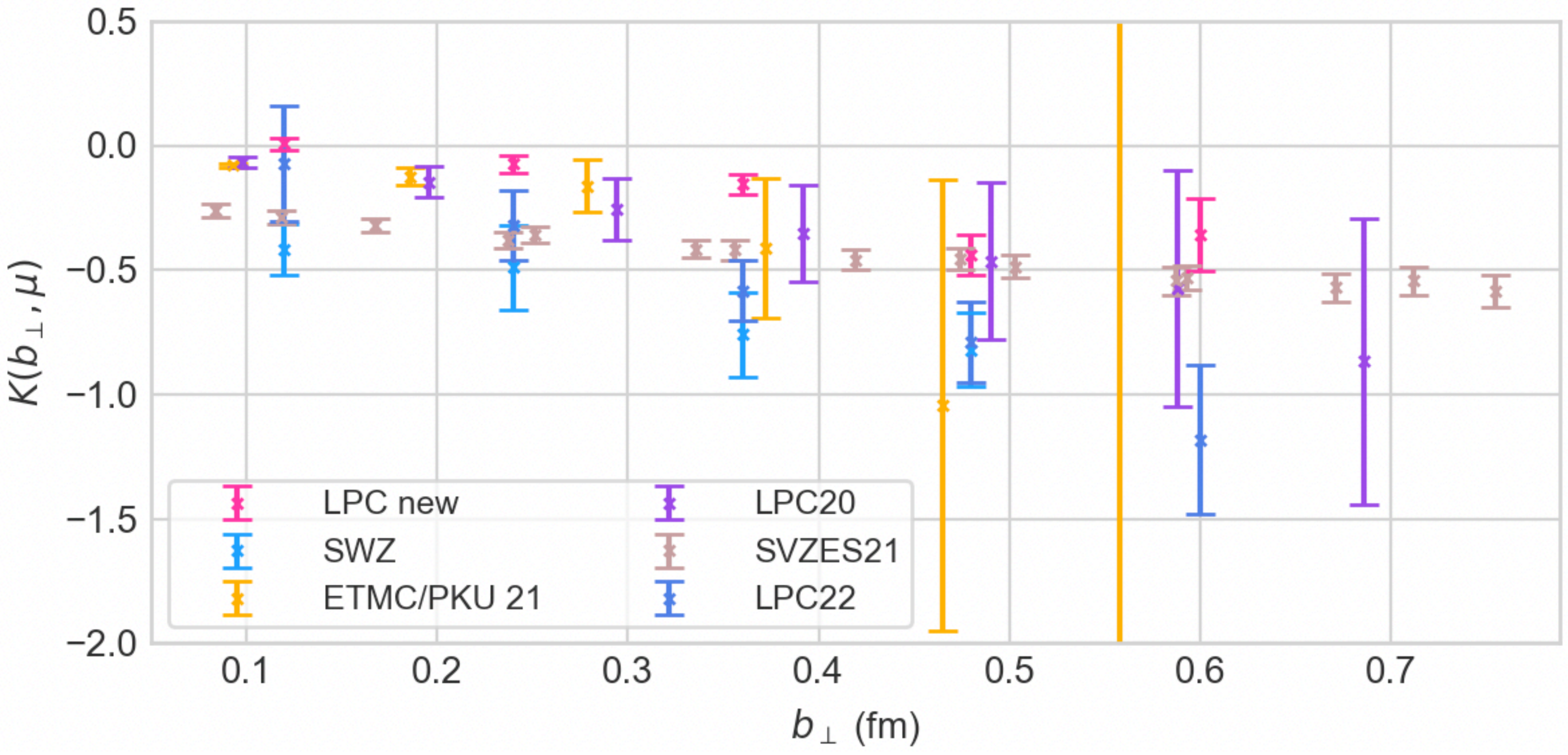




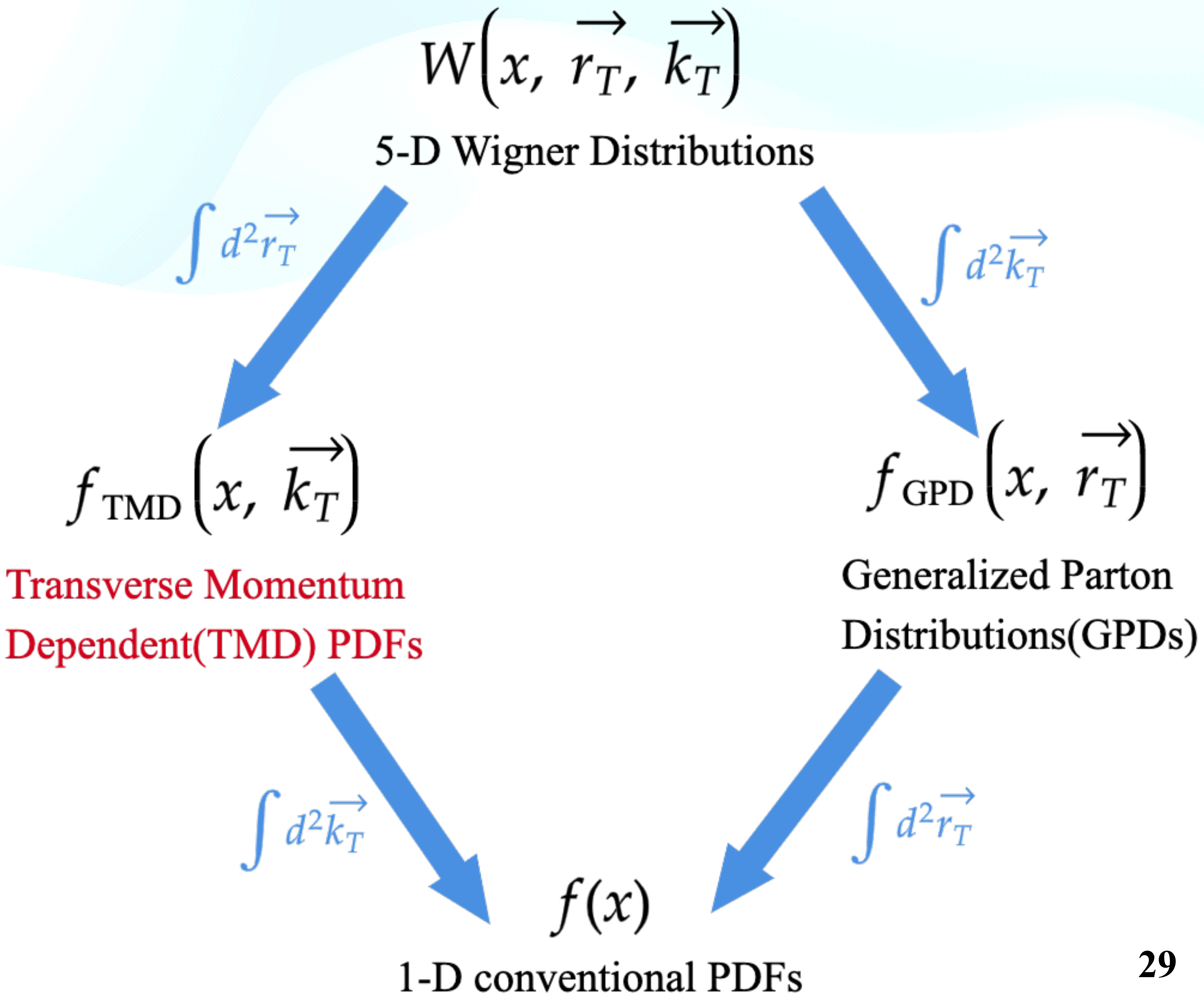
CS kernel

$$K(b_\perp, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{H^\pm(xP_2^z, \mu)\tilde{\Psi}^\pm(x, b_\perp, \mu, P_1^z)}{H^\pm(xP_1^z, \mu)\tilde{\Psi}^\pm(x, b_\perp, \mu, P_2^z)}.$$

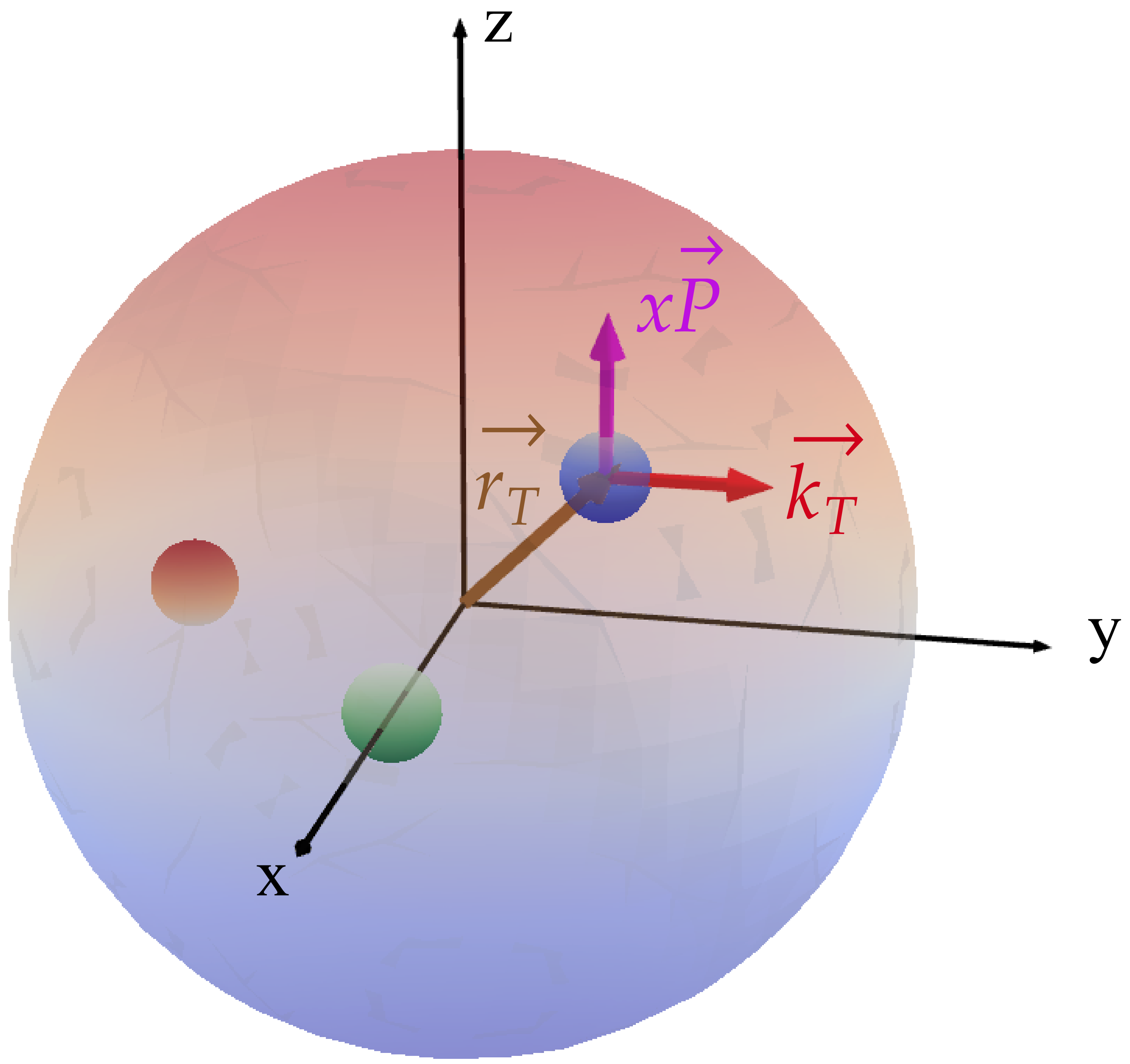
1 loop H from Yong Zhao, mom=12/8, fix x ~ 1/3, without sys err



Background
What are TMDPDFs?



$x\overrightarrow{P}$: **longitudinal momentum of the parton**
$\overrightarrow{k_T}$: **transverse momentum of the parton**
$\overrightarrow{r_T}$: **average position corresponding to the center of the nucleon**



High Energy Physics – Lattice

[Submitted on 4 Nov 2022 (v1), last revised 16 Nov 2022 (this version, v2)]

Unpolarized Transverse-Momentum-Dependent Parton Distributions of the Nucleon from Lattice QCD

Lattice Parton Collaboration: Jin-Chen He, Min-Huan Chu, Jun Hua, Xiangdong Ji, Andreas Schäfer, Yushan Su, Wei Wang, Yibo Yang, Jian-Hui Zhang, Qi-An Zhang

We present a first calculation of the unpolarized proton's isovector transverse-momentum-dependent parton distribution functions (TMDPDFs) from lattice QCD, which are essential to predict observables of multi-scale, semi-inclusive processes in the standard model. We use a $N_f = 2+1+1$ MILC ensemble with valence clover fermions on a highly improved staggered quark sea (HISQ) to compute the quark momentum distributions in large-momentum protons on the lattice. The state-of-the-art techniques in renormalization and extrapolation in correlation distance on the lattice are adopted. The one-loop contributions in the perturbative matching kernel to the light-cone TMDPDFs are taken into account, and the dependence on the pion mass and hadron momentum is explored. Our results are qualitatively comparable with phenomenological TMDPDFs, which provide an opportunity to predict high energy scatterings from the first principles.

Subjects: **High Energy Physics - Lattice (hep-lat)**; High Energy Physics - Phenomenology (hep-ph)

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