Renormalization of quasi transverse-momentum-dependent parton distribution function (quasi TMD PDF) on the lattice

Speaker: Kuan Zhang

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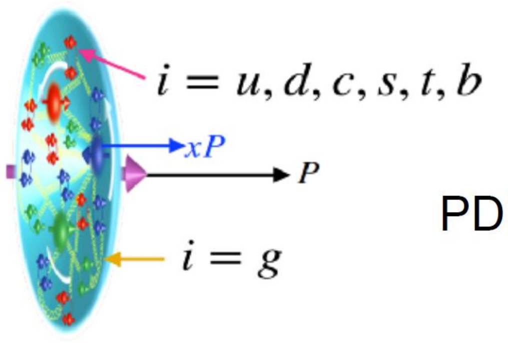
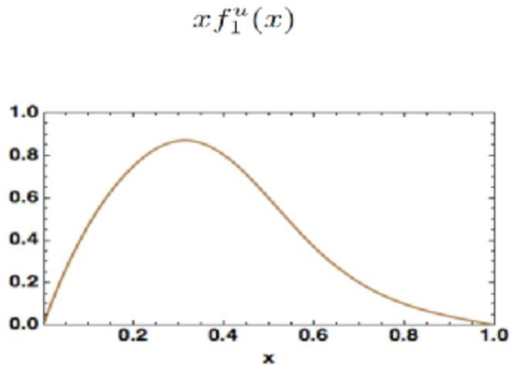
LaMET2022 12.01-12.03

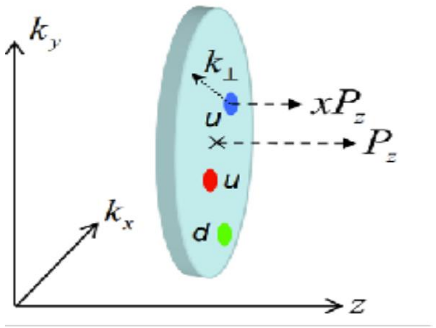
K. Zhang, et.al, LPC, PRL119(2022)082002

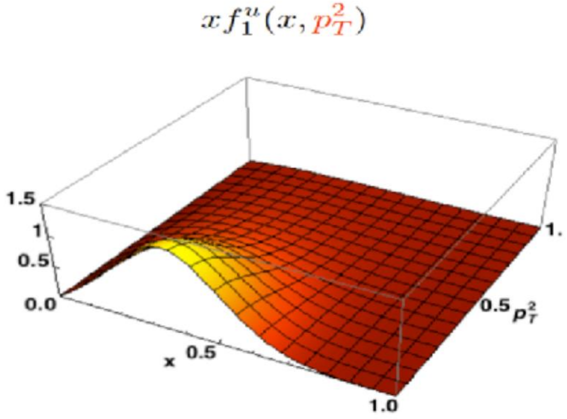
Motivation



PDF







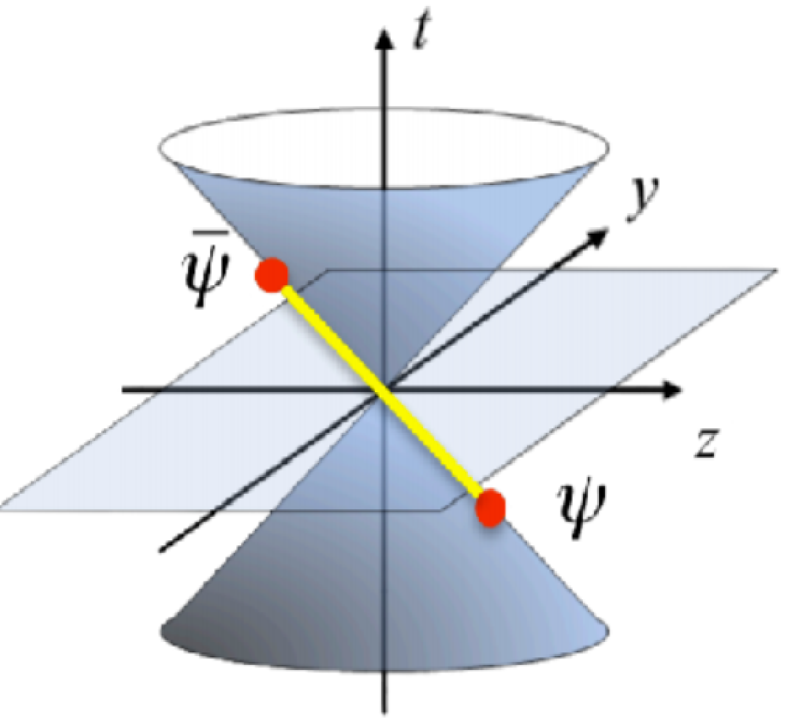
TMD

A. Bacchetta, F. Conti, M. Radici, PRD78 (2008) 074010

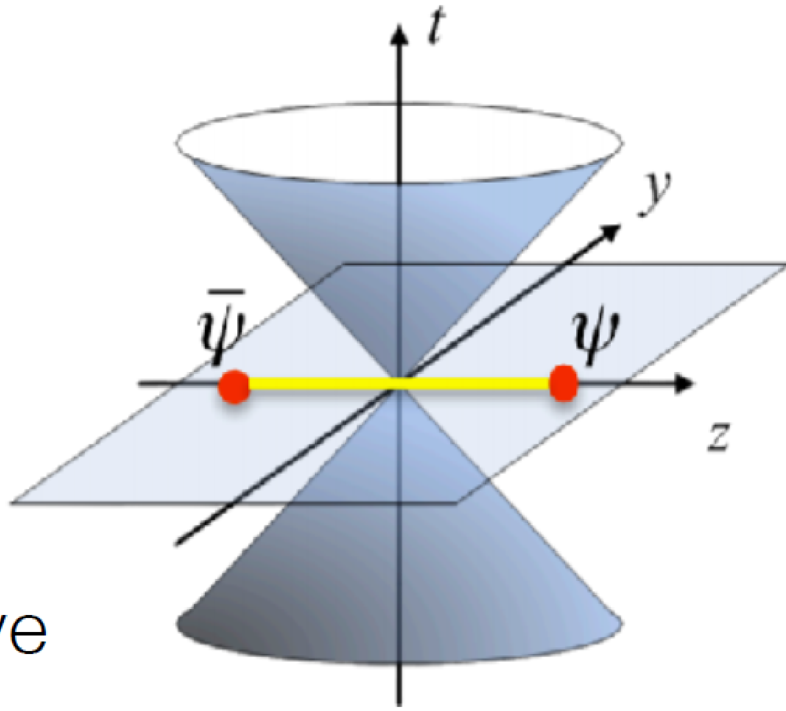
- TMDPDF describes the hadron structure in the transverse direction, and is much less known.

- How to determine it from first principle?

Large Momentum effect theory (LaMET)



Lorentz boost and perturbative matching



Light-cone PDF : Cannot be calculated on the lattice

Quasi-PDF : Directly calculable on the lattice

X. Ji, PRL110, 262002(2013)

Outline

- 1、 Renormalization of quasi PDF operator in RI/MOM scheme
- 2、 Renormalization of quasi TMD-PDF operator
  2.1 RI/MOM scheme
  2.2 Wilson loop + Short distance hadron matrix element (SDR)
- 3、 Numerical details
- 4、 Summary

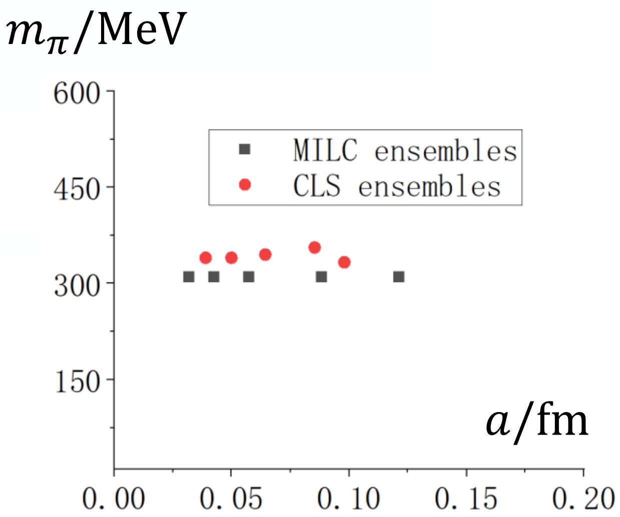
1、Quasi-PDF operator

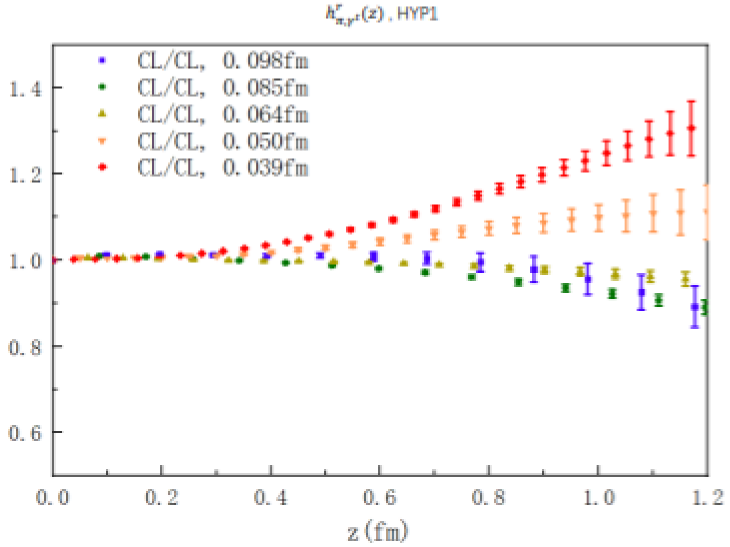
$$O_{\gamma_t}(z) = \bar{q}(0)\gamma_t U_z(0,z)q(z)$$

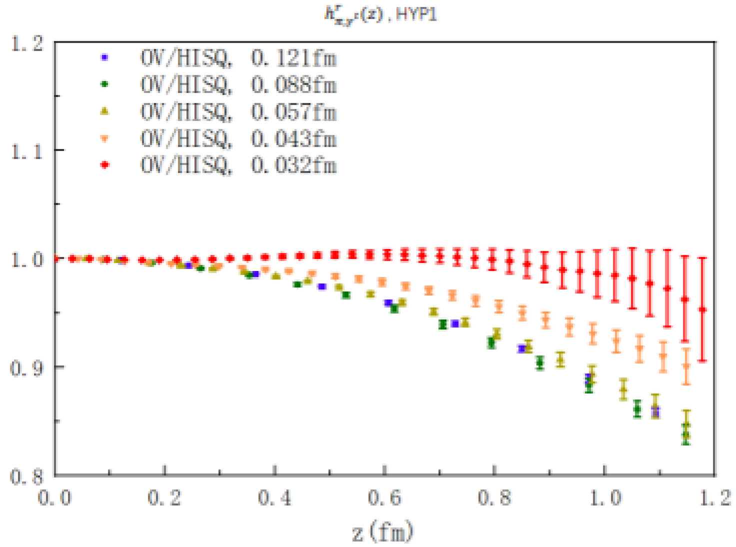
quasi-PDF operator

$$\langle \pi | O_{\gamma_t}(z) | \pi \rangle^{\mathrm{RI/MOM}} \equiv \frac{\langle \pi | O_{\gamma_t}(z) | \pi \rangle}{\langle q | O_{\gamma_t}(z) | q \rangle}$$

RI/MOM renormalization







G. Martinelli, et.al, NPB445, 81(1995)

K. Zhang, et.al, χQCD, PRD104(2021)074501

Hybrid-renormalization

X. Ji. et.al. , NPB964(2021)115311

Self-renormalization

Y.-K. Huo. et.al. ,LPC, NPB969(2021)115443

……

2、 Quasi TMD-PDF on lattice

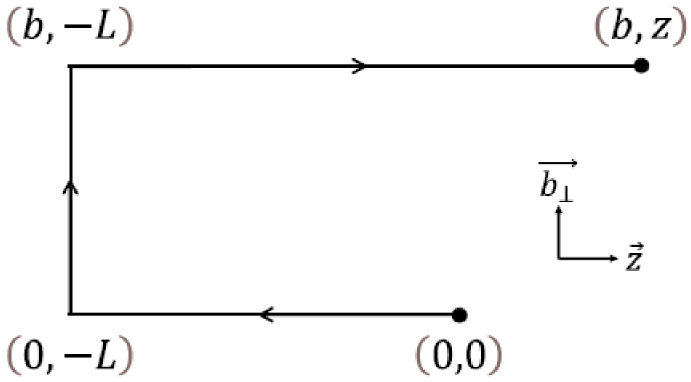
- Matrix element

$$\tilde{h}_{\chi,\gamma_t}(b,z,L,P_z;1/a) = \langle \chi(P_z)|O_{\gamma_t}(b,z,L)|\chi(P_z)\rangle,$$

$$O_\Gamma(b,z,L) \equiv \bar{\psi}(\vec{0}_\perp,0)\Gamma\mathcal{W}(b,z,L)\psi(\vec{b}_\perp,z)$$

$$h_{\chi,\gamma_t}(b,z,P_z;1/a) = \lim_{L\to\infty} \frac{\tilde{h}_{\chi,\gamma_t}(b,z,L,P_z;1/a)}{\sqrt{Z_E(b,2L+z;1/a)}}$$

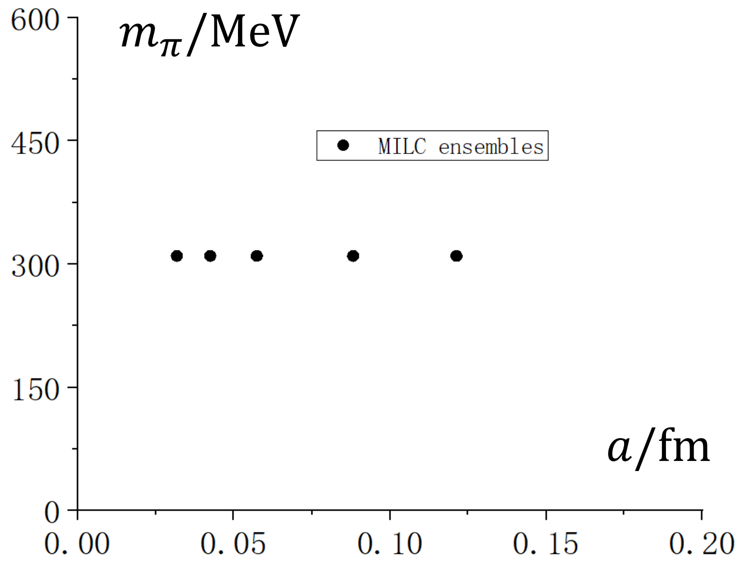
- Staple-shaped link



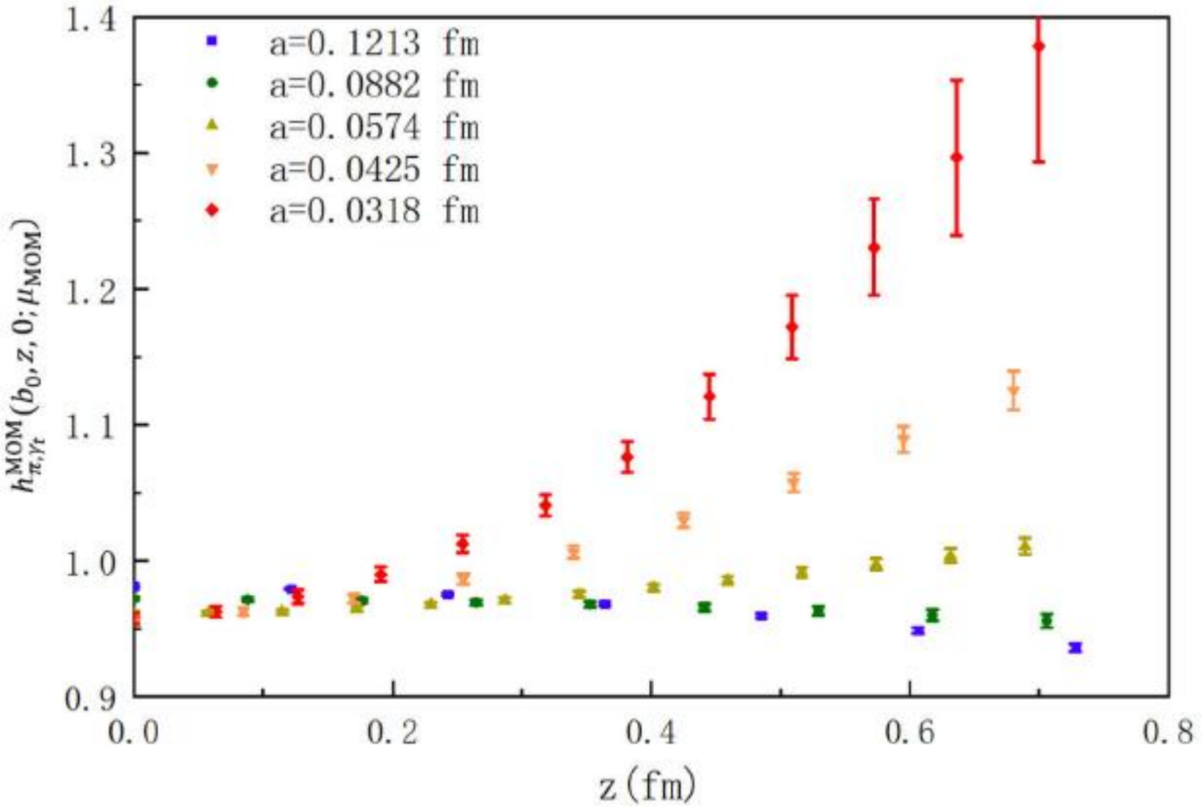
X. Ji, Y. Liu, Y. S. Liu, J.-H. Zhang, and Y. Zhao, Rev. Mod.Phys. 93, 035005 (2021).
X. Ji, Y. Liu, and Y.-S. Liu, Nucl. Phys. B955, 115054 (2020).
M. A. Ebert, I.W. Stewart, and Y. Zhao, J. High Energy Phys. 09 (2019) 037.

Lattice setup

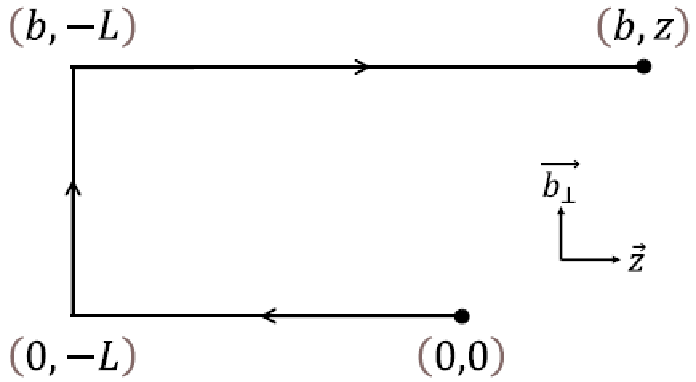
- $\langle\pi(0)|O_{\gamma_t}(b,z)|\pi(0)\rangle$ pion ME (matrix element) in rest frame wall source
- $\langle q(p)|O_{\gamma_t}(b,z)|q(p)\rangle$ $(p_x, p_y, p_z, p_t) = \frac{2\pi}{La}(0,5,0,5)$ $|p|\sim 3GeV$
- 1-step HYP smearing on staple link
- Clover & overlap valence quark



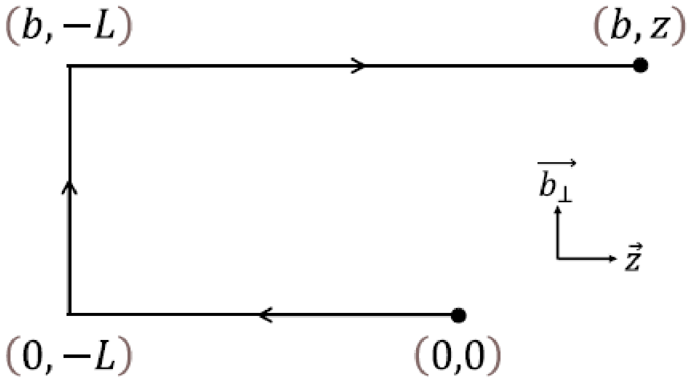
2.1 Staple link ME renormalized in RI/MOM scheme

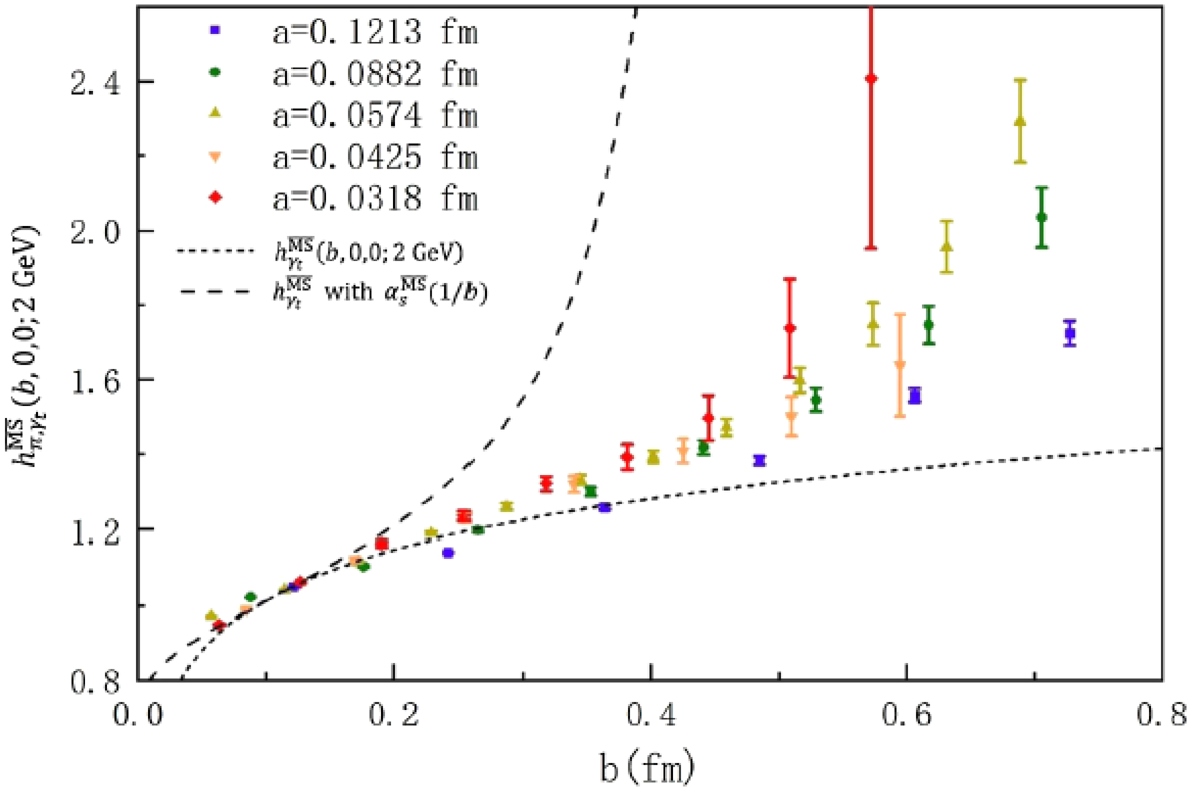


$$
\begin{aligned}
h^{\rm MOM}_{\chi,\Gamma}&(b,z,P_z;p)\\
&=\sum_{\Gamma'}[Z^{\rm MOM}(b,z,p;1/a)]^{-1}_{\Gamma\Gamma'}h_{\chi,\Gamma'}(b,z,P_z;1/a)\\
&=\lim_{L\to\infty}\sum_{\Gamma'}\tilde{Z}^{\rm MOM}(b,z,L,p;1/a)]^{-1}_{\Gamma\Gamma'}\tilde{h}_{\chi,\Gamma'}(b,z,L,P_z;1/a),
\end{aligned}
$$



2.2 Staple link ME renormalized in SDR scheme





$$h^{\mathrm{SDR}}_{\chi,\gamma_t}\left(b,z,P_z;\frac{1}{b_0}\right)=\frac{h_{\chi,\gamma_t}(b,z,P_z;1/a)}{h_{\pi,\gamma_t}(b_0,z_0=0,0,1/a)}$$

$$h^{\overline{\mathrm{MS}}}_{\chi,\gamma_t}(b_0,z_0,0;\mu)=1+\frac{\alpha_s C_F}{2\pi}\left\{\frac{1}{2}+3\gamma_E-3\ln 2+\frac{3}{2}\ln[\mu^2(b_0^2+z_0^2)]-2\frac{z_0}{b_0}\arctan\frac{z_0}{b_0}\right\}+\mathcal{O}(\alpha_s^2)$$

$$h^{\overline{\mathrm{MS}}}_{\chi,\gamma_t}(b,z,P_z;\mu)=h^{\overline{\mathrm{MS}}}_{\gamma_t}(b_0,0,0;\mu)\,h^{\mathrm{SDR}}_{\chi,\gamma_t}\left(b,z,P_z;\frac{1}{b_0}\right)$$

The UV divergence is of a multiplicative structure

M. A. Ebert, I.W. Stewart, and Y. Zhao, J. High Energy Phys. 03 (2020) 099

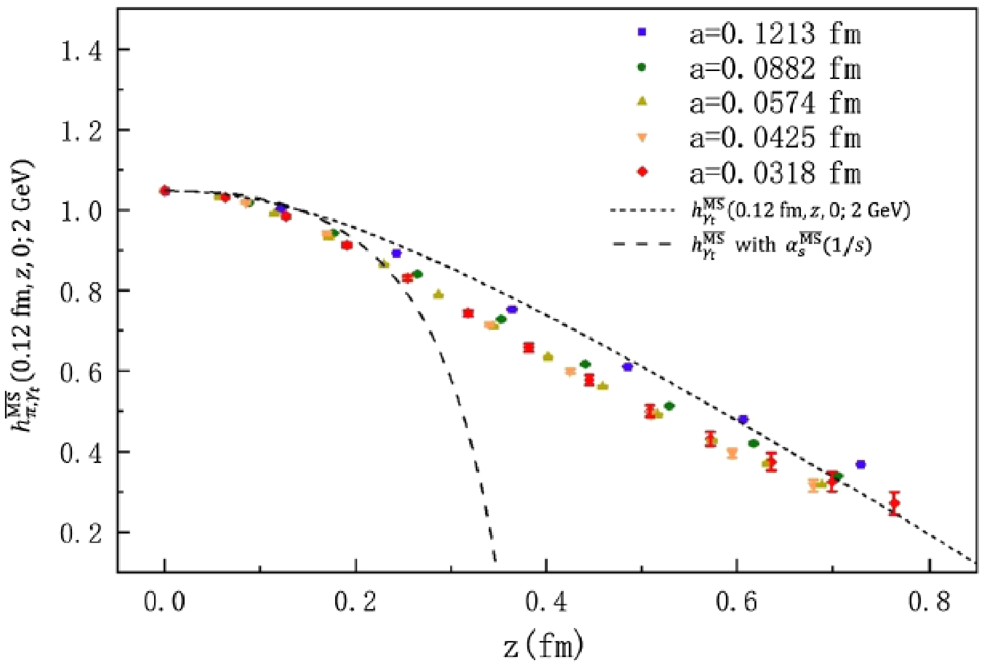
J. R. Green, K. Jansen, and F. Steffens, Phys. Rev. D 101, 074509 (2020).

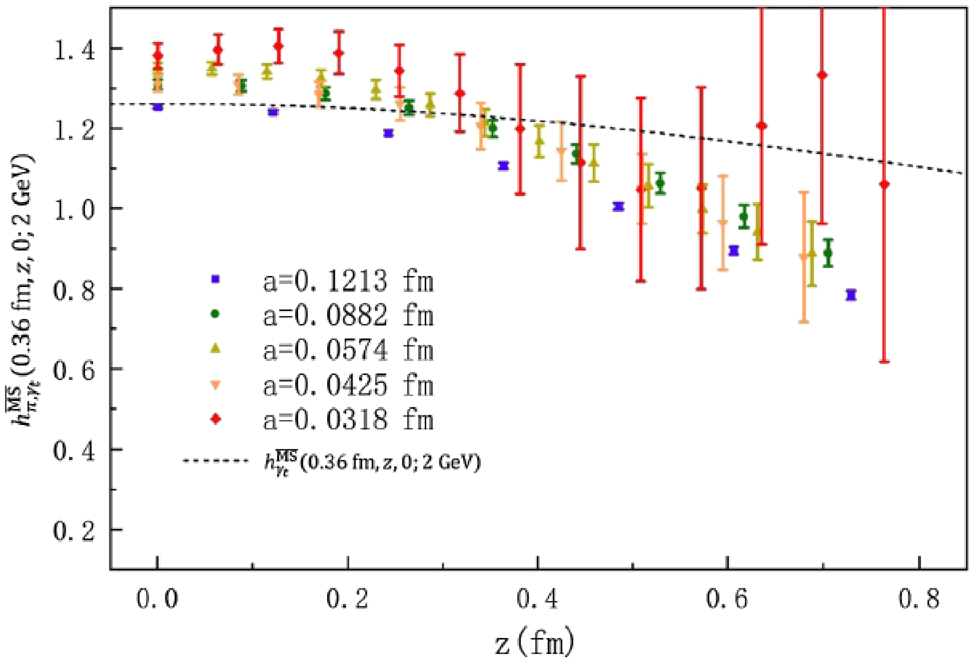
The mixing for nonchiral lattice fermion actions appears to be negligibly small

Y. Ji, J.-H. Zhang, S. Zhao, and R. Zhu, Phys. Rev. D 104, 094510 (2021).

Q.-A. Zhang, et.al, LPC, Phys. Rev. Lett. 125, 192001(2020)

2.2 Staple link ME renormalized in SDR scheme



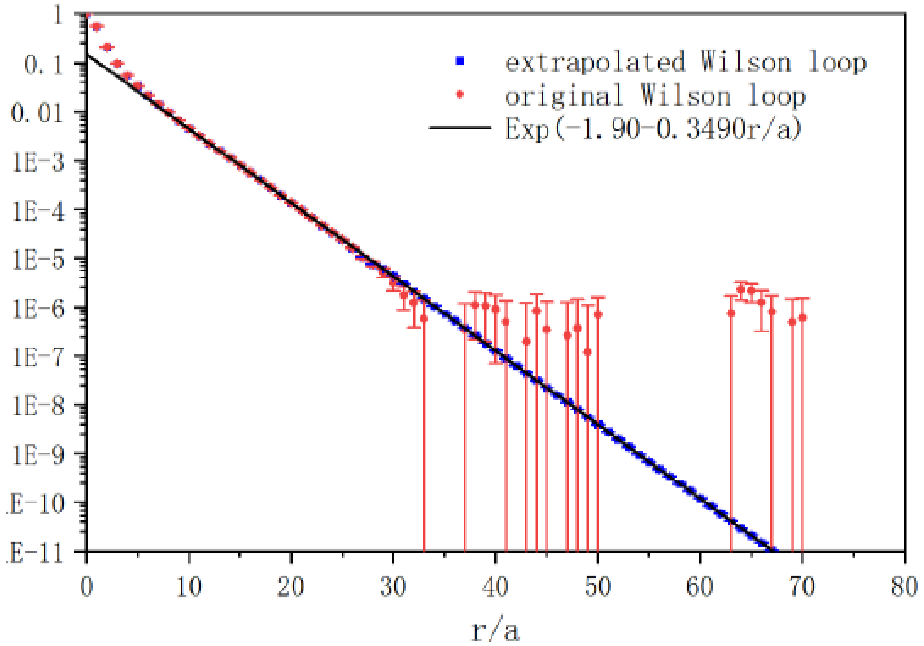


$$h_{\chi,\gamma_t}(b,z,P_z;1/a)=\lim_{L\to\infty}\frac{\tilde{h}_{\chi,\gamma_t}(b,z,L,P_z;1/a)}{\sqrt{Z_E(b,2L+z;1/a)}}$$

$$m_{\text{eff}}(b,r;a)=\ln\frac{Z_E(b,r;a)}{Z_E(b,r+1;a)}$$

3.1 Extrapolation of Wilson loop

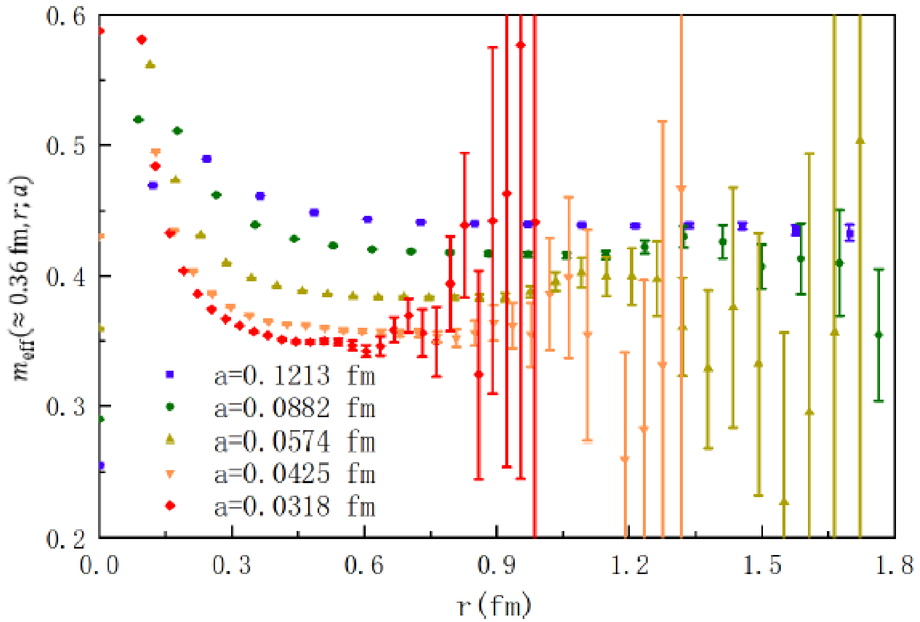
$$Z_E(b,r)\xrightarrow[r\to\infty]{}C(b)e^{-V(b)r}$$

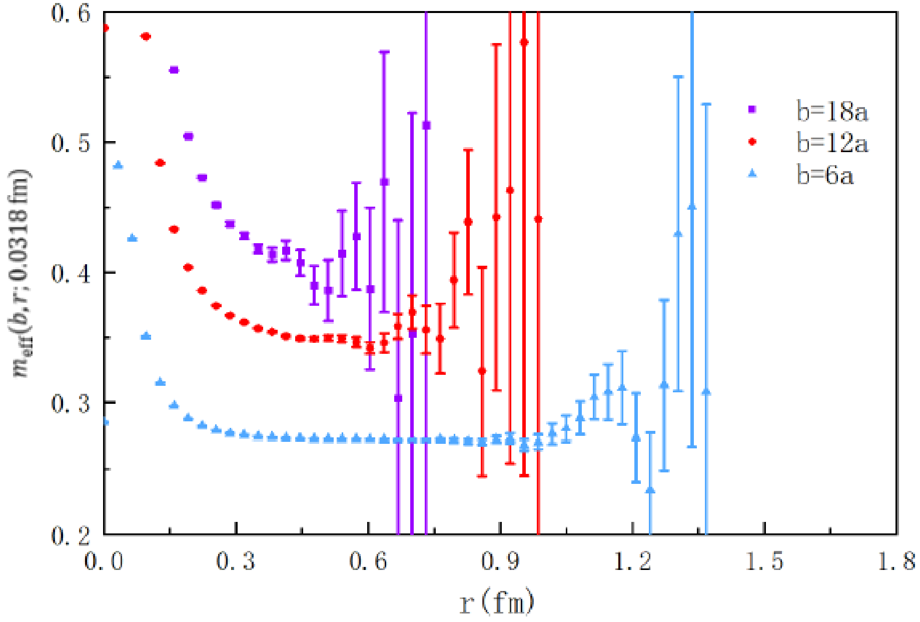


$$e^{c(b;a)-V(b;a)r}$$

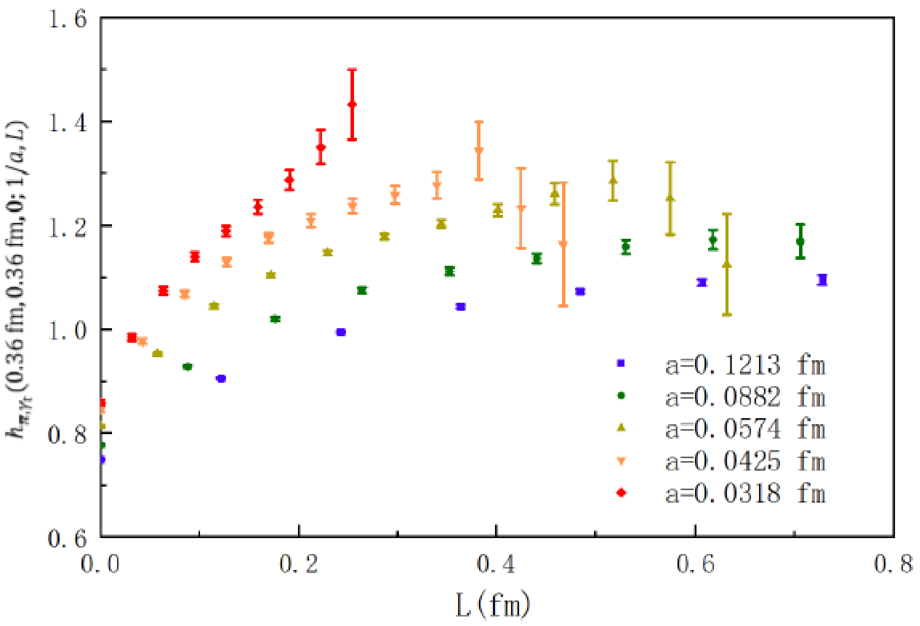
$c(12a;0.0318\text{fm})=-1.90(0.02)$ $V(12a;0.0318\text{fm})=0.3490(0.0015)$

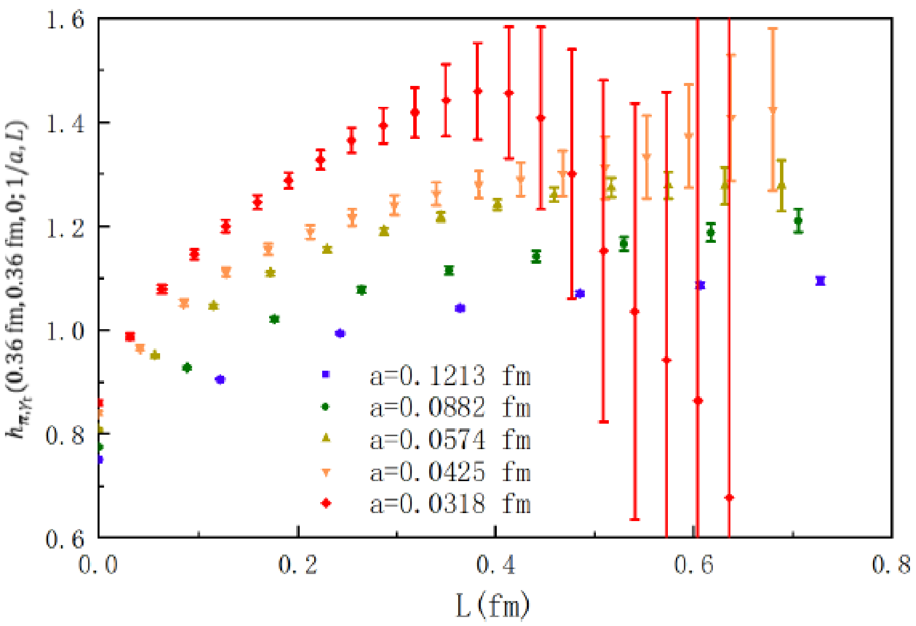
$\chi^2/\text{d.o.f.}=0.63$





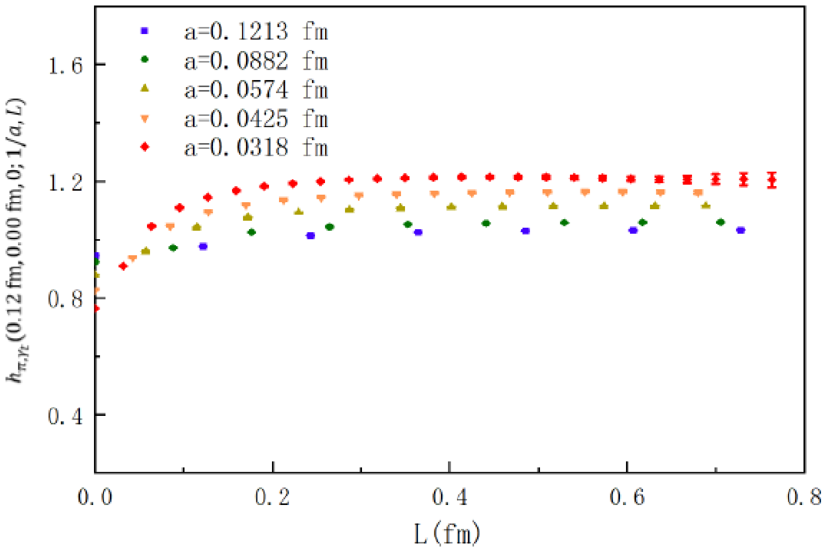
3.1 Extrapolation of Wilson loop

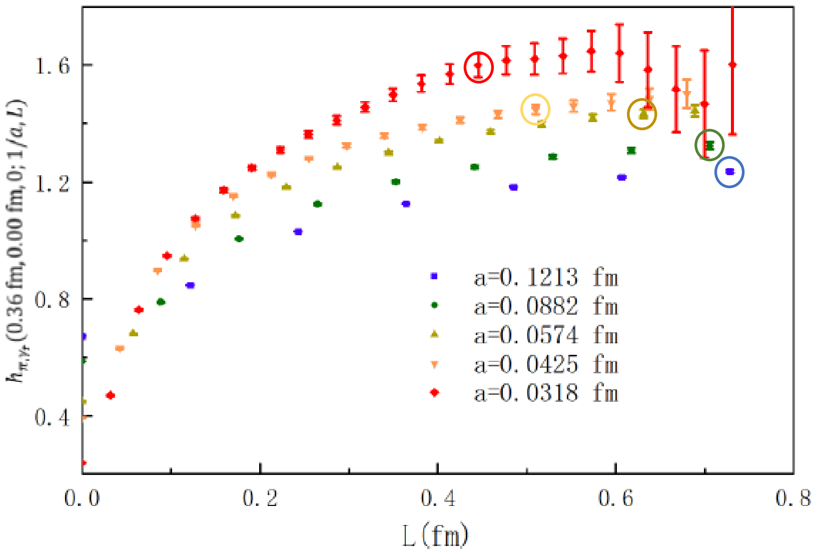


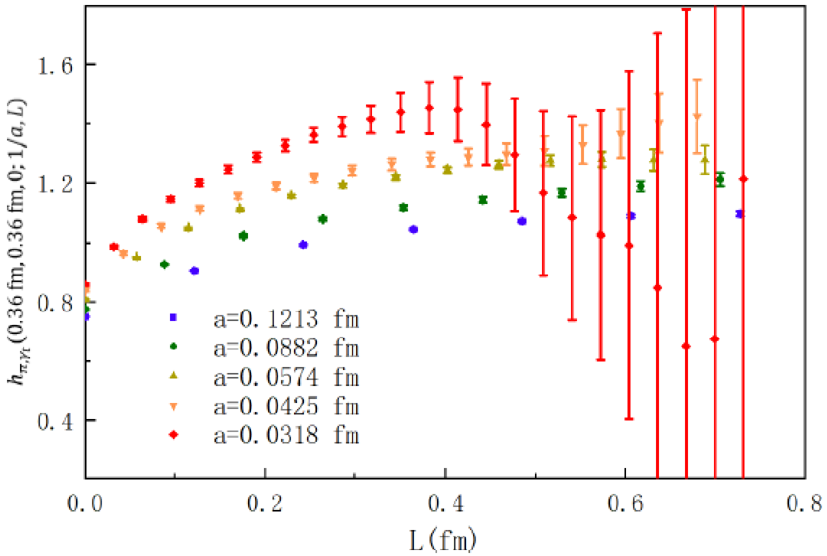


$$h_{\chi,\gamma_t}(b,z,P_z;1/a) = \lim_{L\to\infty} \frac{\tilde{h}_{\chi,\gamma_t}(b,z,L,P_z;1/a)}{\sqrt{Z_E(b,2L+z;1/a)}}$$

3.2 L dependence of the matrix elements

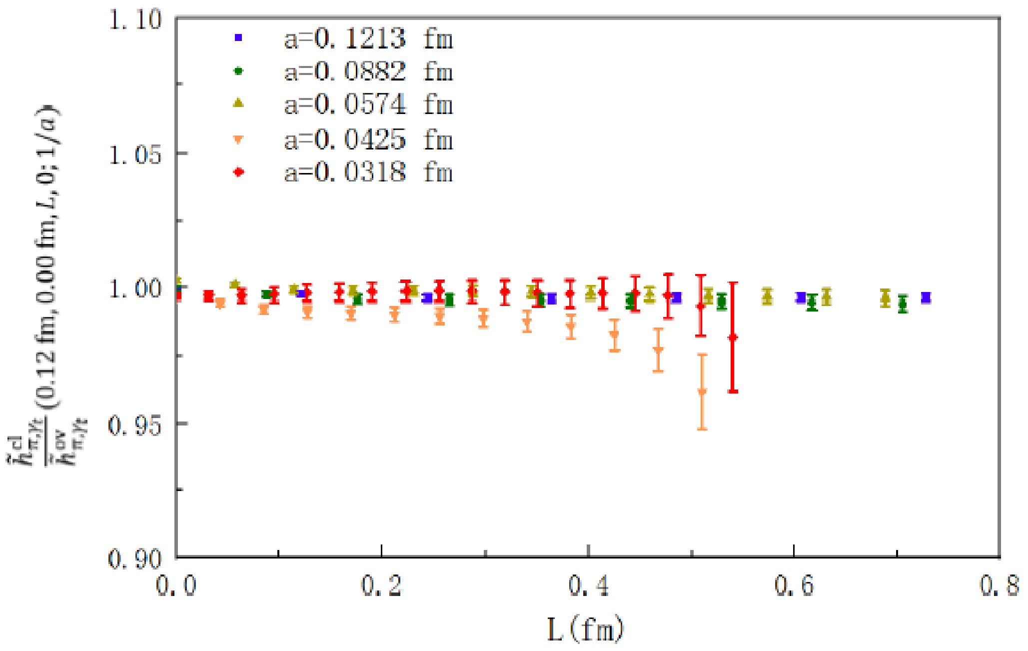


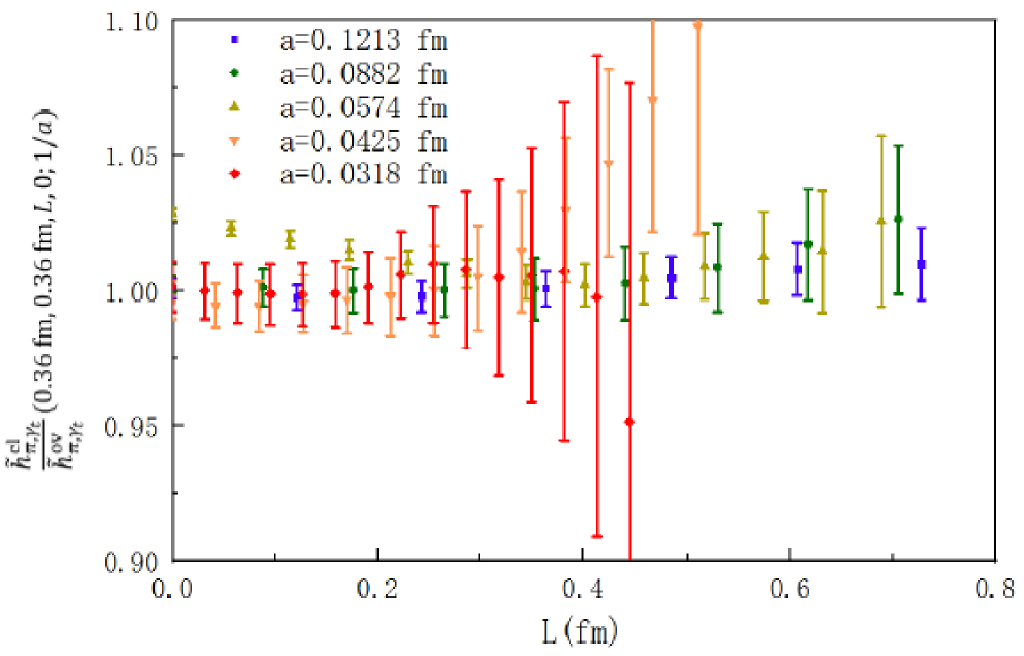




To balance the statistical uncertainty and the systematic uncertainty, we take L = 6a for MILC12, L = 8a for MILC09, L = 11a for MILC06, L = 13a for MILC04, and L = 14a for MILC03 in most cases. For the cases with larger b on the MILC03 ensemble, we set L = b (more specifically, we set L = 16a for b = 16a, and L = 18a for b = 18a).

3.3 Action dependence





3.4 Operator mixing

$$\mathcal{M}_{\mathcal{P}\Gamma}(b,z;1/a)=\frac{\mathrm{Abs}\left[\mathrm{Tr}\left[\mathcal{P}\left\langle q(p)\middle|\bar{q}\Gamma\mathcal{W}(b,z,L)q\middle|q(p)\right\rangle\right]\right]}{\mathrm{Abs}\left[\mathrm{Tr}\left[\Gamma\left\langle q(p)\middle|\bar{q}\Gamma\mathcal{W}(b,z,L)q\middle|q(p)\right\rangle\right]\right]}$$

$M^{\mathrm{cl}}_{\mathcal{P}\Gamma}(4a,4a;a=0.0882\ \mathrm{fm})$

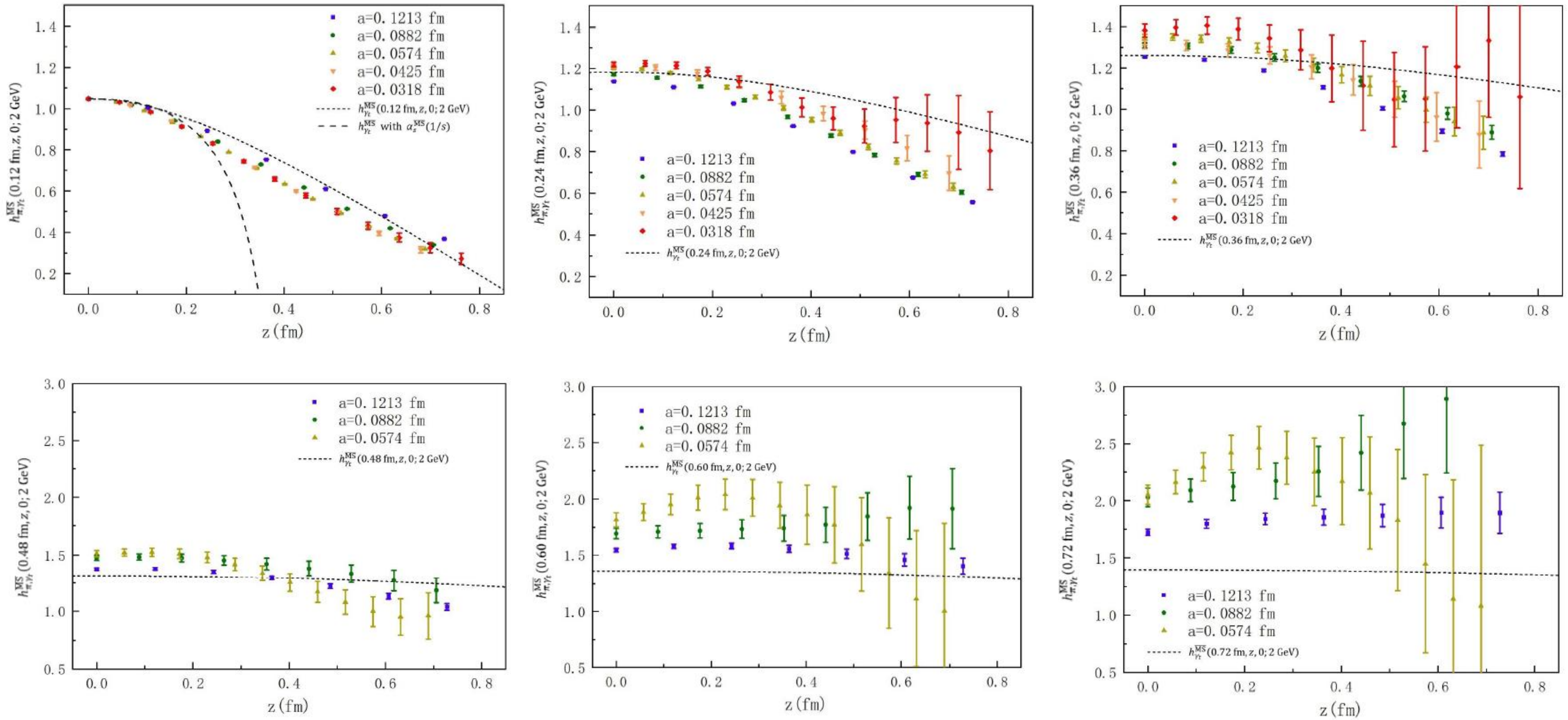
Γ\P	1	γ^t	γ^x	γ^y	γ^z	σ^{tx}	σ^{ty}	σ^{tz}	σ^{xy}	σ^{xz}	σ^{yz}	$\gamma^5\gamma^t$	$\gamma^5\gamma^x$	$\gamma^5\gamma^y$	$\gamma^5\gamma^z$	γ^5
1	1.00	0.05	0.06	0.05	0.07	0.03	0.00	0.11	0.03	0.06	0.11	0.02	0.00	0.02	0.00	0.00
γ^t	0.01	1.00	0.04	0.01	0.06	0.04	0.00	0.15	0.00	0.02	0.00	0.00	0.10	0.07	0.03	0.01
γ^x	0.06	0.05	1.00	0.05	0.05	0.00	0.00	0.02	0.00	0.15	0.02	0.10	0.00	0.10	0.00	0.00
γ^y	0.01	0.01	0.04	1.00	0.05	0.00	0.00	0.00	0.04	0.02	0.15	0.07	0.10	0.00	0.03	0.01
γ^z	0.06	0.05	0.03	0.05	1.00	0.02	0.00	0.00	0.02	0.03	0.00	0.02	0.00	0.02	0.00	0.00
σ^{tx}	0.02	0.03	0.00	0.00	0.02	1.00	0.05	0.04	0.01	0.05	0.00	0.00	0.02	0.07	0.04	0.11
σ^{ty}	0.00	0.00	0.00	0.00	0.00	0.04	1.00	0.05	0.04	0.00	0.05	0.01	0.07	0.01	0.06	0.15
σ^{tz}	0.10	0.15	0.02	0.00	0.00	0.04	0.05	1.00	0.00	0.05	0.01	0.00	0.02	0.07	0.01	0.02
σ^{xy}	0.02	0.00	0.00	0.03	0.02	0.01	0.05	0.00	1.00	0.05	0.04	0.07	0.02	0.00	0.04	0.11
σ^{xz}	0.07	0.02	0.15	0.02	0.04	0.05	0.00	0.04	0.05	1.00	0.04	0.02	0.00	0.01	0.00	0.00
σ^{yz}	0.10	0.00	0.02	0.15	0.00	0.00	0.05	0.01	0.04	0.05	1.00	0.07	0.02	0.00	0.01	0.02
$\gamma^5\gamma^t$	0.02	0.00	0.11	0.07	0.03	0.00	0.01	0.00	0.07	0.04	0.06	1.00	0.04	0.01	0.05	0.00
$\gamma^5\gamma^x$	0.00	0.11	0.00	0.10	0.00	0.02	0.06	0.04	0.02	0.00	0.04	0.05	1.00	0.05	0.04	0.03
$\gamma^5\gamma^y$	0.02	0.07	0.11	0.00	0.03	0.07	0.01	0.06	0.00	0.04	0.00	0.01	0.04	1.00	0.05	0.00
$\gamma^5\gamma^z$	0.00	0.02	0.00	0.02	0.00	0.02	0.06	0.01	0.02	0.00	0.01	0.05	0.04	0.05	1.00	0.16
γ^5	0.00	0.01	0.00	0.01	0.00	0.09	0.06	0.02	0.09	0.00	0.02	0.00	0.04	0.00	0.14	1.00

$M^{\mathrm{ov}}_{\mathcal{P}\Gamma}(4a,4a;a=0.0882\ \mathrm{fm})$

Γ\P	1	γ^t	γ^x	γ^y	γ^z	σ^{tx}	σ^{ty}	σ^{tz}	σ^{xy}	σ^{xz}	σ^{yz}	$\gamma^5\gamma^t$	$\gamma^5\gamma^x$	$\gamma^5\gamma^y$	$\gamma^5\gamma^z$	γ^5
1	1.00	0.00	0.01	0.00	0.01	0.05	0.00	0.19	0.05	0.06	0.19	0.00	0.00	0.00	0.00	0.00
γ^t	0.01	1.00	0.07	0.04	0.08	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.20	0.06	0.05	0.00
γ^x	0.01	0.07	1.00	0.07	0.05	0.00	0.00	0.00	0.00	0.01	0.00	0.19	0.00	0.19	0.00	0.00
γ^y	0.01	0.04	0.07	1.00	0.08	0.00	0.00	0.00	0.00	0.00	0.01	0.06	0.20	0.00	0.05	0.00
γ^z	0.01	0.07	0.03	0.07	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.04	0.00	0.00
σ^{tx}	0.04	0.00	0.00	0.00	0.00	1.00	0.07	0.04	0.02	0.07	0.01	0.00	0.00	0.00	0.00	0.20
σ^{ty}	0.00	0.00	0.00	0.00	0.00	0.06	1.00	0.08	0.06	0.00	0.08	0.00	0.01	0.00	0.01	0.15
σ^{tz}	0.20	0.01	0.00	0.00	0.00	0.04	0.07	1.00	0.01	0.07	0.02	0.00	0.01	0.01	0.00	0.04
σ^{xy}	0.04	0.00	0.00	0.00	0.00	0.02	0.07	0.01	1.00	0.07	0.04	0.00	0.00	0.00	0.00	0.20
σ^{xz}	0.06	0.00	0.01	0.00	0.00	0.08	0.00	0.06	0.08	1.00	0.06	0.01	0.00	0.01	0.00	0.00
σ^{yz}	0.20	0.00	0.00	0.01	0.00	0.01	0.07	0.02	0.04	0.07	1.00	0.01	0.01	0.00	0.00	0.04
$\gamma^5\gamma^t$	0.00	0.01	0.20	0.06	0.05	0.00	0.00	0.00	0.01	0.00	0.01	1.00	0.07	0.04	0.08	0.00
$\gamma^5\gamma^x$	0.00	0.19	0.00	0.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	1.00	0.07	0.05	0.00
$\gamma^5\gamma^y$	0.00	0.06	0.20	0.01	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.04	0.07	1.00	0.08	0.00
$\gamma^5\gamma^z$	0.00	0.04	0.00	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.07	0.03	0.07	1.00	0.01
γ^5	0.00	0.00	0.00	0.00	0.00	0.17	0.06	0.04	0.17	0.00	0.04	0.00	0.00	0.00	0.01	1.00

3.5 Matrix element at larger b

$$h^{\overline{\mathrm{MS}}}_{\chi,\gamma_t}(b,z,P_z;\mu) = h^{\overline{\mathrm{MS}}}_{\gamma_t}(b_0,0,0;\mu)\,h^{\mathrm{SDR}}_{\chi,\gamma_t}\left(b,z,P_z;\frac{1}{b_0}\right)$$



3.6 TMD wave function matrix element

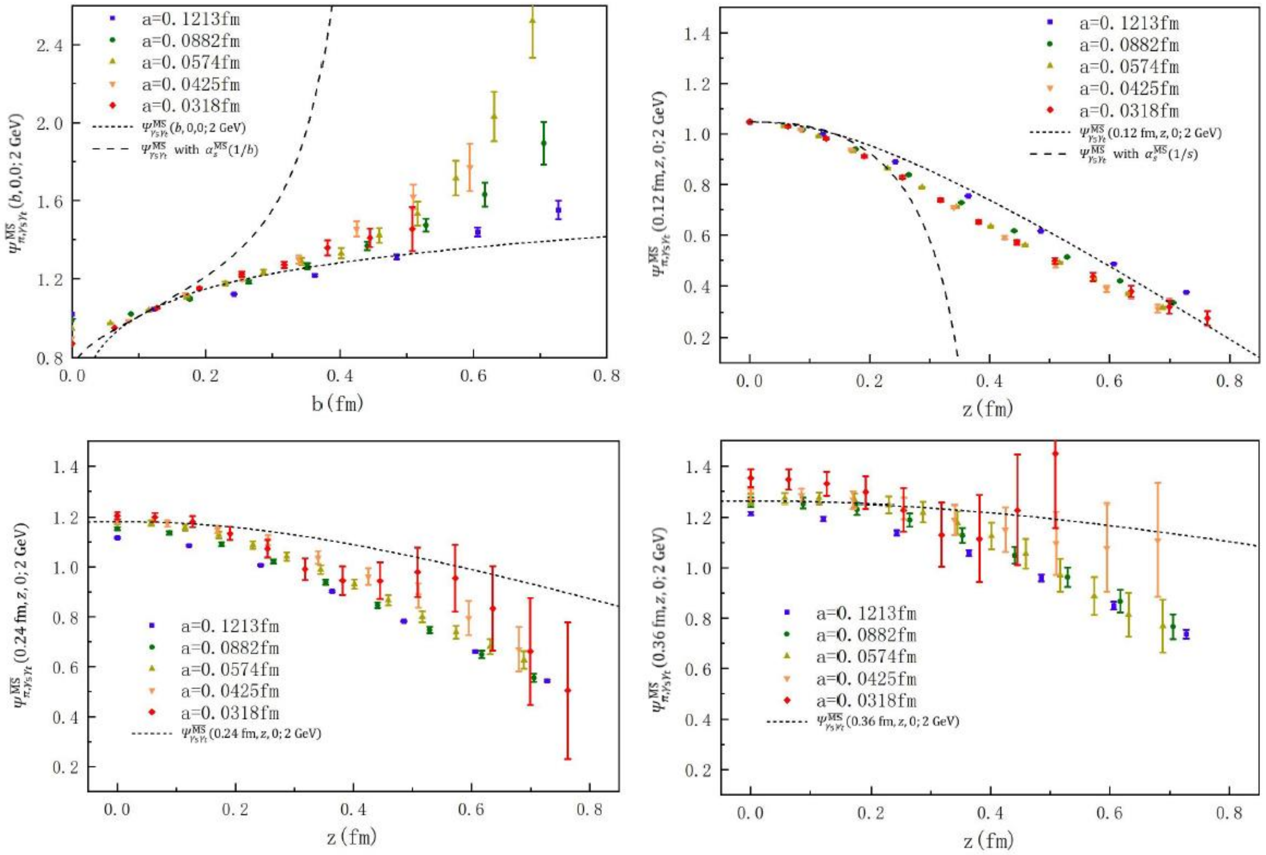
$$\Psi_{\pi,\gamma_5\gamma_t}(b,z,P_z;1/a) = \lim_{L\to\infty} \frac{\langle O_{\gamma_5\gamma_t}(b,z,L)|\pi(P_z)\rangle}{\langle O_{\gamma_5\gamma_t}(0,0,0)|\pi(P_z)\rangle \sqrt{Z_E(b,2L+z;1/a)}},$$

$$\Psi^{\mathrm{SDR}}_{\pi,\gamma_5\gamma_t}(b,z,P_z;\frac{1}{b_0}) = \frac{\Psi_{\pi,\gamma_5\gamma_t}(b,z,P_z;1/a)}{\Psi_{\pi,\gamma_5\gamma_t}(b_0,z_0=0,0,1/a)}.$$

$$\Psi^{\overline{\mathrm{MS}}}_{\gamma_5\gamma_t}(b_0,z_0,0;\mu) = h^{\overline{\mathrm{MS}}}_{\chi,\gamma_t}(b_0,z_0,0;\mu)$$

$$= 1+\frac{\alpha_s C_F}{2\pi}\left\{\frac{1}{2}+3\gamma_E-3\ln 2+\frac{3}{2}\ln[\mu^2(b_0^2+z_0^2)]-2\frac{z_0}{b_0}\arctan\frac{z_0}{b_0}\right\}+\mathcal{O}(\alpha_s^2),$$

$$\Psi^{\overline{\mathrm{MS}}}_{\pi,\gamma_5\gamma_t}(b,z,P_z;\mu)=\psi^{\overline{\mathrm{MS}}}_{\gamma_5\gamma_t}(b_0,0,0;\mu)\Psi^{\mathrm{SDR}}_{\pi,\gamma_t}(b,z,P_z;\frac{1}{b_0}).$$



4、Summary

- The square root of the **Wilson loop** combined with the short distance hadron matrix element provides a successful method to **remove all ultraviolet divergences** of the quasi-TMD operator

- Quasi TMD-PDF renormalized in **RI/MOM** scheme suffers from **residual linear divergence**.