Frame-independent methods to access GPDs from lattice QCD

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in collaboration with:

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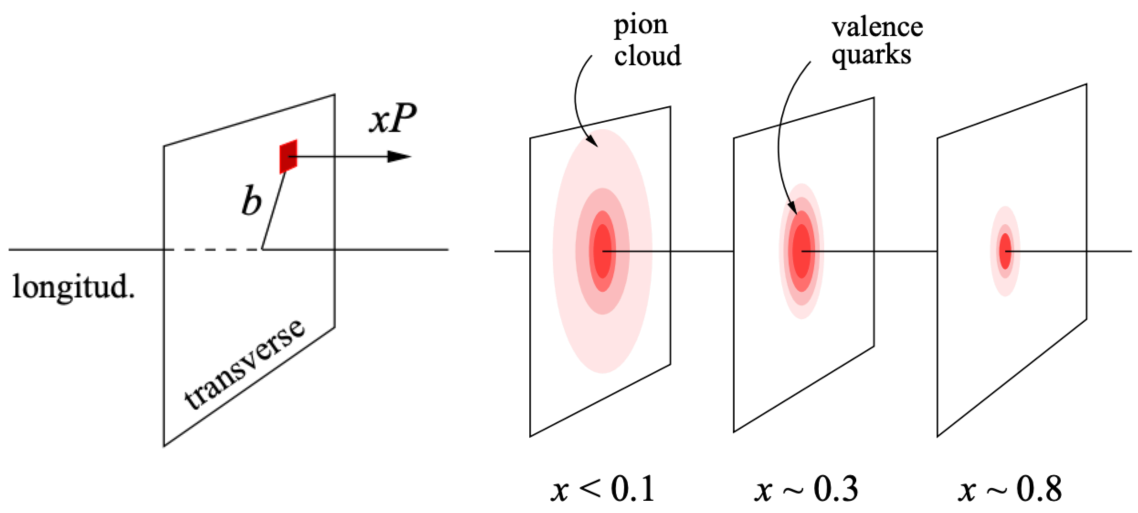
Generalized Parton Distributions

★ Crucial in understanding hadron tomography

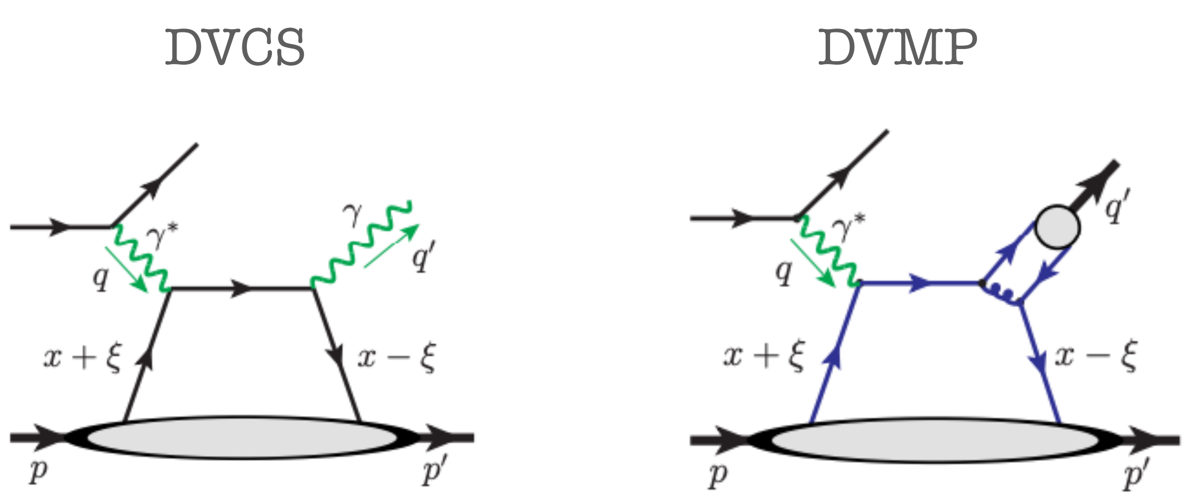


[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

$1_{\text{mom}} + 2_{\text{coord}}$ tomographic images of quark distribution in nucleon at fixed longitudinal momentum

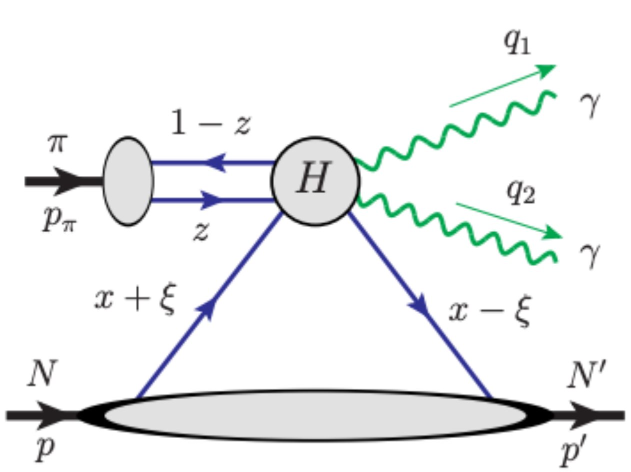
3-D image from FT with respect to longitudinal momentum transfer

★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)



[X.-D. Ji, PRD 55, 7114 (1997)]

★ New class of observables involve a pair of high-transverse mom. particles in the final state



[J. Qiu et al, arXiv:2205.07846]

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★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathcal{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

  (SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs

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- and more challenges …

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★ Essential to complement the knowledge on GPD from lattice QCD

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma\overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle\sim\sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t)-i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\}+\frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\text{ even}}\Big)\Big\}$$

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local operators

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★ **Matrix elements of non-local operators**
(LaMET, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

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Wilson line

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GPDs

Through non-local matrix elements

of fast-moving hadrons

Light-cone GPDs

★ Off-forward matrix elements of non-local light-cone operators

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda' | \bar\psi(-\tfrac{z}{2})\,\gamma^+ \mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})\,|p;\lambda\rangle \Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization in two leading twist GPDs

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar u(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda)$$

★ How can one define GPDs on a Euclidean lattice?

GPDs on the lattice

Off forward correlators with nonlocal (equal-time) operators

[X. Ji, PRL 110 (2013) 262002]

$$\tilde{q}^{\rm GPD}_{\mu}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_f)\,|\,\bar{\Psi}(z)\,\gamma^{\mu}\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_{\mu}$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

Variables of the calculation:

- length of the Wilson line (z)
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→ reduction of power corrections in fwd limit [Radyushkin, PLB 767, 314, 2017]

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- Lorentz non-invariant parametrization
- Typically used in symmetric frame
- A non-symmetric setup may result to different functional form for GPDs compared to the symmetric one

Definition of GPDs in Euclidean lattice

★ Calculation expected to be performed in symmetric frame to extract the "standard" GPDs

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2nd goal:

New definition of Lorentz covariant quasi-GPDs that may have faster convergence to light-cone GPDs

Theoretical setup

[S. Bhattacharya et al., arXiv:2209.05373]

★ Parametrization of matrix elements in Lorentz invariant amplitudes

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + z^{\mu} M A_2 + \frac{\Delta^{\mu}}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu} i\sigma^{z\Delta}}{M}A_6 + \frac{z^{\mu} i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^{\mu} i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Advantages

- Applicable to any kinematic frame and A_i have definite symmetries
- Lorentz invariant amplitudes A_i can be related to the standard H, E GPDs
- Quasi H, E may be redefined (Lorentz covariant) to eliminate $1/P_3$ contributions:

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$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

$$E(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = -A_1 - \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3 + 2A_5 + 2P_{avg,s/a}\cdot zA_6 + 2\Delta_{s/a}\cdot zA_8$$

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★ Proof-of-concept calculation (zero quasi-skewness):

- symmetric frame: $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}$, $\quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2}$ $\quad t^s = -\vec{Q}^2$

- asymmetric frame: $\vec{p}^{\,a}_f = \vec{P}$, $\quad \vec{p}^{\,a}_i = \vec{P} - \vec{Q}$ $\quad t^a = -\vec{Q}^2 + (E_f - E_i)^2$

Matrix element decomposition

Symmetric

$$C_s = \frac{2m^2}{E(E+m)}$$

$$\Gamma_0 = \frac{1}{2}(1+\gamma^0)$$

$$\Gamma_j = \frac{i}{4}(1+\gamma^0)\gamma^5\gamma^j \quad (j=1,2,3)$$

$$\Pi_s^0(\Gamma_0) = C_s\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right)$$

$$\Pi_s^0(\Gamma_1) = i\,C_s\left(\frac{EP_3Q_2}{4m^3}A_1 - \frac{(E+m)P_3Q_2}{2m^3}A_5 - \frac{E\left(P_3^2+m(E+m)\right)zQ_2}{2m^3}A_6\right)$$

$$\Pi_s^0(\Gamma_2) = i\,C_s\left(-\frac{EP_3Q_1}{4m^3}A_1 + \frac{(E+m)P_3Q_1}{2m^3}A_5 + \frac{E\left(P_3^2+m(E+m)\right)zQ_1}{2m^3}A_6\right)$$

Asymmetric

$$C_a = \frac{2m^2}{\sqrt{E_iE_f(E_i+m)(E_f+m)}}$$

$$\Pi_0^a(\Gamma_0) = C_a\left(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1 - \frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3 + \frac{(E_i-E_f)P_3z}{4m}A_4 + \frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5 + \frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6 + \frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\right)$$

$$\Pi_0^a(\Gamma_1) = i\,C_a\left(\frac{(E_f+E_i)P_3Q_2}{8m^3}A_1 + \frac{(E_f-E_i)P_3Q_2}{4m^3}A_3 + \frac{(E_f+m)Q_2z}{4m}A_4 - \frac{(E_f+E_i+2m)P_3Q_2}{4m^3}A_5 - \frac{E_f(E_f+E_i)(E_f+m)Q_2z}{4m^3}A_6 - \frac{E_f(E_f-E_i)(E_f+m)Q_2z}{2m^3}A_8\right)$$

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Matrix element decomposition

Symmetric

$$C_s = \frac{2m^2}{E(E+m)}$$

$$\Gamma_0 = \frac{1}{2}(1+\gamma^0)$$

$$\Gamma_j = \frac{i}{4}(1+\gamma^0)\gamma^5\gamma^j \quad (j=1,2,3)$$

$$\Pi_s^0(\Gamma_0) = C_s\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right)$$

$$\Pi_s^0(\Gamma_1) = i\,C_s\left(\frac{EP_3Q_2}{4m^3}A_1 - \frac{(E+m)P_3Q_2}{2m^3}A_5 - \frac{E\left(P_3^2+m(E+m)\right)zQ_2}{2m^3}A_6\right)$$

$$\Pi_s^0(\Gamma_2) = i\,C_s\left(-\frac{EP_3Q_1}{4m^3}A_1 + \frac{(E+m)P_3Q_1}{2m^3}A_5 + \frac{E\left(P_3^2+m(E+m)\right)zQ_1}{2m^3}A_6\right)$$

Novel feature: z-dependence

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No definite symmetries for Π_μ^a

$$\Pi_0^a(\Gamma_2) = i\,C_a\left(-\frac{(E_f+E_i)P_3Q_1}{8m^3}A_1 - \frac{(E_f-E_i)P_3Q_1}{4m^3}A_3 - \frac{(E_f+m)Q_1z}{4m}A_4 + \frac{(E_f+E_i+2m)P_3Q_1}{4m^3}A_5 + \frac{E_f(E_f+E_i)(E_f+m)Q_1z}{4m^3}A_6 + \frac{E_f(E_f-E_i)(E_f+m)Q_1z}{2m^3}A_8\right)$$

Lorentz-Invariant amplitudes

Symmetric

$$A_1 = \frac{\left(m(E+m)+P_3^2\right)}{E(E+m)}\Pi_0^s(\Gamma_0) - i\,\frac{P_3 Q_1}{2E(E+m)}\Pi_0^s(\Gamma_2) - \frac{Q_1}{2E}\Pi_2^s(\Gamma_3)$$

$$A_5 = -\frac{E}{Q_1}\Pi_2^s(\Gamma_3)$$

$$A_6 = \frac{P_3}{2Ez(E+m)}\Pi_0^s(\Gamma_0) + i\,\frac{\left(P_3^2 - E(E+m)\right)}{EQ_1 z(E+m)}\Pi_0^s(\Gamma_2) + \frac{P_3}{EQ_1 z}\Pi_2^s(\Gamma_3)$$

Asymmetric

$$A_1 = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{2(E_f-E_i)P_3 m^2}{E_f(E_f+m)(E_i+m)Q_1}\frac{\Pi_0^a(\Gamma_2)}{C_a} + \frac{2(E_i-E_f)P_3 m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{C_a}$$
$$+i\,\frac{2(E_i-E_f)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_1^a(\Gamma_0)}{C_a} + \frac{2(E_i-E_f)P_3 m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2(E_f-E_i)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

$$A_5 = \frac{m^2 P_3}{E_f(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{C_a} - \frac{(E_f+E_i)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

$$A_6 = \frac{P_3 m^2}{E_f^2(E_f+m)(E_i+m)z}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{(E_f-E_i-2m)m^2}{E_f^2(E_i+m)Q_1 z}\frac{\Pi_0^a(\Gamma_2)}{C_a} + i\,\frac{(E_i-E_f)P_3 m^2}{E_f^2(E_f+m)(E_i+m)Q_1 z}\frac{\Pi_1^a(\Gamma_0)}{C_a}$$
$$+\frac{(-E_f+E_i+2m)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_1^a(\Gamma_2)}{C_a} + \frac{2(m-E_f)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2P_3 m^2}{E_f^2(E_i+m)Q_1 z}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

- ★ Asymmetric frame equations more complex
- ★ A_i have definite symmetries
- ★ System of 8 independent matrix elements to disentangle the A_i

Lorentz-Invariant amplitudes

Symmetric

$$A_1 = \frac{\left(m(E+m)+P_3^2\right)}{E(E+m)}\Pi_0^s(\Gamma_0) - i\,\frac{P_3Q_1}{2E(E+m)}\Pi_0^s(\Gamma_2) - \frac{Q_1}{2E}\Pi_2^s(\Gamma_3)$$

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$$A_1 = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{2(E_f-E_i)P_3m^2}{E_f(E_f+m)(E_i+m)Q_1}\frac{\Pi_0^a(\Gamma_2)}{C_a} + \frac{2(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{C_a}$$
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$$+\frac{(-E_f+E_i+2m)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_1^a(\Gamma_2)}{C_a} + \frac{2(m-E_f)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2P_3m^2}{E_f^2(E_i+m)Q_1z}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

- ★ Asymmetric frame equations more complex
- ★ A_i have definite symmetries
- ★ System of 8 independent matrix elements to disentangle the A_i

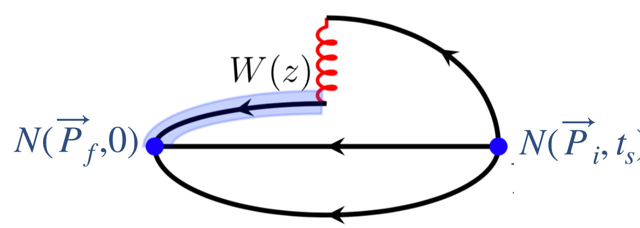
Parameters of calculation

★ Nf=2+1+1 twisted mass (TM) fermions & clover improvement

Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64
Spatial extent:	3 fm

★ Calculation:

- isovector combination
- zero skewness
- T_{sink}=1 fm



frame	P_3 [GeV]	$\mathbf{Q}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
symm	1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
non-symm	1.25	(±2,0,0), (0,±2,0)	0.64	0	8	269	8	17216

★ Computational cost:

- symmetric frame 4 times more expensive than asymmetric frame for same set of $\vec{Q}$ (requires separate calculations at each t)

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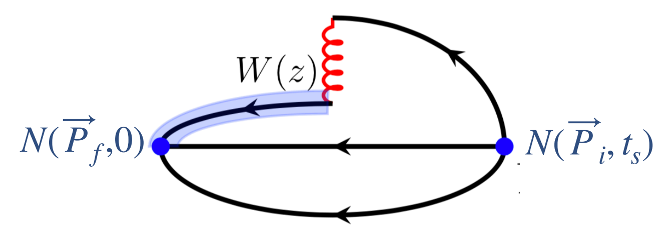

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Small difference: $t^s = -\vec{Q}^2$ $t^a = -\vec{Q}^2 + (E_f - E_i)^2$

$A(-0.64\text{GeV}^2) \sim A(-0.69\text{GeV}^2)$

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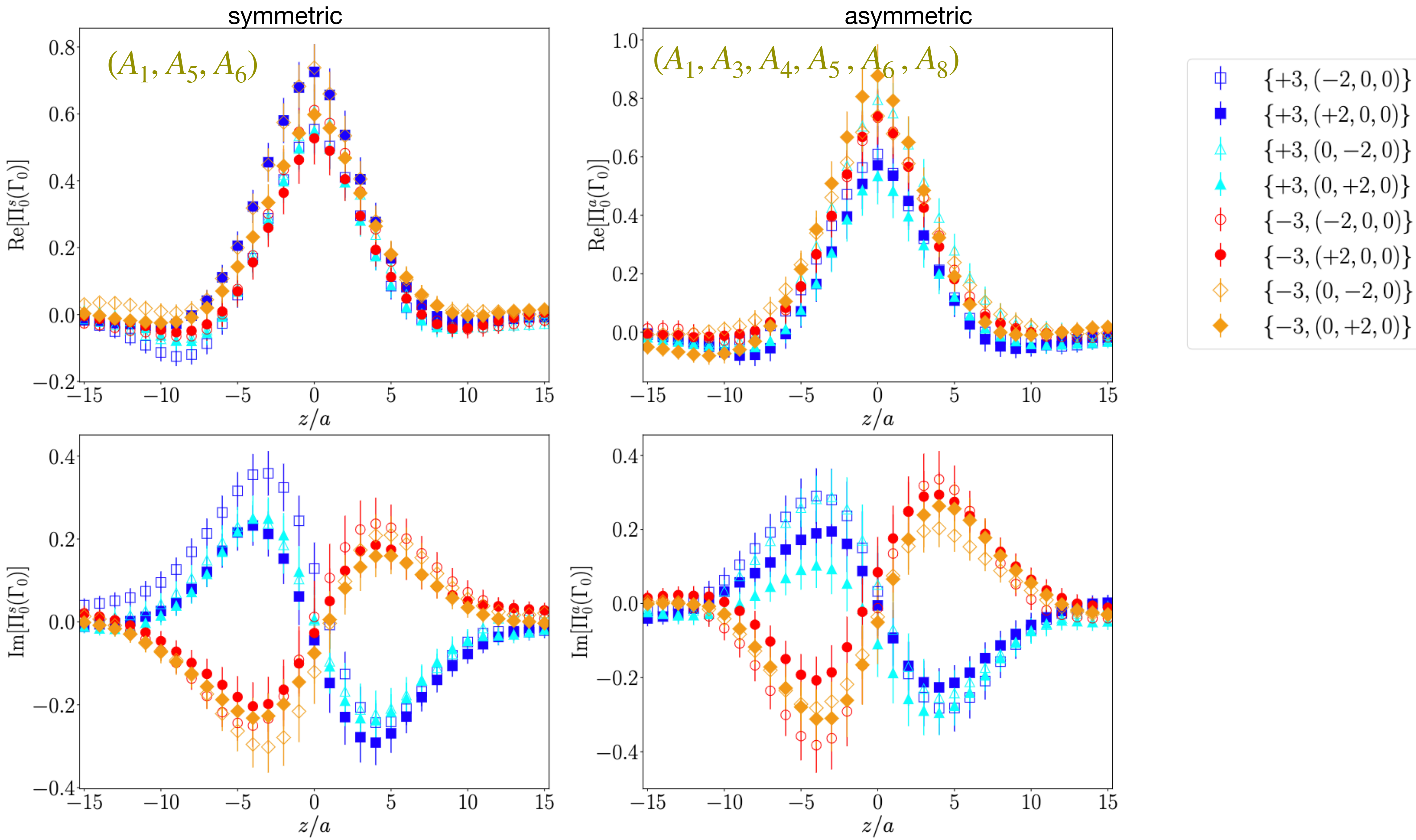
- symmetric frame 4 times more expensive than asymmetric frame for same set of $\vec{Q}$ (requires separate calculations at each t)

Results: matrix elements

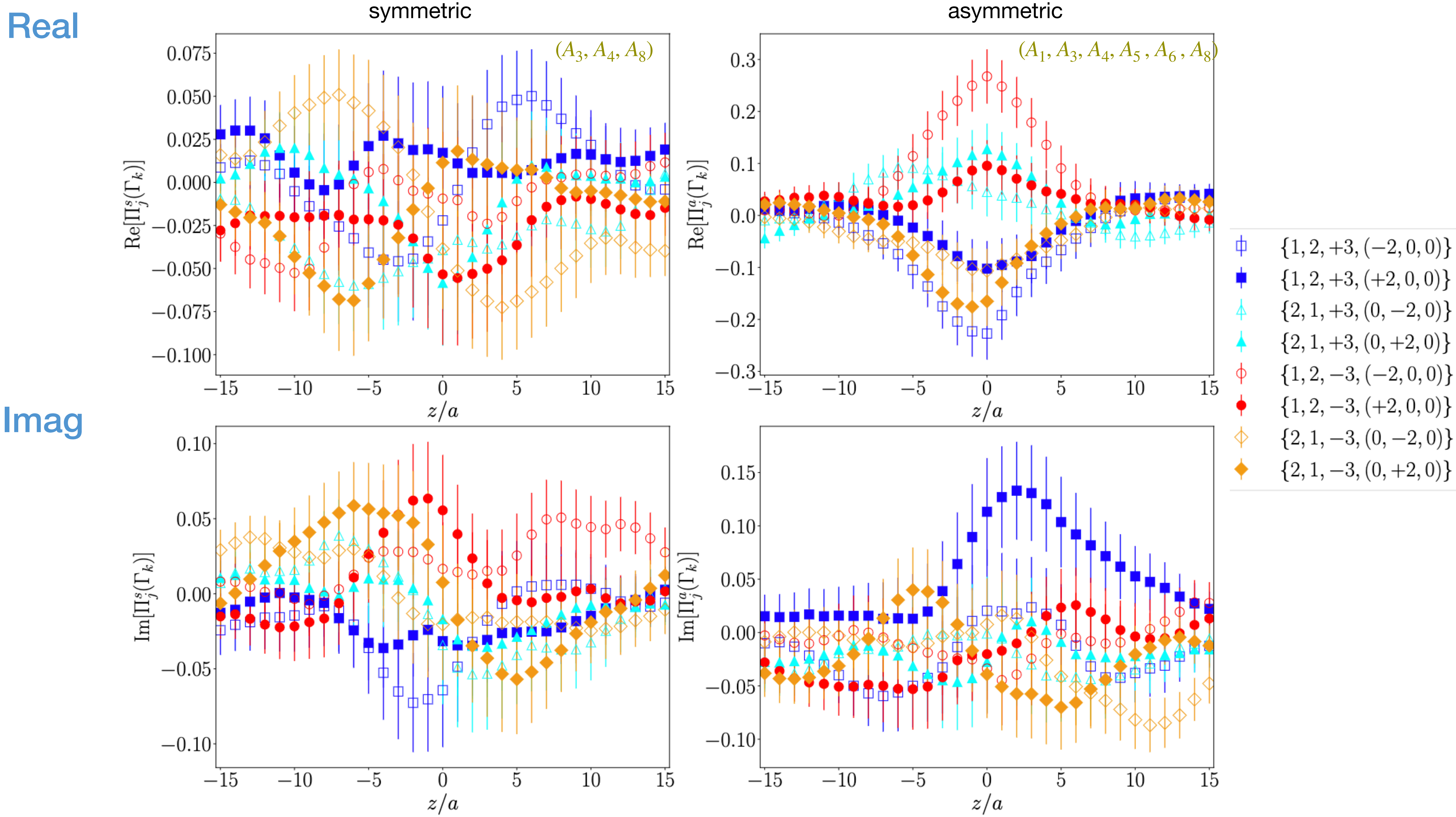
Real

Imag



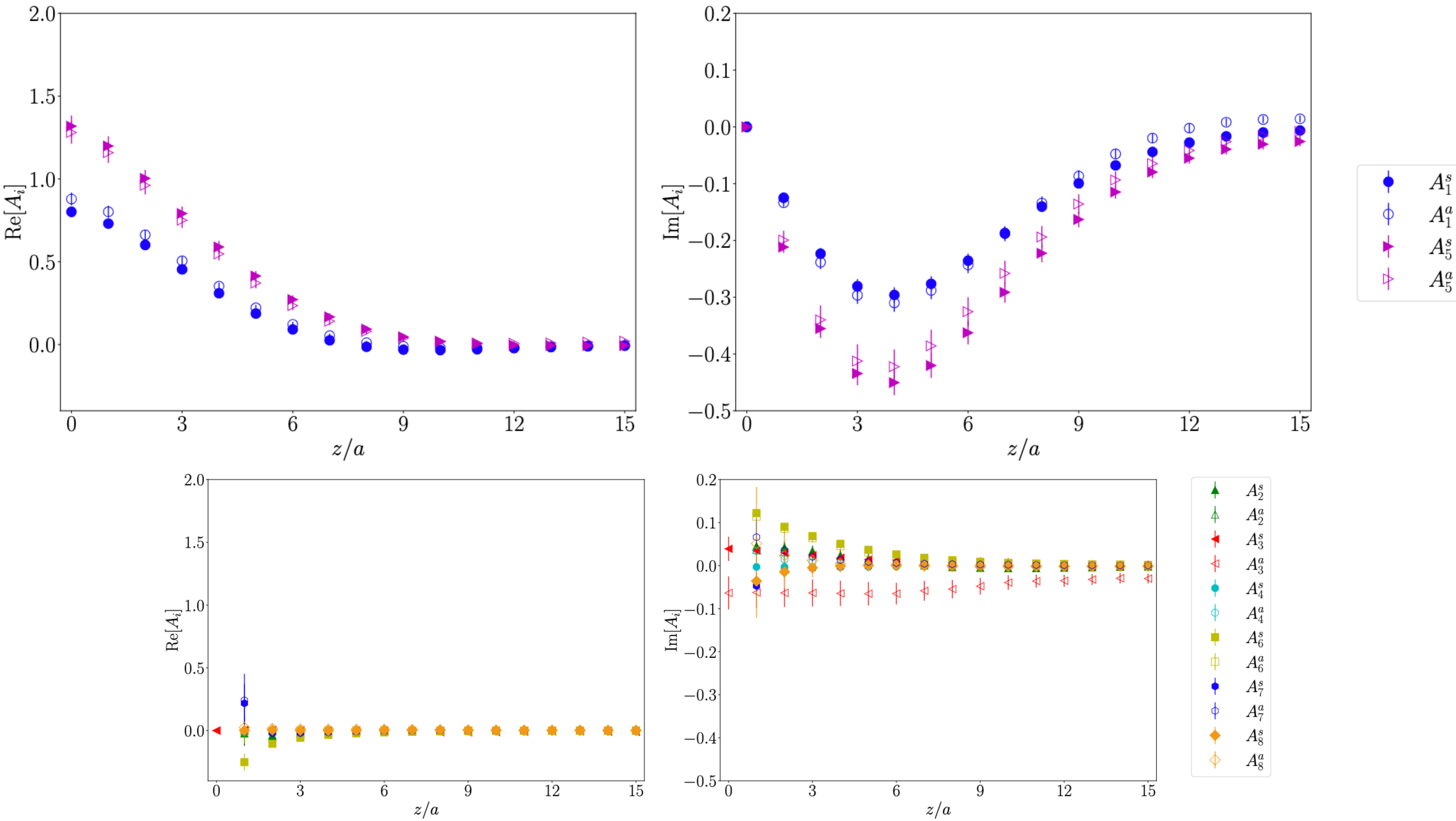
- ★ Lattice data confirm symmetries where applicable **(e.g.,** $\Pi_0^s(\Gamma_0)$ in $\pm P_3, \pm Q, \pm z$)
- ★ ME in asymmetric frame do not have definite symmetries in $\pm P_3, \pm Q, \pm z$
- ★ ME decompose to different A_i
- ★ Multiple ME contribute to the same quantity

Results: matrix elements



- ★ $\Pi_1(\Gamma_2)$ theoretically nonzero
- ★ Noisy contributions lead to challenges in extracting A_i of sub-leading magnitude

Results: A_i



- ★ A_1, A_5 dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of A_1, A_5
- ★ Remaining A_i suppressed (at least for this kinematic setup and $\xi = 0$)

$\mathscr{H}$, $\mathscr{E}$ in terms of A_i

★ Mapping of $\{\mathscr{H}, \mathscr{E}\}$ to A_i using $F^{[\gamma^0]} \sim \left[\gamma^0 H_{Q(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M} E_{Q(0)}(x,\xi,t;P^3)\right]$ in each frame leading to frame dependent relations:

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$$\Pi^s_H = A_1 + \frac{zQ_1^2}{2P_3} A_6$$

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$$\Pi^a_H = A_1 + \frac{Q_0}{P_0} A_3 + \frac{m^2 z Q_0}{2P_0 P_3} A_4 + \frac{z(Q_0^2 + Q_\perp^2)}{2P_3} A_6 + \frac{z(Q_0^3 + Q_0 Q_\perp^2)}{2P_0 P_3} A_8$$

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★ Definition of Lorentz invariant $\mathcal{H}$, $\mathcal{E}$

$(\xi = 0)$

$$\Pi^{\rm impr}_H = A_1$$

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1st approach: extraction of $\{\mathcal{H}^s_0, \mathcal{E}^s_0\}$ using A_i from any frame (universal)

$$\Pi^a_H = A_1 + \frac{Q_0}{P_0} A_3 + \frac{m^2 z Q_0}{2P_0P_3} A_4 + \frac{z(Q_0^2 + Q_\perp^2)}{2P_3} A_6 + \frac{z(Q_0^3 + Q_0 Q_\perp^2)}{2P_0P_3} A_8$$

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1st approach: extraction of $\{\mathscr{H}_0^s, \mathscr{E}_0^s\}$ using A_i from any frame (universal)

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★ Definition of Lorentz invariant $\mathscr{H}$, $\mathscr{E}$

$(\xi = 0)$

$$\Pi_H^{\text{impr}} = A_1$$

$$\Pi_E^{\text{impr}} = -A_1 + 2A_5 + 2zP_3A_6$$

$\mathscr{H}$, $\mathscr{E}$ in terms of A_i

★ Mapping of $\{\mathscr{H}, \mathscr{E}\}$ to A_i using $F^{[\gamma^0]} \sim \left[\gamma^0 H_{Q(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M} E_{Q(0)}(x,\xi,t;P^3)\right]$ in each frame leading to frame dependent relations:

$(\xi = 0)$

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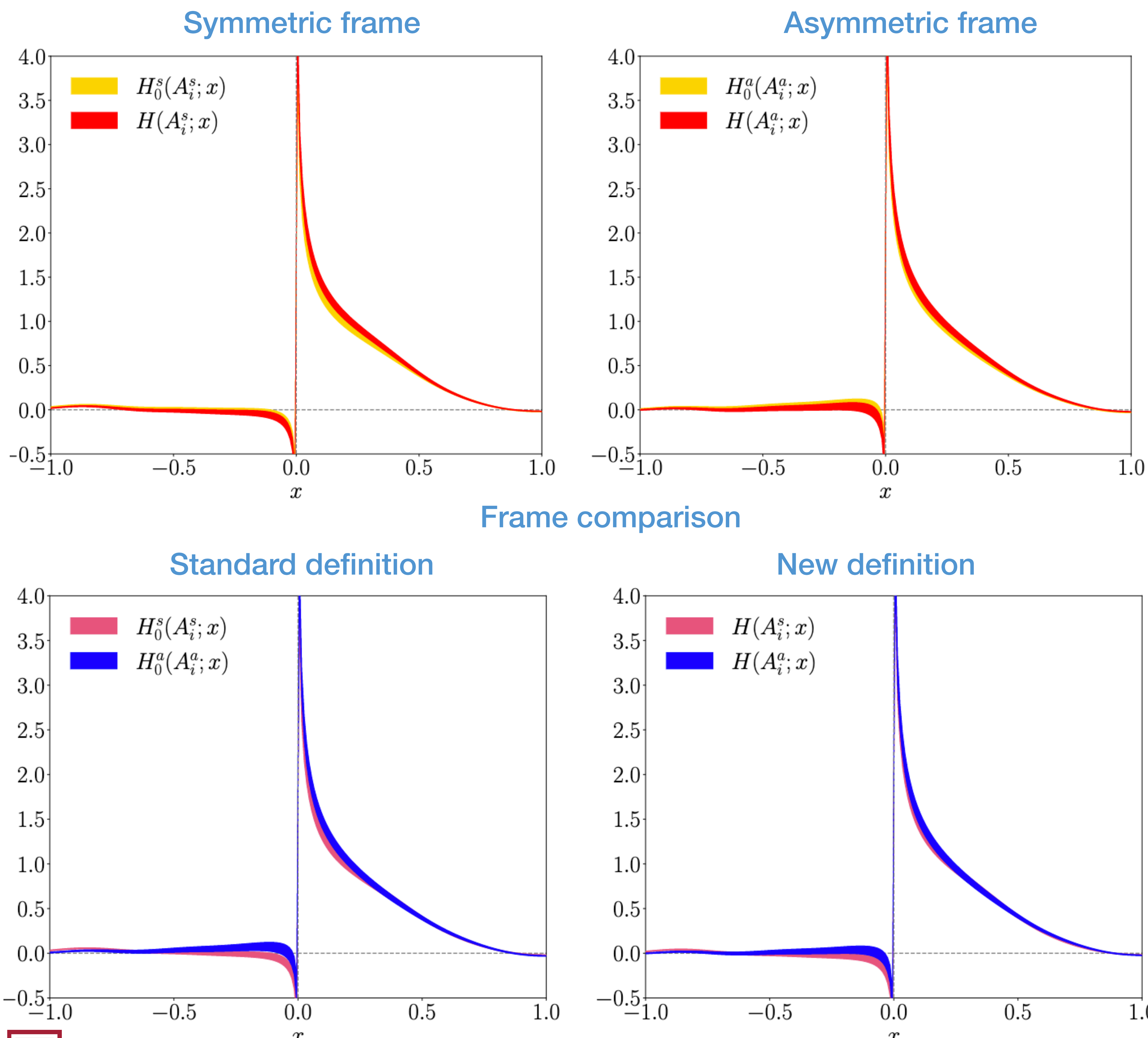
3rd approach: use redefined Lorentz covariant $\{\mathscr{H}, \mathscr{E}\}$ in desired frame

Results: $H - \mathrm{GPD}$

Definition comparison



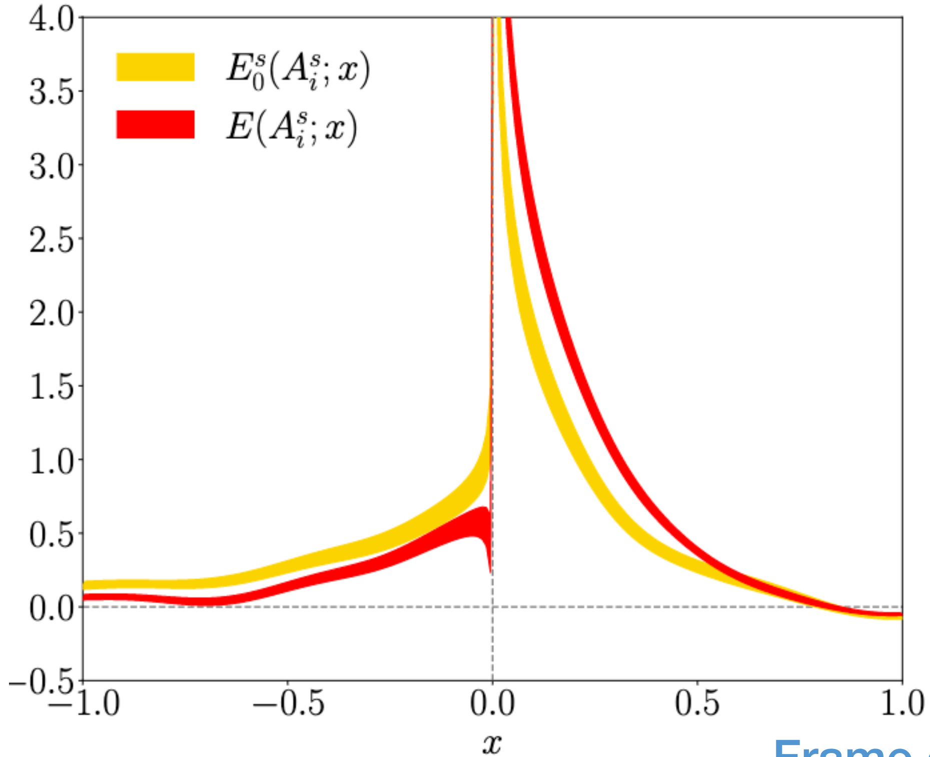
Similar results for H and $\mathscr{H}$ for both frames (agreement not by construction)

Agreement between frames for $\mathscr{H}$ (agreement by construction)

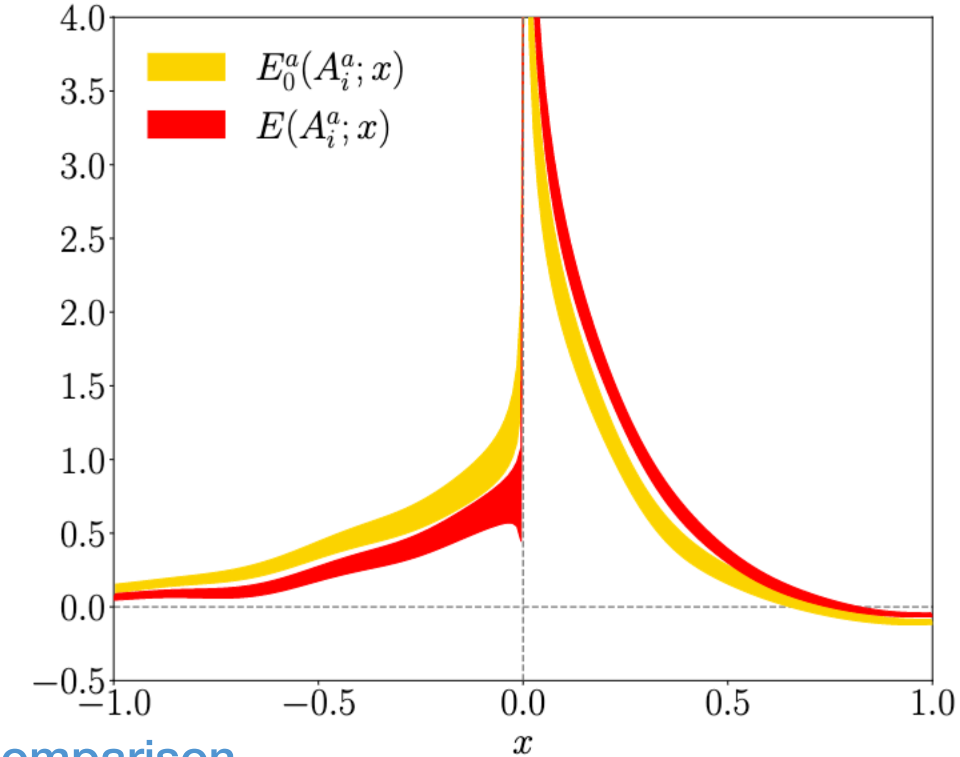
Results: $E - \mathrm{GPD}$

Definition comparison

Symmetric frame



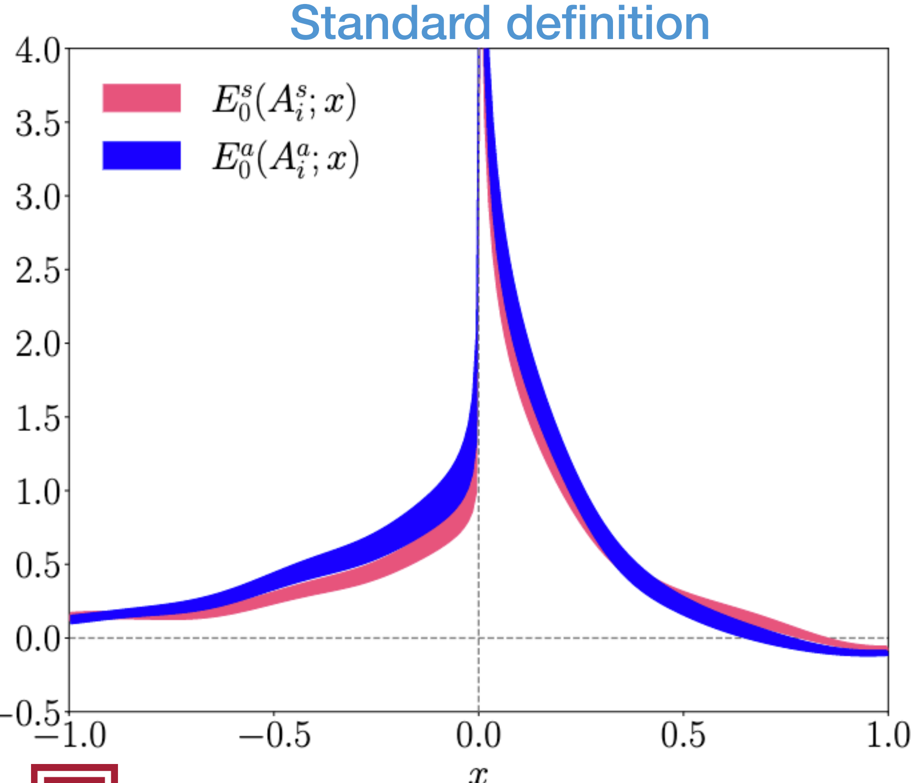
Asymmetric frame



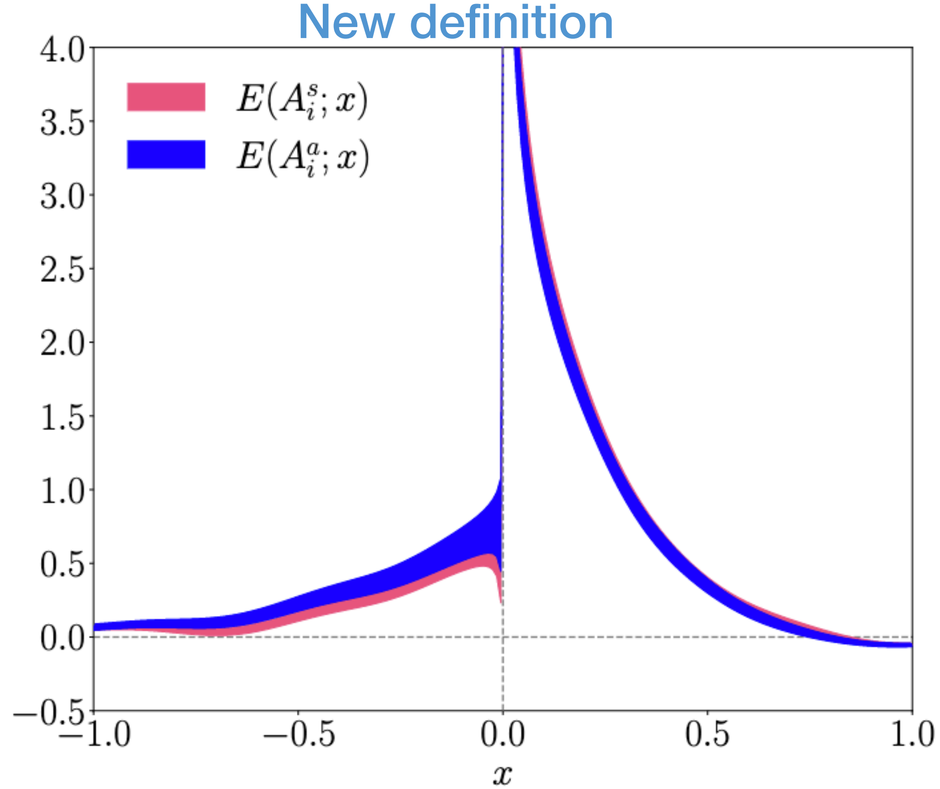
Differences between E and $\mathscr{E}$ for both frames (agreement not by construction)

Frame comparison

Standard definition



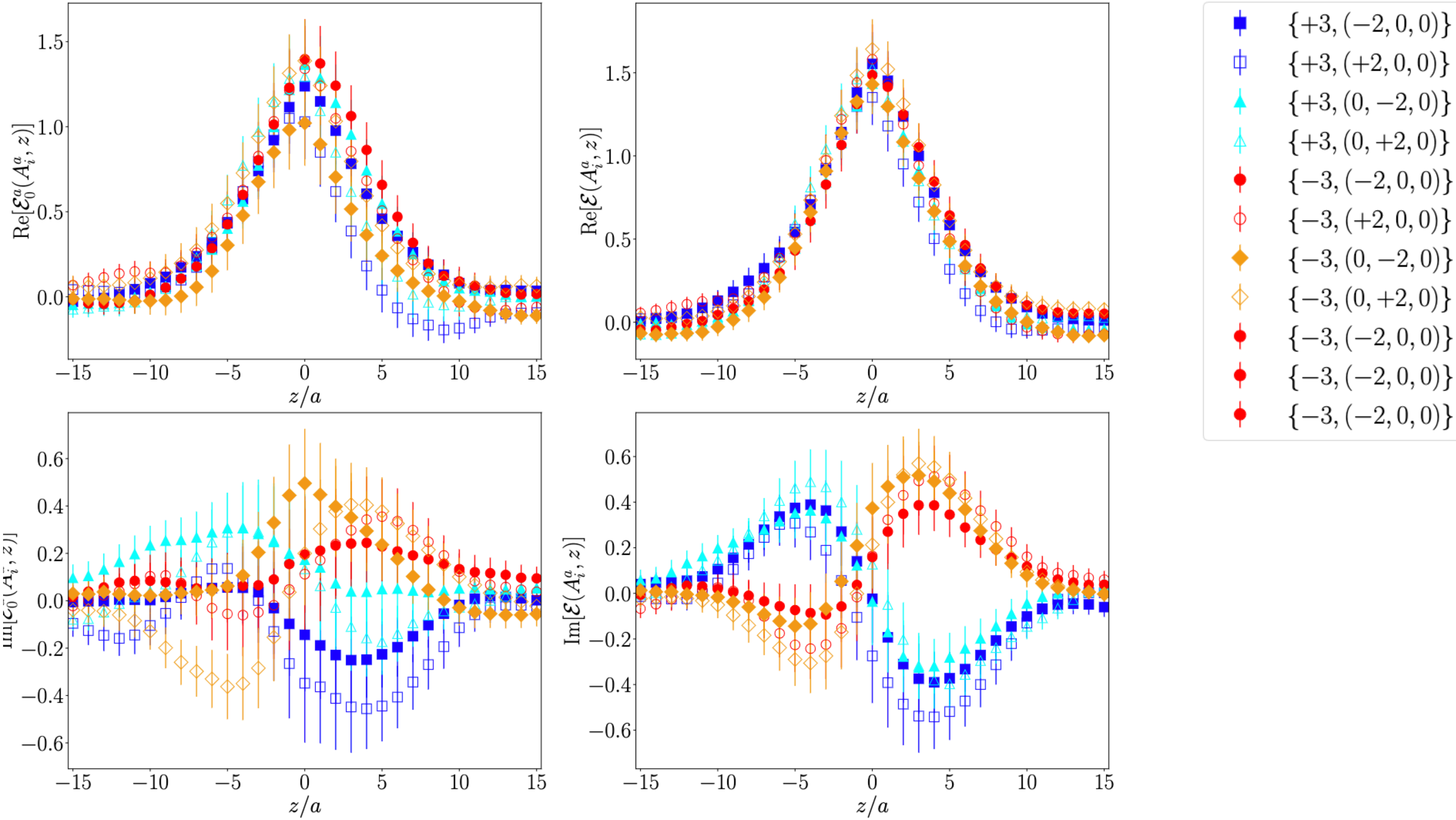
New definition



Agreement reached between frames for improved definition (expected theoretically)

A comment on Lorentz covariant definitions

Example: asymmetric frame



- ★ Lorentz covariant case: more precise data
- ★ Same effect of improvement also for symmetric frame

A comment on Lorentz covariant definitions

Example: asymmetric frame

$$\mathcal{E}_0^a(A_i^a;z) = -\frac{4m^3}{K(E_f+E_i)(E_f+m)(E_i+m)}\Pi_0^a(\Gamma_0) - i\,\frac{4m^3}{KP_3\Delta(E_i+m)}\Pi_0^a(\Gamma_2)\,.$$

$$\mathcal{E}(A_i^a;z) = -\frac{2m^3}{E_f^2(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{K} - i\,\frac{2m^3P_3(E_f+E_i+2m)}{E_f^2\Delta(E_f+m)(E_i+m)}\frac{\Pi_0^a(\Gamma_2)}{K} + \frac{2m^3P_3(E_f+E_i+2m)}{E_f^2(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{K}$$
$$+i\,\frac{2m^3(E_f-E_i)}{E_f^2\Delta(E_i+m)}\frac{\Pi_1^a(\Gamma_0)}{K} + \frac{4m^4P_3}{E_f^2(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{K} - \frac{4m^4}{E_f^2\Delta(E_i+m)}\frac{\Pi_2^a(\Gamma_3)}{K}\,. \qquad ($$

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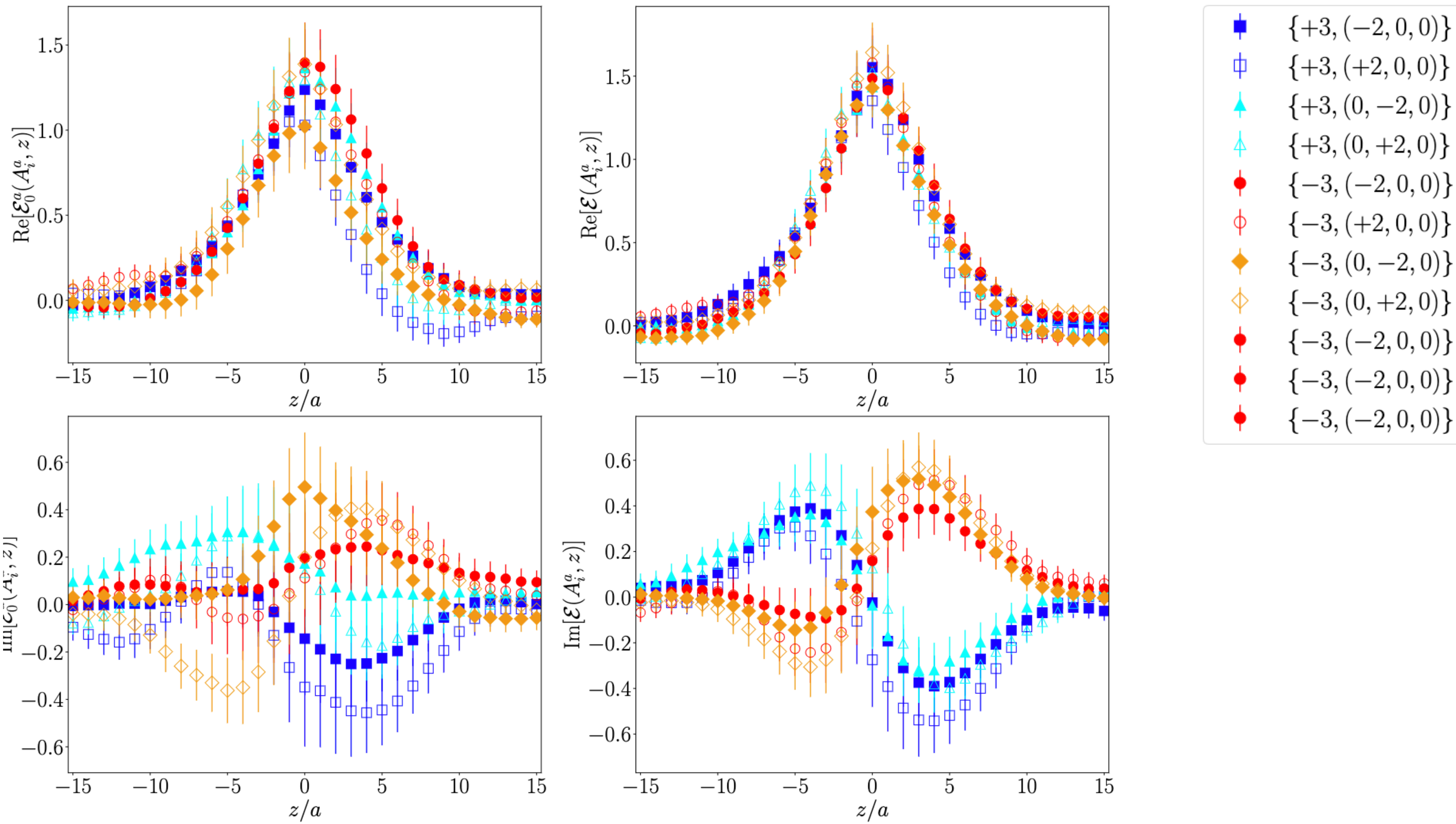

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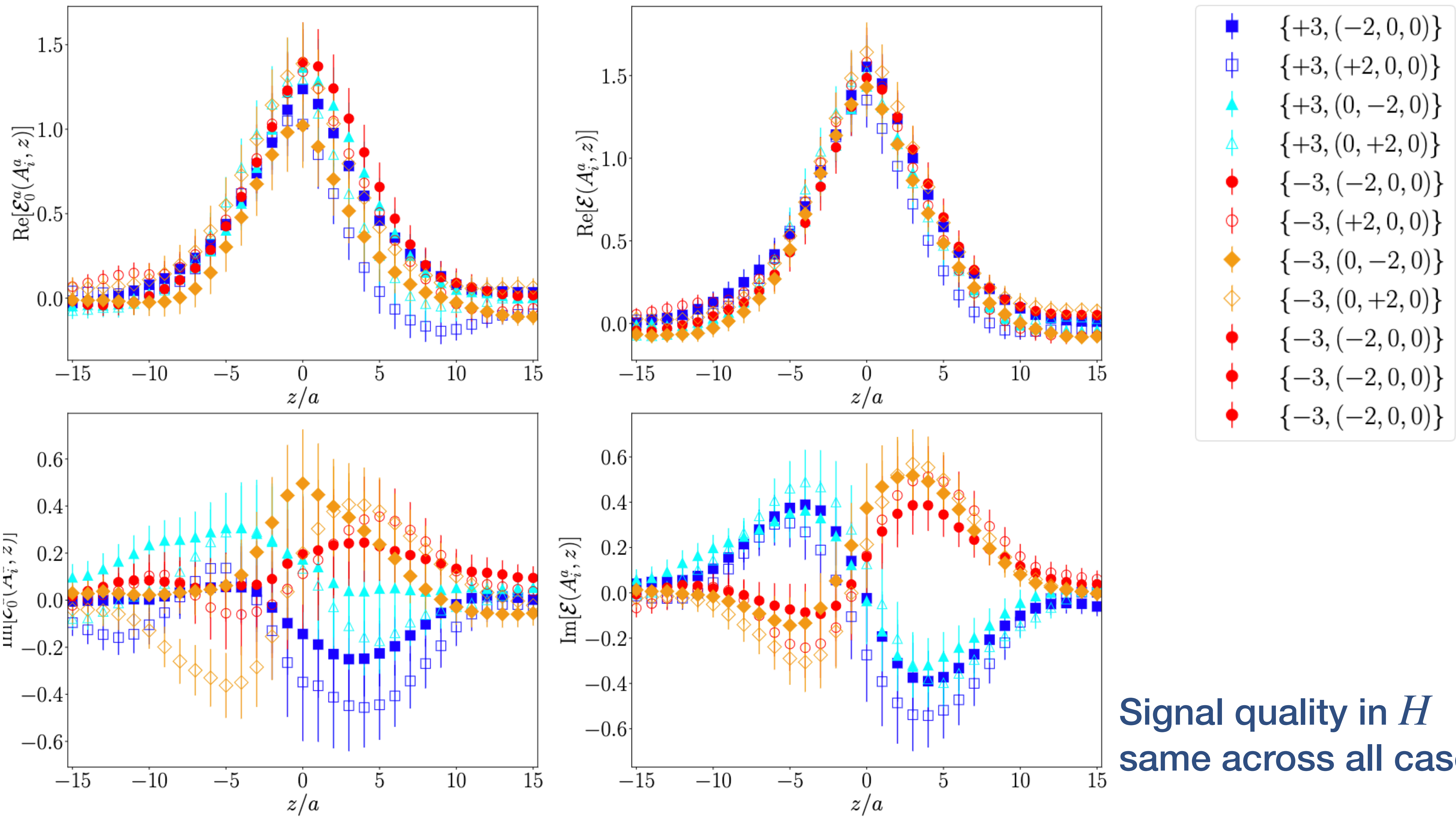


Signal quality in H same across all cases

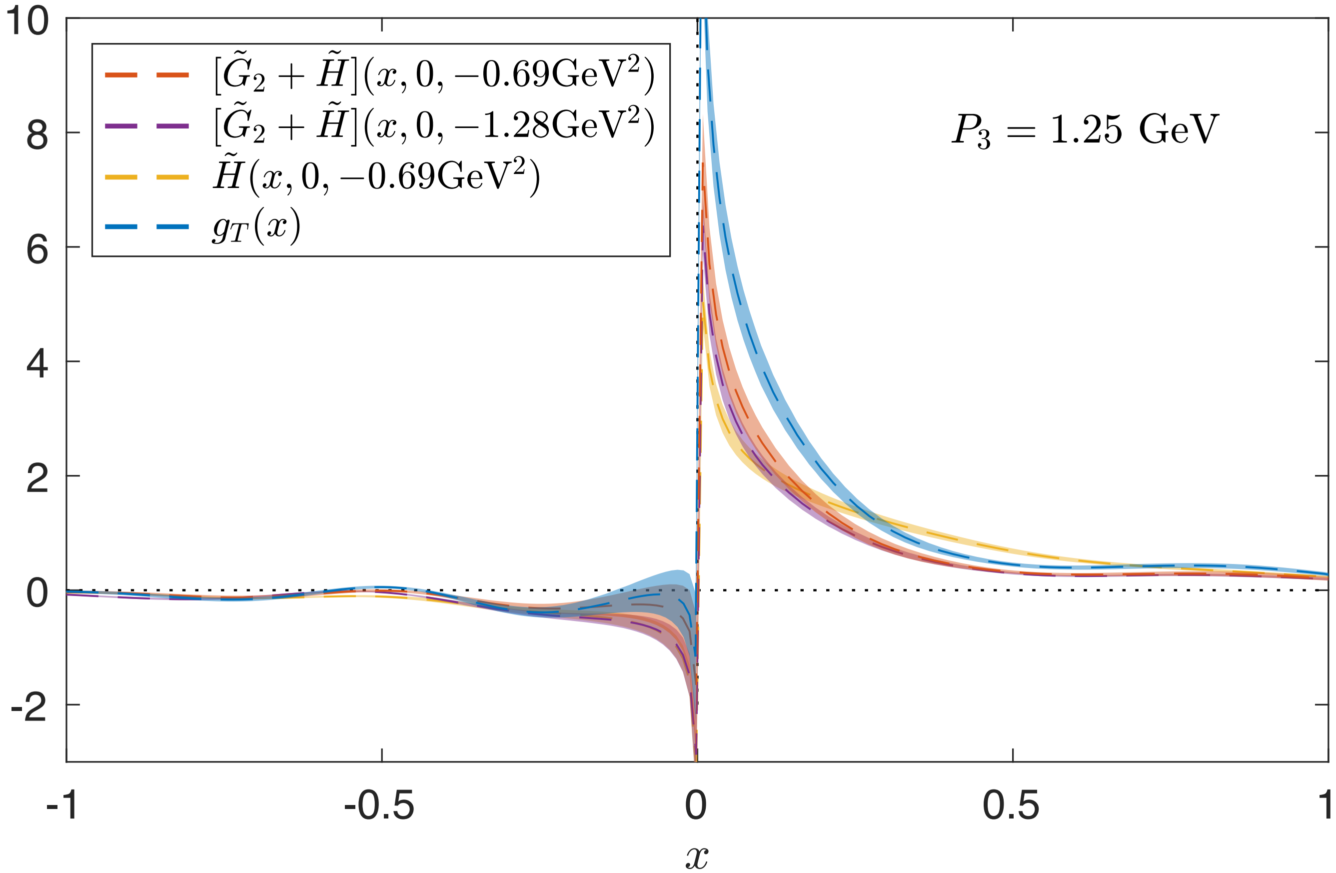
★ Lorentz covariant case: more precise data

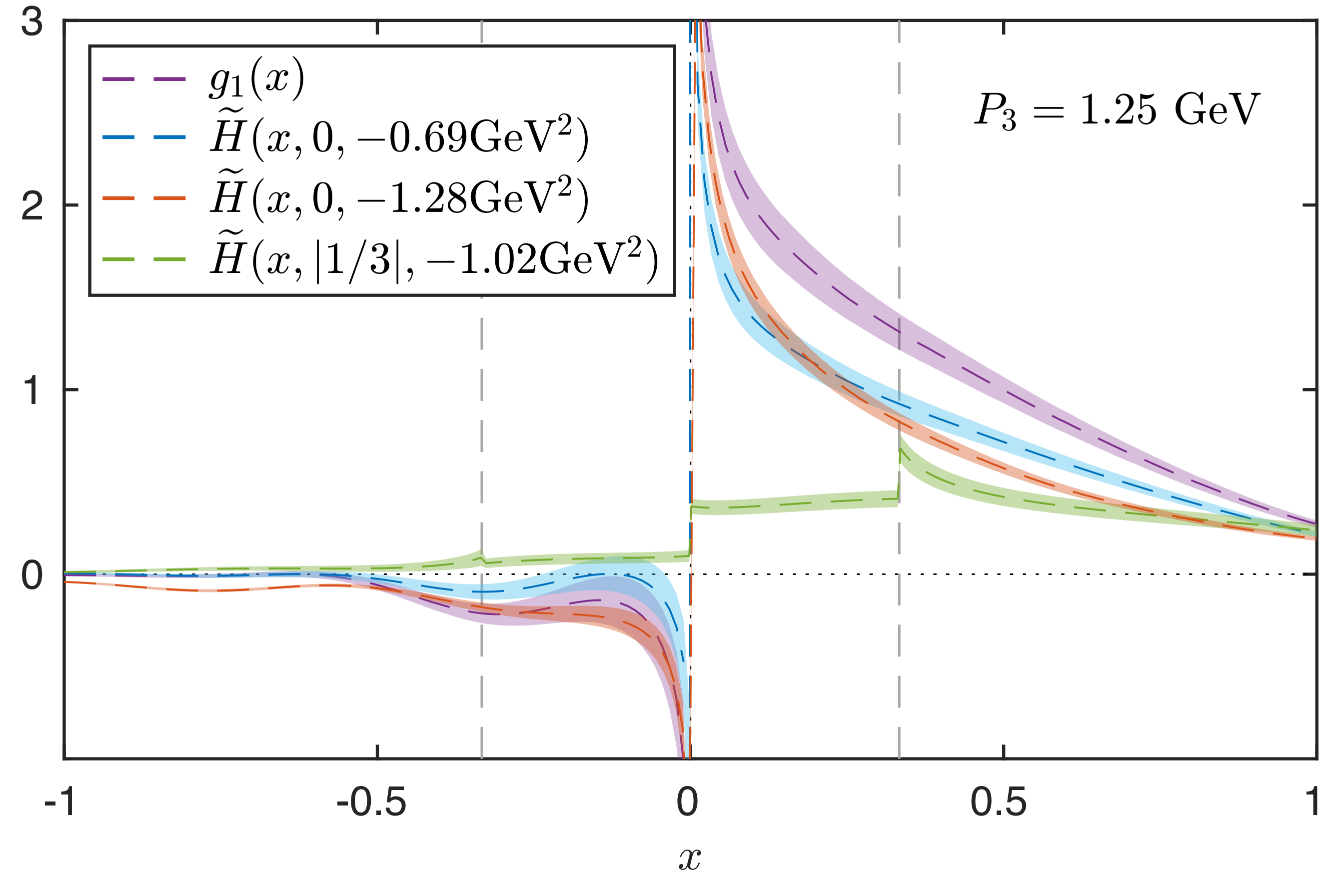
★ Same effect of improvement also for symmetric frame

Possible extensions

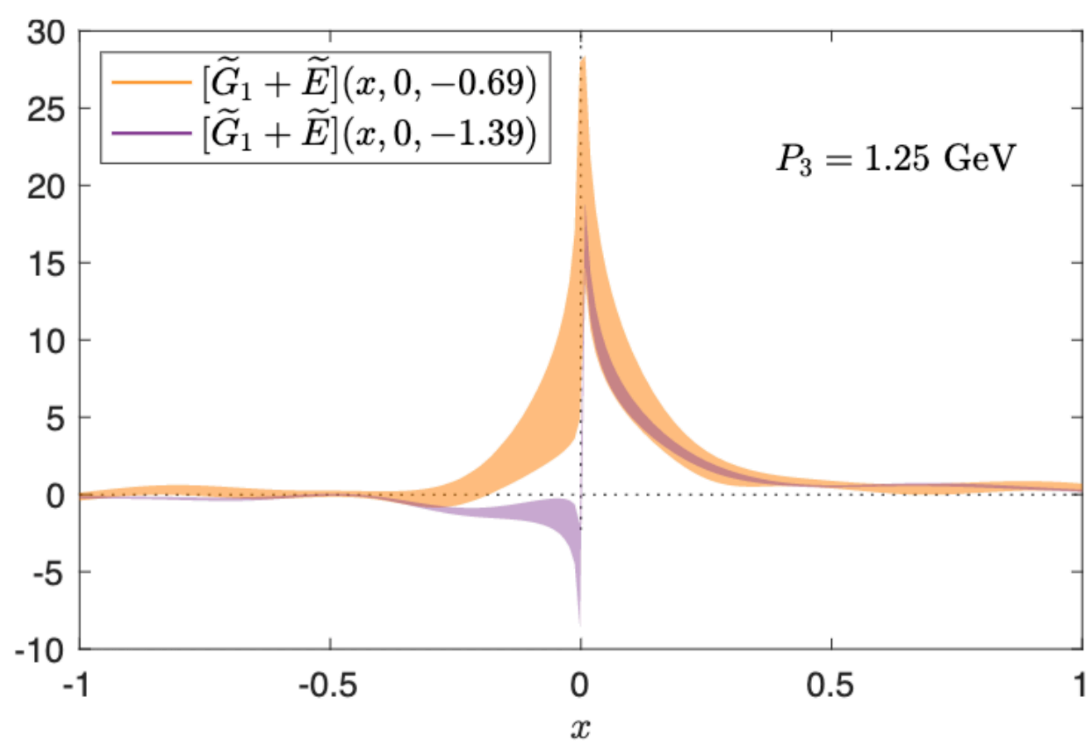
★ Twist-3 GPDs

PRELIMINARY





[S. Bhattacharya et al., PoS LATTICE2021 (2022) 054 arXiv:2112.05538]



★ $g_T(x)$: dominant distribution

★ $\widetilde{H} + \widetilde{G}_2$ similar in magnitude to $\widetilde{H}$

★ $\widetilde{G}_2$ is expected to be small

Summary

★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV

★ New proposal for Lorentz invariant decomposition has great advantages:
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- access to a broad range of t and ξ

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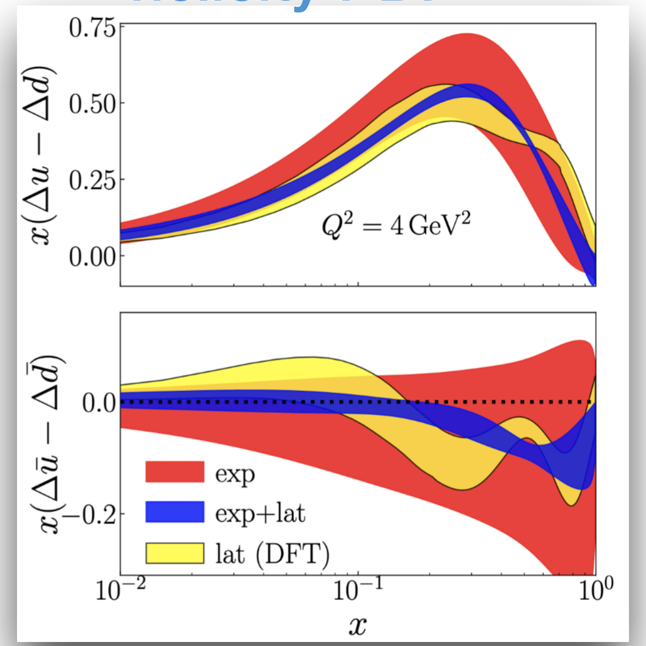
helicity PDF



[JAM & ETMC, PRD 103 (2021) 016003]

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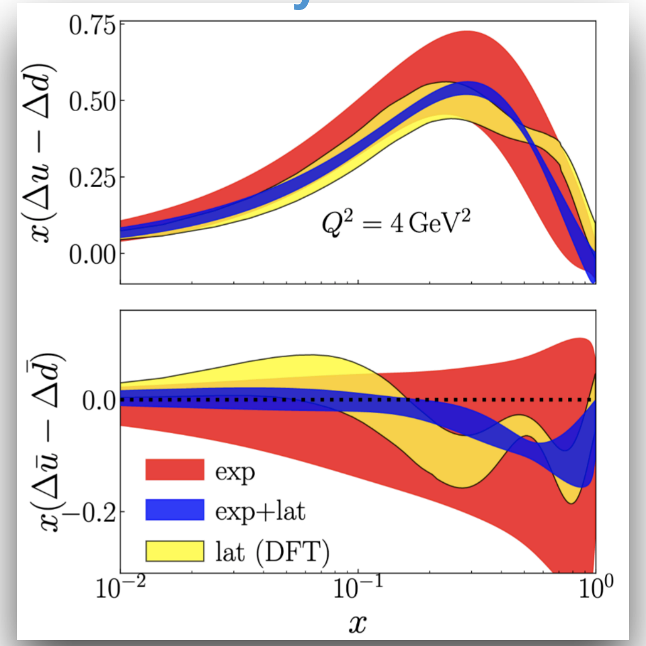

[JAM & ETMC, PRD 103 (2021) 016003]

Thank you

U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

U.S. DEPARTMENT OF ENERGY Office of Science

DOE Early Career Award (NP)
Grant No. DE-SC0020405

BACKUP

Challenges of lattice calculation

★ **Statistical noise increases with P_3, t**

use of momentum smearing method

Challenges of lattice calculation

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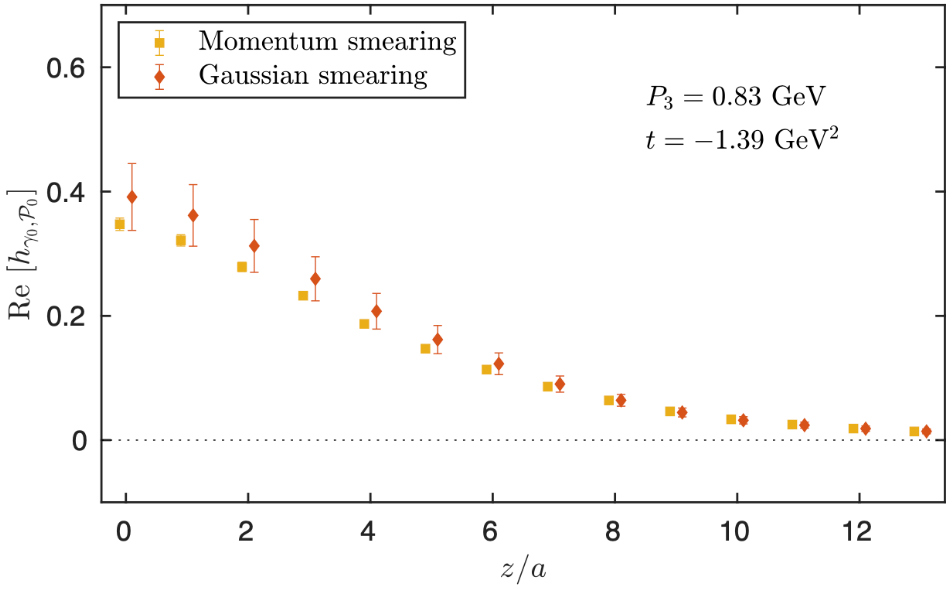

Challenges of lattice calculation

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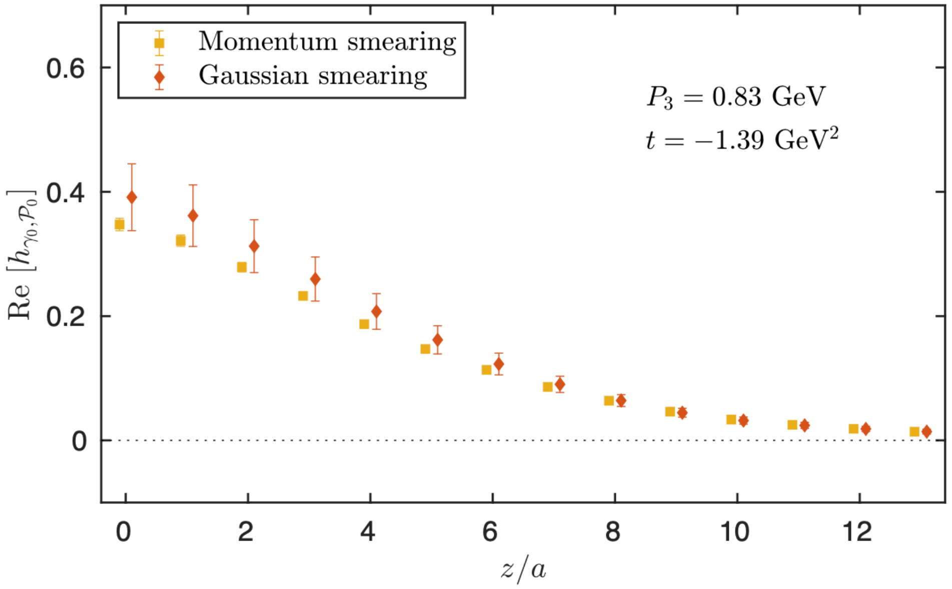

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- Matrix elements decompose into more than one GPDs
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- P_3 must be chosen carefully due to UV cutoff ($a^{-1} \sim 2\,\mathrm{GeV}$)

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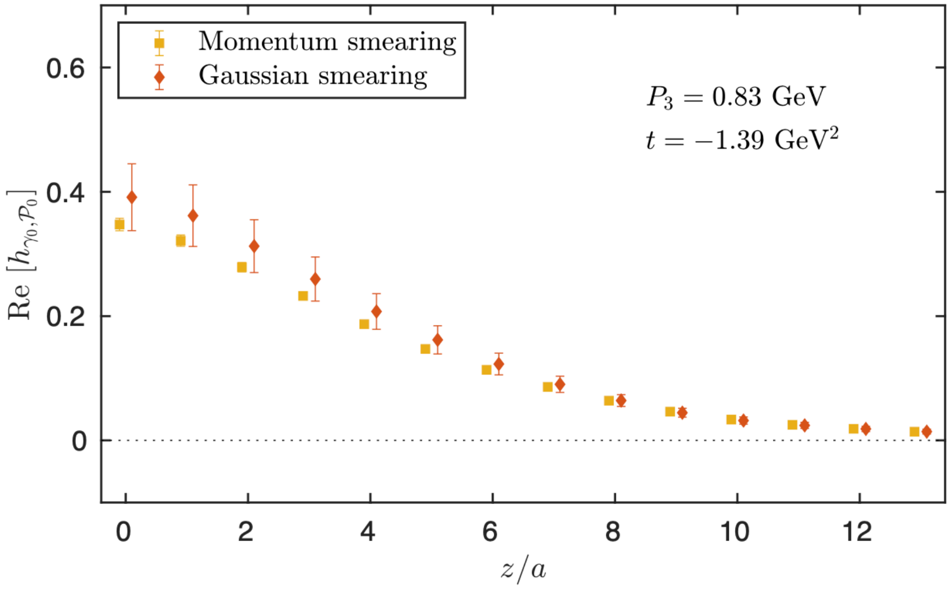

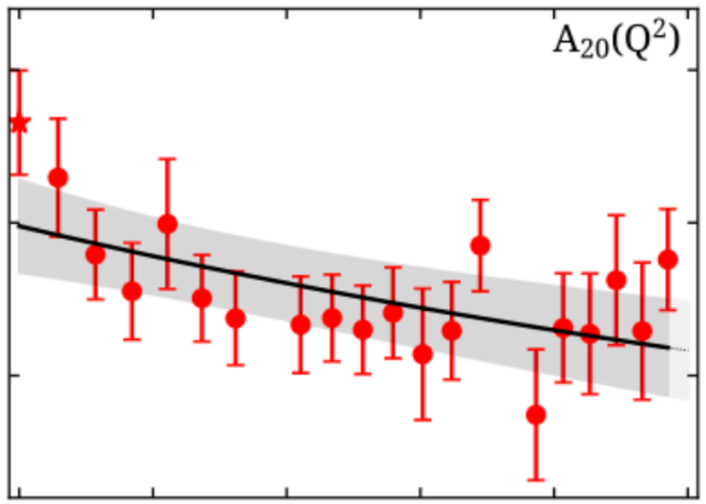

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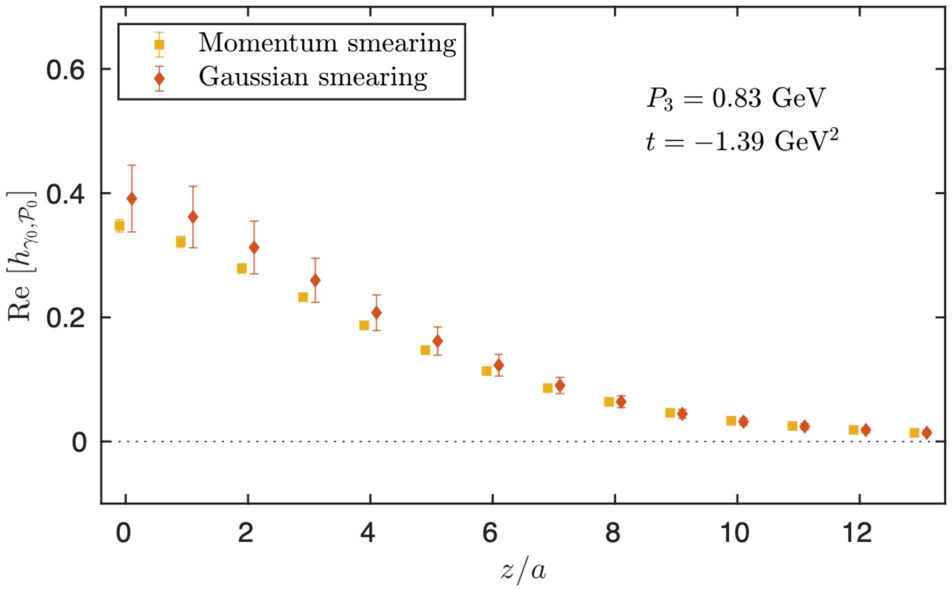

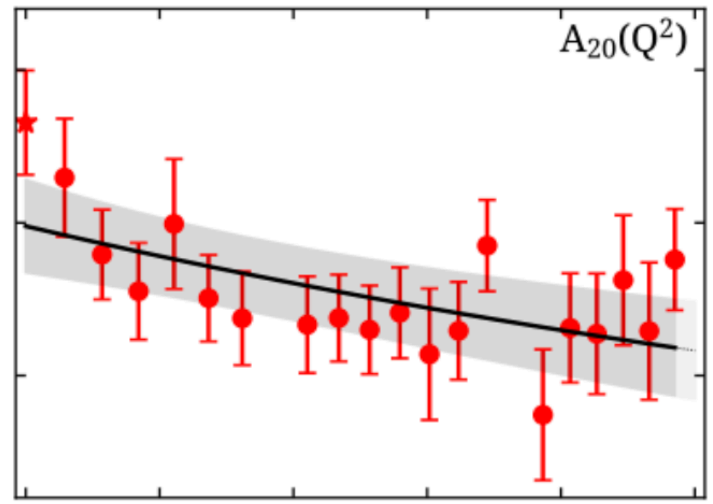

Ref.	m_π(MeV)	P_3(GeV)	$\frac{n}{s}\big\|_{z=0}$
quasi/pseudo [59, 95]	130	1.38	6%
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[M. Constantinou, EPJA 57 (2021) 77]

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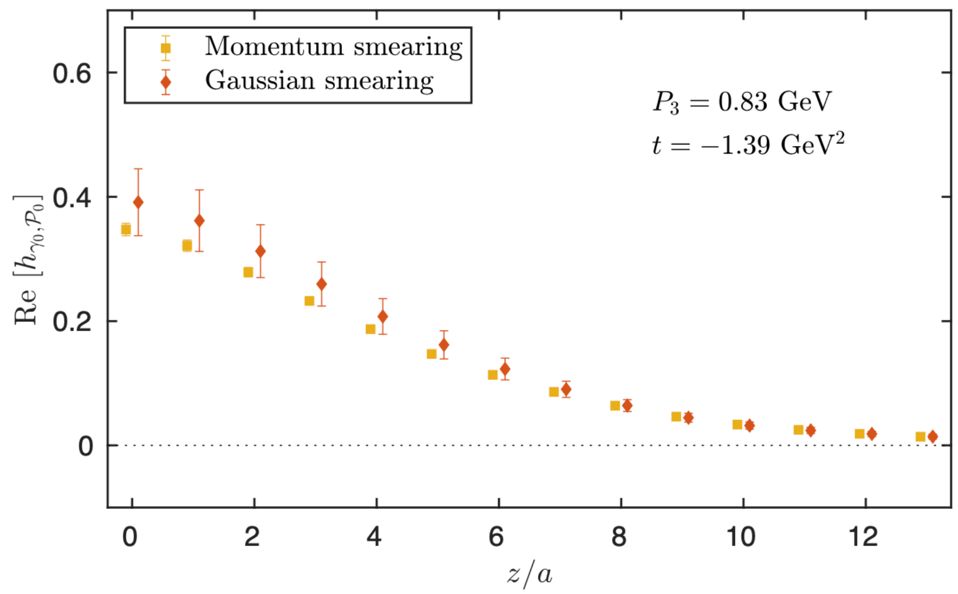

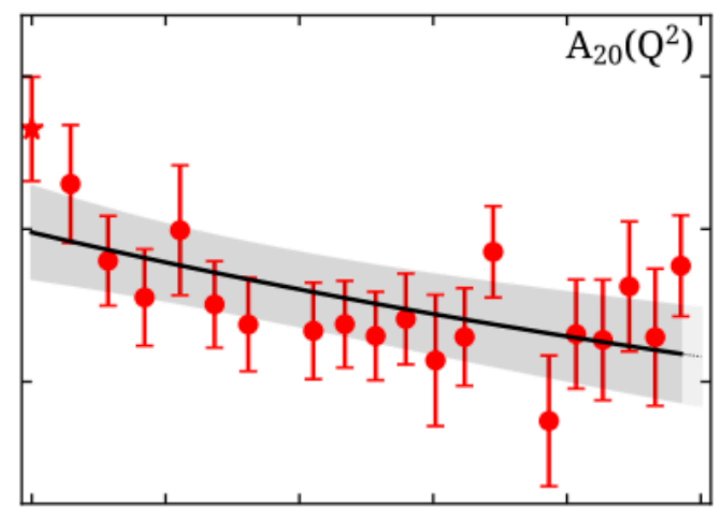

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Further increase of momentum at the cost of credibility

Twist-classification of GPDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

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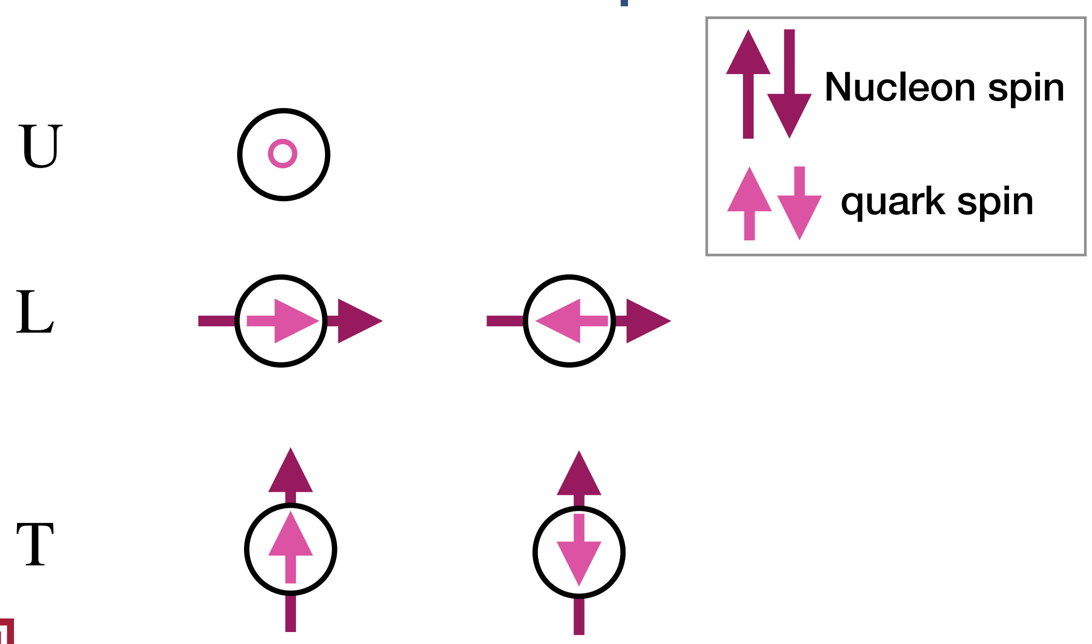
Twist-2 $(f_i^{(0)})$

Nucleon \ Quark	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

Probabilistic interpretation



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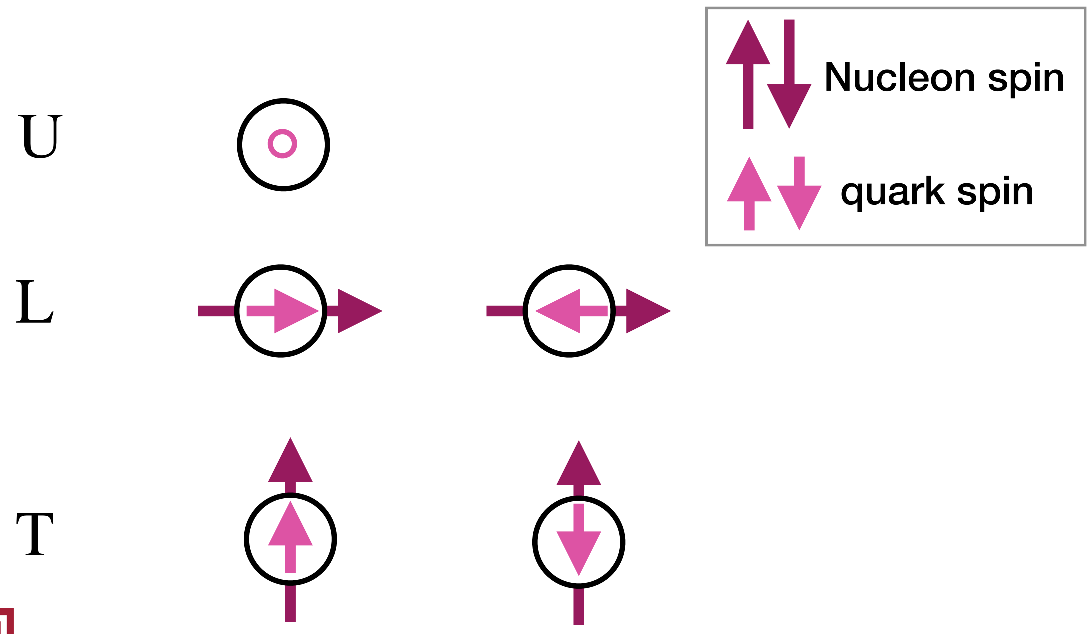
Twist-3 $(f_i^{(1)})$

Nucleon \ $\mathcal{O}$	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

(Selected)

Probabilistic interpretation



- ★ Lack density interpretation, but not-negligible
- ★ Contain info on quark-gluon-quark correlators
- ★ Physical interpretation, e.g., transverse force
- ★ Kinematically suppressed
  Difficult to isolate experimentally
- ★ Theoretically: contain $\delta(x)$ singularities