Extraction of pion GPD from lattice QCD using an asymmetric frame

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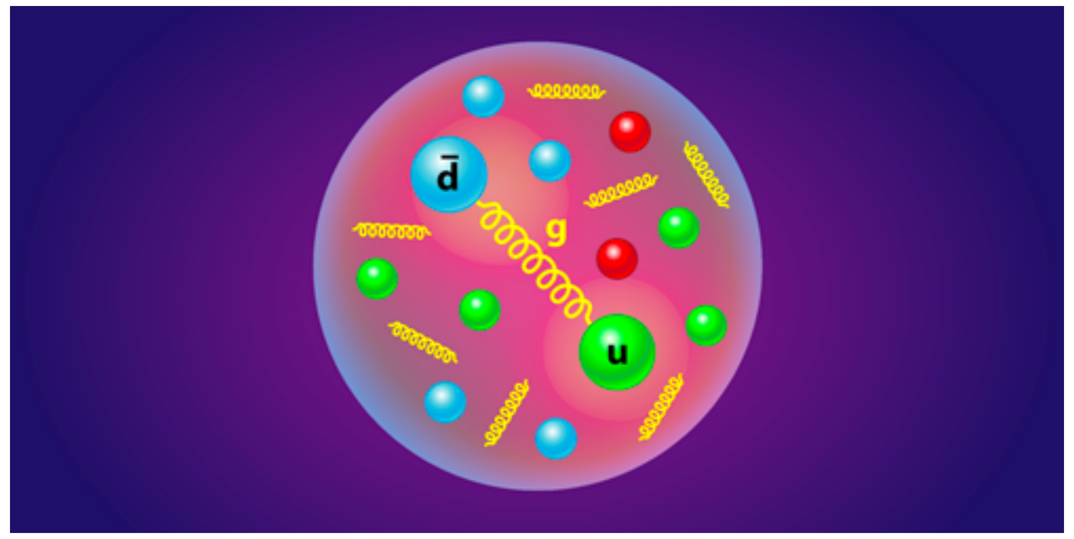
Generalized Parton Distributions (GPDs)

One of the main goals in hadron physics:

Understand 3D nucleon structure

Especially with the EIC launching in the near future



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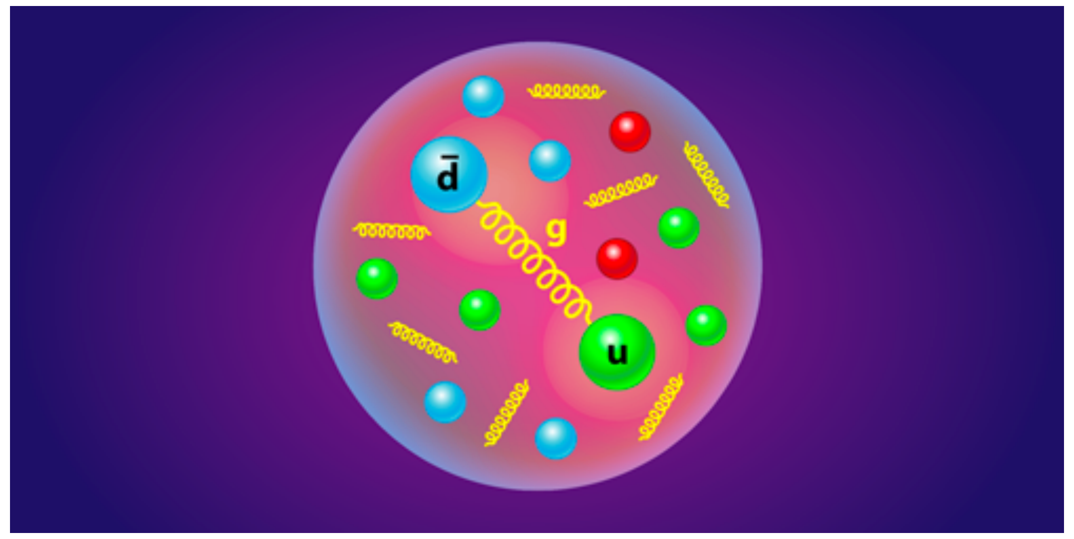

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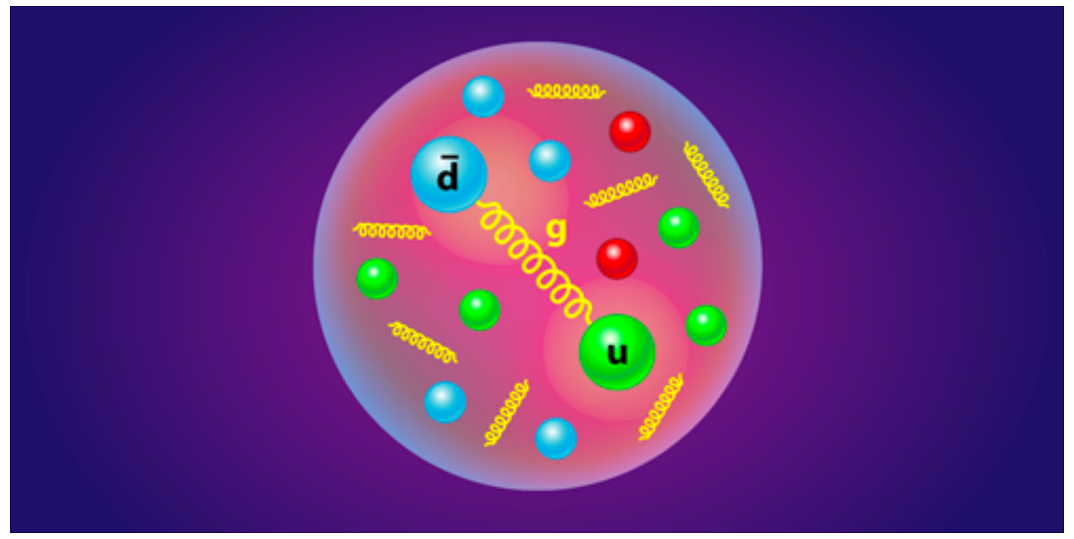

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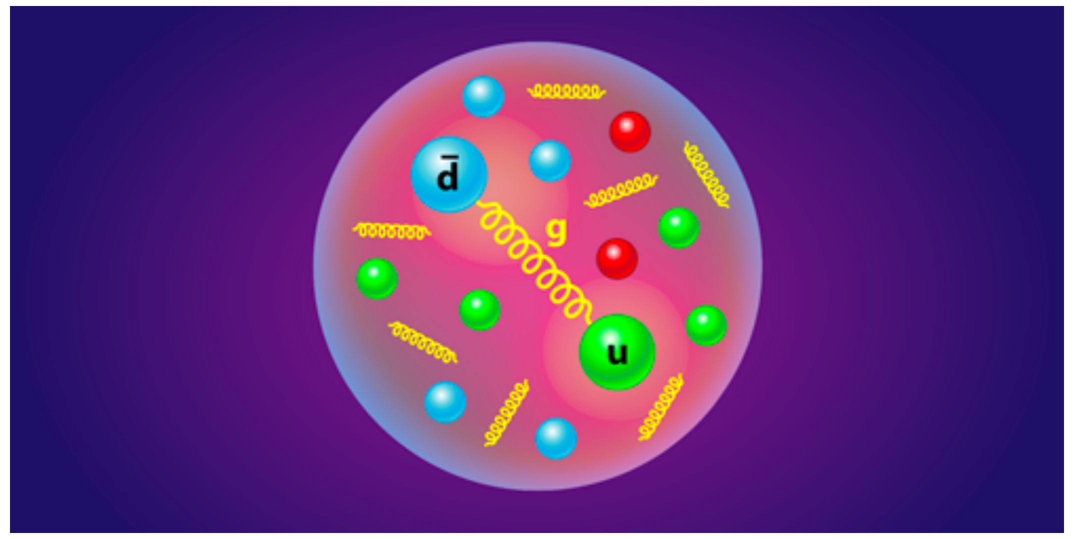

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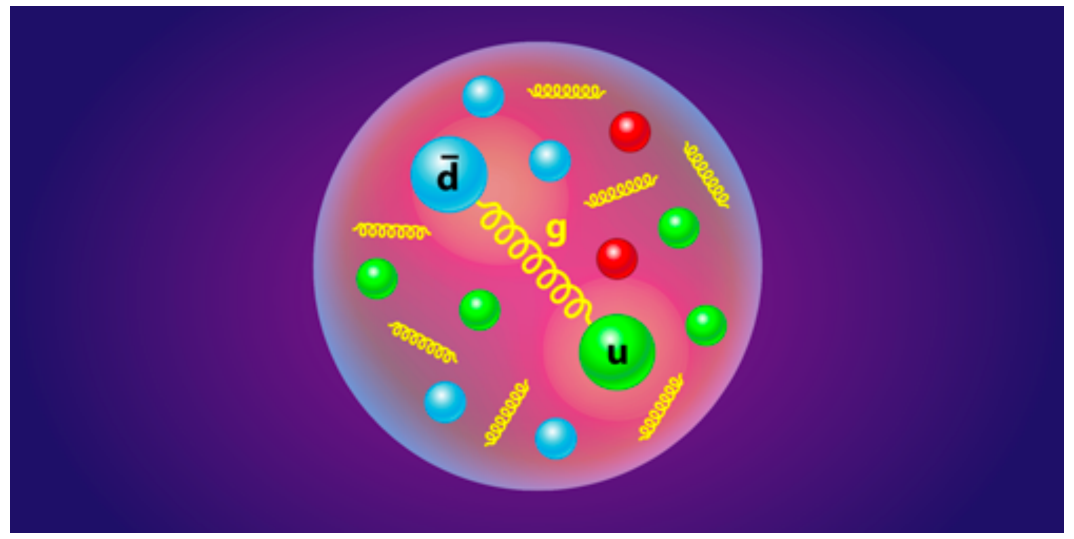

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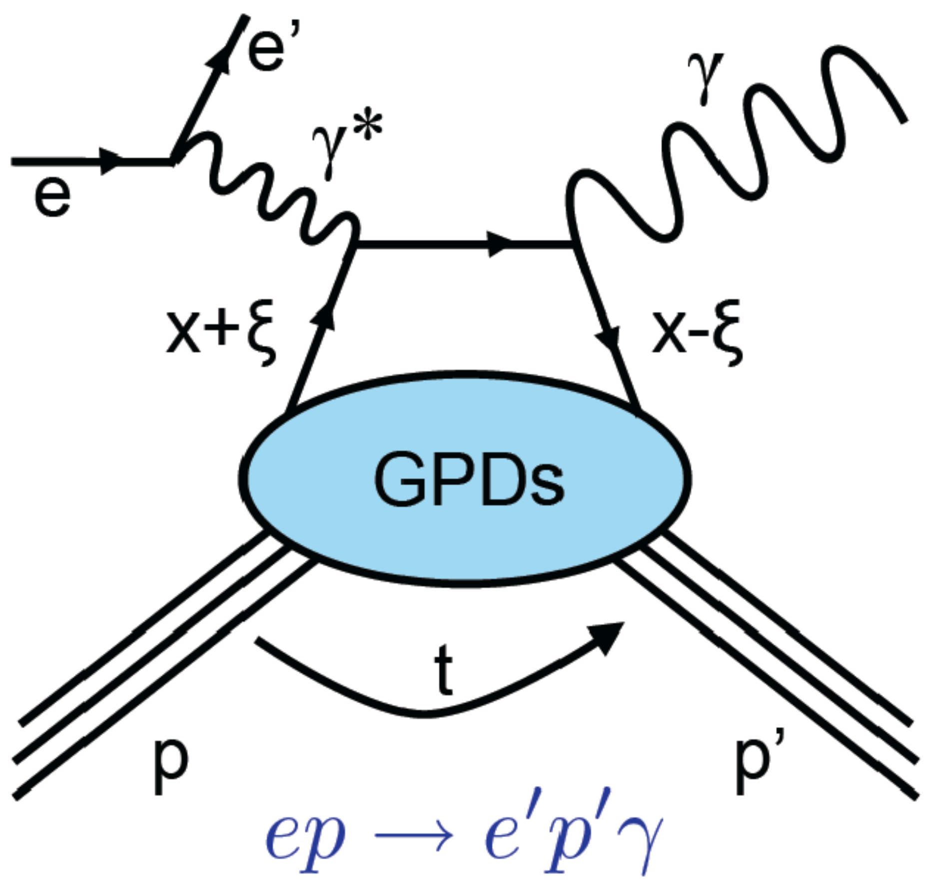
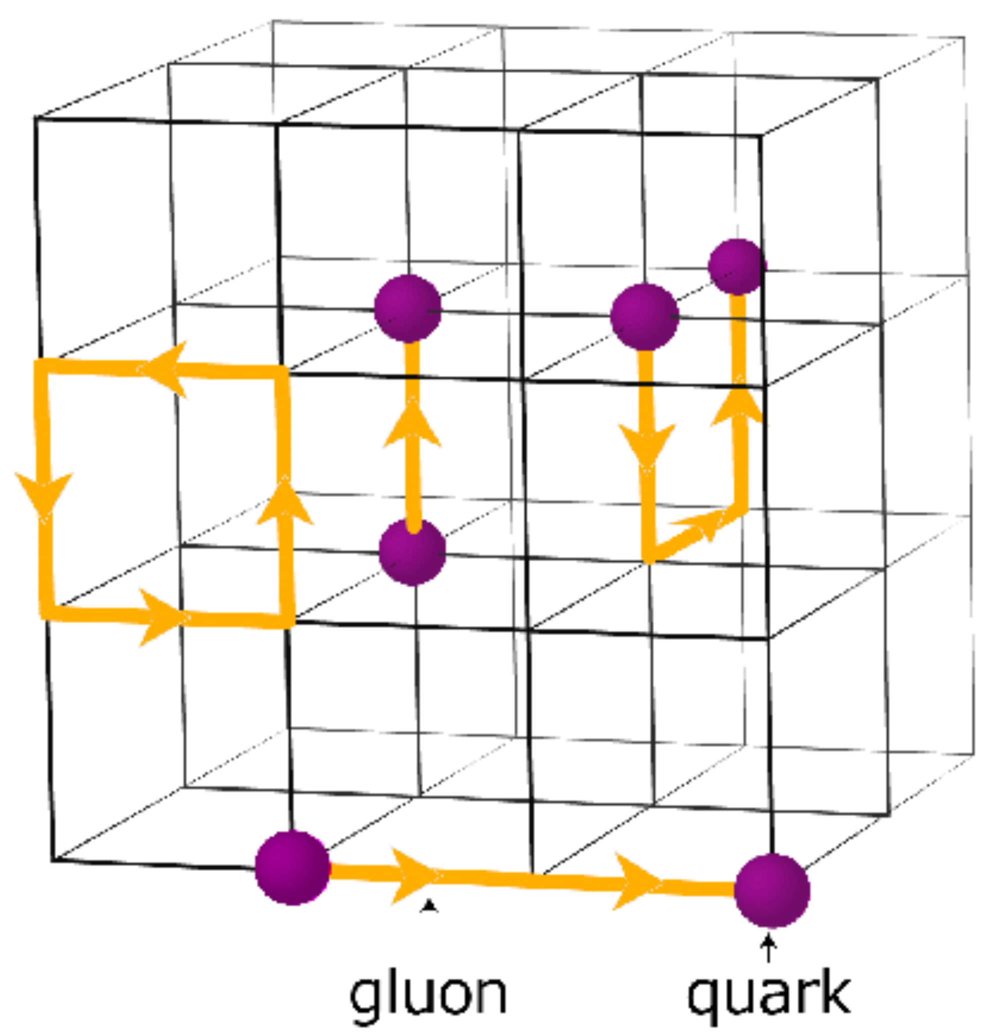
- ❖ Contain information on mechanical properties of hadrons
- ❖ Wealth of information on the hadrons spin
- ❖ Experimentally, we rely on exclusive processes like deeply virtual Compton Scattering (DVCS) - $ep \to eX$ [X.-D. Ji, PRD 55, 7114 (1997)]
- ❖ Exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$ [J. Qui et al., arXiv:2205.07846]



GPDs from Lattice QCD

- ❖ Direct access to partonic distributions impossible in LQCD:
  - ❖ PDFs/GPDs/TMDs are defined on the light cone, that is: $t^2 - \vec{r}^2 = 0$
  - ❖ LQCD is a Euclidean formulation (Wick rotation, $t \rightarrow i\tau$) and light cone: $\tau^2 + \vec{r}^2 = 0$



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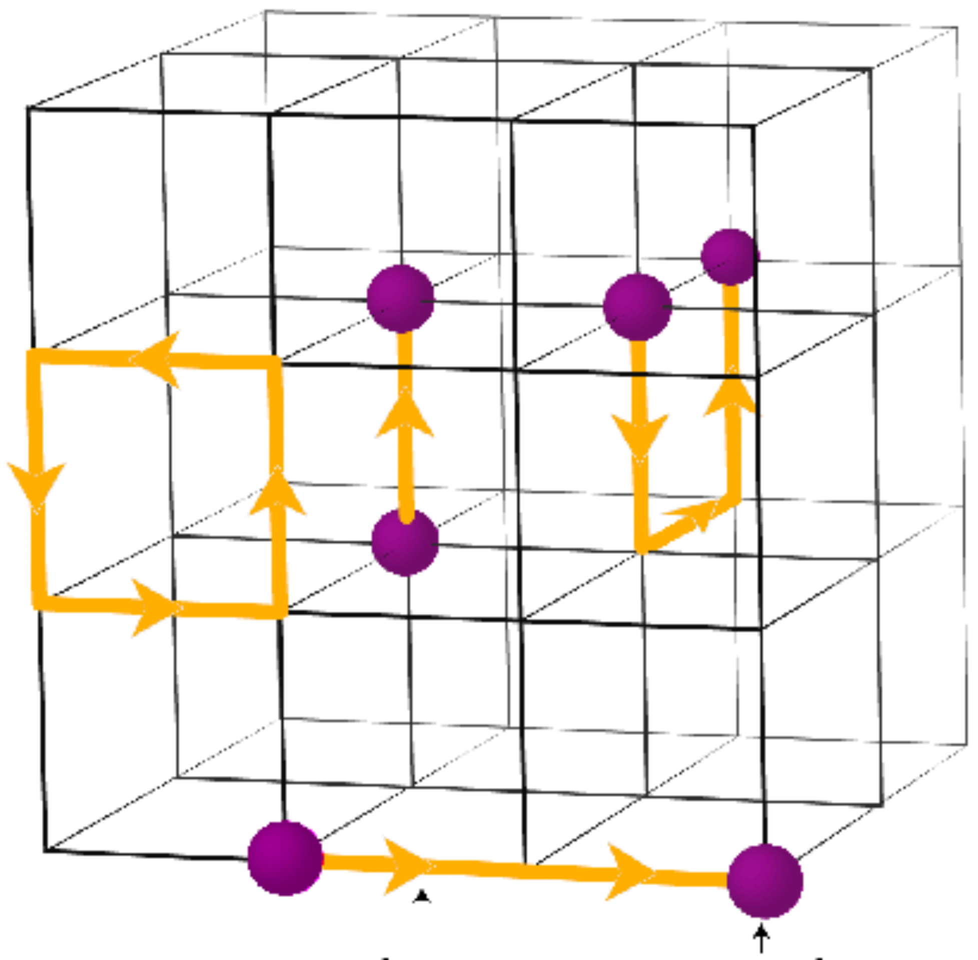

❖ GPD access in Lattice QCD:

- Mellin moments (generalized form factors) $\langle x^{n-1} \rangle = \int_{-1}^{+1} x^{n-1} f(x)\, dx$

- Novel methods (LaMET, pseudo-ITD, and many more)

[Cichy & Constantinou, Adv.High Energy Phys. 2019 (2019) 3036904]

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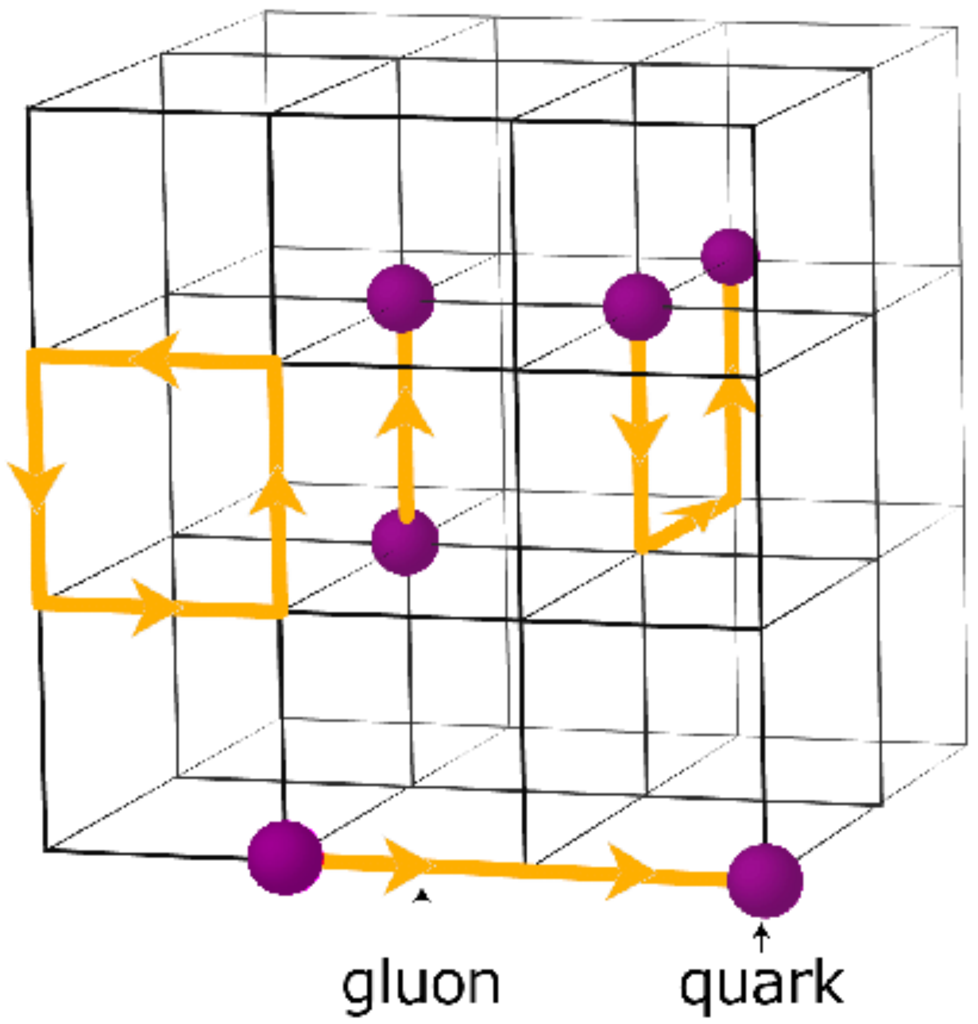

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- Calculation of quasi-GPD in Lattice QCD is very challenging
  - Matrix elements of non-local operators (quarks/gluons spatially separated)
  - Hadron states with momentum boost
  - renormalization more complex and may bring systematic uncertainties
  - introduction of momentum transfer increases noise

→ A lot of computing time

Setup

Lattice Setup:

- $N_f = 2 + 1 + 1$ twisted mass fermions & clover term (ETMC)
- Iwasaki gluons $\beta = 1.778$
- Lattice spacing $a \approx 0.0934$ fm
- $32^3 \times 64$ fm
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Preliminary data

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
asymm	±0.83	(0,0,0)	0	0	4	597	8	19104
asymm	±0.83	(±1,0,0), (0,±1,0)	0.163	0	8	597	8	38208
asymm	±0.83	(±1,±1,0)	0.311	0	16	597	8	76416
asymm	±1.25,	(0,0,0)	0	0	4	648	24	62208
asymm	±1.25	(±1,0,0), (0,±1,0)	0.167	0	8	648	24	124416
asymm	±1.25	(±1,±1,0)	0.327	0	16	648	24	248832
asymm	±1.25	(±2,0,0), (0,±2,0)	0.625	0	8	598	24	114816

- GPDs for all nine combinations of $\vec{\Delta}^2 = 0,1,2$ at the cost of the PDF
- Pion GPDs at same statistics as proton GPDs: four times less expensive (spin-0)

Frame Dependence and Calculations

Almost all of the work in the literature uses the symmetric (Breit) frame.

Here set all of the momentum transfer ($\overrightarrow{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$) to the source: $\overrightarrow{P_i} = P_3\,\hat{z} - \overrightarrow{\Delta}, \qquad \overrightarrow{P_f} = P_3\,\hat{z},$

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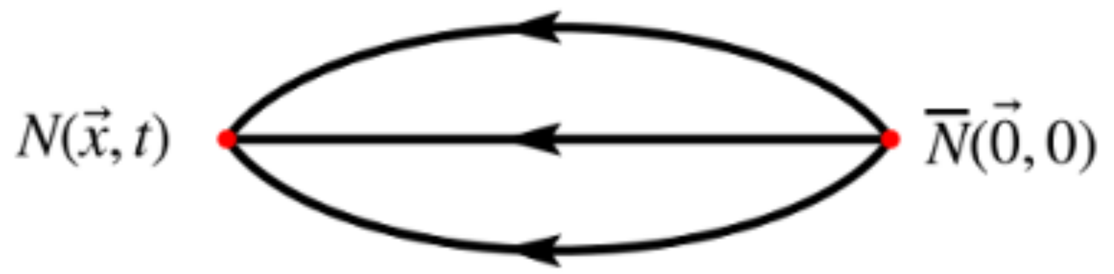
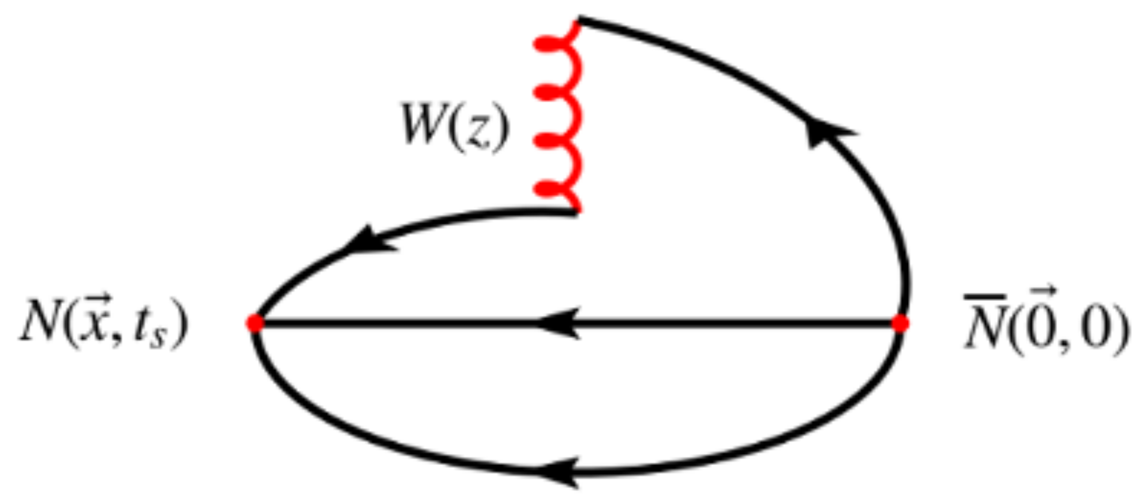
Necessary Steps

1. Calculation of appropriate ratio of the 3-point and 2-point correlation functions:

$$R = \frac{C^{3pt}(t_s, t, p_i, p_f)}{C^{2pt}(t_s, p_f)}\sqrt{\frac{C^{2pt}(t_s - t, p_i)C^{2pt}(t, p_f)C^{2pt}(t_s, p_f)}{C^{2pt}(t_s - t, p_f)C^{2pt}(t, p_i)C^{2pt}(t_s, p_i)}}$$



$\langle N(P) | N(P) \rangle$



$\langle N(P_f) | \bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z{,}0)\Psi(0) | N(P_i) \rangle$

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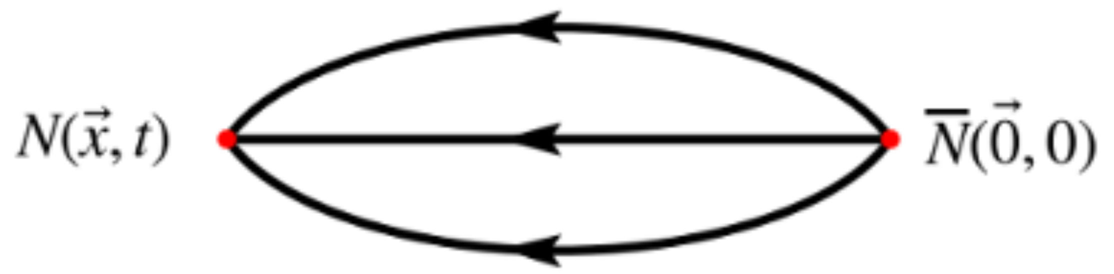

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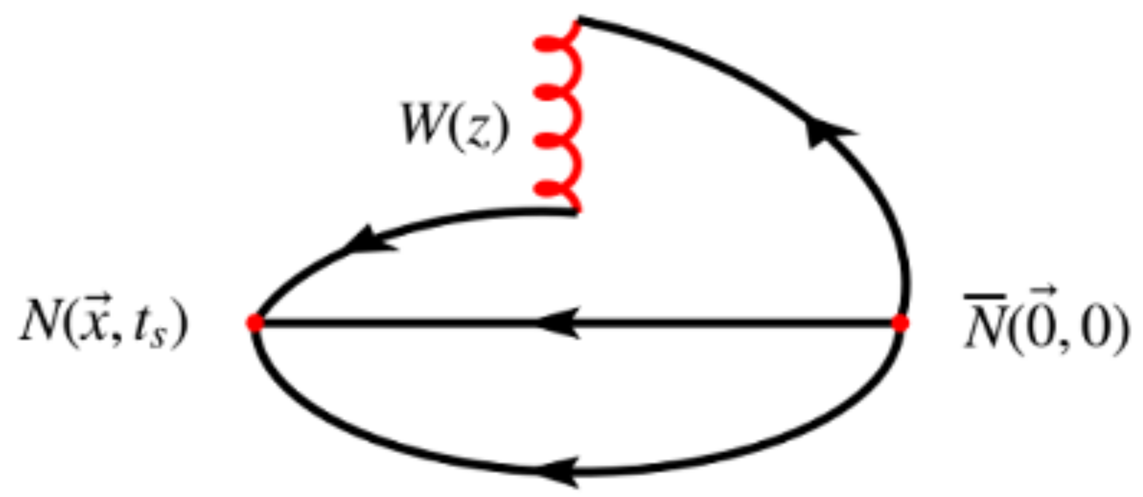

$\langle N(P_f) | \bar{\Psi}(z) \Gamma \mathcal{W}(z,0) \Psi(0) | N(P_i) \rangle$

2. Apply a single-state fit (plateau) to get the ground state of the matrix elements, Π_i^a

$$\longrightarrow \quad F^\mu(z, P, \Delta) = \frac{P^\mu}{m} A_1(z\cdot P, z\cdot\Delta, \Delta^2, z^2) + z^\mu m A_2(z\cdot P, z\cdot\Delta, \Delta^2, z^2) + \frac{\Delta^\mu}{m} A_3(z\cdot P, z\cdot\Delta, \Delta^2, z^2)\,,$$

Reference Martha's Talk

[Bhattacharya et al., arXiv:2209.05373]

Dependent upon 3 linearly-independent Lorentz invariant amplitudes!

$$A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$$

(Based on the idea of: S. Meissner, A. Metz, M. Schlegel, JHEP08(2009)056)

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Strategy

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$$H(z, P, \Delta) = A_1(z \cdot P, z \cdot \Delta, \Delta^2, 0) + \frac{\Delta^+}{P^+} A_3(z \cdot P, z \cdot \Delta, \Delta^2, 0)\,,$$

Standard γ^0 definition

$$\mathcal{H}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) \equiv A_1(z \cdot P, z \cdot \Delta, \Delta^2, z^2) + \frac{z \cdot \Delta}{z \cdot P} A_3(z \cdot P, z \cdot \Delta, \Delta^2, z^2)\,.$$

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6. Renormalize GPDs (RI-MOM, hybrid, ratio, …)

7. Fourier-like transform to x-space

8. Apply matching formalism

Decomposition

$$\left[\mathcal{O}^{\mu}_{v,eucl}\right] = \frac{1}{\sqrt{4E_f E_i}}\left[-i\frac{p^{\mu}}{m}A_1 + imzA_2 - i\frac{\Delta^{\mu}}{m}A_3\right]$$

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Symmetric Frame

$$\Pi_0^s = \frac{1}{\sqrt{4E_f E_i}}\left(\frac{E}{m}A_1\right)$$

$$\Pi_1^s = \frac{-1}{\sqrt{4E_f E_i}}\left(i\frac{\Delta_1}{m}A_3\right)$$

$$\Pi_2^s = \frac{-1}{\sqrt{4E_f E_i}}\left(i\frac{\Delta_2}{m}A_3\right)$$

$$\Pi_3^s = \frac{1}{\sqrt{4E_f E_i}}\left(-i\frac{P_3}{m}A_1 + imzA_2\right)$$

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Non-symmetric Frame

$$\Pi^a_0 = \frac{1}{\sqrt{4E_fE_i}}\left(\frac{E_f+E_i}{2m}A_1 + \frac{E_f-E_i}{m}A_3\right)$$

$$\Pi^a_1 = \frac{-i}{\sqrt{4E_fE_i}}\left(\frac{\Delta_1}{2m}A_1 + \frac{\Delta_1}{m}A_3\right)$$

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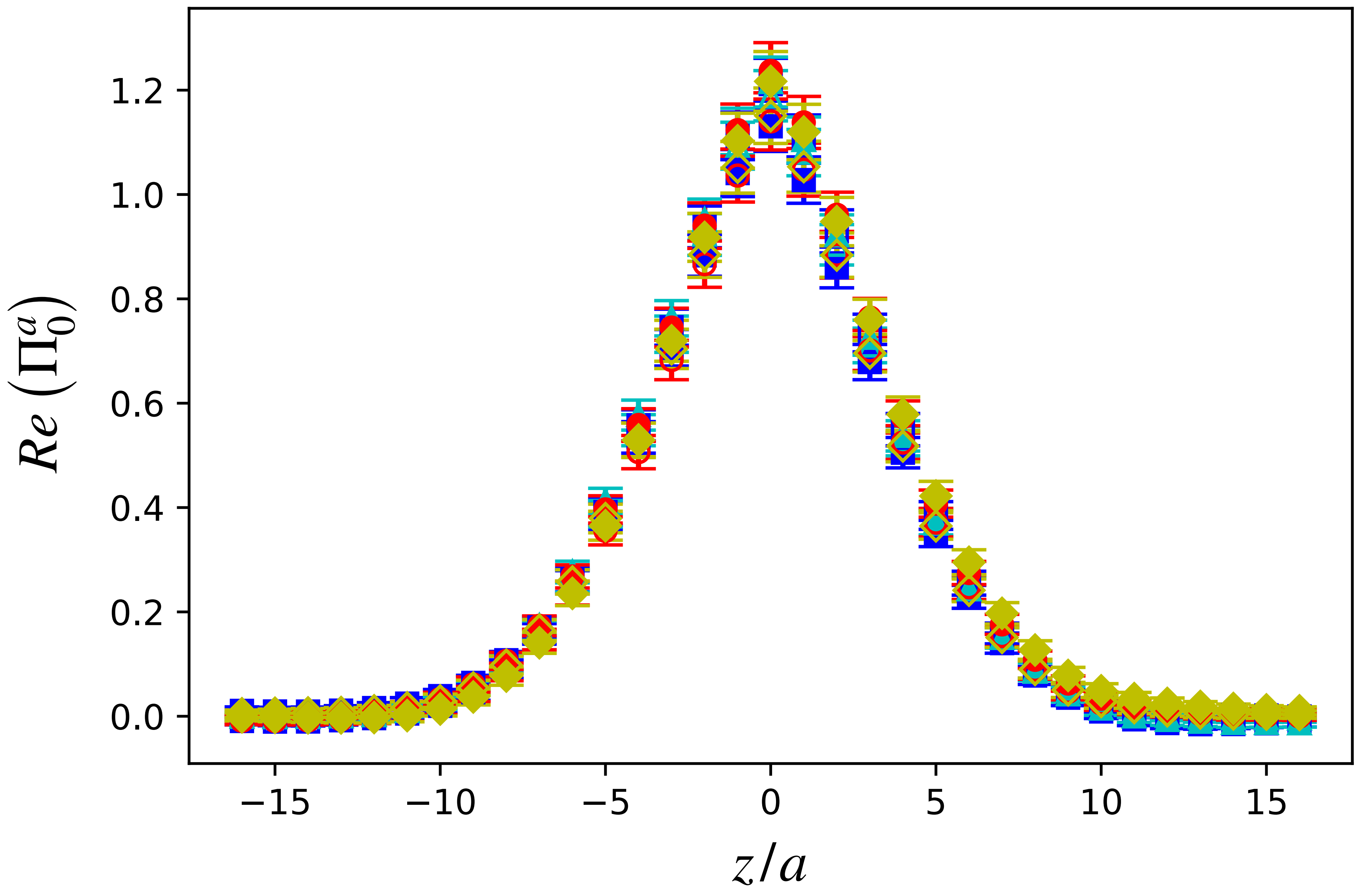
$$\Pi_3^a = \frac{-i}{\sqrt{4E_fE_i}}\left(\frac{P_3}{m}A_1 - mzA_2\right)$$

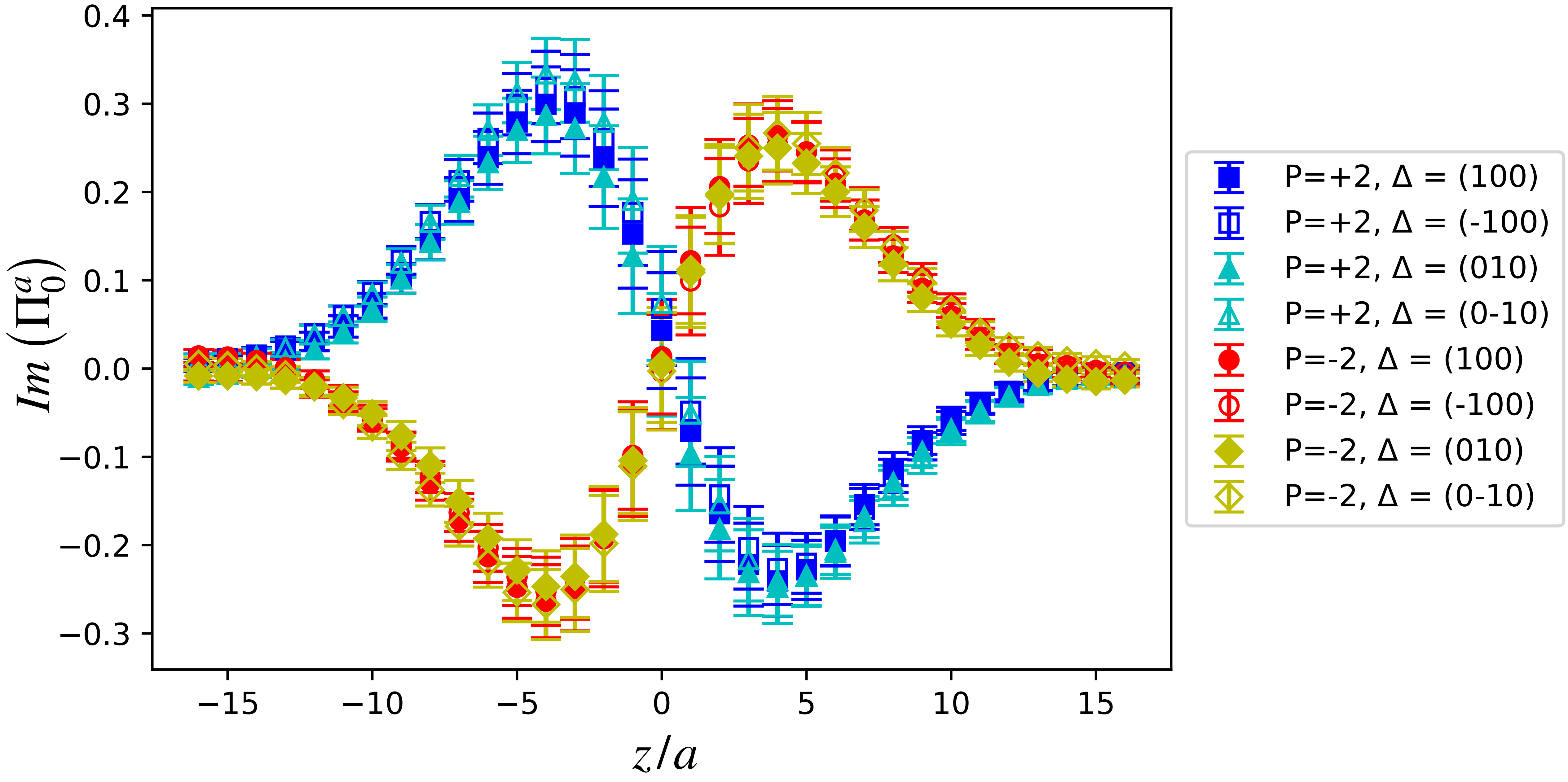
★ Clearly, Π_μ is dependent on the frame, but A_i are frame invariant ★

Matrix Elements: Π_0^a

$P = 0.83\,\mathrm{GeV}$

$-t = 0.163\,\mathrm{GeV}^2$





❖ Clear signal for the matrix elements from all combinations of momenta P, Δ

❖ Signs of P, Δ affect the sign of the imaginary part, but the matrix elements do not have definite symmetry properties

❖ Observed asymmetries appear to be small

Matrix Elements: Π_1^a

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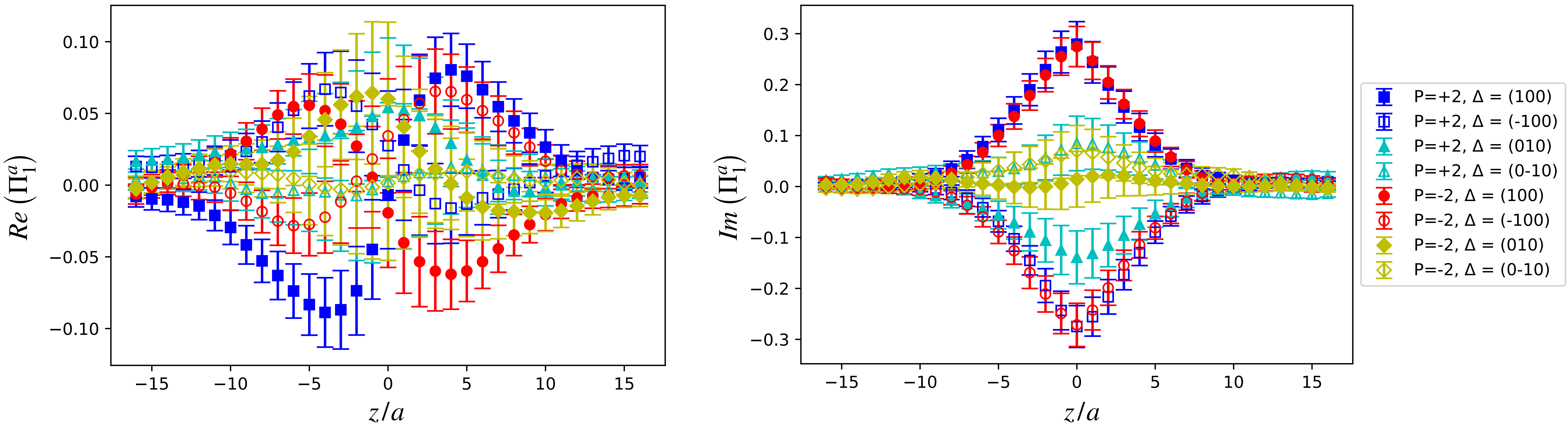

❖ Matrix elements of γ^1 appear to be more noisy and smaller in magnitude (light-cone: twist-3)

❖ Asymmetries appear to be larger than Π_0^a

❖ Role of real and imaginary part in symmetries reversed:

$$\Pi_1^a = \frac{-i}{\sqrt{4E_fE_i}}\left(\frac{\Delta_1}{2m}A_1 + \frac{\Delta_1}{m}A_3\right)$$

Matrix Elements: Π_2^a

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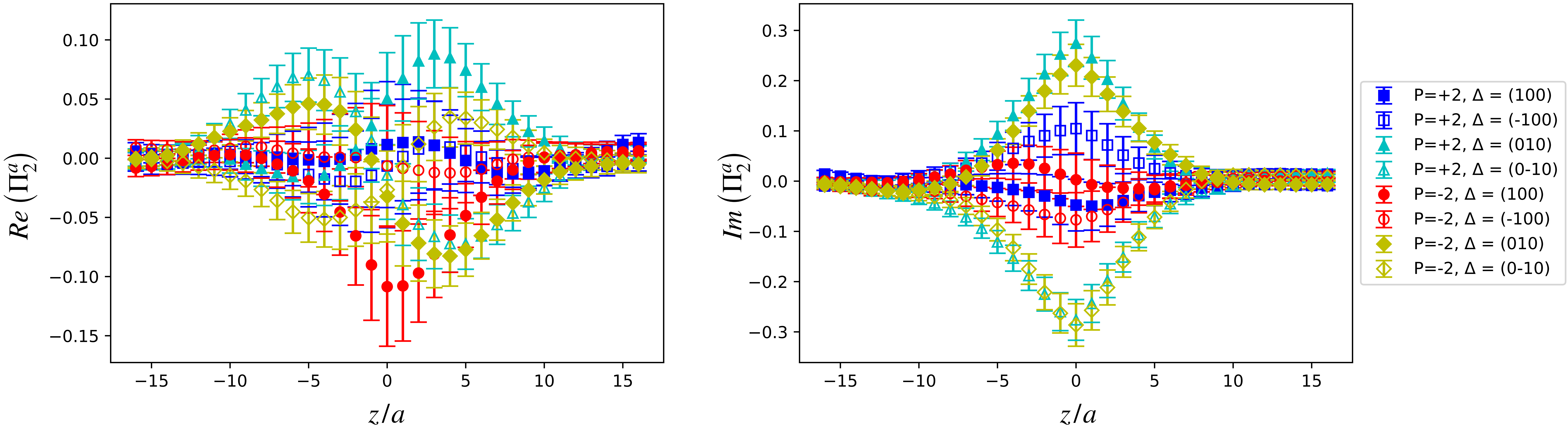


❖ Similar behavior as Π_1^a case

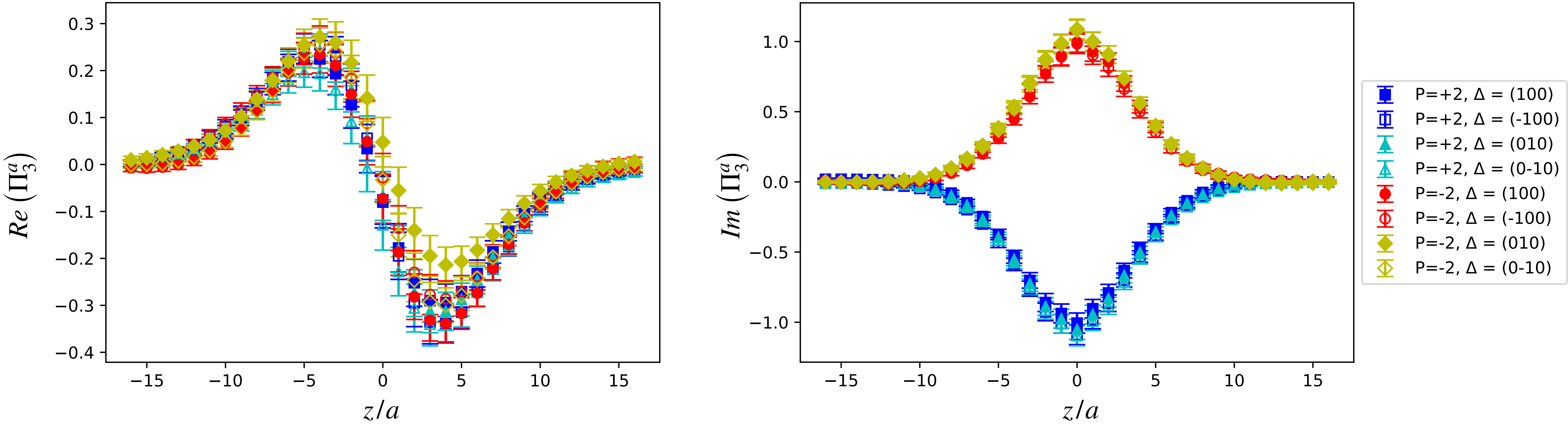
Matrix Elements: Π_3^a

$P = 0.83\,\text{GeV}$

$-t = 0.163\,\text{GeV}^2$

❖ γ^3 operator suffers from finite mixing with scalar operator in lattice regularization
[Constantinou & Panagopoulos, PRD 96 (2017) 5, 054506]

❖ Twisted-mass fermions: mixing between γ^3 and pseudo-scalar (no forward limit)



❖ Good quality of signal (twist-2 in light cone)

❖ Smaller magnitude than Π_0^a due to kinematic factors:

$$\Pi_3^a = \frac{-i}{\sqrt{4E_fE_i}}\left(\frac{P_3}{m}A_1 - mzA_2\right) \qquad \Pi_0^a = \frac{1}{\sqrt{4E_fE_i}}\left(\frac{E_f+E_i}{2m}A_1 + \frac{E_f-E_i}{m}A_3\right)$$

Amplitude Decomposition

Non-symmetric Frame

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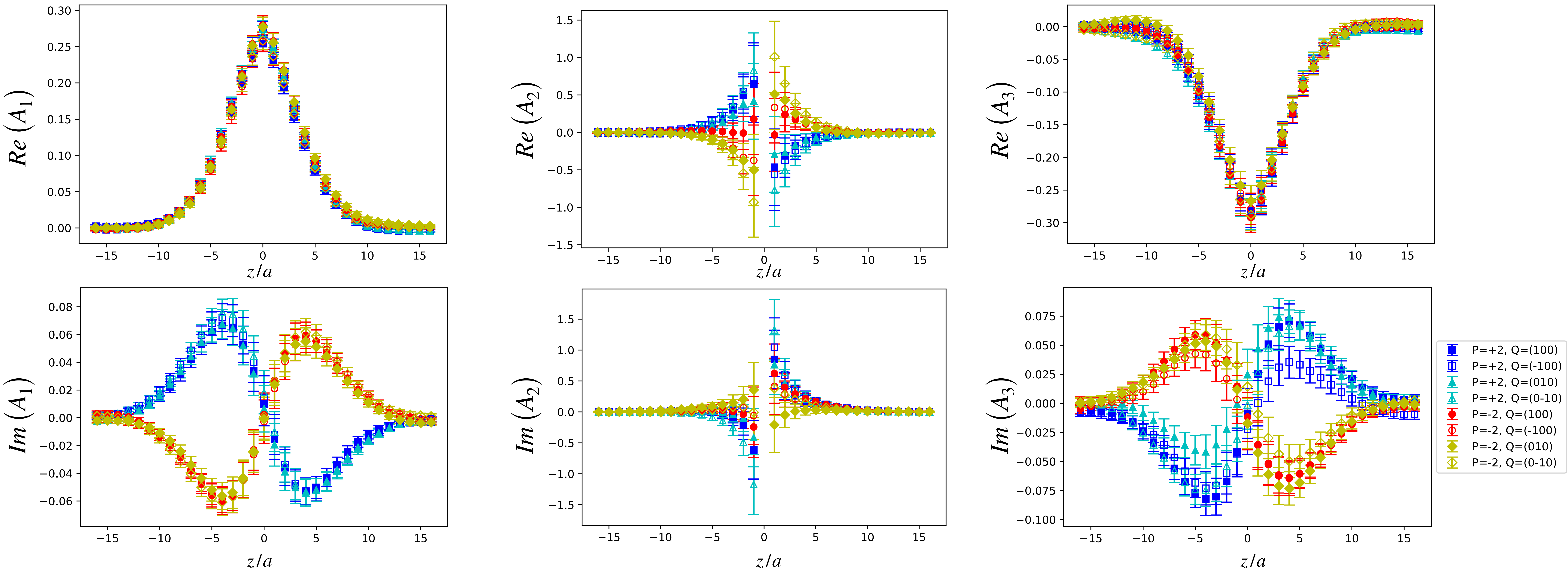
$$\Pi_3^a = \frac{-i}{\sqrt{4E_fE_i}}\left(\frac{P_3}{m}A_1 - mzA_2\right)$$

- ❖ A_1 and A_3 decomposed from Π_0^a and $\Pi_{1,2}^a$
- ❖ A_2 appears in Π_3^a and requires A_1

Amplitudes

$P = 0.83\,\text{GeV}$

$-t = 0.163\,\text{GeV}^2$



❖ A_1 best noise-to-signal ratio followed by A_3

❖ A_2 noisy but not negligible

❖ Amplitudes have definite symmetry properties

Symmetry Properties of A_i

Symmetry properties of the Lorentz invariant amplitudes are as follows: [Bhattacharya et al., arXiv:2209.05373]

- $+A_1^*(-z \cdot P, z \cdot \Delta, \Delta^2, z^2) = A_1(z \cdot P, z \cdot \Delta, \Delta^2, z^2)$
- $-A_2^*(-z \cdot P, z \cdot \Delta, \Delta^2, z^2) = A_2(z \cdot P, z \cdot \Delta, \Delta^2, z^2)$
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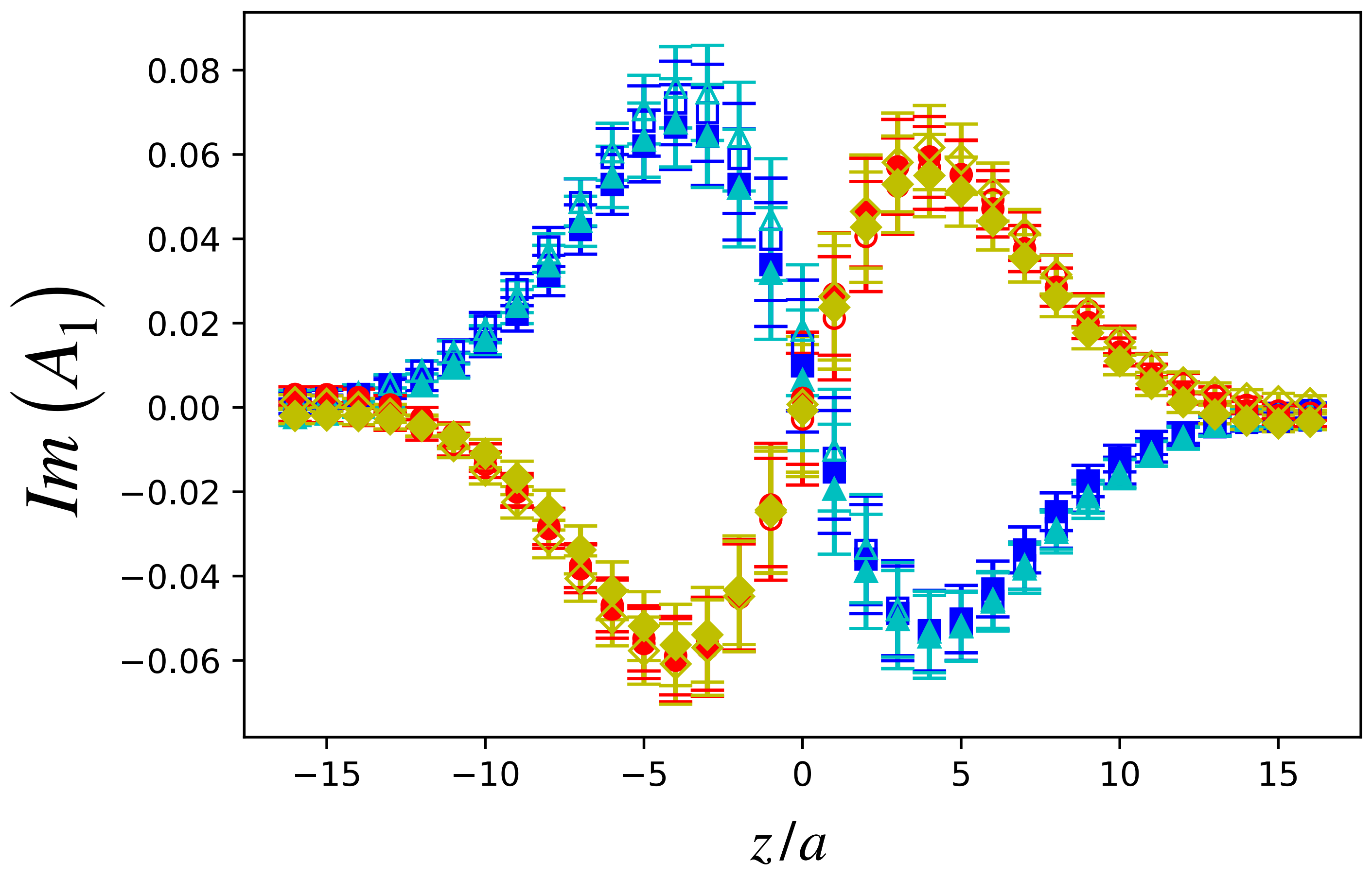

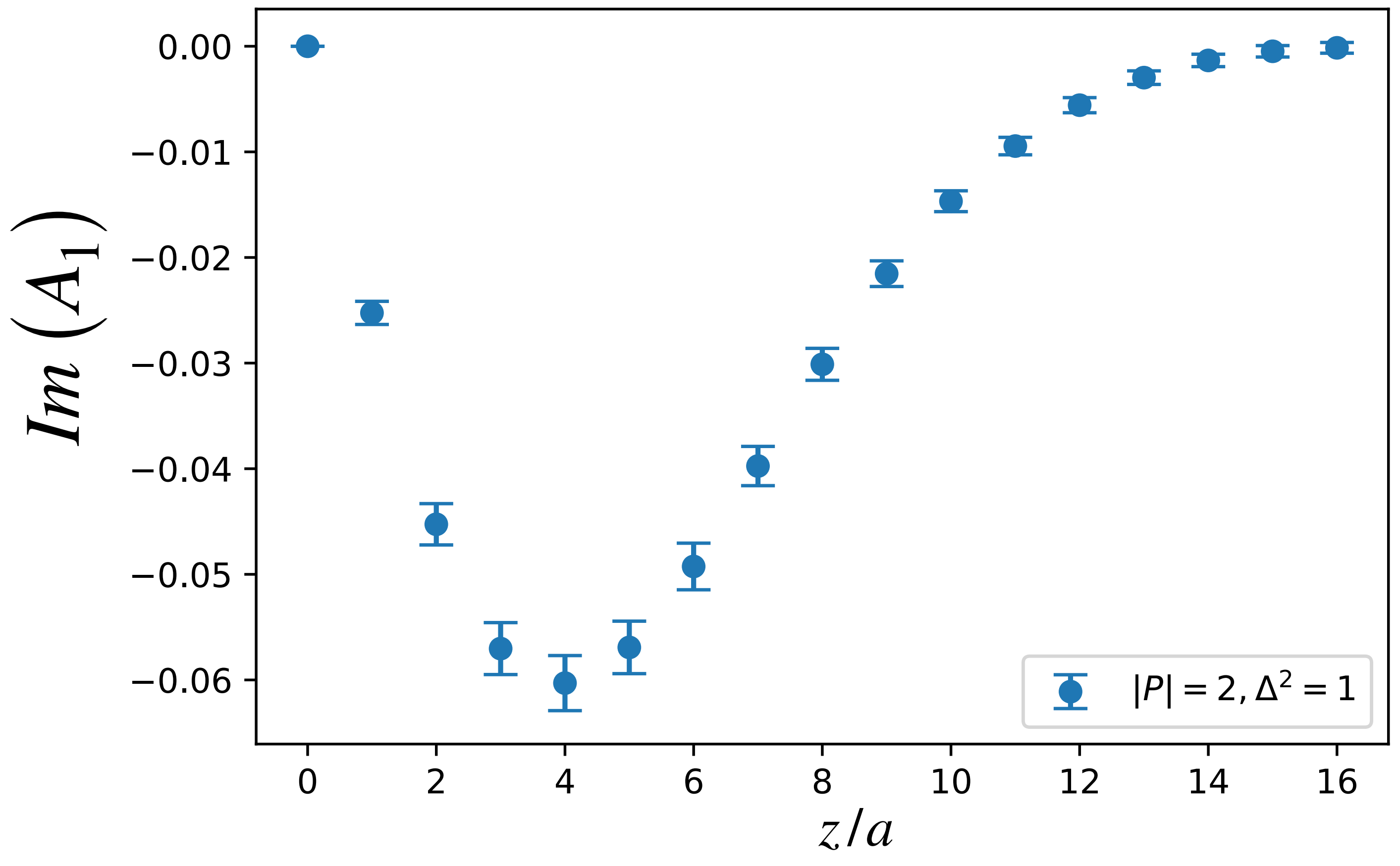

Averaged A_i

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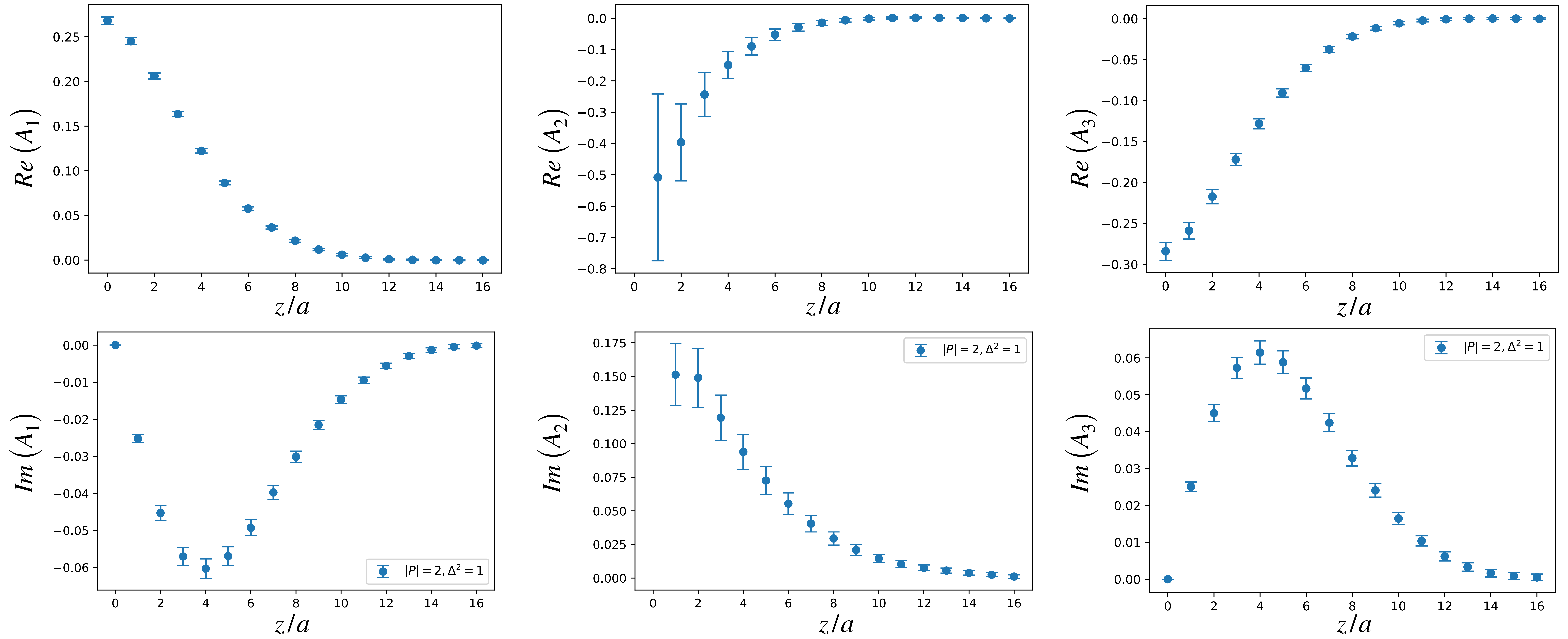
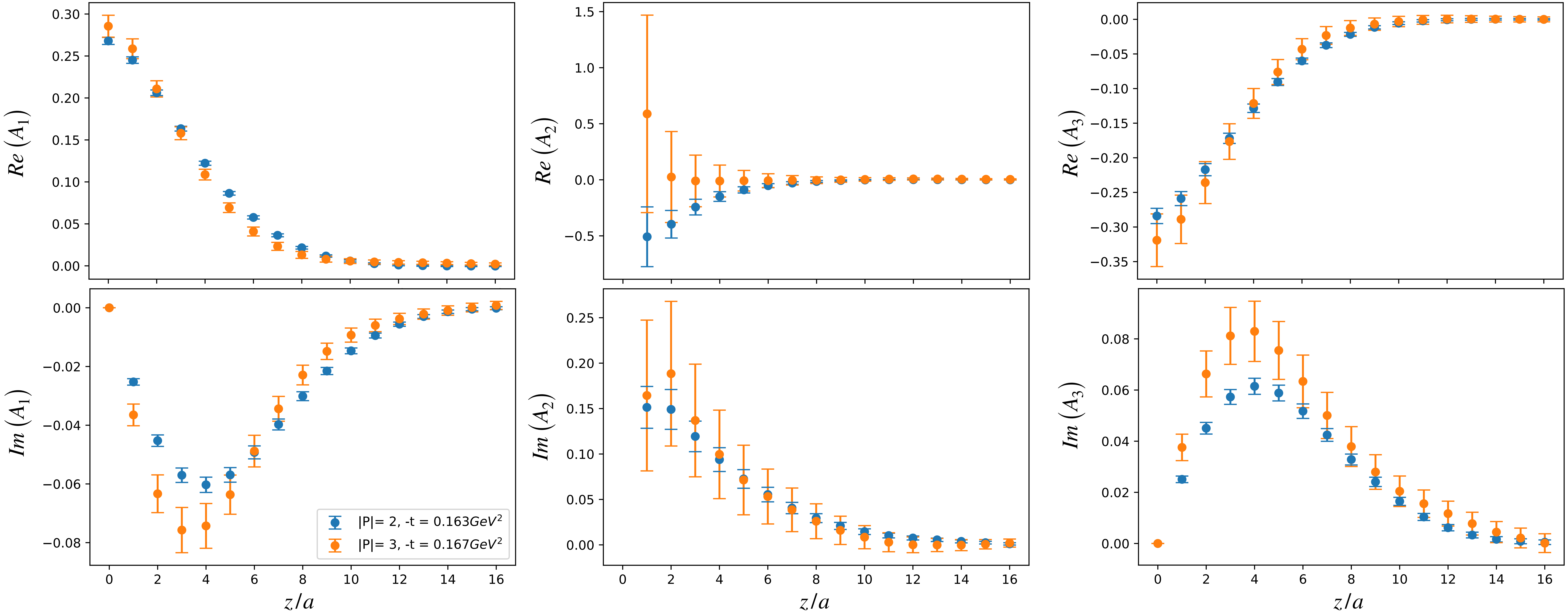


- ❖ Clear signal for all amplitudes (A_2 the least accurate statistically)
- ❖ A_3 has opposite sign than A_1 and A_2 (sign not imposed by decomposition)

Momentum-boost dependence in A_i

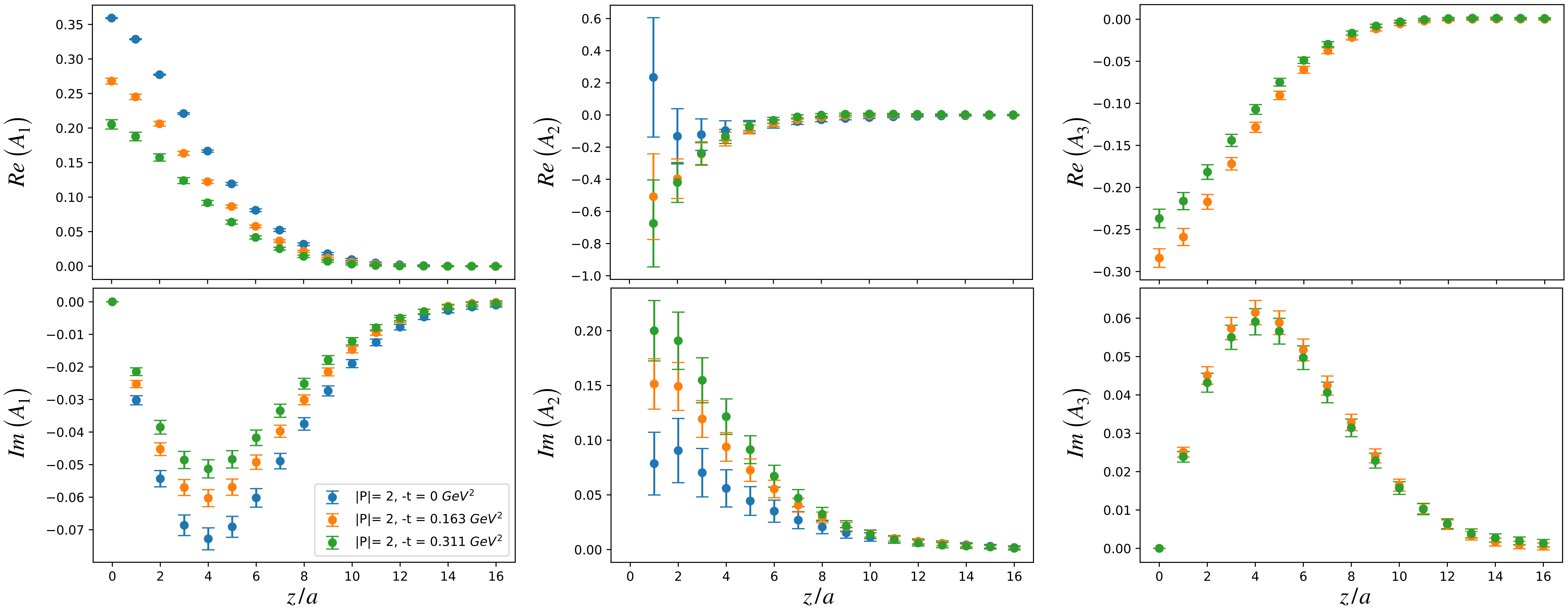


❖ Fixed $\vec{\Delta}$

❖Mild dependence on the momentum boost ($\mathrm{Im}[A_1]$ and $\mathrm{Im}[A_3]$ most notable)

❖ Change in P affects the value of $-t$: $-t = \vec{\Delta}^2 - (E_f - E_i)^2$

Momentum-transfer dependence in A_i



❖ The magnitude of the amplitudes suppresses with increase of $-t$

❖ A_2 suffers from increased noise

❖ A_3 is inaccessible at $-t = 0\ GeV^2$

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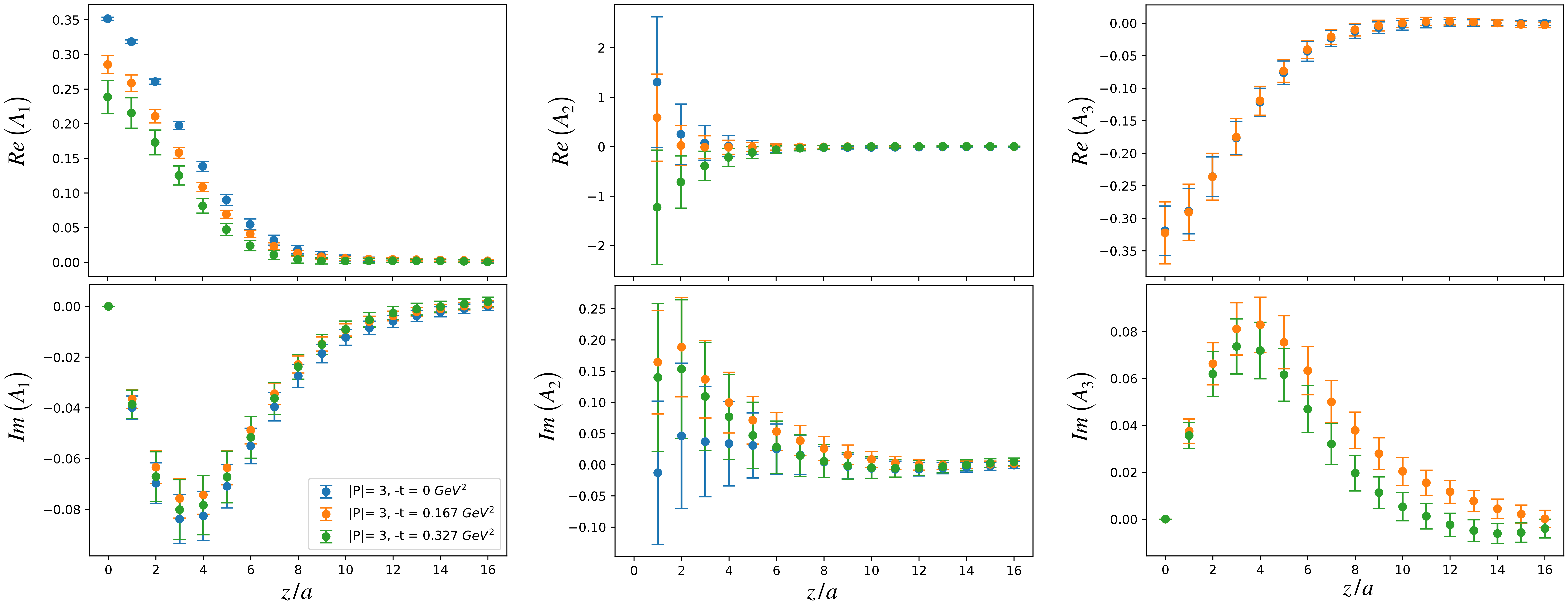
Momentum-transfer dependence in A_i



❖ Increased statistical uncertainties compared to $|P| = 0.83\text{GeV}$

❖ Change of $|P|$ affects the magnitude of the matrix elements

Summary and Future Work

- Quasi-GPDs are intrinsically frame dependent

- Computational challenges can be reduced using a Lorentz-invariant decomposition of matrix elements

- Implementation for pion is feasible

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❖ Include non-zero skewness ($\Delta_3 \neq 0$)

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Thank You!!!

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