Lattice QCD Calculations of Transverse-momentum-dependent Soft Function at one-loop accuracy

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(Lattice Parton Collaboration)

3/12/2022

LaMET 2022

Lattice Parton Collaboration

Outline

- Motivation: Soft function
- Intrinsic soft function in LaMET
- Simulation on lattice
- Numerical results
- Summary

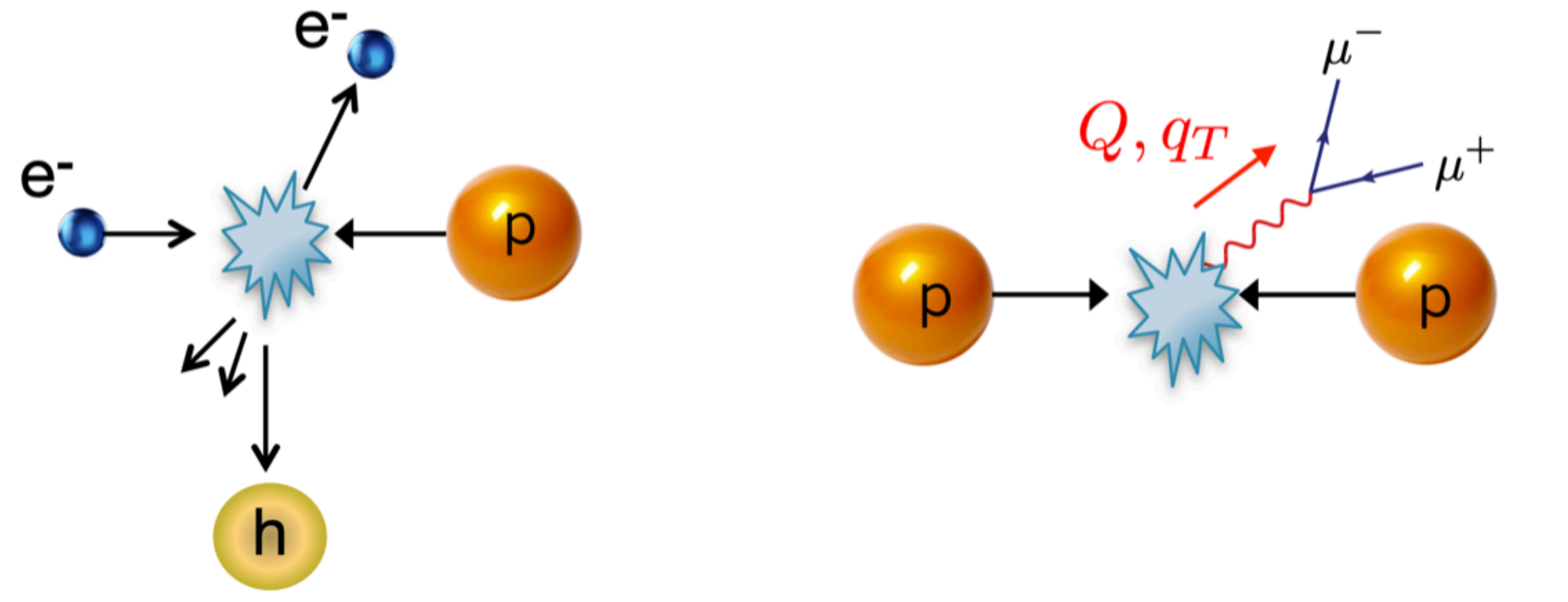
Motivation: TMD process and factorization

Semi-Inclusive DIS

$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$

Drell-Yan

$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$

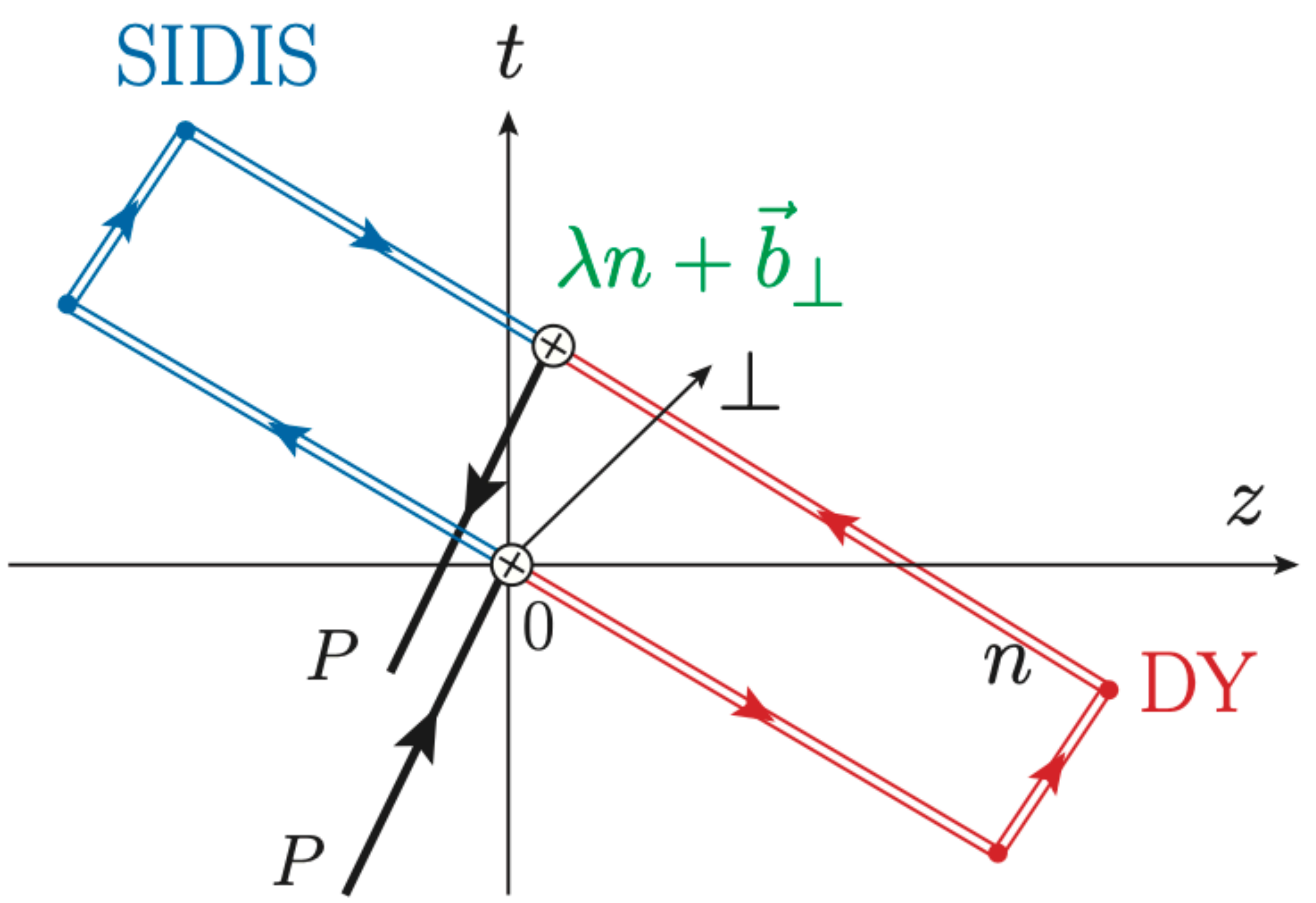


R. Angeles-Martinez et. al, arxiv 2507.05267 (2015)

TMDPDFs as important inputs

light cone TMDPDF

$$f^{\pm}(x, \vec{k}_\perp) = \int \frac{d\lambda}{2\pi} \frac{db_\perp^2}{(2\pi)^2} e^{-i\lambda x + i\vec{k}_\perp \vec{b}_\perp}$$
$$\times \langle P | \bar{q}(\lambda n + \vec{b}_\perp) \Gamma W^{\pm}(\lambda n/2 + \vec{b}_\perp) q(0) | P \rangle$$



- TMD factorization

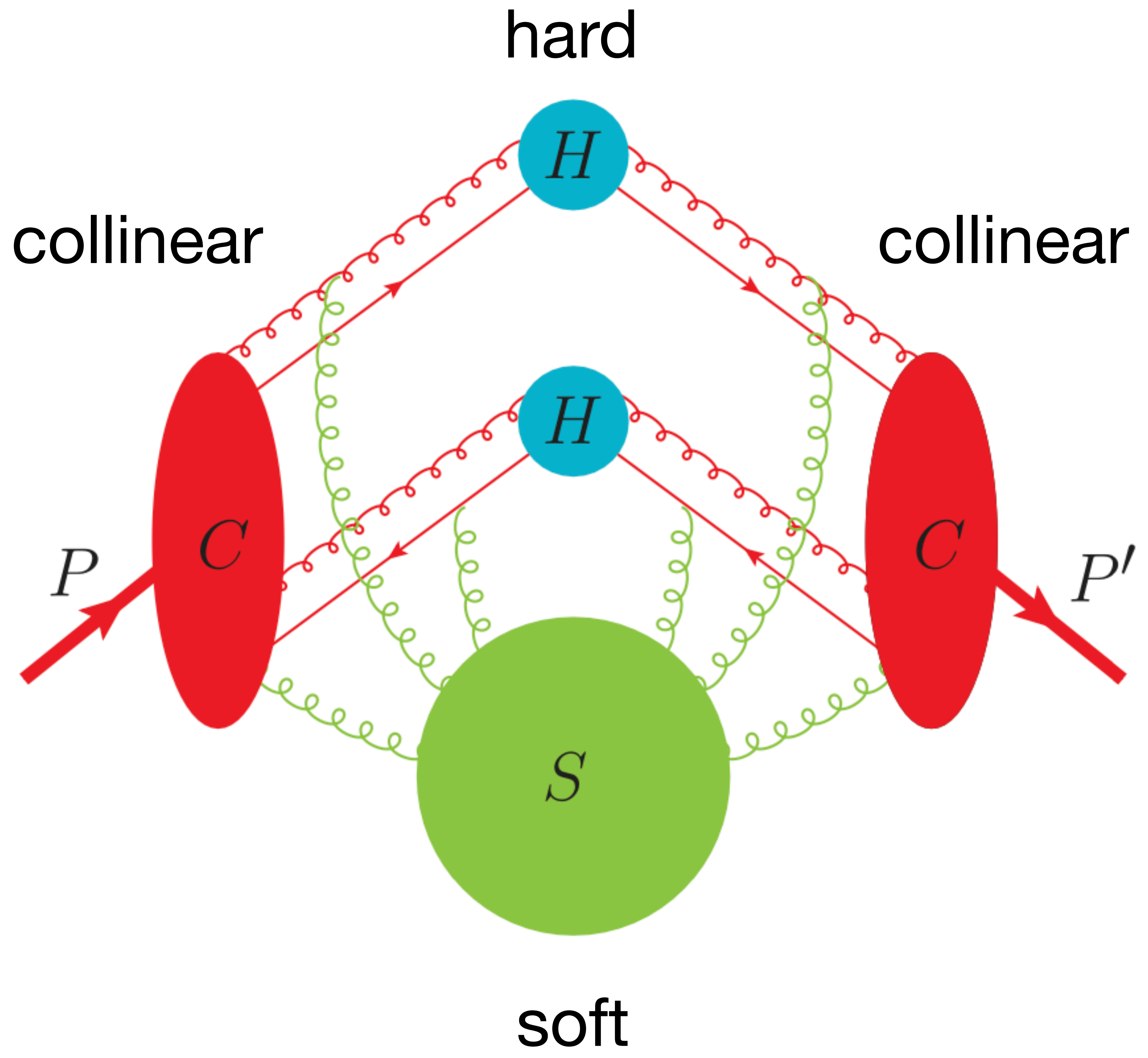
$$\tilde{f}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right) S_{I}^{\frac{1}{2}}\left(b_{\perp}, \mu\right)$$
$$= H^{\pm}\left(x, \zeta^{z}, \mu\right) e^{\left[\frac{1}{2} K\left(b_{\perp}, \mu\right) \ln \frac{\mp\zeta^{z}+i\epsilon}{\zeta}\right]} f^{\pm}\left(x, b_{\perp}, \mu, \zeta\right)$$

- Soft gluon effects

rapidity dependent part $e^{\frac{1}{2}K(b_{\perp},\mu)\frac{\mp\zeta^{z}+i\epsilon}{\zeta}}$

rapidity independent part $S_{\mathrm{I}}^{\frac{1}{2}}(b_{\perp}, \mu)$

Intrinsic soft function

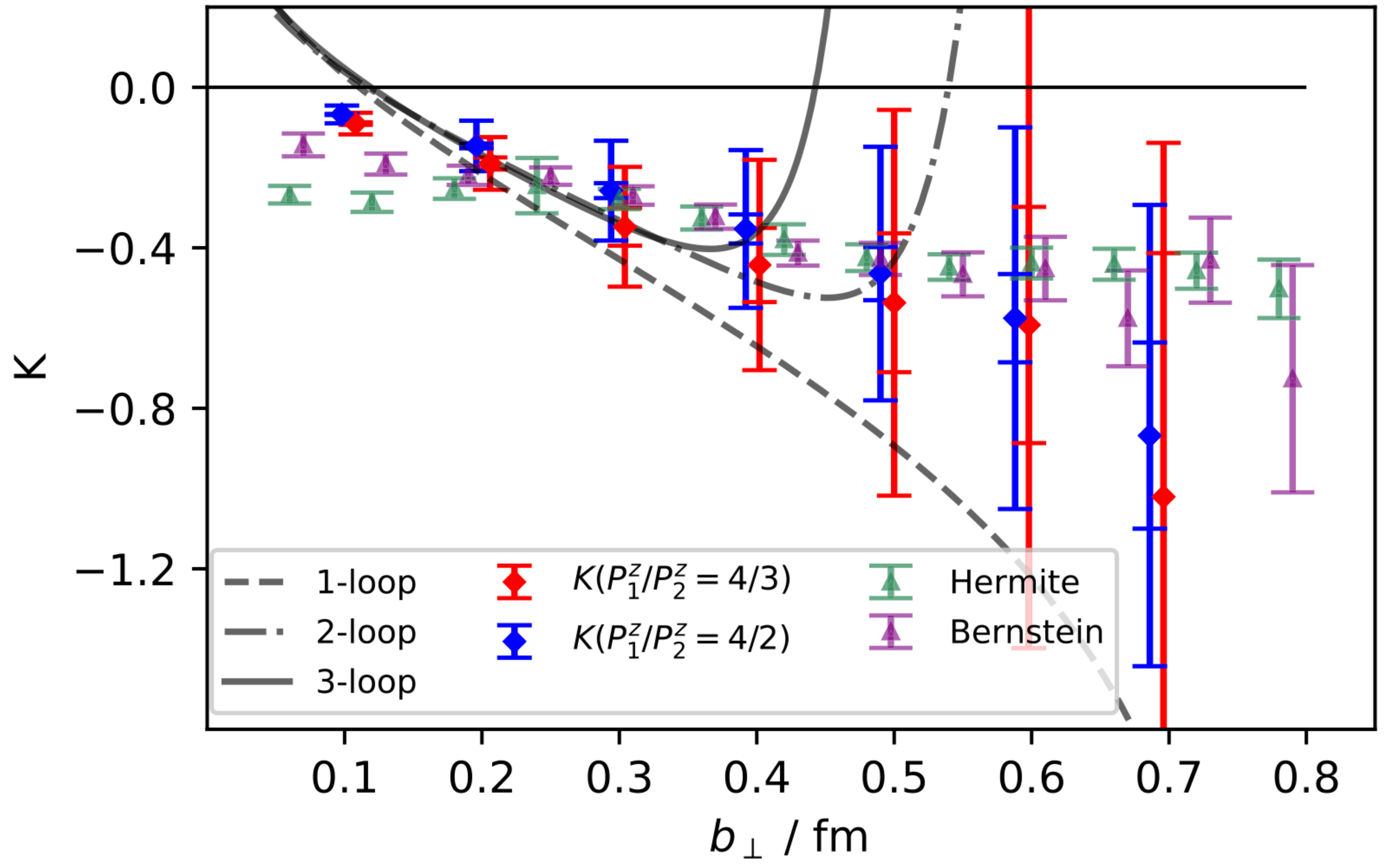


Motivation: Collins-Soper kernel

CS kernel determination

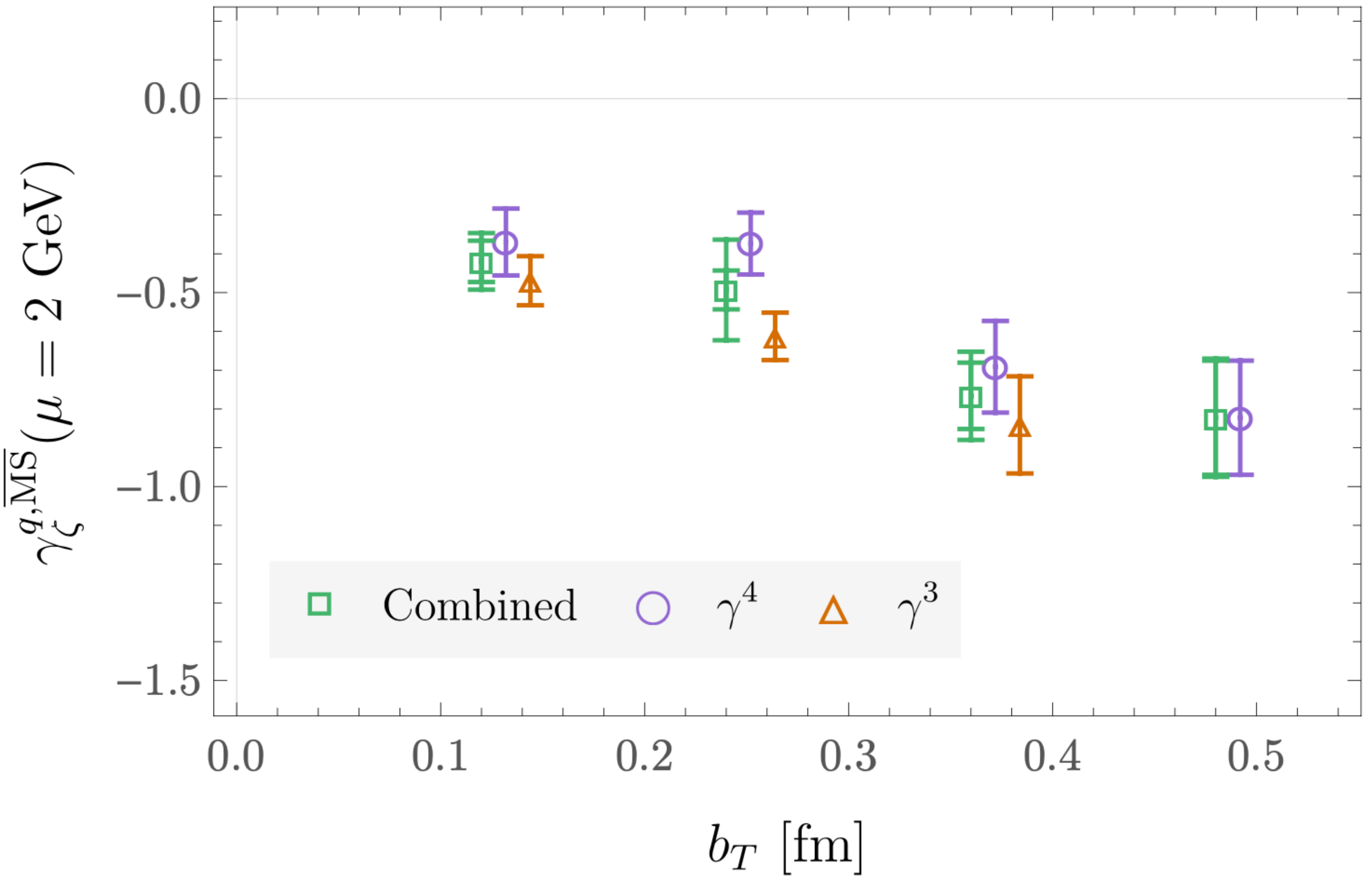
$$K(b_\perp,\mu) = \frac{1}{\ln(P_2^z/P_1^z)} \ln\left[\frac{H^\pm(\zeta_1^z,\bar{\zeta}_1^z)\tilde{\Psi}^\pm(b_\perp,x,\zeta_2^z)}{H^\pm(\zeta_2^z,\bar{\zeta}_2^z)\tilde{\Psi}^\pm(b_\perp,x,\zeta_1^z)}\right]$$

$K(b_\perp,\mu)$ from TMDWFs at **tree level**



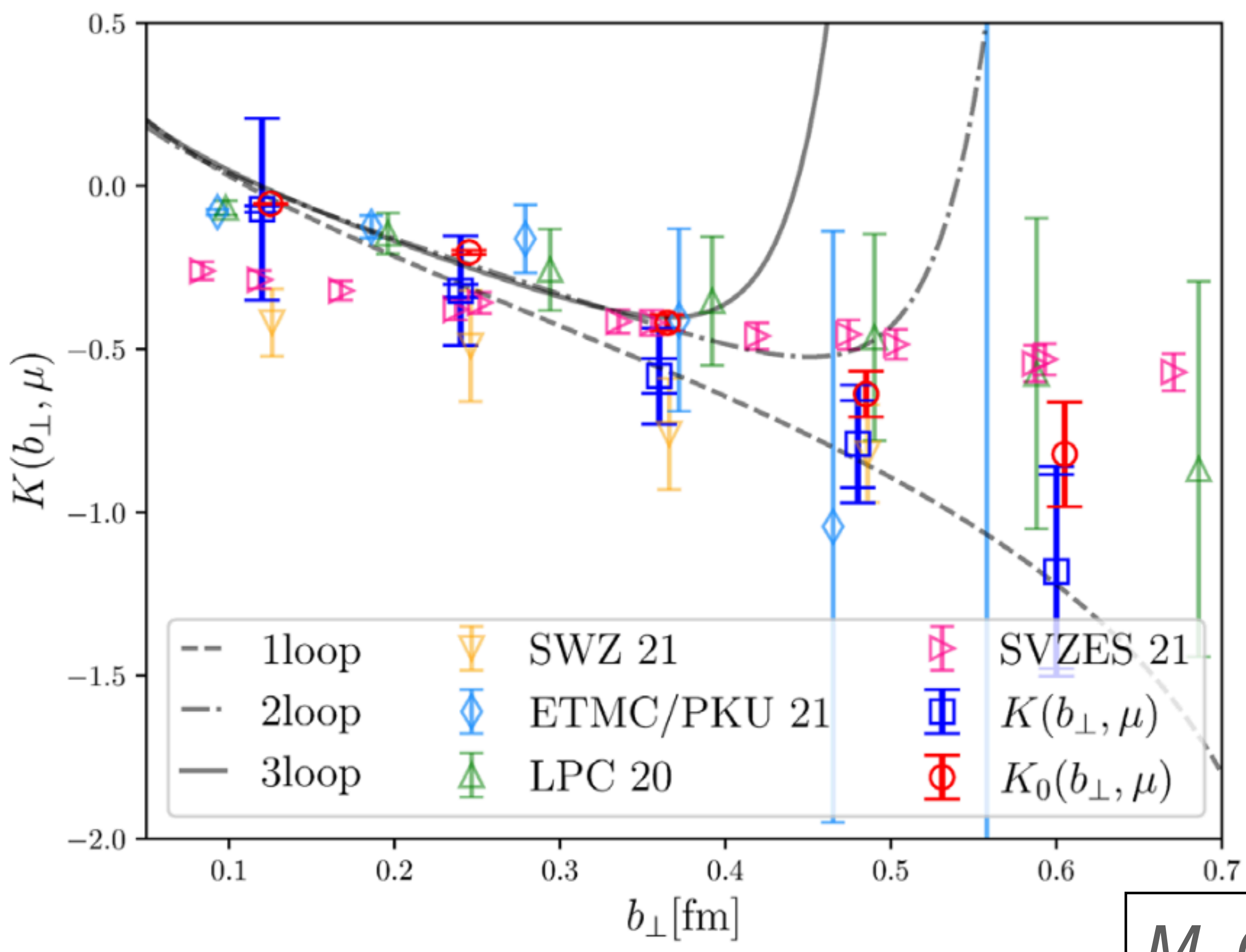
Q. Zhang et al, arxiv 2005.14572 (2020)

$K(b_\perp,\mu)$ from TMDPDFs at **1-loop**



P. Shanahan, M. Wangman, Y. Zhao, arxiv 2017.11903 (2021)

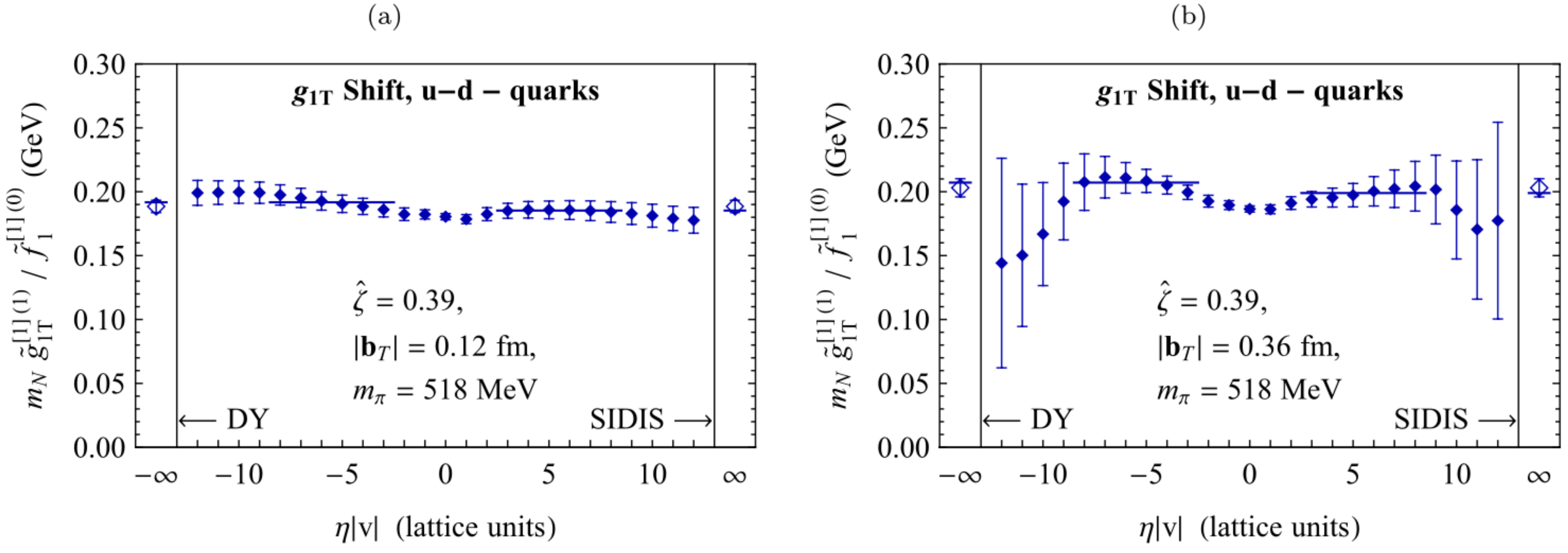
$K(b_\perp,\mu)$ from TMDWFs at **1-loop**



M. Chu et al, arxiv 2204.00200(2022)

TMDPDFs without soft function

appropriate ratio of amplitude to cancel the soft factor



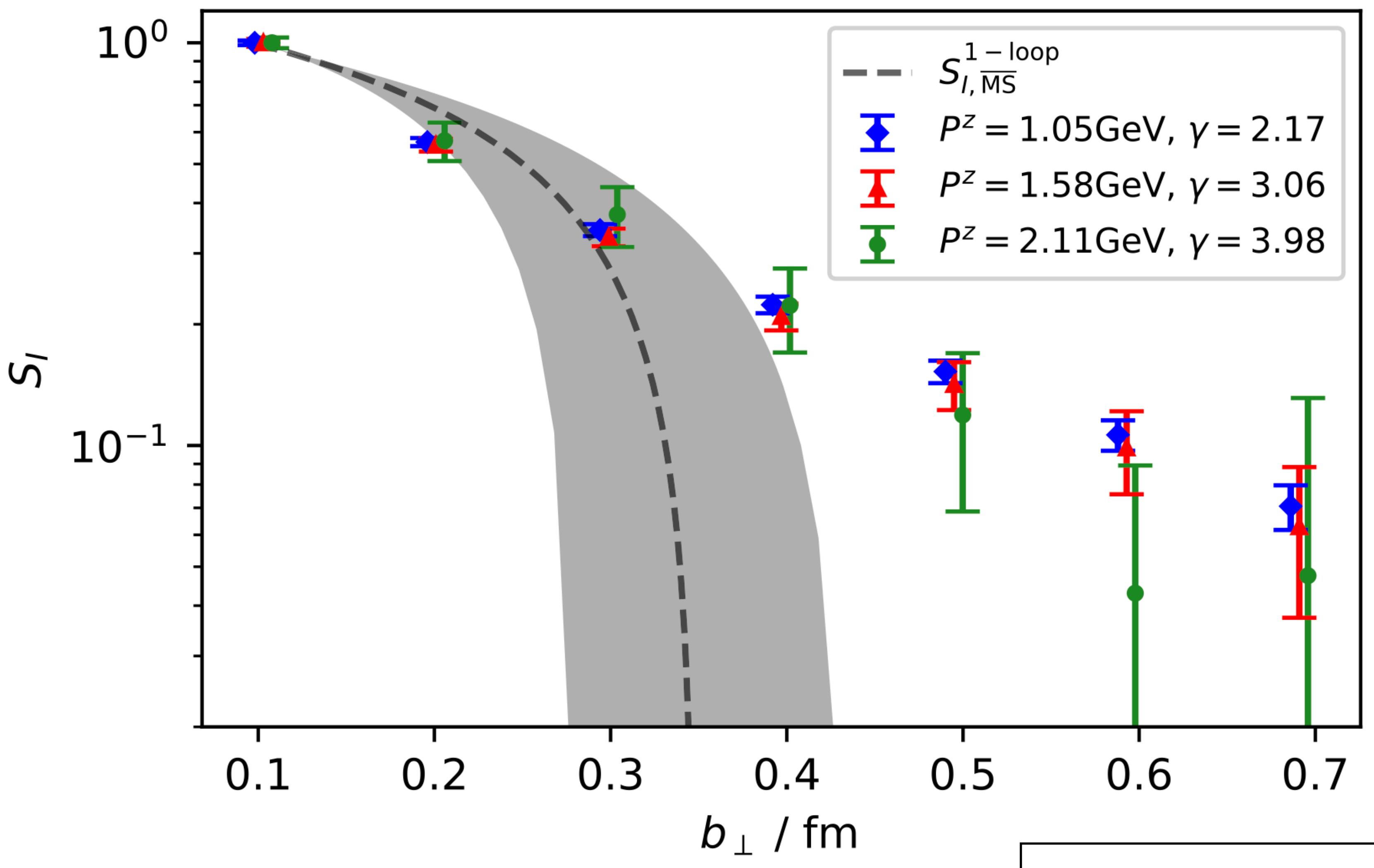
B.U. Musch et al, arxiv 1111.4249 (2011)

quasi beam function

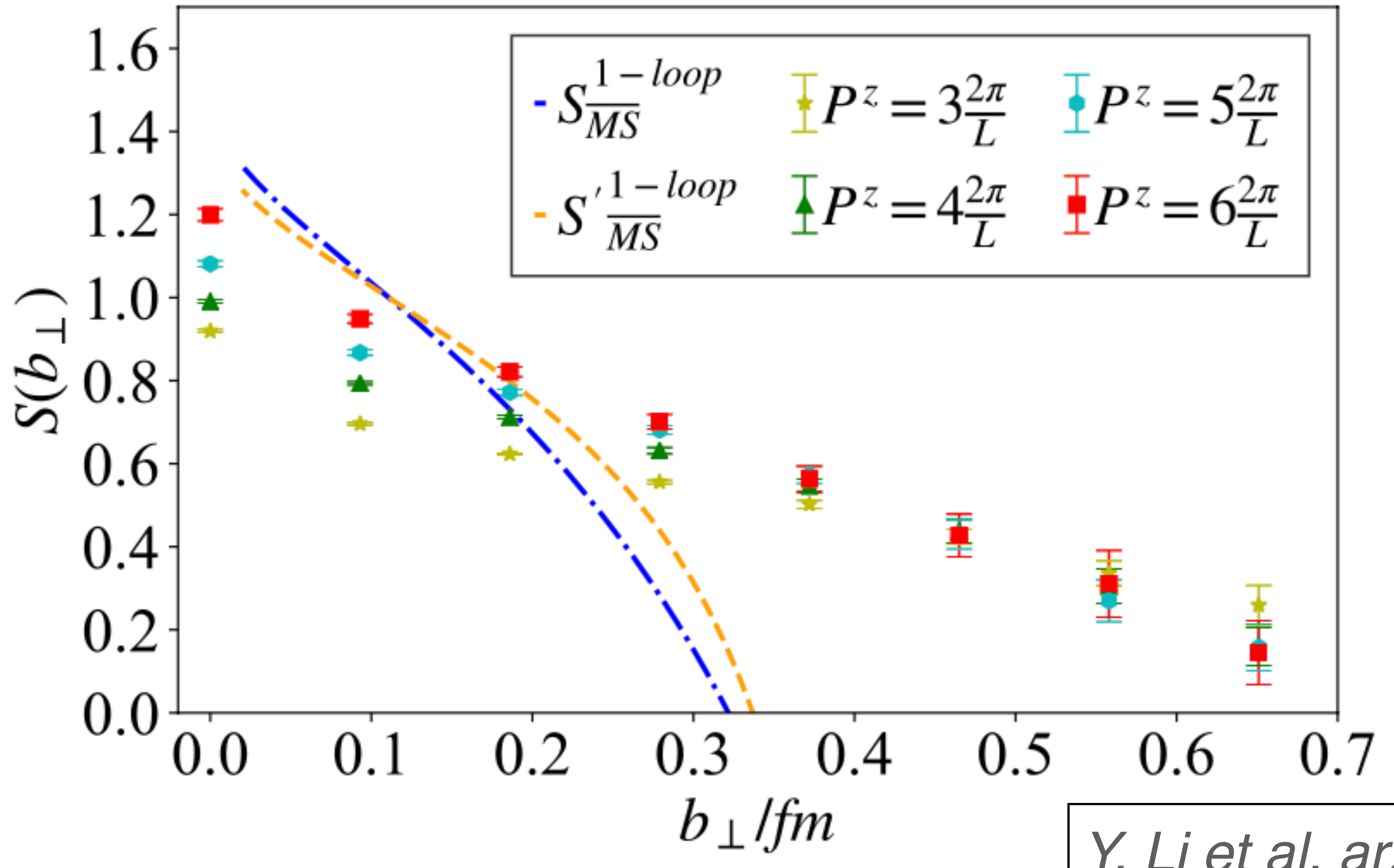
$$\tilde{f}_{q,\Gamma}^{\mathrm{TMD}}(x,\vec{b}_T,\mu,P^z) \equiv \lim_{\eta\to\infty}\int \frac{\mathrm{d}b^z}{2\pi} e^{-\mathrm{i}b^z(xP^z)} Z_{\Gamma\Gamma'}^{\overline{\mathrm{MS}}}(\mu,b^z) \times \frac{2P^z}{N_\Gamma} \boxed{\tilde{B}_q^{\Gamma'}(b^z,\vec{b}_T,\eta,P^z)} \tilde{\Delta}_S^q(b_T,\eta)$$

P. Shanahan et al, arxiv 1911.00800 (2011)

Tree level intrinsic soft function results



Q. Zhang et al, arxiv 2005.14572 (2020)



Y. Li et al, arxiv 2106.13027 (2021)

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Four quark form factor

$$F\left(b_{\perp}, P_1, P_2, \Gamma, \mu\right) = \left\langle P_2 \left| \bar{q}(b_{\perp})\Gamma q(b_{\perp})\bar{q}(0)\Gamma' q(0) \right| P_1 \right\rangle$$

Factorization of form factor

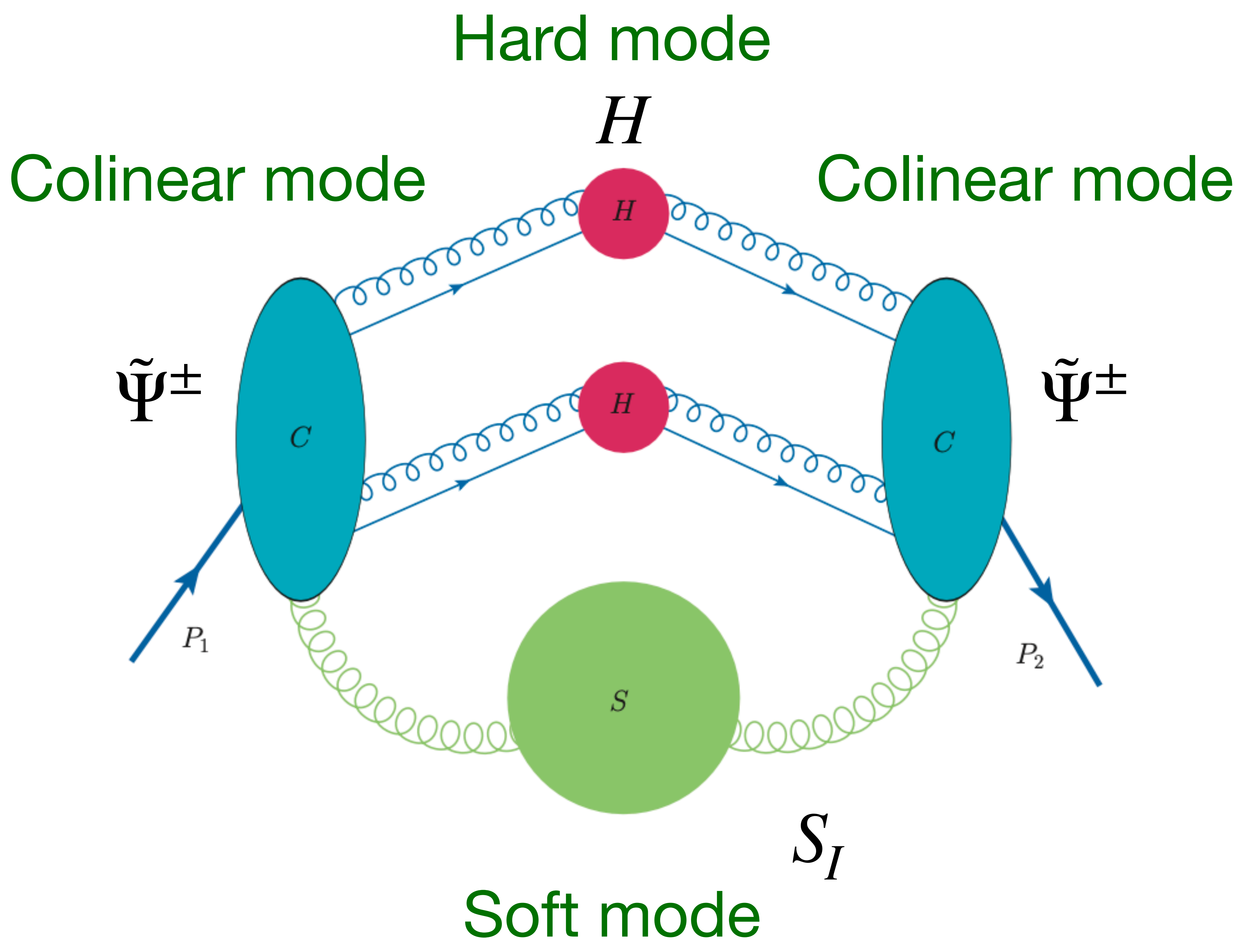
$$F\left(b_{\perp}, P_1, P_2, \Gamma, \mu\right) = \int dx_1 dx_2 H\left(x_1, x_2, \Gamma\right) S_I\left(b_{\perp}, \mu\right) \times \tilde{\Psi}^{\pm *}\left(x_2, b_{\perp}, \zeta^z\right) \tilde{\Psi}^{\pm}\left(x_1, b_{\perp}, \zeta^z\right)$$

Z.F. Deng et al, arxiv 2207.07280(2022)

$F\left(b_{\perp}, P, \Gamma, \mu\right)$: four quark form factor, nonperturbative.

$H\left(x_1, x_2, \Gamma\right)$: Perturbative matching coefficient.

$\tilde{\Psi}^{\pm}(x, b_{\perp}, \zeta)$: quasi-TMDWF, nonperturbative.



the leading order reduced diagram

Normalization

Z.F. Deng et al, arxiv 2207.07280(2022)

$$F(b_\perp, \Gamma, P^z) = \frac{\langle \pi(P_2) | (\bar{q}\Gamma q)|_{b_\perp} (\bar{q}\Gamma q)|_0 | \pi(P_1)\rangle}{f_\pi^2 P_1 \cdot P_2}$$

$$\langle 0 | (\bar{q}\gamma^\mu \gamma_5 q)|_0 | P_1\rangle = -if_\pi P_1^\mu, \langle P_2 | (\bar{q}\gamma_\mu \gamma_5 q)|_0 | 0\rangle = if_\pi P_{2\mu}$$

$$P_2 = (P^z, 0, 0, -P^z), \ \ P_1 = (P^z, 0, 0, P^z)$$

for the denominator

$$\begin{aligned}
&\langle \pi(P_2) | (\bar{q}\gamma^\mu \gamma_5 q)|_0 | 0\rangle \langle 0 | (\bar{q}\gamma_\mu \gamma_5 q)|_0 | \pi(P_1)\rangle \\
&= f_\pi^2 (P_1 \cdot P_2) = 2 f_\pi^2 (P^z)^2 \\
&= 2\langle \pi(P_1) | (\bar{q}\gamma^t \gamma_5 q)|_0 | 0\rangle \langle 0 | (\bar{q}\gamma^t \gamma_5 q)|_0 | \pi(P_1)\rangle \\
&= 2\langle 0 | (\bar{q}\gamma^t \gamma_5 q)|_0 | \pi(P_1)\rangle \langle \pi(P_1) | (\bar{q}\gamma^t \gamma_5 q)|_0 | 0\rangle
\end{aligned}$$

lattice simulation

3pt

$$F(b_\perp, P^z) = \frac{\langle \pi(P_2) | (\bar{q}\Gamma q)|_{b_\perp} (\bar{q}\Gamma q)|_0 | \pi(P_1)\rangle}{2\langle 0 | (\bar{q}\gamma^t \gamma_5 q)|_0 | \pi(P_1)\rangle \langle \pi(P_2) | (\bar{q}\gamma^t \gamma_5 q)|_0 | 0\rangle}$$

local 2pt   local 2pt

Which Γ or combinations of Γ should be used?

Dirac structure

$$F(\Gamma = I) - F(\Gamma = \gamma_5)$$
$$= (\bar\psi_a\psi_b)(\bar\psi_c\psi_d) - (\bar\psi_a\gamma_5\psi_b)(\bar\psi_c\gamma_5\psi_d)$$
$$= \frac{1}{2}\bar\psi_c\gamma^\mu\gamma_5\psi_b\bar\psi_a\gamma_\mu\gamma_5\psi_d - \frac{1}{2}\bar\psi_c\gamma^\mu\psi_b\bar\psi_a\gamma_\mu\psi_d$$

$$\sum F(\Gamma = \gamma^\mu) + F(\Gamma = \gamma^\mu\gamma_5)$$
$$= (\bar\psi_a\gamma^{x,y}\psi_b)(\bar\psi_c\gamma_{x,y}\psi_d) + (\bar\psi_a\gamma^{x,y}\gamma_5\psi_b)(\bar\psi_c\gamma_{x,y}\gamma_5\psi_d)$$
$$= \bar\psi_c\gamma^\mu\gamma_5\psi_b\bar\psi_a\gamma_\mu\gamma_5\psi_d + \bar\psi_c\gamma^\mu\psi_b\bar\psi_a\gamma_\mu\psi_d$$

Fierz rearrangement indicates $F(I) - F(\gamma_5)$ and $\sum F(\gamma^\mu) + F(\gamma^\mu\gamma_5)$ extract leading twist $\gamma^\mu\gamma_5$

UV divergence

Z.F. Deng et al, arxiv 2207.07280(2022)

- The UV divergence in the I and γ_5 form factor. can be removed by the renormalization constant of scalar density operator

$$Z_S = 1 + \frac{\alpha_s C_F}{4\pi}\frac{3}{\epsilon_{\rm UV}}. \quad (59)$$

- There is no UV divergence in the $\gamma_\perp$ and $\gamma_\perp\gamma_5$ form factor. After some simplifications, Eq. (58) gives

$$F(b_\perp, P_1, P_2, \mu) = F^0\Bigg[1 - \frac{\alpha_s C_F}{2\pi}\Bigg(7 - \frac{3}{2}\ln\frac{Q^2\bar Q^2 b_\perp^4}{4e^{-4\gamma_E}} + \frac{1}{2}\ln^2\frac{Q^2 b_\perp^2}{2e^{-2\gamma_E}} + \frac{1}{2}\ln^2\frac{\bar Q^2 b_\perp^2}{2e^{-2\gamma_E}}\Bigg)\Bigg].$$

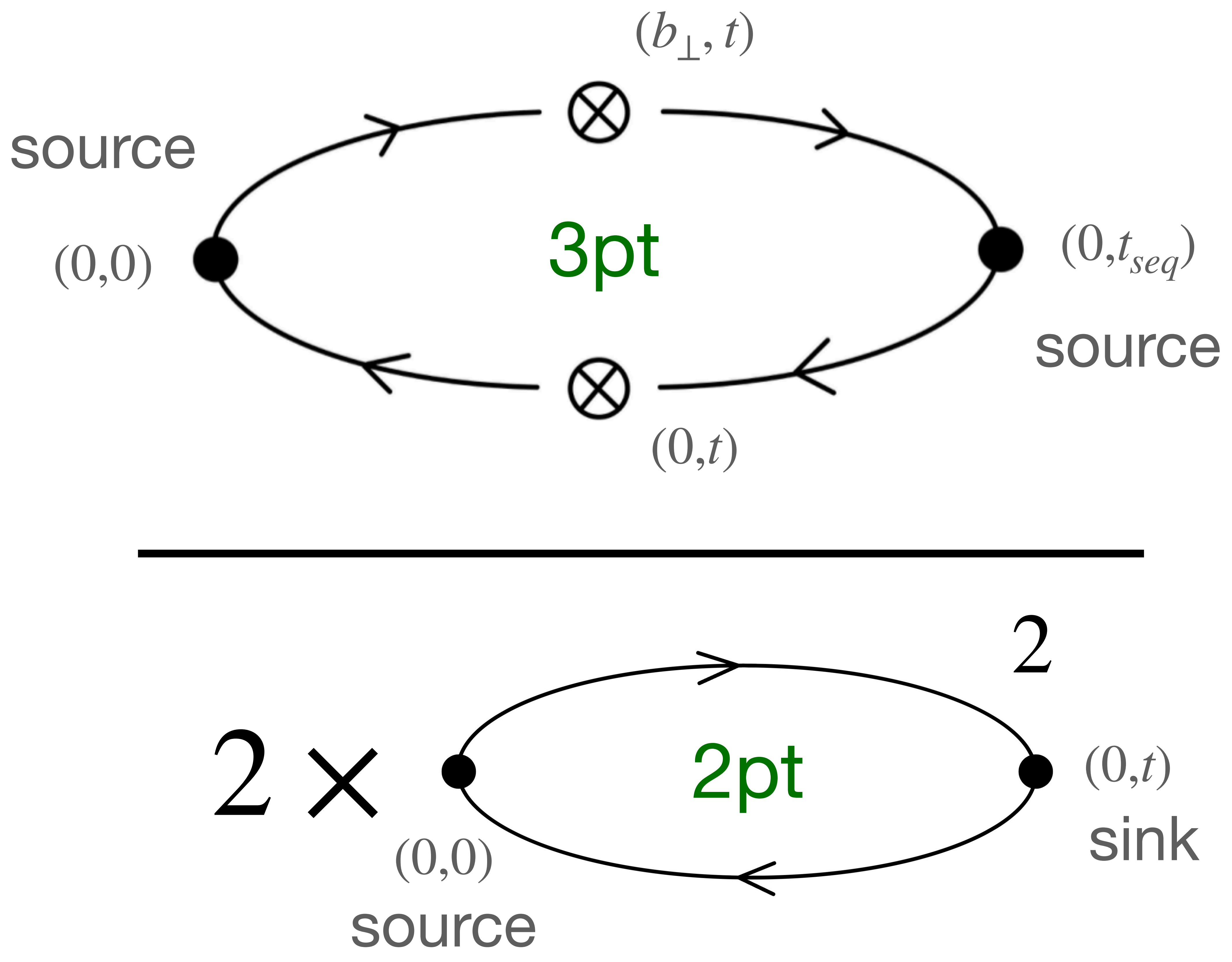
1. $F(\gamma^0)$, $F(\gamma^z)$, $F(\gamma^0\gamma_5)$ and $F(\gamma^z\gamma_5)$ have no contribution in leading order.
2. $F(I)$ and $F(\gamma^5)$ has UV divergence, while $F(\gamma^\perp)$ and $F(\gamma^\perp\gamma^5)$ have not.

Determination on lattice

$$C_3(b_\perp, t_{seq}, t, P)$$
$$= V\langle 0|\bar{q}(0,t_{seq})\gamma_5 q(0,t_{seq})|\pi(P_2)\rangle\langle\pi(P_1)|\bar{q}(0,0)\gamma_5 q(0,0)|0\rangle$$
$$\times \langle\pi(P_2)|(\bar{q}\Gamma q)|_{b_\perp}(\bar{q}\Gamma q)|_0|\pi(P_1)\rangle$$
$$= V\left(\frac{A_w A_{\gamma_5}}{2E}\right)^2 e^{-Et_{seq}}\langle\pi(P_2)|(\bar{q}\Gamma q)|_{b_\perp}(\bar{q}\Gamma q)|_0|\pi(P_1)\rangle$$

$$C_2(t_{seq}, P)$$
$$= V\langle 0|\bar{q}(0,t_{seq})\gamma^t\gamma_5 q(0,t_{seq})|\pi(P_1)\rangle\langle\pi(P_1)|\bar{q}(0,0)\gamma_5 q(0,0)|0\rangle$$
$$= V\frac{A_w A_{\gamma_5}}{2E}e^{-Et_{seq}}\langle 0|(\bar{q}\gamma^t\gamma_5 q)|_0|\pi(P_1)\rangle$$

$$\frac{VC_3(b_\perp, t_{seq}, t, P)}{2C_2^2(t=\frac{t_{seq}}{2})} = \frac{\langle\pi(P_2)|(\bar{q}\Gamma q)|_{b_\perp}(\bar{q}\Gamma q)|_0|\pi(P_1)\rangle}{2\langle 0|(\bar{q}\gamma^t\gamma_5 q)|_0|\pi(P_1)\rangle\langle\pi(P_1)|(\bar{q}\gamma^t\gamma_5 q)|_0|0\rangle}$$
$$= F(b_\perp, \Gamma, P^z)$$



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Simulation on lattice: Lattice setup

MILC configurations

$L^3 \times T$	a (fm)	m_π^{sea} (MeV)	m_π^v (MeV)
$24^3 \times 64$	0.12	310	670
			measurement
			1053

- 2+1+1 flavors of HISQ action (MILC)
- Momenta: 1.72GeV, 2.15GeV, 2.58GeV, 3.01GeV
- Coulomb gauge fixed wall source propagators

CLS configurations

$L^3 \times T$	a (fm)	m_π^{sea} (MeV)	m_π^v (MeV)
$48^3 \times 48$	0.098	333	662
			N_{cfg}
			952

- 2+1 flavors of Symanzik gauge action (CLS)
- Momenta: 1.58GeV, 2.11GeV, 2.64GeV, 3.16GeV
- Coulomb gauge fixed wall source propagators

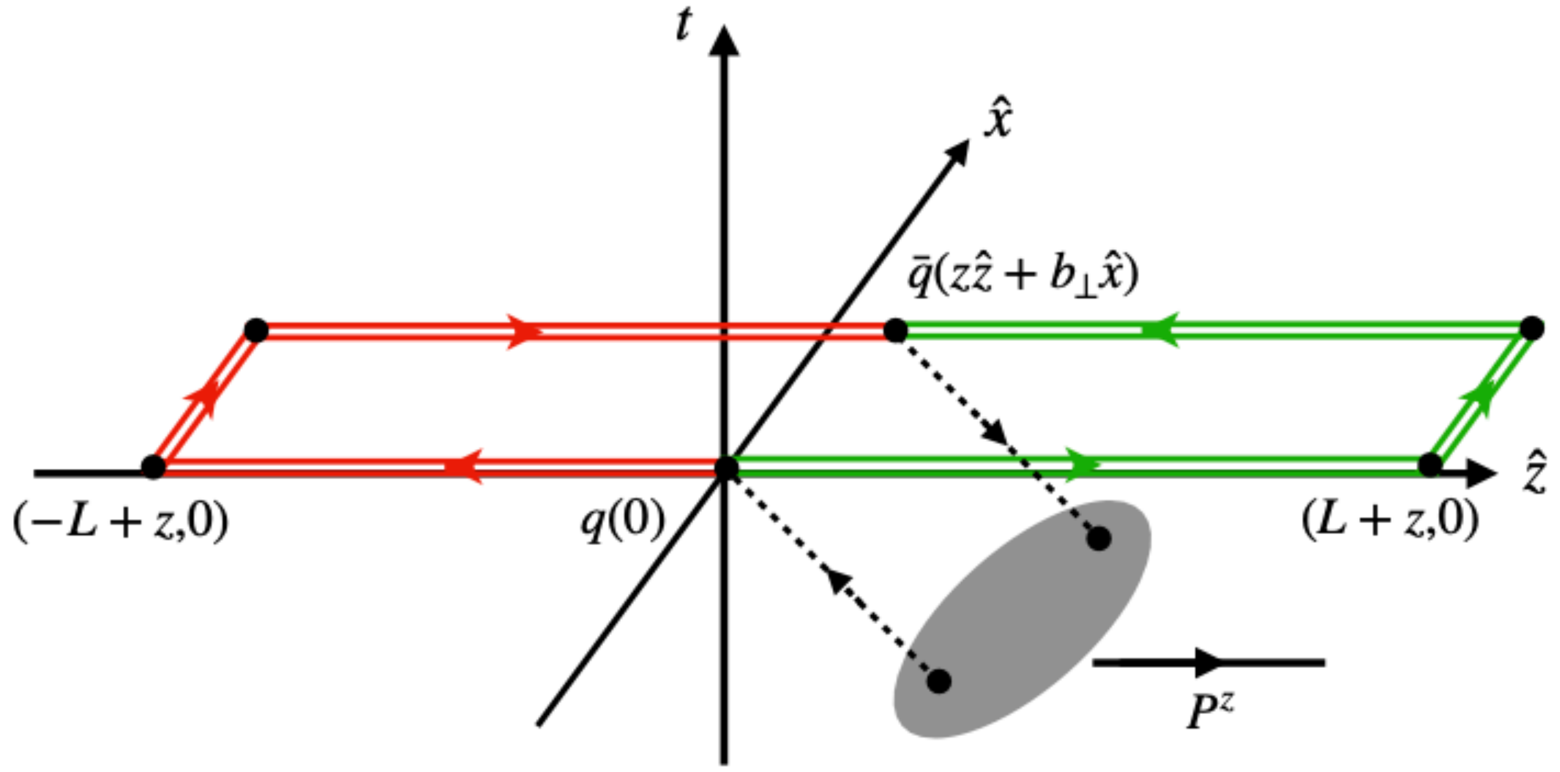
Simulation on lattice: Renormalization

quasi-TMDWF matrix element

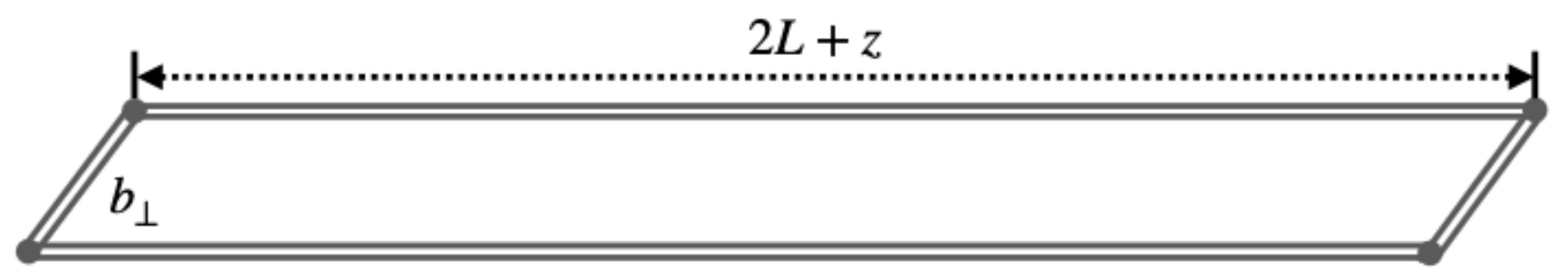
$$\tilde{\Psi}^{\pm}\left(z, b_\perp, \mu, \zeta^z\right) = \frac{\left\langle 0 \left| \bar{q}\left(z\hat{n}_z + b_\perp \hat{n}_\perp\right) \gamma^t \gamma_5 U_{c\pm} q(0) \right| \pi\left(P^z\right) \right\rangle}{\sqrt{Z_E\left(2L \pm z, b_\perp, \mu\right)} Z_O(1/a, \mu, \Gamma)}$$

Wilson loop: linear divergence, pinch pole singularity

MSbar factor: logarithm divergence



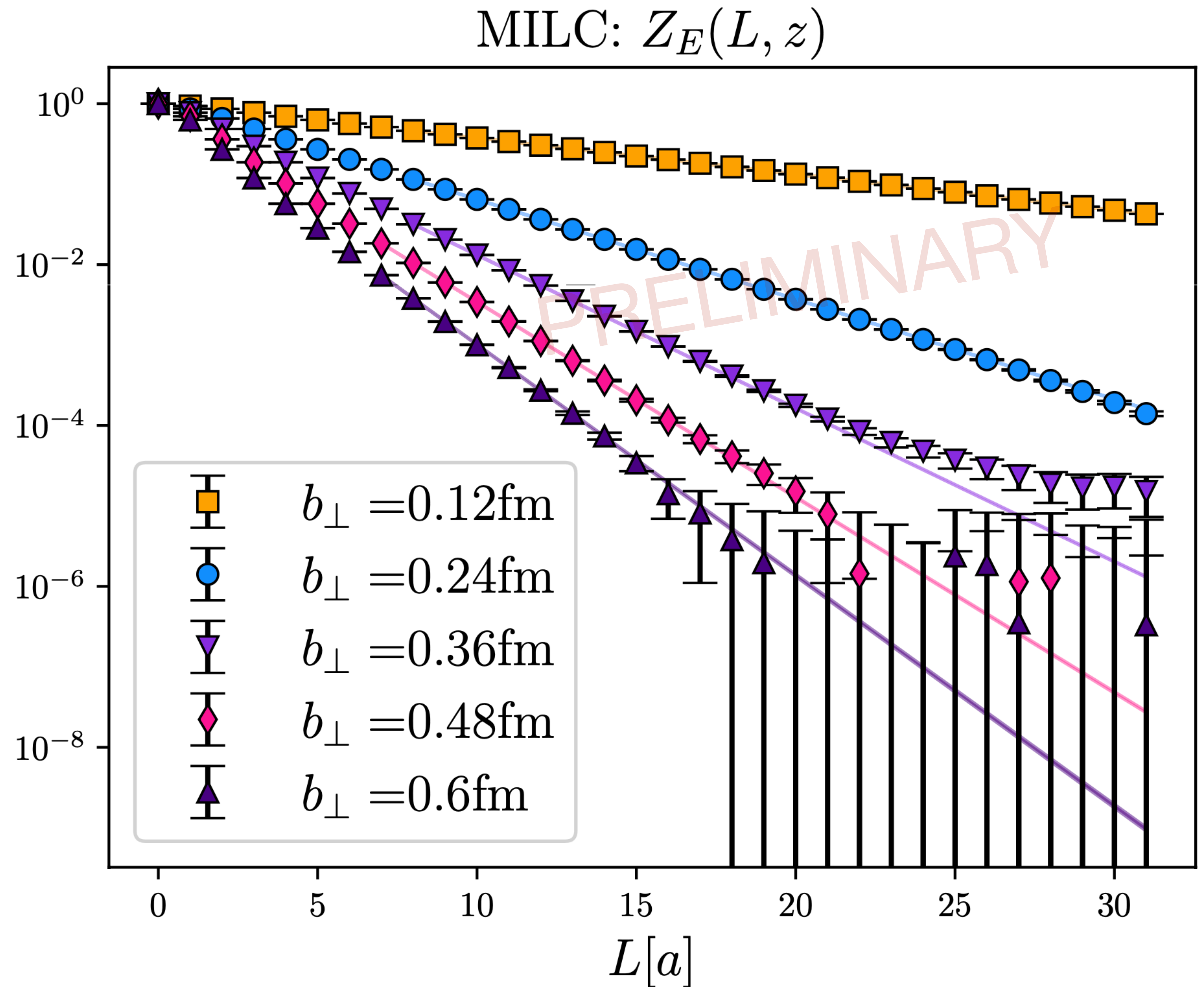
Wilson line in matrix element



Wilson loop

Extrapolation of Wilson loop

$$Z_E = c_0 e^{-EL}(1 + c_1 e^{-\Delta EL})$$

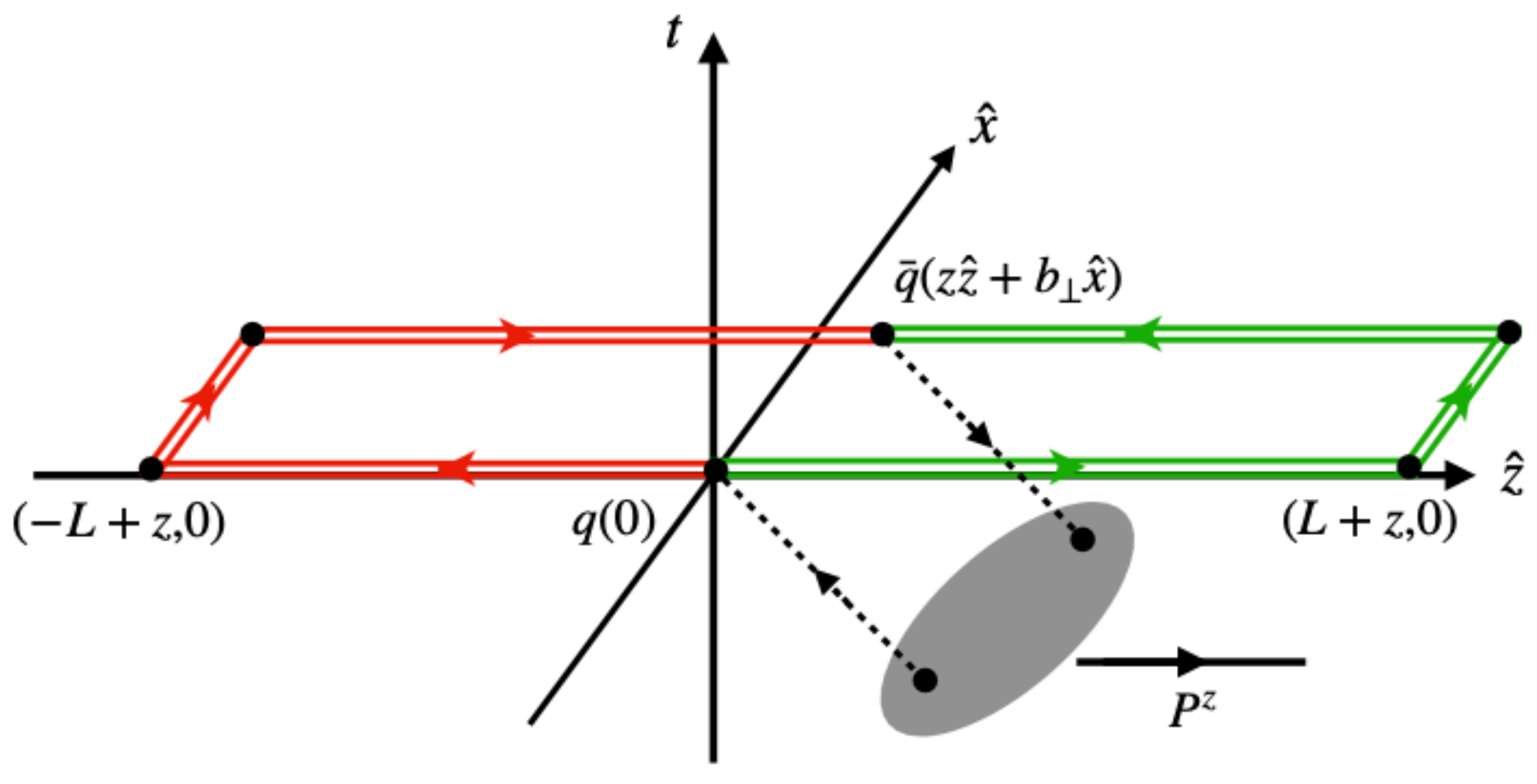


quasi-TMDWF matrix element

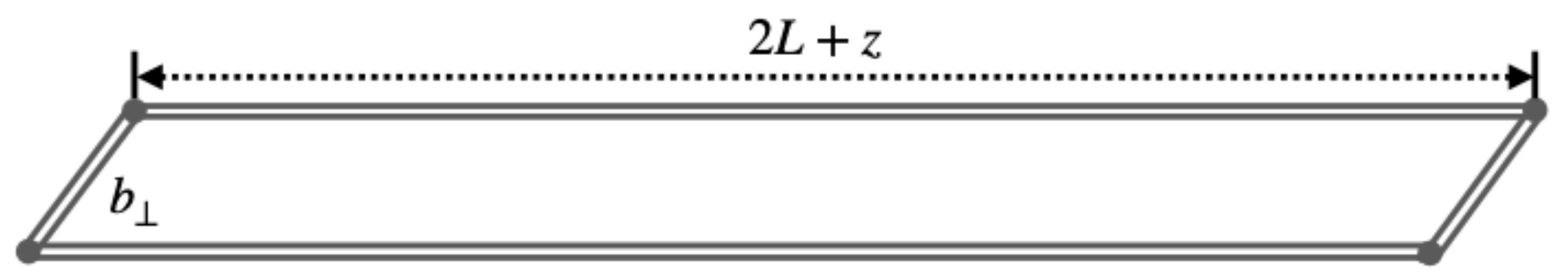
$$\tilde{\Psi}^{\pm}\left(z, b_{\perp}, \mu, \zeta^{z}\right)=\frac{\left\langle 0\left|\bar{q}\left(z \hat{n}_{z}+b_{\perp} \hat{n}_{\perp}\right) \gamma^{t} \gamma_{5} U_{c \pm} q(0)\right| \pi\left(P^{z}\right)\right\rangle}{\sqrt{Z_{E}\left(2 L \pm z, b_{\perp}, \mu\right)} Z_{O}(1 / a, \mu, \Gamma)}$$

Wilson loop: linear divergence, pinch pole singularity

MSbar factor: logarithm divergence



Wilson line in matrix element

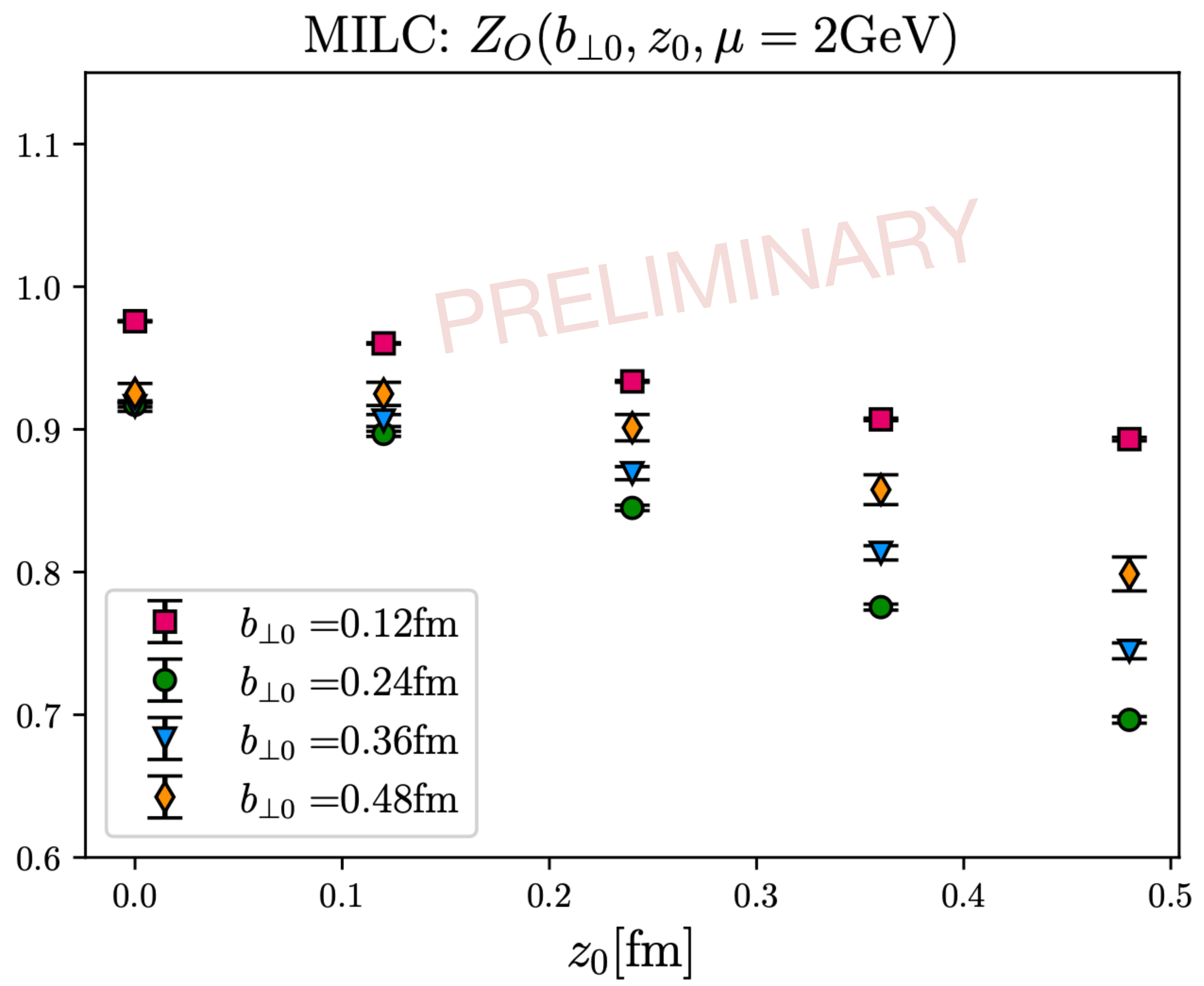


Wilson loop

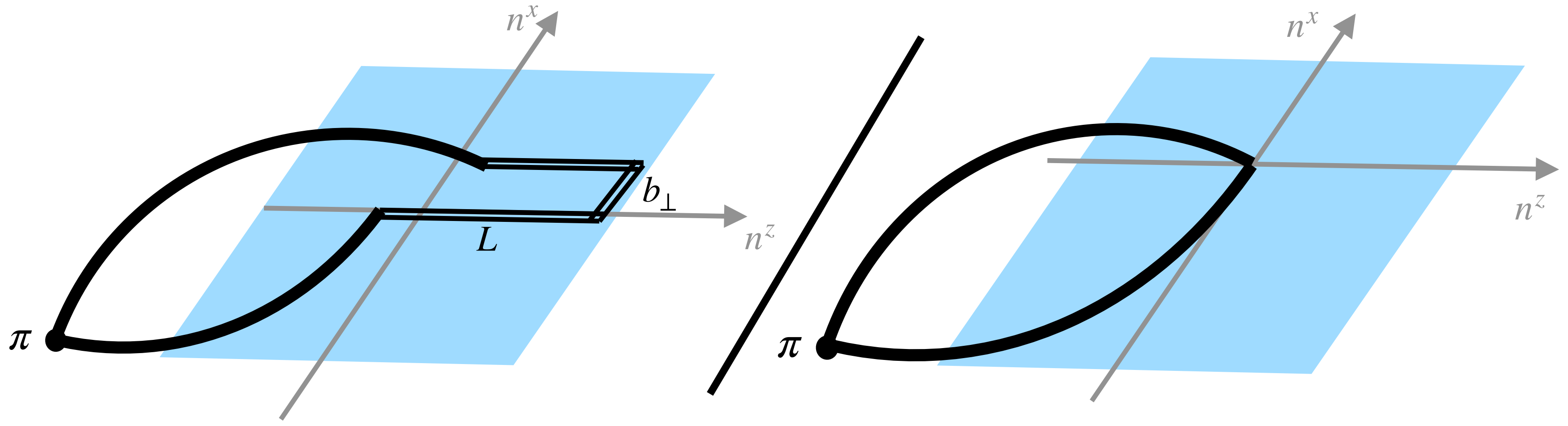
MSbar renormalization factor

$$Z_{O}=\frac{\tilde{\Psi}^{\pm 0}\left(z_{0}, b_{\perp 0}, \mu, P^{z}=0, L\right)}{\sqrt{Z_{E}\left(b_{\perp 0}, 2 L \pm z_{0}\right)} \tilde{\psi}^{\overline{\mathrm{MS}}}\left(b_{\perp 0}, z_{0}, \mu\right)}$$

K. Zhang et al, arxiv 2205.13402(2022)



normalized 2pt

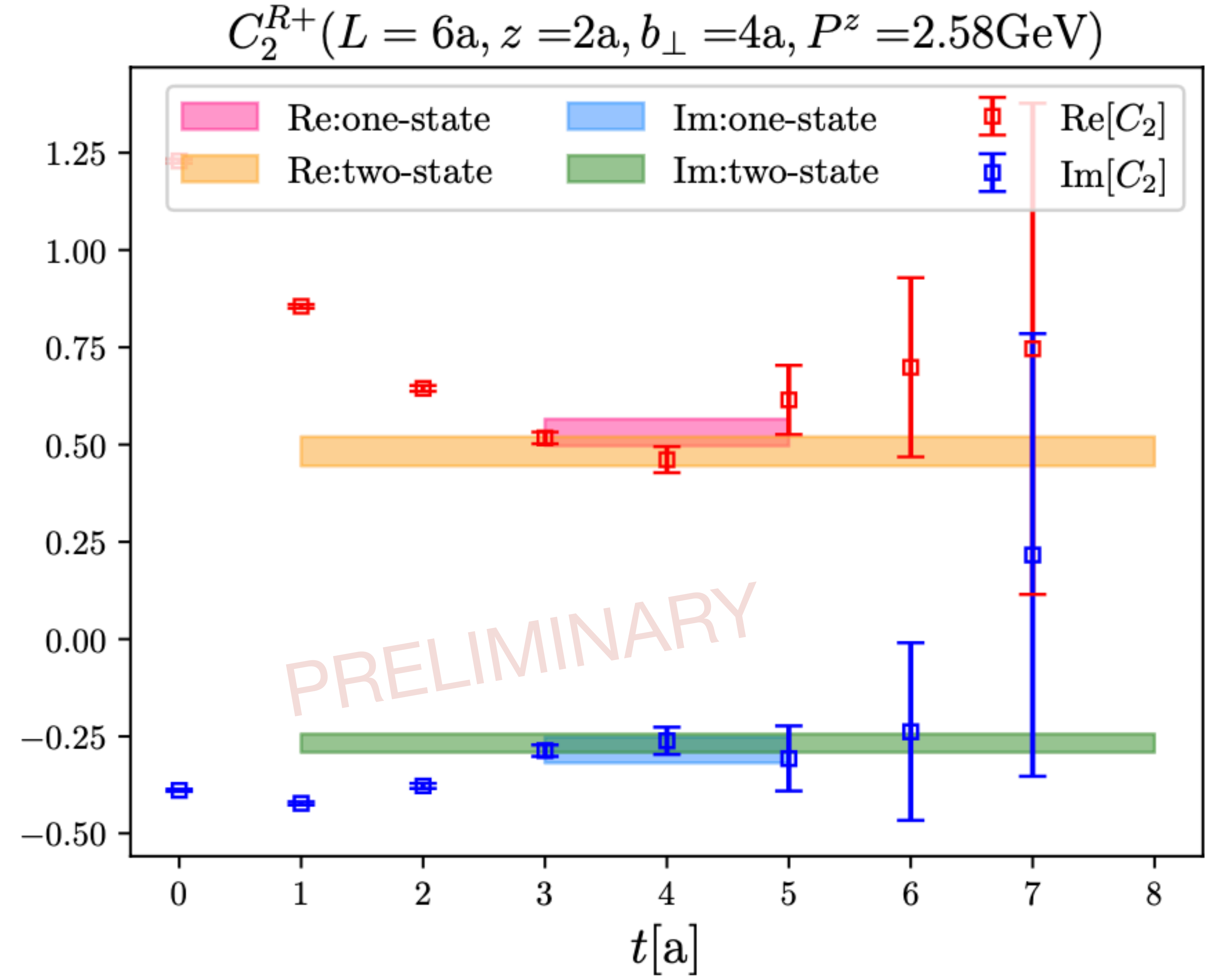
$\bar{C}_2(L,b,z,t) =$ 

two-state fit

$$\bar{C}_2(L,b,z,t) = \frac{\tilde{\Psi}(L,b,z)(1+c_1(L,b,z)e^{-\Delta E t})}{1+c_0(0)e^{-\Delta E t}}$$

1-state fit $\bar{C}_2(L,b,z,t) = \tilde{\Psi}(L,b,z)$

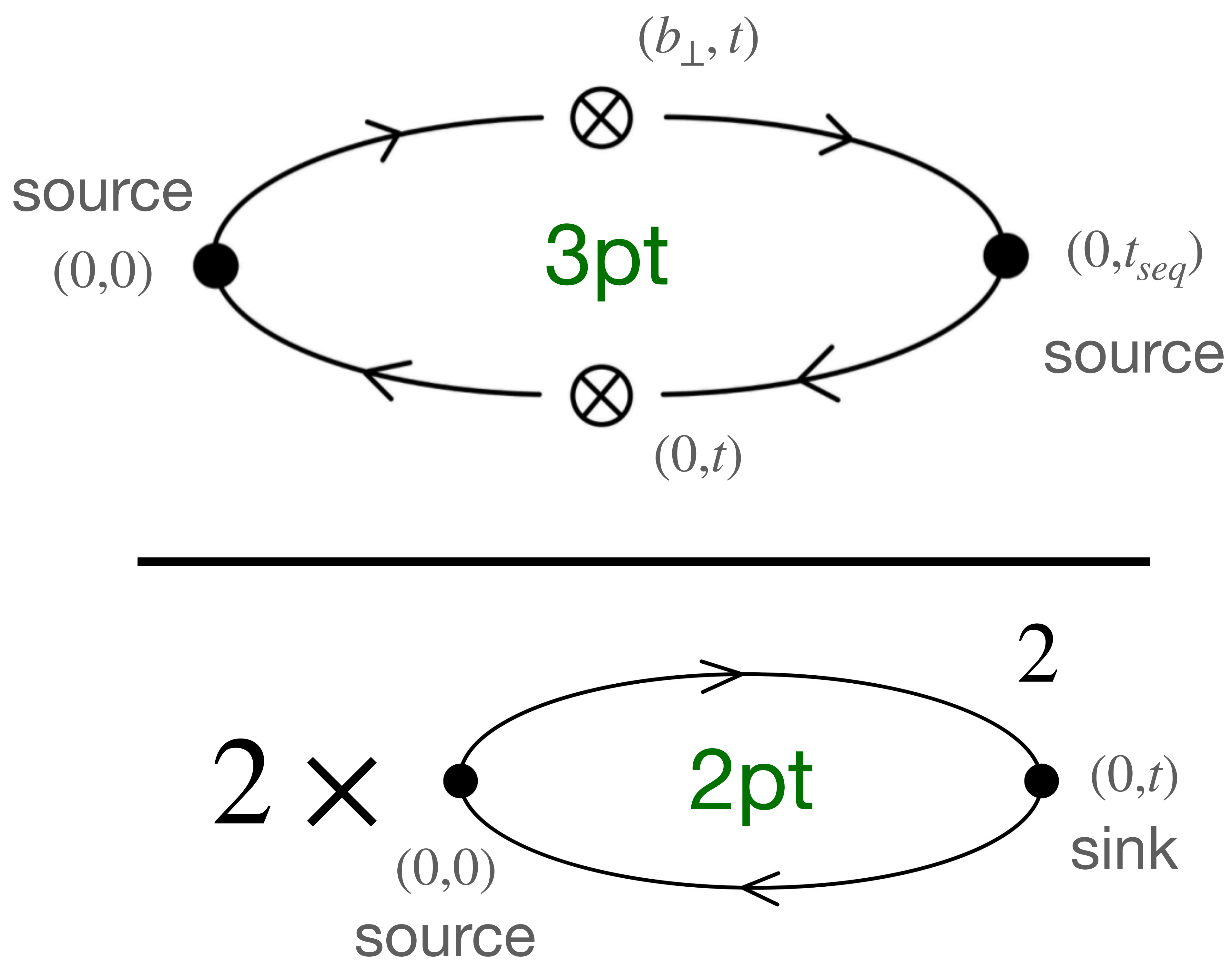
1- and 2-state fit for 2pt



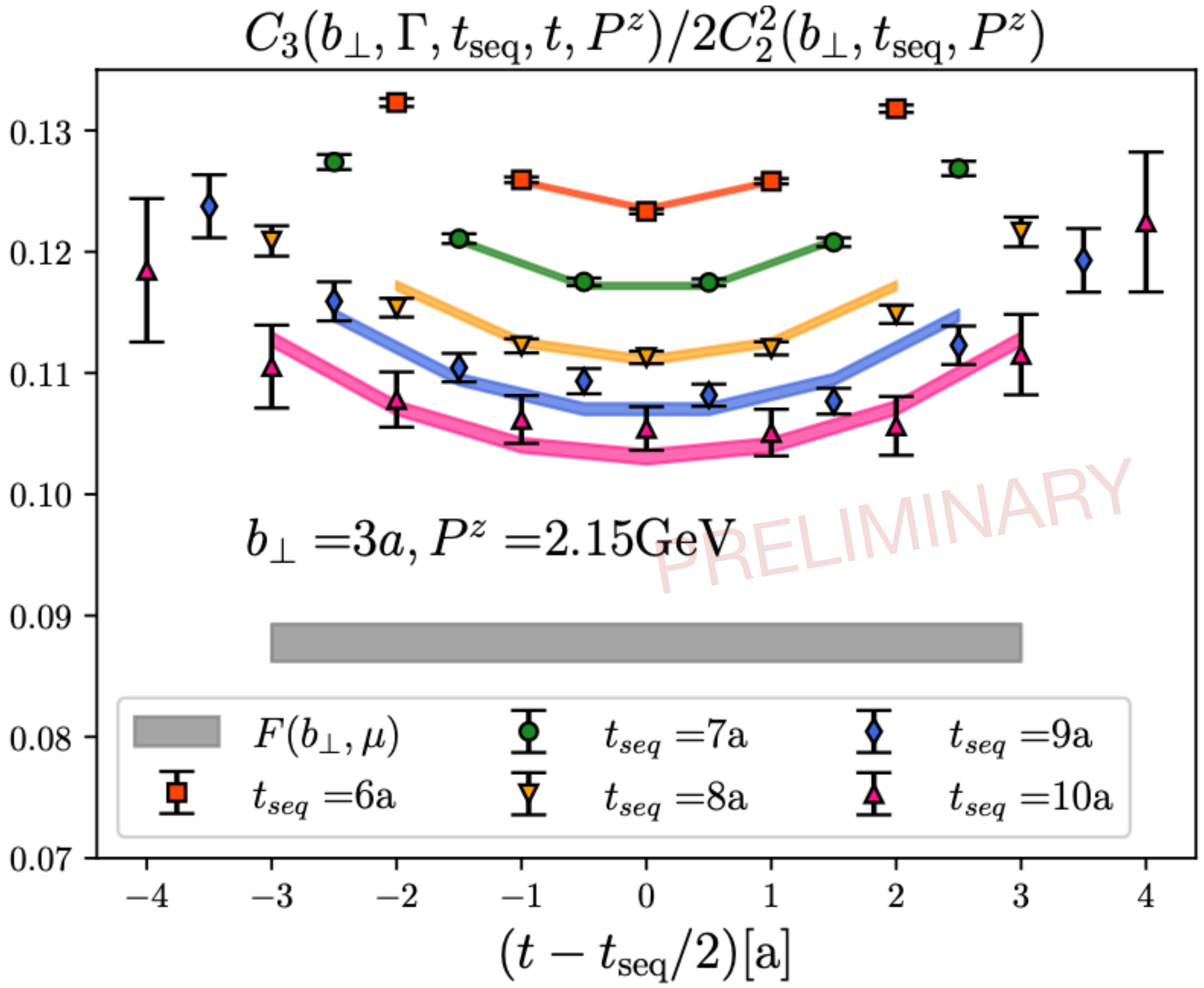
Simulation on lattice: fit for 3pt/2pt

ratio of 3pt/2pt

$$\frac{C_3(b_\perp,\Gamma,t_{\text{sep}},t,P^z)}{2C_2^2(t_{\text{sep}}/2,P^z)} = F(b_\perp,\Gamma,P^z)\frac{1+c_1(e^{-\Delta E t}+e^{-\Delta E(t_{\text{sep}}-t)})}{1+c_2 e^{-\Delta E t_{\text{sep}}/2}}$$



Joint fit with $t_{\text{sep}} = \{6,7,8,9,10\}a$



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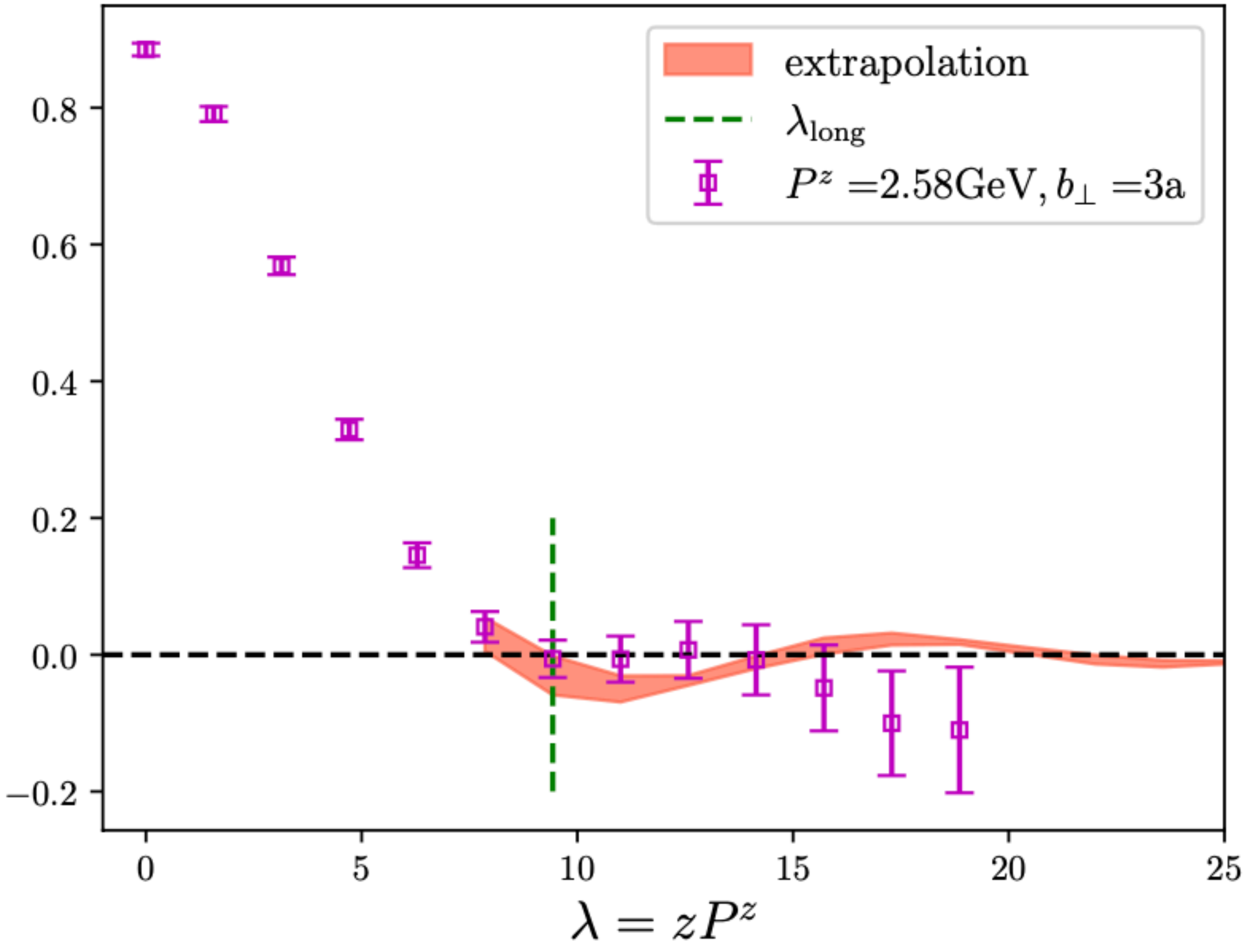
Numerical Result: quasi-TMDWFs

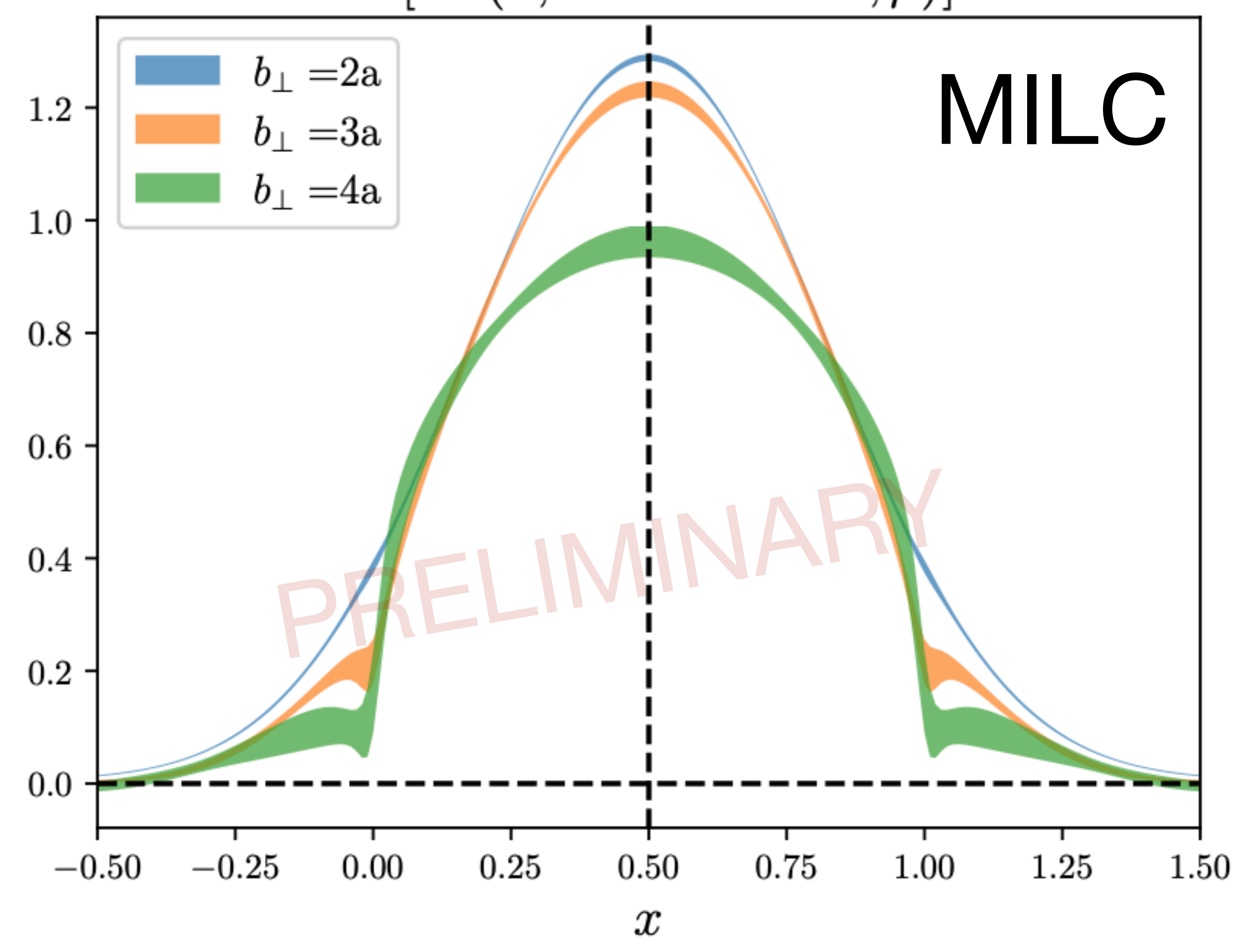
extrapolation

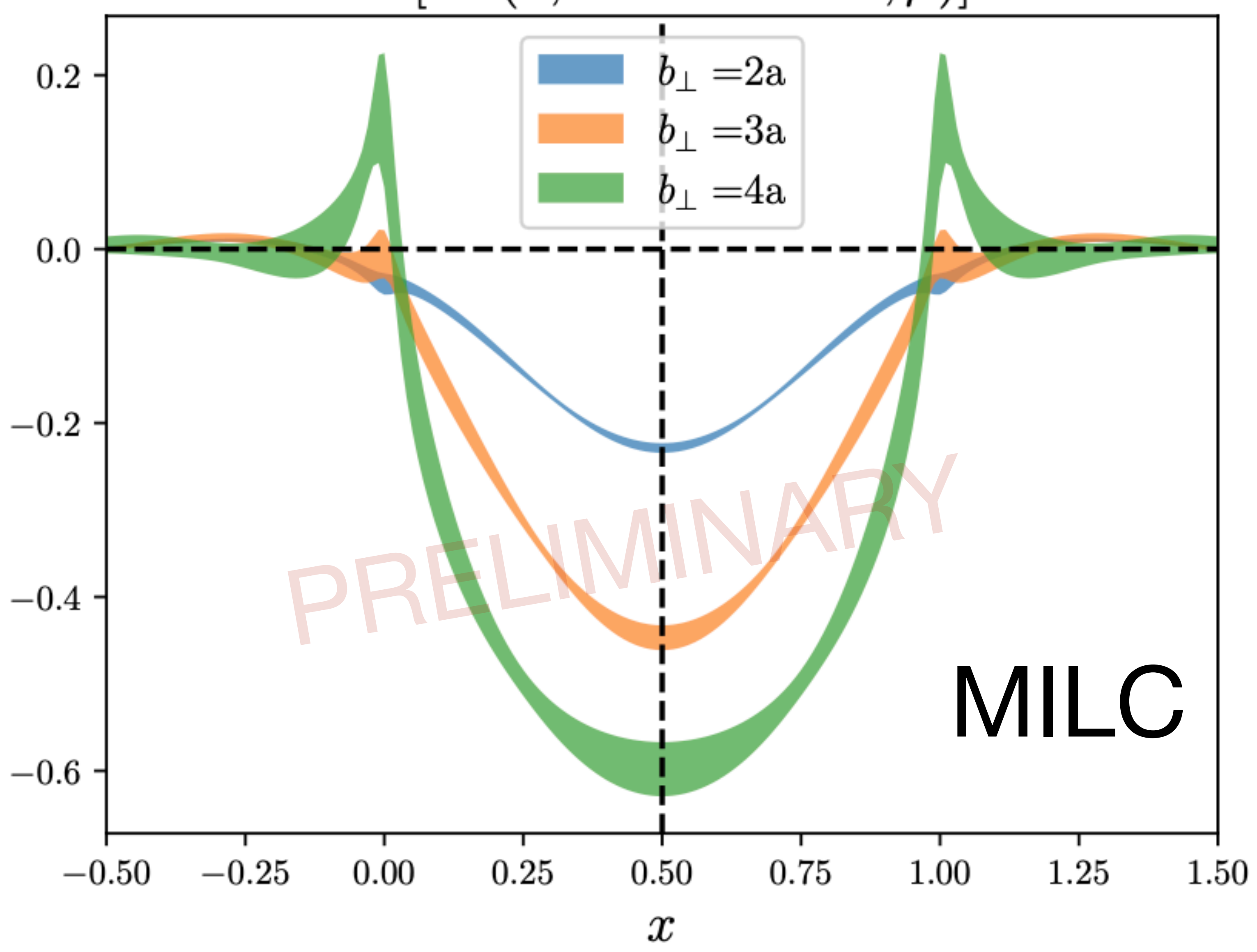
$$\tilde{\Psi}(z, b_\perp, \mu, P^z) = f(b_\perp)\left[\frac{c_1}{(-i\lambda)^d} + e^{i\lambda}\frac{c_2}{(i\lambda)^d}\right] e^{-\frac{\lambda}{\lambda_0}}$$

Joint fit of $b_\perp$s



quasi-TMDWF in momentum space





Numerical Result: intrinsic soft function

Intrinsic soft function matching

$$S_I\left(b_\perp,\mu\right)=\frac{F\left(b_\perp,P^z,\Gamma,\mu\right)}{\int dx_1dx_2H\left(x_1,x_2\right)\tilde{\Psi}^{\pm *}\left(x_2,b_\perp,\zeta^z\right)\tilde{\Psi}^{\pm}\left(x_1,b_\perp,\zeta^z\right)}$$

Infinite P^z limit

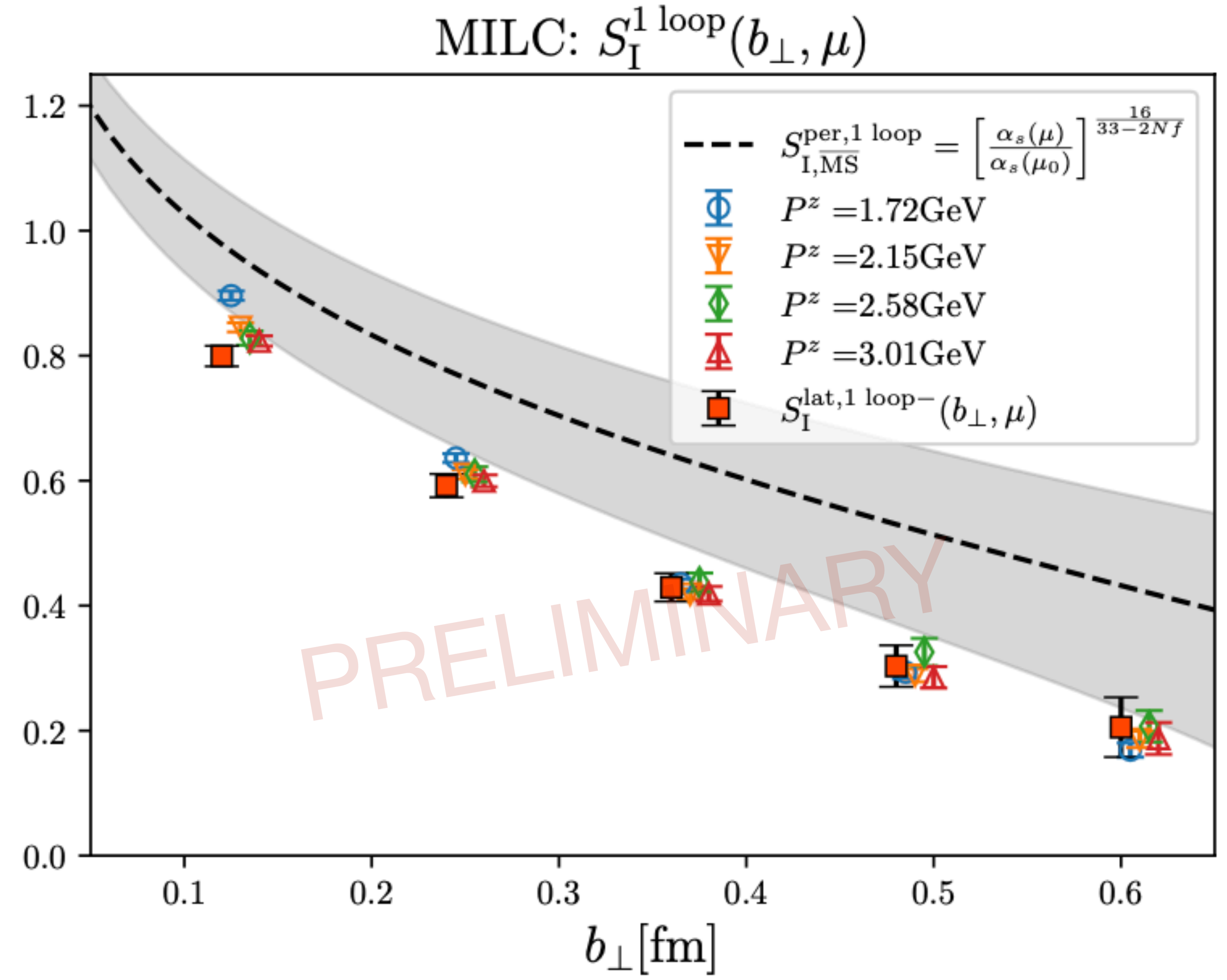
$$S_I(P^z)=S_I(P^z=limit)+\frac{C}{(P^z)^2}$$

Perturbative calculation

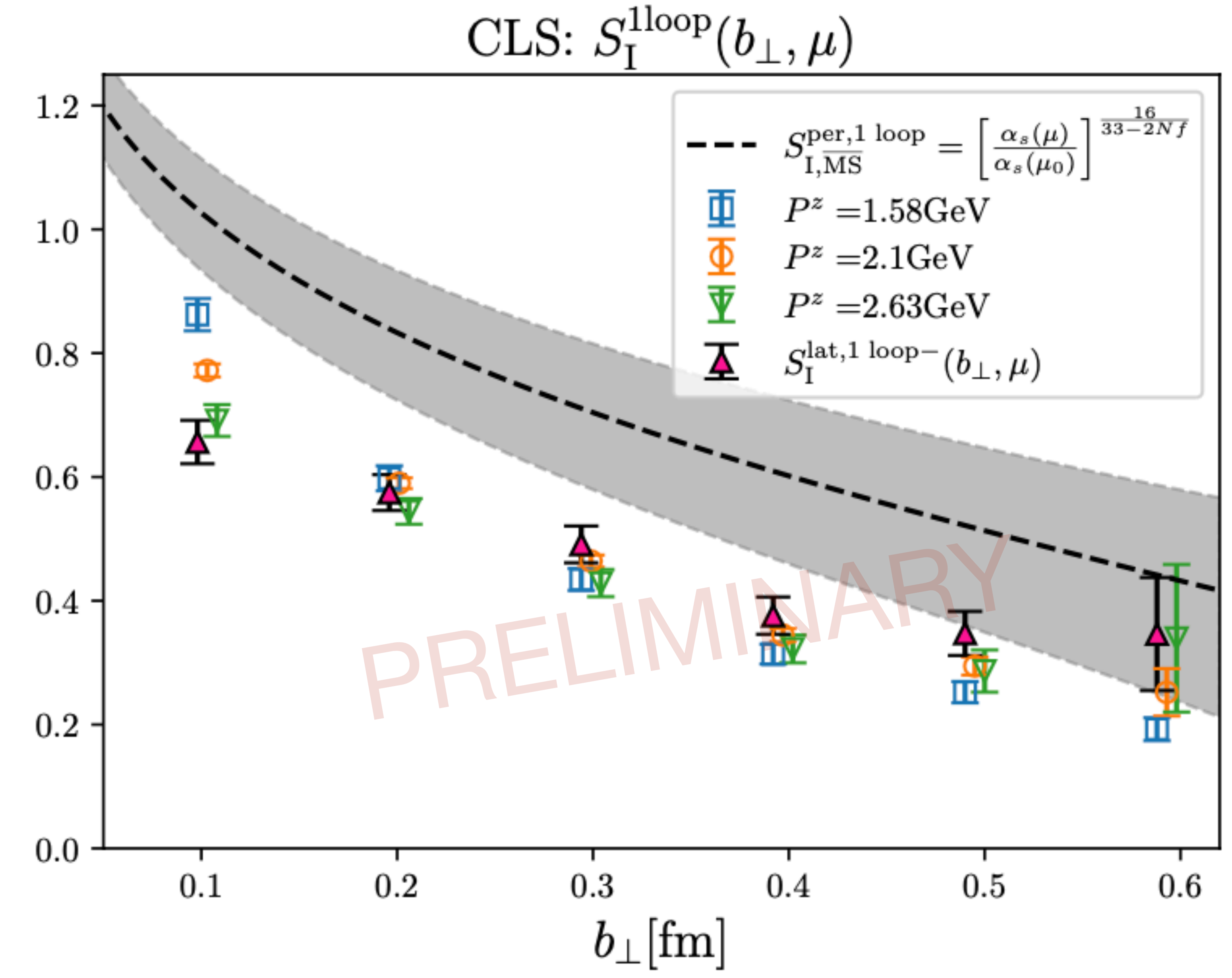
$$S^{\text{per,1 loop}}_{\text{I},\overline{\text{MS}}}=\left[\frac{\alpha_s(\mu=2\text{GeV})}{\alpha_s(\mu_0=1/b^*_\perp)}\right]^{\frac{16}{33-2N_f}}$$

scale $b^*_\perp\in\left[1/\sqrt{2},\sqrt{2}\right]b_\perp$

MILC



CLS



Numerical Result: intrinsic soft function

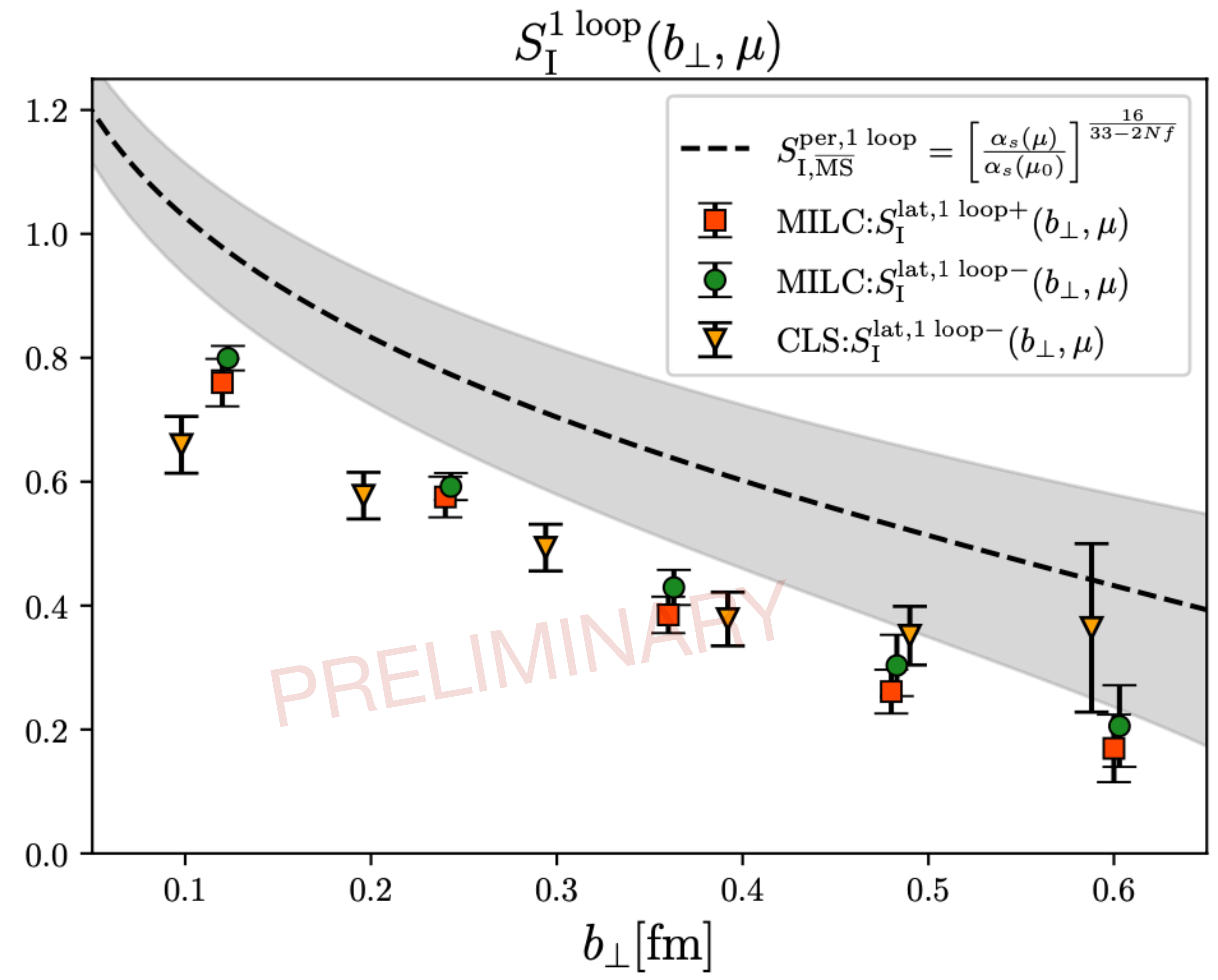
Intrinsic soft function matching

$$S_I\left(b_\perp,\mu\right) = \frac{F\left(b_\perp, P^z, \Gamma, \mu\right)}{\int dx_1 dx_2 H\left(x_1,x_2\right)\tilde{\Psi}^{\pm *}\left(x_2,b_\perp,\zeta^z\right)\tilde{\Psi}^{\pm}\left(x_1,b_\perp,\zeta^z\right)}$$

with systematic uncertainties

$$\sigma_{\rm all} = \sqrt{\sigma^2_{\rm stt} + \sigma^2_{z_{\rm st}+1} + \sigma^2_{z_{\rm st}-1} + \sigma^2_{P^z_{\rm max}-P^z_{\rm limit}}}$$

- $\sigma_{z_{\rm st}\pm 1}$: difference between extrapolation range $[z_{\rm st}, z_{\rm max}]$ and $[z_{\rm st}\pm 1, z_{\rm max}]$
- $\sigma^2_{P^z_{\rm max}-P^z_{\rm limit}}$: difference between $P^z = {\rm limit}$ and the largest used P^z.

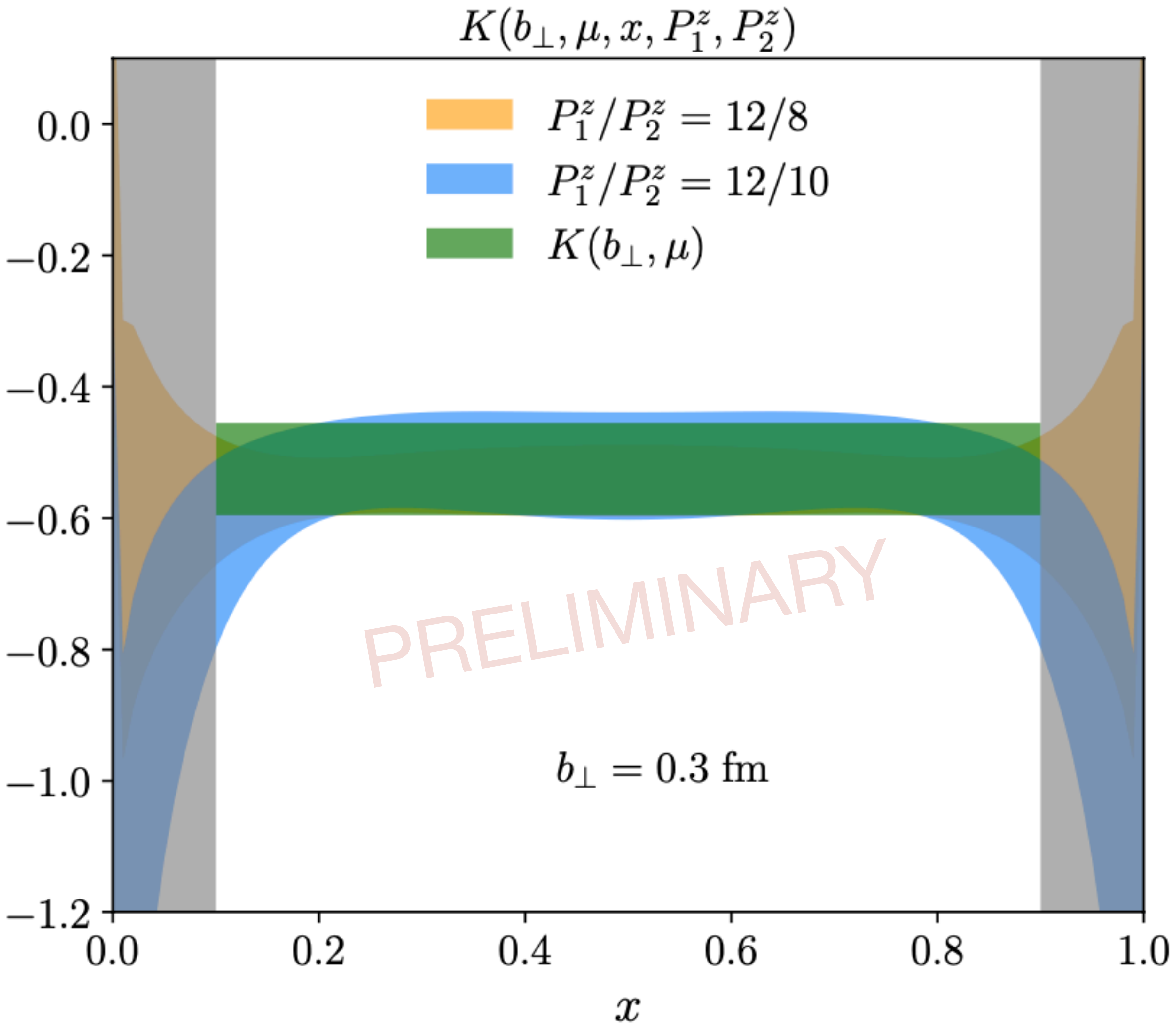


- MILC and CLS perform similar results for intrinsic soft function.
- They are all close to the perturbative calculation.

Numerical Result: CS kernel

Fit for power corrections

$$K(b_\perp, \mu, x, P_1^z, P_2^z) = K(b_\perp, \mu) + A\left[\frac{1}{x^2(1-x)^2(P_1^z)^2} - \frac{1}{x^2(1-x)^2(P_2^z)^2}\right]$$

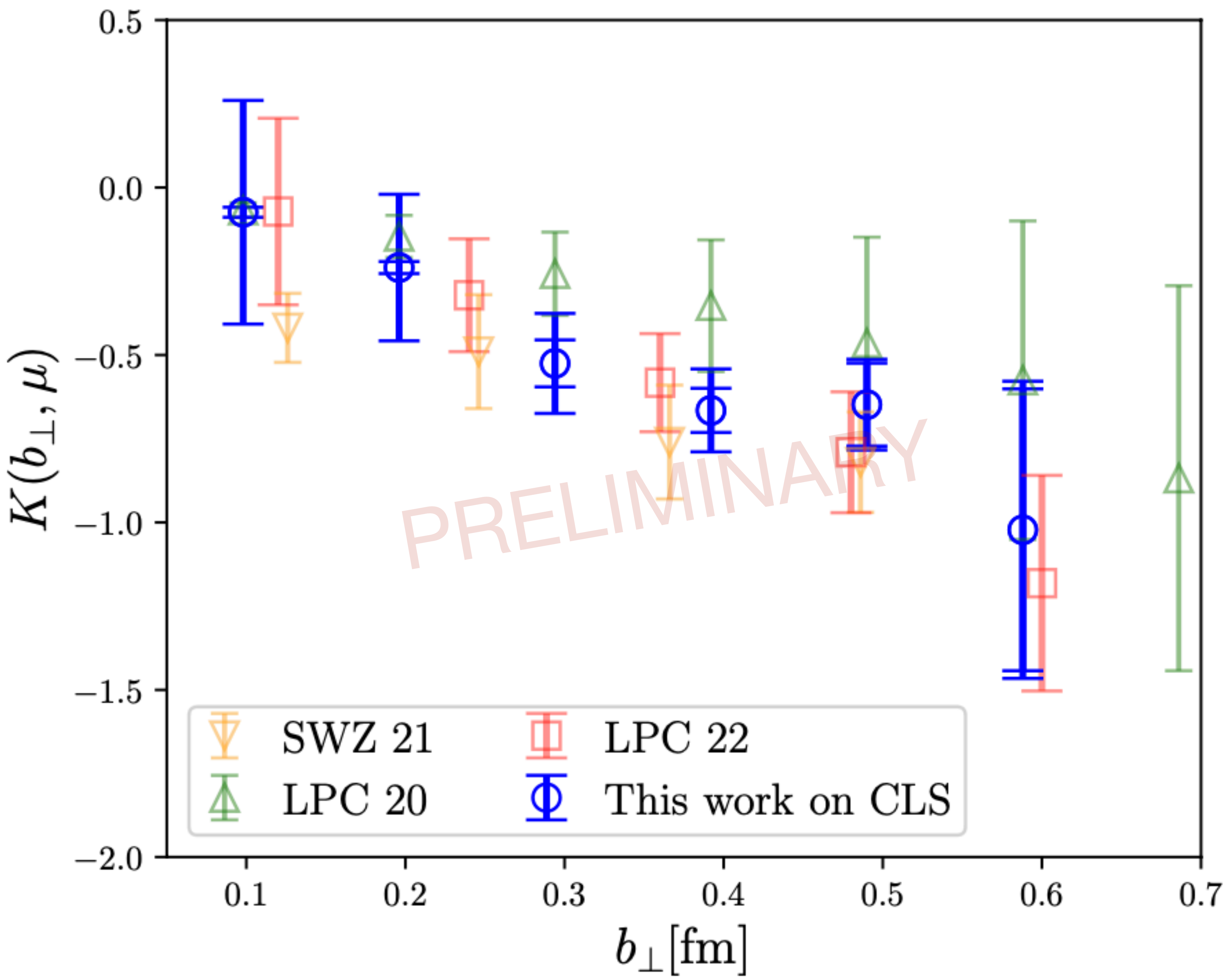


Imaginary part to systematic uncertainties

$$\sigma_{\rm sys} = \sqrt{K(b_\perp, \mu)^2 + {\rm Im}K(b_\perp, \mu)^2} - K(b_\perp, \mu)$$

M. Chu et al, arxiv 2204.00200(2022)

Comparison with previous calculations



- Motivation: Soft function
- Intrinsic soft function in LaMET
- Simulation on lattice
- Numerical results
- Summary

Summary and outlook

- Soft function describes the soft gluon radiation part in TMD factorization.
- This is the first attempt for extracting soft function at 1-loop accuracy.
- This has added evidences for TMD factorization in LaMET.

Thanks for your attention!