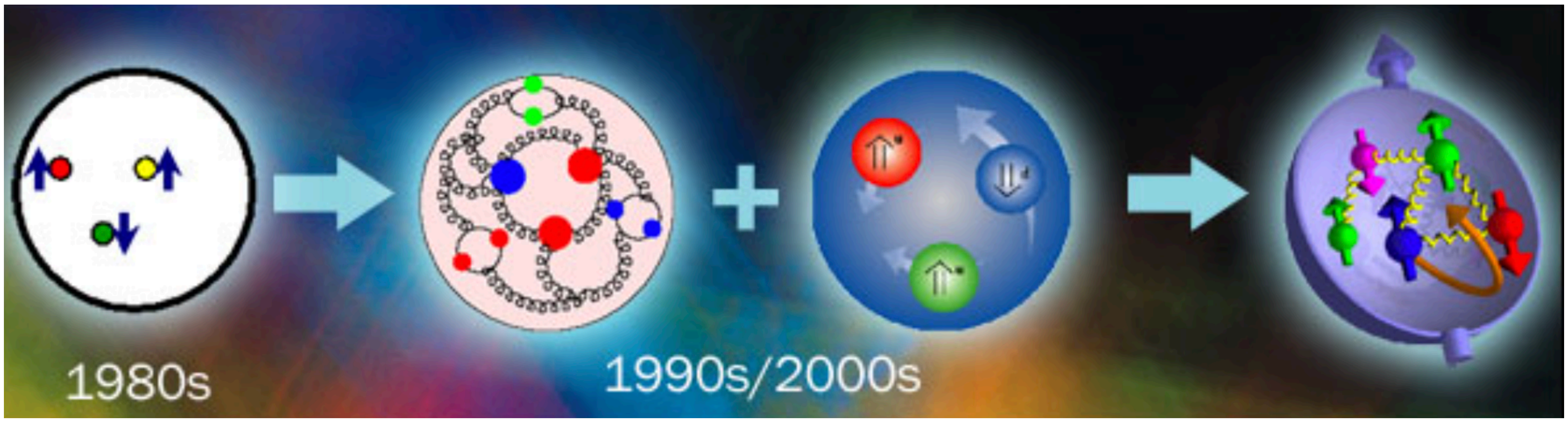
Updates on gluon distributions from HadStruc

Joe Karpie (JLab) part of the HadStruc Collaboration

Jefferson Lab

Gluon Structure

- Why do we want to know gluon distributions?
  - Understanding hadronic properties such as mass, spin, ….



  - Understanding Higgs or top production in high energy collisions
  - Understanding low x physics, gluon saturation

arXiv > nucl-ex > arXiv:1212.1701

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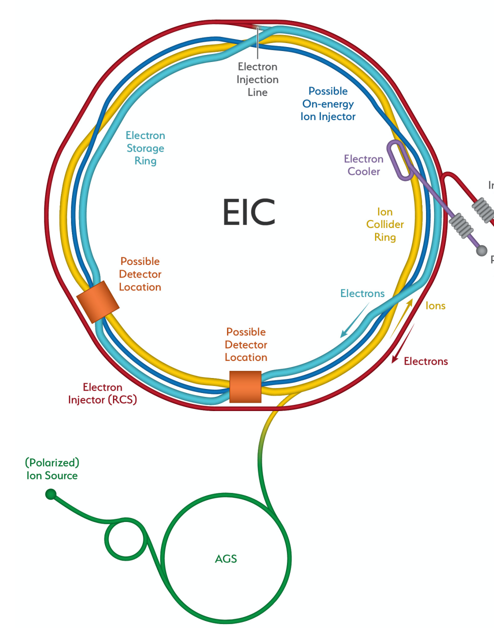
Nuclear Experiment

[Submitted on 7 Dec 2012 (v1), last revised 30 Nov 2014 (this version, v3)]

Electron Ion Collider: The Next QCD Frontier - Understanding the glue that binds us all

A. Accardi, J.L. Albacete, M. Anselmino, N. Armesto, E.C. Aschenauer, A. Bacchetta, D. Boer, W.K. Brooks, T. Burton, N.-B. Chang, W.-T. Deng, A. Deshpande, M. Diehl, A. Dumitru, R. Dupré, R. Ent, S. Fazio, H. Gao, V. Guzey, H. Hakobyan, Y. Hao, D. Hasch, R. Holt, T. Horn, M. Huang, A. Hutton, C. Hyde, J. Jalilian-Marian, S. Klein, B. Kopeliovich, Y. Kovchegov, K. Kumar, K. KumeriČki, M.A.C. Lamont, T. Lappi, J.-H. Lee, Y. Lee, E.M. Levin, F.-L. Lin, V. Litvinenko, T.W. Ludlam, C. Marquet, Z.-E. Meziani, R. McKeown, A. Metz, R. Milner, V.S. Morozov, A.H. Mueller, B. Müller, D. Müller, P. Nadel-Turonski, H. Paukkunen, A. Prokudin, V. Ptitsyn, X. Qian, J.-W. Qiu, M. Ramsey-Musolf, T. Roser, F. Sabatié, R. Sassot, G. Schnell, P. Schweitzer, E. Sichtermann, M. Stratmann, M. Strikman, M. Sullivan, S. Taneja, T. Toll, D. Trbojevic, T. Ullrich, R. Venugopalan, S. Vigdor, W. Vogelsang, C. Weiss, B.-W. Xiao, F. Yuan, Y.-H. Zhang, L. Zheng



Positivity of the PDFs

- In parton model (LO without QCD interactions), $f_i^{\uparrow/\downarrow}(x) \geq 0$
  - Sometimes assumed for PDF analysis
  - For gluons implies,

$$2g(x) = g^{\uparrow}(x) + g^{\downarrow}(x) \qquad\qquad 2\Delta g(x) = g^{\uparrow}(x) - g^{\downarrow}(x)$$

$$|\Delta g(x)| \leq g(x)$$

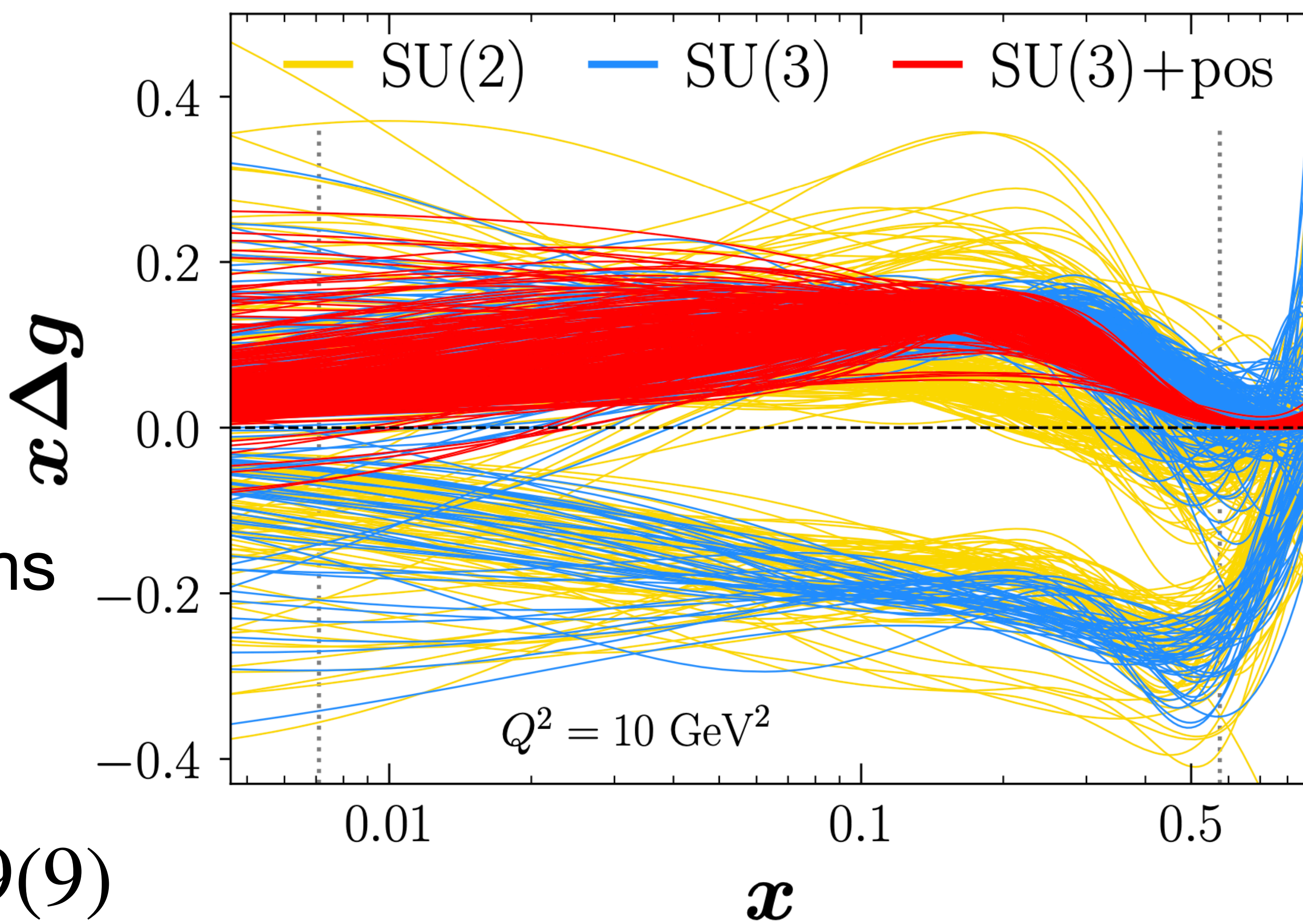
- With interactions, does not have to be true for $\overline{\text{MS}}$ scheme

J. Collins, T. Rogers, N. Sato, Phys Rev D 105 (2022) 7,076010

- How will this effect analyses?

Spinning gluons

Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)



- Positivity removed from JAM helicity gluon PDF

$$|\Delta g| \leq g$$

  - Reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$

- Without constraint: $\Delta G = 0.3(5)$

- Lattice: $\Delta G = 0.251(47)(16)$

  Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
  K-F. Liu arXiv: 2112.08416

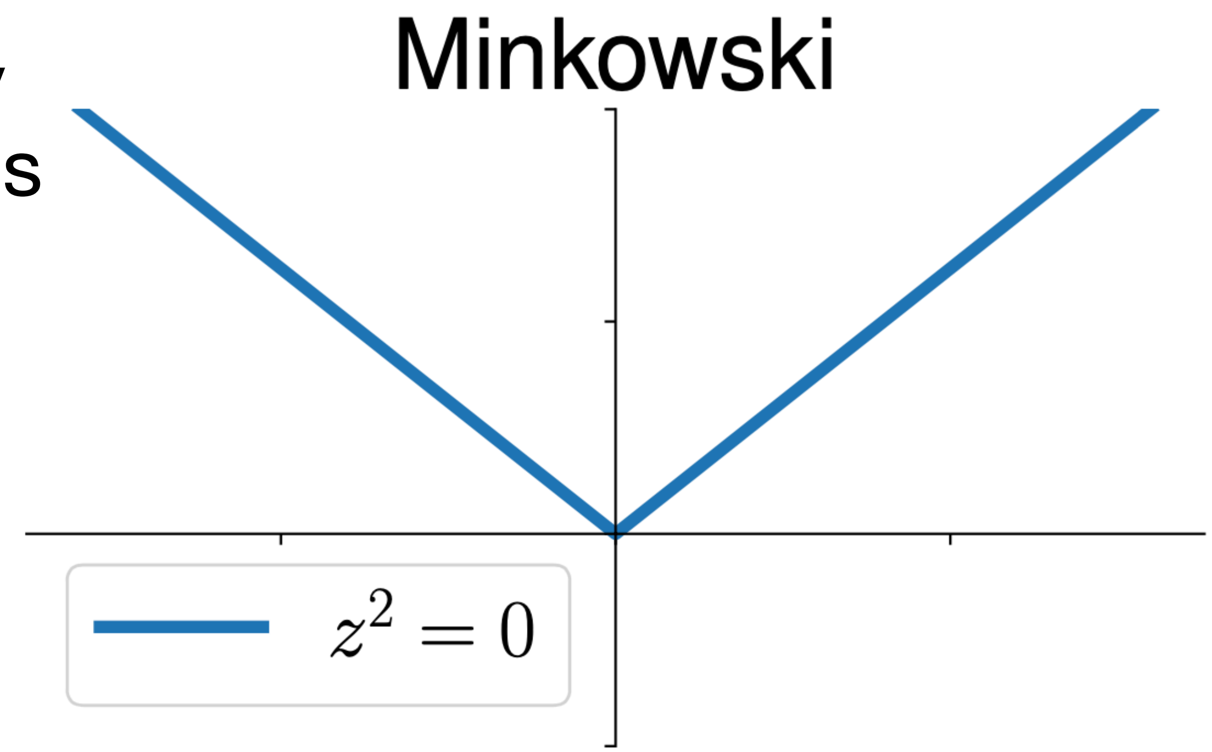
R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

$$J = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

$$\Delta G = \int dx\, \Delta g(x)$$

Parton Distributions and the Lattice

- Parton Distributions are defined by operators with light-like separations



- Use space-like separations

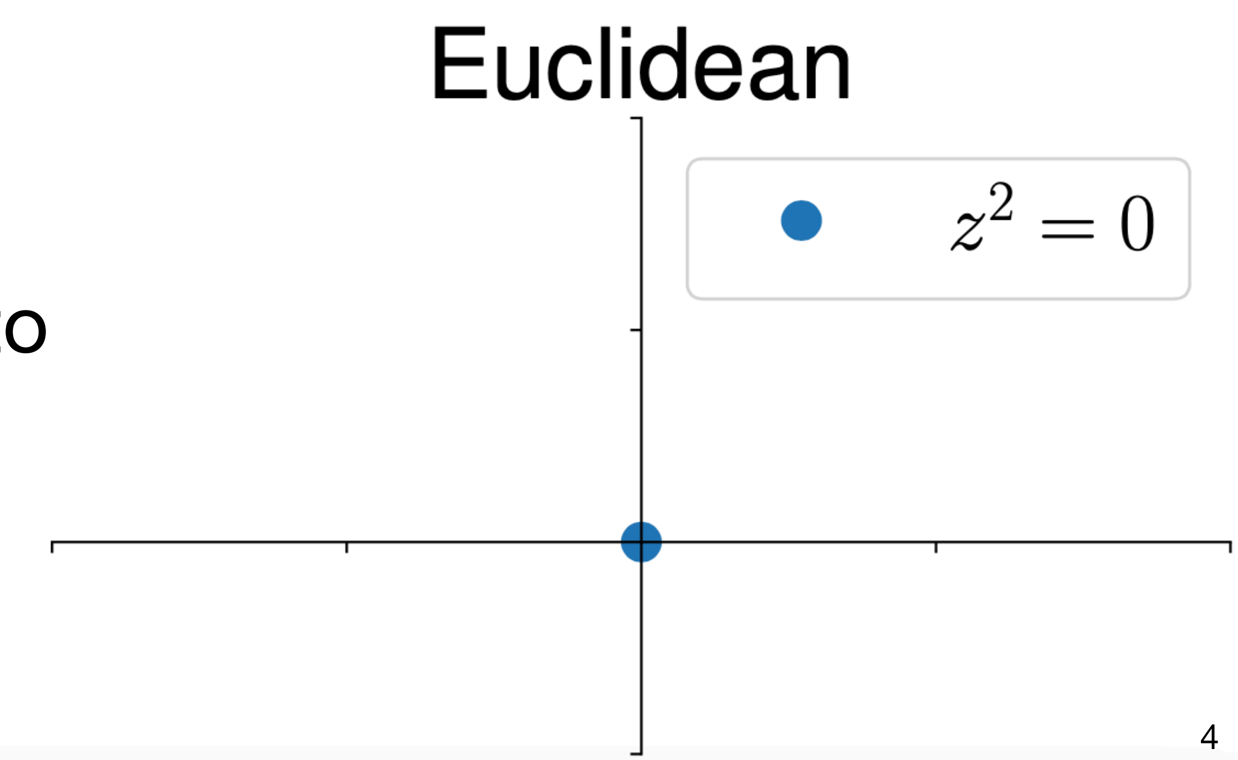
  X. Ji *Phys Rev Lett* 110 (2013) 262002

  - Wilson line operators

$$O_\Gamma^{\mathrm{WL}}(z) = \bar{\psi}(z)\Gamma W(z;0)\psi(0)$$

$$z^2 \neq 0$$

  - Factorizations exist analogous to cross sections



Gluon Matrix Elements

- **General Matrix Element**

$$M^{\mu\alpha;\nu\beta}(z,p,s) = \langle p,s\,|\,\mathrm{Tr}\Big[F^{\mu\alpha}(z)W(z;0)F^{\nu\beta}(0)\Big]\,|\,p,s\rangle$$

    - Assume z is along cardinal direction (eventually lattice axis)

- **Renormalization** Z-Y. Li, Y-Q. Ma, J-W. Qiu. *Phys. Rev. Lett.* 122 (2019) 6, 062002
    - Multiplicatively renormalizable
    - Depends on how many of μ, ν, ρ, σ are in z direction.

- Matrix element has complicated Lorentz decomposition in terms of p^μ, z^μ, s^μ
    - Need to isolate amplitudes with leading twist contributions

The Role of Separation and Momentum

- In quasi-PDF/LaMET and pseudo-PDF/Short distance, separation and momentum swap roles

Factorization Scale:

p_z^2 / z^2

- Describes hard part
- Scale for factorization to PDF
- Scale in power expansion
- Keep away from Λ^2_{QCD}
- Technically only requires single value

NEED!

Dynamical variable:

z / p_z , or $\nu = p \cdot z$

- Describes interesting partonic structure
- Variable for inverse Fourier Transform
- Can take large or small value
- Want as many as are available
- Wider range improves the inverse problem if you really want x-space

WANT!

Difficulty Reaching High Momentum

Difficulty Reaching High Momentum

- **Poor overlap with boosted ground state**
  - Momentum smearing improves overlap with moving states
    G. Bali et al Phys. Rev. D 93 (2016) 9, 094515
  - Distillation from all time slices improves signal
  - GEVP optimizes the overlap with ground state

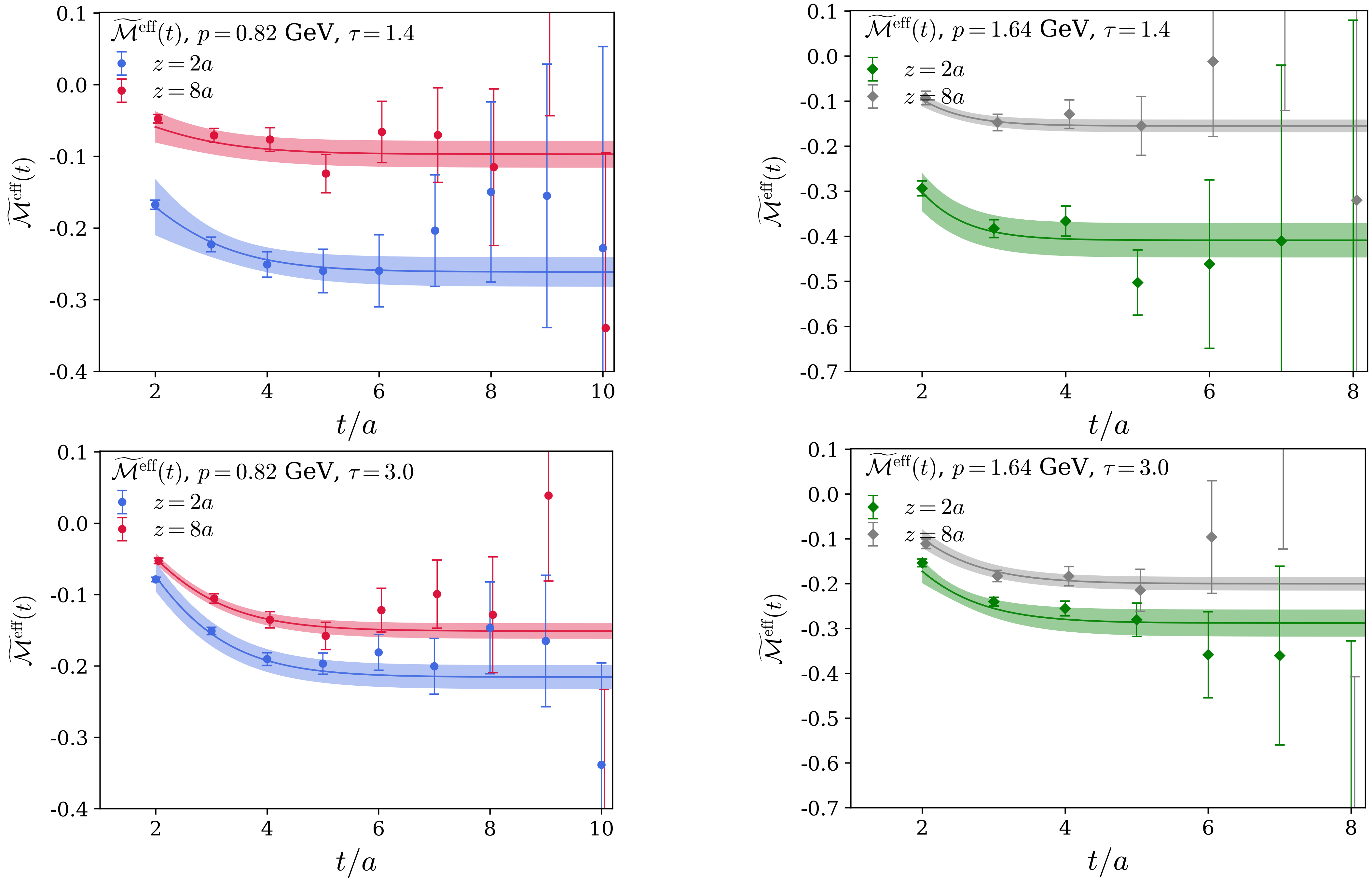
  M. Peardon, et al, Phys. Rev. D 80 (2009) 054506
  C. Egerer et al Phys. Rev. D 103 (2021) 3, 034502

- **Excited state energy gaps collapse**
  - Larger times needed for ground state
  - Summed GEVP techniques can remove lowest states and suppress remaining J. Bulava, M. Donnellan, R. Sommer JHEP 01 (2012) 140
- **Exponentially suppressed signal-to-noise ratio**
  - Without resolution, a 10 GeV calculation may require computers from 2100s to get precision of modern 2.5 GeV calculations!

Fits for matrix elements

$a = 0.094$ fm
$m_\pi = 358$ MeV

- sGEVP eliminates most excited states for t/a > 3
- Fit excited state gap for all matrix elements with same momentum and flow time simultaneously



Spin Averaged matrix element

- Spin averaged combination $\mathscr{M}(\nu, z^3) = \frac{1}{2p_0^2}\left[M_{ti;it} + M_{ij;ij}\right]$
  - Gives **one** amplitude with leading twist contribution

$$i, j = x, y$$

I. Balitsky, W. Morris, A. Radyushkin Phys Rev D 105 (2022) 1, 014008
T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

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T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

- Use **ratio** with finite continuum limit

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)\ \mathcal{M}(0,0)\,|_{p=0,z=0}}{\mathcal{M}(\nu,0)\,|_{z=0}\,\mathcal{M}(0,z^2)\,|_{p=0}}$$

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- Relation to **gluon** and **quark singlet** ITD

$$\langle x \rangle_g \mathfrak{M}(\nu, z^2) = \int_0^1 C^{gg}(u, \mu^2 z^2) I_g(u\nu, \mu^2) + C^{qg}(u, \mu^2 z^2) I_s(u\nu, \mu^2)$$

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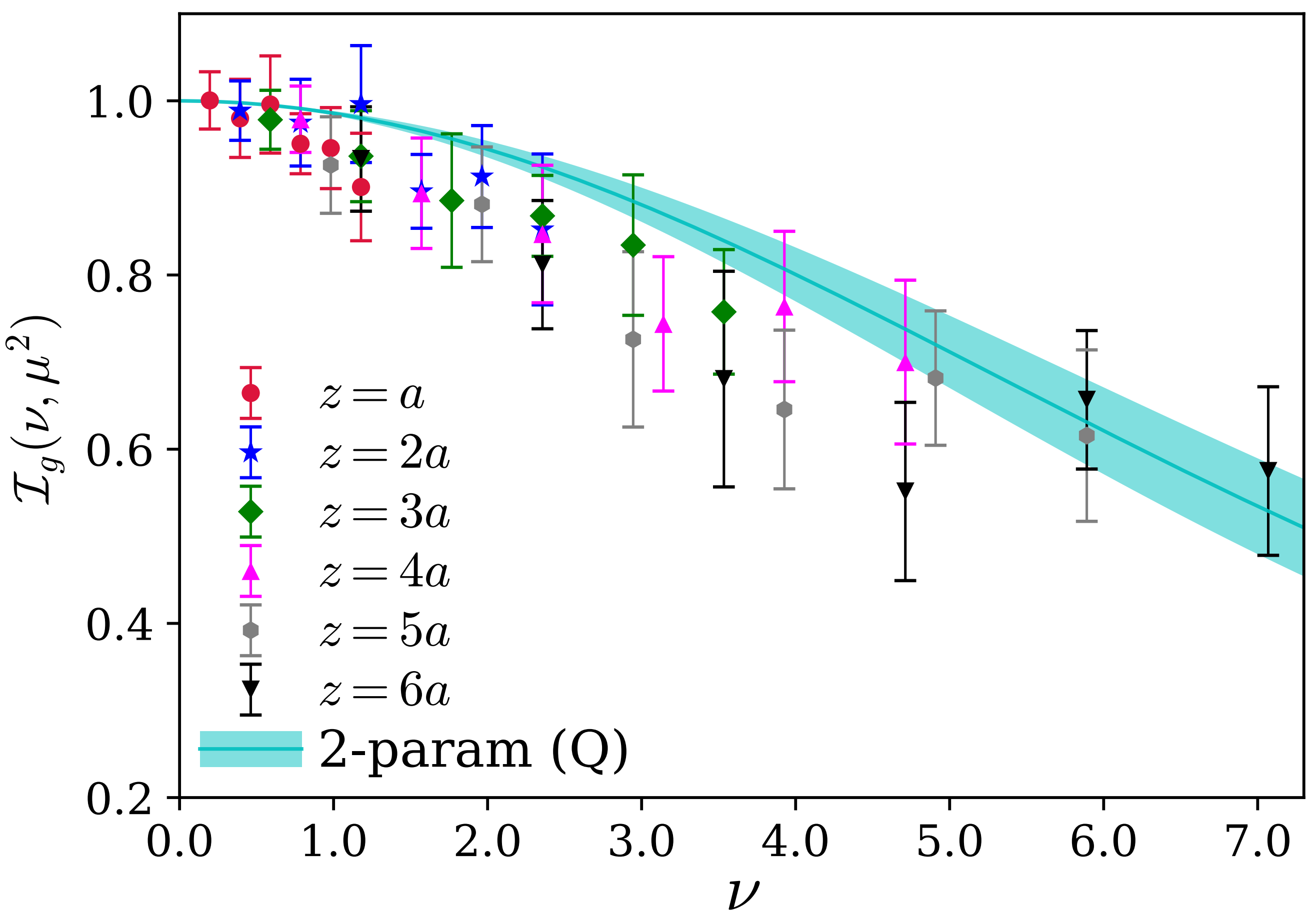
Unpolarized Gluon PDF

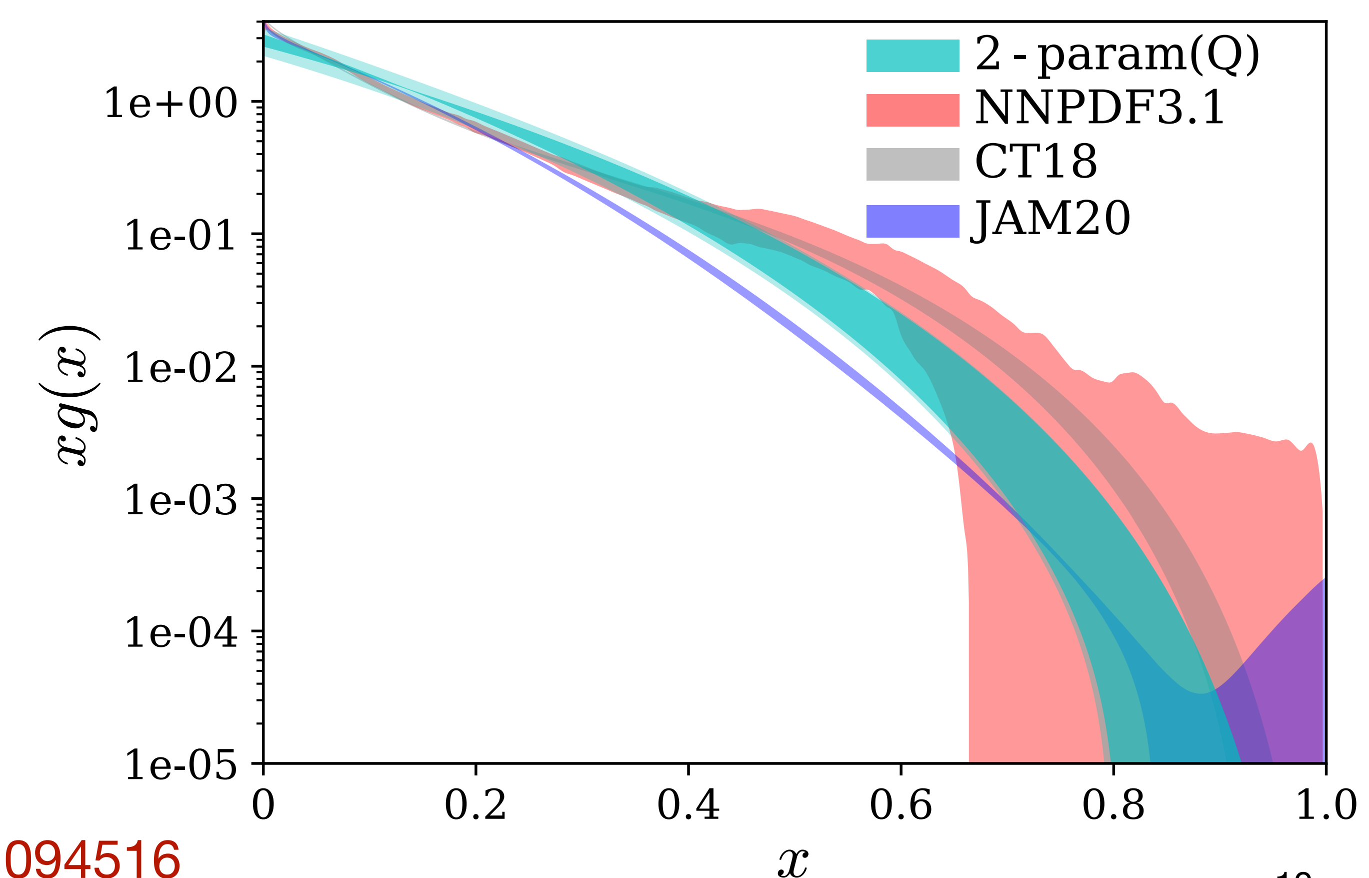


- Extrapolated to $\tau \to 0$ flow time
- Modified by NLO formula

$a = 0.094$ fm

$m_\pi = 358$ MeV

- ITD fit to cosine transform of $xg(x) = x^a(1-x)^b/B(a+1,b+1)$
- Qualitative agreement with global analysis



T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\left[F^{\mu\alpha}(z)W(z;0)\,\widetilde{F}^{\nu\beta}(0)\right]|\,p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
  - Gives **two** amplitudes, one has no leading twist contribution

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- Use ratio with finite continuum limit

$$\widetilde{\mathfrak{M}}(\nu,z^2) = i\frac{\left[\widetilde{\mathcal{M}}(z,p)/p_z p_0\right]/Z_L(z/a)}{\mathcal{M}(0,z^2)/m^2}$$

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- Relation to gluon and quark singlet ITD

$$\langle x\rangle_g\,\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(u,\mu^2z^2)\widetilde{I}_g(u\nu,\mu^2) + \widetilde{C}^{qg}(u,\mu^2z^2)\widetilde{I}_s(u\nu,\mu^2)$$

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Neglected for now

$$\langle x\rangle_g\,\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(u,\mu^2z^2)\widetilde{I}_g(u\nu,\mu^2) + \widetilde{C}^{qg}(u,\mu^2z^2)\widetilde{I}_s(u\nu,\mu^2)$$

Lorentz decomposition

I. Balitsky, W. Morris, A. Radyushkin
JHEP 02 (2022) 193

$$\widetilde{M}^{(2)}_{\mu\alpha;\lambda\beta}(z,p) = (sz)\big(g_{\mu\lambda}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\lambda - g_{\alpha\lambda}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{pp}$$
$$+(sz)\big(g_{\mu\lambda}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\lambda - g_{\alpha\lambda}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{zz}$$
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$$+(sz)\big(g_{\mu\lambda}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\lambda - g_{\alpha\lambda}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{pz}$$
$$+(sz)\big(p_\mu z_\alpha - p_\alpha z_\mu\big)\big(p_\lambda z_\beta - p_\beta z_\lambda\big)\widetilde{\mathcal{M}}_{ppzz}$$
$$+(sz)\big(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\big)\widetilde{\mathcal{M}}_{gg}$$

$$\widetilde{M}^{(1)}_{\mu\alpha;\lambda\beta}(z,p) = \Big(g_{\mu\lambda}s_\alpha p_\beta - g_{\mu\beta}s_\alpha p_\lambda - g_{\alpha\lambda}s_\mu p_\beta + g_{\alpha\beta}s_\mu p_\lambda\Big)\widetilde{\mathcal{M}}_{sp}$$
$$+\Big(g_{\mu\lambda}p_\alpha s_\beta - g_{\mu\beta}p_\alpha s_\lambda - g_{\alpha\lambda}p_\mu s_\beta + g_{\alpha\beta}p_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{ps}$$
$$+\Big(g_{\mu\lambda}s_\alpha z_\beta - g_{\mu\beta}s_\alpha z_\lambda - g_{\alpha\lambda}s_\mu z_\beta + g_{\alpha\beta}s_\mu z_\lambda\Big)\widetilde{\mathcal{M}}_{sz}$$
$$+\Big(g_{\mu\lambda}z_\alpha s_\beta - g_{\mu\beta}z_\alpha s_\lambda - g_{\alpha\lambda}z_\mu s_\beta + g_{\alpha\beta}z_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{zs}$$
$$+(p_\mu s_\alpha - p_\alpha s_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{pspz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(p_\lambda s_\beta - p_\beta s_\lambda)\widetilde{\mathcal{M}}_{pzps}$$
$$+(s_\mu z_\alpha - s_\alpha z_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{szpz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(s_\lambda z_\beta - s_\beta z_\lambda)\widetilde{\mathcal{M}}_{pzsz}$$

Want: $M_{\Delta g}(\nu, z^2) = \big[\widetilde{\mathcal{M}}^{(+)}_{sp} - \nu\,\widetilde{\mathcal{M}}_{pp}\big]$

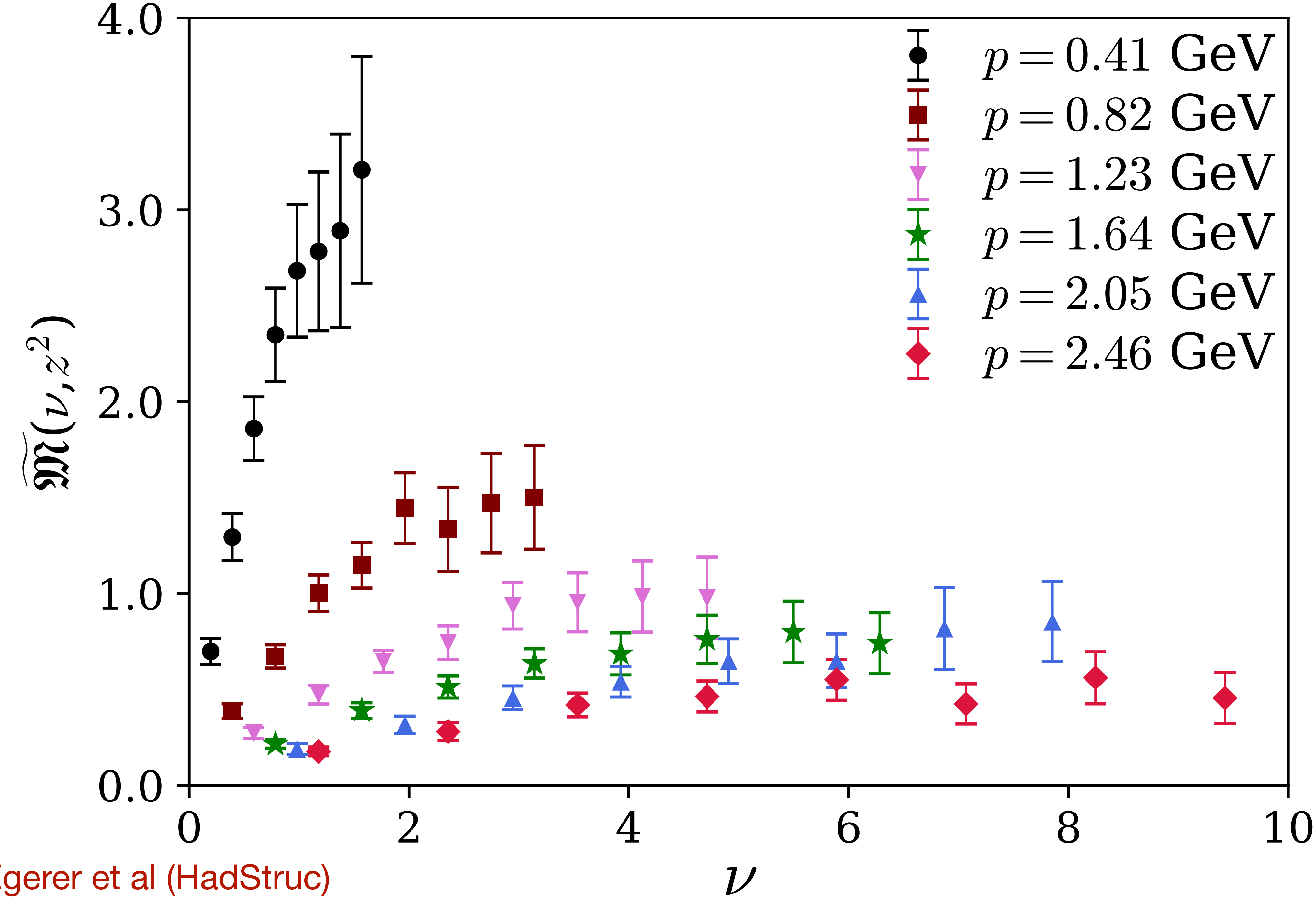
Can get: $\widetilde{\mathcal{M}}(z,p) = \big[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\big]$

$$= M_{\Delta g} - \frac{m^2 z^2}{\nu}\widetilde{\mathcal{M}}_{pp}$$

$$= M_{\Delta g} - \frac{m^2}{p_z^2}\nu\,\widetilde{\mathcal{M}}_{pp}$$

Helicity Gluon Matrix Element

- Large contamination from $\frac{m^2}{p_z^2}\nu\widetilde{\mathcal{M}}_{pp}$ will need to be removed

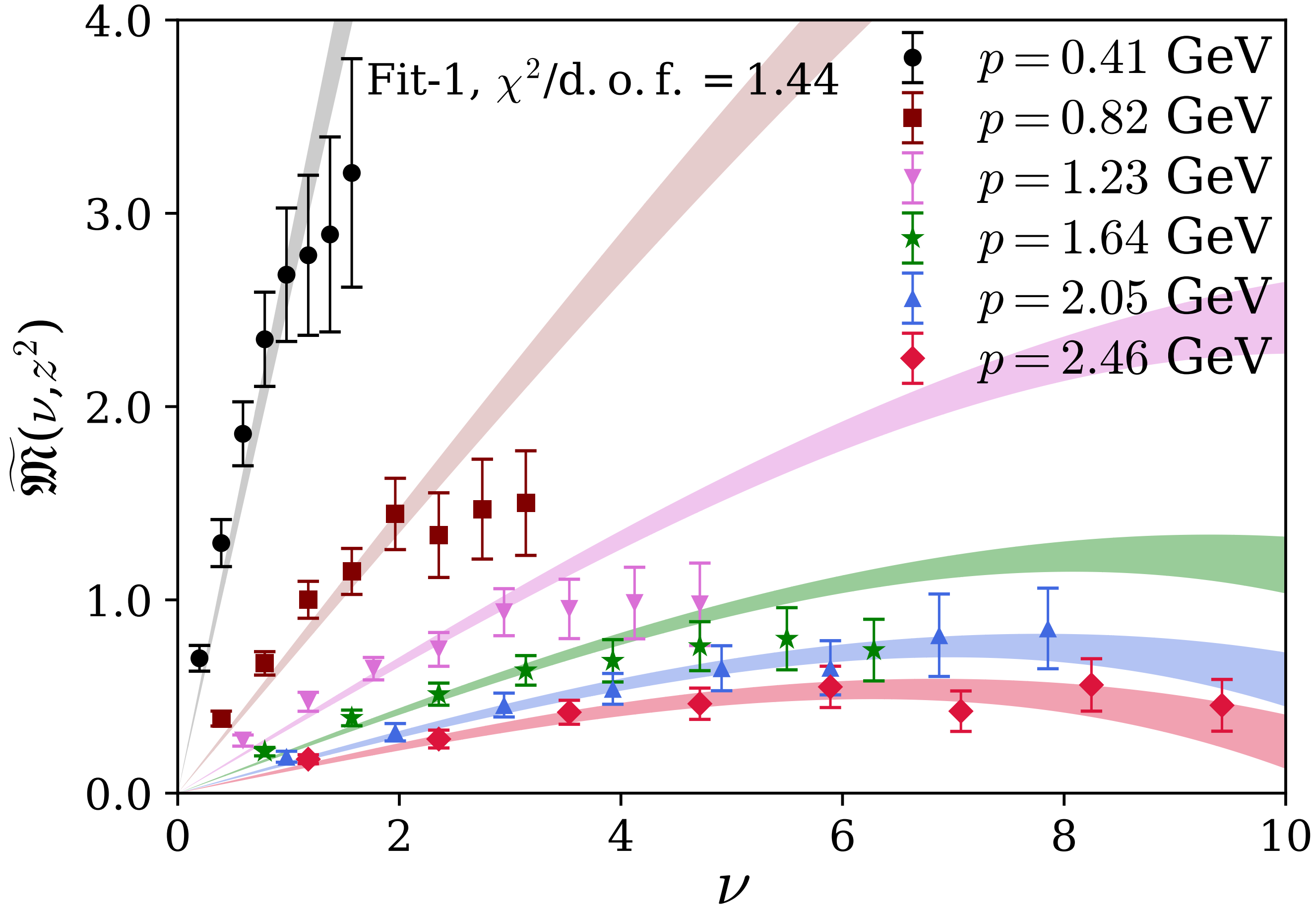


$a = 0.094$ fm
$m_\pi = 358$ MeV

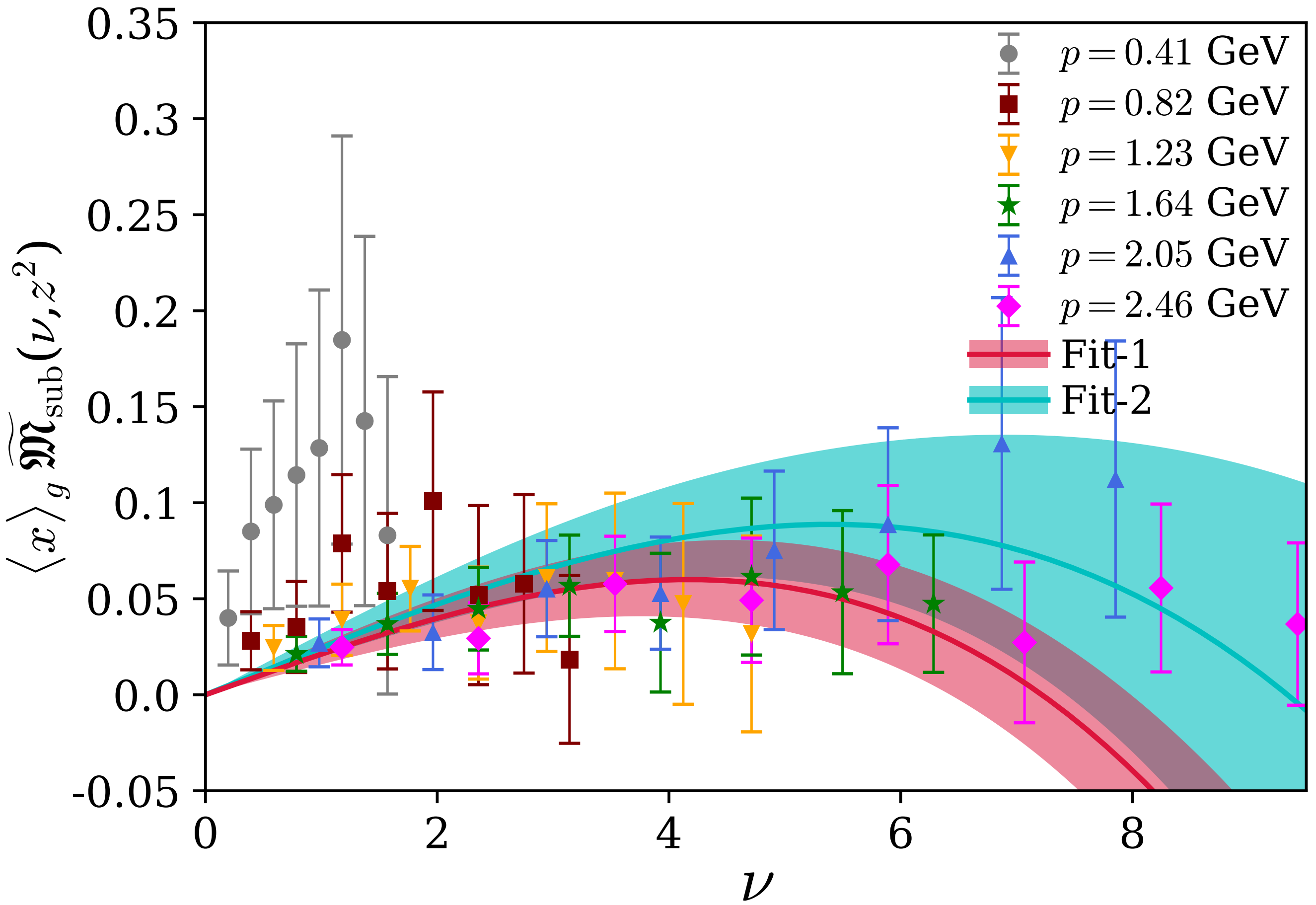
C. Egerer et al (HadStruc)
arXiv: 2207.08733

Correcting Helicity Gluon Results

- Model both terms



- Subtract rest frame



C. Egerer et al (HadStruc) arXiv: 2207.08733

- Model with Neural Network

T. Khan, T. Liu, R. Sufian arXiv:2211.15587

What can we do with this new data?

Spinning gluons

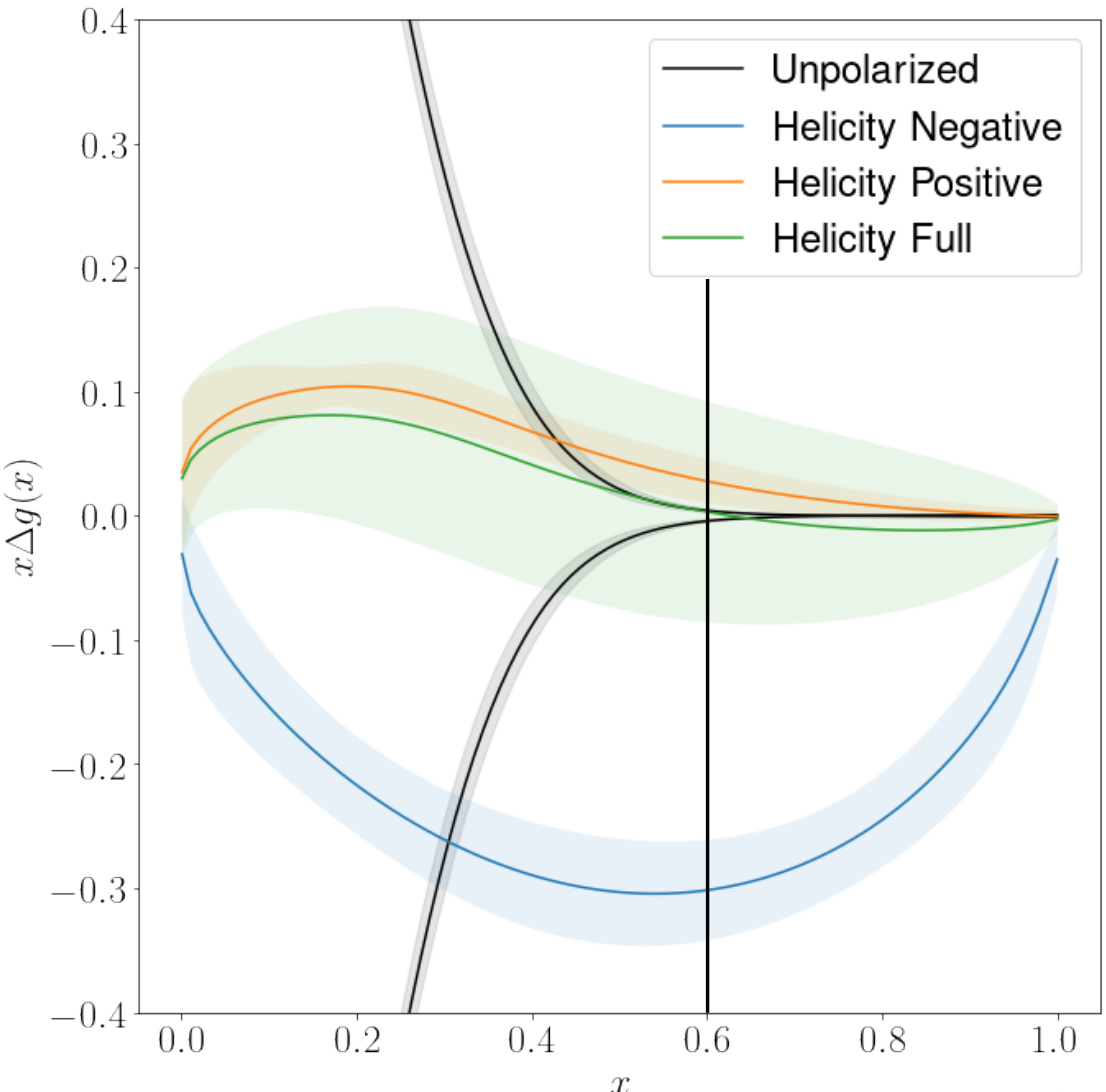
Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)

- Positivity removed from JAM helicity gluon PDF

$$|\Delta g| \leq g$$

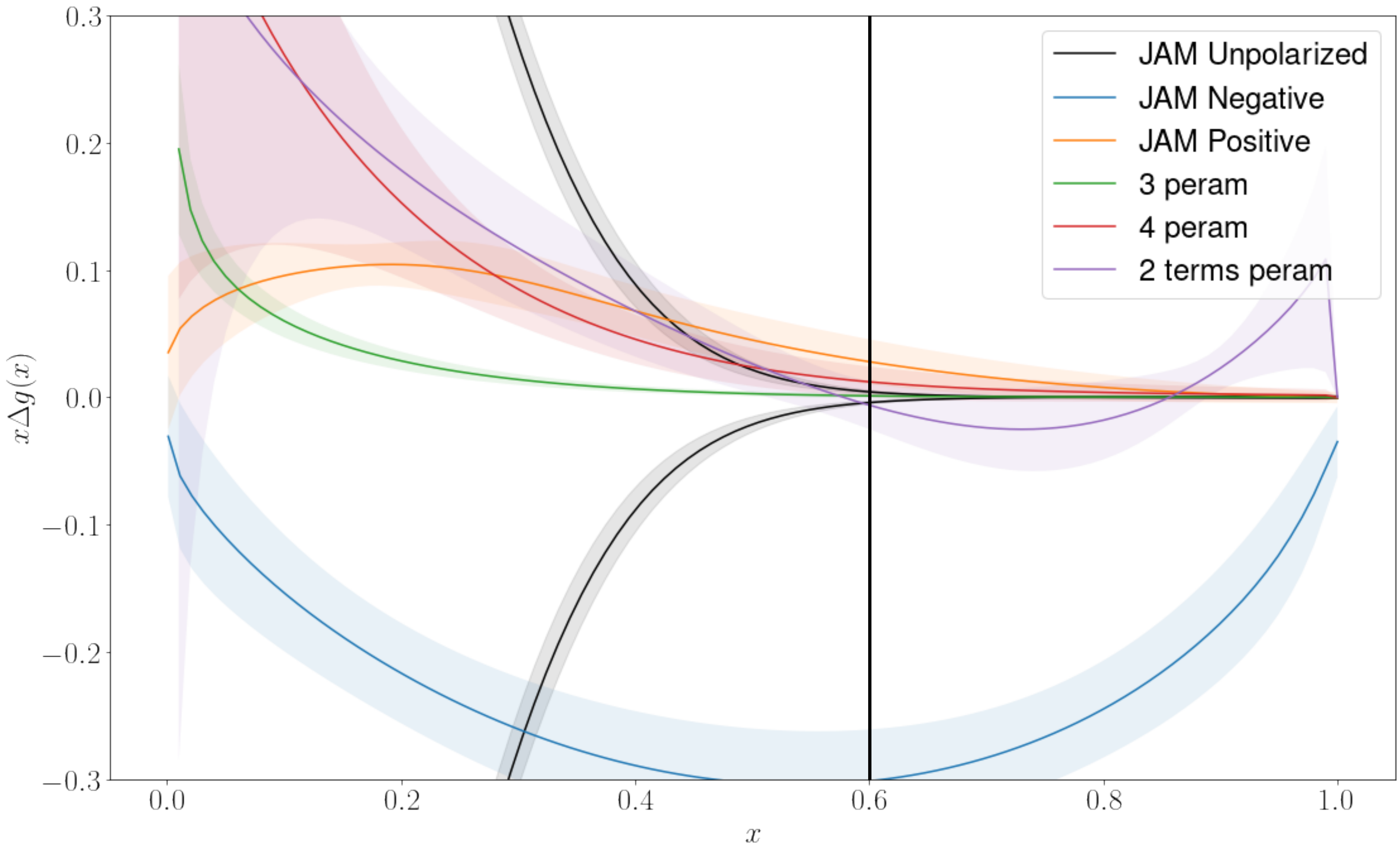
- Data only came from lower x < 0.6

- Lattice results are sensitive over a range of x, specifically the larger x region



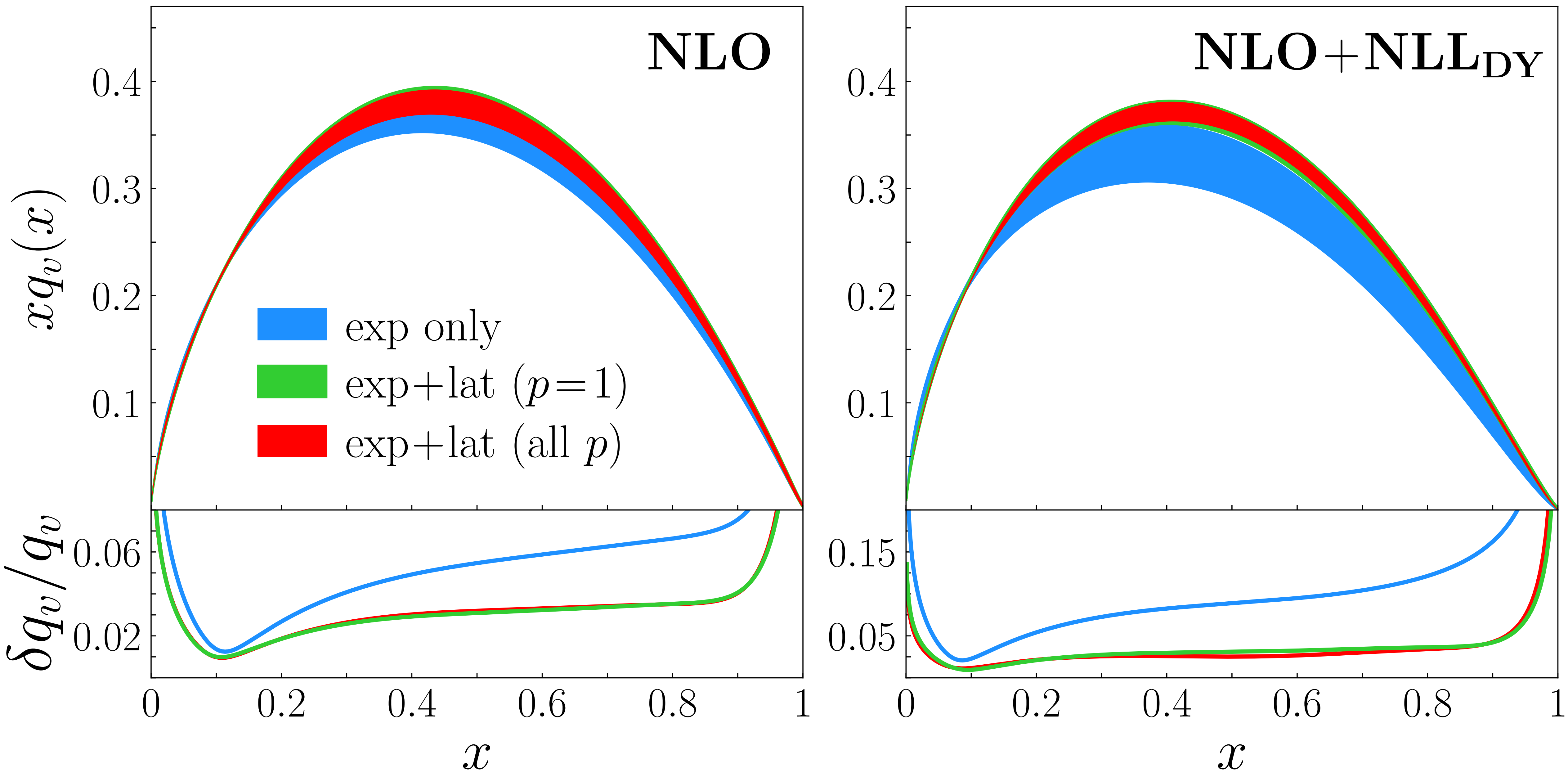
Fitting Lattice Alone

- Fitting subtracted lattice data alone leaves huge model dependence on x space PDF



Combining Lattice and Experiment

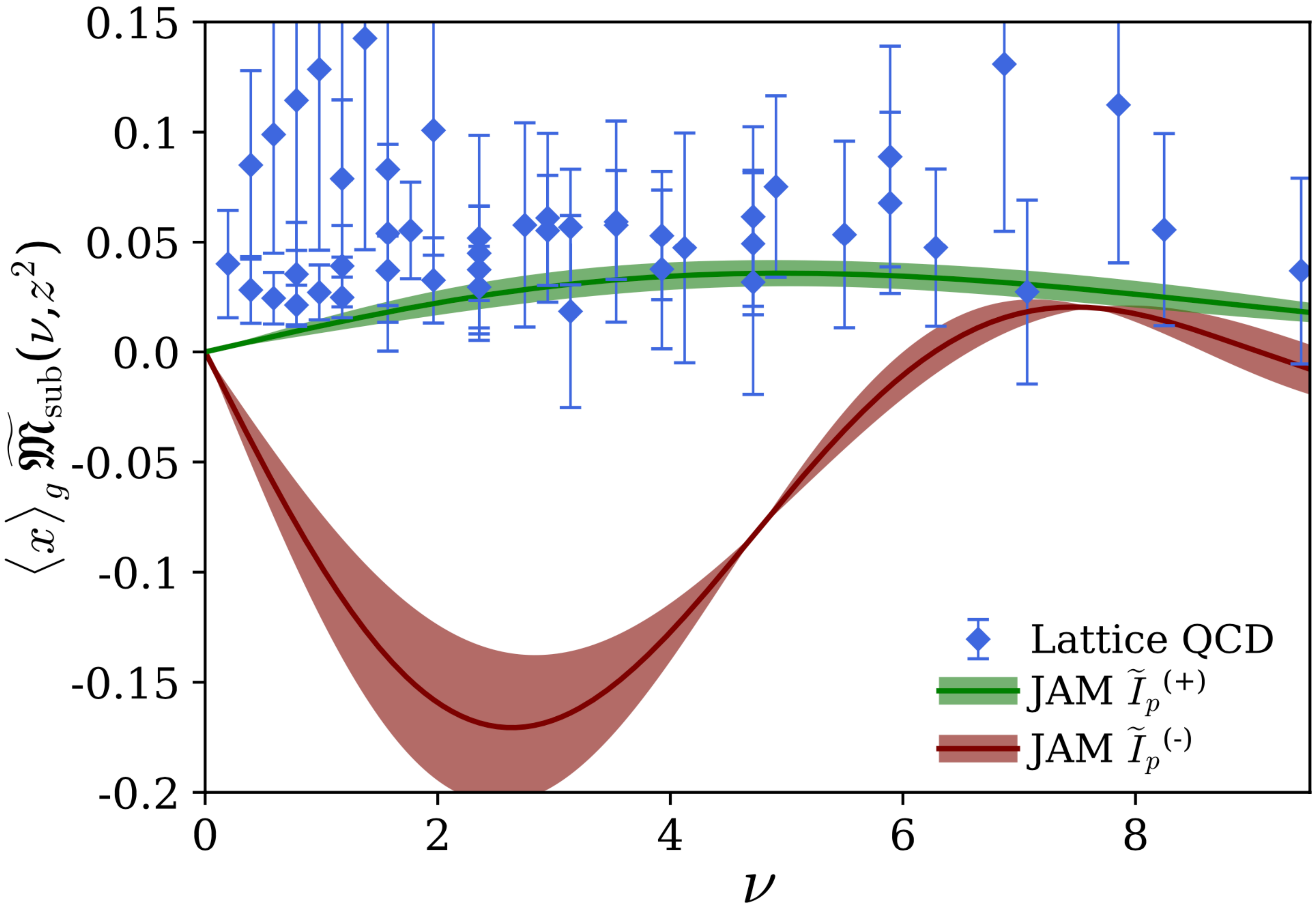
- Simultaneously fit Lattice and Experimental pion PDF data
- Each gives unique information complementing each other



P. Barry et al (HadStruc and JAM), *Phys. Rev. D* 105 (2022) 11, 114051

Combining Lattice and Experiment

Can lattice data affect phenomenological polarized gluon analysis?



$$\Delta G = \int d\nu \, I_g(\nu)$$

- The positive and negative solutions without positivity constraints plotted in ν space

Y. Zhou et al Phys. Rev. D 105, 074022 (2022)
C. Egerer et al (HadStruc) arXiv:2207.08733

- Only positive band consistent with lattice data

Conclusions

- Gluon x or ν dependent structure requires state-of-the-art calculations
  - Use of **distillation** with large number of configurations
  - **Summed GeVP** to control excited states
  - **Wilson flow** to improve signal
- Future work towards gluon 3D distributions, higher twist PDFs,…. is possible given enough resources to find signal
- Possibly impact phenomenological PDF analyses
  - **Today:** with polarized gluon PDF
  - **Future:** JLab 12GeV and EIC data on PDF and on new distributions

Thank you and the organizers!