## First Glimpse into the Kaon Gluon Parton Distribution Using Lattice QCD

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## Overview

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## Introduction

- Most of kaon's mass arises due to the dynamics of QCD and not through the Higgs Mechanism. Studying its structure is important to understand more generally the emergence of hadronic mass.
- Experimental efforts, e.g the EIC, plan to probe the kaon structure. Theoretical input is needed.




## Specifications on the Lattice

- The kaon's gluon PDF was calculated through the pseudo-PDF method at lattice spacings $a \approx 0.12 \mathrm{fm}$, and $a \approx 0.15 \mathrm{fm}$ and pion mass of about 310 Mev .
- We label these ensembles as a12m310 and a15m310 respectively.
- Used clover valance fermions on ensembles with $N_{f}=2+1+1$ HISQ generated by the MILC Collaboration. Bazavov et al. Phys. Rev. D 82,074501 (2010).
- HYP smearing was used to to reduce statistical fluctuations.
- Applied Gaussian momentum smearing was to reach higher boost momenta.


## Gluon Operator

The Unpolarized gluon operator is given by

$$
\begin{equation*}
\mathcal{O}(z) \equiv \sum_{i \neq z, t} \mathcal{O}\left(F^{t i}, F^{t i} ; z\right)-\sum_{i, j \neq z, t} \mathcal{O}\left(F^{i j}, F^{i j} ; z\right) \tag{1}
\end{equation*}
$$

where $\mathcal{O}\left(F^{\mu \nu}, F^{\alpha \beta} ; z\right)=F_{\nu}^{\mu}(z) U(z, 0) F_{\beta}^{\alpha}(0)$, the Wilson link length is denoted by $z$, and the field strength $F_{\mu \nu}$ is given by

$$
\begin{equation*}
F_{\mu \nu}=\frac{i}{8 a^{2} g_{0}}\left(\mathcal{P}_{[\mu, \nu]}+\mathcal{P}_{[\nu,-\mu]}+\mathcal{P}_{[-\mu,-\nu]}+\mathcal{P}_{[-\nu, \mu]}\right) \tag{2}
\end{equation*}
$$

Balitsky et al. Phys. Lett. B 808, 135621 (2020)

## 2-point Correlator

Calculate and fit the two-point correlator.

$$
\begin{align*}
C_{\Phi}^{2 p t}\left(P_{z} ; t\right) & =\int d^{3} y e^{-i y \cdot P_{z}}\left\langle\chi_{\Phi}(\vec{y}, t) \mid \chi_{\Phi}(\overrightarrow{0}, 0)\right\rangle \\
& =\left|A_{\Phi, 0}\right|^{2} e^{-E_{\Phi, 0} t}+\left|A_{\Phi, 1}\right|^{2} e^{-E_{\Phi, 1} t}+\ldots \tag{3}
\end{align*}
$$

where $P_{z}$ is the meson momentum in the z-direction, $\chi_{\Phi}=\bar{q}_{1} \gamma_{5} q_{2}$ is the pseudoscalar-meson interpolation operator, and $t$ is the Euclidean time.

## 3-point Correlator

Calculate and fit the 3-point correlator.

$$
\begin{align*}
& C_{\Phi}^{3 \mathrm{pt}}\left(z, P_{z} ; t_{\text {sep }}, t\right) \\
& =\int d^{3} y e^{-i y \cdot P_{z}}\left\langle\chi_{\Phi}\left(\vec{y}, t_{\text {sep }}\right)\right| \mathcal{O}(z, t)\left|\chi_{\Phi}(\overrightarrow{0}, 0)\right\rangle \\
& =\left|A_{\Phi, 0}\right|^{2}\langle 0| \mathcal{O}|0\rangle e^{-E_{\Phi, 0} t_{\text {sep }}} \\
& +\left|A_{\Phi, 0}\right|\left|A_{\Phi, 1}\right|\langle 0| \mathcal{O}|1\rangle e^{-E_{\Phi, 1}\left(t_{\text {sep }}-t\right)} e^{-E_{\Phi, 0} t} \\
& +\left|A_{\Phi, 0}\right|\left|A_{\Phi, 1}\right|\langle 1| \mathcal{O}|0\rangle e^{-E_{\Phi, 0}\left(t_{\text {sep }}-t\right)} e^{-E_{\Phi, 1} t} \\
& +\left|A_{\Phi, 1}\right|^{2}\langle 1| \mathcal{O}|1\rangle e^{-E_{\Phi, 1} t_{\text {sep }}}+\ldots \tag{4}
\end{align*}
$$

where $t_{\text {sep }}$ is the source-sink time separation, and $t$ is the gluon-operator insertion time, and $z$ the Wilson-line length.

## Ground State Extraction

Find the best $t_{\text {sep }}$ range for a reliable ground-state estimation.


The behavior of $R^{\text {ratio }}\left(z, P_{z} ; t_{\text {sep }}, t\right)=\frac{C^{3 p t}\left(z, P_{z} ; t_{\text {sep }}, t\right)}{C^{2 p t}\left(P_{z} ; t_{\text {sep }}\right)}$ supports our choice.

## Ground State Extraction

Similar results for ensemble a15m310


## RpITD

Construct loffe-time pseudo-distribution (pITD)

$$
\begin{equation*}
\mathcal{M}\left(\nu, z^{2}\right)=\left\langle 0\left(P_{z}\right)\right| \mathcal{O}(z)\left|0\left(P_{z}\right)\right\rangle \tag{5}
\end{equation*}
$$

Where the loffe-time is given by $\nu=z P_{z}$
The reduced pITD (RpITD) was constructed to remove the divergences in the pITD.

$$
\begin{equation*}
\mathscr{M}\left(\nu, z^{2}\right)=\frac{\mathcal{M}\left(z P_{z}, z^{2}\right) / \mathcal{M}\left(0 \cdot P_{z}, 0\right)}{\mathcal{M}\left(z \cdot 0, z^{2}\right) / \mathcal{M}(0 \cdot 0,0)} . \tag{6}
\end{equation*}
$$

## RpITD

The RpITD for both the a12m310 and a15m310 ensembles have consistent values.


Kaon RpITDs for the a12m310 and a15m310 ensembles at boost momentum $P_{z} \approx 1.3 \mathrm{GeV}$ as a function of $z$

## Relating the RpITD to the Gloun PDF $g(x)$

The RpITD can be related to the gluon PDF through.

$$
\begin{equation*}
\mathscr{M}\left(\nu, z^{2}\right)=\int_{0}^{1} d x \frac{x g\left(x, \mu^{2}\right)}{\langle x\rangle_{g}} R_{g g}\left(x \nu, z^{2} \mu^{2}\right) \tag{7}
\end{equation*}
$$

where $\mu$ is the renormalization scale in the $\overline{\mathrm{MS}}$ scheme and and $R_{g g}$ is the gluon-in-gluon matching kernel.
Perform a fit based on the phenomenological motivated form, also used by JAM to obtain the pion PDF.

$$
\begin{equation*}
f_{g}(x, \mu)=\frac{x g(x, \mu)}{\langle x\rangle_{g}(\mu)}=\frac{x^{A}(1-x)^{C}}{B(A+1, C+1)} \tag{8}
\end{equation*}
$$

## RpITD

RpITD with reconstructed fit bands from ensembles a15m310(left) and a12m310(right).



We observe almost no $z^{2}$ dependence.

## RpITD

RpITD with reconstructed fit bands ensembles a12m310(left) kaon and a12m310(right) pion.
The RpITDs are similar in agreement with previous works, where the ratio was found to be around 1 .
Roberts et al. Prog.Part. Nucl. Phys. 120, 103883 (2021)



Fan et al. Phys. Lett. B 823, 136778 (2021)

## Final Results

For a $15 \mathrm{~m} 310 \chi^{2} /$ dof $\approx 1.8$, and for a $12 \mathrm{~m} 310 \chi^{2} /$ dof $\approx 1$.
Our result at $a \approx 0.12 \mathrm{fm}$, is consistent with the DSE results within one-sigma for $x>0.15$.
Cui et al. Eur. Phys. J. C 80, 1064 (2020)



## Summary and Outlook

- Calculated the klaon gluon PDF through the pseudo-PDF method.
- Perform calculation at additonal lattice spacing 0.09 fm .
- Perform calculation at a lighter pion mass.
- Improve systematic uncertainties associated with lattice spacing, finite volume effects, and unphysical pion mass.

