Continuum-Physical Nucleon Gluon PDF

Based on: arXiv:2210.09985v1 [hep-lat] 18 Oct 2022

SPEAKER: WILLIAM GOOD[1,2]

COLLABORATORS: ZHOUYOU FAN[1], HUEY-WEN LIN[1,2]

2 DECEMBER 2022

[1]DEPARTMENT OF PHYSICS AND ASTRONOMY, MICHIGAN STATE UNIVERSITY

[2]DEPARTMENT OF COMPUTATIONAL MATHEMATICS, SCIENCE AND ENGINEERING, MICHIGAN STATE UNIVERSITY

Background and Introduction

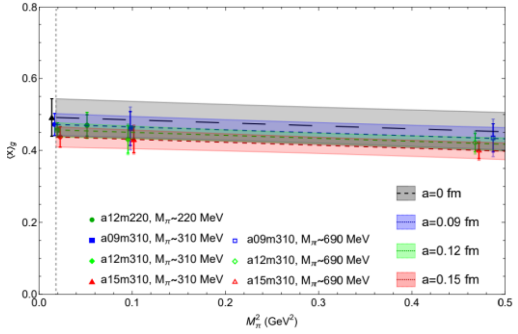
The gluon PDF is an important input for high energy scattering experiments

Phenomenological studies of the gluon PDF still see issues in the large-x range

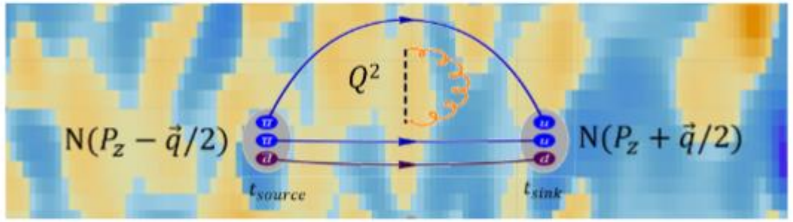
Gluon PDFs are difficult on the lattice due to large noise

We present the first physical-continuum limit results of the unpolarized nucleon gluon PDF

Methodology Overview



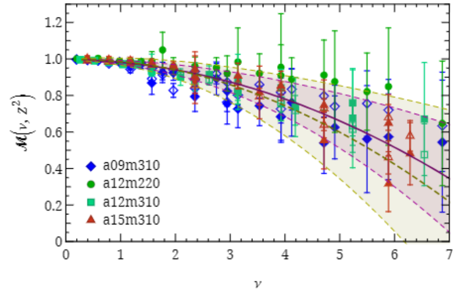
$\langle x \rangle_g$ obtained from other work



Correlator measurements on the lattice

Find MEs from 2pt and 3pt correlator analysis

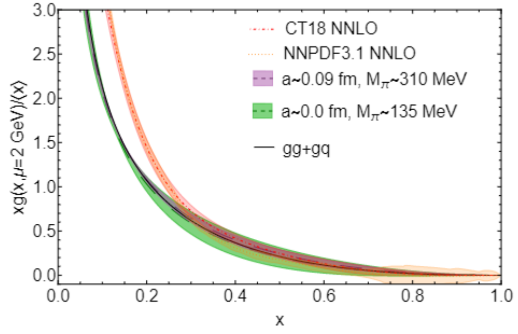
Get Reduced pseudo-Ioffe Time Distribution (RpITD)



Extrapolate to the physical-continuum limit RpITD

Fit $xg(x)/\langle x \rangle_g$

Final nucleon gluon PDF $xg(x)$



LaMET2022

General Lattice Setup

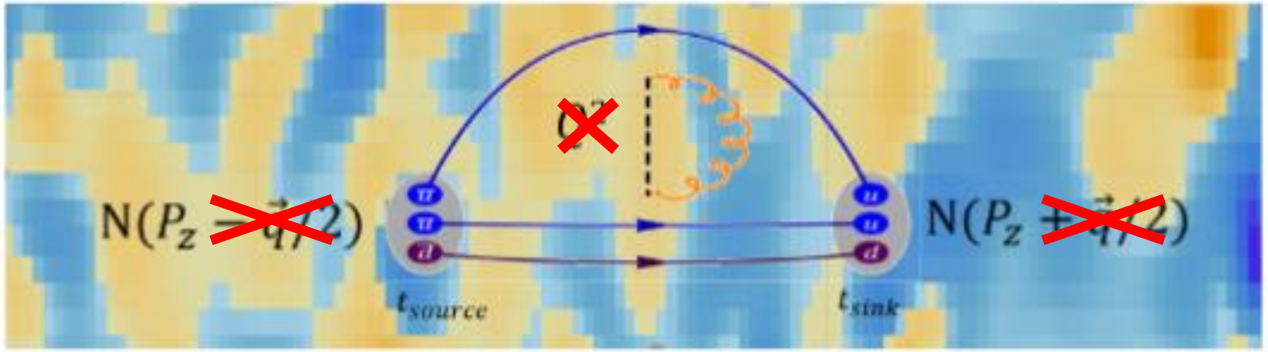
Calculation carried out with $\mathrm{N_f} = 2 + 1 + 1$ highly improved staggered quarks (HISQ) generated by MILC collaboration (Follana et al. PRD 75:054502, 2007.)

Five steps of hypercubic (HYP) smearing on gauge links

Wilson-clover fermions used in valence sector

Used lattice spacings $a \approx 0.15, 0.12, 0.09$ fm and pion masses $M_\pi \approx 220, 310$ MeV

Valence quark masses tuned to reproduce light and strange pion masses $M_l \approx 220, 310$ and $M_s \approx 700$ MeV



More Ensemble Details

Ensemble	a09m310	a12m220	a12m310	a15m310
a (fm)	0.0888(8)	0.1184(10)	0.1207(11)	0.1510(20)
$L^3 \times T$	$32^3 \times 96$	$32^3 \times 64$	$24^3 \times 64$	$16^3 \times 48$
M_π^{val} (GeV)	0.313(1)	0.2266(3)	0.309(1)	0.319(3)
$M_{\eta_s}^{\text{val}}$ (GeV)	0.698(7)	N/A	0.6841(6)	0.687(1)
P_z (GeV)	$[0, 3.05]$	$[0, 2.29]$	$[0, 2.14]$	$[0, 2.56]$
N_{cfg}	1009	957	1013	900
$N_{\text{meas}}^{\text{2pt}}$	387,456	1,466,944	324,160	259,200
t_{sep}	$[6, 10]$	$[6, 10]$	$[5, 9]$	$[4, 8]$

Correlator Definitions

$$C_N^{\mathrm{2pt}}(P_z;t) = \langle 0|\Gamma \int d^3y\, e^{-iyP_z} \chi(\vec{y},t)\chi(\vec{0},0)|0\rangle$$

$$\Gamma = \frac{1}{2}(1+\gamma_4) \qquad \chi(\vec{y},t) = \epsilon^{lmn}[u(y)^{l^T} i\gamma_4\gamma_2\gamma_5 d^m(y)]u^n(y)$$

$$C_N^{\mathrm{3pt}}(z,P_z;t_{\mathrm{sep}},t) =$$

$$\langle 0|\Gamma \int d^3y\, e^{-iyP_z} \chi(\vec{y},t_{\mathrm{sep}})\mathcal{O}_g(z,t)\chi(\vec{0},0)|0\rangle$$

$$\mathcal{O}(z) \equiv \sum_{i\neq z,t} \mathcal{O}(F^{ti},F^{ti};z) - \frac{1}{4}\sum_{i,j\neq z,t} \mathcal{O}(F^{ij},F^{ij};z)$$

Balitsky et al, PLB 808:135621, 2020.

$$\mathcal{O}(F^{\mu\nu},F^{\alpha\beta};z) = F^{\mu}_{\nu}(z)U(z,0)F^{\alpha}_{\beta}(0)$$

2pt and 3pt Analysis

Applied Gaussian momentum smearing on the quark field to improve signal up to 3.0 GeV

For fitting, take correlators to be of form:

$$C_N^{2pt}(P_z, t) = \left|A_{N,0}\right|^2 e^{-E_{N,0}t} + \left|A_{N,1}\right|^2 e^{-E_{N,1}t} + \cdots$$

and

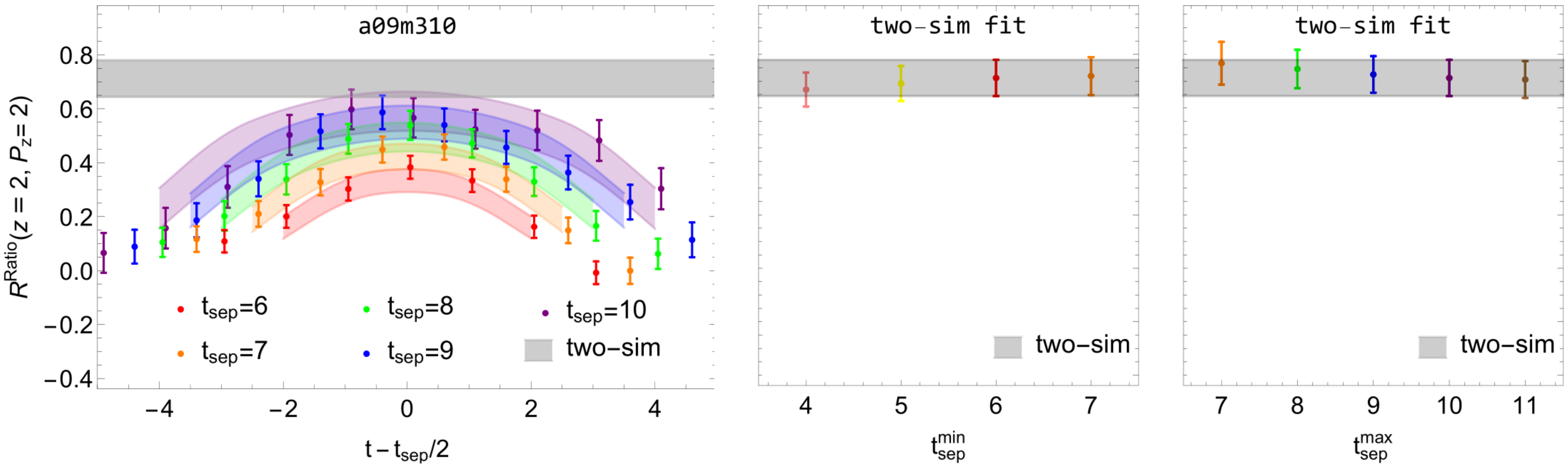
$$C_N^{3pt}(z, P_z, t, t_{sep}) =$$
$$\left|A_{N,0}\right|^2 \langle 0|O_g|0\rangle e^{-E_{N,0}t_{sep}} + \left|A_{N,0}\right|\left|A_{N,1}\right|\langle 0|O_g|1\rangle e^{-E_{N,1}(t_{sep}-t)} e^{-E_{N,0}t} +$$
$$\left|A_{N,0}\right|\left|A_{N,1}\right|\langle 1|O_g|0\rangle e^{-E_{N,0}(t_{sep}-t)} e^{-E_{N,1}t} + \left|A_{N,1}\right|^2 \langle 1|O_g|1\rangle e^{-E_{N,1}t_{sep}} + \cdots$$

Correlator Ratio Plots and Fits

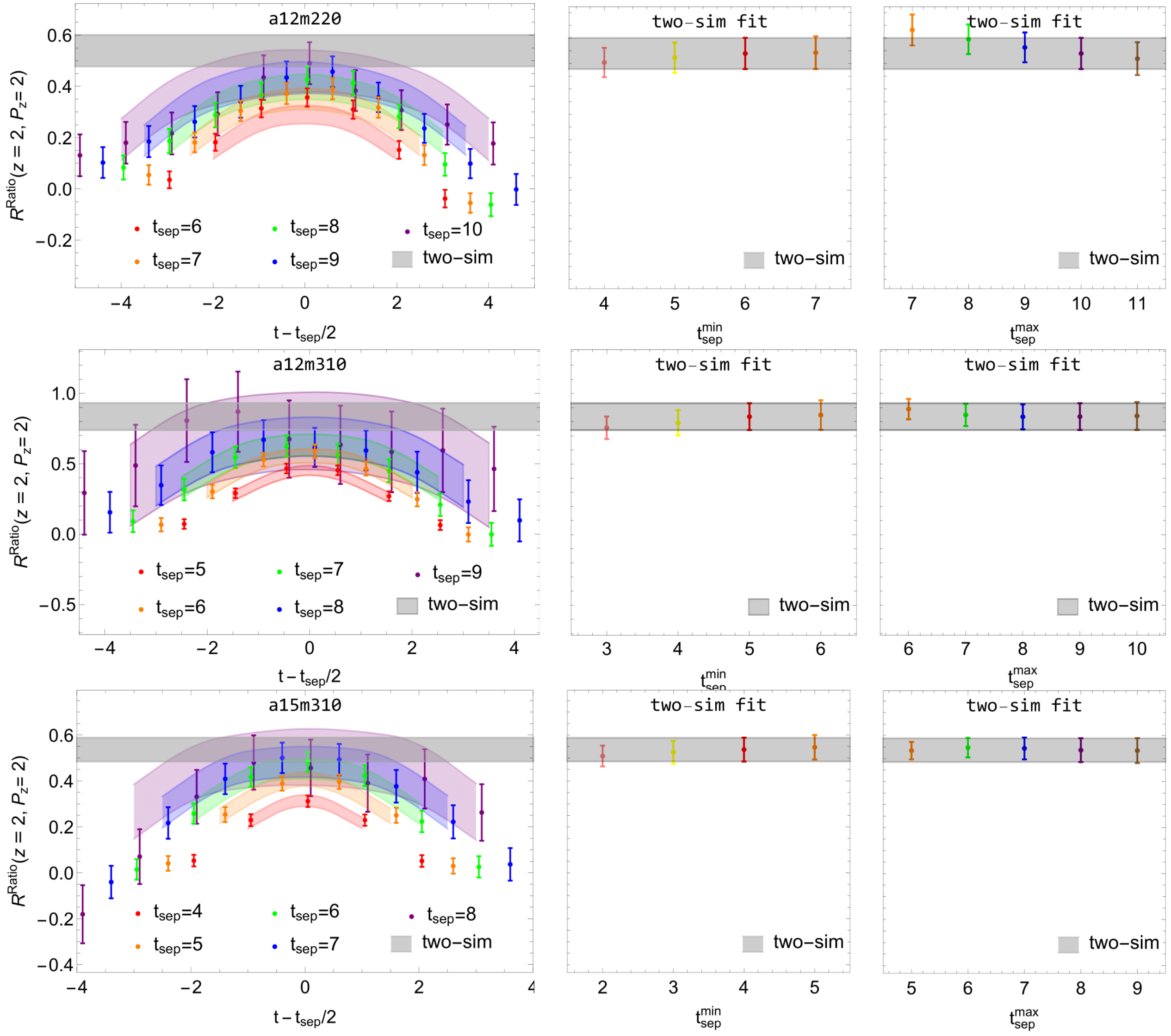
Plot the ratio:

$$R_N(z, P_z, t, t_{sep}) = \frac{C_N^{3pt}(z, P_z, t, t_{sep})}{C_N^{2pt}(P_z, t)}$$

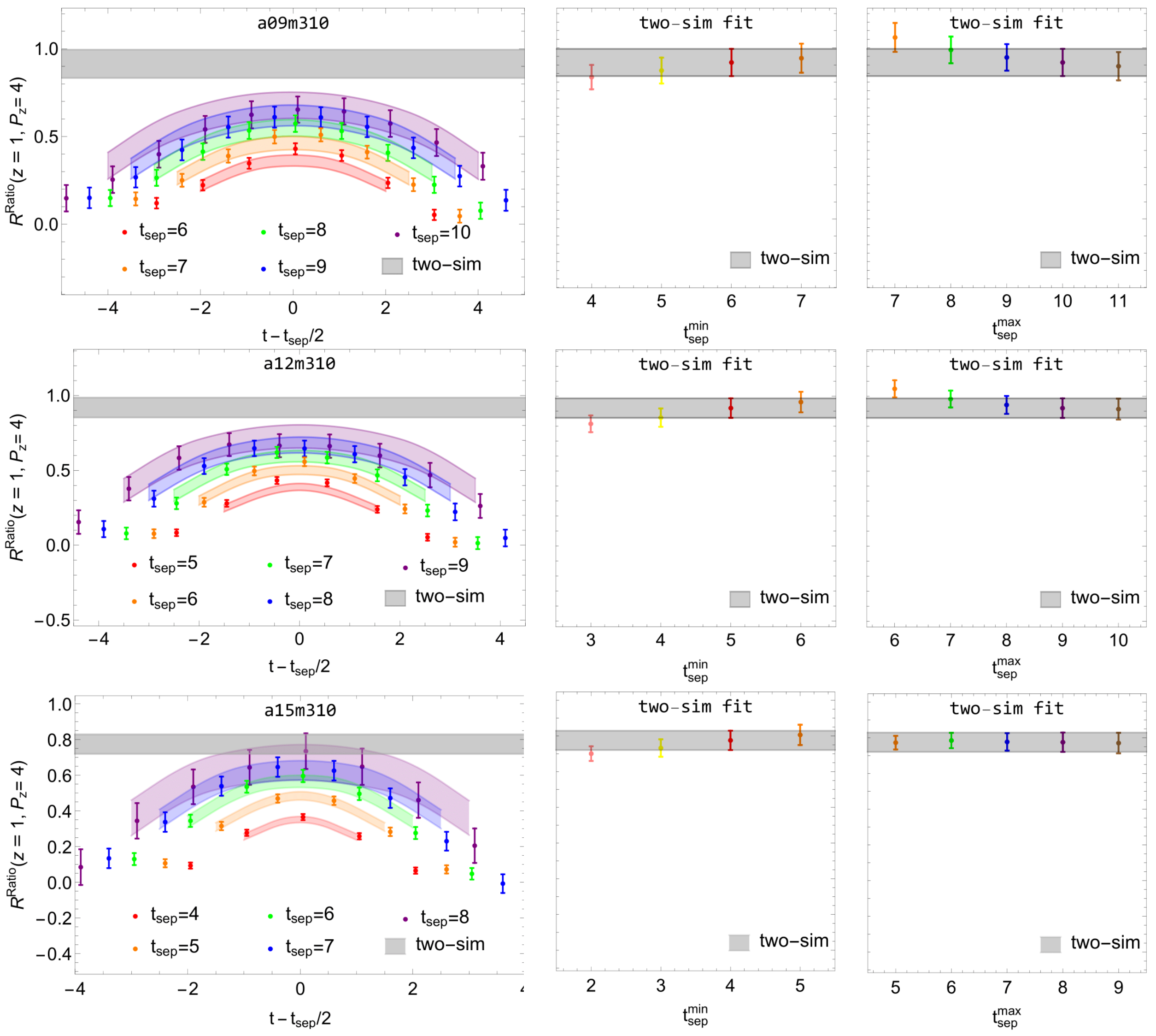
Simultaneously fit 2pt and 3pt to obtain $\langle 0 | O_g | 0 \rangle$ matrix elements (MEs)



Other Ratio Plots



Light



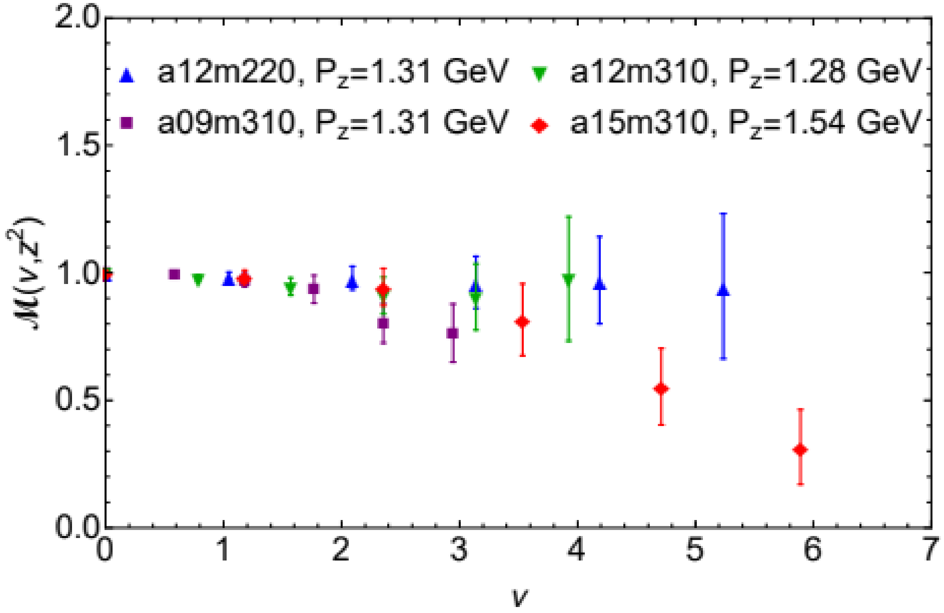
Strange

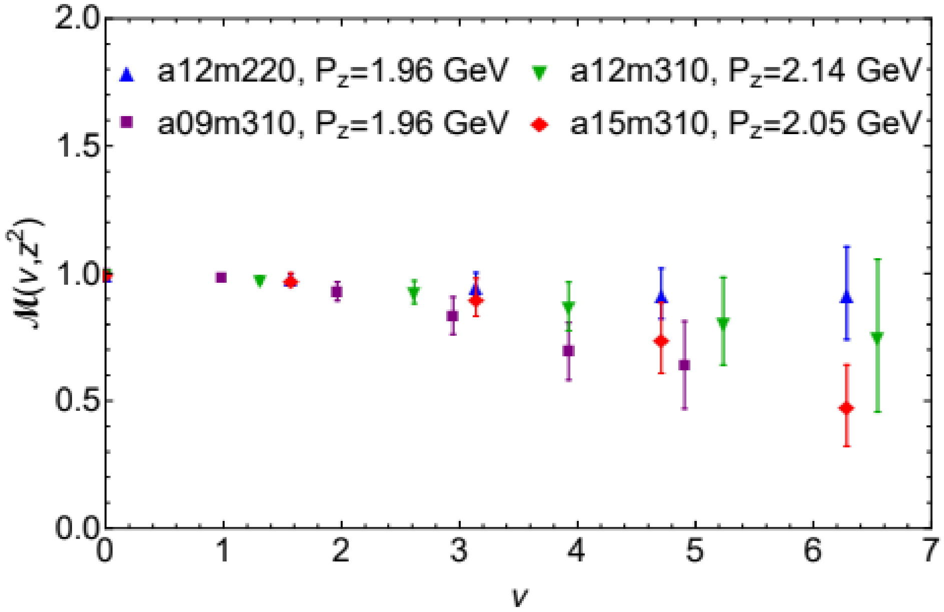
Reduced Pseudo Ioffe Time Distribution (RpITD)

Take the double ratio of the MEs (everyone's favorite)

$$\mathscr{M}(\nu, z^2) = \frac{\mathcal{M}(zP_z, z^2)/\mathcal{M}(0 \cdot P_z, 0)}{\mathcal{M}(z \cdot 0, z^2)/\mathcal{M}(0 \cdot 0, 0)}$$

$(M(\nu, z^2) = \langle 0(P_z) | O_g(z) | 0(P_z) \rangle)$ $\quad (\nu = zP_z)$

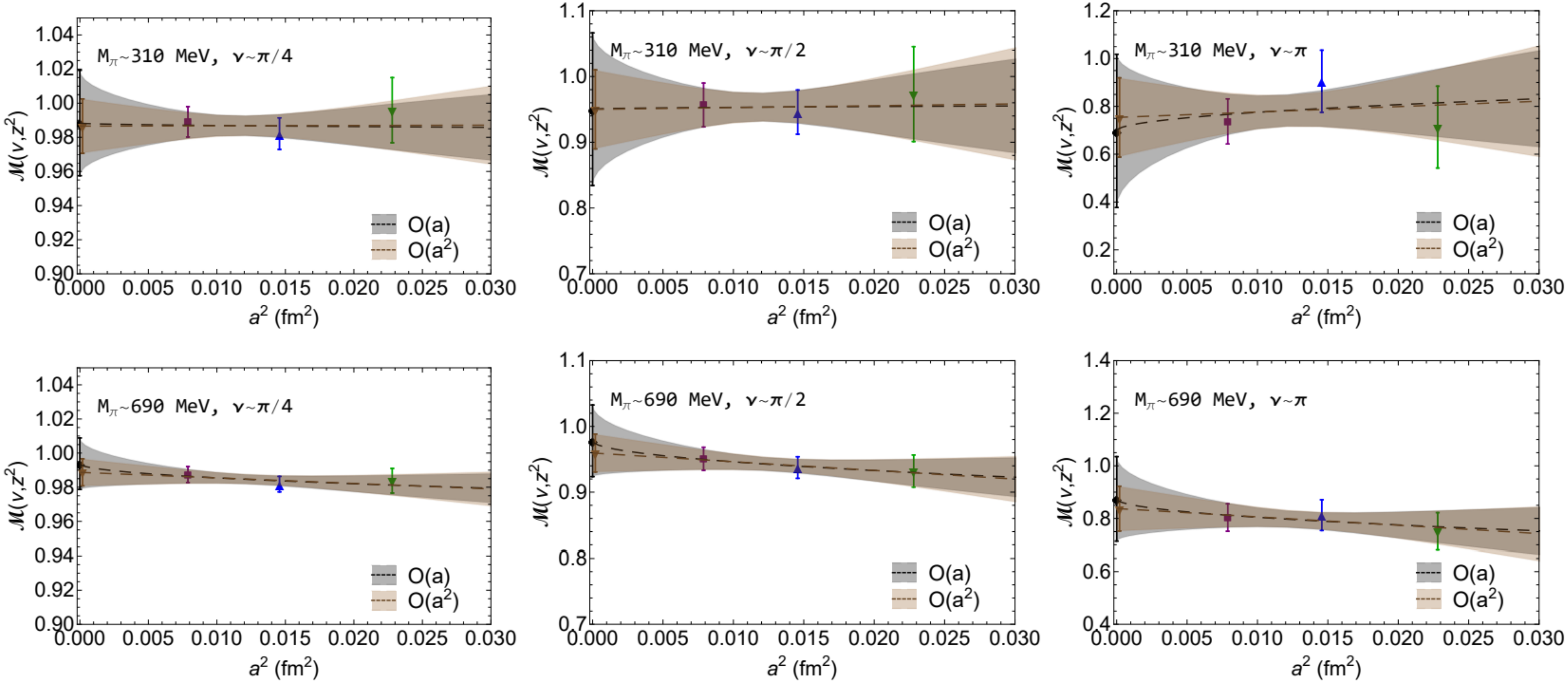




Continuum RpITD Data

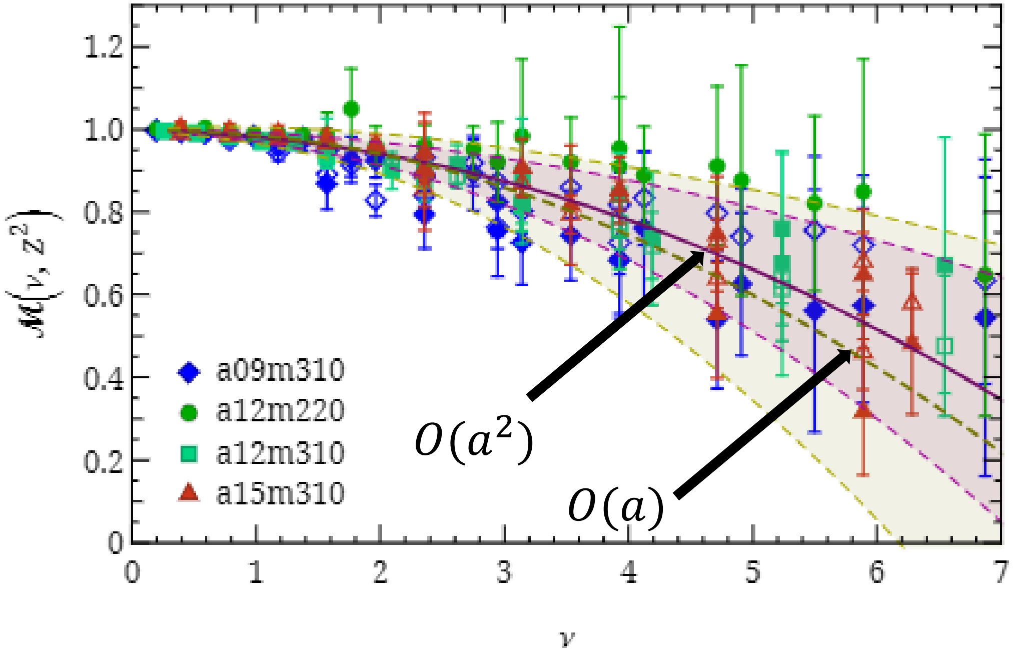
Assumed fit form n = 1, 2 $$\mathscr{M}(\nu, z^2, a, M_\pi) = \mathscr{M}^{\text{cont}} + c_a a^n$$

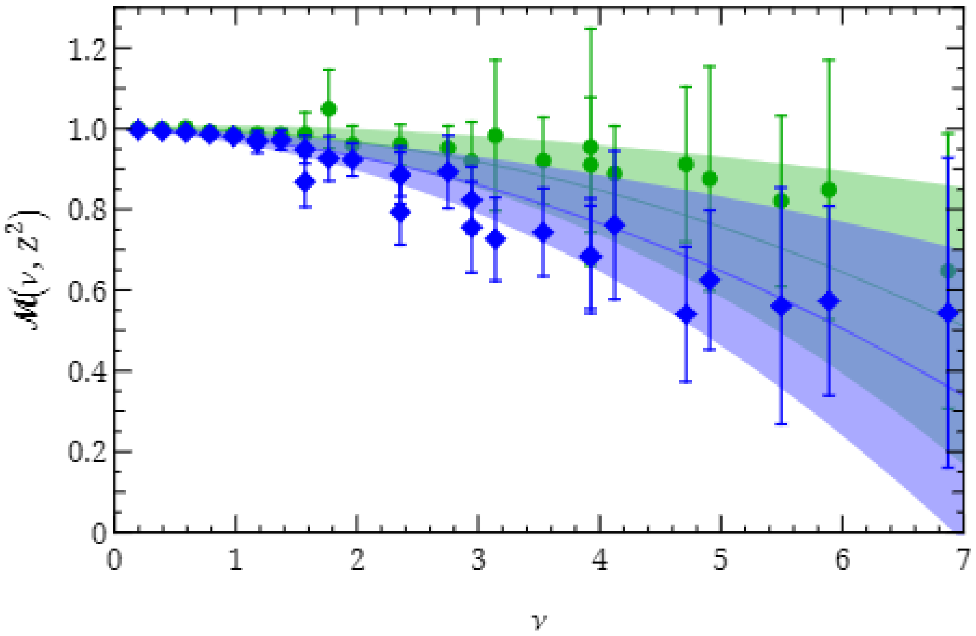
Choose constant P_z and change z to get $\nu = \frac{\pi}{4}, \frac{\pi}{2}, \pi$



Extrapolate to Physical-Continuum limit

$$\mathscr{M}(\nu, z^2, a, M_\pi) = \left(\sum_{k=0}^{k_{\max}} \lambda_k(a, M_\pi)\nu^k + c_z(a, M_\pi)z^2\right) \times \left(1 + c_a a^2 + c_M(M_\pi^2 - (M_\pi^{\mathrm{phys}})^2\right)$$





Phenomenological Fit Form

Gluon matching kernel R_{gg} connects the RpITD to the PDF as shown

Balitsky et al, PLB 808:135621, 2020.

$$\mathscr{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x,\mu^2)}{\langle x \rangle_g} R_{gg}(x\nu, z^2\mu^2)$$

Use a typical global analysis fit form

$B(A+1, C+1)$ is beta function (integral of numerator)

$$f_g(x,\mu) = \frac{xg(x,\mu)}{\langle x \rangle_g(\mu)} = \frac{x^A(1-x)^C}{B(A+1,C+1)}$$

Minimize

$$\chi^2(\mu, a, M_\pi) = \sum_{\nu,z} \frac{(\mathscr{M}^{\text{fit}}(\nu,\mu,z^2,a,M_\pi) - \mathscr{M}^{\text{lat}}(\nu,z^2,a,M_\pi))^2}{\sigma^2_{\mathscr{M}}(\nu,z^2,a,M_\pi)}$$

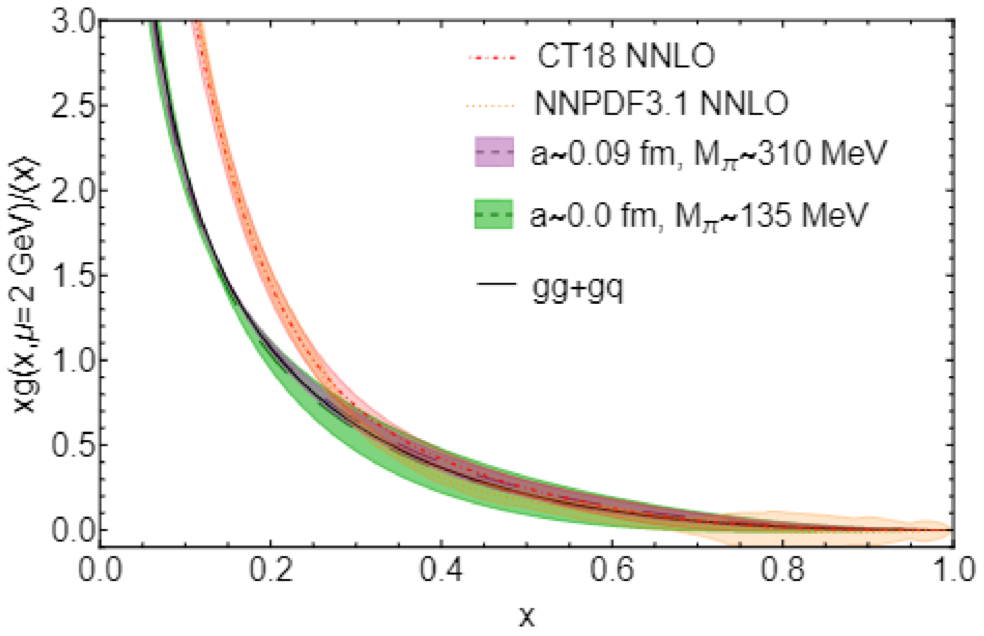
Results

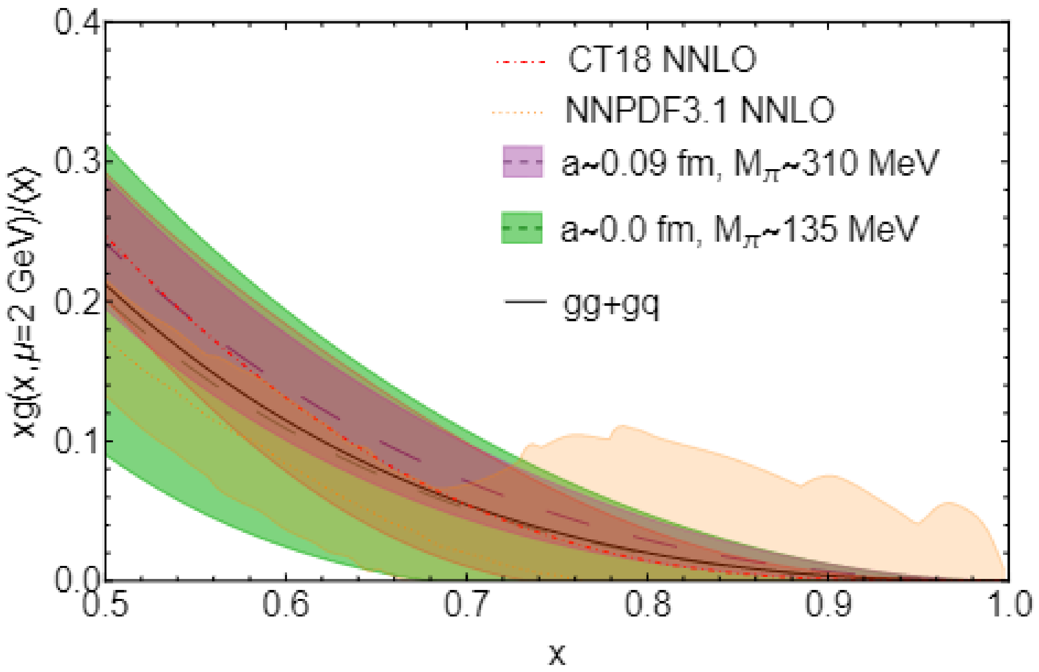
Ball et al. EPJ 77(10):663, 2017.
Hou et al. PRD 103(1):014013, 2021.
Fan et al. IJMPA 36(13):2150080, 2021.

Compare to global analysis PDFs

Add quark term from CT18 global fit

$$\frac{P_z}{P_0} \int_0^1 dx \frac{x q_S(x,\mu^2)}{\langle x \rangle_g} R_{gq}(x\nu, z^2\mu^2)$$



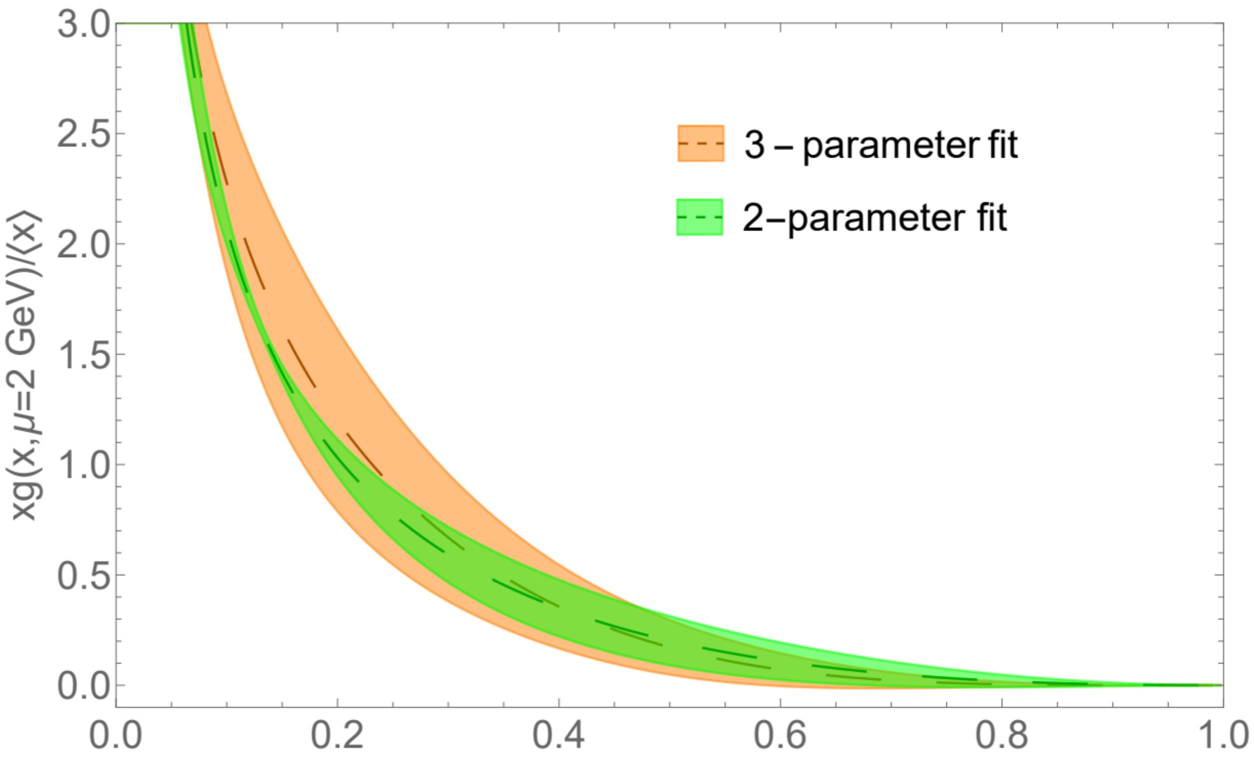


One Last Consideration

Try 3 parameter fit

$$f_{g,3}(x,\mu) = \frac{x^A(1-x)^C(1+Dx)}{B(A+1,C+1)+DB(A+2,C+1)}$$

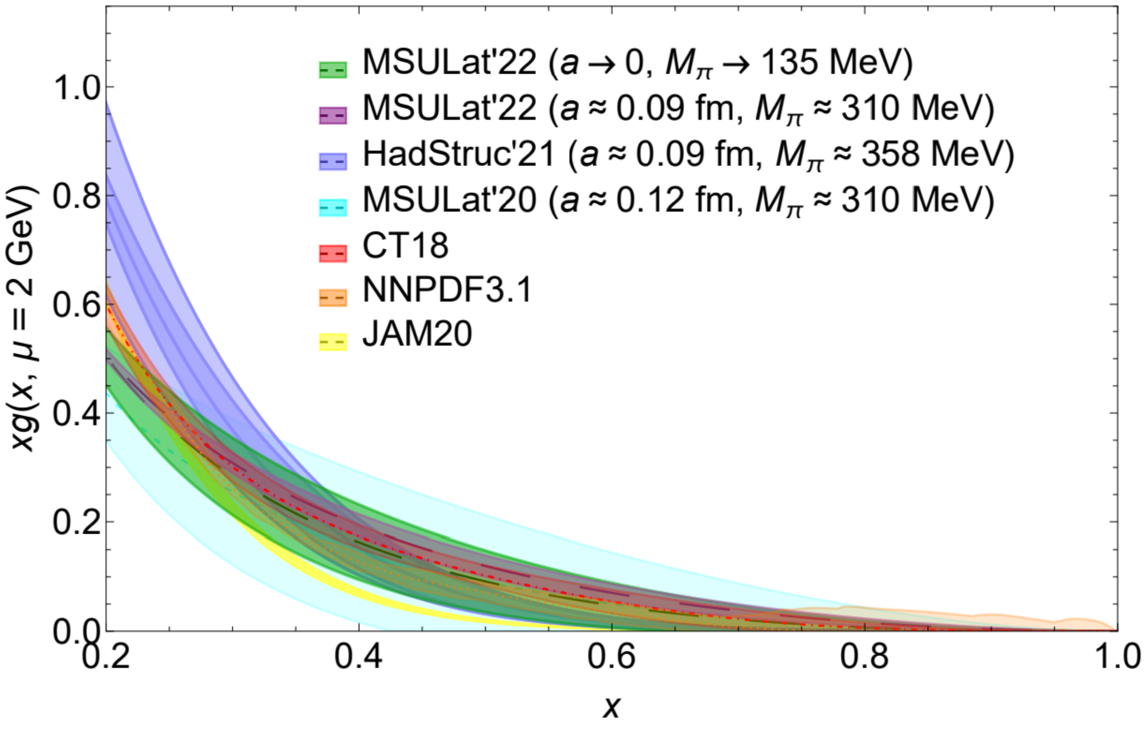
Similar results, more error

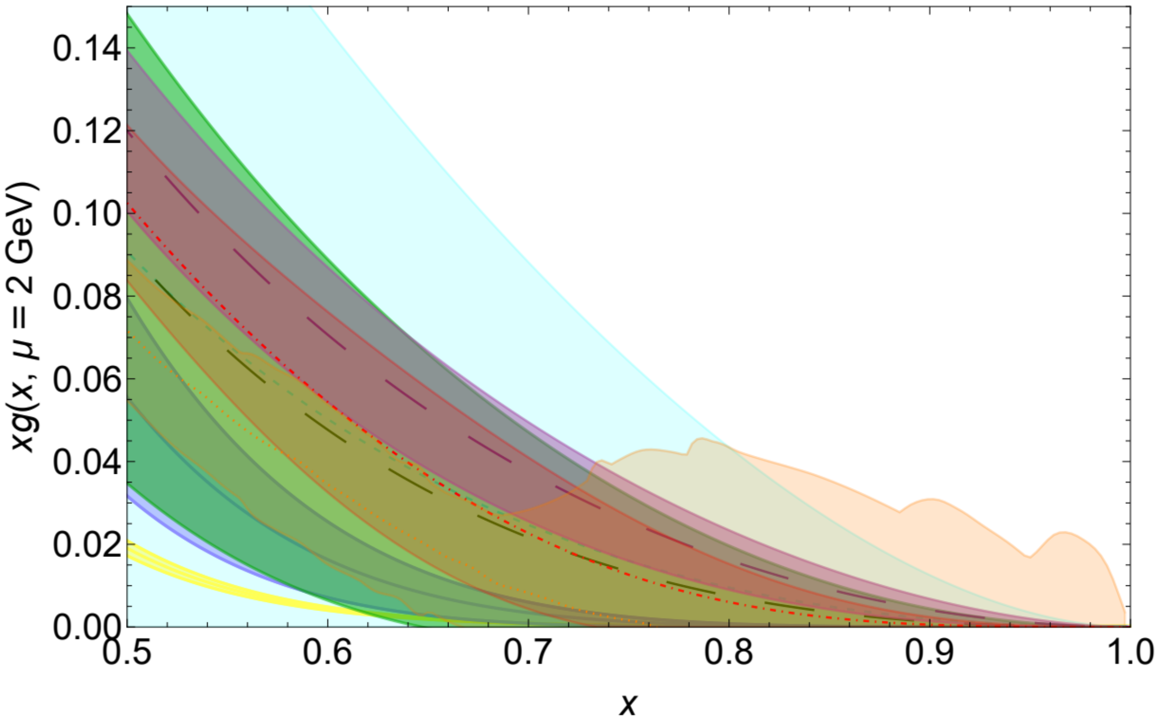


Final PDF Results

Multiply through by $\langle x \rangle$ from Fan et al. arXiv:2208.00980v1 [hep-lat] 2022

Ball et al. EPJ 77(10):663, 2017.
Hou et al. PRD 103(1):014013, 2021.
Fan et al. IJMPA 36(13):2150080, 2021.
Kahn et al. PRD 104(9):094516, 2021.
Moffat et al. PRD 104(1):016015, 2021.





Summary and Conclusion

Explained the analysis of the correlator data to get the RpITDs

Explored continuum and continuum-physical extrapolations for the RpITD

Matched the RpITD to gluon PDF, showing some differences from global analysis and HadStruc'21 lattice calculations

Noise in the calculation implies more data is needed

Need to better understand systematics

- i.e., differences in lattice calculations

Thank You!

Backup: Gluon Momentum Fractions

Ours: $\langle x \rangle_g = 0.492(52)_{Stat+NPR} + (49)_{mixing}$

NNPDF3.1

HadStruc: $\langle x \rangle_g = 0.427(92)$

Alexandrou et al. 101.094513, 2020.