SOUTH CHINA NORMAL UNIVERSITY 华南师范大学

Lattice Parton Collaboration

Lattice calculation of TMDWF by LaMET

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CONTENTS

1 Introduction

2 Process: Soft function & CS kernel

3 Numerical calculation on LQCD

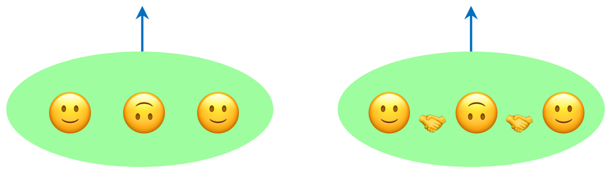
4 Numerical results

5 Summary

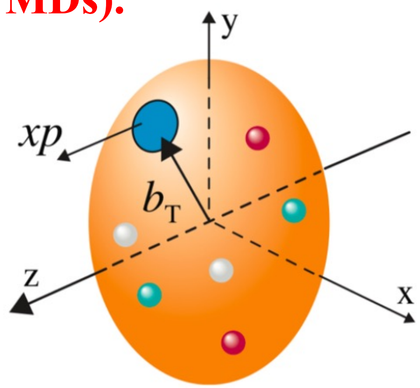
Introduction

Why TMDs ?

- After several decades of research into the inner structures of hadrons, our understanding of structures has expanded from 1-dimensional (PDF, DA…) to 3-dimensional (TMDs).

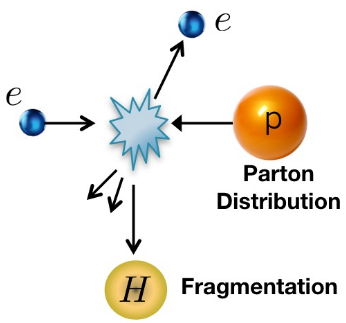


PDFs TMDs

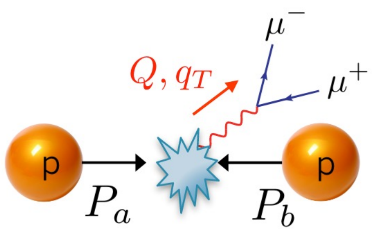


- TMDs are playing an increasing important role in understanding high-energy scattering

Semi-Inclusive DIS

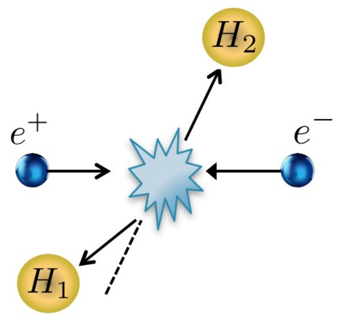


Drell-Yan



$q_T \ll Q$

Dihadron in e^+e^-



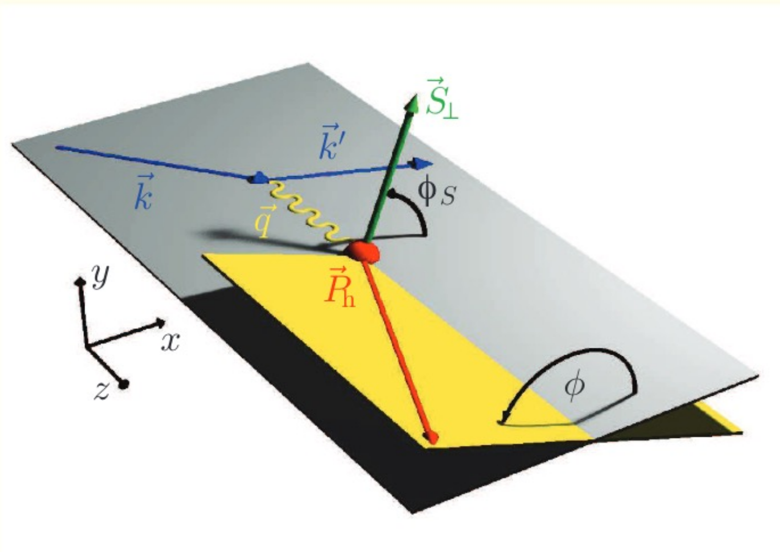
Introduction

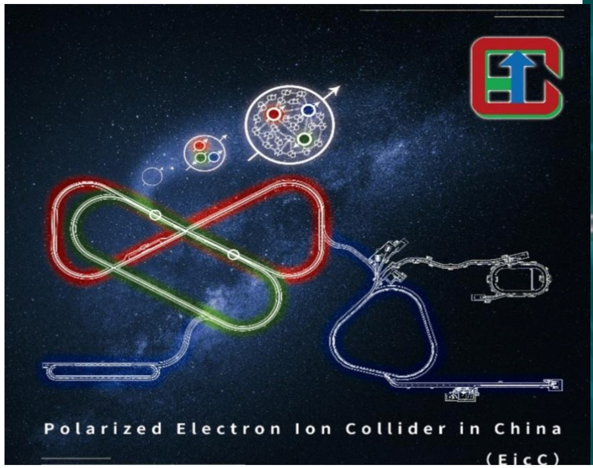
Why TMDs ?

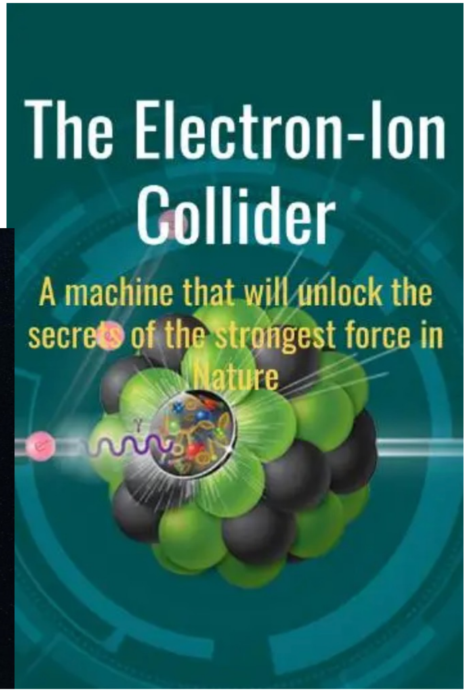
➢ **Targets for the next generation of high energy colliders.**

Semi-inclusive hadron production in deep inelastic scattering (SIDIS)

Transverse momentum dependent (TMD) quark distributions of nucleons



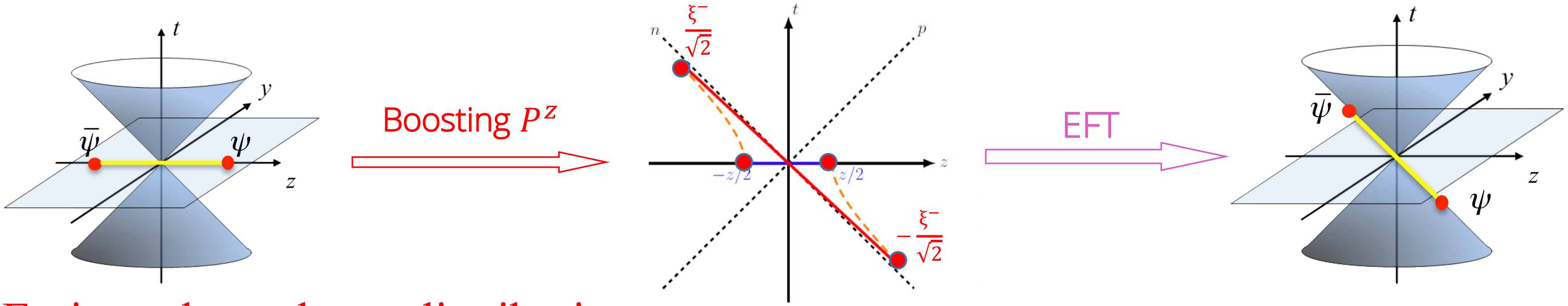




Introduction

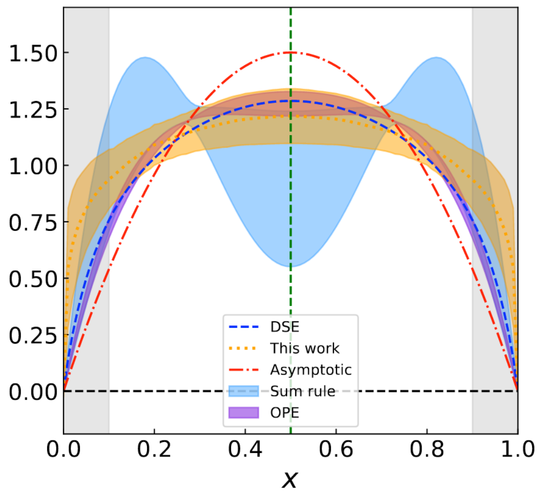
Why LaMET ?

➢ Large Momentum Effective Theory:

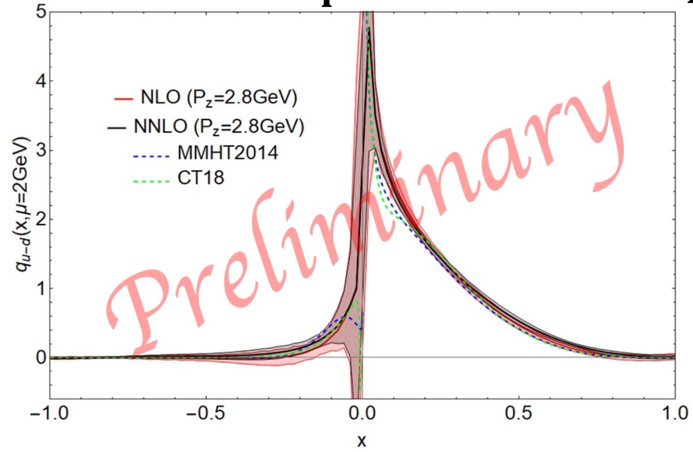


- Entire x dependence distributions

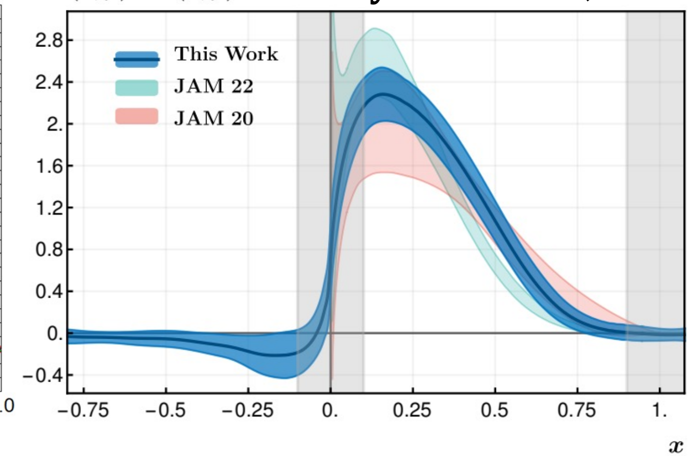
π LCDA PRL129(2022)



Proton unpolarized PDF

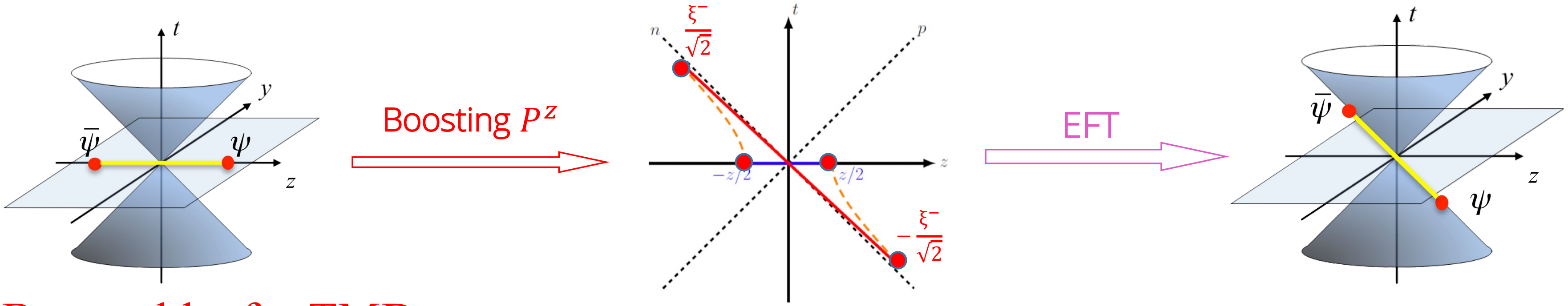


Proton transversity PDF 2208, 08008



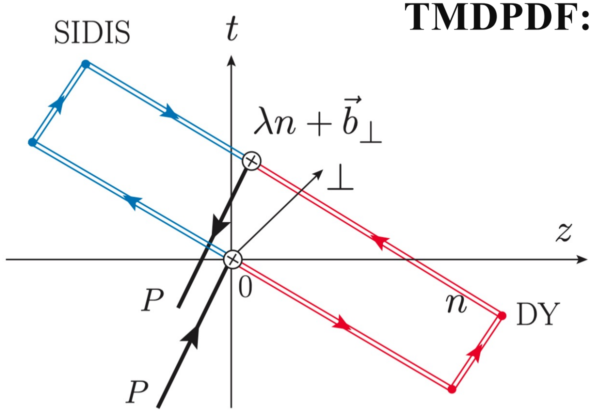
Introduction

- **Large Momentum Effective Theory:**

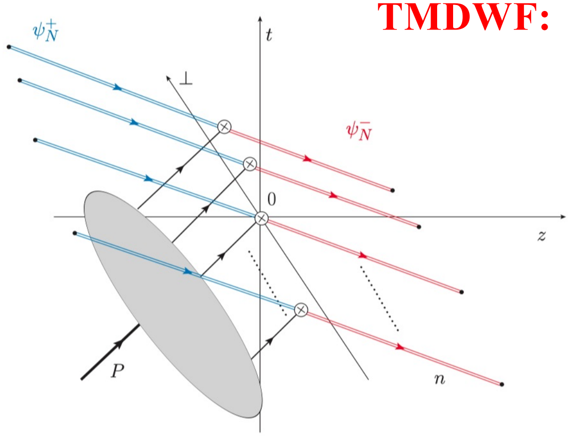


- Be capable for TMDs

TMDPDF:

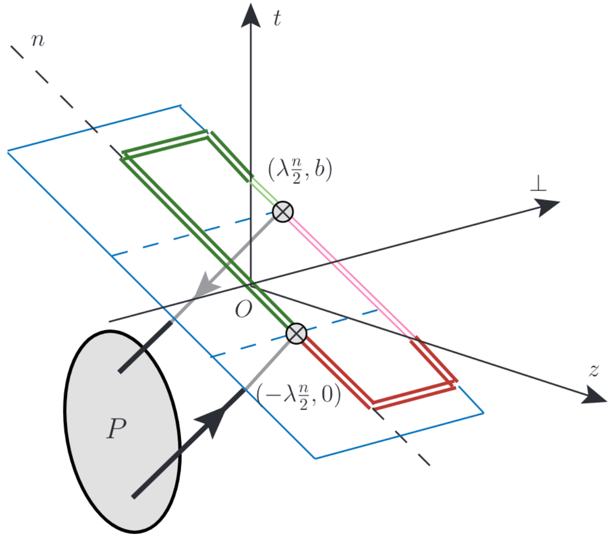


TMDWF:



Introduction

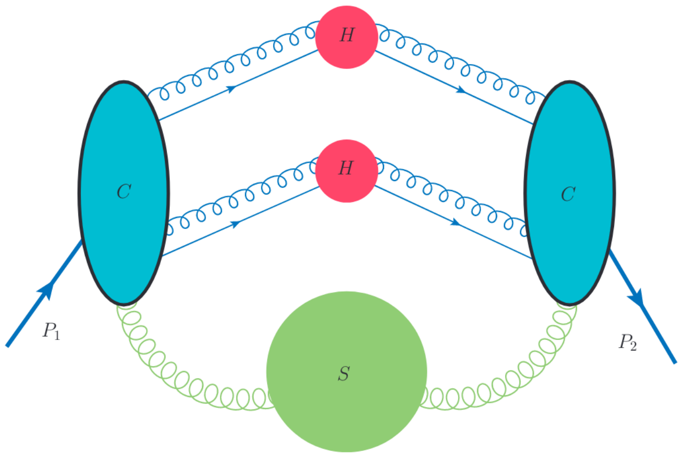
For TMDWFs



- Nonperturbative nature of hadrons in phenomenological research on exclusive processes

- Platform for extracting soft function

- More stable for extracting Collins-Soper kernel



Process: SF & CS K

Lattice Calculation

- (LPC) M.H.Chu et.al. PRD.106, 034509 (2022)
  CS kernel from quasi-TMDWFs (1-loop)
- P. Shanahan et.al. PRD.104, 114502(2021)
  CS kernel from quasi-TMDPDFs (1-loop)
- M. Schlemmer et.al. JHEP.08,004(2021)
  CS kernel by different TMDs
- L.Yuan, X.Feng et.al. PRL. 128, 062002 (2022)
  Twists' effects on soft function
- (LPC) Q.A. Zhang et.al. PRL. 125, 192001 (2020)
  Soft function and CS kernel (First)

Process: SF & CS K

- Y.S.Su et.al. 2209.01236 (2022)
  Longitudinal momentum logarithms resummation

Theoretical Support

- (LPC) K.Zhang PRL.129,082002 (2022)
  Renormalization of TMDs on lattice

- Z.F.Deng et.al. JHEP.09,046(2022)
  TMDWF and one-loop soft function in LaMET

- X.D.Ji, Y.Z.Liu PRD. 105, 076014 (2021)
  TMDWFs' calculation in LaMET

- X.D.Ji et.al. RMP.93, 035005(2021)
  An overview on on LaMET

Calculation on LQCD

MILC ensemble

$L^3 \times T$	$a(fm)$	$m_\pi^{sea}(MeV)$	$m_\pi^{v}(MeV)$
$24^3 \times 64$	0.121	310	670
			measurement
			1053×4

- 2+1+1 flavors of HISQ action (MILC)
- Momenta: 1.72GeV, 2.15GeV, 2.58GeV, 3.01GeV
- Coulomb gauge fixed wall source propagators

CLS ensemble

$L^3 \times T$	$a(fm)$	$m_\pi^{sea}(MeV)$	$m_\pi^{v}(MeV)$
$48^3 \times 48$	0.098	333	662
			measurement
			952×4

- 2+1 flavors of Symanzik gauge action (CLS)
- Momenta: 1.58GeV, 2.11GeV, 2.64GeV, 3.16GeV
- Coulomb gauge fixed wall source propagators

Calculation on LQCD

➢ **Multiplicative factorization of quasi-TMDWF in LaMET**

$$\tilde{\Psi}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right) S_{I}^{\frac{1}{2}}\left(b_{\perp}, \mu\right)$$
$$= H^{\pm}\left(x, \zeta^{z}, \mu\right) \exp\left[\frac{1}{2} K\left(b_{\perp}, \mu\right) \ln \frac{\pm\zeta^{z} + i\epsilon}{\zeta}\right] \Psi^{\pm}\left(x, b_{\perp}, \mu, \zeta\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^{2}}{x\zeta_{z}}, \frac{M^{2}}{(P^{z})^{2}}, \frac{1}{b_{\perp}^{2}\zeta_{z}}\right)$$

$\tilde{\Psi}^{\pm}(x, b_{\perp}, \mu, \zeta_{z})$: **Quasi-TMDWF, calculable on Euclidean lattice**

$S_{r}(b_{\perp}, \mu)$: **Intrinsic soft function, non-canceling soft gluon radiation**

$H^{\pm}(\zeta_{z}, \bar{\zeta}_{z}, \mu^{2})$: **Matching coefficient, perturbative, up to 1-loop result yet**

$K(b_{\perp}, \mu)$: **Collins-Soper kernel, evolution for rapidity scale**

$\Psi^{\pm}(x, b_{\perp}, \mu, \zeta)$: **TMDWF, physical observable**

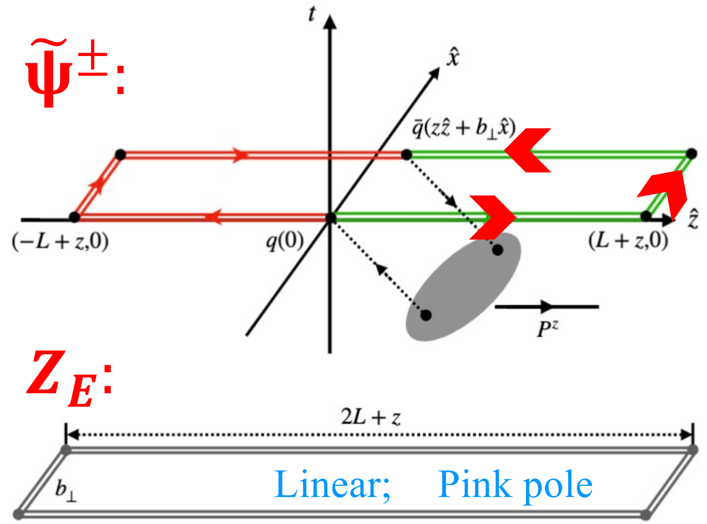
Calculation on LQCD

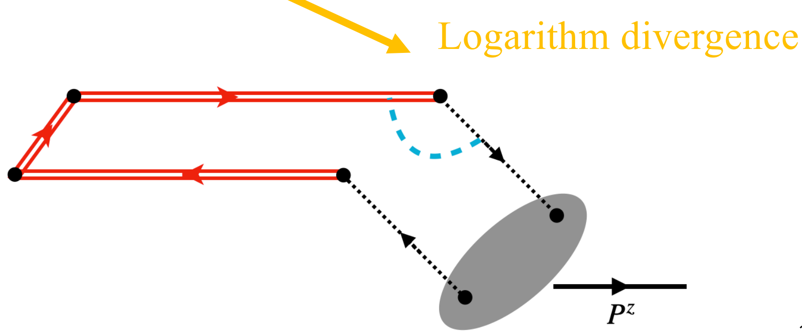
- **Quasi TMDWF in Euclidean lattice:**

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \int \frac{P^z dz}{2\pi} e^{ixzP^z} \times \frac{\langle 0 | \bar{q}(z\hat{n}_z + b_\perp \hat{n}_\perp)\gamma^t \gamma_5 U_{c\pm} q(0) | \pi(P^z)\rangle}{\sqrt{Z_E(2L \pm z, b_\perp, \mu)} Z_O(1/a, \mu, \Gamma)}.$$

- **Staple-shaped gauge-link:**

$$U_{c\pm} = U_z^\dagger(z\hat{n}_z + b_\perp \hat{n}_\perp; L) U_\perp(\pm L\hat{n}_z + z\hat{n}_z; b_\perp) \times U_z(0\hat{n}_z; \pm L + z).$$





Calculation on LQCD

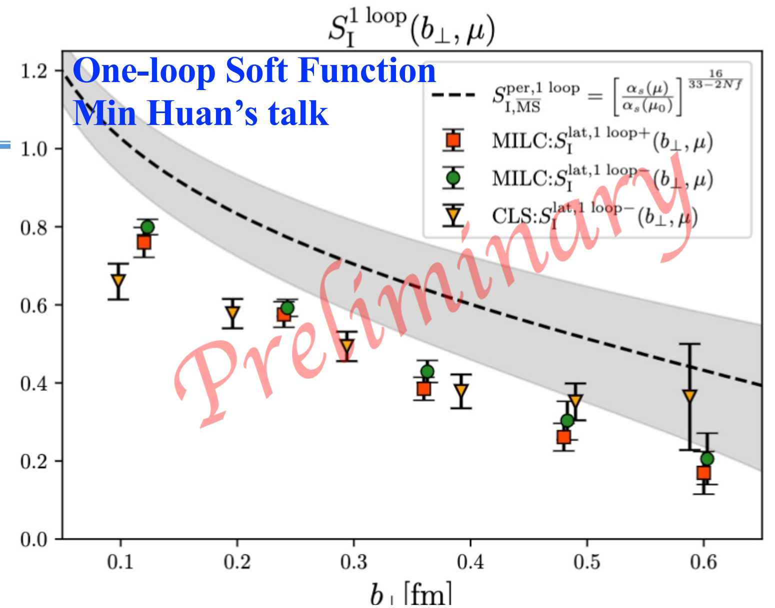
➤ **Soft Function**

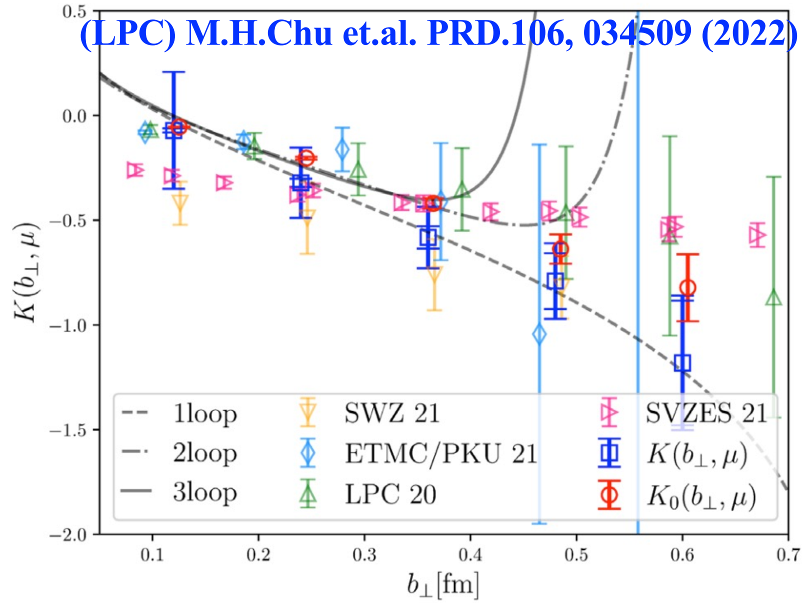
$$F(b_\perp, P_1, P_2, \Gamma, \mu) = \frac{\langle P_2|\bar{q}(b_\perp)\Gamma q(b_\perp)\bar{q}(0)\Gamma' q(0)|P_1\rangle}{\langle 0|\bar{q}(0)\gamma^\mu\gamma^5 q(0)|P_1\rangle\langle P_2|\bar{q}(0)\gamma_\mu\gamma^5 q(0)|0\rangle}$$

$$S_I(b_\perp, \mu) = \frac{F(b_\perp, P_1, P_2, \Gamma, \mu)}{\int dx_1 dx_2 H(x_1, x_2, \Gamma)\tilde{\Psi}^{\pm *}(x_2, b_\perp, \zeta^z)\tilde{\Psi}^{\pm}(x_1, b_\perp, \zeta^z)}$$

➤ **Collins-Soper kernel**

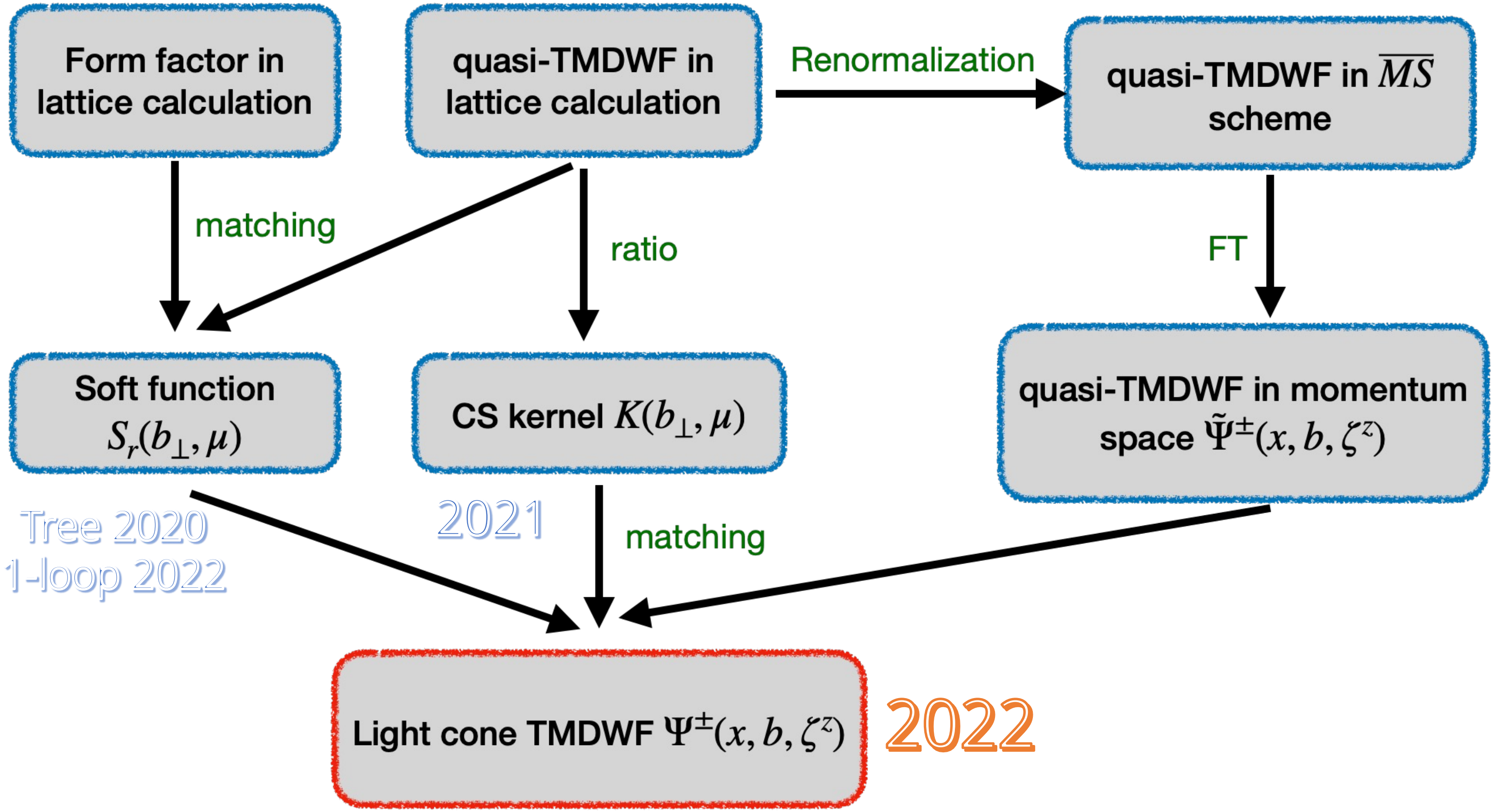
$$K(b_\perp, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{H^\pm(xP_2^z, \mu)\tilde{\Psi}^\pm(x, b_\perp, \mu, P_1^z)}{H^\pm(xP_1^z, \mu)\tilde{\Psi}^\pm(x, b_\perp, \mu, P_2^z)}$$





Calculation on LQCD

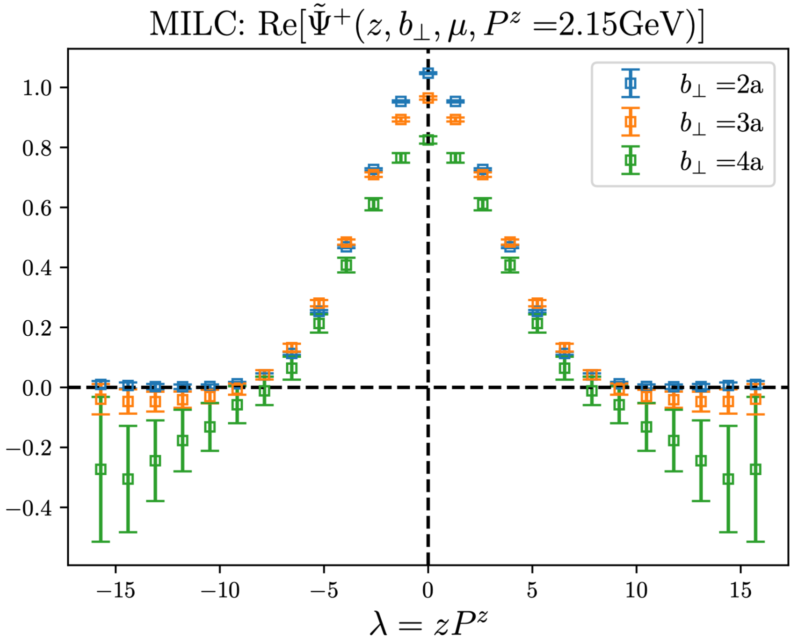
$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z)\, S_I^{\frac{1}{2}}(b_\perp, \mu) = H^{\pm}(x, \zeta^z, \mu)\, \mathbf{exp}\left[\frac{1}{2} K(b_\perp, \mu) \ln \frac{\pm\zeta^z + i\epsilon}{\zeta}\right] \Psi^{\pm}(x, b_\perp, \mu, \zeta)$$



Numerical results

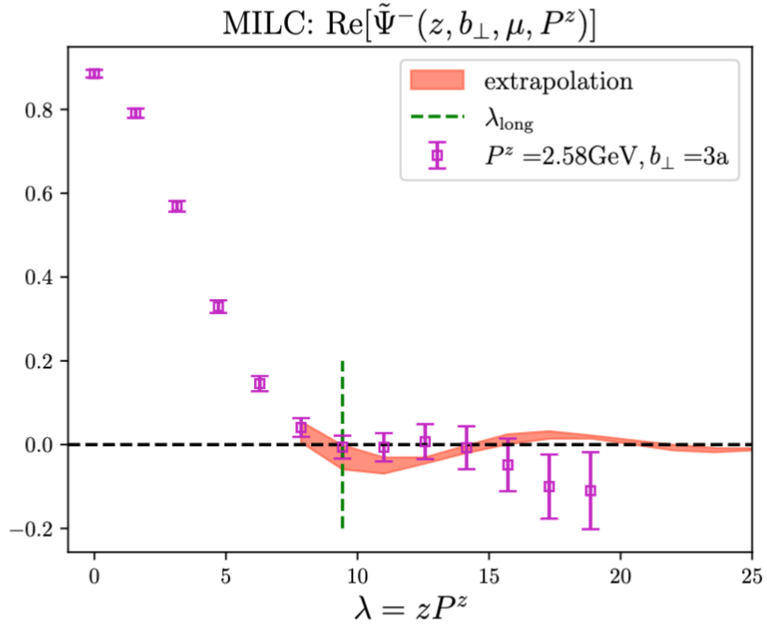
$$\tilde{\Psi}^{\pm}(x,b_\perp,\mu,\zeta^z) = \lim_{L\to\infty}\int \frac{P^z dz}{2\pi} e^{ixzP^z} \frac{\langle 0|\bar{q}(z\hat{n}_z + b_\perp \hat{n}_\perp)\gamma^t\gamma_5 U_{c\pm} q(0)|\pi(P^z)\rangle}{\sqrt{Z_E(2L\pm z, b_\perp,\mu)}Z_O(1/a,\mu,\Gamma)}$$

- **Quasi TMDWF in coordinate space**



- **Follow the hybrid scheme, we adopt an extrapolation at large λ**

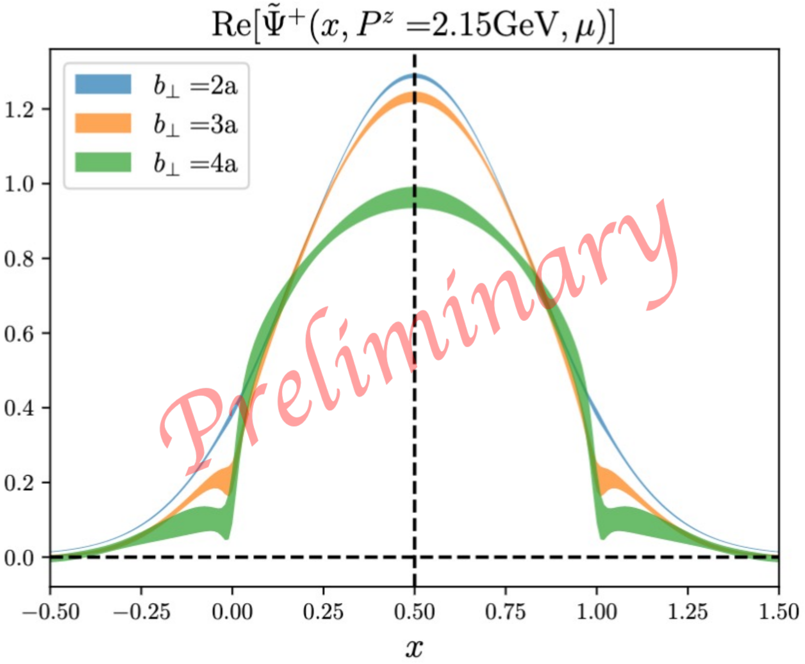
$$\tilde{\Psi}(z,b_\perp,\mu,P^z) = f(b_\perp)\left[\frac{c_1}{(-i\lambda)^d} + e^{i\lambda}\frac{c_2}{(i\lambda)^d}\right] e^{-\frac{\lambda}{\lambda_0}}$$

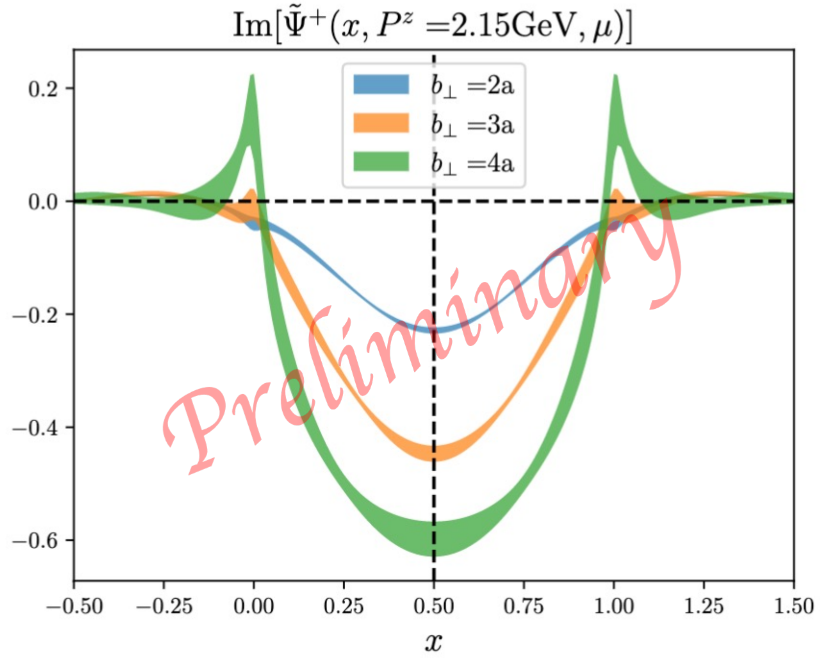


Numerical results

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \int \frac{P^z dz}{2\pi} e^{ixzP^z} \frac{\langle 0|\bar{q}(z\hat{n}_z + b_\perp \hat{n}_\perp)\gamma^t\gamma_5 U_{c\pm} q(0)|\pi(P^z)\rangle}{\sqrt{Z_E(2L \pm z, b_\perp, \mu)} Z_O(1/a, \mu, \Gamma)}$$

➢ **Quasi TMDWF in momentum space**





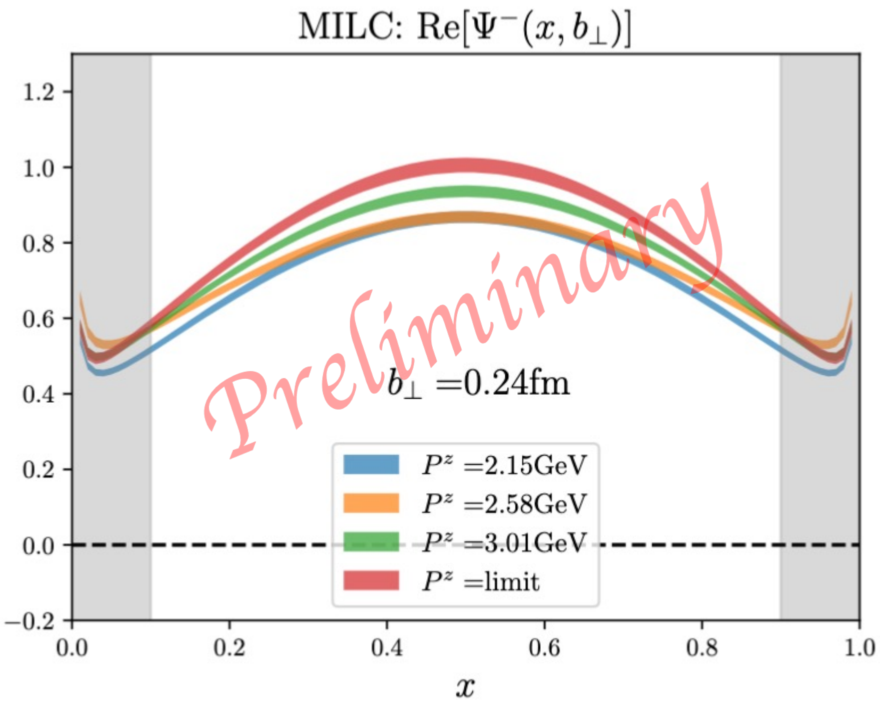
Numerical results

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z)\, S_I^{\frac{1}{2}}(b_\perp, \mu) = H^{\pm}(x, \zeta^z, \mu)\, \mathbf{exp}\left[\frac{1}{2} K(b_\perp, \mu) \ln \frac{\pm\zeta^z + i\epsilon}{\zeta}\right] \Psi^{\pm}(x, b_\perp, \mu, \zeta)$$

$$H^{\pm}(x, \zeta^z, \mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\frac{5\pi^2}{6} - 4 + l_{\pm} + \bar{l}_{\pm} - \frac{1}{2}(l_{\pm}^2 + \bar{l}_{\pm}^2)\right)$$

$$l_{\pm} = \ln[(-x\zeta^z \pm i\epsilon)/\mu^2]$$

- **Pz dependence of TMDWF after mathing**

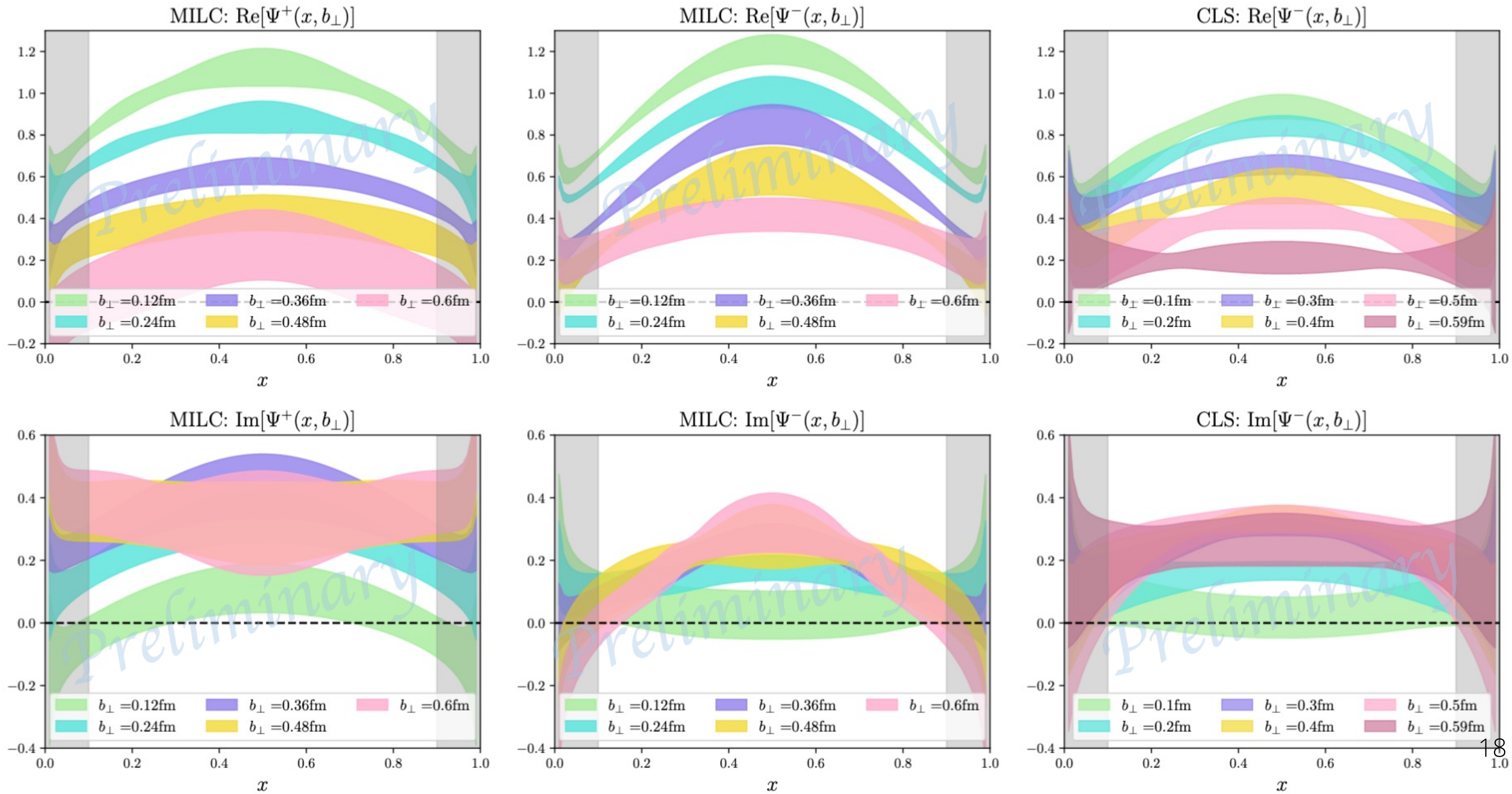


- **Pz extrapolation:**

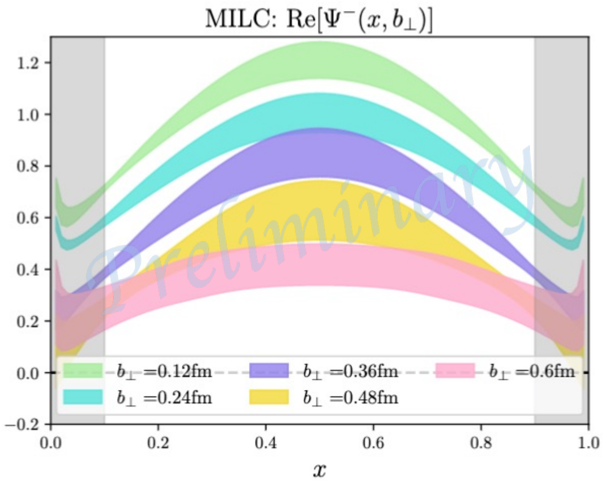
$$\Psi^{\pm}(x, P_z) = \Psi^{\pm}(x, P_z \to \infty) + \frac{c_2(x)}{P_z^2} + \mathcal{O}\left(\frac{1}{P_z^4}\right)$$

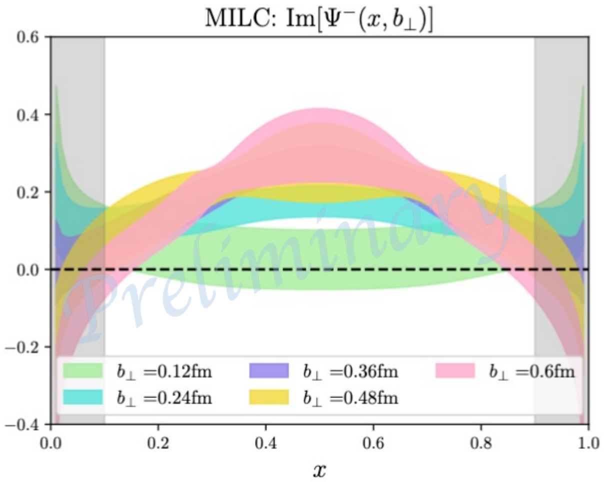
Numerical results

TMDWFs extracted by 3 cases:

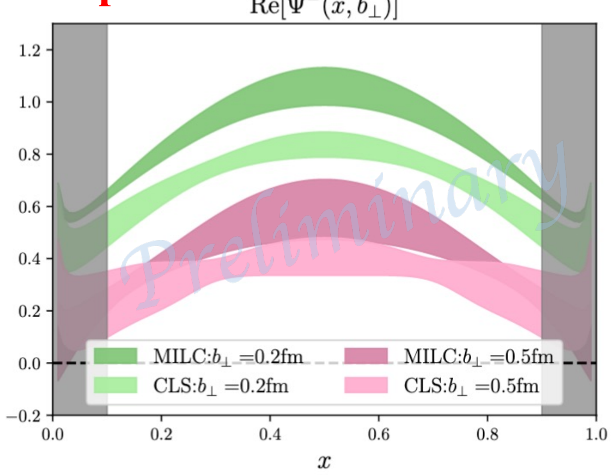


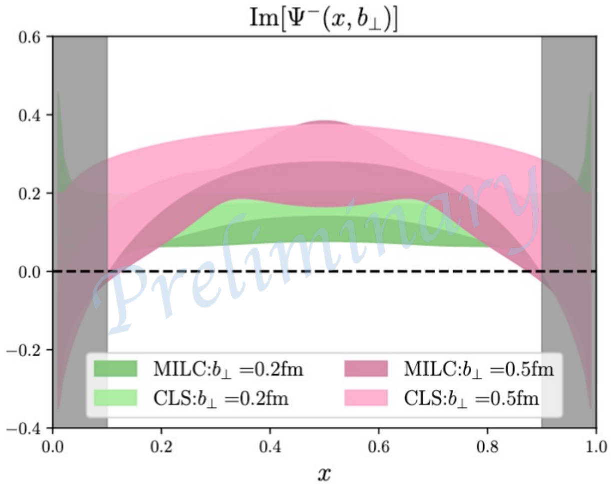
Numerical results





Comparison





- The real part of the TMDWF decreases as $b_\perp$ increases
- The imaginary part increases with $b_\perp$ and then stabilises around 0.25
- The results from MILC and CLS show same trend with small difference

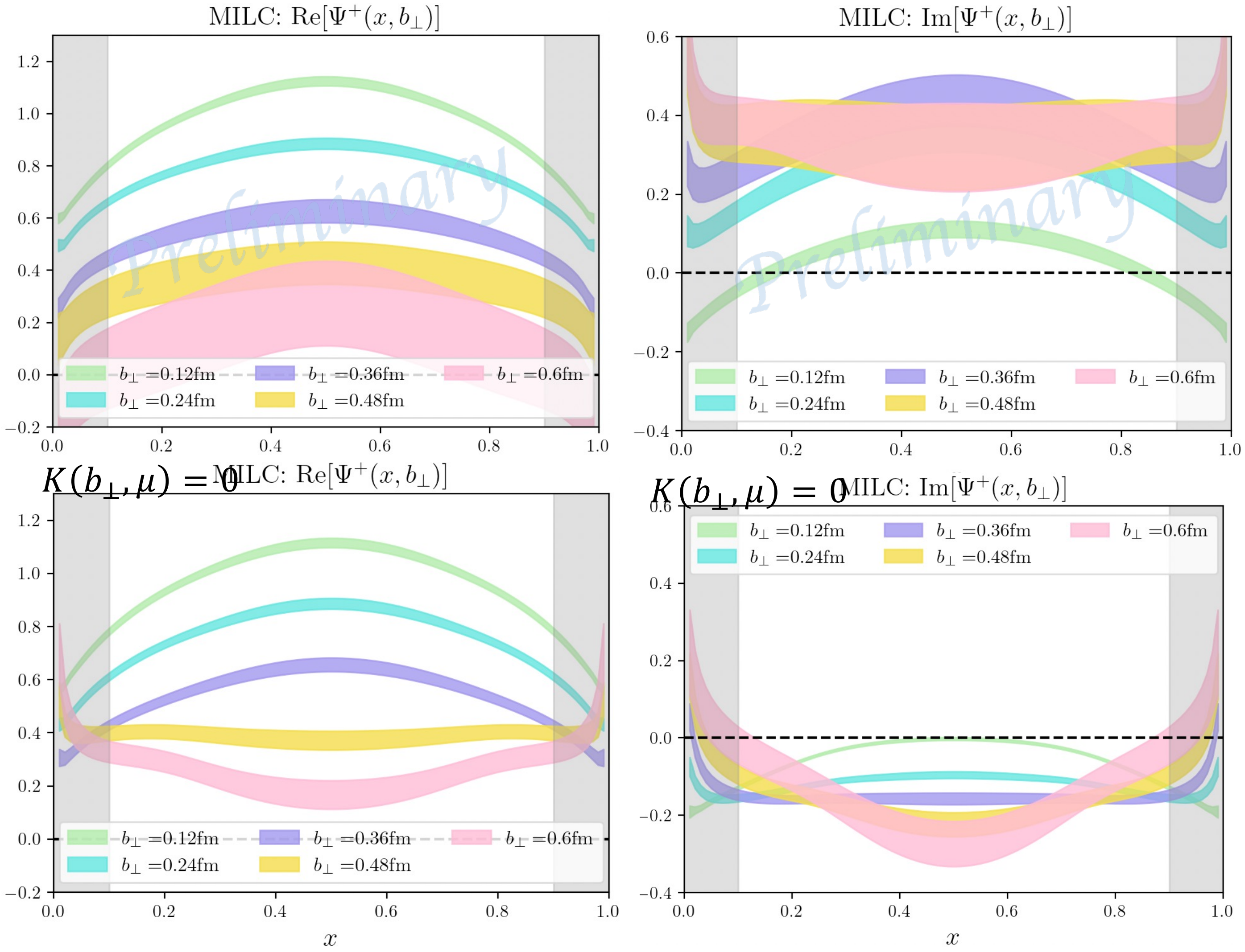
Summary

- Calculation on transverse momentum dependent wave function(TMDWF) is indispensable
- We calculate the one-loop intrinsic soft function and TMDWF with LaMET on MILC and CLS ensembles
- The MILC and CLS results show good agreement, but discrete errors are still relatively significant in current results

Thanks for your attention!

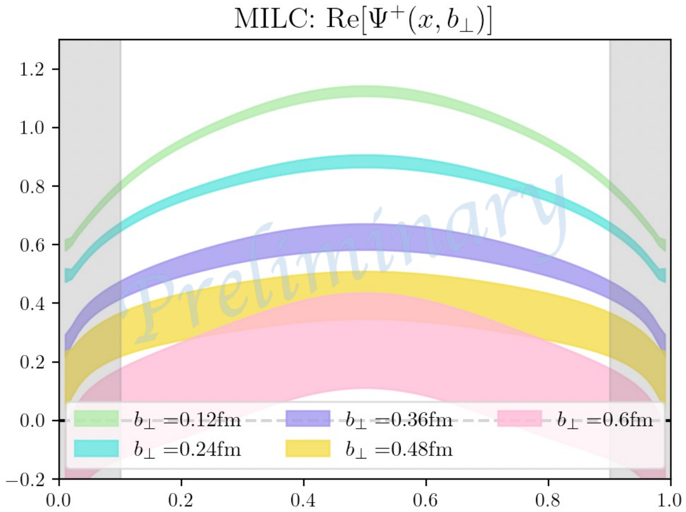
Backup slides

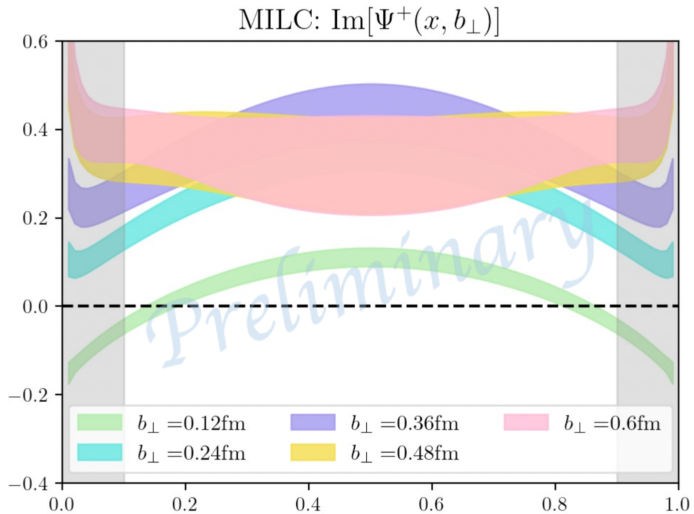
$$\tilde{\Psi}^{\pm}\left(x, b_\perp, \mu, \zeta^z\right) S_I^{\frac{1}{2}}\left(b_\perp, \mu\right) = H^{\pm}\left(x, \zeta^z, \mu\right) \exp\left[\frac{1}{2} K\left(b_\perp, \mu\right) \ln \frac{\pm\zeta^z + i\epsilon}{\zeta}\right] \Psi^{\pm}\left(x, b_\perp, \mu, \zeta\right)$$



Backup slides

$$\tilde{\Psi}^{\pm}(x,b_\perp,\mu,\zeta^z)\, S_I^{\frac{1}{2}}(b_\perp,\mu) = H^{\pm}(x,\zeta^z,\mu)\, \mathbf{exp}\left[\frac{1}{2}K(b_\perp,\mu)\ln\frac{\pm\zeta^z+i\epsilon}{\zeta}\right]\Psi^{\pm}(x,b_\perp,\mu,\zeta)$$

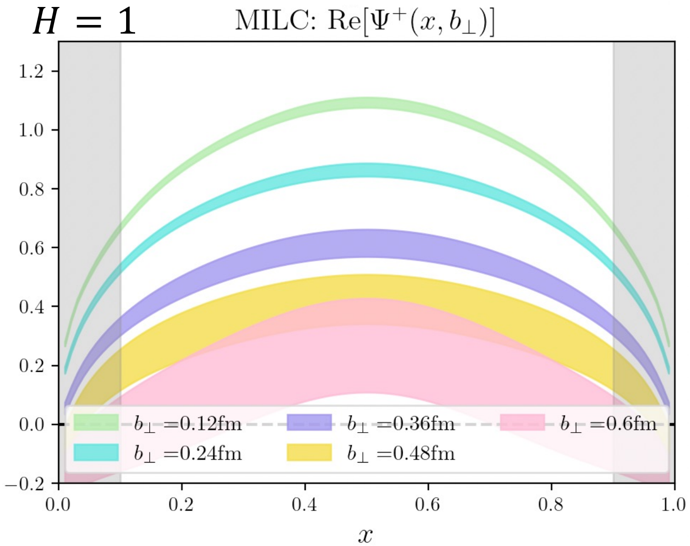




$$H^{\pm}(x,\zeta^z,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\frac{5\pi^2}{6} - 4 + l_\pm + \bar{l}_\pm - \frac{1}{2}\left(l_\pm^2 + \bar{l}_\pm^2\right)\right),$$

$$l_\pm = \ln[(-x\zeta^z \pm i\epsilon)/\mu^2]$$

$H = 1$



$H = 1$