Unpolarized gluon PDF for the proton using the twisted mass formulation

Joseph Delmar

Department of Physics, Temple University

In collaboration with
C. Alexandrou, K. Cichy, M Constantinou, K. Hadjiyiannakou

LaMET 2022 - Chicago, USA

December 2, 2022

Outline

1 Introduction
- Motivation
- gPDFs on the Lattice

2 Calculation of gluon PDFs
- The Pseudo-PDF Approach
- Lattice Setup

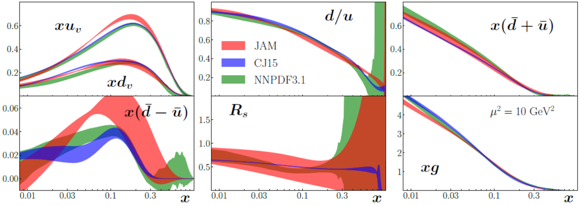
3 Results
- Matrix Elements
- Double Ratio and Interpolation
- ITD Development
- Reconstructed Pseudo-PDF
- Ongoing Work
- Summary - Future Work

4 Acknowledgements

Motivation

- Gluon contributions to physical quantities play a critical role in hadron structure
- Gluon contributions can be large, eg. gluon momentum fraction ≈ 40%
- Dedicated experimental efforts to understand gluonic structure of hadron

[Moffat et al, PRD 104, 016015 (2021)]
[Ball et al, EPJC 77, 663 (2017)]
[Accardi et al, PRD 93, 114017 (2016)]



- Lattice studies of gPDFs can assist in constraining global analysis

gPDFs on the Lattice

- Several challenges in extracting reliable results
  - purely disconnected diagram
  - at least an order of magnitude more statistics than quark counterparts
  - unavoidable mixing with quark singlet PDFs
- x-dependence of gluon PDFs even more challenging
- Inverse problem in reconstruction of x-dependence due to limited lattice data

Pseudo-PDF Approach (in a nutshell)

- Matrix elements of **non-local operators** and **momentum-boosted proton states**
  - Several choices for the form of the gluon operator consisting of two field-strength tensors, separated by spatial distance z, and two straight Wilson lines, connecting points $0 \to z$ and $z \to 0$

$$M_{\mu i;\nu j}(P,z) = \langle N(P)|F_{\mu i}(z)W(z,0)F_{\nu j}(0)W(0,z)|N(P)\rangle$$

  - Choice of indices for $F_{\mu\nu}$ not unique
  - This operator avoids finite mixing under renormalization
    - must subtract vacuum expectation value

$$\mathcal{O} = \frac{1}{2}\sum_{i\neq 3} F_{i3}(x+z\hat{z})W(x+z\hat{z},x)F_{i3}(x) - \sum_{i\neq j\neq 3} F_{ij}(x+z\hat{z})W(x+z\hat{z},x)F_{ij}(x)$$

- Matrix elements extracted from ratio of 2pt- and 3pt- functions
- Ground state from plateau fit

$$\frac{C^{3pt}(t,\tau,0,\vec{P})}{C^{2pt}(t,0,\vec{P})} \overset{0<<\tau<<t}{=} \mathcal{M}(\nu,z^2)$$

Pseudo-PDF Approach (in a nutshell)

- $\mathfrak{M}$ - Euclidean, renormalized in ratio scheme, scale $1/z$
  - want: Minkowski, renormalized in $\overline{\mathrm{MS}}$, scale μ
  - we need matching to the light-cone, scheme conversion and evolution
- Form the double ratio (reduced ITD) with zero-momentum and local matrix elements for renormalization and to reduce higher twist contributions [Orginos et al., Phys.Rev.D 96 (2017) 9, 094503]

$$\mathfrak{M}(\nu, z^2) \equiv \left(\frac{M(\nu, z^2)}{M(\nu, 0)|_{z=0}} \right) \Big/ \left(\frac{M(0, z^2)|_{p=0}}{M(0, 0)|_{p=0,z=0}} \right)$$

- Scale evolution and apply matching kernel on ITD
  - neglect mixing with quark singlet
  - normalize with $\langle x \rangle_g$

$$\mathcal{Q}(\nu, z^2, \mu^2) = \mathfrak{M} + \frac{\alpha_s N_c}{2\pi} \int_0^1 du\, \mathfrak{M}(u\nu, z^2) \left\{ \ln\left(\frac{z^2 \mu^2 e^{2\gamma_E}}{4} \right) B(u) + 4 \left[\frac{u + \ln(\bar{u})}{\bar{u}} \right]_+ + \frac{2}{3} \left[1 - u^3 \right]_+ \right\}$$

- Reconstruct x-dependence (Backus-Gilbert, fitting ansatz, Fourier tansform, etc.)

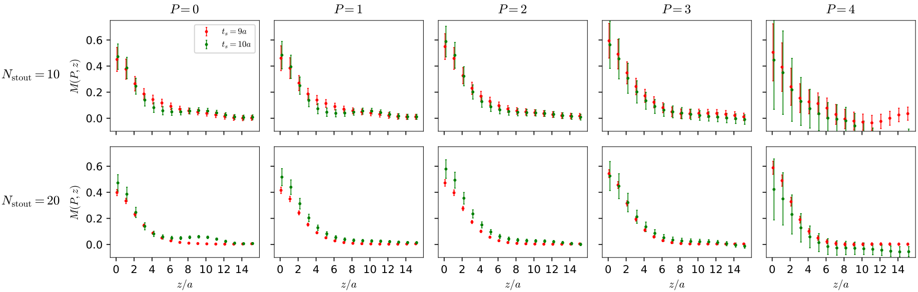
$$\mathcal{Q}(\nu, z^2, \mu^2) = \int_0^1 dx\, cos(x\nu)\, x\, g(x, \mu^2)$$

Lattice Parameters and Statistics

- N_f=2+1+1 ensemble of twisted-mass clover fermions and Iwasaki improved gluons
  - $m_\pi = 260$ MeV
  - $a = 0.093$ fm
  - $L^3 \times T = 32^3 \times 64$
  - $Lm_\pi = 4$
- Stout smearing ($\omega = 0.129$)
  - field-strength tensor: 10, 20 steps
  - Wilson line: 0, 10 steps
- Momentum smearing (optimized value $\xi = 0.6$) used for $P = 2, 3, 4$ [Bali et al, PRD 93, 094515 (2016)]
- Excited states:
  - Numerical results relatively good up to $t_s = 10a$
- Statistics
  - Average over all 6 spatial directions of Wilson line / momentum ($\pm x$, $\pm y$, $\pm z$)
  - Statistics much higher than quark PDFs

$\|\mathbf{P_3}\|$ $[\frac{2\pi}{L}]$	$\|\mathbf{P_3}\|$ [GeV]	$N_{\rm confs}$	$N_{\rm src}$	N_{dir}	Total statistics
0 to 4	0 - 1.67	1,134	200	6	1,360,800

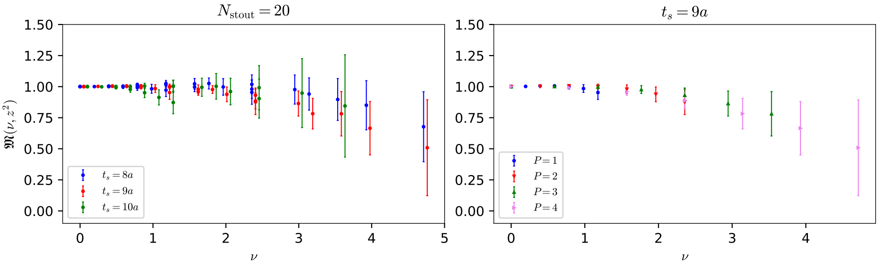
Matrix Elements: Excited States Contamination and Effect of N_{stout}



- Various values of t_s and two stout steps for the gluon operator
- Statistical errors increase with momentum boost and t_s
- MEs have expected behavior (higher boosts decay faster to 0)
- Final results use $N_{\mathrm{stout}}=20$

Double Ratio (Reduced ITD)

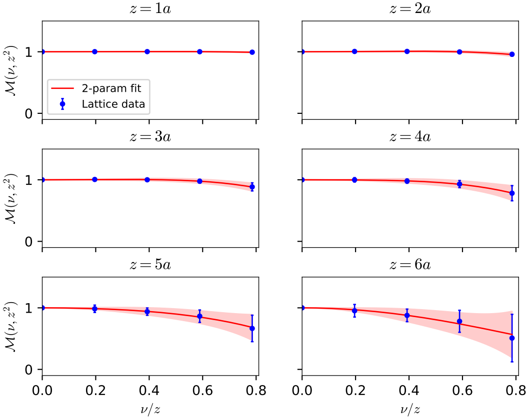
- Double-ratio analysis to narrow down t_s



- Values at different t_s are compatible within uncertainties
- Final results use $t_s = 9a$
  - $z_{max} = 6a = 0.568$ fm motivated by signal
- Lattice data form a smooth function
  - Must interpolate for evolution and matching

Interpolation of Double Ratio

- We interpolate the double-ratio at each z to get a continuous function for the integration
    - interpolation done with linear and 2nd-order polynomial fits

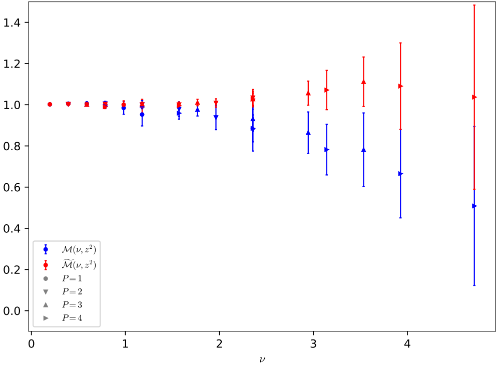


- 2nd-order polynomial fits prove to be the most suitable for evolution and matching
- Choice of fit mostly irrelevant below $z = 4a$

ITD Development

- Apply the evolution kernel to the reduced matrix elements to the scale $\mu = 2$ GeV ahead of final conversion to $\overline{\mathrm{MS}}$ scheme and matching to the light-cone

$$\tilde{\mathfrak{M}}(\nu, z^2, \mu^2) = \mathfrak{M} + \frac{\alpha_s N_c}{2\pi} \int_0^1 du \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) B(u) \mathfrak{M}(u\nu, z^2)$$

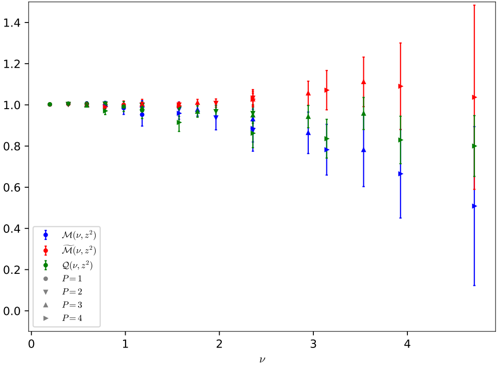


- Data from different (P, z) pairs fall on a universal curve
- We find good agreement up to $z = 6a$

ITD Development

- Apply the matching kernel to convert to $\overline{\mathrm{MS}}$ scheme and match to the light-cone

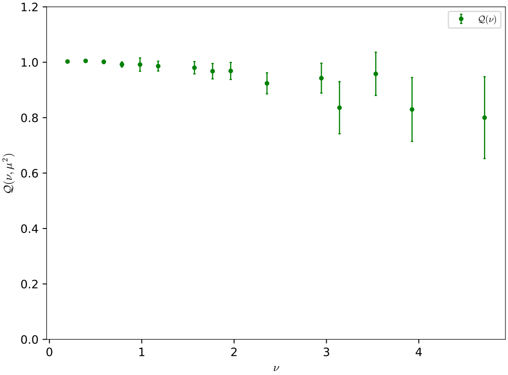
$$\mathcal{Q}(\nu, z^2, \mu^2) = \tilde{\mathfrak{M}}(\nu, z^2, \mu^2) + \frac{\alpha_s N_c}{2\pi} \int_0^1 du\, L(u) \mathfrak{M}(u\nu, z^2)$$



- We continue to find good agreement between common values of Ioffe time from different combinations of momenta and Wilson line lengths
- Matching effects in opposite direction of evolution

ITD Development

- Average over common ν for final pseudo-ITD

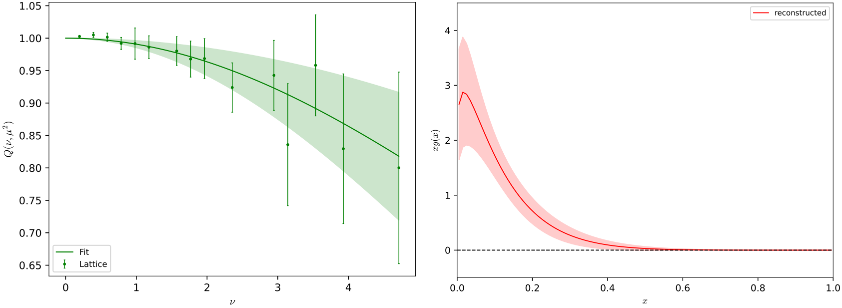


- No information remains regarding initial (P, z) pairs

PDF Reconstruction

- The fit is chosen by the minimization of

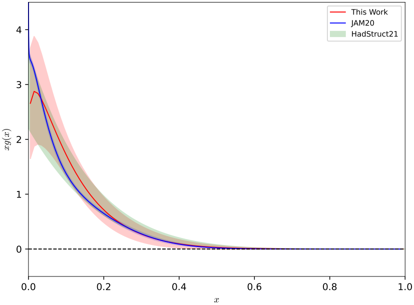
$$\chi^2 = \sum_{\nu=0}^{\nu_{max}} \frac{\left(Q(\nu,\mu^2) - \int_0^1 dx \cos(\nu x)\, N\, x^a (1-x)^b\right)^2}{\sigma_Q^2(\nu,\mu^2)}$$



- PDF is normalized using gluon momentum fraction $\langle x\rangle_g^{\overline{\text{MS}}}(\mu{=}2\,\text{GeV}){=}0.427(92)$ [Alexandrou et al, PRD 101, 094513 (2020)]
- Other reconstruction methods (naive Fourier-transform, Backus-Gilbert method) have proven less suitable [Bhat et al, PRD 103, 034510 (2021)]

Comparison with Other Works

- Comparison with lattice results from HadStruc collaboration [Khan et al, PRD 104, 094516 (2021)]
  - $m_\pi = 358$ MeV, $a = 0.094$, $L^3 \times T = 32^3 \times 64$
- JAM20 global analysis [Moffat et al, PRD 104, 016015 (2021)], $\langle x \rangle_g = 0.40(1)$



- We find agreement between all results
- This work: $\nu_{max} = 4.71$
- HadStruc: $\nu_{max} = 7.07$

Quark Singlet Contribution

- All calculations neglect the mixing between quark-singlet and gluon PDFs (singlet-quark PDFs challenging to calculate)
- A proper analysis requires inclusion of the singlet-quark contributions
- The mixing for non-local operators appears in the matching equations:

$$\mathfrak{M}(\nu, z^2, \mu^2) = \frac{\mathcal{I}_g(\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} - \frac{\alpha_s N_c}{2\pi} \int_0^1 du \, \frac{\mathcal{I}_g(u\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} \left\{ \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4} \right) B_{gg}(u) + 4 \left[\frac{u + \ln(\bar{u})}{\bar{u}} \right]_+ + \frac{2}{3}\left[1 - u^3\right]_+ \right\} - \frac{\alpha_s C_F}{2\pi} \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4} \right) \int_0^1 dw \, \frac{\mathcal{I}_S(w\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} \mathfrak{B}_{gq}(w)$$

- We attempt to extend our calculation by incorporating the single-quark PDFs calculated on the same ensemble

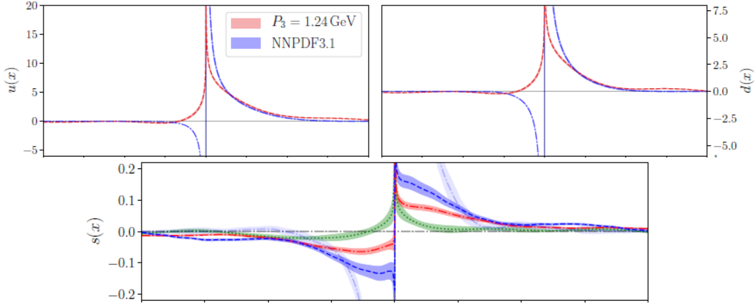
PHYSICAL REVIEW D **104,** 054503 (2021)

Flavor decomposition of the nucleon unpolarized, helicity, and transversity parton distribution functions from lattice QCD simulations

Constantia Alexandrou,[1,2] Martha Constantinou,[3] Kyriakos Hadjiyiannakou,[1,2] Karl Jansen,[4] and Floriano Manigrasso[1,5,6]

Disconnected Matrix Elements

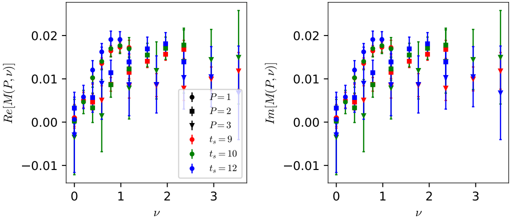
- Published work of [C. Alexandrou et al., PRD 104 (2021) 5, 054503] uses quasi-PDFs method



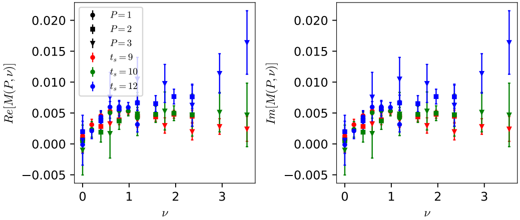
- Here, we combine the light-quark and strange-quark looks with the two point functions produced for the gluon PDF to increase statistics of disconnected diagram

Disconnected Matrix Elements

- Light and strange disconnect matrix elements for $t_s = 9, 10, 12$



light-quark

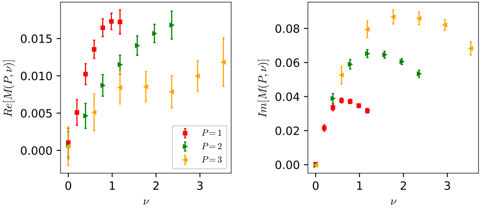


strange-quark

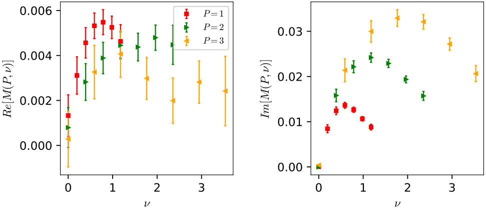
- Large errors: compatibility between data on different t_s values

Disconnected Matrix Elements

- Focus on matrix elements for $t_s = 9$ for the pseudo-PDFs analysis



light-quark

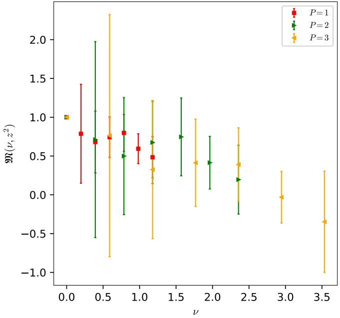


strange-quark

- Dependence on momentum expected in matrix elements (ME)
- tiny strange-quark ME but non-zero signal achieved with advanced methods like hierarchical probing [A. Stathopoulos et al., arXiv:1302.4018]

Strange Quark Double Ratio

- Light-quark PDFs require inclusion of the connected diagram (in progress)
- Strange quark PDF is obtained exclusively from the disconnected diagram
- Reduced-ITD for strange quark contributions:



- Signal quality decays fast with increase of P and z.
- Study of correlations between matrix elements on-going

Summary - Future Work

Summary

- We presented a calculation of gluon PDFs using stout smearing to suppress the statistical noise
- We employ the pseudo-ITD method to overcome difficulties with direct evaluation of renormalization functions
- We study excited-state effects using various $T_{\rm sink}$ values
- Comparison of the unpolarized gluon PDF with other lattice results and global analysis shows compativle results
- First results on the quark singlet PDFs using pseudo-ITD (on-going)

Future work

- Completion of the analysis and elimination of mixing with singlet quark PDFs
- Address sources of systematic effects (excited states, ν dependence of fits)
- Study discretization effects: new ensemble at finer lattice spacing ($a = 0.08$ fm) in production

Acknowledgements

- This research is financial support by the U.S. Department of Energy, Office of Nuclear Physics, Early Career Award under Grant No. DE-SC0020405
- The calculations have been partly carried out on HPC resources supported in part by the National Science Foundation through major research instrumentation grant number 1625061 and by the US Army Research Laboratory under contract number W911NF-16-2-0189
- Computations for this work were carried out in part on facilities of the USQCD Collaboration, which are funded by the Office of Science of the U.S. Department of Energy.

THANK YOU