Connecting Euclidean to lightcone correlations: From forward to non-forward kinematics

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OUTLINE

- Introduction
- Theoretical framework
- Perturbative matching
- Summary and outlook

Introduction

☐ Generalized parton observables (3D structure)

$$W(x, \mathbf{k_T}, \mathbf{b_T})$$

5D Wigner Distributions

$\int d^2\mathbf{b_T}$ ↙ ↘ $\int d^2\mathbf{k_T}$

$$f_{\rm TMD}(x, \mathbf{k_T})$$

Transverse Momentum Dependent (TMD) PDFs

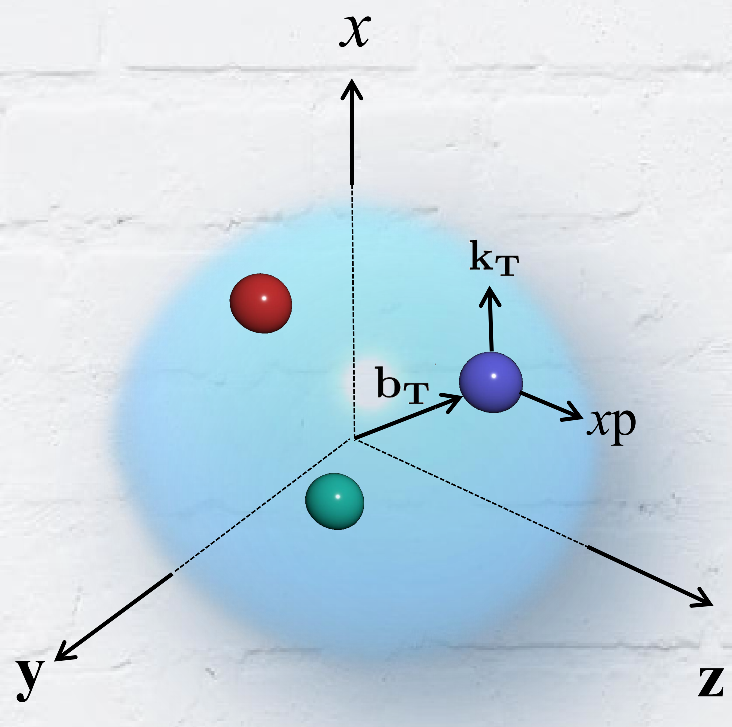
$$f_{\rm GPD}(x, \mathbf{b_T})$$

Generalized Parton Distributions (GPDs)

$\int d^2\mathbf{k_T}$ ↘ ↙ $\int d^2\mathbf{b_T}$

$$f(x)$$

1D Conventional PDFs



b_T denotes the impact parameter of the struck quark with respect to the center of momentum of the hadron

Introduction

◻ Theoretically, the unpolarized quark GPDs are defined as

$$F(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \left\langle p'' \middle| \bar{\psi}\left(-\frac{z}{2}\right)\gamma^+ L\left(-\frac{z}{2},\frac{z}{2}\right)\psi\left(\frac{z}{2}\right) \middle| p' \right\rangle_{z^+=0,\vec{z}_\perp=0}$$

$$= \frac{1}{2p^+}\left[H(x,\xi,t)\bar{u}(p'')\gamma^+ u(p') + E(x,\xi,t)\bar{u}(p'')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p')\right]$$

x	$\Delta^\mu = p''^\mu - p'^\mu$	$t = \Delta^2$	$\xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$
momentum fraction	momentum transfer	momentum transfer squared	skewness

$f_{\mathrm{GPD}}(x, \mathbf{b_T})$ → ($\mathbf{b_T} \leftrightarrow \Delta_{\mathrm{T}}$, FT) → $H(x,0,t), E(x,0,t)$ ← ($\xi = 0$) ← $H(x,\xi,t), E(x,\xi,t)$

Impact Parameter Distributions

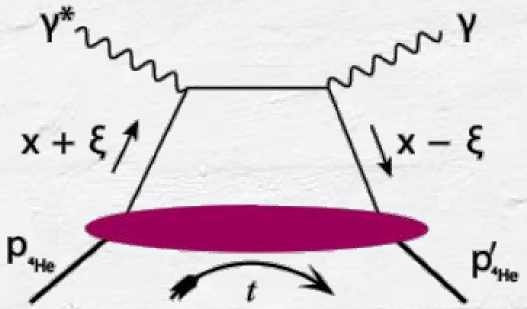
$H(x,0,t), E(x,0,t)$ ↓ $\int dx$ → Form Factors (FFs)

$H(x,\xi,t), E(x,\xi,t)$ ↓ $\int_{-1}^{1} dx x^{n-1}$ (Mellin moments) → Generalized form factors (GFFs)

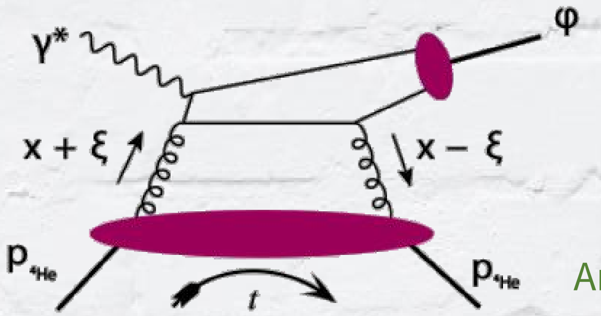
Introduction

◻ Experimentally, GPDs can be accessed in exclusive processes

- deeply virtual Compton scattering (DVCS) Ji, PRD 55 (1997)
- deeply virtual meson production (DVMP) Kriesten, PRD 101 (2020)



DVCS



DVMP

Armstrong et al, arxiv: 1708.00888

◻ Limitations in Global fit

- only limited data
- complicated kinematic dependence and no reliable framework (QCD models) for extracting 3D parton distributions

Introduction

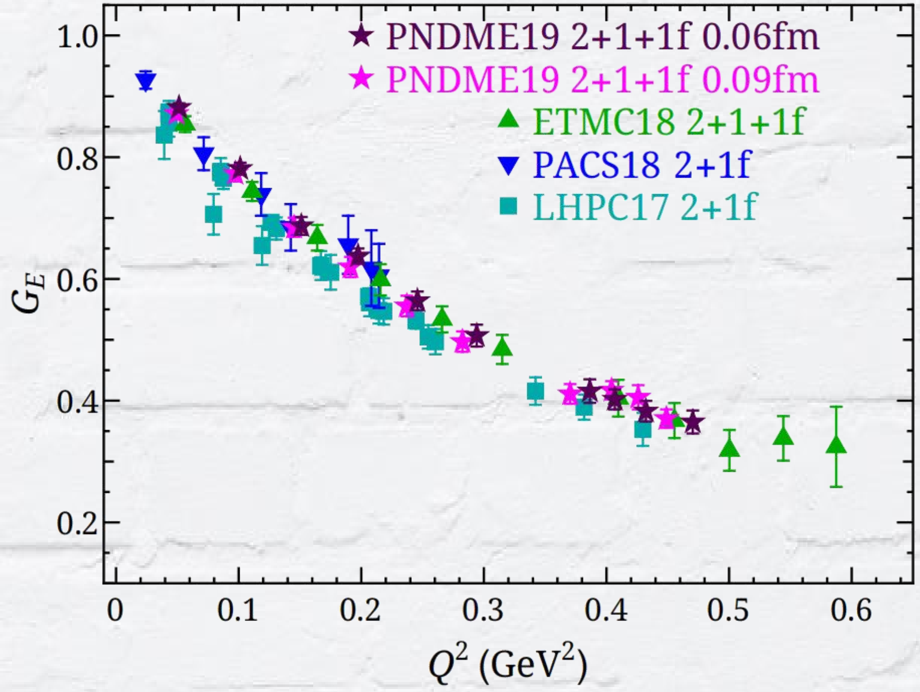
□ Extracting nucleon GPDs using lattice QCD

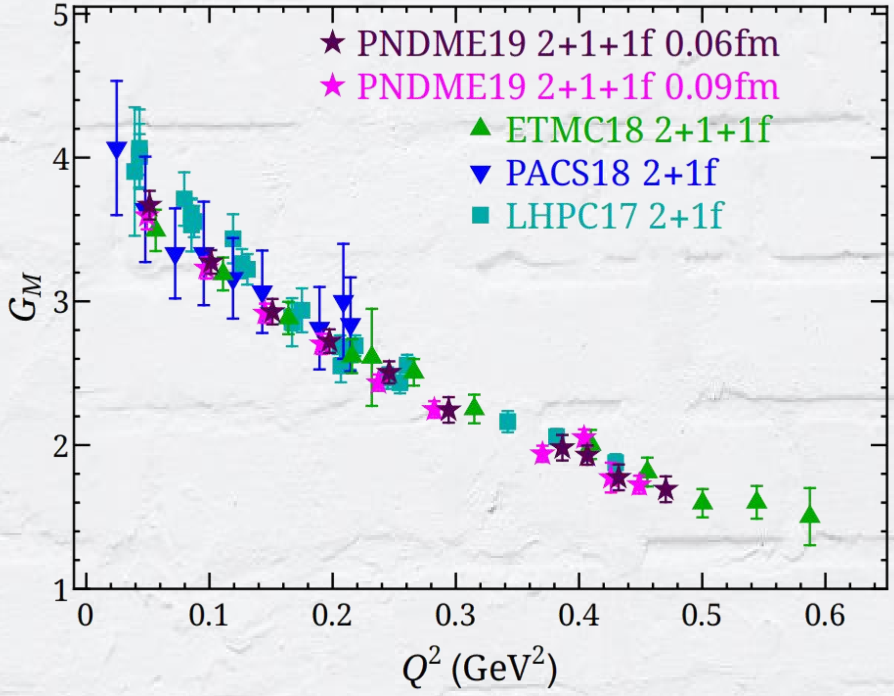
- Mellin moments of the GPDs (FFs)

$$\langle N(p_f)|V_\mu^+(x)|N(p_i)\rangle = \bar{u}^N \left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{2M_N}F_2(q^2)\right] u_N e^{iq\cdot x}$$

$$G_E(q^2) = F_1(q^2) + q^2 F_2(q^2)/(2M_N)^2$$

$$F_1(q^2) + F_2(q^2)$$





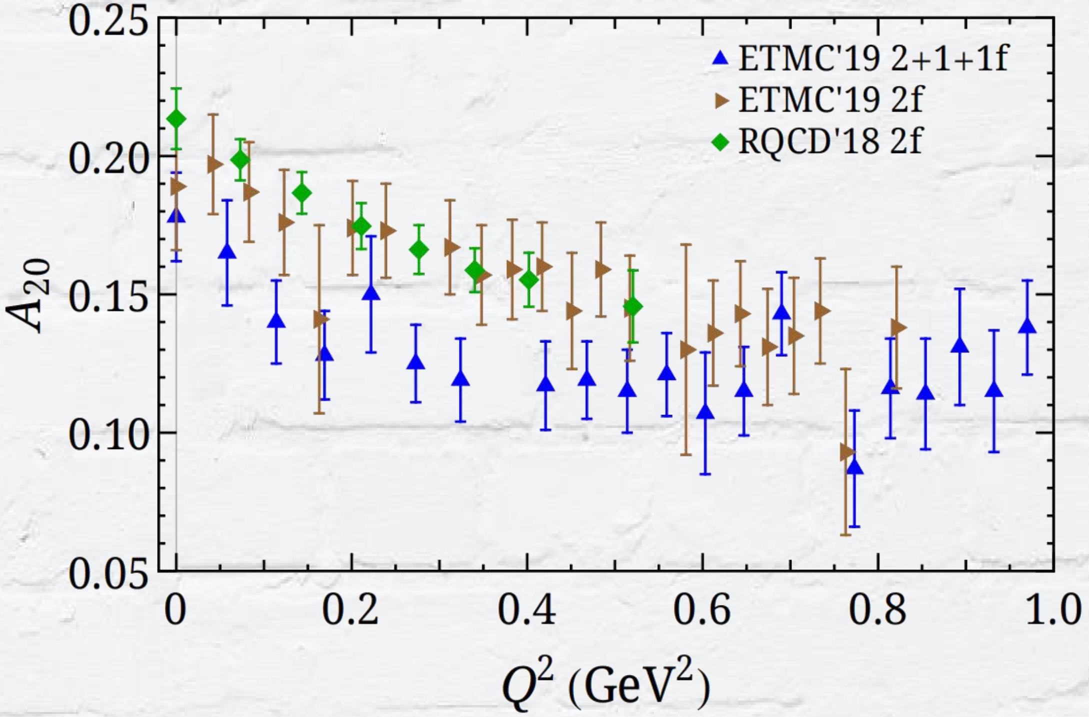
Constantinou et al, PPNP 121 (2021)

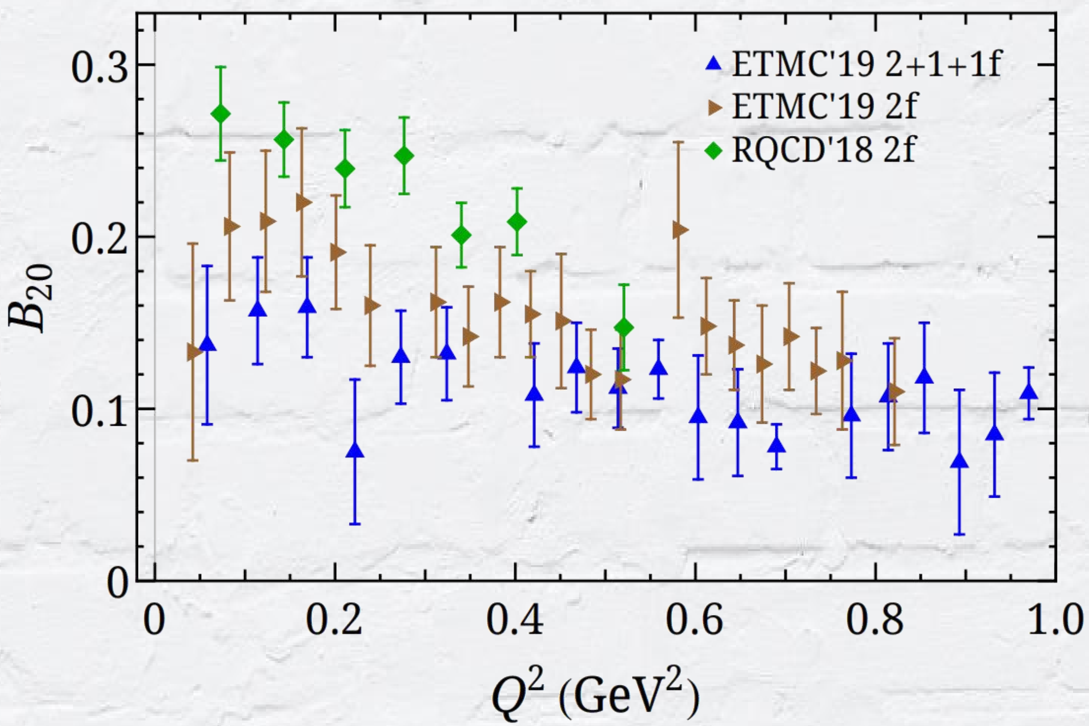
Introduction

☐ Extracting nucleon GPDs using lattice QCD

- Mellin moments of the GPDs (GFFs)

$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\Big[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}\Big]u_N(p,s)$$





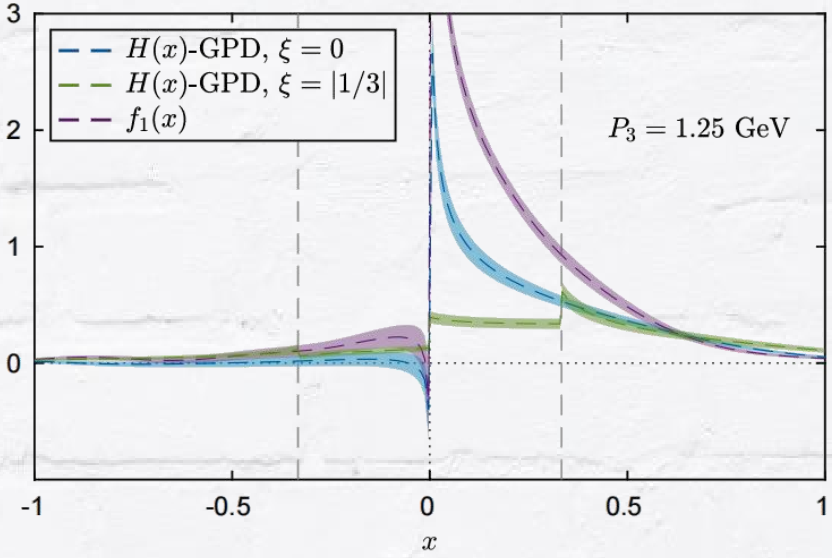
Constantinou et al, PPNP 121 (2021)

Introduction

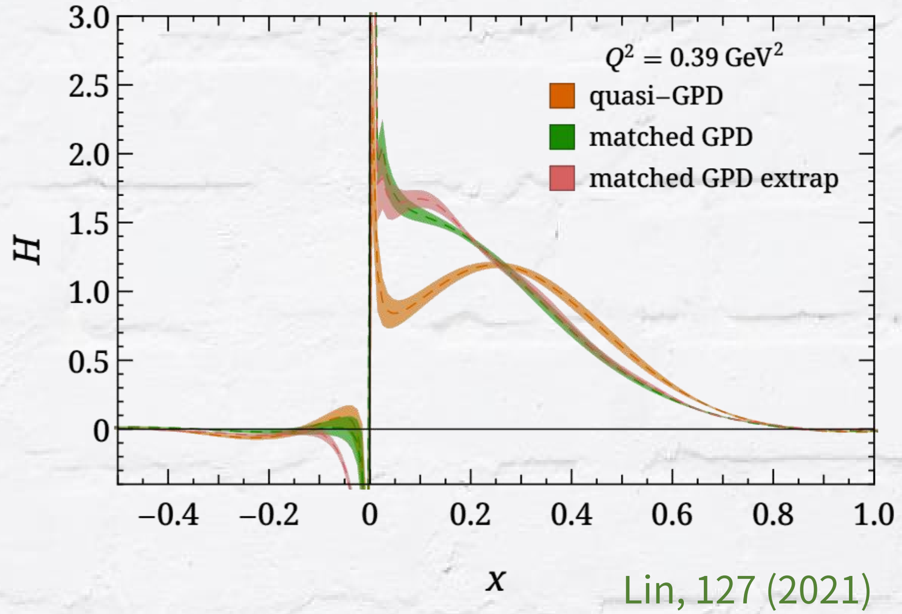
☐ Extracting nucleon GPDs using lattice QCD

- Large-momentum effective theory (LaMET)

$$F(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \left\langle p'' \left| \bar{\psi}\left(-\frac{z}{2}\right) \gamma^+ L\left(-\frac{z}{2},\frac{z}{2}\right) \psi\left(\frac{z}{2}\right) \right| p' \right\rangle_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2p^+}\left[H(x,\xi,t)\bar{u}(p'')\gamma^+ u(p') + E(x,\xi,t)\bar{u}(p'')\frac{i\sigma^{+\nu}\Delta_\nu}{2M} u(p') \right]$$



ETMC collaboration, PRL 125 (2020)



Lin, 127 (2021)

Nucleon helicity GPDs can be seen in Ref. Lin, PLB 824 (2022);

Studying pion GPDs in asymmetric frames (Bhattacharya, arXiv:2209.05373; See Martha's talk and Joshua's talk).

Motivation and Goal

- Only quark GPDs in non-singlet case without mixing.
- Renormalization and matching using RI/MOM scheme.
- To have a unified framework for perturbative matching including flavor non-singlet and singlet case, both in coordinate, pseudo and momentum space.
- In a state-of-the-art scheme.
- Provide a manual for extracting all leading-twist GPDs, PDFs and DAs from lattice QCD.

Theoretical framework

- **Factorization formula:** Quark and gluon quasi-distributions can mix with each other,

$$\begin{pmatrix} O_q \\ O_g \end{pmatrix} = \begin{pmatrix} C_{qq} & C_{qg} \\ C_{gq} & C_{gg} \end{pmatrix} \otimes \begin{pmatrix} O_q^{l.t.} \\ O_g^{l.t.} \end{pmatrix}$$

$$O_q(z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \left[C_{qq}(\alpha, \beta, z_{12}^2) O_q^{l.t.}(z_{12}^\alpha, z_{21}^\beta) + C_{qg}(\alpha, \beta, z_{12}^2) O_g^{l.t.}(z_{12}^\alpha, z_{21}^\beta) \right]$$

- **Spatial nonlocal operator:**

$$O_q(z_1, z_2) = \frac{1}{2} \left[\bar{\psi}_i(z_1) \Gamma L(z_1, z_2)^{ij} \psi_j(z_2) - (z_1 \leftrightarrow z_2) \right] \qquad \Gamma = \gamma^t, \gamma^z \gamma_5, \gamma^t \gamma^\perp \gamma_5.$$

$$O_{g,u}(z_1, z_2) = g^{ij} \mathbf{F}_{ij} \,,$$

$$O_{g,h}(z_1, z_2) = \epsilon^{ij} \mathbf{F}_{ij} \,,$$

$$\mathbf{F}_{ij} = \frac{1}{\mathbf{z_{12}}^2} F^a_{z_{12} i}(z_1) [z_1, z_2]^{ab} F^b_{j z_{12}}(z_2)$$

$$O_{g,t}(z_1, z_2) = \frac{1}{2} \left[\mathbf{F}_{ij} + \mathbf{F}_{ji} \right] - \frac{1}{d-2} g_T^{ij} \mathbf{F}_\alpha{}^\alpha,$$

$$\{i, j, \alpha = 1, 2\}.$$

l.t. stands for the leading-twist projection which acts as the generating function of leading-twist local operators

Theoretical framework

□ Sandwiched between quark or gluon external states (GPDs)

	$O_q/O_q^{l.t.}$	$O_g/O_g^{l.t.}$
$\vert q\rangle$	Quark in quark (Non-singlet case) $C_{qq}^{(1)} = \frac{\langle q\vert O_q\vert q'\rangle^{(1)} - \langle q\vert O_q^{l.t.}\vert q'\rangle^{(1)}}{\langle q\vert O_q^{l.t.}\vert q'\rangle^{(0)}}$	Gluon in quark $C_{gq}^{(1)} = \frac{\langle q\vert O_g\vert q'\rangle^{(1)} - \langle q\vert O_g^{l.t.}\vert q'\rangle^{(1)}}{\langle q\vert O_g^{l.t.}\vert q'\rangle^{(0)}}$
$\vert g\rangle$	Quark in gluon $C_{qg}^{(1)} = \frac{\langle g\vert O_q\vert g'\rangle^{(1)} - \langle g\vert O_q^{l.t.}\vert g'\rangle^{(1)}}{\langle g\vert O_q^{l.t.}\vert g'\rangle^{(0)}}$	Gluon in gluon $C_{gg}^{(1)} = \frac{\langle g\vert O_g\vert g'\rangle^{(1)} - \langle g\vert O_g^{l.t.}\vert g'\rangle^{(1)}}{\langle g\vert O_g^{l.t.}\vert g'\rangle^{(0)}}$

Quark in gluon: gluon matrix element of the quark quasi-GPD operator

- PDFs (forward) $\langle q|O_q|q\rangle$
- DAs $\langle q\bar{q}'|O_q|0\rangle$

Theoretical framework

- The matching coefficients for the GPDs **H(x,ξ,t)** and **E(x,ξ,t)** should be same. Liu et al, PRD 100.034006 (2019).
- **Fourier transformation** $\tilde{H}_{q/g}(x,\xi,\frac{\mu}{P_z}) = \frac{1}{N}\int\frac{dz_1}{2\pi}\int\frac{dz_2}{2\pi}e^{-\frac{i}{2}P_z[(\xi+x)z_1+(\xi-x)z_2]}\langle q\,|O_{q/g}(z_1,z_2)|\,q'\rangle$

Quasi-GPDs Quasi-LF correlations

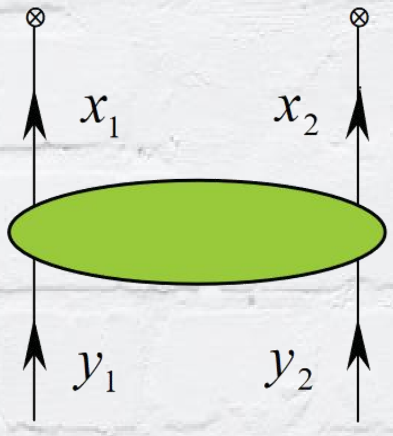
- Pseudo space

$$\mathscr{P}(x,\xi,\mu^2 z^2) = \int_{-1}^{1} dy\, K(t_1,t_2\,|\,y_1,y_2;\mu^2 z^2)H(y,\xi),\ K(t_1,t_2\,|\,y_1,y_2) = \begin{pmatrix} K_{qq} & K_{qg} \\ K_{gq} & K_{gg} \end{pmatrix}$$

Pseudo-GPDs

- Momentum space

$$\tilde{H}(x,\xi,\frac{\mu}{P_z}) = \int_{-1}^{1} dy\, \mathcal{K}(x_1,x_2\,|\,y_1,y_2;\frac{\mu}{P_z})H(y,\xi),\ \mathcal{K}(x_1,x_2\,|\,y_1,y_2) = \begin{pmatrix} \mathcal{K}_{qq} & \mathcal{K}_{qg} \\ \mathcal{K}_{gq} & \mathcal{K}_{gg} \end{pmatrix}$$



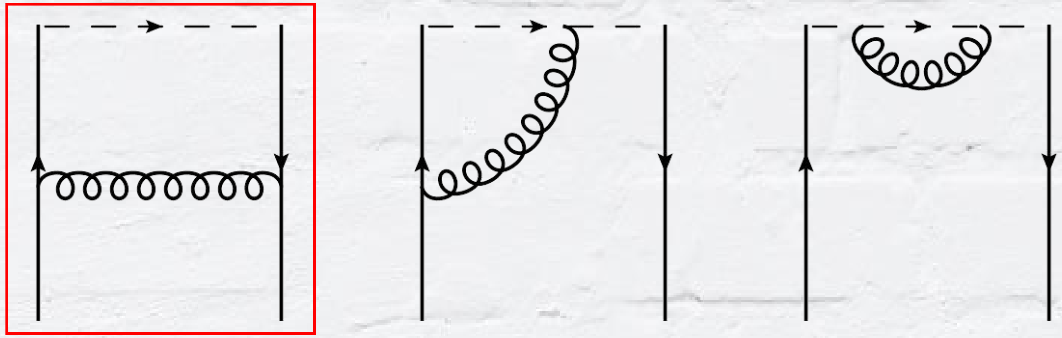
$$\mathrm{X}_1 = \frac{\xi+\mathrm{X}}{2},\quad \mathrm{X}_2 = \frac{\xi-\mathrm{X}}{2}\quad (\mathrm{X}=x,y,t)\,.$$

Belitsky et al, Phys.Rept. 418 (2005).

Matching coefficient calculations

☐ Quark in quark (In coordinate space)

- **MSbar scheme:**



$$C_{qq}^{\overline{\rm MS}}(\alpha,\beta,\mu^2 z_{12}^2) = \delta(\alpha)\delta(\beta) + 2a_s C_F\Big\{ \Big(A_{1,\Gamma} + [\bar\alpha/\alpha]_+ \delta(\beta) + [\bar\beta/\beta]_+ \delta(\alpha)\Big)(\mathrm{L_z}-1) + \mathrm{A}_{2,\Gamma} - 2\,[\ln(\alpha)/\alpha]_+ \delta(\beta) - 2\,[\ln(\beta)/\beta]_+ \delta(\alpha) \Big\} + 2a_s C_F(-2\mathrm{L_z}+2)\delta(\alpha)\delta(\beta)\,,$$

$$\mathrm{L_z} = \ln\frac{4e^{-2\gamma_E}}{-\mu^2 z_{12}^2}\,,$$

$$A_{1,\gamma^t} = 1\,,\quad A_{1,\gamma^z\gamma_5} = 1\,,\quad A_{1,\gamma^t\gamma^\perp\gamma_5} = 0$$

$$A_{2,\gamma^t} = 2\,,\quad A_{2,\gamma^z\gamma_5} = 4\,,\quad A_{2,\gamma^t\gamma^\perp\gamma_5} = 0$$

The matching kernels are consistent with Ref. Radyushkin, PRD 100 (2019).

Matching coefficient calculations

- **Ratio scheme:** divide by the same operator matrix element at zero momentum.

Radyushkin, PRD 98 (2018)

$$\tilde{h}_{qq}(0,\mu^2 z_{12}^2) = Z_{qq}^R(\mu^2 z_{12}^2) = 1 + 2a_s C_F\left(-A_{3,\Gamma}\,\mathrm{L_z} + \mathrm{A}_{4,\Gamma}\right)$$

$$A_{3,\gamma^t} = 3/2\,, \qquad A_{3,\gamma^z\gamma_5} = 3/2\,, \qquad A_{3,\gamma^t\gamma^\perp\gamma_5} = 2\,,$$
$$A_{4,\gamma^t} = 5/2\,, \qquad A_{4,\gamma^z\gamma_5} = 7/2\,, \qquad A_{4,\gamma^t\gamma^\perp\gamma_5} = 2\,.$$

Specifically, the perturbative ratio matching kernel at 1-loop level gives

$$C_{qq}^R(\alpha,\beta,\mu^2 z_{12}^2) = \delta(\alpha)\delta(\beta) + 2a_s C_F\left\{\left(A_{1,\Gamma} + \frac{\bar{\alpha}}{\alpha}\delta(\beta) + \frac{\bar{\beta}}{\beta}\delta(\alpha)\right)(\mathrm{L_z}-1) + \mathrm{A}_{2,\Gamma} - 2\frac{\ln\alpha}{\alpha}\delta(\beta) - 2\frac{\ln\beta}{\beta}\delta(\alpha)\right\}_+$$

- **Problem:** introduce undesired IR effects at large distances (LaMET)
- **Ratio-hybrid scheme:** Ji et al, NPB 964 (2021)

$$C_{qq}^H(\alpha,\beta,\mu^2 z_{12}^2, z_{12}^2/z_s^2) = C_{qq}^R(\alpha,\beta,\mu^2 z_{12}^2)\,\theta\left(z_s - |z_{12}|\right) + C_{qq}^{\overline{\mathrm{MS}}}(\alpha,\beta,\mu^2 z_{12}^2)\,e^{-\delta m|z|}\,\mathrm{Z}_h(z_s)\,\theta\left(|z_{12}| - z_s\right)$$

$$= C_{qq}^R(\alpha,\beta,z_{12}^2) + 2a_s C_F\, A_{3,\Gamma}\ln\frac{z_{12}^2}{z_s^2}\delta(\alpha)\delta(\beta)\,\theta\left(|z_{12}| - z_s\right)$$

Matching coefficient calculations

☐ Quark in quark (In pseudo space)

$$K(t_1,t_2\,|\,y_1,y_2;\mu^2z^2) = \int_0^1 d\alpha \int_0^1 d\beta\, C_{qq}(\alpha,\beta,\mu^2z^2)\,\delta(t_1-\bar{\alpha}y_1-\bar{\alpha}\beta y_2)$$

Ji and Belitsky, NPB 894 (2015)

- **MSbar scheme:**

$$K_{qq}^{\overline{\rm MS}}(t_1,t_2\,|\,y_1,y_2) = \delta(t_1-y_1) + k_{qq}^{(1)}(t_1,t_2\,|\,y_1,y_2)$$

$$k_{qq,u}^{(1)} = 2a_sC_F\left\{\frac{|t_1|}{y_1(t_1+t_2)} + \frac{|t_2|}{y_2(t_1+t_2)} - \frac{|t_1-y_1|}{y_1y_2}\right\}(\mathrm{L_z}+1) + k_{qq,t}^{(1)}\,,$$

$$k_{qq,h}^{(1)} = k_{qq,u}^{(1)} + 4a_sC_F\left\{\frac{|t_1|}{y_1(t_1+t_2)} + \frac{|t_2|}{y_2(t_1+t_2)} - \frac{|t_1-y_1|}{y_1y_2}\right\},$$

$$k_{qq,t}^{(1)} = 2a_sC_F\left\{\left(\frac{|t_1|}{y_1(y_1-t_1)} + \frac{t_1}{y_1|t_1-y_1|}\right)(\mathrm{L_z}-1) + \left(\frac{|t_1|}{t_1(t_1-y_1)} - \frac{1}{|t_1-y_1|}\right)\ln\frac{(t_1-y_1)^2}{y_1^2} + (t_1\to t_2,\ y_1\to y_2)\right\}_+$$

$$+\,2a_sC_F\,(-2\,\mathrm{L_z}+2)\,\delta(t_1-y_1)$$

- **Ratio scheme:**

$$K_{qq}^{R}(t_1,t_2\,|\,y_1,y_2) = K_{qq}^{\overline{\rm MS}}(t_1,t_2\,|\,y_1,y_2) - 2a_sC_F\,(-A_{3,\Gamma}\,\mathrm{L_z} + \mathrm{A}_{4,\Gamma})\,\delta(t_1-y_1)$$

Matching coefficient calculations

□ Quark in quark (In momentum space)

- **Fourier transformation:** $\mathcal{K}\left(x_1, x_2 \,|\, y_1, y_2; \frac{\mu}{P_z}\right) = P_z \int_{-1}^{1} dt_1 \int_{-1}^{1} dt_2 \int \frac{dz_1}{2\pi} \int \frac{dz_2}{2\pi}\, e^{iP_z[(x_1-t_1)z_1+(x_2-t_2)z_2]}\, K\left(t_1, t_2 \,|\, y_1, y_2; \mu^2 z^2\right).$
- **MSbar scheme:**

$$\mathcal{K}_{qq}^{\overline{\rm MS}}(x_1,x_2\,|\,y_1,y_2) = \delta(x_1-y_1) + \mathscr{k}_{qq}^{(1)}(x_1,x_2\,|\,y_1,y_2)$$

$$\mathscr{k}_{qq,u}^{(1)} = 2a_s C_F\left\{\frac{|x_1|}{y_1(y_1+y_2)}\ln\frac{4P_z^2x_1^2}{\mu^2 e} + \frac{|x_2|}{y_2(y_1+y_2)}\ln\frac{4P_z^2x_2^2}{\mu^2 e} - \frac{|x_1-y_1|}{y_1y_2}\ln\frac{4P_z^2(x_1-y_1)^2}{\mu^2 e}\right\} + \mathscr{k}_{qq,t}^{(1)},$$

$$\mathscr{k}_{qq,h}^{(1)} = \mathscr{k}_{qq,u}^{(1)} + 4a_s C_F\left\{\frac{|x_1|}{y_1(y_1+y_2)} + \frac{|x_2|}{y_2(y_1+y_2)} - \frac{|x_1-y_1|}{y_1y_2}\right\},$$

$$\mathscr{k}_{qq,t}^{(1)} = 2a_s C_F\left\{\frac{|x_1|}{y_1(y_1-x_1)}\ln\frac{4P_z^2x_1^2}{\mu^2 e} + \frac{|x_2|}{y_2(y_2-x_2)}\ln\frac{4P_z^2x_2^2}{\mu^2 e} + \left(\frac{x_1}{y_1}+\frac{x_2}{y_2}\right)\frac{1}{|x_1-y_1|}\ln\frac{4P_z^2(x_1-y_1)^2}{\mu^2 e}\right\}.$$

Having same results in Ref. Ma et al, JHEP (2022)130, they calculate directly in momentum space.

- **Ratio scheme && Ratio-hybrid scheme:**

$$\mathcal{K}_{qq}^{R}(x_1,x_2\,|\,y_1,y_2) = \mathcal{K}_{qq}^{\overline{\rm MS}}(x_1,x_2\,|\,y_1,y_2) - 2a_s C_F\, A_{3,\Gamma}\,\frac{1}{|x_1-y_1|}$$

$$\mathcal{K}_{qq}^{H}(x_1,x_2\,|\,y_1,y_2) = \mathcal{K}_{qq}^{R}(x_1,x_2\,|\,y_1,y_2) + 2a_s C_F\, A_{3,\Gamma}\left[-\frac{1}{|x_1-y_1|} + \frac{2{\rm Si}((x_1-y_1)\lambda_s)}{\pi(x_1-y_1)}\right]_+$$

Chou and Chen, PRD 106 (2022)
yesterday Chen' talk

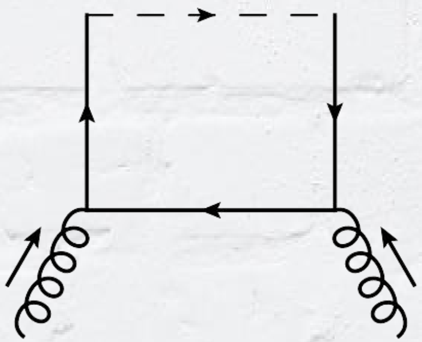
Matching coefficient calculations

In coordinate space and in $\overline{\text{MS}}$ scheme

☐ Quark in gluon

$$C_{qg}^{(1)}(\alpha,\beta,\mu^2 z_{12}^2) = 4i a_s T_F N_f \, \mathbf{z}_{12} \, B_\Gamma \, \mathrm{L_z}\,,$$

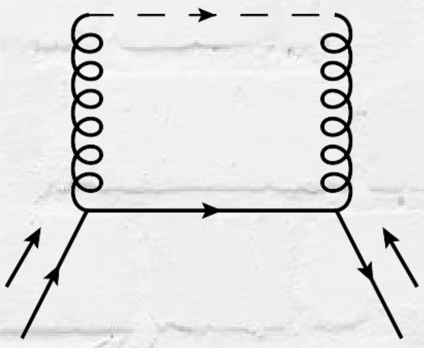
$$B_{\gamma^0} = \bar{\alpha}\bar{\beta} + 3\alpha\beta\,, \qquad B_{\gamma^z\gamma_5} = \bar{\alpha}\bar{\beta} - \alpha\beta.$$



☐ Gluon in quark

$$C_{gq}^{(1)}(\alpha,\beta,\mu^2 z_{12}^2) = \frac{-2i a_s C_F}{\mathbf{z}_{12}} \left\{ \Big(\delta(\alpha)\delta(\beta) + D_1\Big)\,(\mathrm{L_z}+1) + D_2 - 2\Big(\delta(\alpha)+\delta(\beta)\Big) \right\},$$

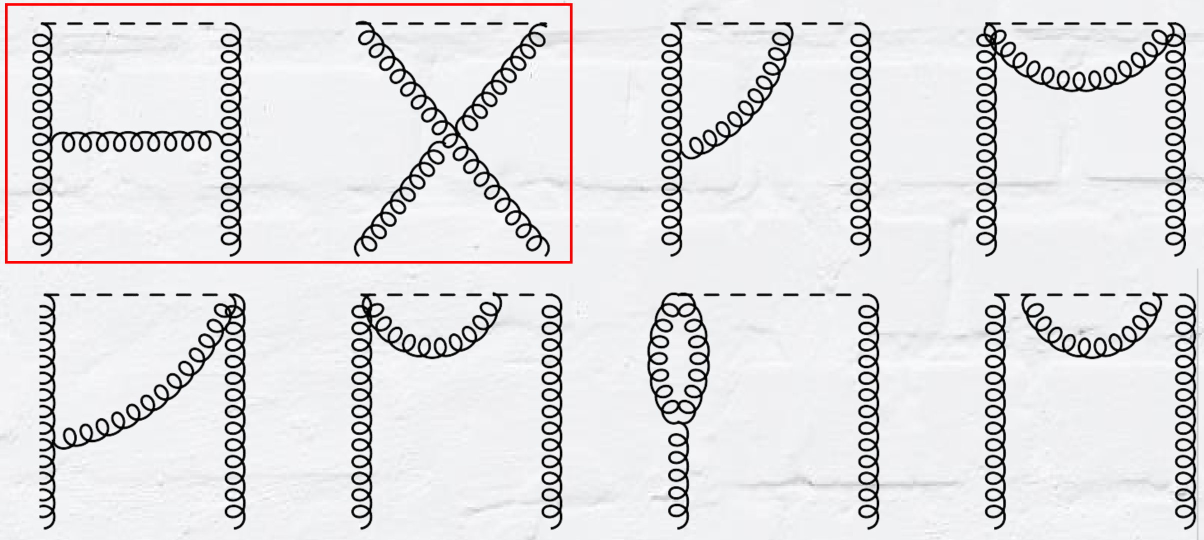
$$D_{1,u} = 2\,, \qquad D_{1,h} = -2\,, \qquad D_{2,u} = 6\,, \qquad D_{2,h} = 4\,.$$



- Transversity distributions do not mix.
- Mixing terms should ensure dimension consistency

Matching coefficient calculations

In coordinate space and in $\overline{\text{MS}}$ scheme



☐ Gluon in gluon

$$C_{gg}^{(1)}(\alpha,\beta,\mu^2 z_{12}^2) = 2a_s C_A\Big\{\Big(E_1 + \left[\bar{\alpha}^2/\alpha\right]_+\delta(\beta) + \left[\bar{\beta}^2/\beta\right]_+\delta(\alpha)\Big)(\mathrm{L_z}-1) + E_2 - 2\left[\ln(\alpha)/\alpha\right]_+\delta(\beta)$$

$$- 2\left[\ln(\beta)/\beta\right]_+\delta(\alpha)\Big\} + 2a_s C_A\left(-3\,\mathrm{L_z}+2\right)\delta(\alpha)\delta(\beta)\,,$$

$$E_{1,u} = 4(1-\alpha-\beta+3\alpha\beta)\,, \qquad E_{1,h} = 4(1-\alpha-\beta)\,, \qquad E_{1,t} = 0,$$

$$E_{2,u} = 3E_{1,u} + 8\alpha\beta\,, \qquad E_{2,h} = 3/2\,E_{1,h}\,, \qquad E_{2,t} = 2(1+\alpha+\beta-2\alpha\beta).$$

The evolution kernels are all consistent with Ref. Belitsky et al, Phys.Rept. 418 (2005).

Matching coefficient calculations

◻ Reduction to PDFs (forward limit)

- Factorization formula $\tilde{q}(x,\mu,P_z) = \int_{-1}^{1} dy\, F\,(x,y,\mu,P_z)\, q(y,\mu) + h.t.$
- The non-forward GPDs matching kernels reduce to the PDFs matching kernels when the skewness ξ=0, namely, $F(x,y) = \mathcal{K}(x,-x\,|\,y,-y).$

◻ Reduction to DAs

- Factorization formula $\tilde{\phi}(x,P_z) = \int_{0}^{1} dy\, V\,(x,y,\mu,P_z)\, \phi(y,\mu) + h.t.$
- Taking limit $V(x,y) = \mathcal{K}(x,1-x\,|\,y,1-y).$

Complete matching for PDFs in RI/MOM scheme in Ref. Liu et al, PRD 100.074509 (2019);
Matching for non-singlet the meson DAs in Ref. Liu et al, PRD 99, 094036 (2019);

Summary and outlook

- GPDs plays an important role in the detailed understanding of the inner 3D structure of nucleon.
- Lattice QCD calculations can provide great help to extract GPDs.
- We provide a unified framework for perturbative matching connecting Euclidean to lightcone correlations
  - Both for non-singlet and singlet (GPDs, PDFs, DAs)
  - In coordinate, pseudo and momentum space
  - In a state-of-the-art scheme (ratio and hybrid scheme).
- Follow-up: two-loop level

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Thank you for listening !

Introduction

□ Generalized parton observables (3D structure)

$$W(x, \mathbf{k_T}, \mathbf{b_T})$$

5D Wigner Distributions

$\int d^2\mathbf{b_T}$ $\int d^2\mathbf{k_T}$

$$f_{\mathrm{TMD}}(x, \mathbf{k_T})$$

Transverse Momentum Dependent (TMD) PDFs

$$f_{\mathrm{GPD}}(x, \mathbf{b_T})$$

Generalized Parton Distributions (GPDs)

$\int d^2\mathbf{k_T}$ $\int d^2\mathbf{b_T}$

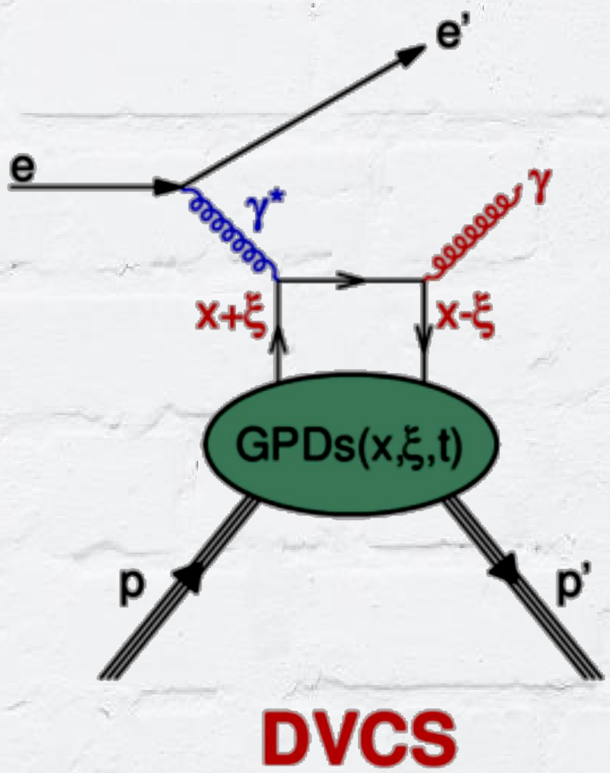
$$f(x)$$

1D Conventional PDFs

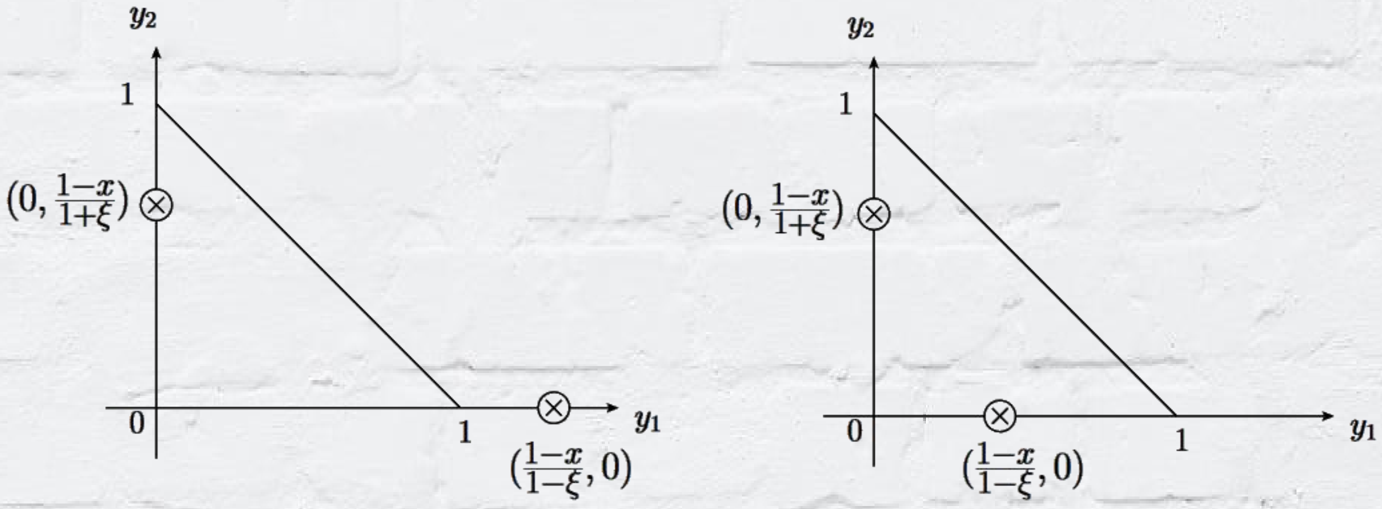
notation b_T:The unfortunate overlapping use of the notation b_T for two distinct objects in the TMD PDF and GPD literatures should be noted. In the GPD literature, b_T denotes the impact parameter of the struck quark with respect to the center of momentum of the hadron; it is Fourier conjugate to the momentum transfer Δ_T . In the TMD PDF literature, b_T denotes the Fourier conjugate to the transverse momentum k_T of the struck quark, corresponding to a relative separation in the relevant quark bilinear operator. Put succinctly, the relation between the conventions in the two literatures is akin to the relation between center-of-mass and relative coordinates.

Constantinou et al — PPNP 121 (2021)

Introduction



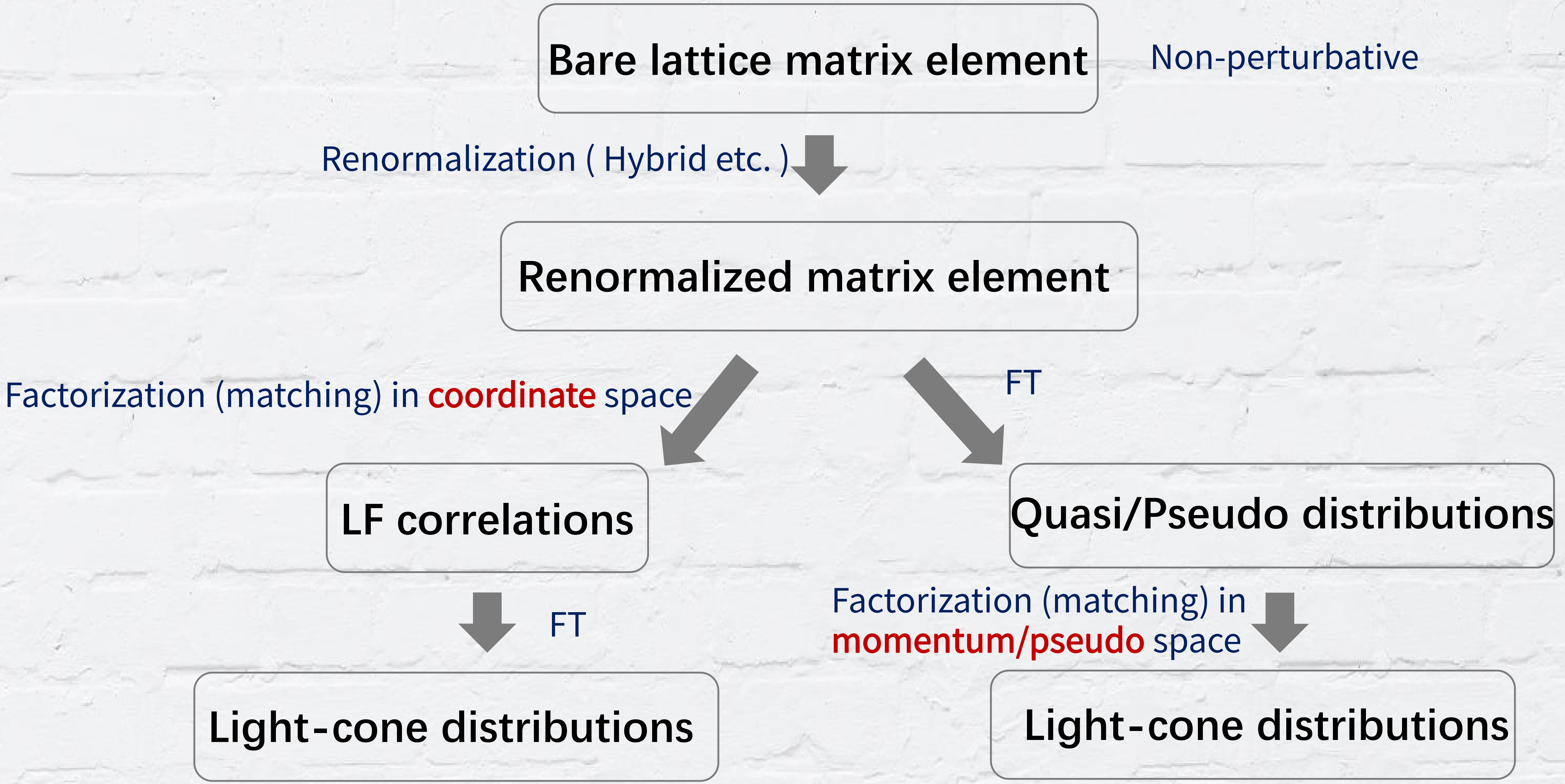
$eA \rightarrow e' A\gamma$



Integration in ERBL (left) and DGLAP (right) regions: The singularities are denoted by cross

Liu et al — PRD 100 (2019)

Theoretical framework



Theoretical framework

□ Nonsinglet quark light-cone GPD v.s quasi-GPD

$$F(x,\xi,t)=\frac{1}{2P^+}\int\frac{dz_1^-}{2\pi}\int\frac{dz_2^-}{2\pi}e^{-iP^+((\xi+x)z_1^-+(\xi-x)z_2^-)}\langle P_1,S_1|\bar{\psi}(z_1^-)\gamma^+L(z_1^-,z_2^-)\psi(z_2^-)|P_2,S_2\rangle$$

LF correlation in Minkowski space

$$\tilde{F}(x,\xi,t)=\frac{1}{2P^z}\int\frac{dz_1}{2\pi}\int\frac{dz_2}{2\pi}e^{-iP^z((\xi+x)z_1+(\xi-x)z_2)}\langle P_1,S_1|\bar{\psi}(z_1)\gamma^0L(z_1,z_2)\psi(z_2)|P_2,S_2\rangle$$

Quasi-LF correlation in Euclidean space

□ Factorization formula

In coordinate space

$$\tilde{h}(P_1,P_2,z_1,z_2)=\int_0^1 d\alpha d\beta\, C(\alpha,\beta,z_{12}^2)h(P_1,P_2,z_1^-,z_2^-)+h.t..$$

Quasi-LF correlation — LF correlation

FT ↓ — ↓ FT

In momentum space

$$\tilde{F}(x,\xi,t)=\int_{-1}^1 dy\,K\left(\frac{\xi+x}{2},\frac{\xi-x}{2},\frac{\xi+y}{2},\frac{\xi-y}{2}\right)F(y,\xi,t)+h.t.$$

Quasi-GPD — Light-cone GPD