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TMDWFs and Soft functions at one-loop in LaMET

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LaMET2022

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1. Introduction

- The exploration of underlying structures of hadrons has always been one of the most important frontiers in particle and nuclear physics.

- The TMD wave functions play an important role in theoretical analyses of B meson weak decays[1-4].

- In LaMET, one can construct the directly computable hadron matrix elements with non-local operators, named as quasi-distributions, on the lattice[5-8].

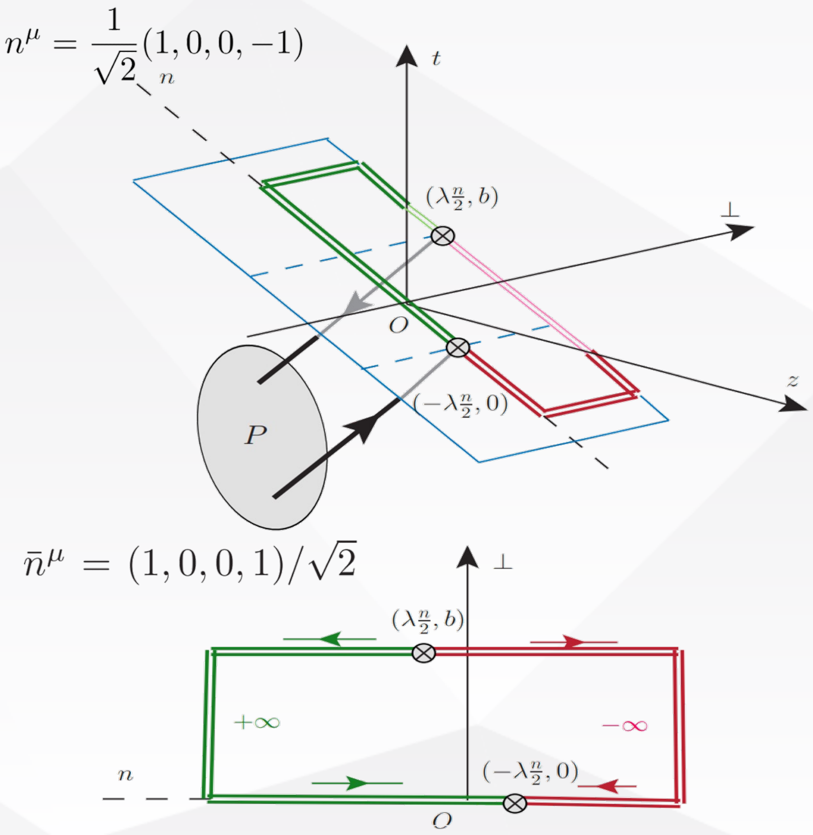
1. H. n. Li and H. L. Yu, Phys. Rev. D 53, 2480-2490 (1996).
2. Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D 63, 054008 (2001), Phys. Lett. B 504, 6-14 (2001) .
3. C. D. Lu, K. Ukai and M. Z. Yang, Phys. Rev. D 63, 074009 (2001).
4. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606, 245- 321 (2001).
5. X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
6. X. Ji, Sci. China Phys. Mech. Astron. 57, 1407-1412 (2014).
7. K. Cichy and M. Constantinou, Adv. High Energy Phys. 2019, 3036904 (2019).
8. X. Ji, Y. S. Liu, Y. Liu, J. H. Zhang and Y. Zhao, Rev. Mod. Phys. 93, 035005 (2021).

$$\psi^{\pm}(x, b_\perp, \mu, \delta^-) = \frac{1}{-i f_\pi P^+} \int \frac{d(\lambda P^+)}{2\pi} e^{-i(x-\frac{1}{2})P^+\lambda} \times \left\langle 0 \left| \overline{\Psi}^{\pm}_n(\lambda n/2 + b)\, \gamma^+ \gamma^5 \Psi^{\pm}_n(-\lambda n/2) \right| P \right\rangle |_{\delta^-},$$

$$P^\mu = (P^z, 0, 0, P^z) \qquad b^\mu = (0, \vec{b}_\perp, 0)$$

$$\langle 0 | \overline{\psi}(0)\, \gamma^\mu \gamma^5 \psi(0) | \pi \rangle = -i f_\pi P^\mu$$

$$\Psi^{\pm}_n(\xi)|_{\delta^-} = \mathcal{P} e^{ig \int_0^{\pm\infty} ds\, n\cdot A(\xi + sn) e^{-\frac{\delta^-}{2}|s|}} \psi(\xi)$$



The Wilson line structure in the TMDWF, where the red line show the Wilson line in the "−" direction, and the green line show the Wilson line in the "+" direction. All the Wilson lines are in the n− ⊥ plane which show in blue, and the two end points of Wilson line are given by the position (+, ⊥).

1. Introduction

$$\psi^{\pm}\left(x, b_{\perp}, \mu, \delta^{-}\right)=\frac{1}{-i f_{\pi} P^{+}} \int \frac{d\left(\lambda P^{+}\right)}{2 \pi} e^{-i\left(x-\frac{1}{2}\right) P^{+} \lambda} \times\left\langle 0\left|\overline{\Psi}_{n}^{\pm}(\lambda n / 2+b) \gamma^{+} \gamma^{5} \Psi_{n}^{\pm}(-\lambda n / 2)\right| P\right\rangle\Big|_{\delta^{-}},$$

Light-front TMDWFs

$$\tilde{\Psi}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right)=\lim_{L \rightarrow \infty} \frac{1}{-i f_{\pi}} \int \frac{d \lambda}{2 \pi} e^{-i\left(x-\frac{1}{2}\right)\left(-P^{z}\right) \lambda} \times \frac{\left\langle 0\left|\overline{\Psi}_{\mp n_{z}}\left(\frac{\lambda n_{z}}{2}+b\right) \gamma^{z} \gamma^{5} \Psi_{\mp n_{z}}\left(-\frac{\lambda n_{z}}{2}\right)\right| P\right\rangle}{\sqrt{Z_{E}\left(2 L, b_{\perp}, \mu\right)}},$$

quasi TMDWFs

2. TMDWF and factorization

$$\psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \delta^-) = \frac{1}{2P^+}\int \frac{d(\lambda P^+)}{2\pi} e^{-i(x-\frac{1}{2})P^+\lambda}$$
$$\times \left\langle 0 \left| \overline{\Psi}^{\pm}_{n}(\lambda n/2 + b)\gamma^+\gamma^5\Psi^{\pm}_{n}(-\lambda n/2) \right| q\bar{q} \right\rangle |_{\delta^-},$$

the quark pair is chosen to have the same J^{PC} with the pion

$$\tilde{\Psi}^{\pm}_{q\bar{q}}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty}\int \frac{d\lambda}{4\pi} e^{-i(x-\frac{1}{2})(-P^z)\lambda}$$
$$\times \frac{\langle 0|\overline{\Psi}_{\mp n_z}(\frac{\lambda n_z}{2}+b)\gamma^z\gamma^5\Psi_{\mp n_z}(-\frac{\lambda n_z}{2})|q\bar{q}\rangle}{\sqrt{Z_E(2L, b_\perp, \mu)}}.$$

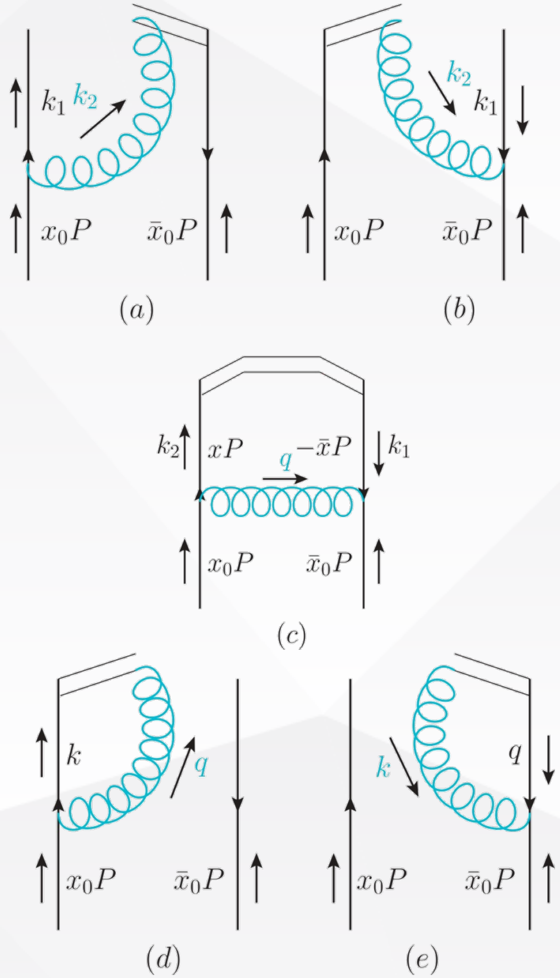


Diagram of wave function. Self-energies of external lines are not shown.

Light-front TMDWFs:

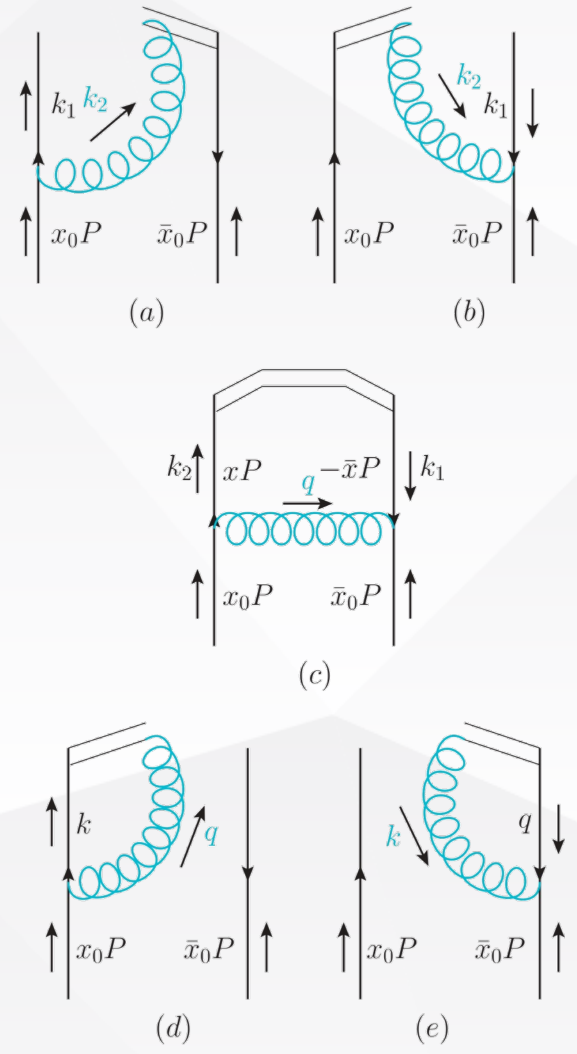
$$\psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \delta^-) = \delta(x - x_0) + \frac{\alpha_s C_F}{2\pi}\Big[f(x, x_0, b_\perp, \mu)\Big]_+$$
$$+ \frac{\alpha_s C_F}{2\pi}\delta(x - x_0)\left[L_b\left(\frac{3}{2} + \ln\frac{-\delta^{-2} \mp i0}{4\bar{x}xP^{+2}}\right) + \frac{1}{2}\right],$$

Rapidity divergence

$$f(x, x_0, b_\perp, \mu) = \left[\left(\frac{x}{x_0(x - x_0)} - \frac{x}{x_0}\right)\left(\frac{1}{\epsilon_{\rm IR}} + L_b\right) + \frac{x}{x_0}\right]\theta(x_0 - x) + \{x \to 1 - x, x_0 \to 1 - x_0\}.$$

$$L_b = \ln\frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}}$$

$$\Psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \zeta) = \lim_{\delta^- \to 0}\frac{\psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \delta^-)}{\sqrt{S^{\pm}(b_\perp, \mu, \delta^- e^{2y_n}, \delta^-)}}$$



X. Ji, et. al., Large-momentum effective theory, Rev. Mod. Phys. 93 (2021) 035005.

2. TMDWF and factorization

$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-) = \frac{1}{N_c}\mathrm{tr}\langle 0|\mathcal{T} W_{\bar n}^{-\dagger}(b_\perp)|_{\delta^+} W_n^{\pm}(b_\perp)|_{\delta^-} \times W_n^{\pm\dagger}(0)|_{\delta^-} W_{\bar n}^{-}(0)|_{\delta^+}|0\rangle .$$

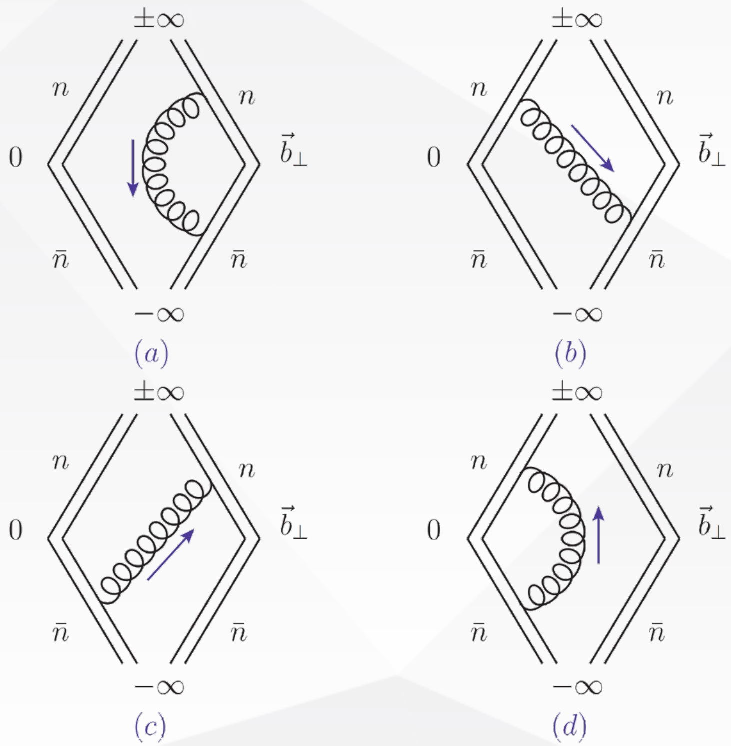
$$\begin{aligned}
S^{(a)\pm} &= S^{(d)\pm} \\
&= -\mu_0^{2\epsilon} i g^2 C_F \int \frac{d^dq}{(2\pi)^d}\frac{\bar n^\mu}{q^- + i\frac{\delta^+}{2}}\frac{n_\mu}{q^+ \pm i\frac{\delta^-}{2}}\frac{1}{q^2+i\epsilon} \\
&= \frac{\alpha_s C_F}{4\pi}\Bigg[-\frac{2}{\epsilon_{\rm UV}^2} + \frac{2}{\epsilon_{\rm UV}}\ln\frac{\mp\delta^-\delta^+ - i0}{2\mu^2} - \ln^2\left(\frac{\mp\delta^-\delta^+ - i0}{2\mu^2}\right) - \frac{\pi^2}{2}\Bigg],
\end{aligned}$$

$$\begin{aligned}
S^{(b)\pm} &= S^{(c)\pm} \\
&= \mu_0^{2\epsilon} i g^2 C_F \int \frac{d^dq}{(2\pi)^d}\frac{\bar n^\mu}{q^- + i\frac{\delta^+}{2}}\frac{n_\mu}{q^+ \pm i\frac{\delta^-}{2}}\frac{e^{-iq\cdot b}}{q^2+i\epsilon} \\
&= \frac{\alpha_s C_F}{4\pi}\Bigg[L_b^2 + 2L_b\ln\frac{\mp\delta^-\delta^+ - i0}{2\mu^2} + \ln^2\left(\frac{\mp\delta^-\delta^+ - i0}{2\mu^2}\right) + \frac{2\pi^2}{3}\Bigg].
\end{aligned}$$

$$\mu = \mu_0 e^{(\ln(4\pi)-\gamma_E)/2}$$

$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-) = 1 + \frac{\alpha_s C_F}{2\pi}\left(L_b^2 + 2L_b\ln\frac{\mp\delta^-\delta^+ - i0}{2\mu^2} + \frac{\pi^2}{6}\right)$$

M.G. Echevarría, et. al., Phys. Lett. B 726 (2013) 795.
M.G. Echevarria , et. al., JHEP 07 (2012) 002.



One-loop diagrams for the soft funtion. Diagram (a)(d) give the virtual diagram, and diagram (b)(c) give the real diagram.

2. TMDWF and factorization

$$\Psi^{\pm}_{\bar{q}q}(x,b_\perp,\mu,\zeta)=\delta(x-x_0)+\frac{\alpha_s C_F}{2\pi}[f(x,x_0,b_\perp,\mu)]_+$$
$$+\frac{\alpha_s C_F}{2\pi}\delta(x-x_0)\left\{-\frac{L_b^2}{2}+L_b\left(\frac{3}{2}+\ln\frac{\mu^2}{\pm\sqrt{\zeta\bar{\zeta}-i0}}\right)+\frac{1}{2}-\frac{\pi^2}{12}\right\},$$

where $\bar{\zeta} = 2(\bar{x}P^+)^2 e^{2y_n}$.

$$f(x,x_0,b_\perp,\mu)=\left[\left(\frac{x}{x_0(x-x_0)}-\frac{x}{x_0}\right)\left(\frac{1}{\epsilon_{\rm IR}}+L_b\right)\right.$$
$$\left.+\frac{x}{x_0}\right]\theta(x_0-x)+\{x\to 1-x, x_0\to 1-x_0\}.$$

X. Ji and Y. Liu, Phys. Rev. D 105 (2022) 076014.

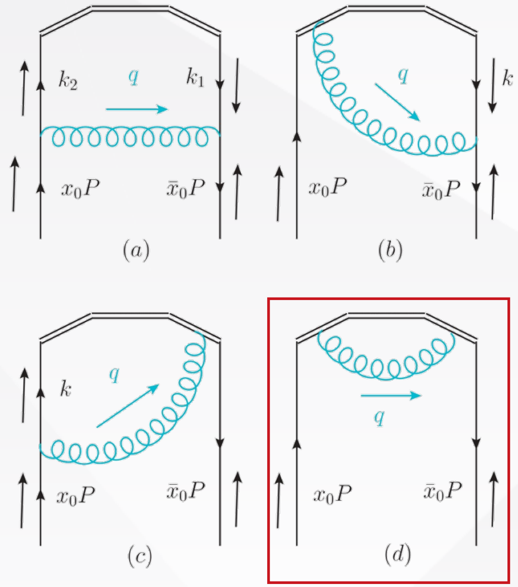
2. TMDWF and factorization

$$\tilde{\Psi}^{\pm}_{q\bar{q}}(x, b_\perp, \mu, \zeta^z) = \delta(x - x_0) + \frac{\alpha_s C_F}{2\pi}[f(x, x_0, b_\perp, \mu)]_+ + \frac{\alpha_s C_F}{2\pi}\delta(x - x_0)A^{\pm}\left(x, \mu, \zeta^z, \bar{\zeta}^z\right),$$

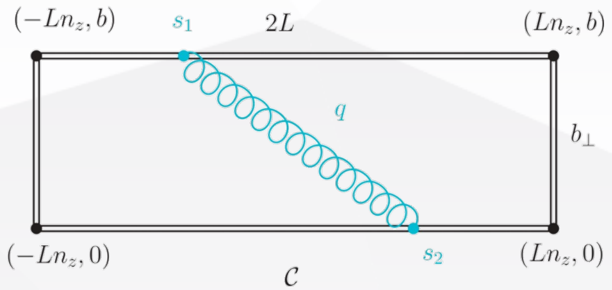
$$\bar{\zeta}^z = (2\bar{x}P \cdot n_z)^2$$

$$A^{\pm}\left(x, \mu, \zeta^z, \bar{\zeta}^z\right) = -\frac{L_b^2}{2} + \frac{5}{2}L_b - \frac{3}{2} - \frac{\pi^2}{2} + \left[-\frac{1}{4}\ln^2\frac{-\zeta^z \pm i0}{\mu^2} + \frac{1}{2}(1 - L_b)\ln\frac{-\zeta^z \pm i0}{\mu^2} + \{\zeta^z \to \bar{\zeta}^z\}\right]$$

X. Ji and Y. Liu, Phys. Rev. D 105 (2022) 076014.



One-loop diagrams for the quasi TMDWF.



One-loop diagrams for the Wilson loop.

2. TMDWF and factorization

$$F(b_\perp, P_1, P_2, \mu) = \frac{\langle P_2 | (\bar{\psi}_a \Gamma \psi_b)(b)(\bar{\psi}_c \Gamma' \psi_d)(0) | P_1 \rangle}{f_\pi^2 P_1 \cdot P_2}$$

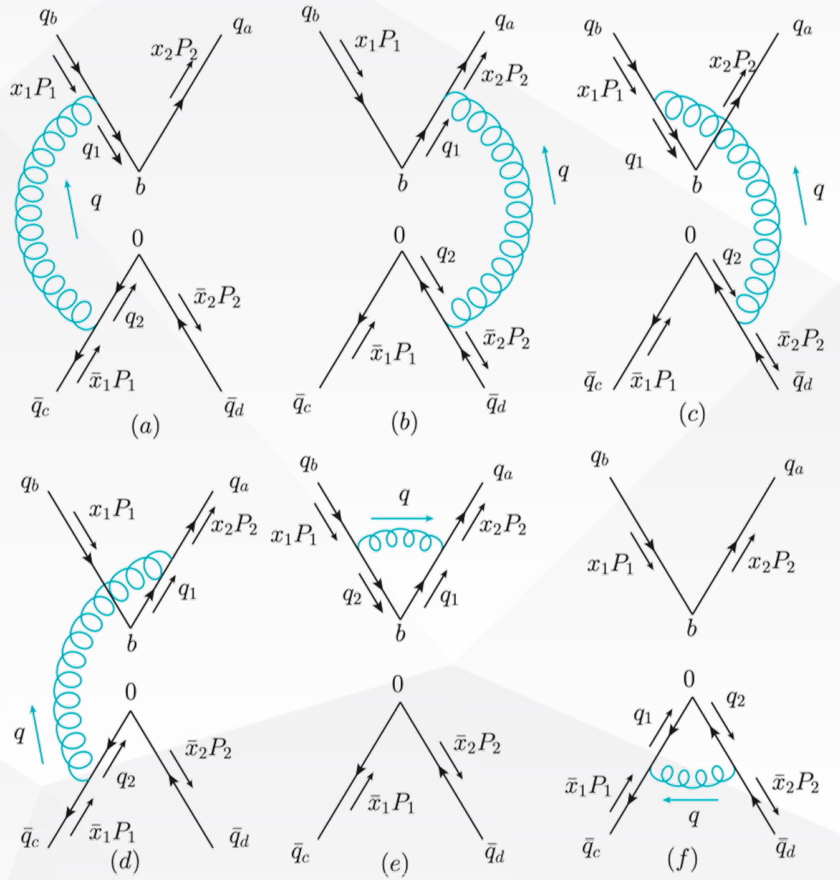
$$\Gamma = \Gamma' = I,\ \gamma_5 \text{ or } \gamma_\perp \text{ and } \gamma_\perp \gamma_5$$

$$\langle 0 | \bar{\psi}(0) \gamma^\mu \gamma^5 \psi(0) | P_1 \rangle = -i f_\pi P_1^\mu$$
$$\langle P_2 | \bar{\psi}(0) \gamma_\mu \gamma^5 \psi(0) | 0 \rangle = i f_\pi P_{2\mu}$$

$$\frac{\langle \bar{q}_d(\bar{x}_2 P_2) q_a(x_2 P_2) | (\bar{\psi}_a \Gamma \psi_b)(b)(\bar{\psi}_c \Gamma \psi_d)(0) | q_b(x_1 P_1) \bar{q}_c(\bar{x}_1 P_1) \rangle}{4 P_1 \cdot P_2}$$

$$\langle 0 | \bar{\psi}_c \gamma^\mu \gamma^5 \psi_b | q_b(x_1 P_1) \bar{q}_c(\bar{x}_1 P_1) \rangle |_{\text{tree}} = 2 P_1^\mu,$$
$$\langle \bar{q}_d(\bar{x}_2 P_2) q_a(x_2 P_2) | \bar{\psi}_a \gamma_\mu \gamma^5 \psi_d | 0 \rangle |_{\text{tree}} = 2 P_{2\mu}.$$

$$P_1^\mu = (P^z, 0, 0, P^z) \text{ and } P_2^\mu = (P^z, 0, 0, -P^z)$$



One-loop Feynman diagrams to the form factor. The quark self-energy corrections are not shown.

2. TMDWF and factorization

$\Gamma = I, \gamma_5$

$$F(b_\perp, P_1, P_2, \mu) = F^0\Big\{1 - \frac{\alpha_s C_F}{2\pi}\Big[L_b^2 + L_b\Big(\ln\frac{4Q^2\bar{Q}^2}{\mu^4} - 3\Big) + \frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} + \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + 1\Big]\Big\}.$$

$\Gamma = \gamma_\perp,\ \gamma_\perp\gamma_5$

$$F(b_\perp, P_1, P_2, \mu) = F^0\Big[1 - \frac{\alpha_s C_F}{2\pi}\Big(7 - \frac{3}{2}\ln\frac{Q^2\bar{Q}^2 b_\perp^4}{4e^{-4\gamma_E}} + \frac{1}{2}\ln^2\frac{Q^2 b_\perp^2}{2e^{-2\gamma_E}} + \frac{1}{2}\ln^2\frac{\bar{Q}^2 b_\perp^2}{2e^{-2\gamma_E}}\Big)\Big].$$

$$F^0 = \begin{cases} \dfrac{1}{4N_c}, & \text{for} \quad \Gamma = I \\ -\dfrac{1}{4N_c}, & \text{for} \quad \Gamma = \gamma_5,\ \gamma_\perp \text{ or } \gamma_\perp\gamma_5. \end{cases}$$

The form factor is an infrared-safe quantity at one-loop order!

2. TMDWF and factorization

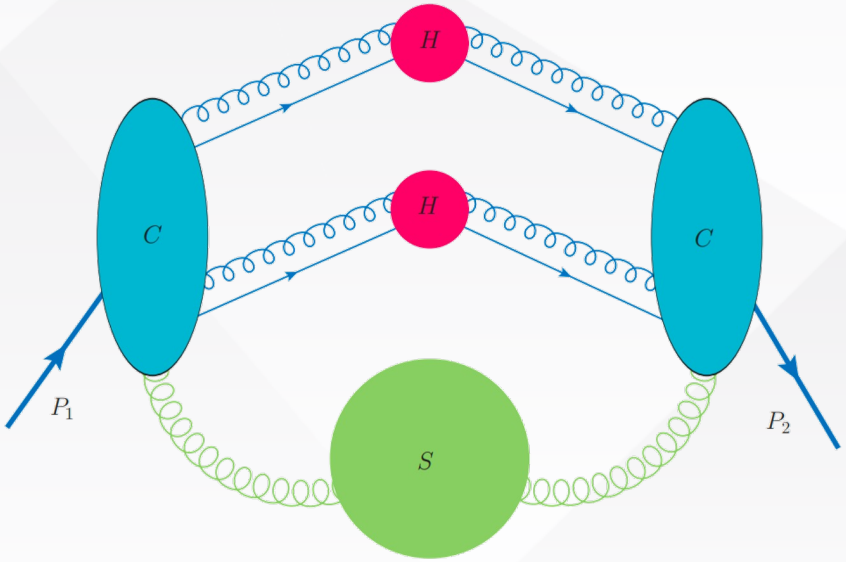
$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 \overset{\text{H}}{H_F}(Q^2, \bar{Q}^2, \mu^2)$$

$$\times \left[\frac{\overset{\text{C}}{\psi^{\pm}_{\bar{q}q}(x_2, b_\perp, \mu, \delta'^{+})}}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^{+}, \delta^{-})}}\right]^{\dagger} \left[\frac{\overset{\text{C}}{\psi^{\pm}_{\bar{q}q}(x_1, b_\perp, \mu, \delta'^{-})}}{\sqrt{S^{\pm}(b_\perp, \mu, \delta^{+}, \delta'^{-})}}\right]$$

$$\times \underset{\text{S}}{\frac{S^{\pm}(b_\perp, \mu, \delta^{+}, \delta^{-})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^{+}, \delta^{-}) S^{\pm}(b_\perp, \mu, \delta^{+}, \delta'^{-})}}}$$

$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 H(x_1, x_2) \overset{\text{S}}{S_r}(b_\perp, \mu)$$

$$\times \underset{\text{C}}{\tilde{\Psi}^{\dagger}_{q\bar{q}}(x_2, b_\perp, \mu, \zeta^z_2)} \underset{\text{C}}{\tilde{\Psi}_{q\bar{q}}(x_1, b_\perp, \mu, \zeta^z_1)}$$



The leading-power reduced diagram for the large-momentum form factor of a meson. Two H denote the two hard cores separated in the transverse space by $b_\perp$, C are collinear sub-diagrams and S denotes the soft sub-diagram.

2. TMDWF and factorization

For $\Gamma = I$ or $\Gamma = \gamma_5$, we have the hard kernel

$$H_F(Q^2, \bar{Q}^2) = H_F^{(0)}\left[1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} - \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + \frac{\pi^2}{6} - 2\right)\right].$$

For $\Gamma = \gamma_\perp$ or $\Gamma = \gamma_\perp\gamma_5$, the hard kernel is calculated as

$$H_F(Q^2, \bar{Q}^2) = H_F^{(0)}\left[1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\frac{4Q^2\bar{Q}^2}{\mu^4} - \frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} - \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + \frac{\pi^2}{6} - 8\right)\right].$$

$$H_F^{(0)} = \begin{cases} \frac{1}{4N_c}, & \Gamma = I \\ -\frac{1}{4N_c}, & \Gamma = \gamma_5,\ \gamma_\perp \text{ or } \gamma_\perp\gamma_5. \end{cases}$$

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2. TMDWF and factorization

For $\Gamma = I$ or $\Gamma = \gamma_5$, the matching kernel is then derived as:

$$H(x_1,x_2) = H^{(0)}\left\{1+\frac{\alpha_s C_F}{8\pi}\left[4\pi^2+8+\ln^2\left(\frac{-\zeta_1^z \pm i0}{\mu^2}\right)+\ln^2\left(\frac{-\bar{\zeta}_1^z \pm i0}{\mu^2}\right)+\ln^2\left(\frac{-\zeta_2^z \mp i0}{\mu^2}\right)\right.\right.$$
$$\left.\left.+\ln^2\left(\frac{-\bar{\zeta}_2^z \mp i0}{\mu^2}\right)-\frac{1}{2}\ln^2\left(\frac{\zeta_1^z\zeta_2^z}{\mu^4}\right)-\frac{1}{2}\ln^2\left(\frac{\bar{\zeta}_1^z\bar{\zeta}_2^z}{\mu^4}\right)-2\ln\frac{\zeta_1^z\zeta_2^z\bar{\zeta}_1^z\bar{\zeta}_2^z}{\mu^8}\right]\right\}$$
$$= H^{(0)}\left\{1+\frac{\alpha_s C_F}{2\pi}\left[2+\pi^2+\frac{1}{2}\ln^2\left(-\frac{x_2}{x_1}\mp i0\right)\right.\right.$$
$$\left.\left.+\frac{1}{2}\ln^2\left(-\frac{\bar{x}_2}{\bar{x}_1}\mp i0\right)-\ln\frac{16x_1x_2\bar{x}_1\bar{x}_2P^{z4}}{\mu^4}\right]\right\}.$$

For $\Gamma = \gamma_\perp$ or $\Gamma = \gamma_\perp\gamma_5$, we have:

$$H(x_1,x_2) = H^{(0)}\left\{1+\frac{\alpha_s C_F}{8\pi}\left[4\pi^2-16+\ln^2\left(\frac{-\zeta_1^z \pm i0}{\mu^2}\right)+\ln^2\left(\frac{-\bar{\zeta}_1^z \pm i0}{\mu^2}\right)+\ln^2\left(\frac{-\zeta_2^z \mp i0}{\mu^2}\right)\right.\right.$$
$$\left.\left.+\ln^2\left(\frac{-\bar{\zeta}_2^z \mp i0}{\mu^2}\right)-\frac{1}{2}\ln^2\left(\frac{\zeta_1^z\zeta_2^z}{\mu^4}\right)-\frac{1}{2}\ln^2\left(\frac{\bar{\zeta}_1^z\bar{\zeta}_2^z}{\mu^4}\right)+\ln\frac{\zeta_1^z\zeta_2^z\bar{\zeta}_1^z\bar{\zeta}_2^z}{\mu^8}\right]\right\}$$
$$= H^{(0)}\left\{1+\frac{\alpha_s C_F}{2\pi}\left[\pi^2-4+\frac{1}{2}\ln^2\left(-\frac{x_2}{x_1}\mp i0\right)\right.\right.$$
$$\left.\left.+\frac{1}{2}\ln^2\left(-\frac{\bar{x}_2}{\bar{x}_1}\mp i0\right)+\frac{1}{2}\ln\frac{16x_1\bar{x}_1x_2\bar{x}_2P^{z4}}{\mu^4}\right]\right\}.$$

2. TMDWF and factorization

$$H(x_1, x_2) = \frac{H_F(Q^2, \bar{Q}^2, \mu^2)}{\left[H_1^{\pm}\left(\zeta_2^z, \bar{\zeta}_2^z, \mu\right)\right]^{\dagger}\left[H_1^{\pm}\left(\zeta_1^z, \bar{\zeta}_1^z, \mu\right)\right]},$$

where $\zeta_i^z = (2x_i P\cdot n_z)^2$, $\bar{\zeta}_i^z = (2\bar{x}_i P\cdot n_z)^2$, and the condition $\zeta_1^z\zeta_2^z = \zeta_1\zeta_2$ is used.

$$H_1^{\pm}\left(\zeta^z, \bar{\zeta}^z, \mu\right) = 1 + \frac{\alpha_s C_F}{2\pi}\left\{-\frac{5\pi^2}{12} - 2 + \frac{1}{2}\left[\ln\frac{-\zeta^z \pm i0}{\mu^2} - \frac{1}{2}\ln^2\frac{-\zeta^z \pm i0}{\mu^2} + \{\zeta^z \to \bar{\zeta}^z\}\right]\right\}.$$

$$\widetilde{\Psi}^{\pm}_{\bar{q}q}\left(x, b_\perp, \mu, \zeta^z\right) S_r^{\frac{1}{2}}\left(b_\perp, \mu\right) = H_1^{\pm}\left(\zeta^z, \bar{\zeta}^z, \mu\right) \times e^{\frac{1}{2}\ln\frac{\mp\zeta^z + i0}{\zeta} K_1(b_\perp, \mu)} \Psi^{\pm}_{\bar{q}q}\left(x, b_\perp, \mu, \zeta\right)$$

Matching

Min-Huan Chu's talk today (session I)

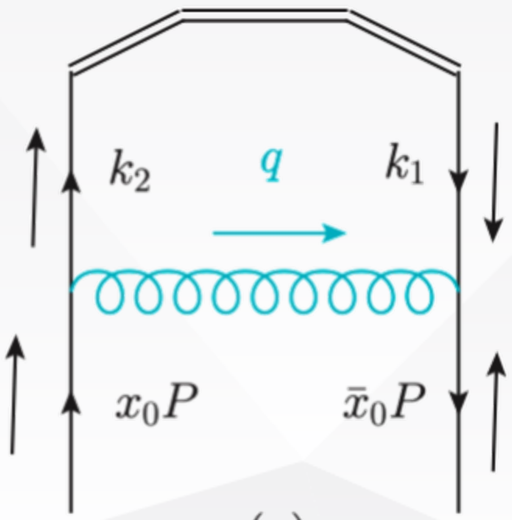
3. Expansion by regions

$$q^\mu = (q^+, q^-, q_\perp)$$

Expansion by regions:

- ✓ **Hard:** $q^\mu \sim (Q, Q, Q)$
- ✓ **Collinear:** $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ **Soft:** $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

$$\tilde{\psi}_{\bar{q}q}^{\pm(1,a)} = \mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l \gamma^5 v) \int \frac{d^d q}{(2\pi)^4} \frac{\frac{D-2}{P^z}[(\bar{x}_0 P + q)^2 q^l - (x_0 P - q)^2 q^l - P^l q^2]}{[(\bar{x}_0 P + q)^2 + i\epsilon][(x_0 P - q)^2 + i\epsilon](q^2 + i\epsilon)} e^{-iq\cdot b_\perp} \delta\left[(x - x_0)P^z + q^z\right]$$



(a)

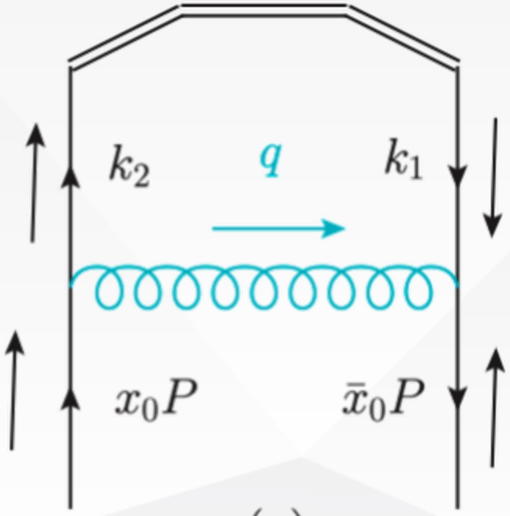
Expansion by regions:

- ✓ **Hard:** $q^\mu \sim (Q, Q, Q)$
- ✓ Collinear: $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ Soft: $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

$$\tilde{\psi}^{\pm(1,a)}_{\bar{q}q} = \mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l \gamma^5 v) \int \frac{d^d q}{(2\pi)^4} \frac{\frac{D-2}{P^z}[(\bar{x}_0 P + q)^2 q^l - (x_0 P - q)^2 q^l - P^l q^2]}{[(\bar{x}_0 P + q)^2 + i\epsilon][(x_0 P - q)^2 + i\epsilon](q^2 + i\epsilon)} e^{-iq\cdot b_\perp} \delta\left[(x - x_0)P^z + q^z\right].$$

Highly Oscillation

$$\Lambda_{\rm QCD} \ll 1/b_\perp \ll P^z$$



(a)

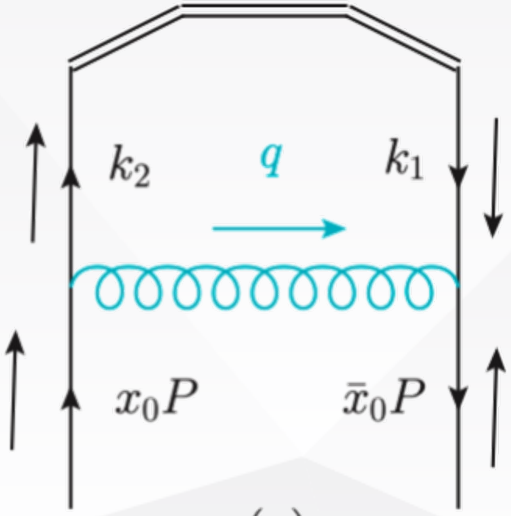
3. Expansion by regions

Expansion by regions:

- ✓ Hard: $q^{\mu} \sim (Q, Q, Q)$
- ✓ Collinear: $q^{\mu} \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ Soft: $q^{\mu} \sim (\Lambda, \Lambda, \Lambda)$

$$\tilde{\psi}_{\bar{q}q}^{\pm(1,a)} = \mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l \gamma^5 v) \int \frac{d^d q}{(2\pi)^4} \frac{\frac{D-2}{P^z}[(\bar{x}_0 P + q)^2 q^l - (x_0 P - q)^2 q^l - P^l q^2]}{[(\bar{x}_0 P + q)^2 + i\epsilon][(x_0 P - q)^2 + i\epsilon](q^2 + i\epsilon)} e^{-iq\cdot b_\perp} \delta\left[(x - x_0)P^z + q^z\right].$$

$$\frac{\Lambda^4 * \Lambda^2}{\Lambda * \Lambda * \Lambda^2} \sim \Lambda^2$$, Power suppress!



(a)

Expansion by regions:

✓ Hard: $q^\mu \sim (Q, Q, Q)$

✓ Collinear: $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$

✓ Soft: $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

$$\frac{\Lambda^4 * \Lambda^2}{\Lambda^2 * \Lambda^2 * \Lambda^2} \sim 1$$

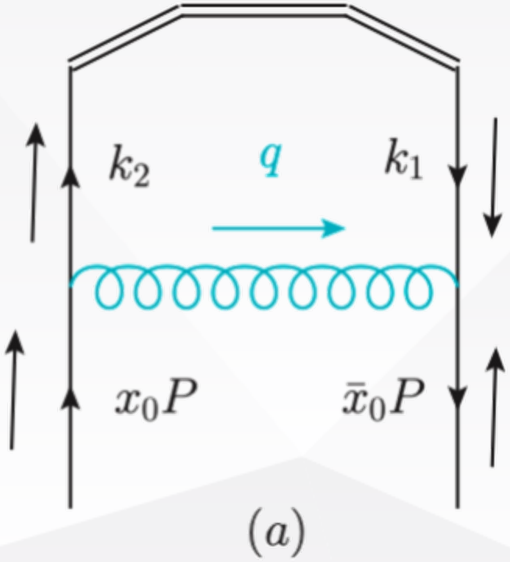
Leading power!

$$\tilde{\psi}^{\pm(1,a)}_{\bar{q}q} = \mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l \gamma^5 v) \int \frac{d^d q}{(2\pi)^4} \frac{\frac{D-2}{P^z}[(\bar{x}_0 P + q)^2 q^l - (x_0 P - q)^2 q^l - P^l q^2]}{[(\bar{x}_0 P + q)^2 + i\epsilon][(x_0 P - q)^2 + i\epsilon](q^2 + i\epsilon)} e^{-iq\cdot b_\perp} \delta\left[(x - x_0)P^z + q^z\right].$$

⬇

$$\mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l \gamma^5 v) \int \frac{d^d q}{(2\pi)^d} \frac{D-2}{P^z} \frac{-P^l q_\perp^2}{[(\bar{x}_0 P + q)^2 + i\epsilon][(x_0 P - q)^2 + i\epsilon](q^2 + i\epsilon)} e^{-iq\cdot b} \sqrt{2}\delta\left[(x - x_0)P^+ + q^+\right]$$

Light-front result



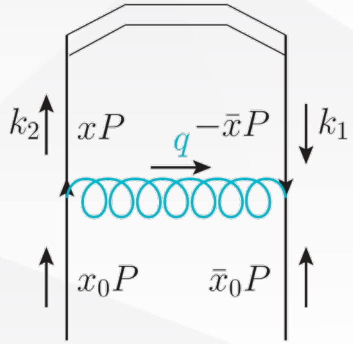
Collinear mode in quasi TMDWFs = TMDWFs

3. Expansion by regions

$$F^{(1,a)} = \mu_0^{2\epsilon}\frac{ig^2C_F}{4N_cP_1\cdot P_2}\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b}$$
$$\times\frac{1}{[(q+x_1P_1)^2+i\epsilon][(q-\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)}$$
$$\times c_\Gamma\bar{u}_a(x_2P_2)\gamma_\nu\gamma_5 v_d(\bar{x}_2P_2)$$
$$\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}-\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(\not{q}+x_1\not{P}_1)\gamma_\mu u_b(x_1P_1)$$
$$= \mu_0^{2\epsilon}\frac{ig^2C_F}{4P_1\cdot P_2}\int\frac{d^dq}{(2\pi)^d}e^{iq\cdot b}$$
$$\times\frac{1}{[(q-x_1P_1)^2+i\epsilon][(q+\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)}$$
$$\times(-H_F^{(0)})\bar{u}_a(x_2P_2)\gamma_\nu\gamma_5 v_d(\bar{x}_2P_2)$$
$$\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}+\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(\not{q}-x_1\not{P}_1)\gamma_\mu u_b(x_1P_1).$$

Fierz transformation

$$F^{(1,a)} = H_F^{(0)}\mu_0^{2\epsilon}\frac{ig^2C_F}{2P_1^+}\int\frac{d^dq}{(2\pi)^d}e^{iq\cdot b}$$
$$\times\frac{1}{(q-x_1P_1)^2+i\epsilon][(q+\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)}$$
$$\times$$
$$\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}+\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(x_1\not{P}_1-\not{q})\gamma_\mu u_b(x_1P_1)$$
$$= H_F^{(0)}\times\int dx\psi_{\bar{q}q}^{(1,c)}(x).$$



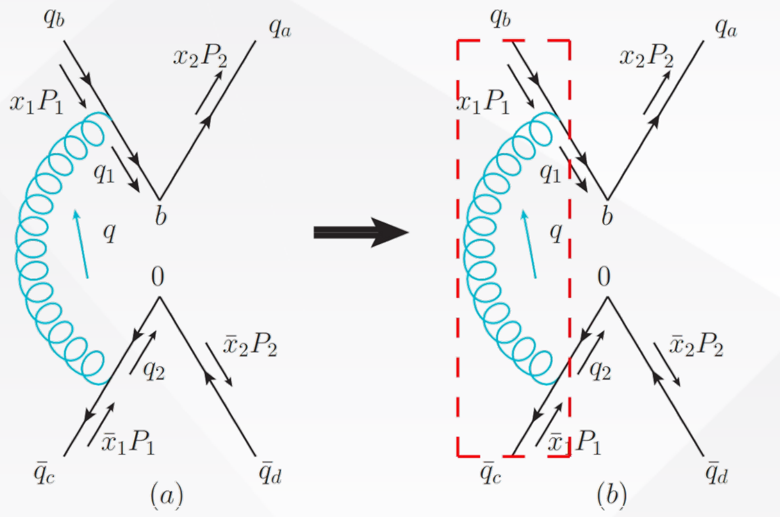


FIG. 6: Factorization of form factor shown in Fig. 5 (a). Only collinear mode contributes in this diagram, while both hard and soft contributions are power suppressed.

$$F^{(1,a)} = H_F^{(0)}\otimes\psi_{\bar{q}q}^{(1,c)}\otimes(\psi_{\bar{q}q}^{(0)})^\dagger\times\left(\frac{1}{S}\right)^{(0)}$$

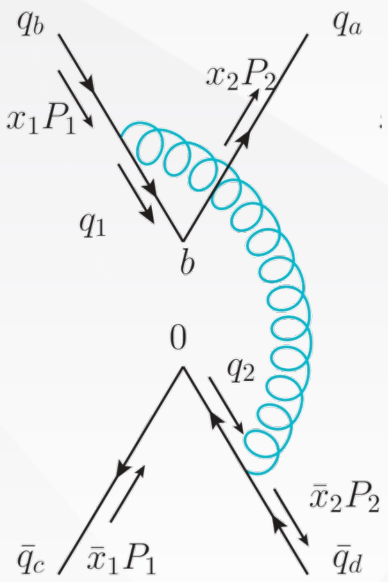
Hard Collinear Soft

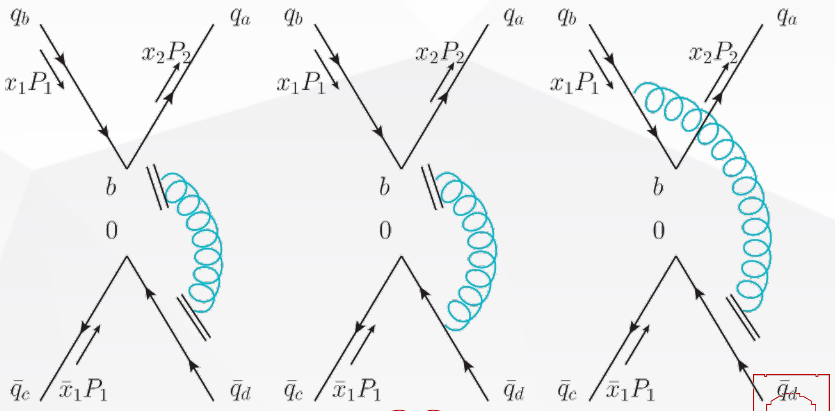
$$F^{(1,b)} = H_F^{(0)}\otimes\psi_{\bar{q}q}^{(0)}\otimes(\psi_{\bar{q}q}^{(1,c)})^\dagger\times\left(\frac{1}{S}\right)^{(0)}$$

3. Expansion by regions

$$F^{(1,c)}|_{soft} = H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(1,b)}$$

$$F^{(1,c)}|_{collinear} = \begin{cases} H_F^{(0)} \otimes \psi_{\bar{q}q}^{(1,a)}|_{\text{collinear}} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)} & q \,//\, P_1 \\ H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(1,a)})^\dagger|_{\text{collinear}} \times \left(\frac{1}{S}\right)^{(0)} & q \,//\, P_2 \end{cases}$$





$$H_F^{(1,e)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)}$$

$$+H_F^{(0)} \otimes \psi_{\bar{q}q}^{(1,d)} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)}$$

$$+H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(1,d)})^\dagger \times \left(\frac{1}{S}\right)^{(0)}$$

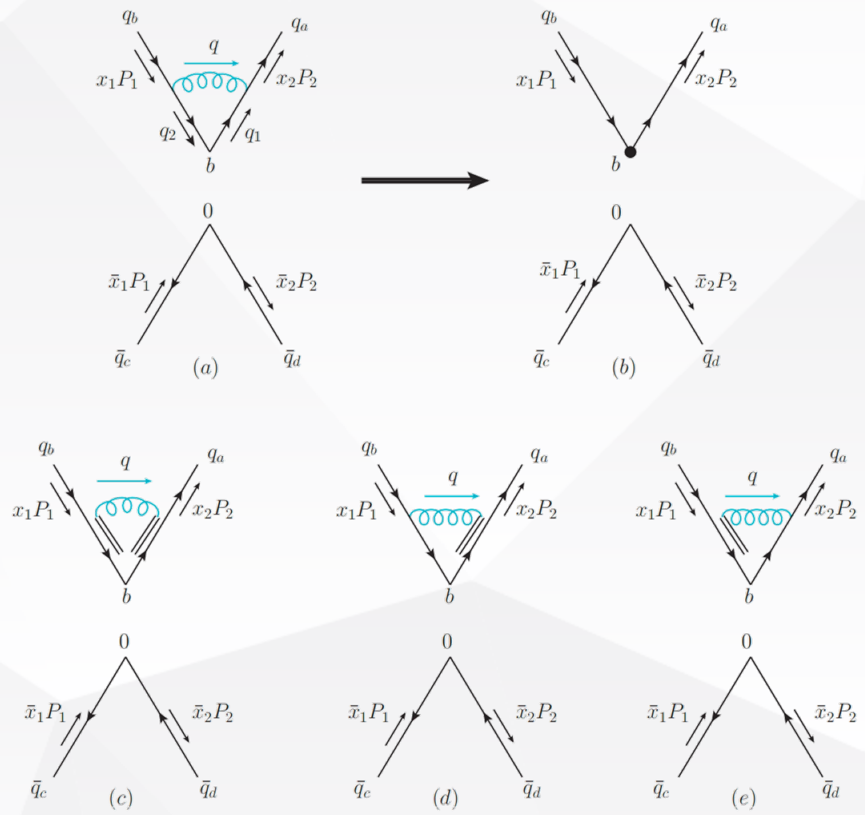
$$+H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(1,d)}$$

$$H_F(Q^2, \bar{Q}^2) = H^{Sud}(-Q^2) H^{Sud}(-\bar{Q}^2)$$

$$Q^2 = x_1 x_2 P_1 \cdot P_2$$

$$\bar{Q}^2 = \bar{x}_1 \bar{x}_2 P_1 \cdot P_2$$

J. Collins and T.C. Rogers, Phys. Rev. D 96 (2017) 054011.



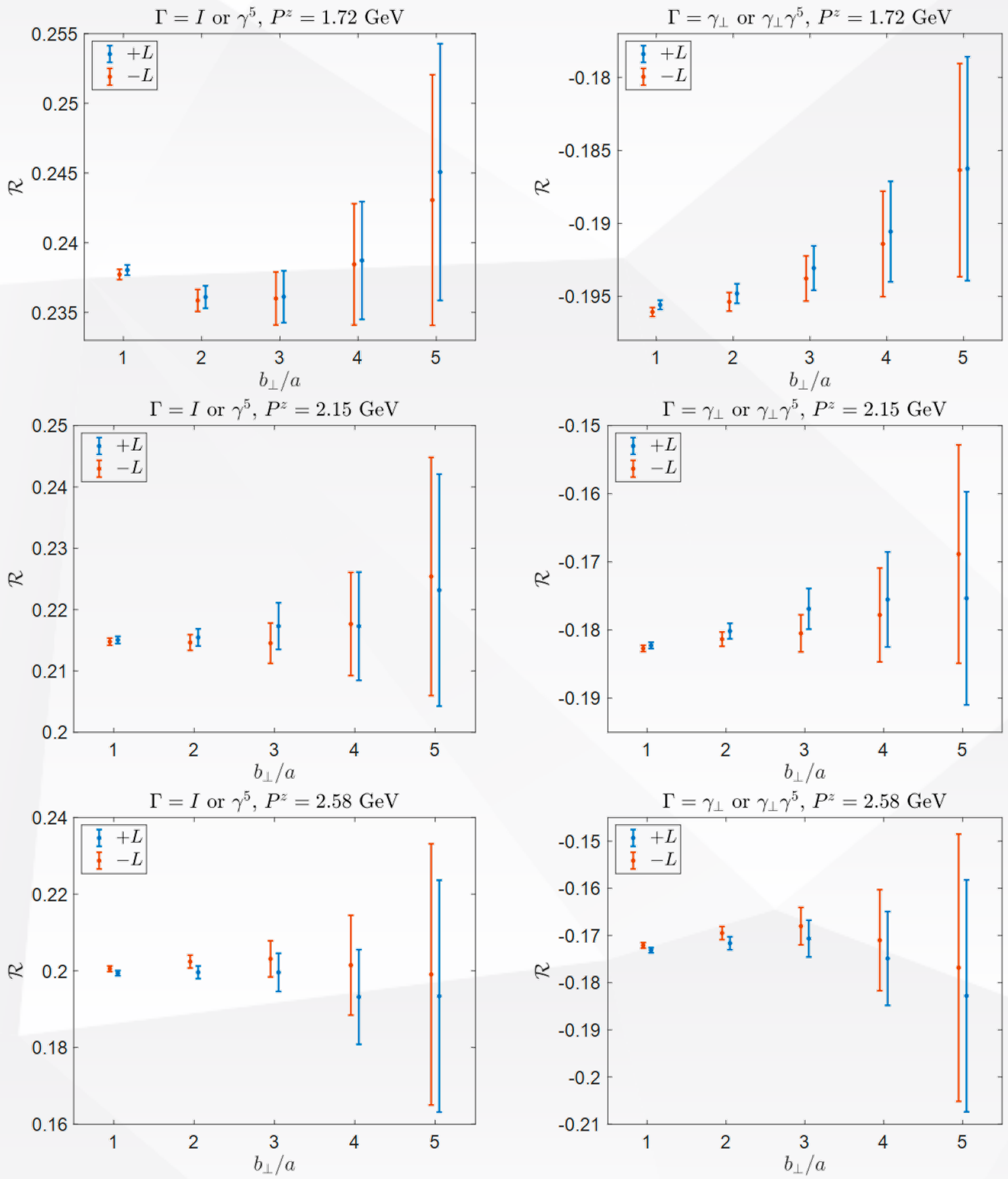
4. Lattice results

$$S_r(b_\perp, \mu) = \frac{F(b_\perp, P_1, P_2, \mu)}{\mathcal{H}},$$

where the denominator term is

$$\mathcal{H} = \int dx_1 dx_2 H(x_1, x_2) \tilde{\Psi}^\dagger(x_2, b_\perp, P^z, \zeta_2^z) \tilde{\Psi}(x_1, b_\perp, P^z, \zeta_1^z).$$

4. Lattice results



The lattice data on quasi-TMDWFs from LPC.

LPC collaboration, Phys. Rev. D 106 (2022) 034509.

$$S_r(b_\perp, \mu) = \frac{F(b_\perp, P_1, P_2, \mu)}{\mathcal{H}}$$

$a = 0.12\,\mathrm{fm}$

$$\mathcal{H} = \int dx_1 dx_2 H(x_1, x_2) \times \tilde{\Psi}^\dagger(x_2, b_\perp, P^z, \zeta_2^z)\tilde{\Psi}(x_1, b_\perp, P^z, \zeta_1^z)$$

$$\mathcal{R} = \frac{\mathcal{H}_1 - \mathcal{H}_0}{\mathcal{H}_0}$$

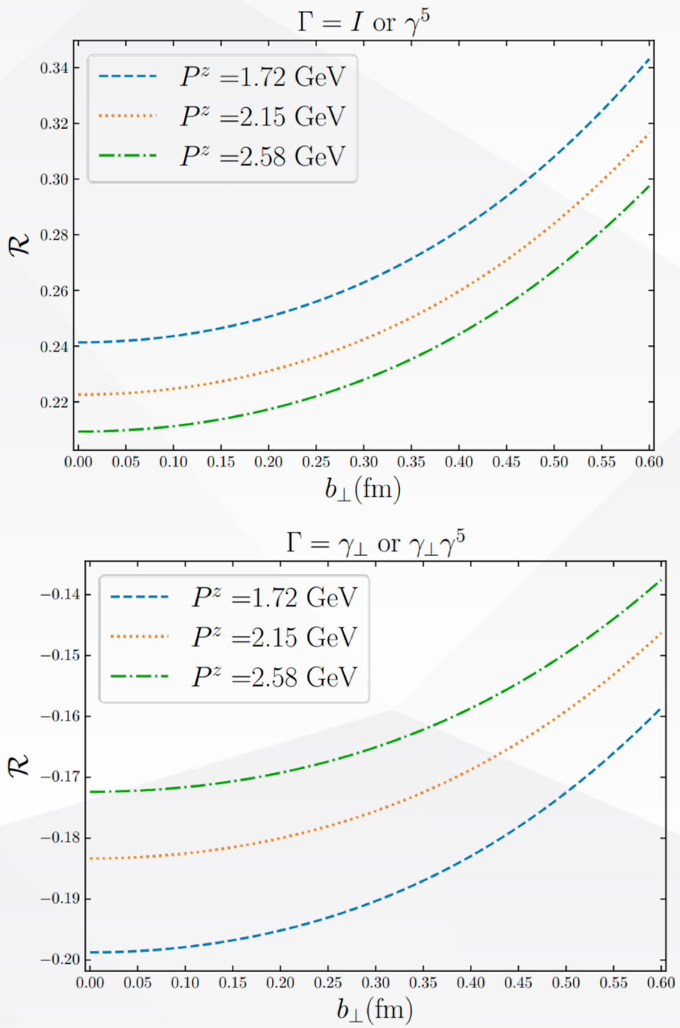
4. Lattice results

As a comparison, we adopt a phenomenological model for quasi-TMDWF $\tilde{\Psi}'(x, b_\perp)$

$$\tilde{\Psi}'(x,b_\perp) = 6x(1-x)\left[1+\frac{3a_2^\pi}{2}\left(5(2x-1)^2-1\right)\right]\exp\left[-\frac{x(1-x)b_\perp^2}{\alpha^2}\right],$$

where the longitudinal and transverse distributions are entangled. We choose $\alpha = 0.197\,\text{fm}$, and the Gegenbauer moments $a_2^\pi = 0.25$

C.-D. Lu, et. al., Phys. Rev. D 75 (2007) 094020.



a phenomenological model for quasi-TMDWFs

5.Summary

1. In LaMET, the TMDWFs and soft functions can be extracted from the simulation of a four-quark form factor.

2. The one-loop TMD factorization of the form factor can be proofed in expansion by regions approach.

3. The perturbative corrections of soft functions depend on the operator to define the form factor, but are less sensitive to the transverse separation.

4. These results will be helpful to precisely extract the soft functions and TMD wave functions from the first-principle in future.

Thanks for your attention!