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UNIVERSITY

A Brief History
of a Big Idea
for a Small Thing

Calvin W. Johnson, SDSU

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Shell Model 75 Symposium, July 19, 2024

Welcome!

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In the beginning….

Discovery of the atomic nucleus 1911

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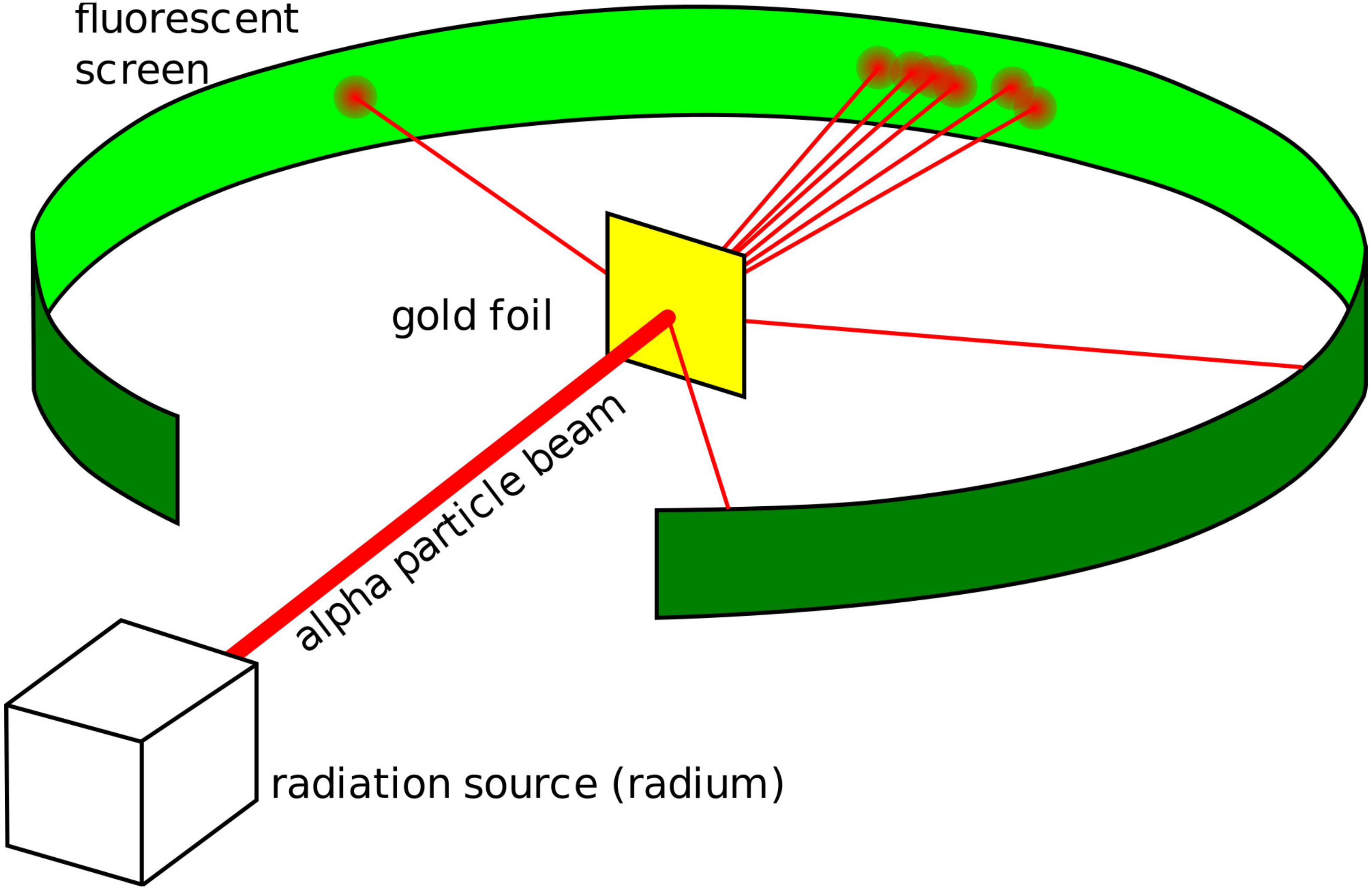

Discovery of the atomic nucleus 1911

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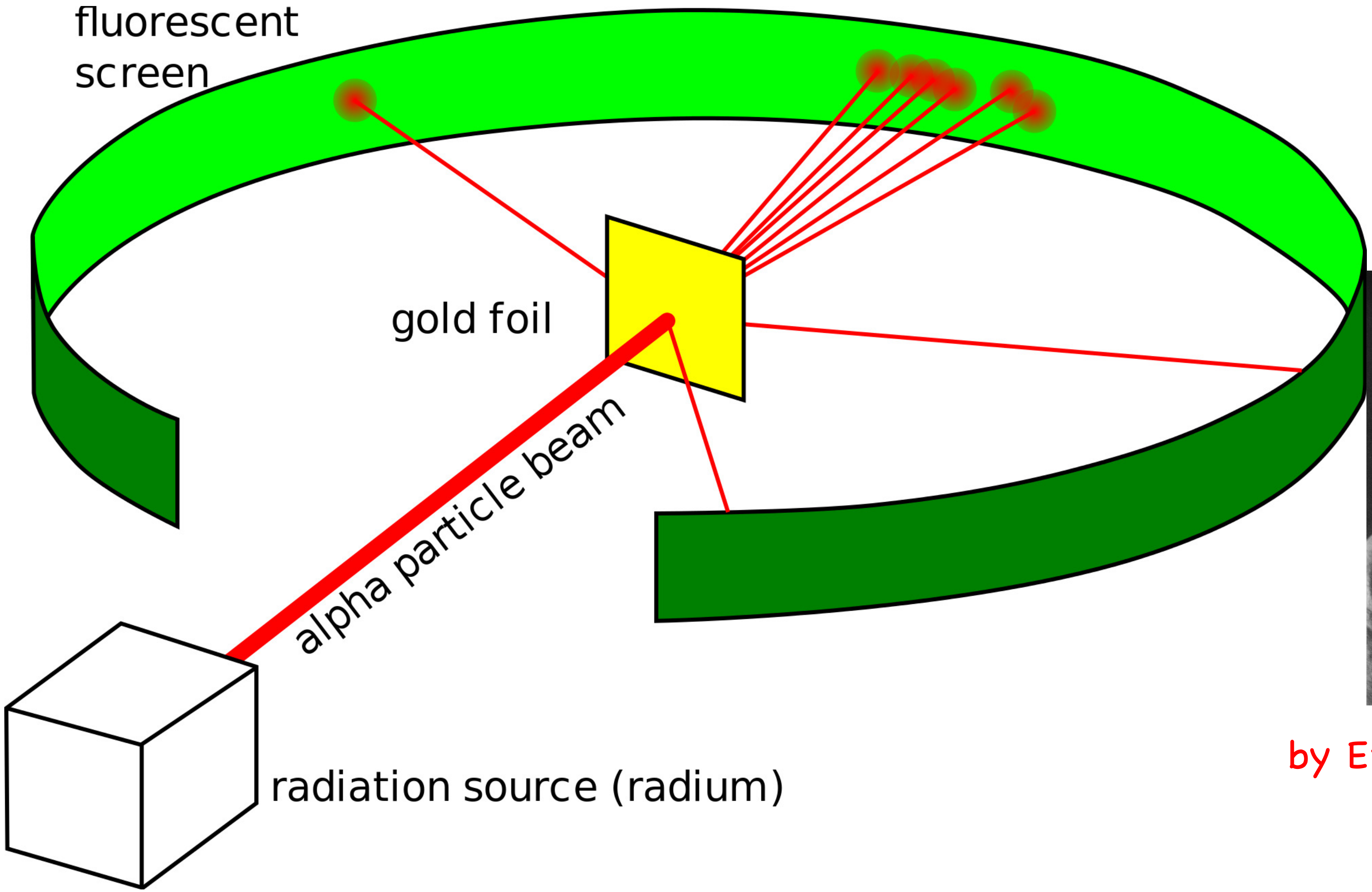
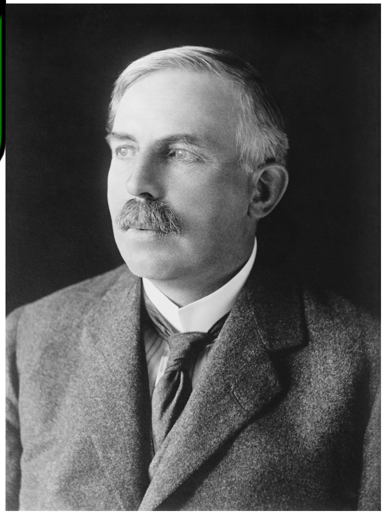
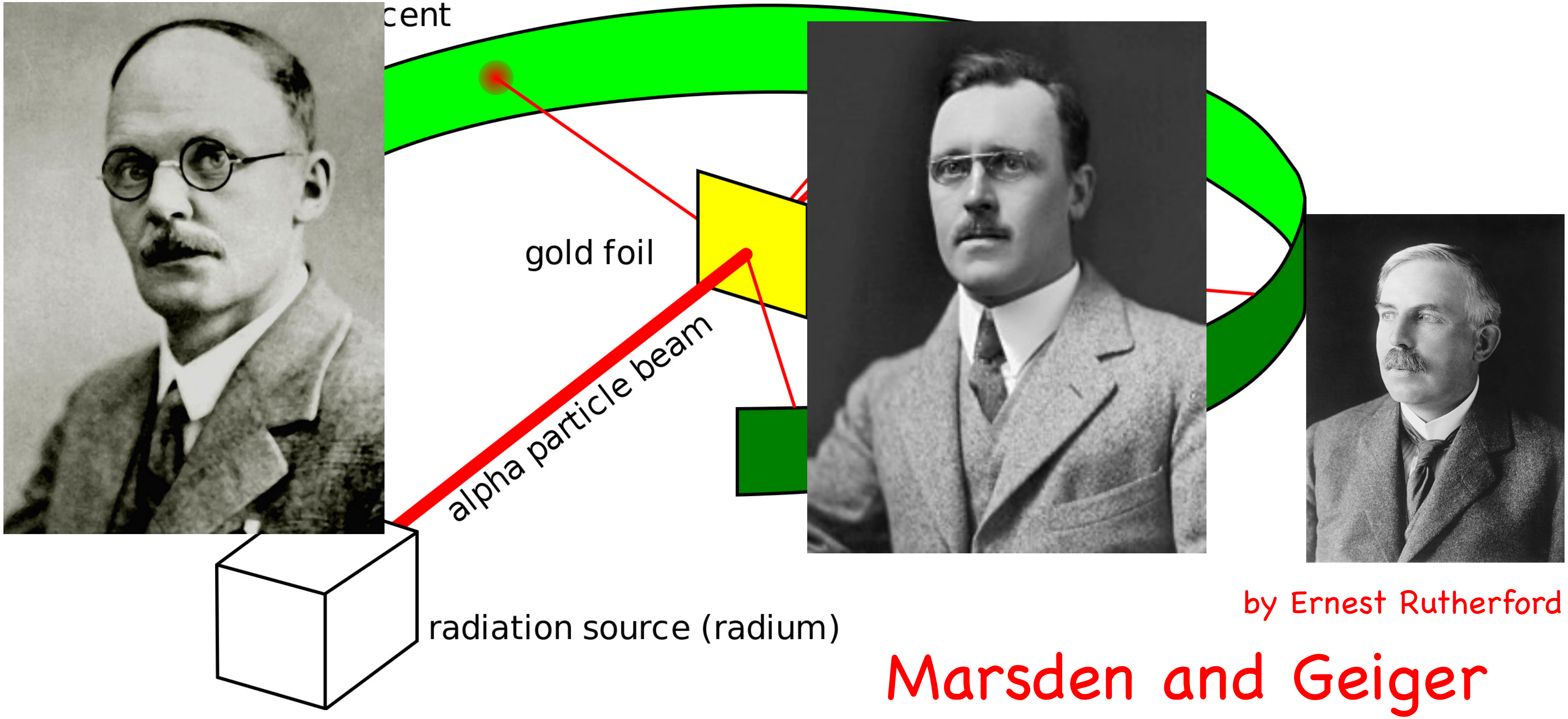




by Ernest Rutherford

Discovery of the atomic nucleus 1911



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Marsden and Geiger

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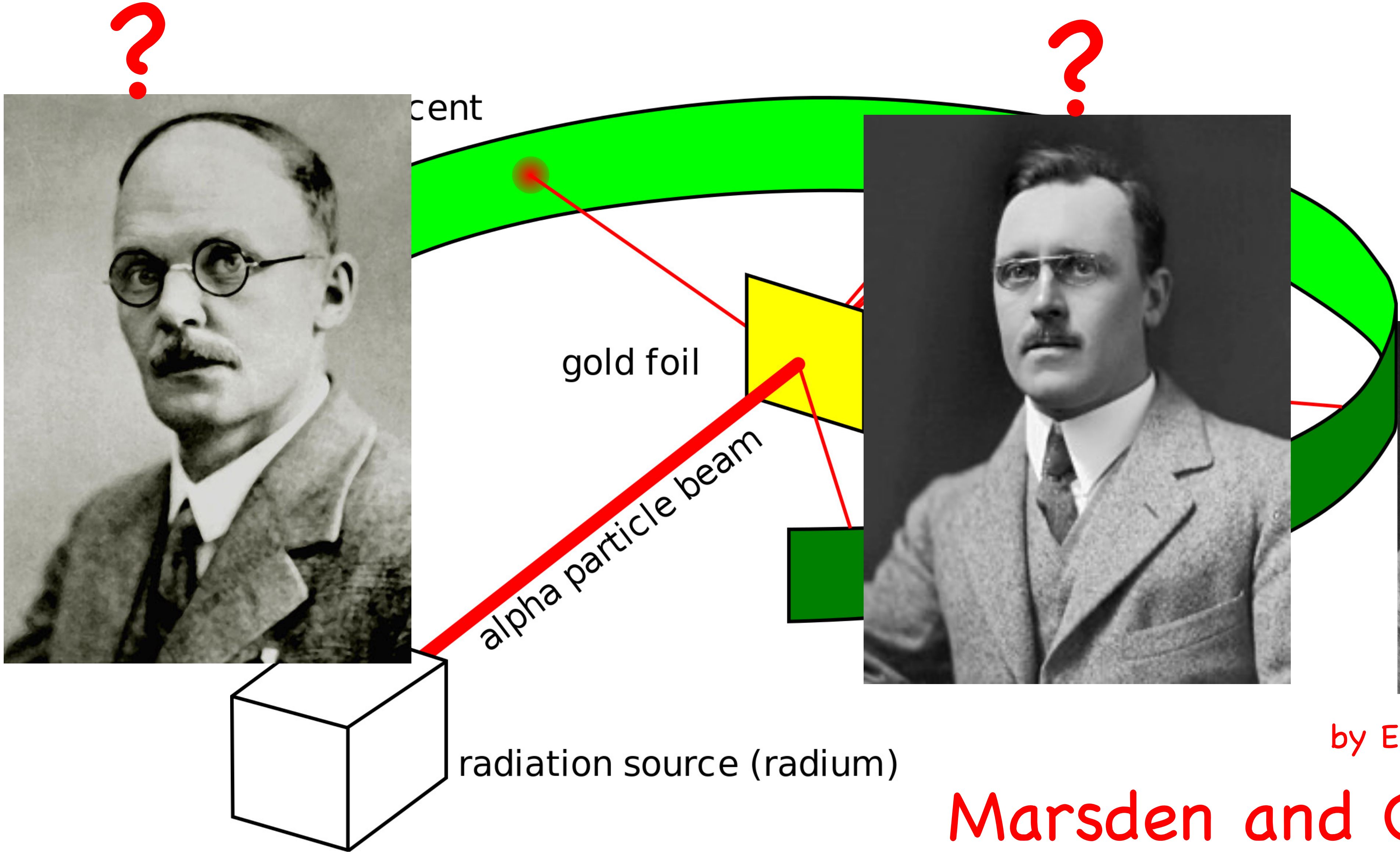

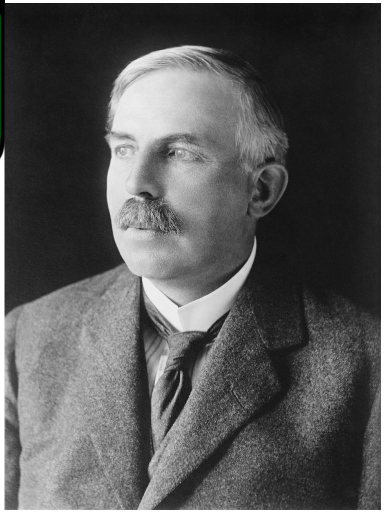

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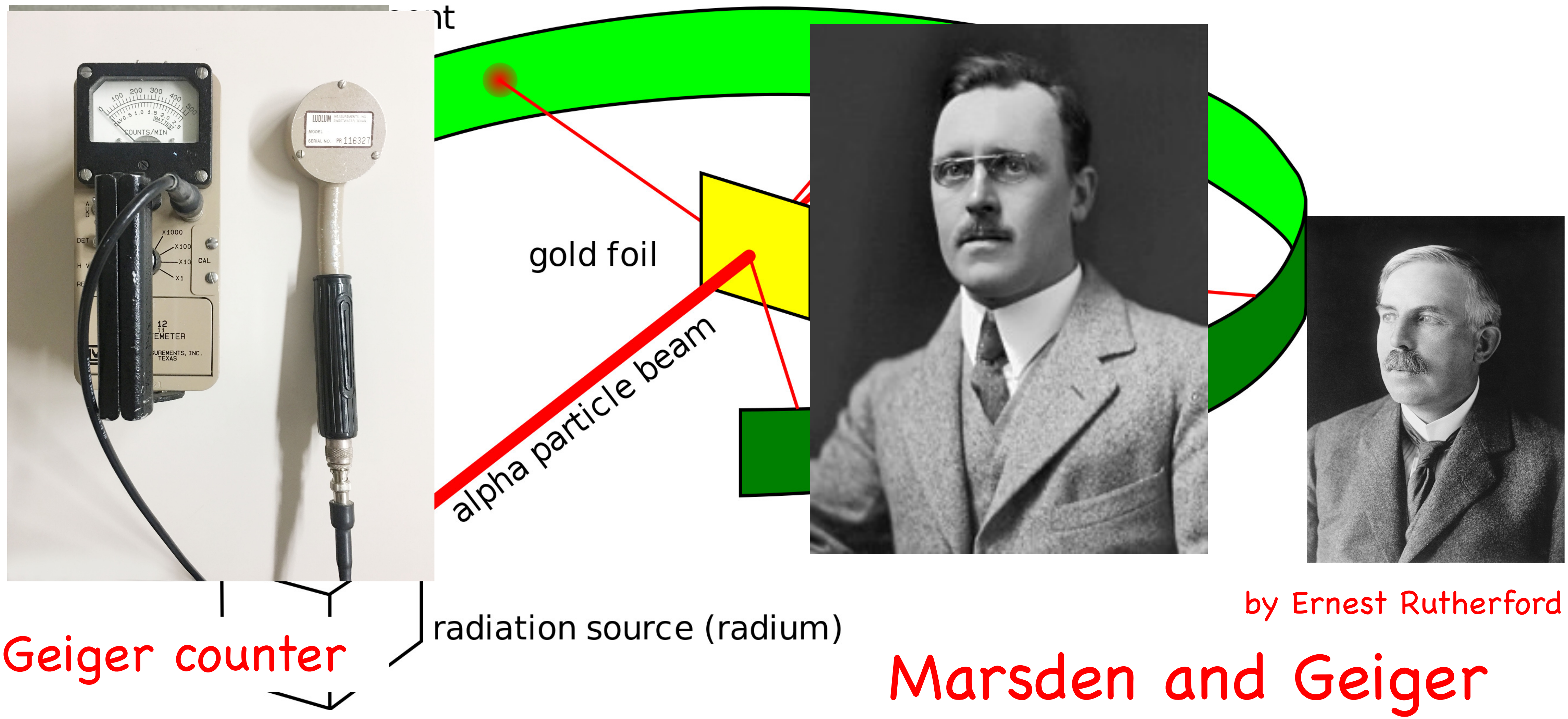
Discovery of the atomic nucleus 1911

San Diego State University



Geiger counter

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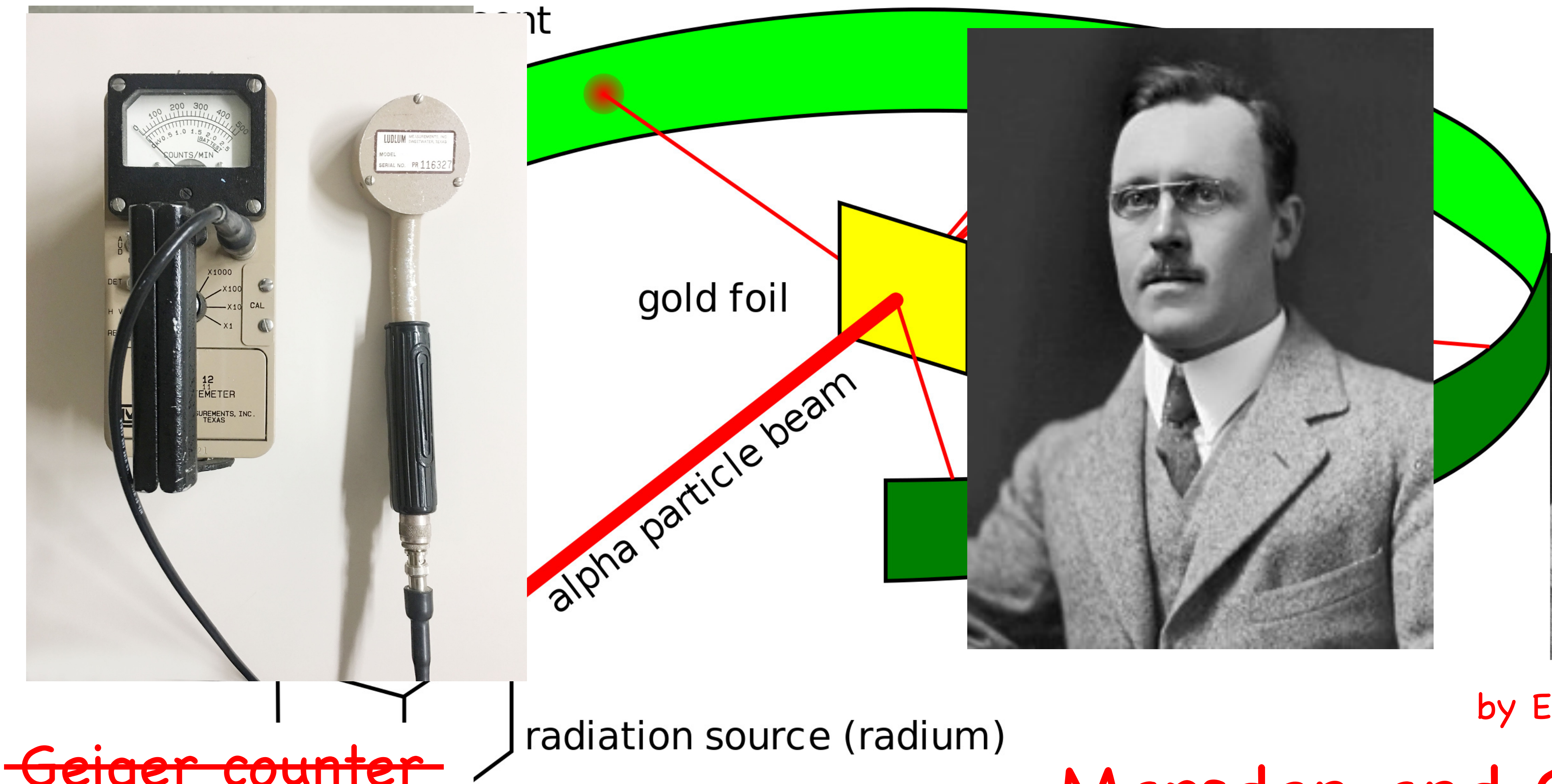
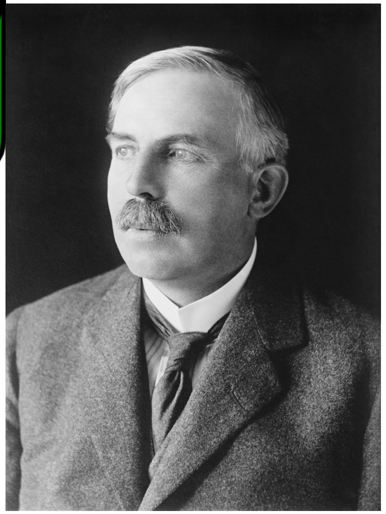
Discovery of the atomic nucleus 1911





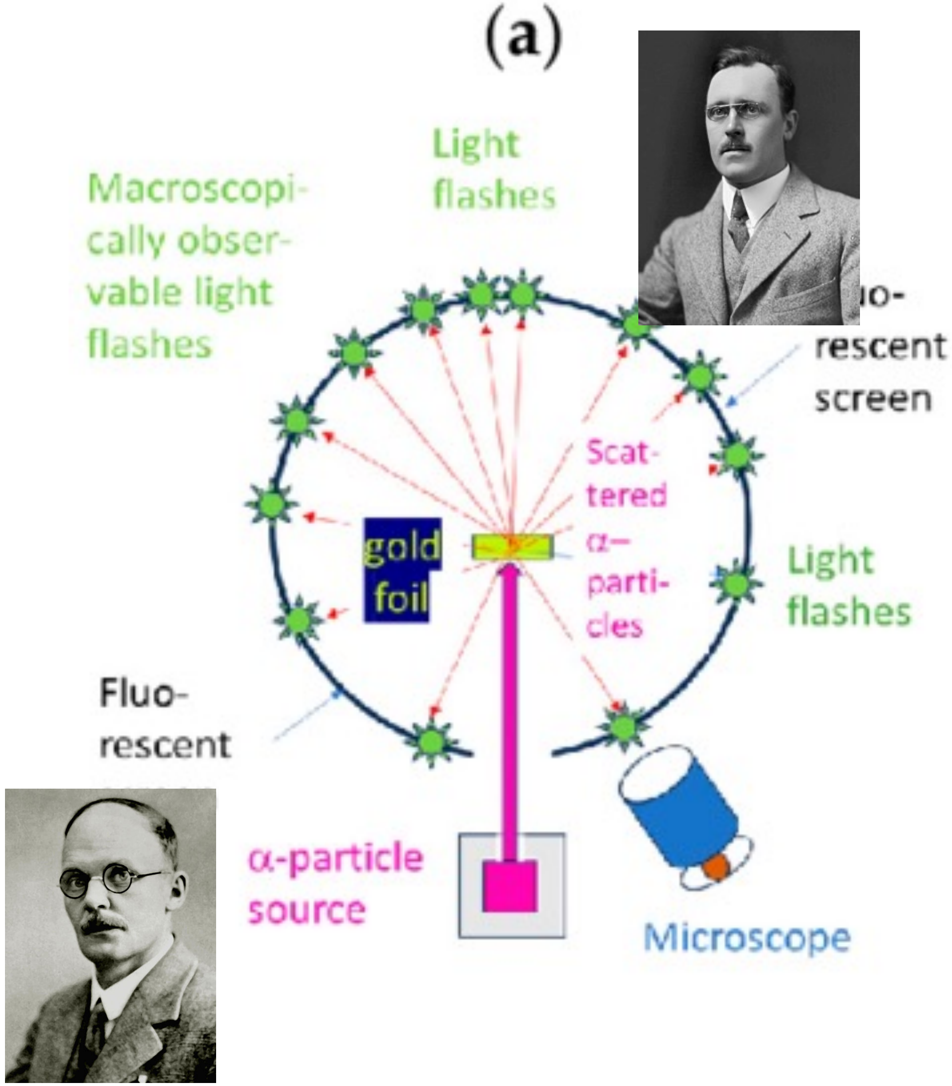
by Ernest Rutherford

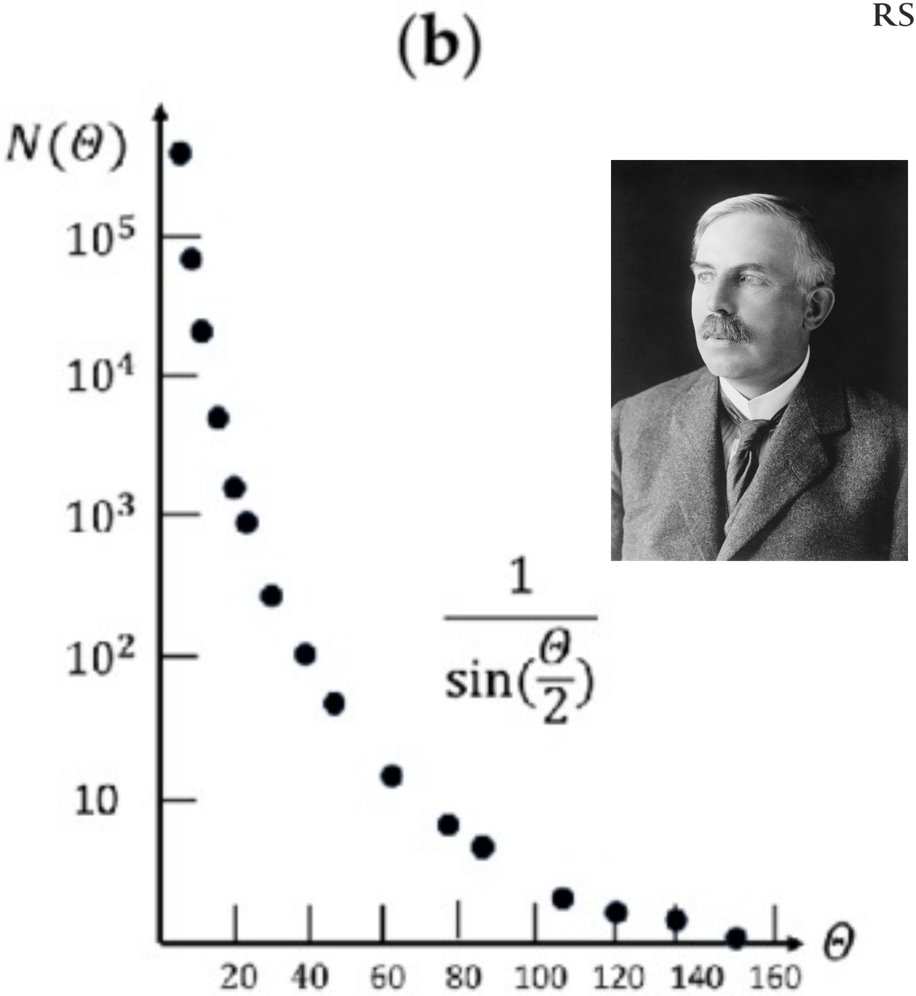
~~Geiger counter~~

Geiger-Mueller counter

Marsden and Geiger

Discovery of the atomic nucleus 1911





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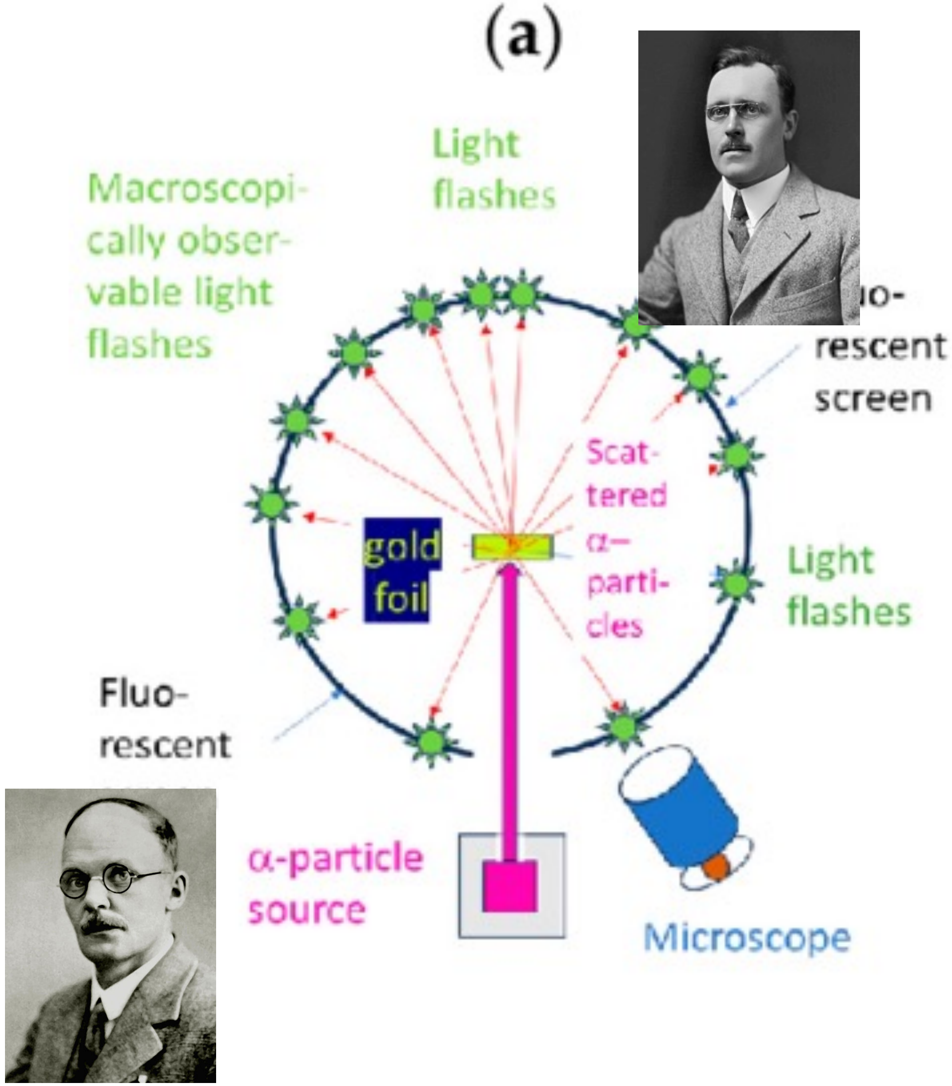
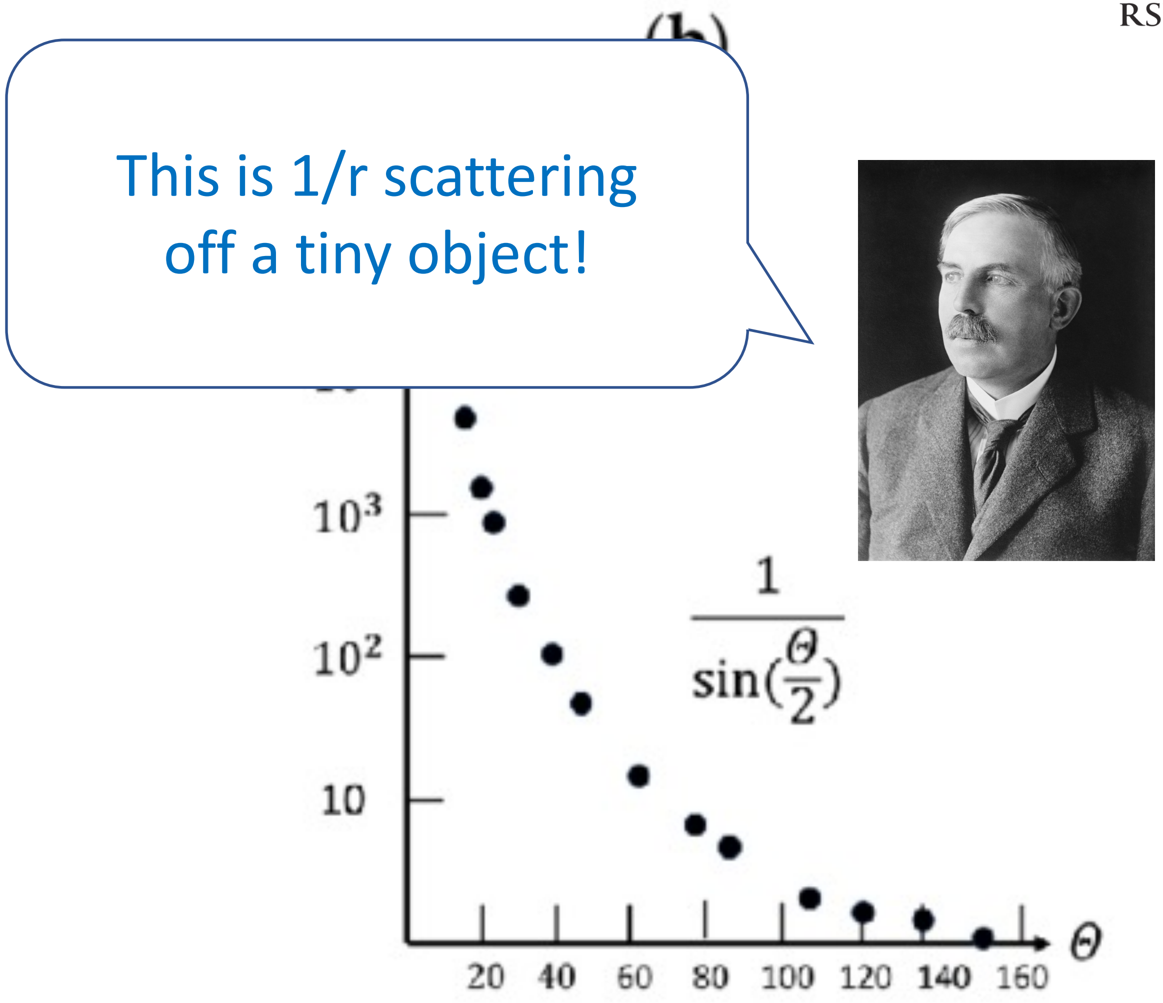
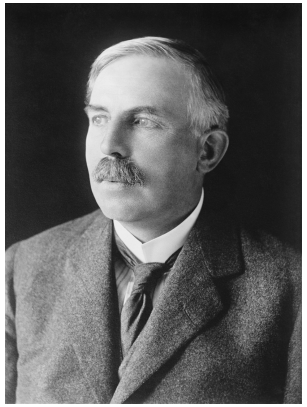




This is 1/r scattering off a tiny object!

Discovery of the atomic nucleus 1911

It’s like a tiny fly inside a massive English cathedral!



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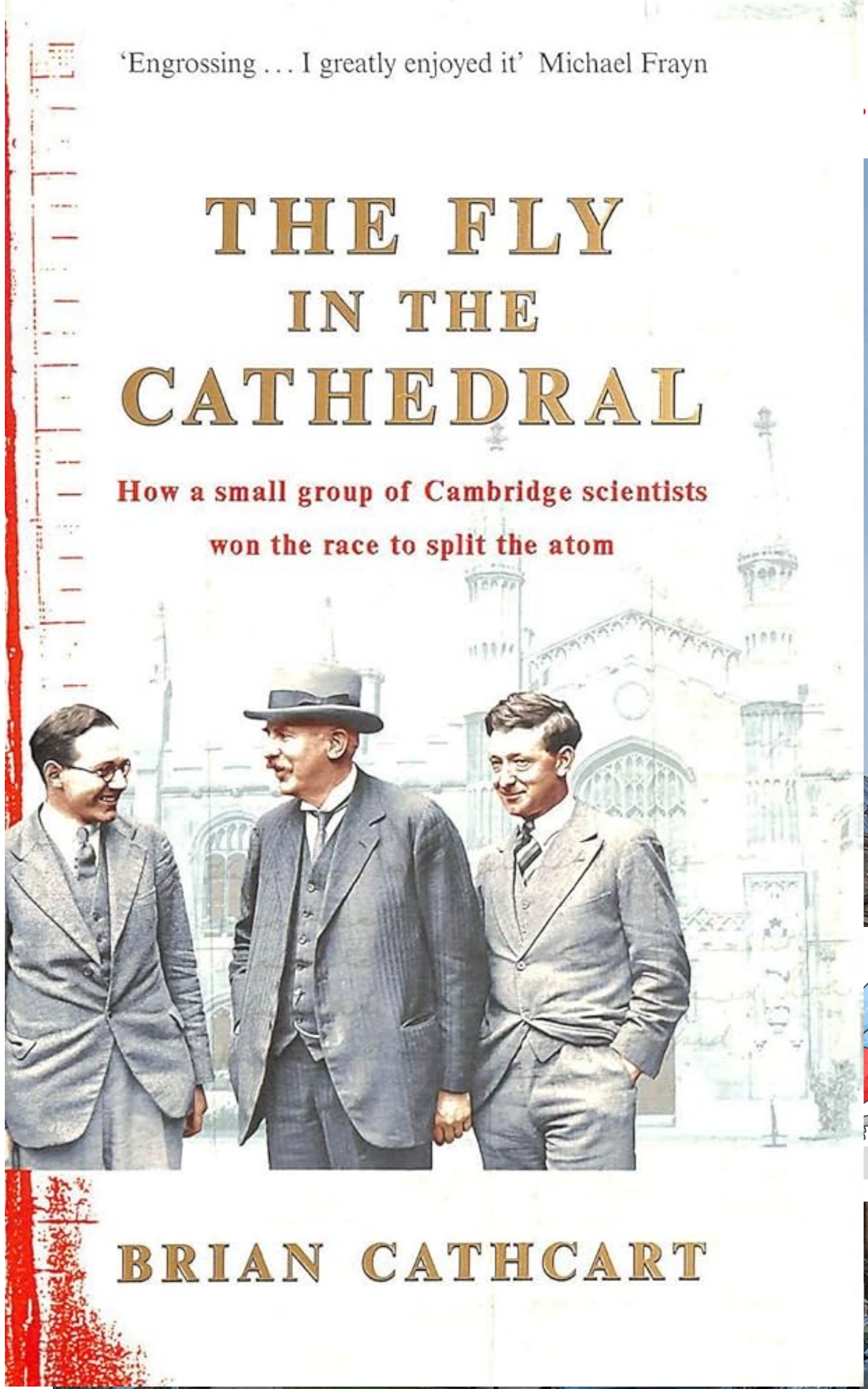


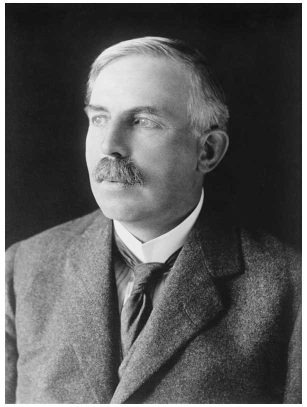
'Engrossing . . . I greatly enjoyed it' Michael Frayn

THE FLY
IN THE
CATHEDRAL

How a small group of Cambridge scientists
won the race to split the atom

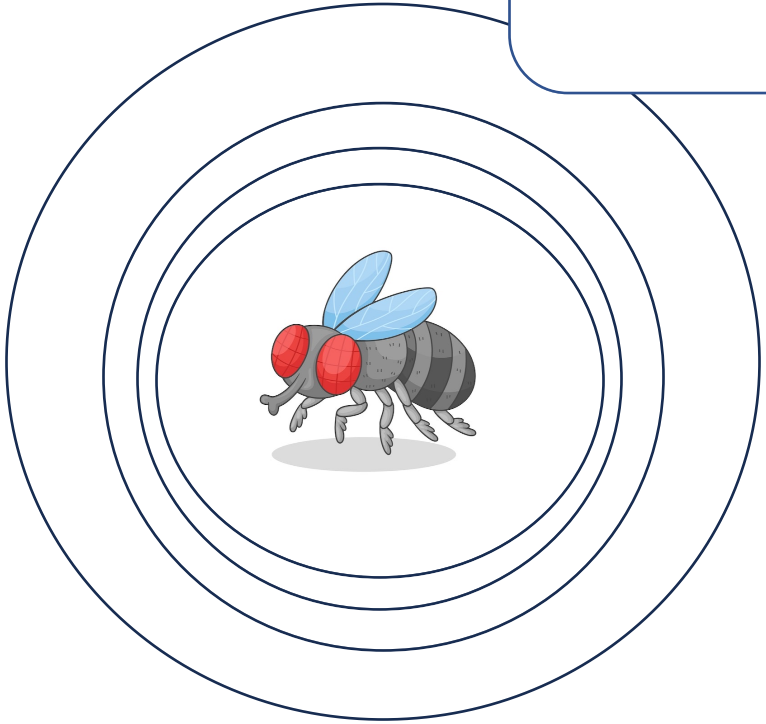
BRIAN CATHCART

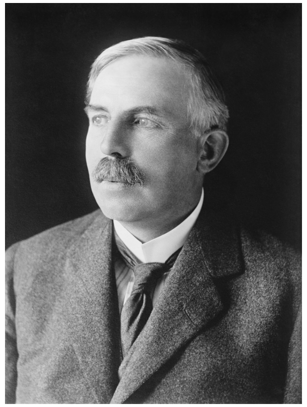
It's like a tiny fly inside a massive English cathedral!



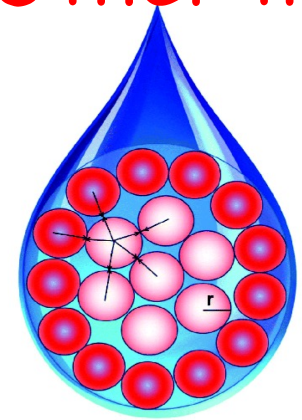
Discovery of the atomic nucleus 1911

What do you mean, “shell model”?



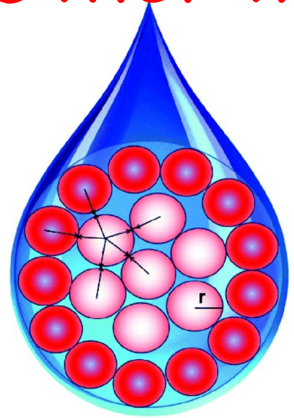


Other nuclear milestones



1928: Gamow proposes liquid drop model

Other nuclear milestones



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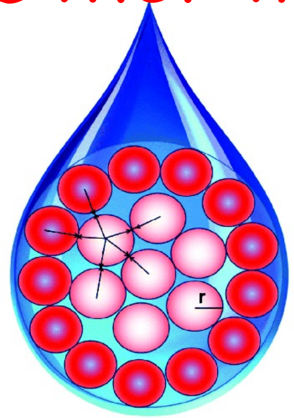
1932: Chadwick discovers neutron

Other nuclear milestones



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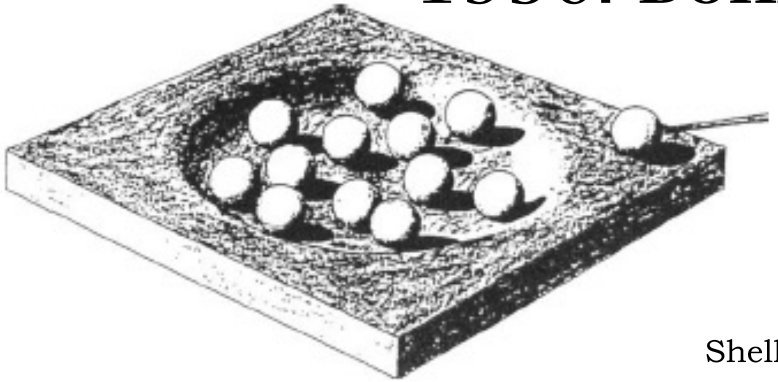
1936: Bohr proposes compound nucleus



Other nuclear milestones

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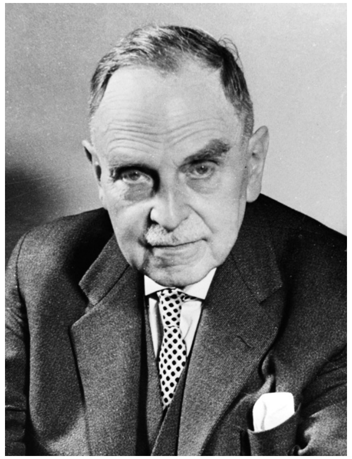
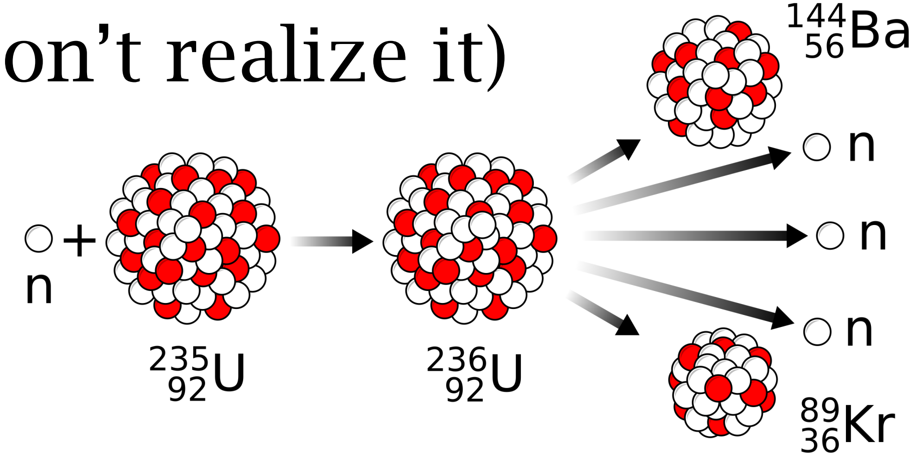


1938: Hahn and Strassmann
'discover' fission (but don't realize it)



Other nuclear milestones

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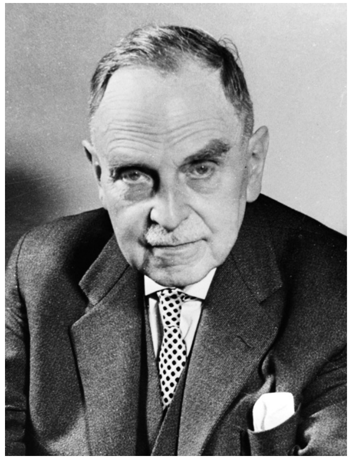

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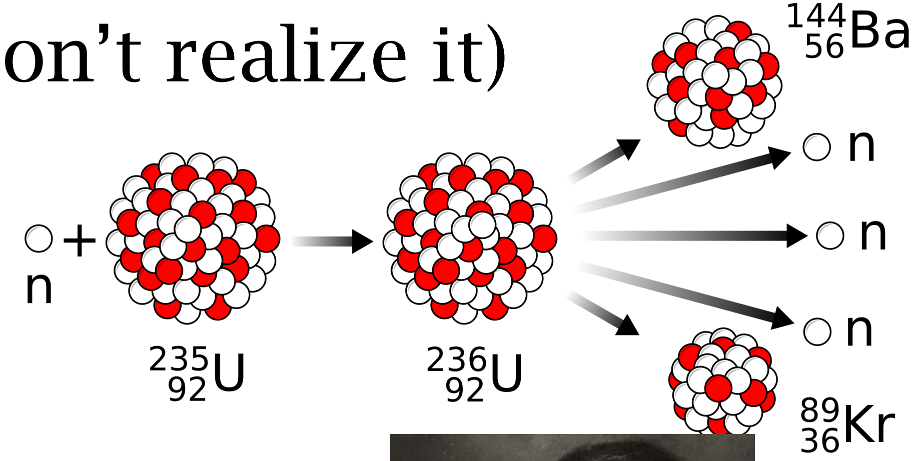
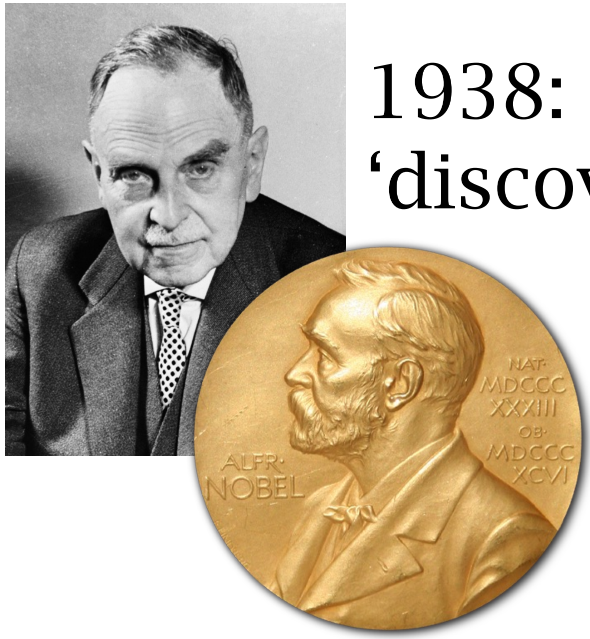


1939: Meitner and Frisch explain fission

Other nuclear milestones



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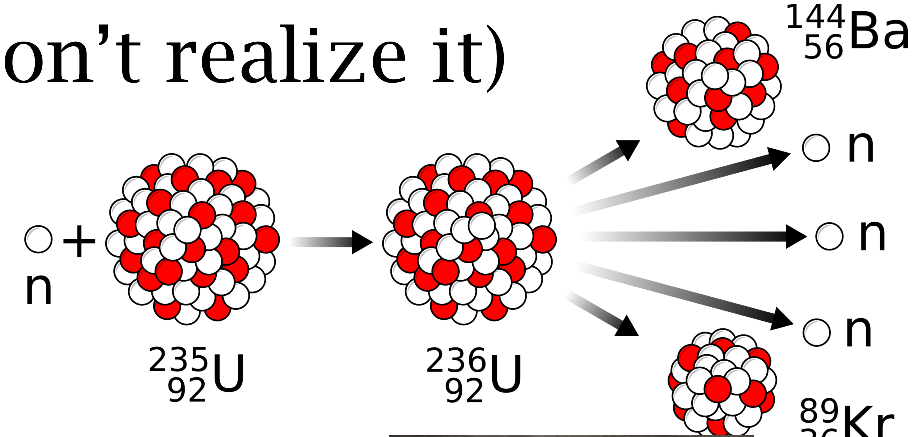
1944



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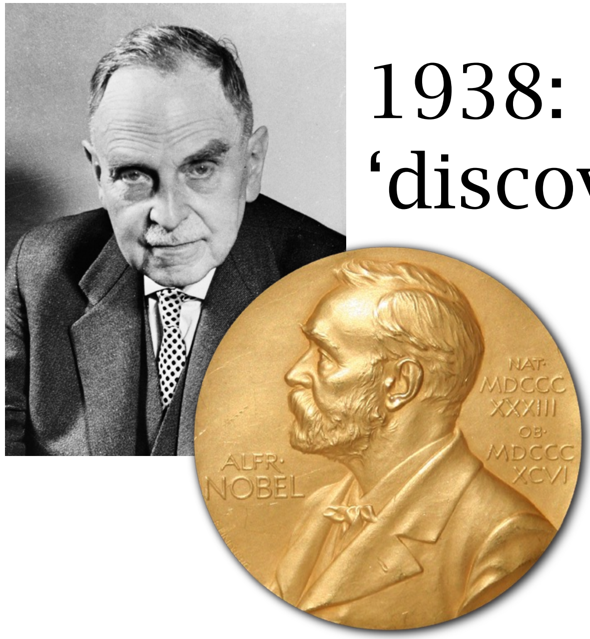

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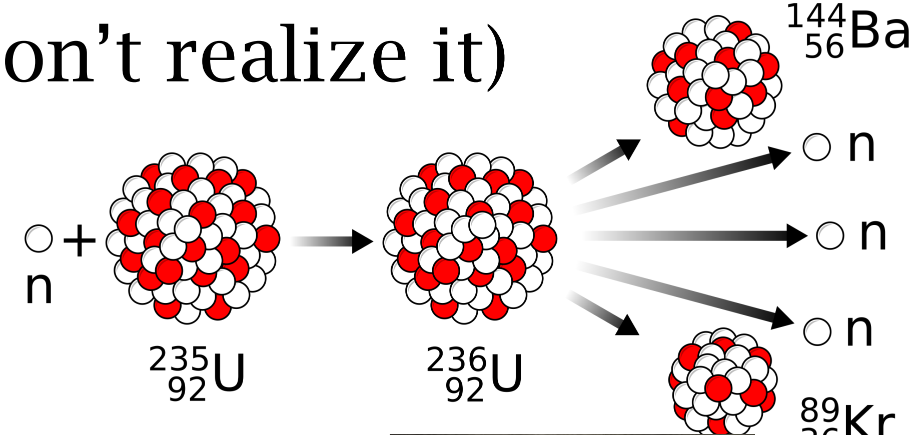

1939: Meitner and Frisch explain fission

PERIODIC TABLE OF ELEMENTS

Chemical Group Block

1

PubChem

Atomic Number — 17 — 35.45 — Atomic Mass, u
Symbol — Cl
Name — Chlorine
Chemical Group Block — Halogen

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 1.0080 **H** Hydrogen Nonmetal																	2 4.00260 **He** Helium Noble Gas
2	3 7.0 **Li** Lithium Alkali Metal	4 9.012183 **Be** Beryllium Alkaline Earth Me...											5 10.81 **B** Boron Metalloid	6 12.011 **C** Carbon Nonmetal	7 14.007 **N** Nitrogen Nonmetal	8 15.999 **O** Oxygen Nonmetal	9 18.9984... **F** Fluorine Halogen	10 20.180 **Ne** Neon Noble Gas
3	11 22.989... **Na** Sodium Alkali Metal	12 24.305 **Mg** Magnesium Alkaline Earth Me...											13 26.981... **Al** Aluminum Post-Transition M...	14 28.085 **Si** Silicon Metalloid	15 30.973... **P** Phosphorus Nonmetal	16 32.07 **S** Sulfur Nonmetal	17 35.45 **Cl** Chlorine Halogen	18 39.9 **Ar** Argon Noble Gas
4	19 39.0983 **K** Potassium Alkali Metal	20 40.08 **Ca** Calcium Alkaline Earth Me...	21 44.95591 **Sc** Scandium Transition Metal	22 47.867 **Ti** Titanium Transition Metal	23 50.9415 **V** Vanadium Transition Metal	24 51.996 **Cr** Chromium Transition Metal	25 54.93804 **Mn** Manganese Transition Metal	26 55.84 **Fe** Iron Transition Metal	27 58.93319 **Co** Cobalt Transition Metal	28 58.693 **Ni** Nickel Transition Metal	29 63.55 **Cu** Copper Transition Metal	30 65.4 **Zn** Zinc Transition Metal	31 69.723 **Ga** Gallium Post-Transition M...	32 72.63 **Ge** Germanium Metalloid	33 74.92159 **As** Arsenic Metalloid	34 78.97 **Se** Selenium Nonmetal	35 79.90 **Br** Bromine Halogen	36 83.80 **Kr** Krypton Noble Gas
5	37 85.468 **Rb** Rubidium Alkali Metal	38 87.62 **Sr** Strontium Alkaline Earth Me...	39 88.90584 **Y** Yttrium Transition Metal	40 91.22 **Zr** Zirconium Transition Metal	41 92.90637 **Nb** Niobium Transition Metal	42 95.95 **Mo** Molybdenum Transition Metal	43 96.90636 **Tc** Technetium Transition Metal	44 101.1 **Ru** Ruthenium Transition Metal	45 102.9055 **Rh** Rhodium Transition Metal	46 106.42 **Pd** Palladium Transition Metal	47 107.868 **Ag** Silver Transition Metal	48 112.41 **Cd** Cadmium Transition Metal	49 114.818 **In** Indium Post-Transition M...	50 118.71 **Sn** Tin Post-Transition M...	51 121.760 **Sb** Antimony Metalloid	52 127.6 **Te** Tellurium Metalloid	53 126.9045 **I** Iodine Halogen	54 131.29 **Xe** Xenon Noble Gas
6	55 132.90... **Cs** Cesium Alkali Metal	56 137.33 **Ba** Barium Alkaline Earth Me...		72 178.49 **Hf** Hafnium Transition Metal	73 180.9479 **Ta** Tantalum Transition Metal	74 183.84 **W** Tungsten Transition Metal	75 186.207 **Re** Rhenium Transition Metal	76 190.2 **Os** Osmium Transition Metal	77 192.22 **Ir** Iridium Transition Metal	78 195.08 **Pt** Platinum Transition Metal	79 196.96... **Au** Gold Transition Metal	80 200.59 **Hg** Mercury Transition Metal	81 204.383 **Tl** Thallium Post-Transition M...				5 209.98... **At** Astatine Halogen	86 222.01... **Rn** Radon Noble Gas
7	87 223.01... **Fr** Francium Alkali Metal	88 226.02... **Ra** Radium Alkaline Earth Me...		104 267.1... **Rf** Rutherfordium Transition Metal	105 268.1... **Db** Dubnium Transition Metal	106 269.1... **Sg** Seaborgium Transition Metal	107 270.1... **Bh** Bohrium Transition Metal	108 269.1... **Hs** Hassium Transition Metal	109 277.1... **Mt** Meitnerium Transition Metal	110 282.1... **Ds** Darmstadtium Transition Metal	111 282.1... genium ion Metal	112 286.1... Copernicium Transition Metal	113 286.1... **Nh** Nihonium Post-Transition M...				17 294.2... **Ts** ennessine Halogen	118 295.2... **Og** Oganesson Noble Gas
				57 138.9055 **La** Lanthanum Lanthanide	58 140.116 **Ce** Cerium Lanthanide	59 140.90... **Pr** Praseodymium Lanthanide	60 144.24 **Nd** Neodymium Lanthanide	61 144.91... **Pm** Promethium Lanthanide	62 150.4 **Sm** Samarium Lanthanide	63 151.964 **Eu** Europium Lanthanide	64 157.2 **Gd** Gadolinium Lanthanide	65 158.92... **Tb** Terbium Lanthanide	66 162.500 **Dy** Dysprosium Lanthanide				0 173.05 **Yb** Ytterbium Lanthanide	71 174.9668 **Lu** Lutetium Lanthanide
				89 227.02... **Ac** Actinium Actinide	90 232.038 **Th** Thorium Actinide	91 231.03... **Pa** Protactinium Actinide	92 238.0289 **U** Uranium Actinide	93 237.04... **Np** Neptunium Actinide	94 244.06... **Pu** Plutonium Actinide	95 243.06... **Am** Americium Actinide	96 247.07... **Cm** Curium Actinide	97 247.07... **Bk** Berkelium Actinide	98 251.07... **Cf** Californium Actinide				02 259.1... **No** Nobelium Actinide	103 266.1... **Lr** Lawrencium Actinide

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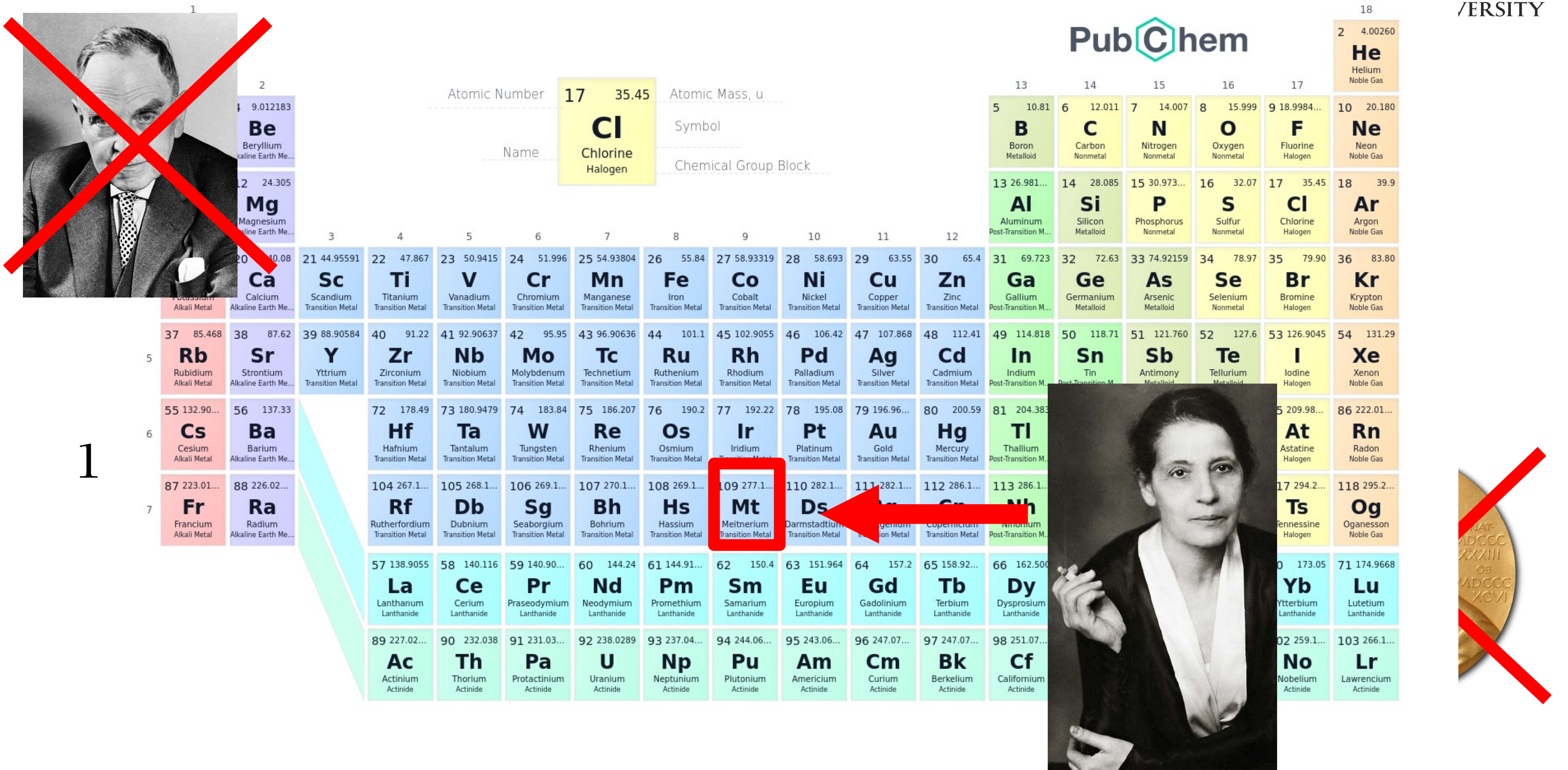

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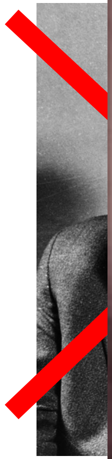

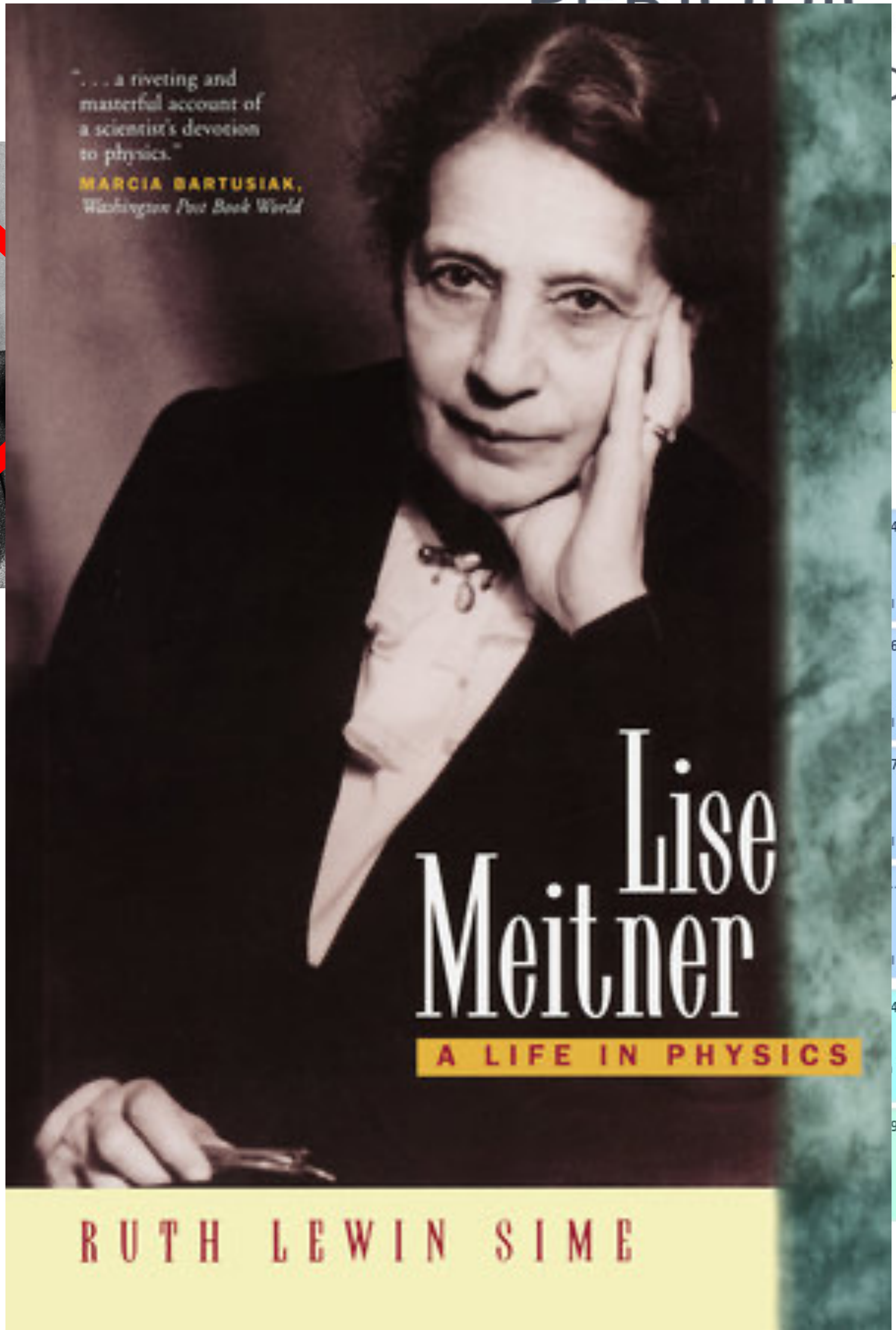

"... a riveting and masterful account of a scientist's devotion to physics."
MARCIA BARTUSIAK,
Washington Post Book World

Lise Meitner
A LIFE IN PHYSICS
RUTH LEWIN SIME

.45 Atomic Mass, u
Symbol
Chemical Group Block

PubChem

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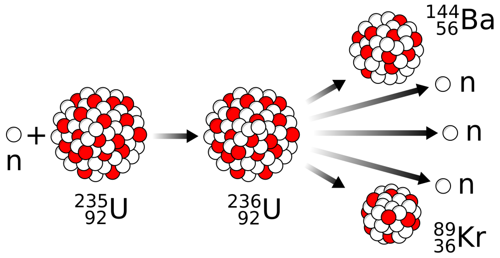
76 190.2 Os Osmium Transition Metal	77 192.22 Ir Iridium	78 195.08 Pt Platinum Transition Metal	79 196.96... Au Gold Transition Metal	80 200.59 Hg Mercury Transition Metal	81 204.383 Tl Thallium Post-Transition M...				209.98... At Astatine Halogen	86 222.01... Rn Radon Noble Gas
108 269.1... Hs Hassium Transition Metal	109 277.1... Mt Meitnerium Transition Metal	110 282.1... Ds Darmstadtium Transition Metal	111 282.1... genium ion Metal	112 286.1... Transition Metal	113 286.1... Post-Transition M...				294.2... Ts ennessine Halogen	118 295.2... Og Oganesson Noble Gas
61 144.91... Pm Promethium Lanthanide	62 150.4 Sm Samarium Lanthanide	63 151.964 Eu Europium Lanthanide	64 157.2 Gd Gadolinium Lanthanide	65 158.92... Tb Terbium Lanthanide	66 162.500 Dy Dysprosium Lanthanide				173.05 Yb Ytterbium Lanthanide	71 174.9668 Lu Lutetium Lanthanide
93 237.04... Np Neptunium Actinide	94 244.06... Pu Plutonium Actinide	95 243.06... Am Americium Actinide	96 247.07... Cm Curium Actinide	97 247.07... Bk Berkelium Actinide	98 251.07... Cf Californium Actinide				259.1... No Nobelium Actinide	103 266.1... Lr Lawrencium Actinide

The birth of the shell model

Our story so far: by the early 1940s, a lot of nuclear properties seemed well described by the liquid drop model….



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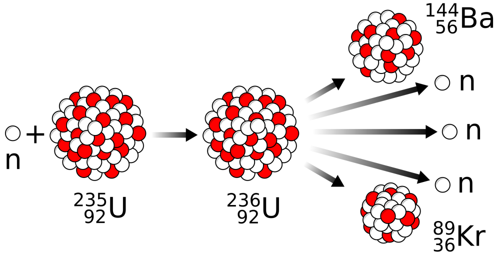

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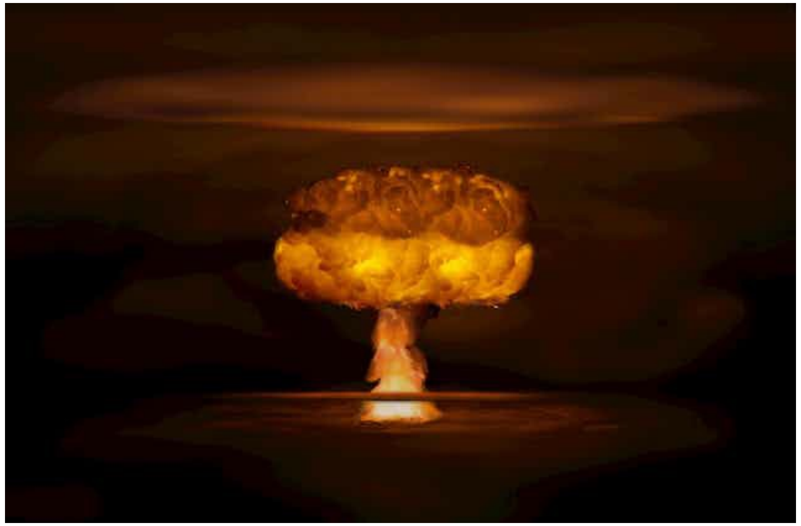
….which led to this…



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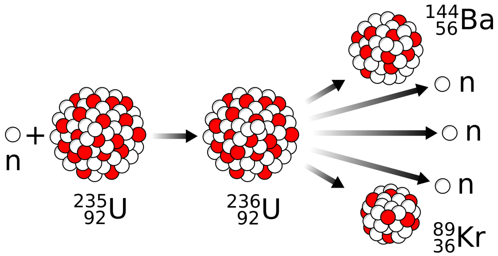

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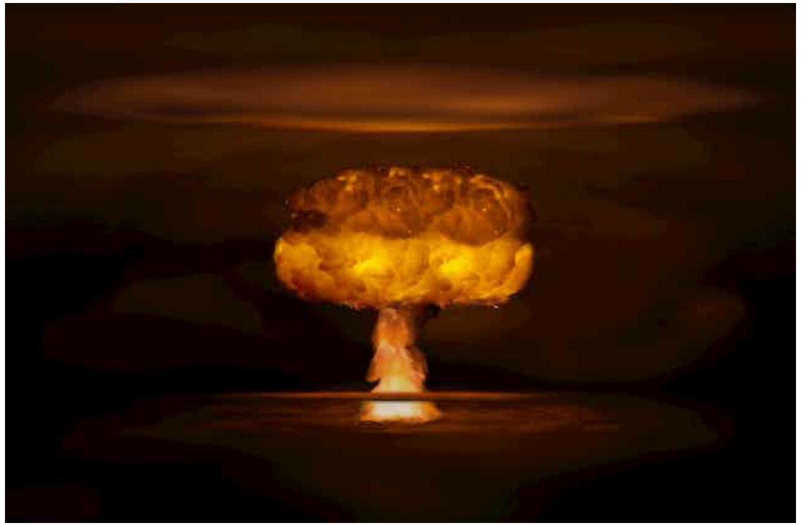
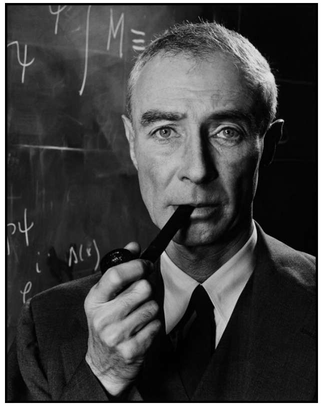


….developed here.

The birth of the shell model

San Diego State
University

As the movie *Oppenheimer* showed (kind of), many physicists were recruited for the Manhattan project












The birth of the shell model

SAN DIEGO STATE UNIVERSITY

One of these was Maria Goeppert Mayer.

The birth of the shell model

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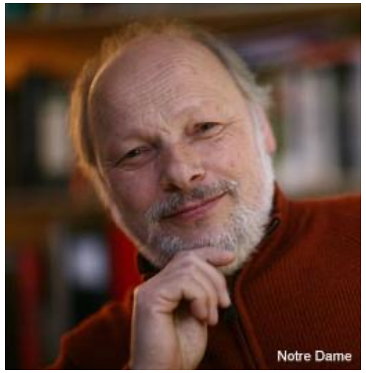
I'll have more history to report in a little while



Michael Wiescher

The birth of the shell model

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Her Ph.D dissertation (1930, Gottingen) was on two-photon absorption.

The birth of the shell model

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In 1942 she worked on separation of fissile material at Columbia.
In 1945 she joined Teller at Los Alamos to work on opacities.

The birth of the shell model

One of these was Maria Goeppert Mayer.

In 1946, Goeppert Mayer went to the newly formed Argonne National Lab, protesting “I don’t know anything about nuclear physics.”

Through her work on fission, though, she noticed unusual patterns in binding energies.

One o

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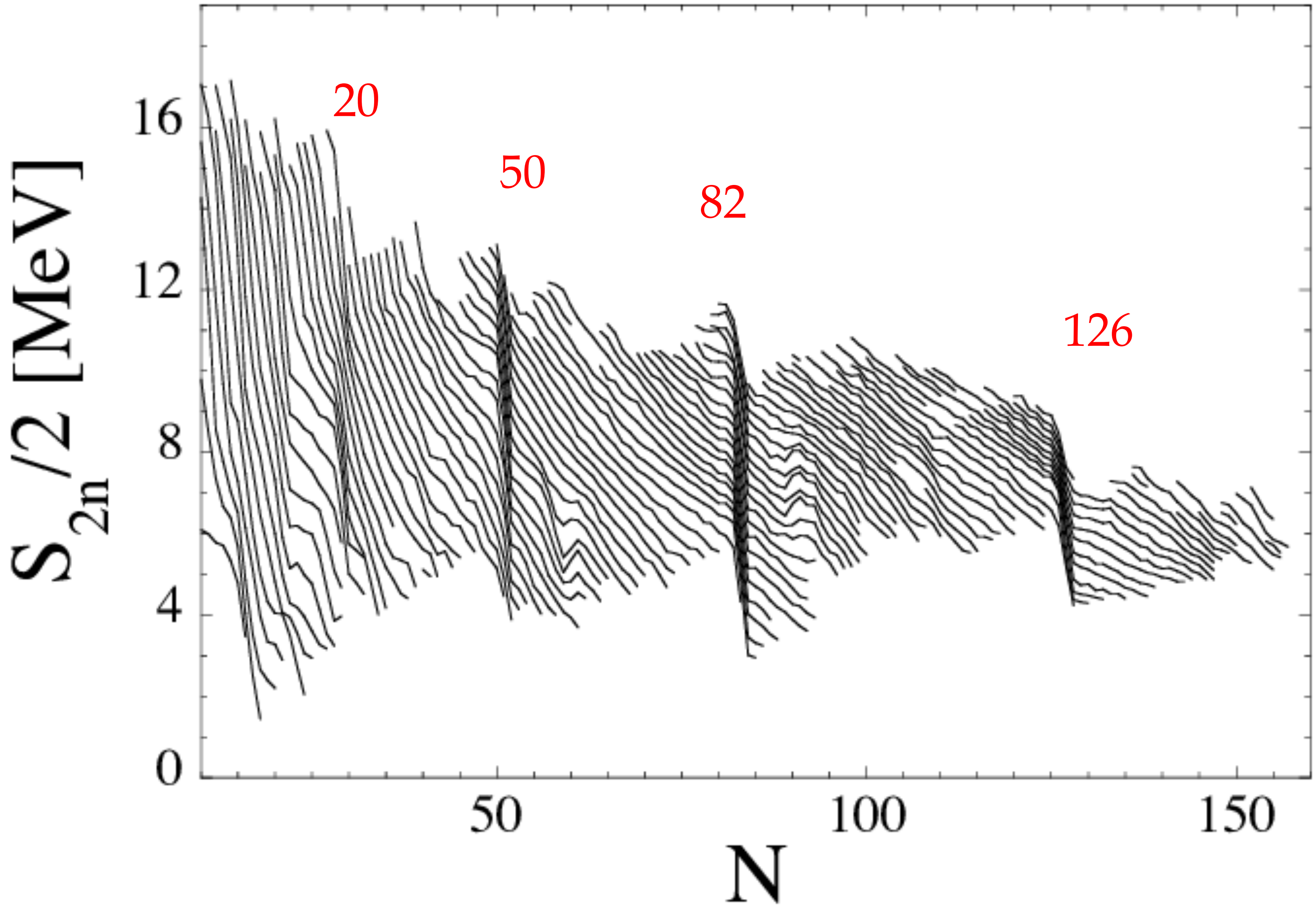

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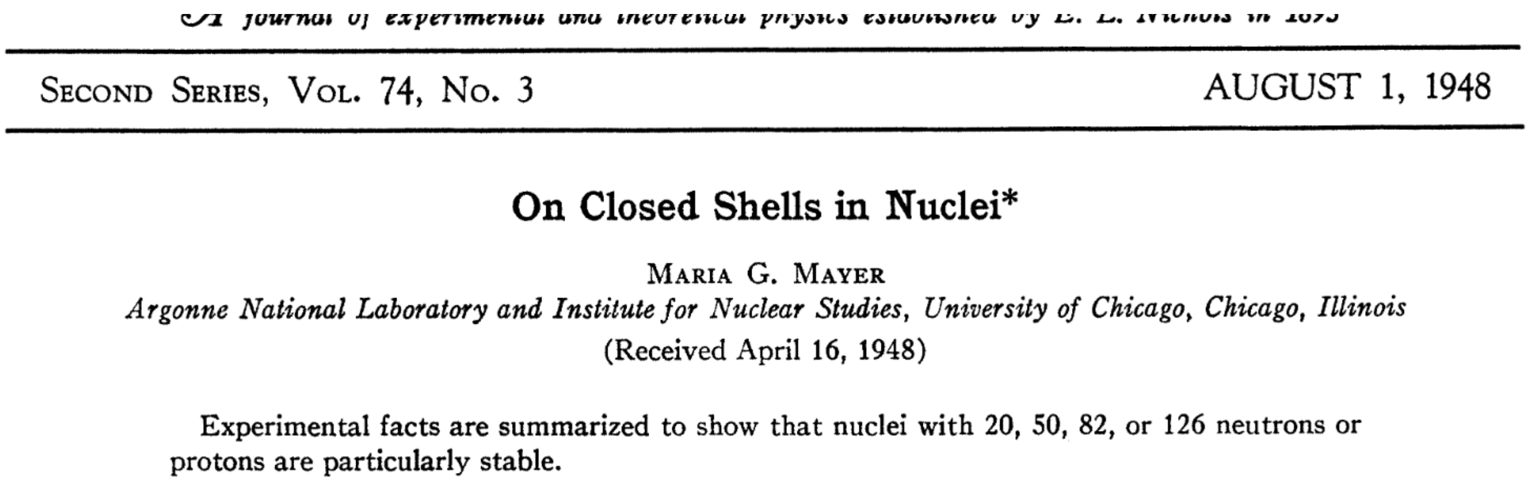

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

Second Series, Vol. 74, No. 3 AUGUST 1, 1948

On Closed Shells in Nuclei*

Maria G. Mayer
Argonne National Laboratory and Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received April 16, 1948)

Experimental facts are summarized to show that nuclei with 20, 50, 82, or 126 neutrons or protons are particularly stable.

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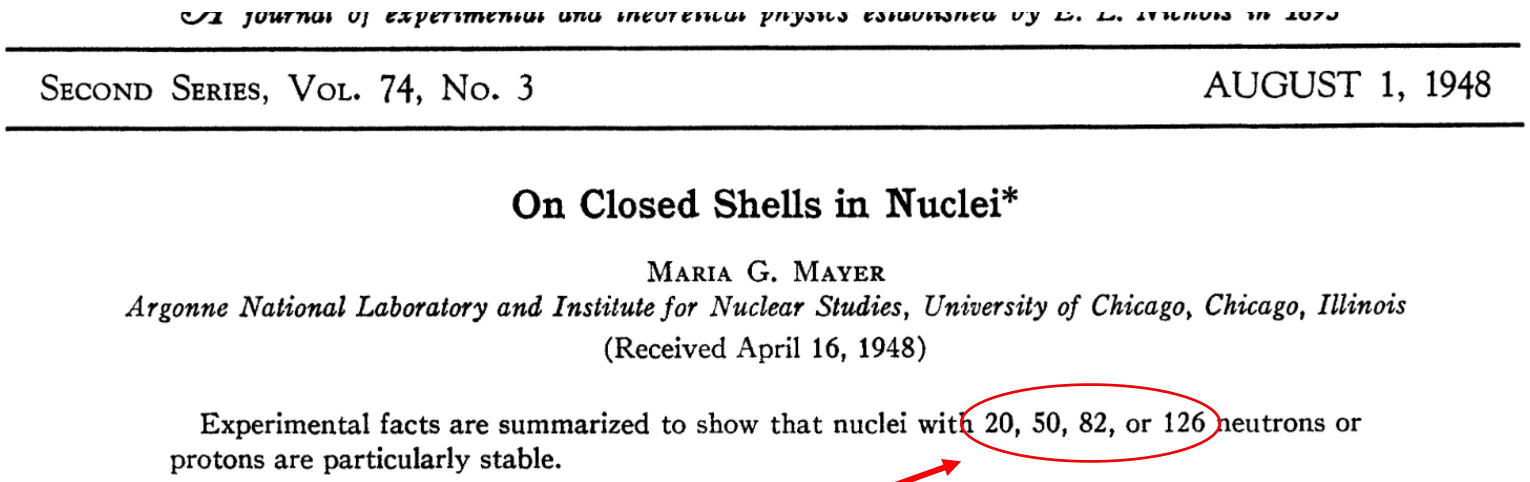

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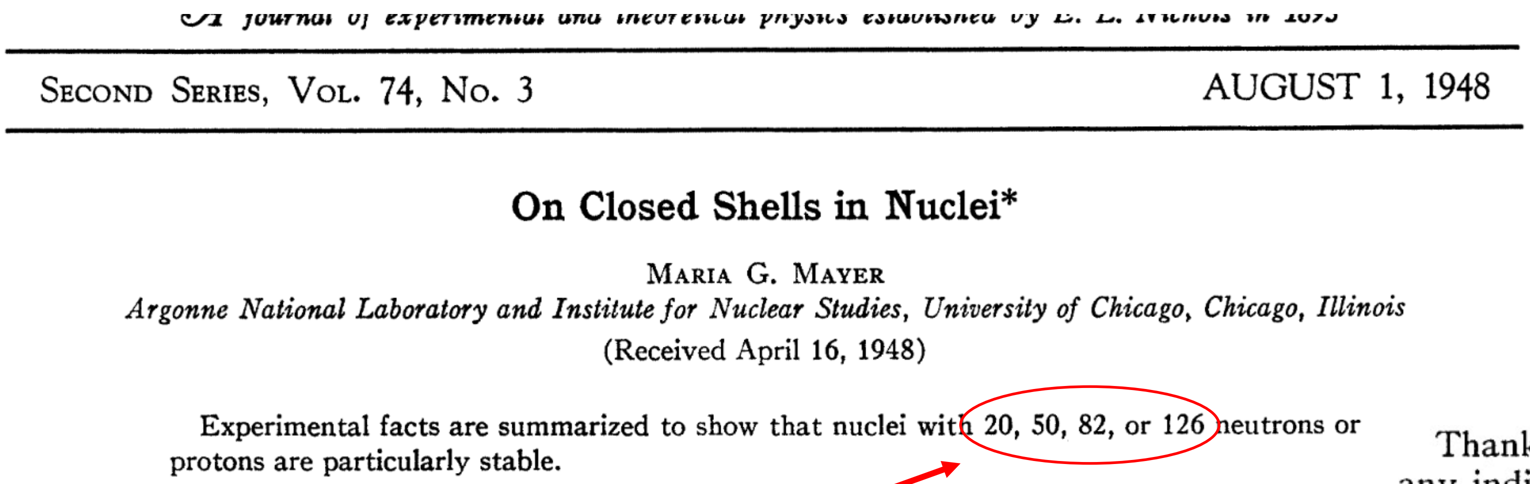

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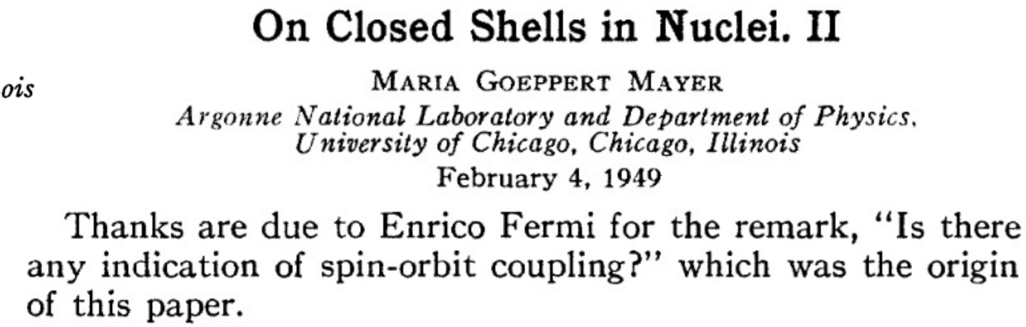

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MARIA GOEPPERT MAYER
Argonne National Laboratory and Department of Physics, University of Chicago, Chicago, Illinois
February 4, 1949

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Magic numbers!

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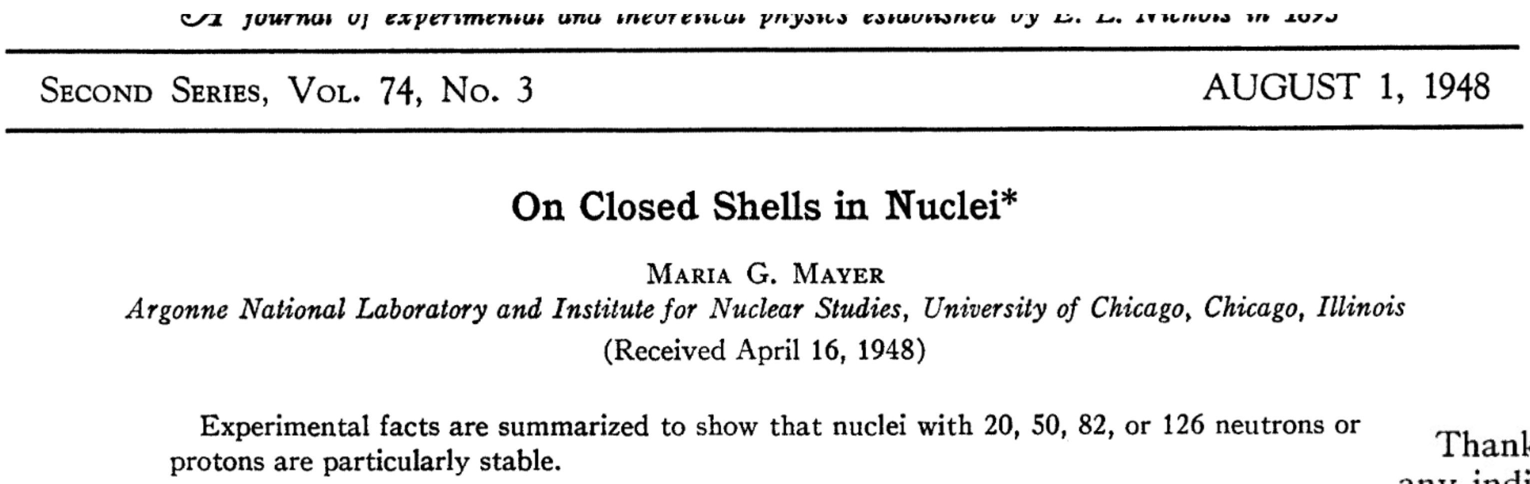

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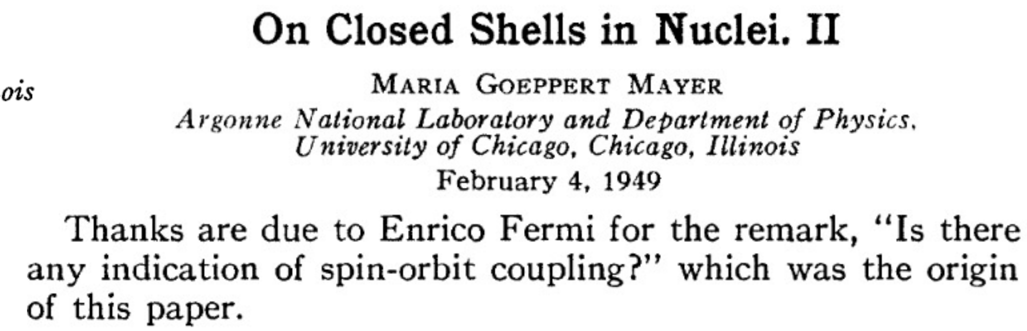

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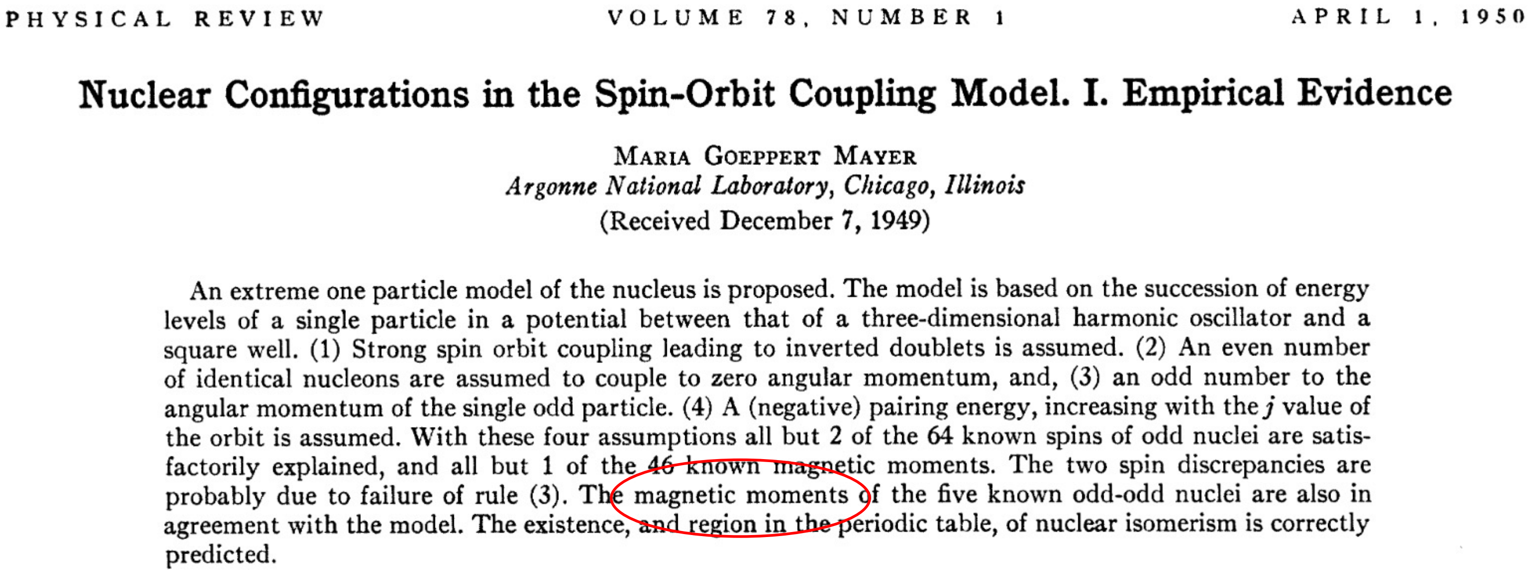


PHYSICAL REVIEW VOLUME 78, NUMBER 1 APRIL 1, 1950

Nuclear Configurations in the Spin-Orbit Coupling Model. I. Empirical Evidence

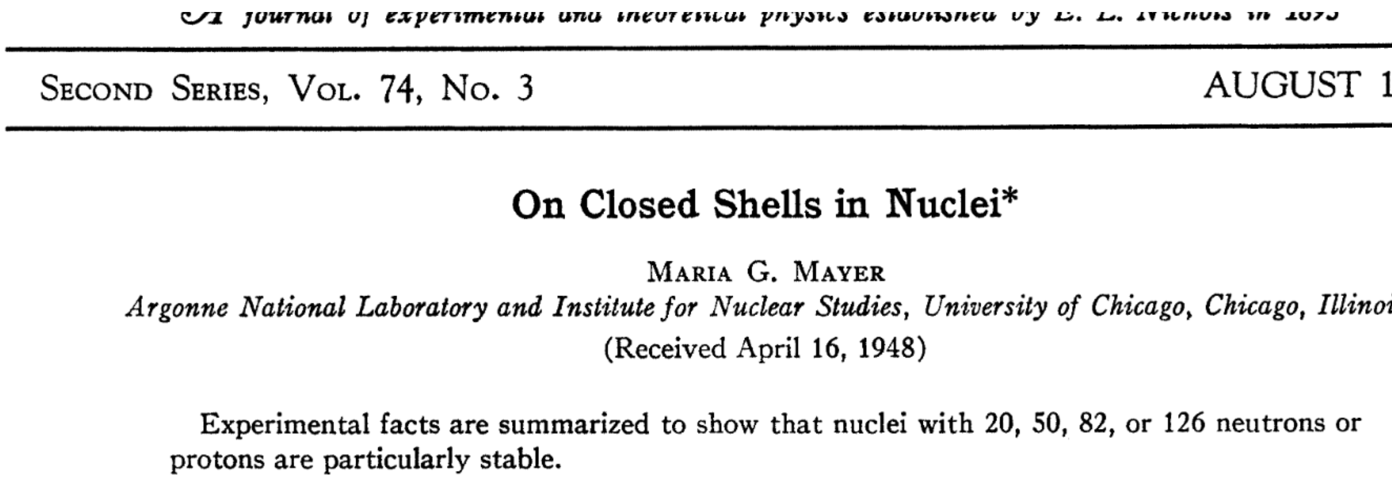
MARIA GOEPPERT MAYER
Argonne National Laboratory, Chicago, Illinois

(Received December 7, 1949)

An extreme one particle model of the nucleus is proposed. The model is based on the succession of energy levels of a single particle in a potential between that of a three-dimensional harmonic oscillator and a square well. (1) Strong spin orbit coupling leading to inverted doublets is assumed. (2) An even number of identical nucleons are assumed to couple to zero angular momentum, and, (3) an odd number to the angular momentum of the single odd particle. (4) A (negative) pairing energy, increasing with the *j* value of the orbit is assumed. With these four assumptions all but 2 of the 64 known spins of odd nuclei are satisfactorily explained, and all but 1 of the 46 known magnetic moments. The two spin discrepancies are probably due to failure of rule (3). The magnetic moments of the five known odd-odd nuclei are also in agreement with the model. The existence, and region in the periodic table, of nuclear isomerism is correctly predicted.

The birth of the shell model

SAN DIEGO STATE UNIVERSITY



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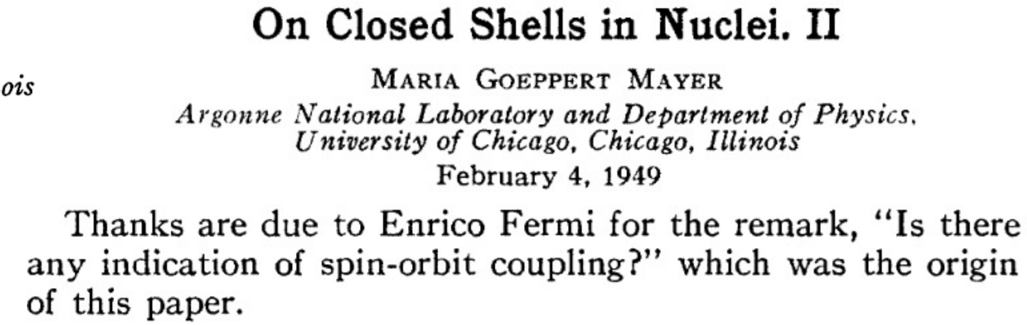

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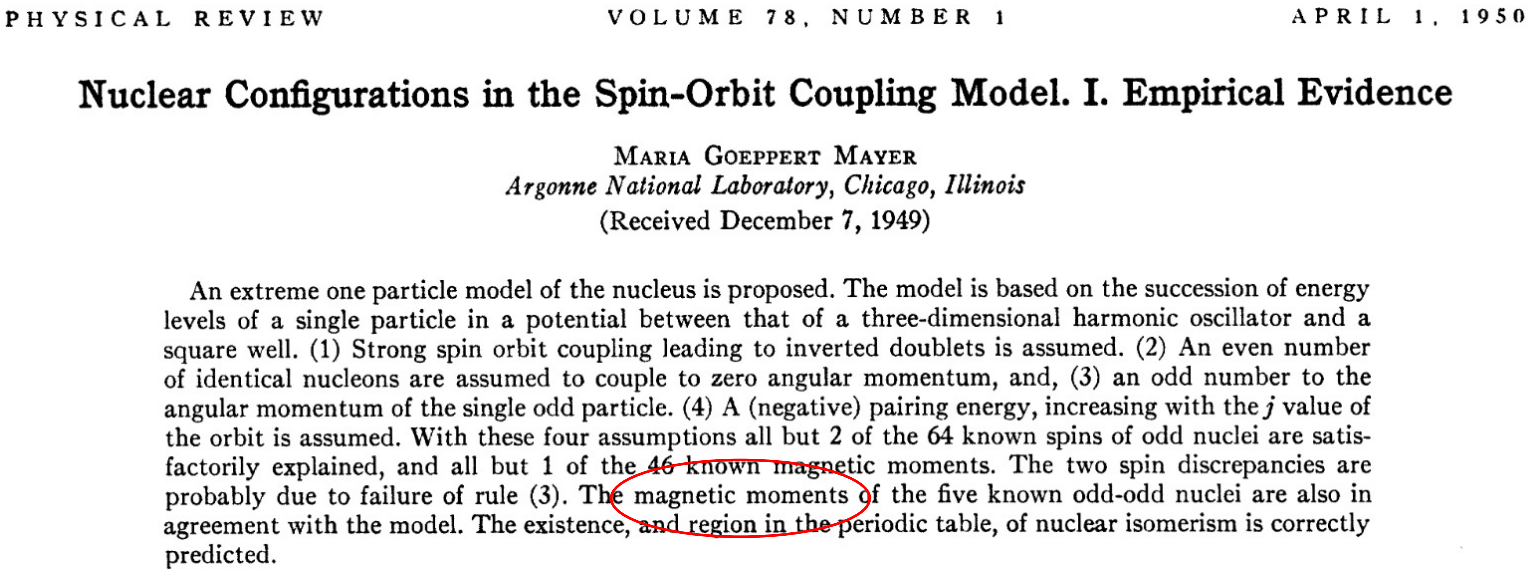

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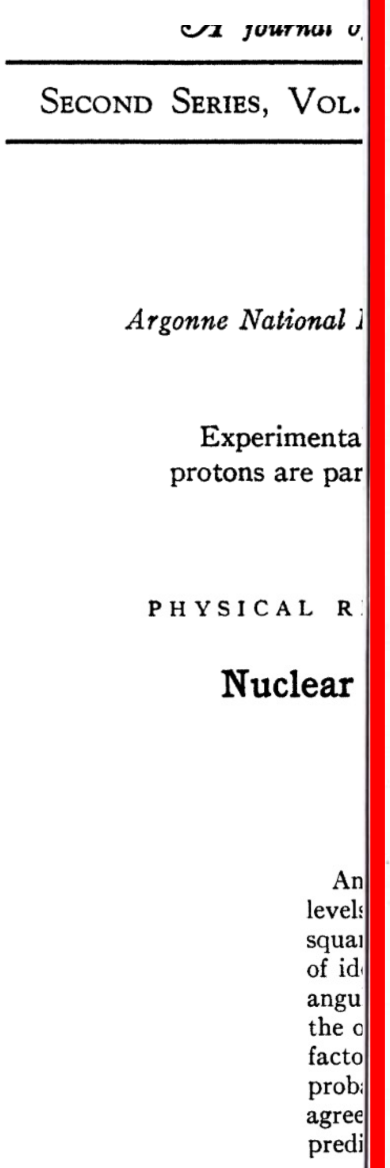

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British, French Back GIs in Soviet Block

WORLD RENOWNED—Dr. Maria Goeppert Mayer, 57, holds the slide rule she uses in the study of nuclear physics that won her a Nobel Prize today. She is a University of California professor here.

S.D. Mother Wins Nobel Physics Prize

Dr. Mayer 1st Woman in U.S., 2nd in History So Honored

By FRANK HOGAN

Dr. Maria Goeppert Mayer, 57, a research physicis at the University of California here, today was name a 1963 Nobel Prize winner in physics.

The red-haired college professor, mother of two, i the first woman residing in America to win a Nob physics prize, the second woman in history.

She received the news in a phone call from Sweden at 4 this morning. The caller informed her she had been cited by the Swedish Royal Academy of Science for her work in determining the nature of the shell of the atom's nucleus.

Will Share Award

Mrs. Mayer will share the $51,158 prize with a West German scientist, J. Hans D. Jensen of Heidelberg, who made a similar but independent study, and Eugene Wigner of Princeton University, who helped lay the groundwork for the present advanced study in nuclear physics.

Wigner will get half the prize. Mrs. Mayer and Jensen will divide the rest.

Second on Campus Here

Mrs. Mayer is the second Nobel Prize winner on the university's San Diego campus. The other is Dr. Harold C. Urey of 7890 Torrey Lane, professor of chemistry at large, who won the Nobel chemistry award in 1934.

Mother of a married daughter and a grown son, Mrs. Mayer is the wife of Dr. Joseph E. Mayer, a chemistry professor at the University of California here. They joined the local faculty in 1960, and live at 2345 Via Sienna, La Jolla.

Excited and insisting the award caught her by sur-

State Dept. Fires Officer Over 'Leak'

WASHINGTON (UPI) — Ott F. Otepka was dismissed today from his job as a Stat Department security office on charges of conduct "un becoming an officer of th Department of State."

Otepka, 48, has been under suspension since Sept. 23, accused among othe things of giving confidentia information to the Senat Internal Security subcommittee.

A letter removing him from the ranks of department employes was delivered today.

Signed by John Ordway, chief of the department's personnel division, it replied to Otepka's 12-page-long letter of Oct. 14, requesting that charges against him be dismissed, and upheld the charges.

Otepka had been chief security risk evaluator for the department at a salary o $16,900-a-year.

A qualified source s that ...pka's case w... or- ... the de- cision to dismiss him was

Diem Brother Turned Over To Viet Rebels

U.S. Consulate Told Leader Will Get Trial

Written From NEWS SERVICES

Ngo Dinh Can, boss of central Viet Nam during the regime of his brother, slain President Ngo Dinh Diem, was turned over to South Viet Nam's revolutionary government today after he sought asylum at the U.S. consulate at Hue.

U.S. officials said they were assured that Can would not be lynched and would receive the due process of law.

There were these other developments:

First — The revolutionary government named former Vice President Nguyen Ngoc Tho premier and appointed a 15-man cabinet.

Second — The children of Mrs. Ngo Dinh Nhu arrived in Rome today. Mrs. Nhu, in Los Angeles, said she will fly there tomorrow and then may return to the United States.

Third—Dr. Tran Van Chuong, former South Vietnamese ambassador to the United States and Mrs. Nhu's father, said in Fresno that the new government has the backing of the Vietnamese people and that the United States will lose no prestige because of the fall of Diem and his brother, Nhu.

Fourth—U.S. recognition of the new government is expected quickly, according to Washington officials.

Can, who dropped from sight when the coup overthrew Diem's regime last weekend went to the U.S. consulate last night seeking refuge.

Consular officials notified the revolutionary government and today Can was flown 400 miles in a U.S. military plane to Saigon and turned over to officials of the military junta.

Buddhist leaders, students and others in Hue long have charged that Can ruled his bailiwick as a bloody tyrant.

Meantime, Maj. Gen. Duong Van Minh, 47, leader of the coup, said in a proclamation that a republican form of government will be retained although the junta will be the country's final authority.

Viet Cong War

Minh, who assumed the position of chief of state, said his committee of generals will give its chief attention to direction of the war against the Communist Viet Cong.

Until revision of the 1956 constitution, Minh said, legislative and executive power will center in the junta but will be exercised by the provisional government. He explained that the junta will retain active authority over national defense, security and finances.

In Rome, Mrs. Nhu's brother-in-law, Archbishop Ngo Dinh Thuc of Hue, met her three younger children — a boy 15 and two girls 11 and 4 — at the airport.

Convoys Sent To Join U.S. In Showdown

Red Armored Cars Box In Yanks Trying Break-Out

Written From NEWS SERVICES

BERLIN—The British and French sent convoys out to join a Russian-blockaded U.S. Army convoy on the Autobahn leading to Berlin today in an Allied challenge to the Soviet Union.

The Russians allowed the French and British convoys to pass a checkpoint outside Berlin en route to their rendezvous at the border 110 miles away with the Americans. Both were halted for 30 to 50 minutes before being allowed to proceed.

Incident 'Quite Serious'

The Americans have been held up since 9 a.m. yesterday at the Russians' Marienborn checkpoint just across from West Germany in a dispute over clearance procedures.

The Russians brought up armored cars and 100 armed soldiers to hem in the Americans this morning in an incident which the State Department has termed "quite serious."

GIs Headed Off

The American convoy attempted to run the blockade early this morning but the Soviets used the armored cars to head it off and box it in.

In Washington, the White House said today that President Kennedy is deeply concerned over the blockade and that officials are trying to resolve the impasse through discussions here, in Moscow and in Bonn.

Kennedy summoned Secretary of State Dean Rusk; Llewellyn Thompson, top depart-

CHECK POINT

STOP!

U.S. ARMY

NEXT UP—Lt. Baker, left, leader of 56-man U.S. convoy waiting to cross border from West to East Germany at Helmstedt, supervises unloading of food. The convoy will move to West Berlin if another U.S. convoy ahead of it—held by the Soviets almost 24 hours—is allowed to pass.—(UPI) Telephoto

Reds Warned By Johnson To Watch Step

ROTTERDAM, The Netherlands (UPI)—Vice President Johnson warned the Russians today that although the Soviet Union can win American friendship it "can also provoke our hostility."

Johnson and his wife,

Early Turnout Points to Record

By RALPH BENNETT
EVENING TRIBUNE Political Writer

The doors to 1,095 polling places in the city swung open this morning and the voters started to file in.

Under skies of flawless blue, it soon was evident that a record vote was being cast.

Twenty-five per cent of the voters cast ballots in the five hours. Election officers said

Last night the most intensive and expensive political campaign in the city's history came to an end.

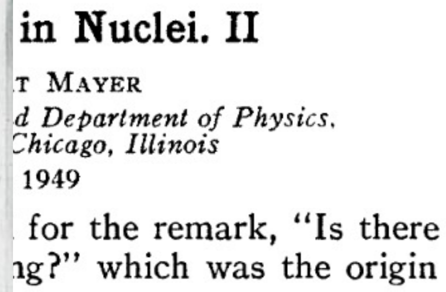
The mayor candidates — Councilman Frank Curran, 50, an insurance man, and Murray D. Goodrich, 61, a scrap metal dealer and surplus store owner—closed their campaigns.

Television Debate

WEATHERMAN NOW VOTING FOR SUNSHINE

Weatherman A. W. Anderson has delayed until tomorrow the rains he had originally forecast for election day today.

Yesterday, he said a weak storm front would bring showers today. But, he said, the storm has stalled over Oregon and now won't get here until late tonight.



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SAN DIEGO STATE
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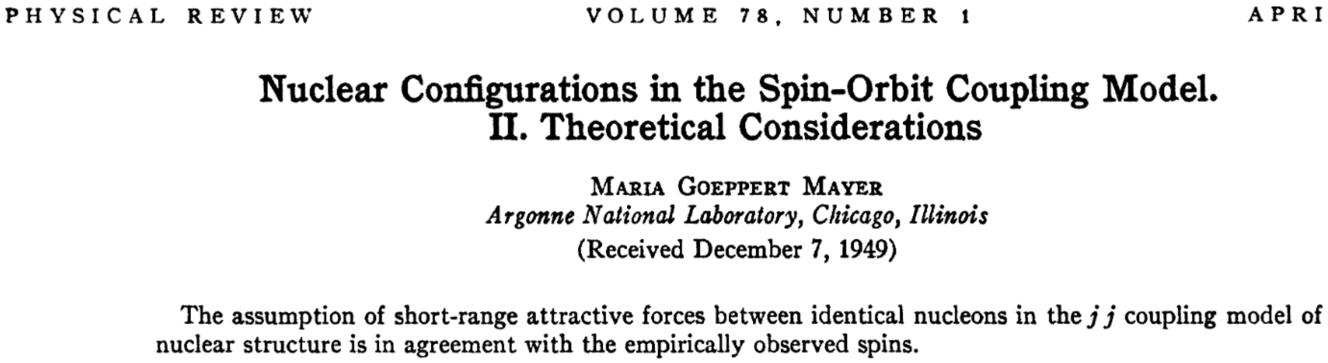
The birth of the shell model



PHYSICAL REVIEW VOLUME 78, NUMBER 1 APRIL 1, 1950

Nuclear Configurations in the Spin-Orbit Coupling Model. II. Theoretical Considerations

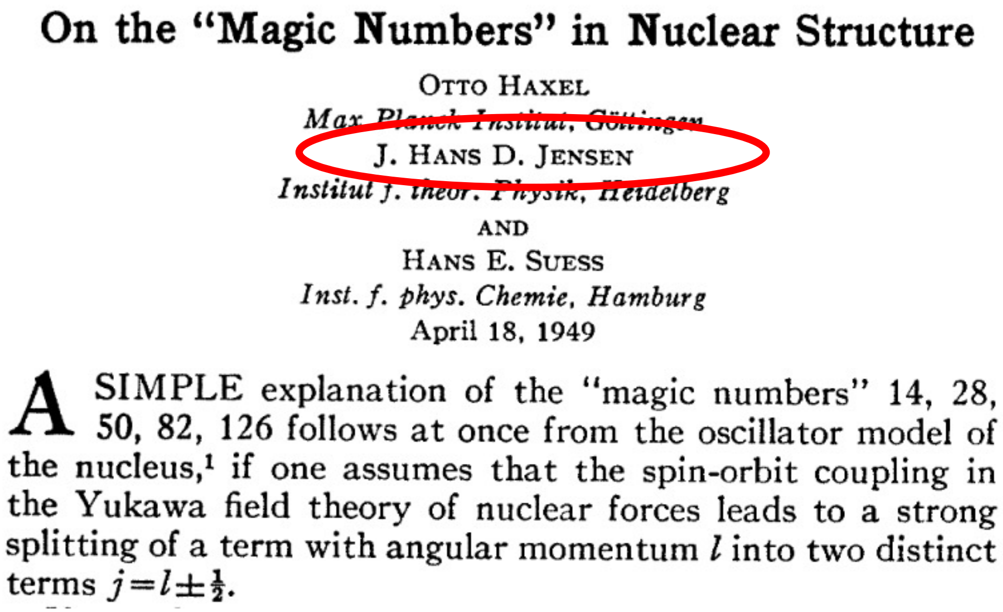
MARIA GOEPPERT MAYER
Argonne National Laboratory, Chicago, Illinois

(Received December 7, 1949)

The assumption of short-range attractive forces between identical nucleons in the $j\,j$ coupling model of nuclear structure is in agreement with the empirically observed spins.

Pairing!

The birth of the shell model



On the "Magic Numbers" in Nuclear Structure

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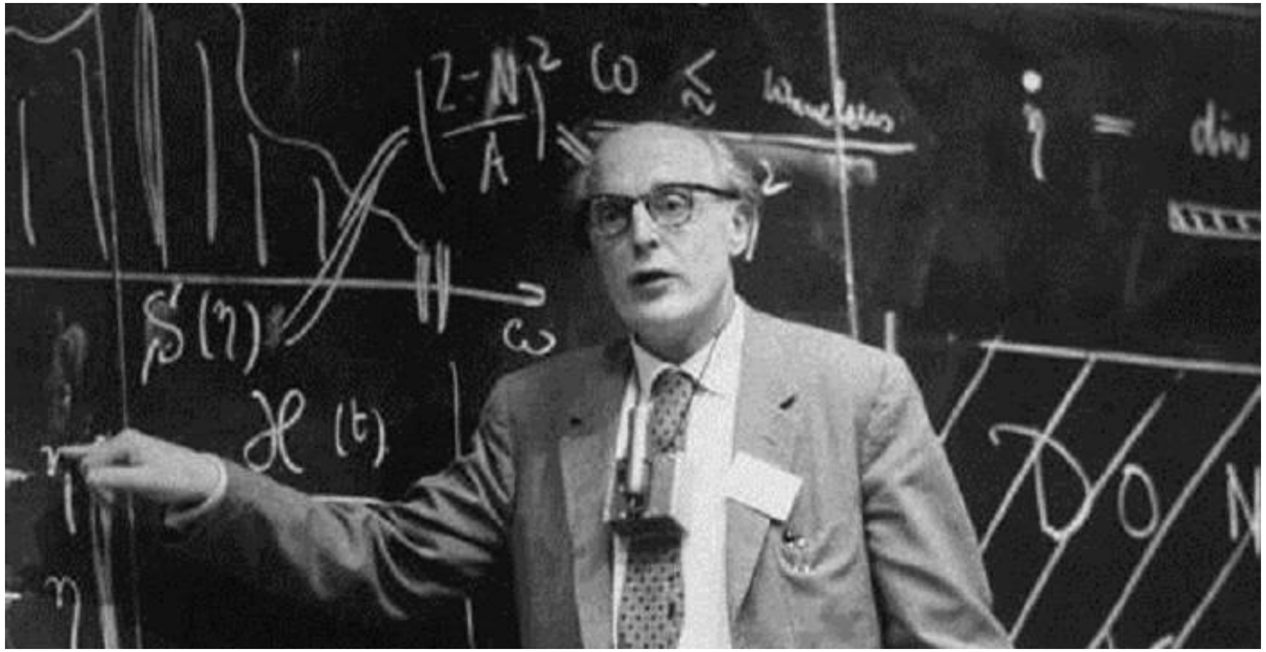
Max Planck Institut, Göttingen
J. Hans D. Jensen
Institut f. theor. Physik, Heidelberg
and
Hans E. Suess
Inst. f. phys. Chemie, Hamburg
April 18, 1949

A SIMPLE explanation of the "magic numbers" 14, 28, 50, 82, 126 follows at once from the oscillator model of the nucleus,[1] if one assumes that the spin-orbit coupling in the Yukawa field theory of nuclear forces leads to a strong splitting of a term with angular momentum l into two distinct terms $j=l\pm\frac{1}{2}$.



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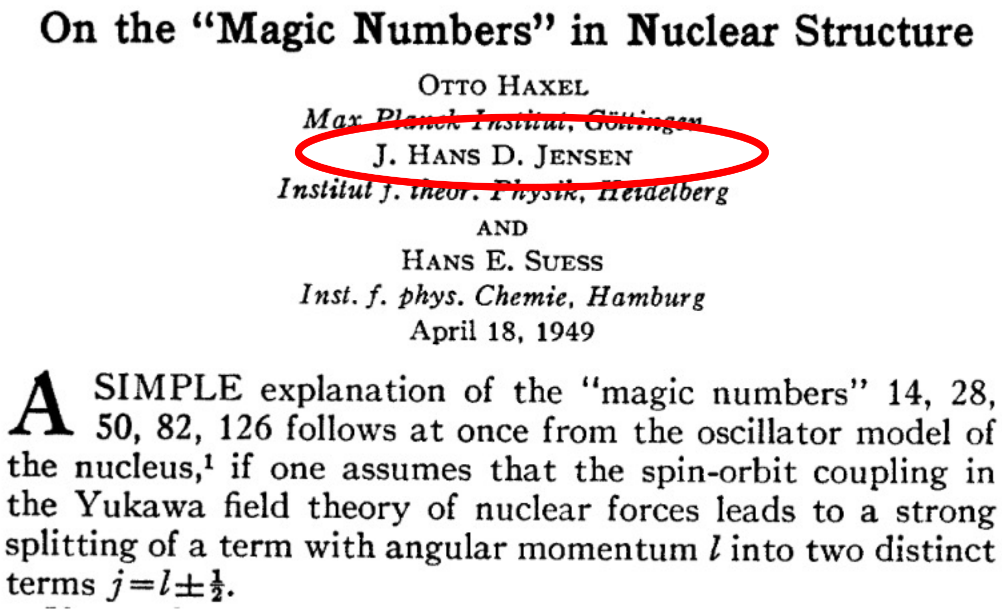

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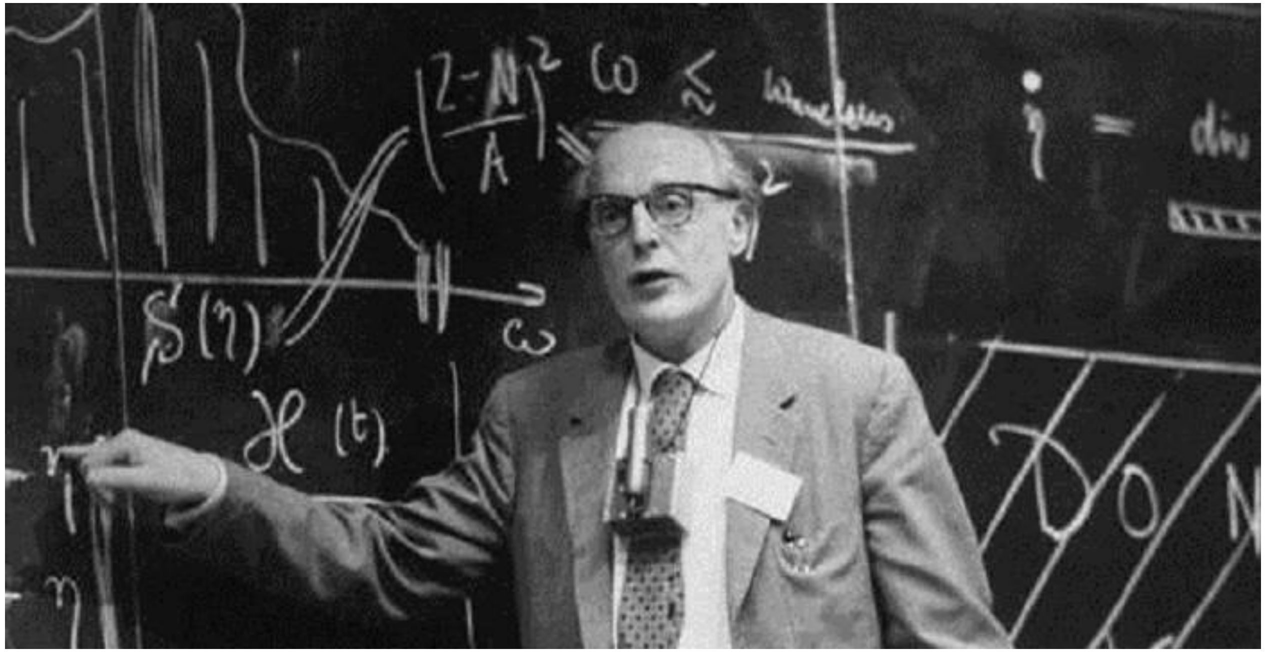

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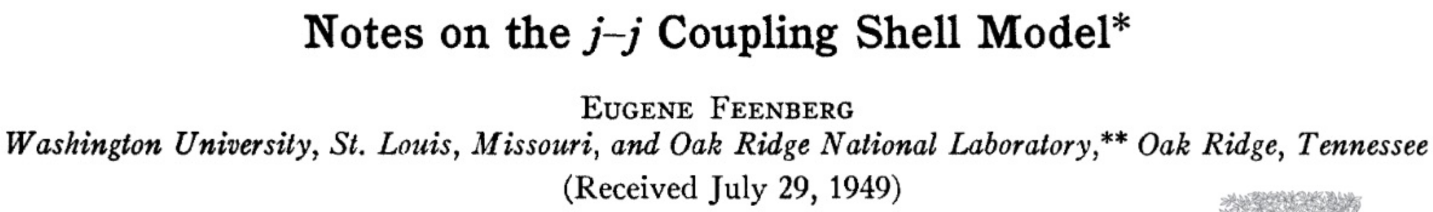
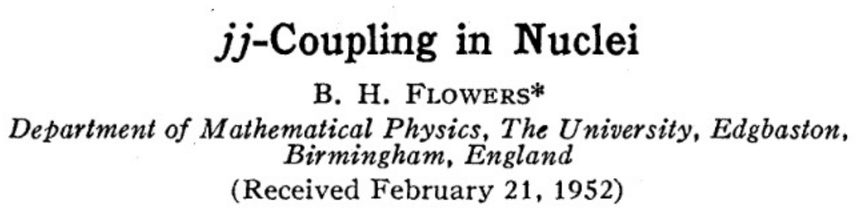


Notes on the *j–j* Coupling Shell Model*

EUGENE FEENBERG
*Washington University, St. Louis, Missouri, and Oak Ridge National Laboratory,*** *Oak Ridge, Tennessee*

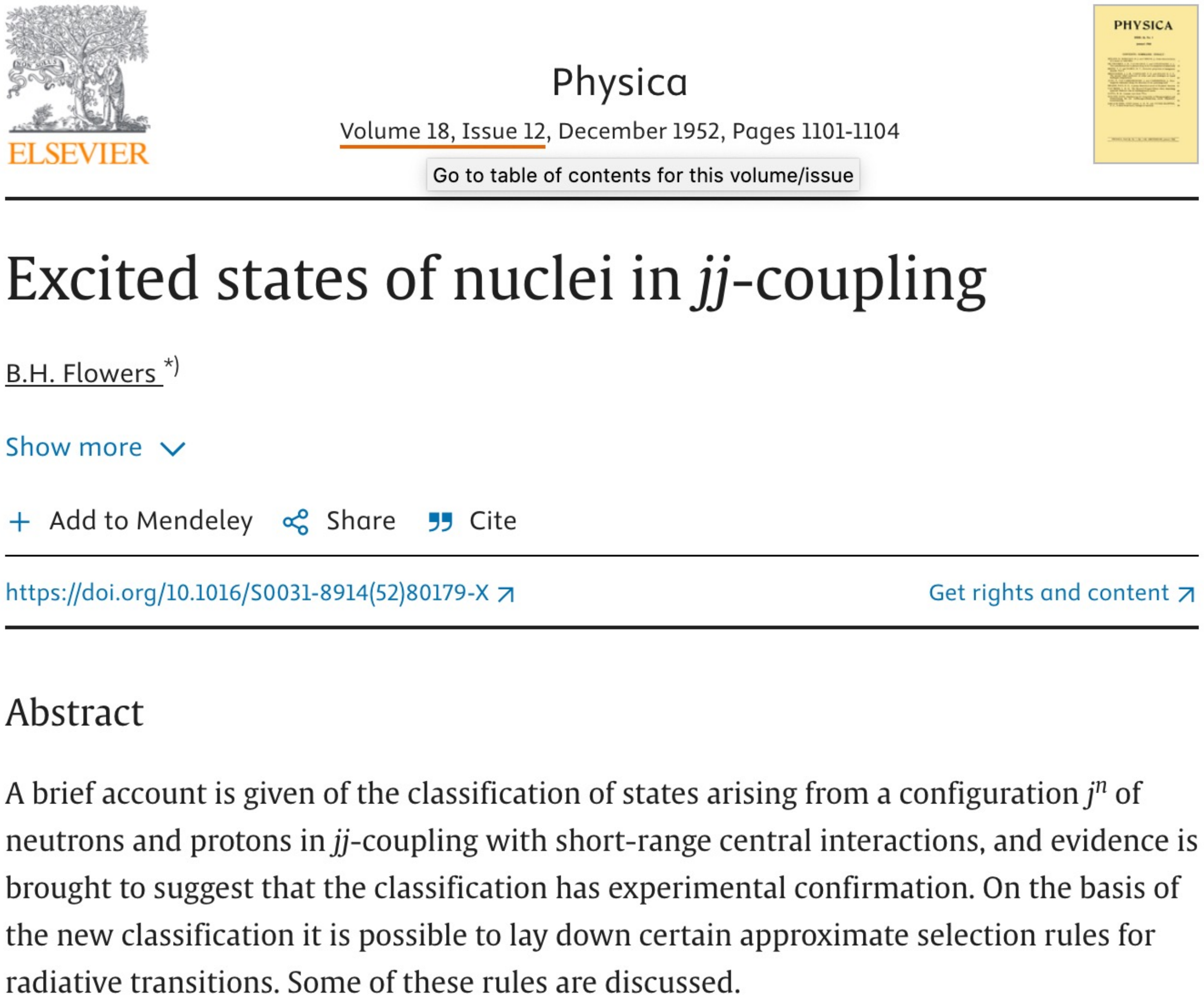
(Received July 29, 1949)



jj-Coupling in Nuclei

B. H. FLOWERS*
Department of Mathematical Physics, The University, Edgbaston, Birmingham, England

(Received February 21, 1952)



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Physica

Volume 18, Issue 12, December 1952, Pages 1101-1104

Go to table of contents for this volume/issue

Excited states of nuclei in *jj*-coupling

B.H. Flowers *)

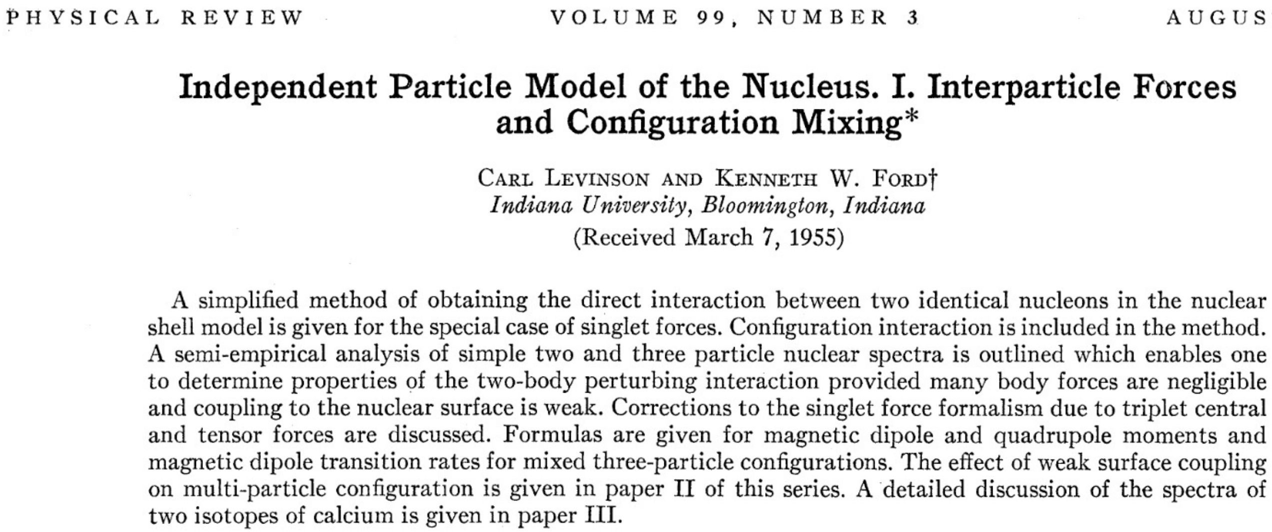
https://doi.org/10.1016/S0031-8914(52)80179-X

Abstract

A brief account is given of the classification of states arising from a configuration j^n of neutrons and protons in *jj*-coupling with short-range central interactions, and evidence is brought to suggest that the classification has experimental confirmation. On the basis of the new classification it is possible to lay down certain approximate selection rules for radiative transitions. Some of these rules are discussed.

The birth of the shell model

SAN DIEGO STATE UNIVERSITY



PHYSICAL REVIEW VOLUME 99, NUMBER 3 AUGUST 1, 1955

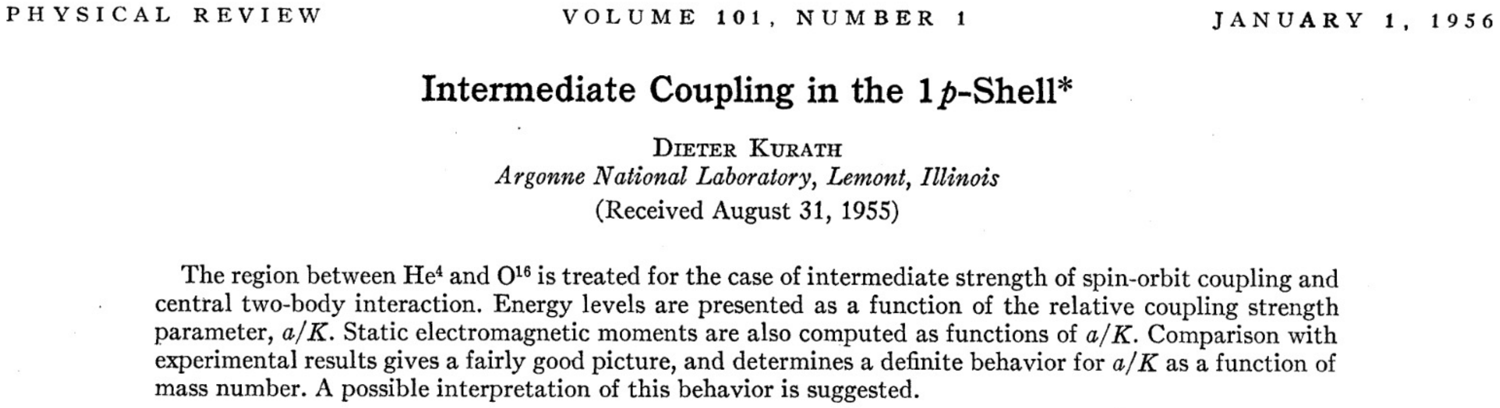
Independent Particle Model of the Nucleus. I. Interparticle Forces and Configuration Mixing*

CARL LEVINSON AND KENNETH W. FORD†
Indiana University, Bloomington, Indiana

(Received March 7, 1955)

A simplified method of obtaining the direct interaction between two identical nucleons in the nuclear shell model is given for the special case of singlet forces. Configuration interaction is included in the method. A semi-empirical analysis of simple two and three particle nuclear spectra is outlined which enables one to determine properties of the two-body perturbing interaction provided many body forces are negligible and coupling to the nuclear surface is weak. Corrections to the singlet force formalism due to triplet central and tensor forces are discussed. Formulas are given for magnetic dipole and quadrupole moments and magnetic dipole transition rates for mixed three-particle configurations. The effect of weak surface coupling on multi-particle configuration is given in paper II of this series. A detailed discussion of the spectra of two isotopes of calcium is given in paper III.

Early configuration-interaction



PHYSICAL REVIEW VOLUME 101, NUMBER 1 JANUARY 1, 1956

Intermediate Coupling in the 1p-Shell*

DIETER KURATH
Argonne National Laboratory, Lemont, Illinois

(Received August 31, 1955)

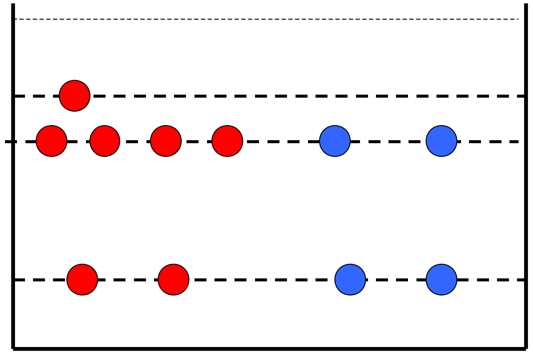
The region between He^4 and O^{16} is treated for the case of intermediate strength of spin-orbit coupling and central two-body interaction. Energy levels are presented as a function of the relative coupling strength parameter, a/K. Static electromagnetic moments are also computed as functions of a/K. Comparison with experimental results gives a fairly good picture, and determines a definite behavior for a/K as a function of mass number. A possible interpretation of this behavior is suggested.

The birth of the shell model

Independent-particle model:

A single shell-model ‘configuration’

$$|\Psi\rangle = |(0s)^4(0p_{3/2})^2\rangle$$



The birth of the shell model

Independent-particle model:

A single shell-model 'configuration'

$$|\Psi\rangle = |(0s)^4(0p_{3/2})^2\rangle$$

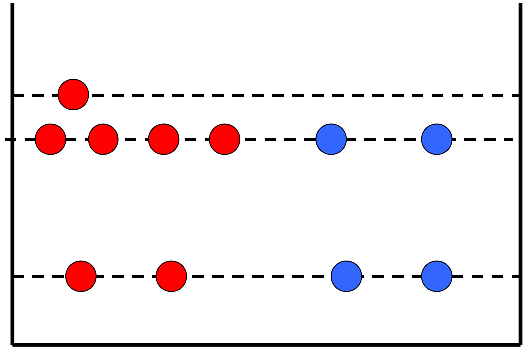
Configuration-interaction:

A superposition of different configurations

$$|\Psi\rangle = c_1|(0s)^4(0p_{3/2})^2\rangle + c_2|(0s)^4(0p_{3/2})^1(0p_{1/2})^1\rangle + \ldots.$$

The birth of the shell model

Model excited states as independent particles moving in mean-field, but one or more particles in a higher orbit = “particle-hole excitation”

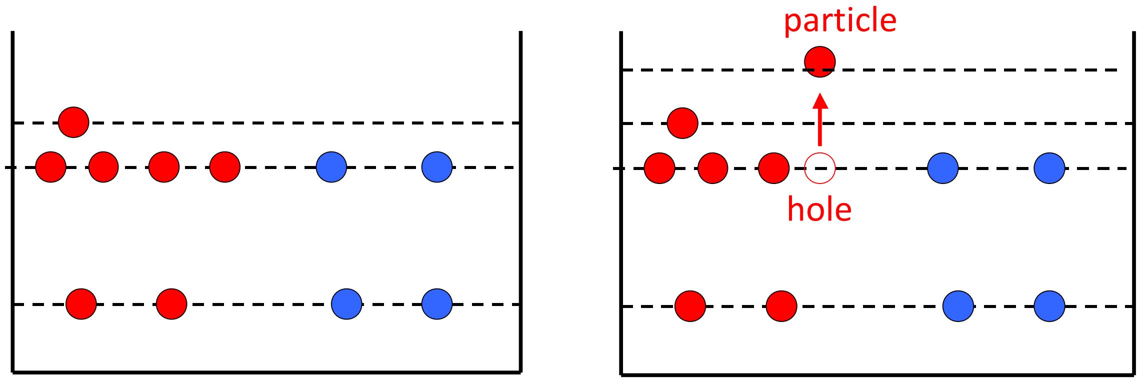


original configuration

The birth of the shell model

SAN DIEGO STATE UNIVERSITY

Model excited states as independent particles moving in mean-field, but one or more particles in a higher orbit = "particle-hole excitation"

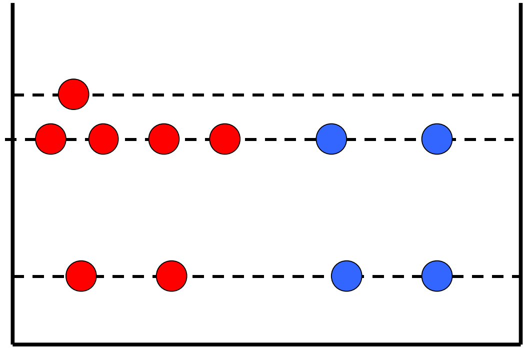


original configuration

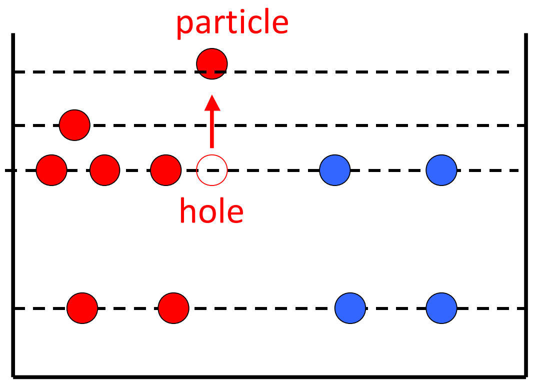
one-particle, one-hole (1p1h) excitation

The birth of the shell model

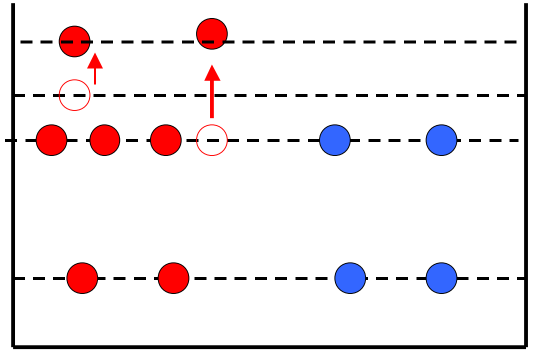
Model excited states as independent particles moving in mean-field, but one or more particles in a higher orbit = "particle-hole excitation"



original configuration



one-particle, one-hole
(1p1h) excitation



two-particle, two-hole
(2p2h) excitation

Configuration-interaction:

A superposition of different configurations

$$|\Psi\rangle = c_1 |(0s)^4 (0p_{3/2})^2\rangle + c_2 |(0s)^4 (0p_{3/2})^1 (0p_{1/2})^1\rangle + \ldots.$$

Configuration-interaction method

SAN DIEGO STATE UNIVERSITY

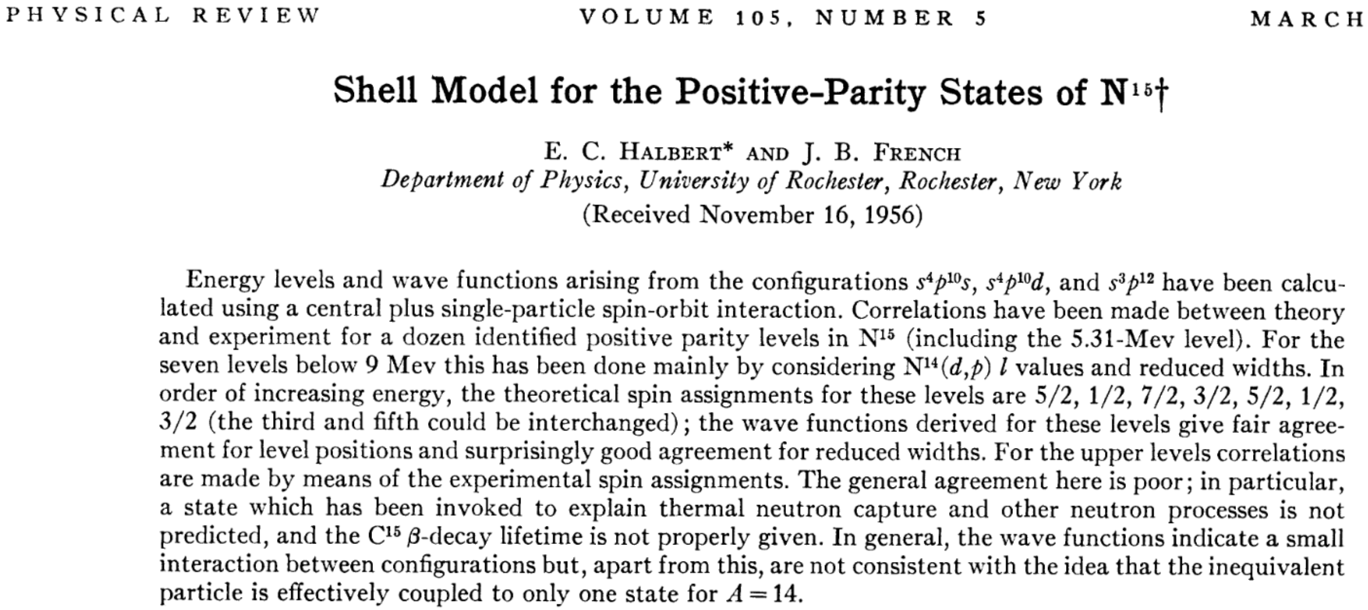
$$\hat{\mathbf{H}}\left|\Psi\right\rangle = E\left|\Psi\right\rangle$$

$$\left|\Psi\right\rangle = \sum_{\alpha} c_{\alpha}\left|\alpha\right\rangle \qquad H_{\alpha\beta} = \left\langle\alpha\middle|\hat{\mathbf{H}}\middle|\beta\right\rangle$$

$$\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha}$$

The birth of the shell model

SAN DIEGO STATE
UNIVERSITY



PHYSICAL REVIEW VOLUME 105, NUMBER 5 MARCH 1, 1957

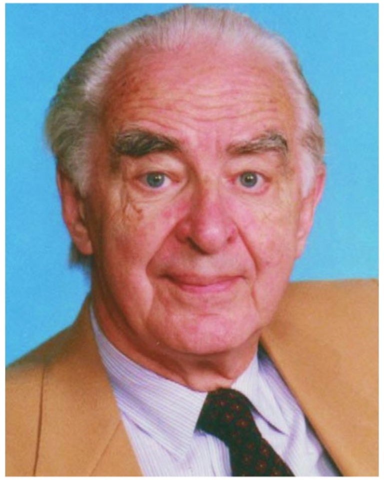
Shell Model for the Positive-Parity States of N^{15}†

E. C. HALBERT* AND J. B. FRENCH
Department of Physics, University of Rochester, Rochester, New York

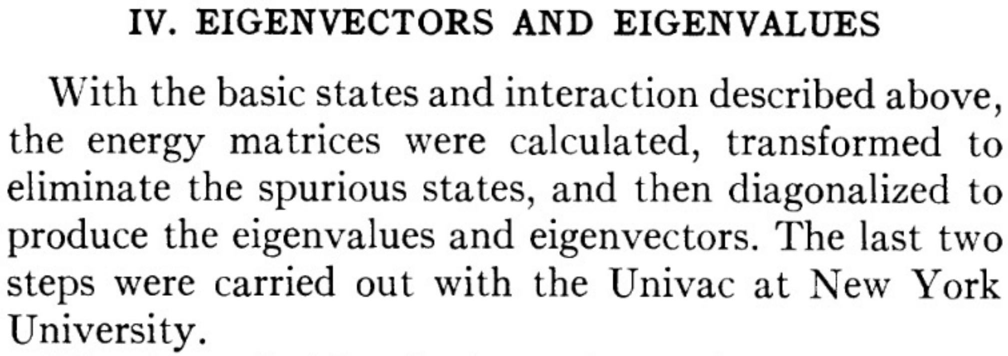
(Received November 16, 1956)

Energy levels and wave functions arising from the configurations $s^4p^{10}s$, $s^4p^{10}d$, and s^3p^{12} have been calculated using a central plus single-particle spin-orbit interaction. Correlations have been made between theory and experiment for a dozen identified positive parity levels in N^{15} (including the 5.31-Mev level). For the seven levels below 9 Mev this has been done mainly by considering $N^{14}(d,p)$ l values and reduced widths. In order of increasing energy, the theoretical spin assignments for these levels are 5/2, 1/2, 7/2, 3/2, 5/2, 1/2, 3/2 (the third and fifth could be interchanged); the wave functions derived for these levels give fair agreement for level positions and surprisingly good agreement for reduced widths. For the upper levels correlations are made by means of the experimental spin assignments. The general agreement here is poor; in particular, a state which has been invoked to explain thermal neutron capture and other neutron processes is not predicted, and the C^{15} β-decay lifetime is not properly given. In general, the wave functions indicate a small interaction between configurations but, apart from this, are not consistent with the idea that the inequivalent particle is effectively coupled to only one state for $A=14$.

Edith Halbert



J. Bruce French

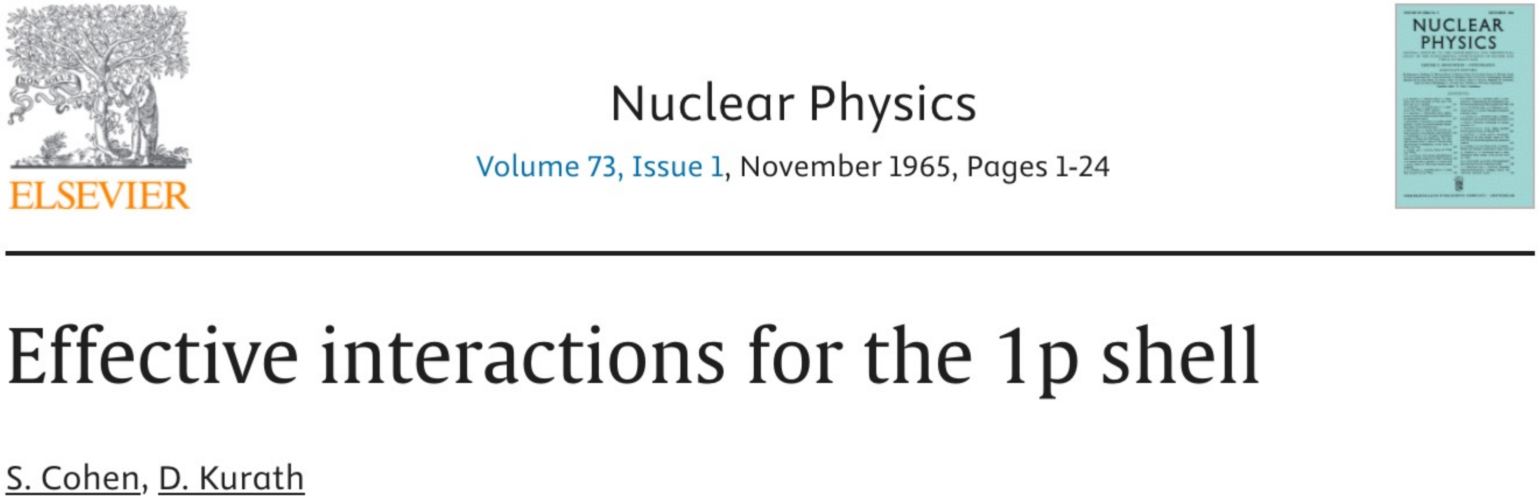


IV. EIGENVECTORS AND EIGENVALUES

With the basic states and interaction described above, the energy matrices were calculated, transformed to eliminate the spurious states, and then diagonalized to produce the eigenvalues and eigenvectors. The last two steps were carried out with the Univac at New York University.

1950's

The birth of the shell model



ELSEVIER

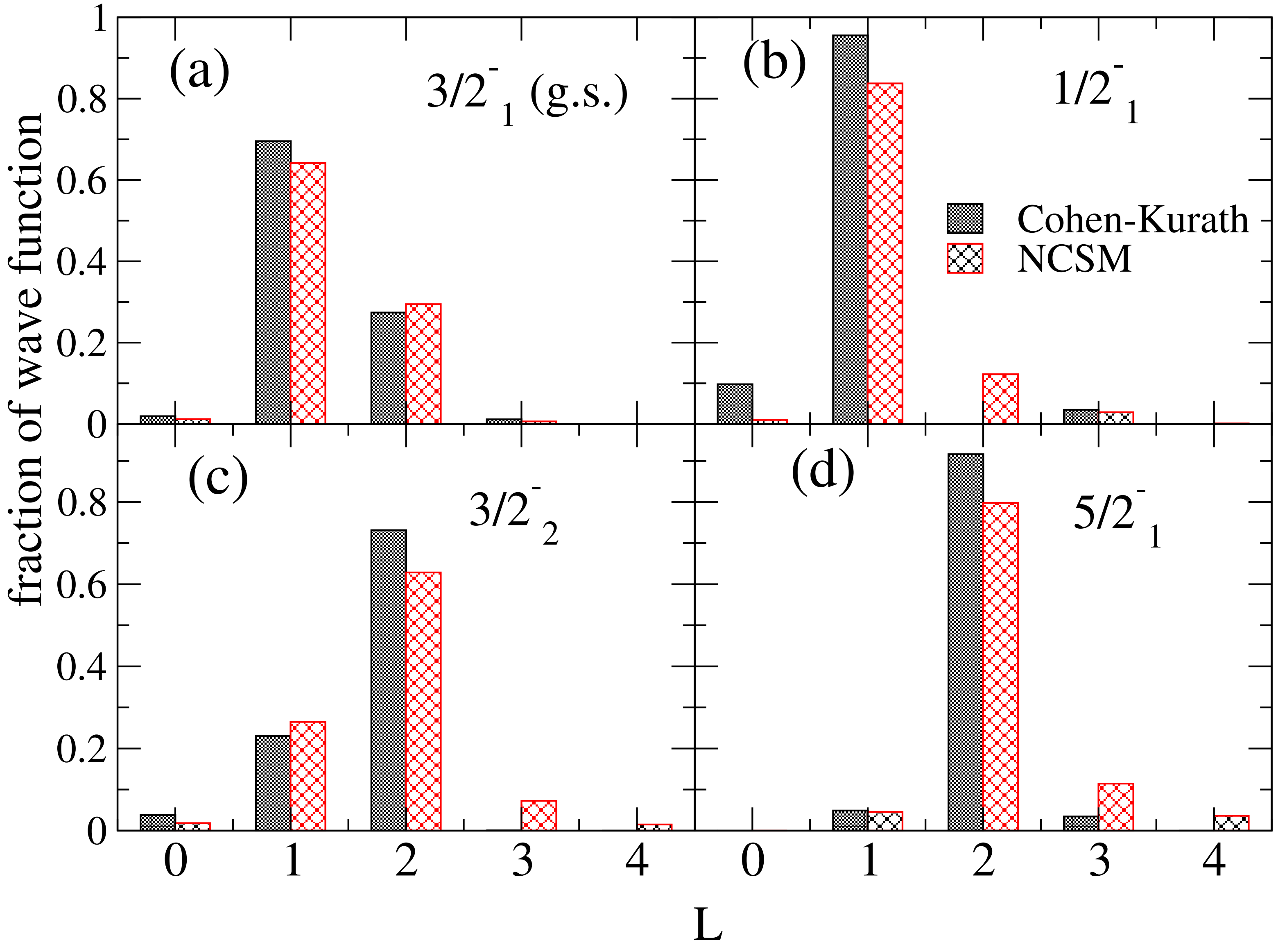
Nuclear Physics

Volume 73, Issue 1, November 1965, Pages 1-24

Effective interactions for the 1p shell

S. Cohen, D. Kurath

1960s


fraction of wave function
L
(a)
$3/2^-_1$ (g.s.)
(b)
$1/2^-_1$
(c)
$3/2^-_2$
(d)
$5/2^-_1$
▨ Cohen-Kurath
▨ NCSM
1
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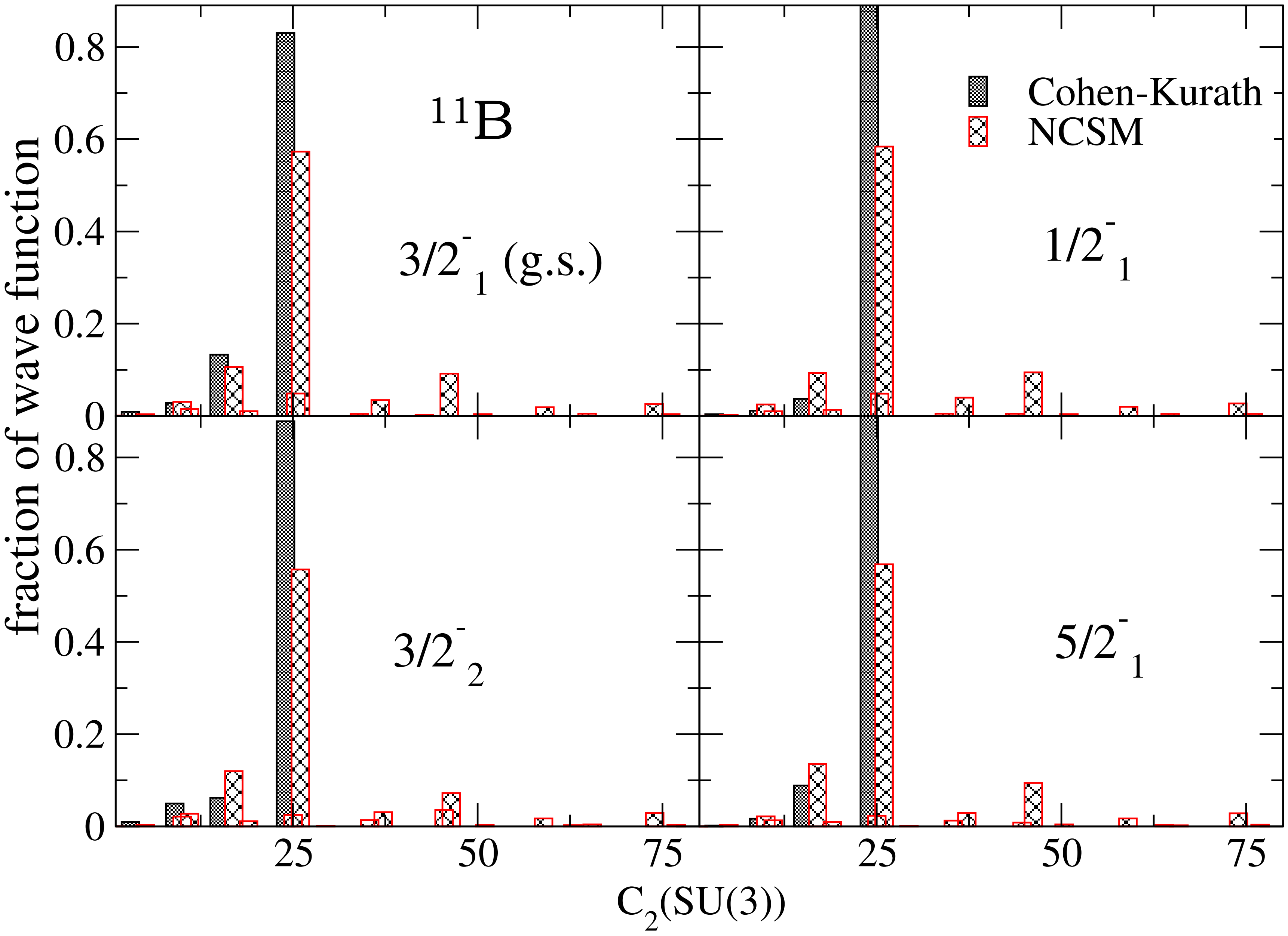
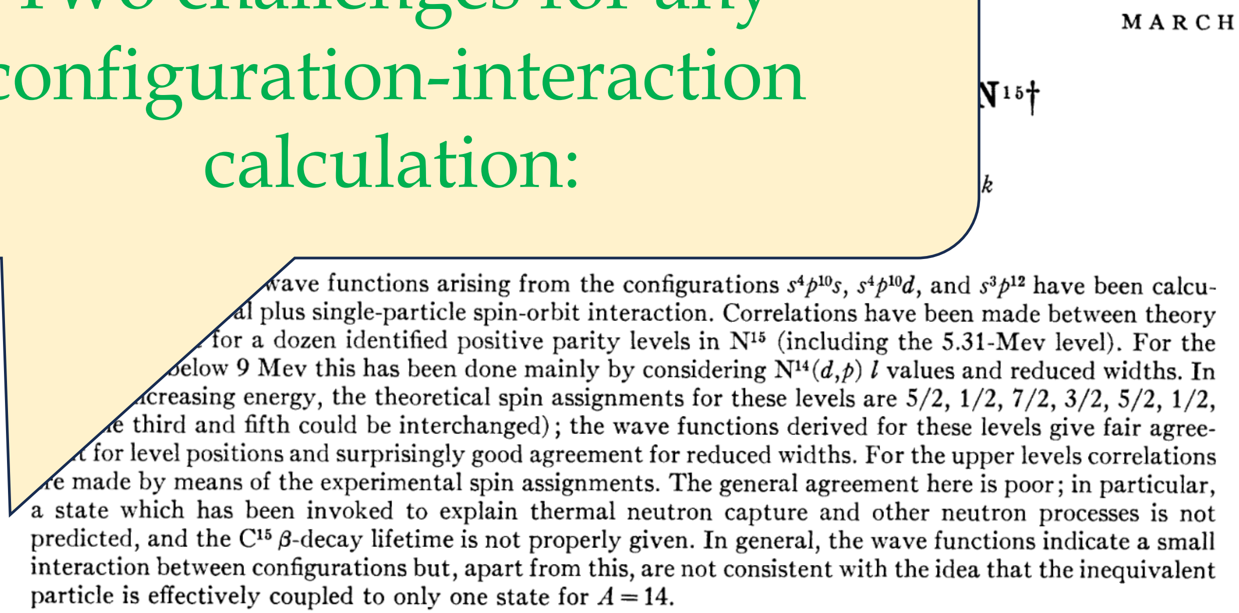


SAN DIEGO STATE UNIVERSITY

The birth of the shell model

Two challenges for any configuration-interaction calculation:


MARCH 1, 1957

$N^{15}\dagger$

k

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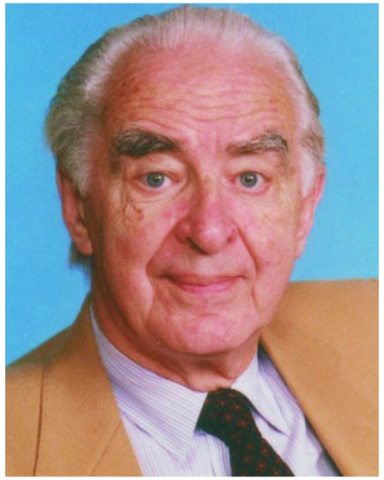
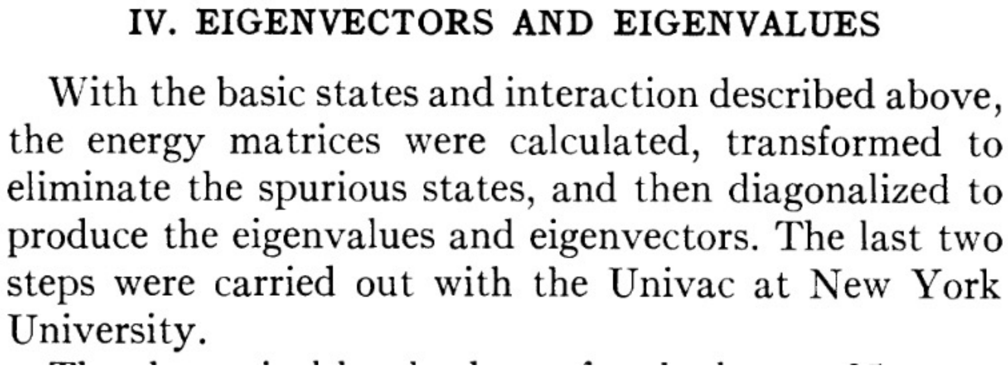
Edith Halbert


J. Bruce French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1950’s

The birth of the shell model

San Diego State
University

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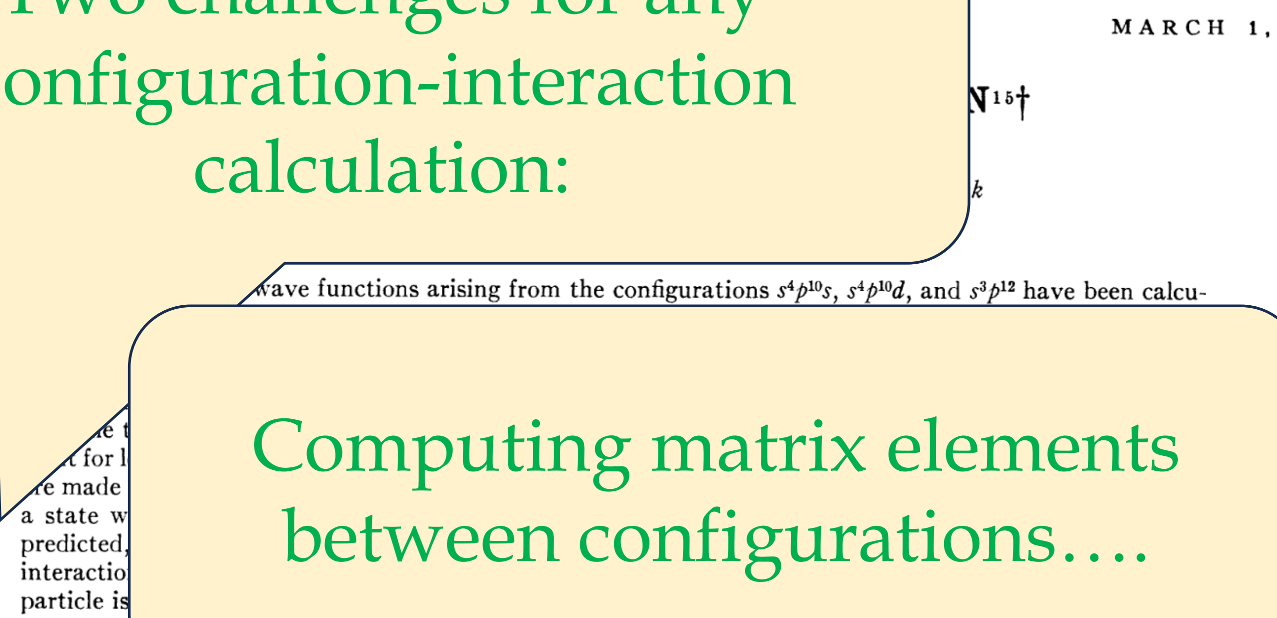

MARCH 1, 1957

N^{15}†

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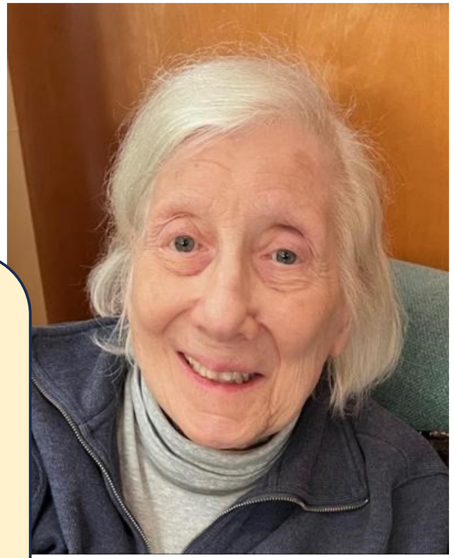
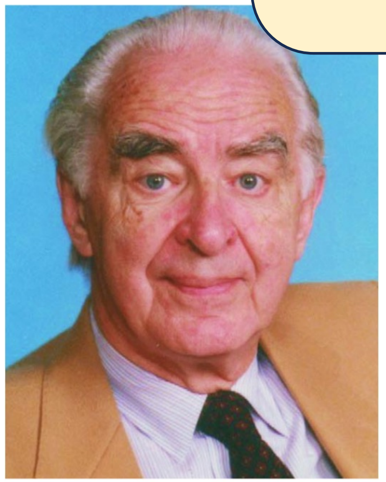
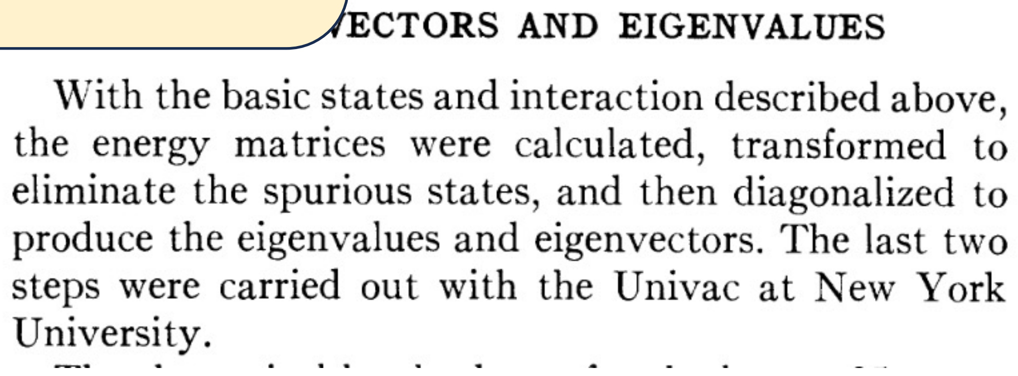
Computing matrix elements between configurations….



Edith Halbert



J. Bruce French



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1950's

The birth of the shell model

SAN DIEGO STATE UNIVERSITY

Two challenges for any configuration-interaction calculation:

MARCH 1, 1957

N^{15}†

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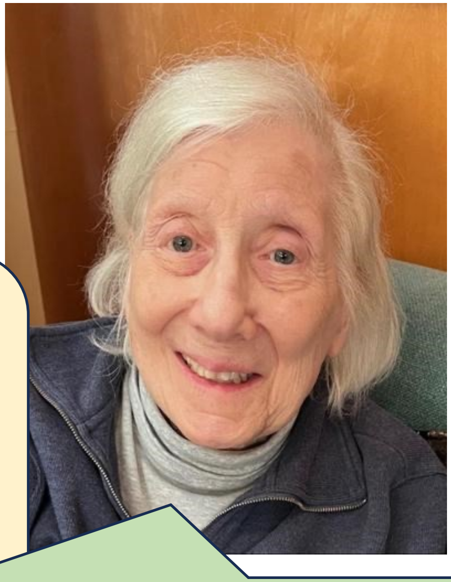
Computing matrix elements between configurations….



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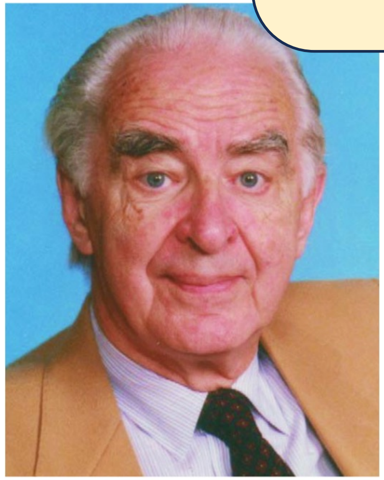

J. Bruce French

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…and finding the eigenvalues and eigenvectors of the matrix

1950's

Configuration-interaction method

$$\hat{\mathbf{H}}|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi\rangle = \sum_{\alpha} c_{\alpha}|\alpha\rangle$$

$$\boxed{H_{\alpha\beta} = \langle\alpha|\hat{\mathbf{H}}|\beta\rangle}$$

Computing this!

$$\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha}$$

Configuration-interaction method

SAN DIEGO STATE UNIVERSITY

$$\hat{\mathbf{H}}|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi\rangle = \sum_{\alpha} c_{\alpha}|\alpha\rangle$$

$$\boxed{H_{\alpha\beta} = \langle\alpha|\hat{\mathbf{H}}|\beta\rangle}$$

Computing this!

…and solving this!

$$\boxed{\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha}}$$

Configuration-interaction method: the early days

SAN DIEGO STATE UNIVERSITY

Many early calls relied upon
"coefficients of fractional parentage"
→ think "generalized Clebsch-Gordan coefficients"

$$|(j)^n \ ; J\rangle = \sum_K c_K [|(j)^{n-1} \ ; K\rangle \otimes |j\rangle]_J$$

1960s

Configuration-interaction method: the early days

SAN DIEGO STATE UNIVERSITY

Many early calculations relied upon
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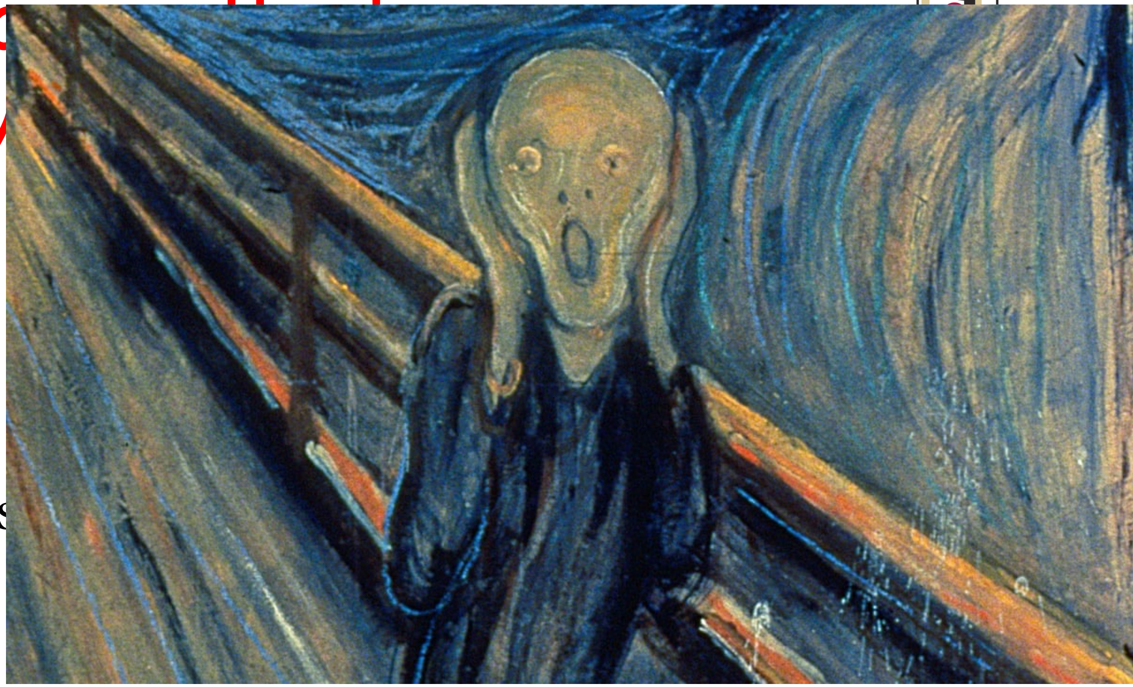
1960s

Configuration-interacti
the early day



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→ think “generalized Clebsch-Gordan coefficients

$$|(j)^{n}\ ;J\rangle = \sum_{K} \boxed{c_K}\,[|(j)^{n-1}\ ;K\rangle \otimes |j\rangle]_J$$

CFP

1960s

Configuration-interaction method

Today we can find all eigenpairs of a real, symmetric matrix using the Householder algorithm (1958).

But this scales as **(dimension)^3**

$$\boxed{\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha}}$$

The birth of the shell model

Entering the 1970s, the shell-model faced
two challenges

The birth of the shell model

SAN DIEGO STATE
UNIVERSITY

Entering the 1970s, the shell-model faced
two challenges

Dimensionality and Intruders

The challenge of dimensionality

$1d_{5/2}$		
$0g_{9/2}$	10	[50]
$1p_{1/2}$	2	
$0f_{5/2}$	6	
$1p_{3/2}$	4	
$0f_{7/2}$	8	
$0d_{3/2}$	4	[20]
$1s_{1/2}$	2	
$0d_{5/2}$	6	
$0p_{1/2}$	2	[8]
$0p_{3/2}$	4	
$0s_{1/2}$	2	[2]

Number of many-body states goes like

single-particle states

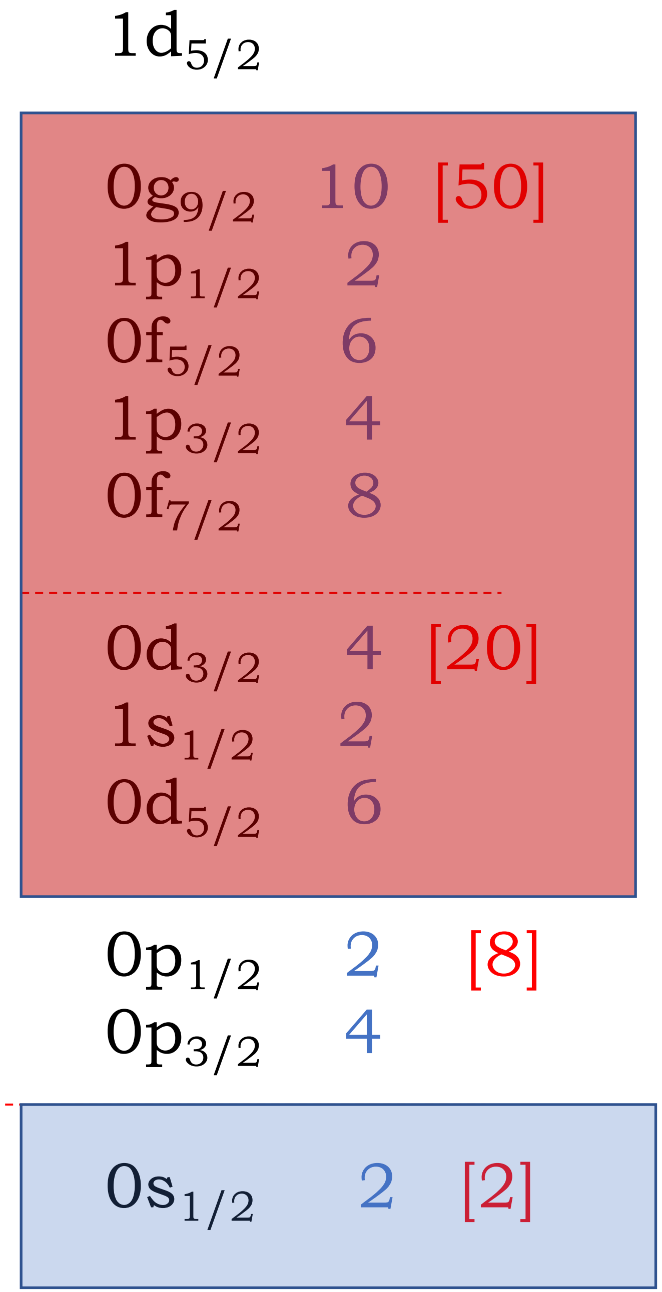
particles

$$\binom{N_s}{N_p} = \frac{N_s!}{N_p!(N_s - N_p)!}$$

This exhibits **exponential** scaling!

(In the fixed-J_z, or M-scheme, the actual dimensions are less because of the selection rule. But the scaling still holds)

The challenge of dimensionality



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$0s_{1/2}$ 2 [2]

Number of many-body states goes like

\# single-particle states

\# particles

$$\binom{N_s}{N_p} = \frac{N_s!}{N_p!(N_s - N_p)!}$$

This exhibits **exponential** scaling!

p-shell: max dim = 84

(In the fixed-J_z, or M-scheme, the actual dimensions are less because of the selection rule. But the scaling still holds)

The challenge of dimensionality

$1d_{5/2}$		
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Number of many-body states goes like

single-particle states

$$\binom{N_s}{N_p} = \frac{N_s!}{N_p!(N_s - N_p)!}$$

particles

sd-shell: max dim = 93,000

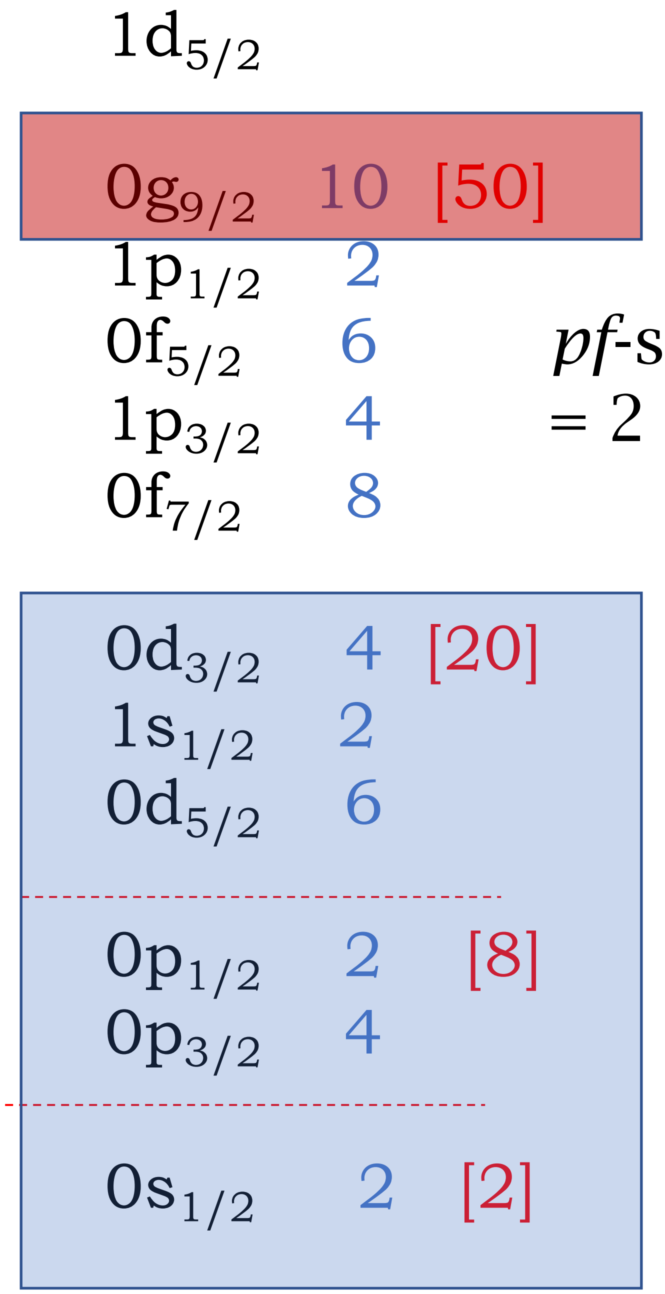
This exhibits **exponential** scaling!

p-shell: max dim = 84

(In the fixed-J_z, or M-scheme, the actual dimensions are less because of the selection rule. But the scaling still holds)

The challenge of dimensionality

SAN DIEGO STATE UNIVERSITY



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pf-shell: max dim
= 2 billion

Number of many-body states goes like

\# single-particle states

$$\binom{N_s}{N_p} = \frac{N_s!}{N_p!(N_s-N_p)!}$$

\# particles

sd-shell: max dim
= 93,000

This exhibits **exponential** scaling!

p-shell: max dim
= 84

(In the fixed-J_z, or M-scheme, the actual dimensions are less because of the selection rule. But the scaling still holds)

The challenge of dimensionality

SAN DIEGO STATE UNIVERSITY

Not everything scales **exponentially**.

For example, the number of terms in a two-body Hamiltonian has **polynomial** scaling

$$\hat{H} = \sum_{ijkl} V_{ijkl} \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_l \hat{a}_k$$

$$\# \text{ terms} \sim (N_s)^4$$

Number of many-body states goes like

single-particle states

particles

$$\binom{N_s}{N_p} = \frac{N_s!}{N_p!(N_s - N_p)!}$$

This exhibits **exponential** scaling!

(In the fixed-J_z, or M-scheme, the actual dimensions are less because of the selection rule. But the scaling still holds)

The challenge of dimensionality

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Number of many-body states goes like

single-particle states

$$\binom{N_s}{N_p} = \frac{N_s!}{N_p!(N_s - N_p)!}$$

particles

$\hat{H} =$

ter

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the actual dimensions are

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The **exponential scaling** of the configuration-interaction basis motivates alternate methods, such as coupled clusters, which have **polynomial scaling**

The challenge of dimensionality

Not everything scales **exponentially**.

For example, the number of terms in a two-body Hamiltonian has **polynomial** scaling

But configuration-interaction still has many advantages, such as adaptability and ease in generating excited states

The **exponential scaling** of the configuration-interaction basis motivates alternate methods, such as coupled clusters, which have **polynomial scaling**

$\hat{H} =$

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The challenge of dimensionality

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$0g_{9/2}$	10	[50]
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pf-shell: max dim = 2 billion

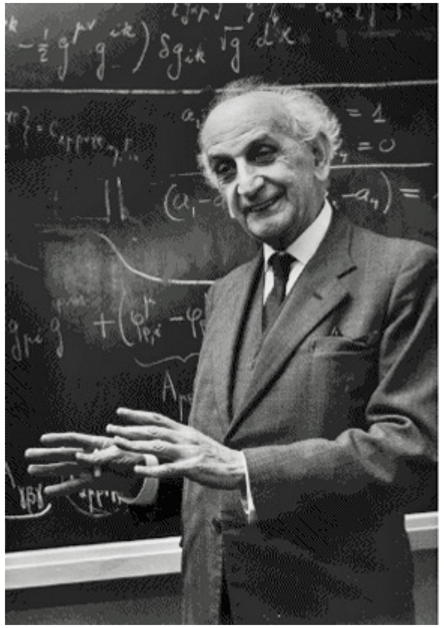
sd-shell: max dim = 93,000

p-shell: max dim = 84

With the Householder algorithm, we can diagonalize dimensions of up to a few thousand… how do we handle dimensions of *2 billion?!*

The challenge of dimensionality

There is another way!



(Cornelius Lanczos)

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The challenge of dimensionality

SAN DIEGO STATE
UNIVERSITY

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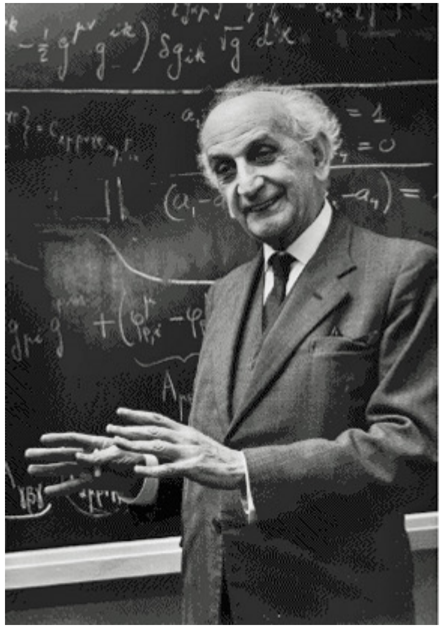

(Cornelius Lanczos)

The Lanczos algorithm (you're welcome) and related power and Arnoldi methods seek to find only the extremal eigenvalues, even in huge dimensions

The challenge of dimensionality

SAN DIEGO STATE UNIVERSITY

There is another way!

Today we can do dimensions of up to 35 billion! (arXiv:2402.12606)

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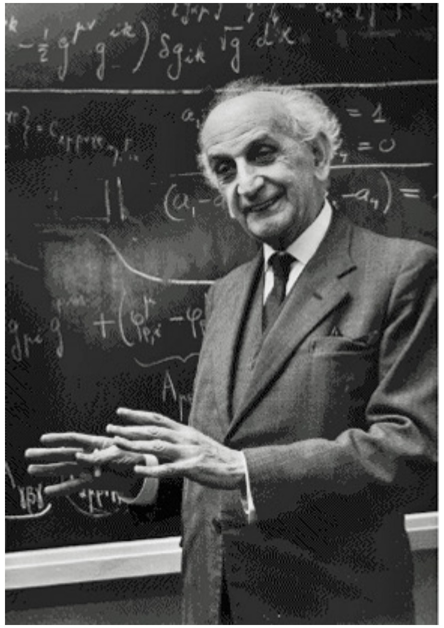

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Anna McCoy

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SAN DIEGO STATE UNIVERSITY

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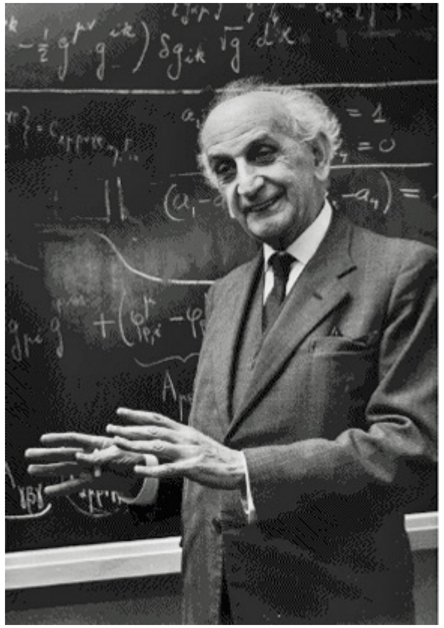

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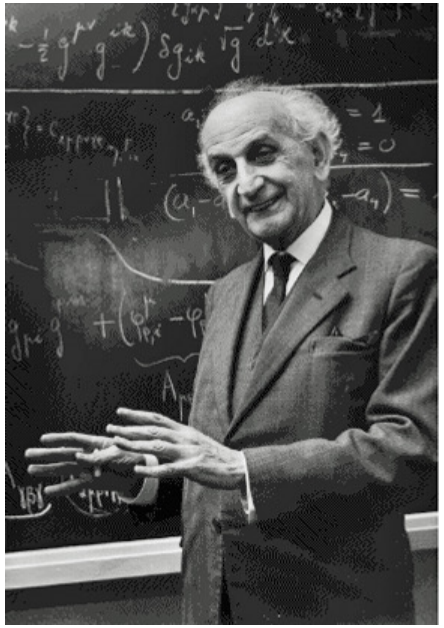
The Lanczos algorithm (you're welcome) and related power and Arnoldi methods seek to find only the extremal eigenvalues, even in huge dimensions

Or even more!

Frederic Nowacki

The challenge of dimensionality

Whitehead *et al* (Adv. Nucl. Phys **9**, 123 (1977)) introduced both the Lanczos algorithm and a simple bit representation of Slater determinants.



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Product wavefunction ("Slater Determinant")

$$\Psi(\vec{r}_1,\vec{r}_2,\vec{r}_3\ldots) = \phi_{\enclose{circle}{n_1}}(\vec{r}_1)\phi_{\enclose{circle}{n_2}}(\vec{r}_2)\phi_{\enclose{circle}{n_3}}(\vec{r}_3)\ldots\phi_{\enclose{circle}{n_N}}(\vec{r}_N)$$

Each many-body state can be *uniquely* determined by a list of "occupied" single-particle states = "occupation representation"

$$\left|\alpha\right\rangle = \hat{a}^{+}_{n_1}\hat{a}^{+}_{n_2}\hat{a}^{+}_{n_3}\ldots\hat{a}^{+}_{n_N}\left|0\right\rangle$$

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n_i	1	2	3	4	5	6	7
α=1	1	0	0	1	1	0	1
α=2	1	0	1	0	0	1	1
α=3	0	1	1	1	0	1	0

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$$|\Psi\rangle = \sum_{\mu\nu} c_{\mu\nu}|p_\mu\rangle|n_\nu\rangle$$

"bit representation" convenient for digital computers

$$|01101000\ldots\rangle|10010100\ldots\rangle$$

Product wavefunctio

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Each many-body stat
by a list of “occupied
= “occupation repres

Working in the M-scheme, one doesn’t need coefficients of fractional parentage!

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0	1	1	0	1
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1	1	0	1	0

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$$\nu\left|p_\mu\right\rangle\left|n_\nu\right\rangle$$

$$.\rangle\left|10010100...\right\rangle$$

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Product wavefunctio

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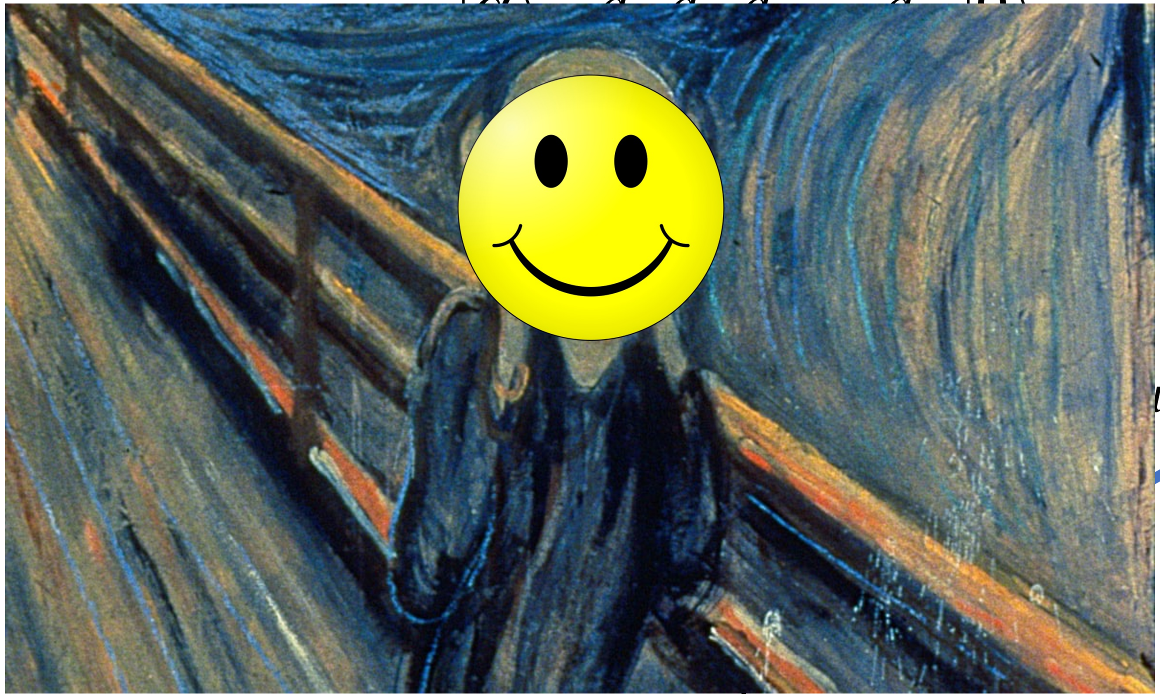


$\nu |p_\mu\rangle |n_\nu\rangle$

$.\rangle |10010100\ldots\rangle$

Configuration-interaction method …today

San Diego State University

$$\hat{\mathbf{H}}|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi\rangle = \sum_{\alpha} c_{\alpha}|\alpha\rangle$$

$$\boxed{H_{\alpha\beta} = \langle\alpha|\hat{\mathbf{H}}|\beta\rangle}$$

Can compute quickly using bit manipulation

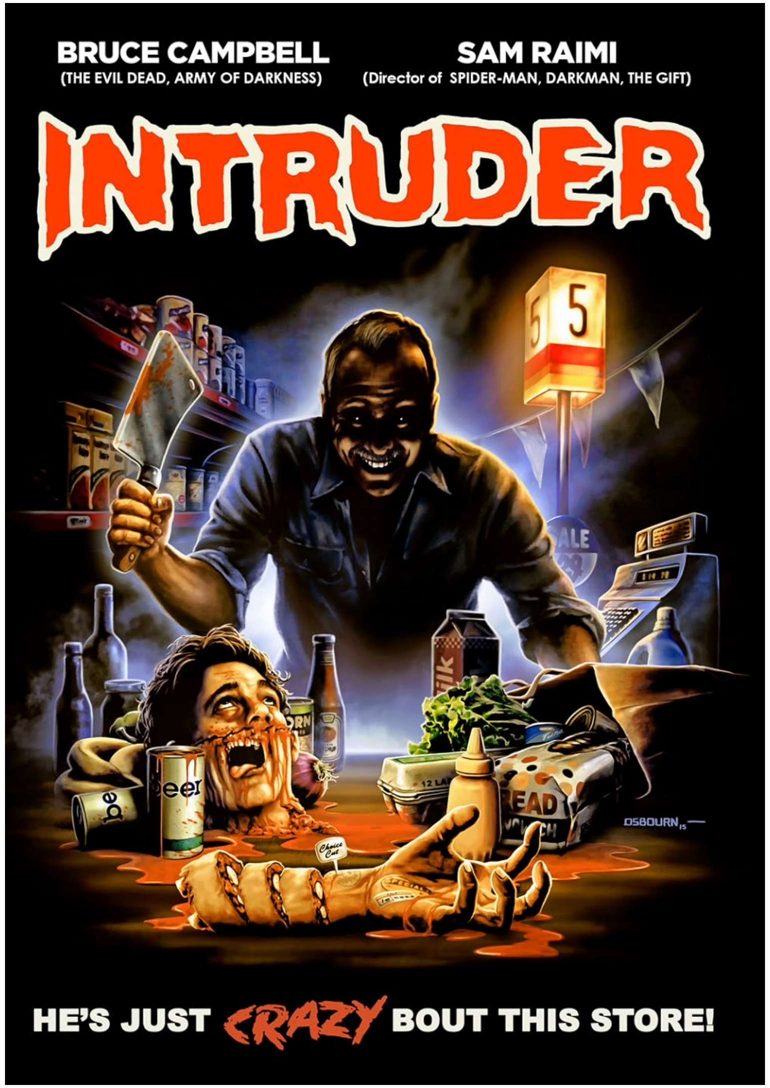
… and solve for low eigenstates with Lanczos

$$\boxed{\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha}}$$

The challenge of intruders

What about the other problem…intruders?

The challenge of intruders

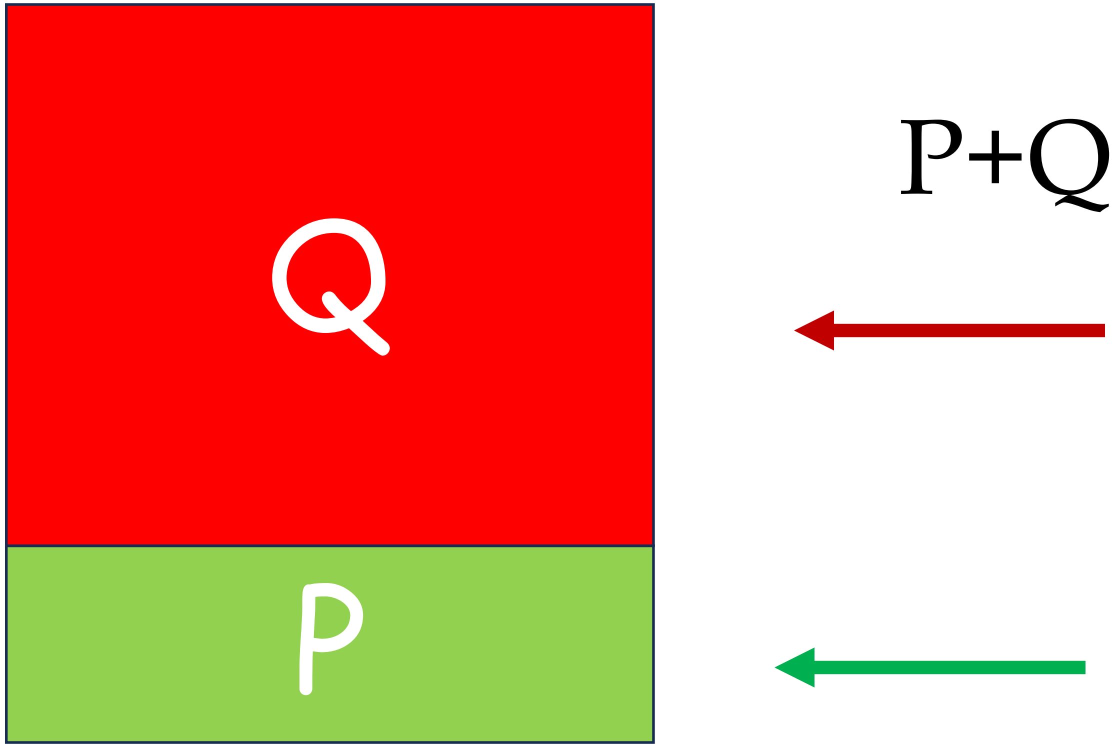


What about the other problem…intruders?

Shell Model 75 Symposium, July 19, 2024

The challenge of intruders

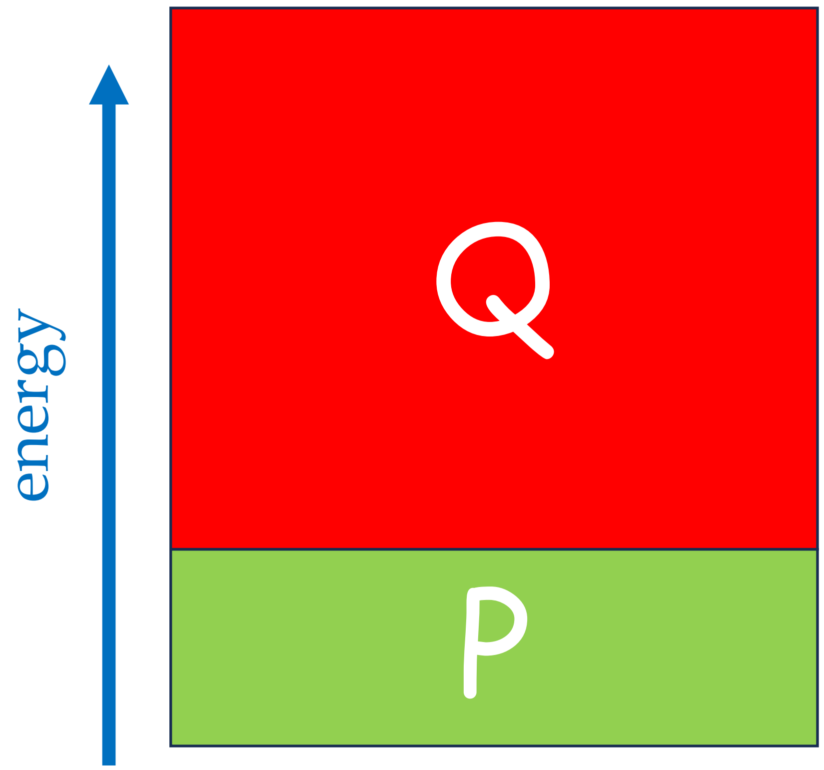
$$P+Q = 1$$



“excluded” space

model space

The challenge of intruders



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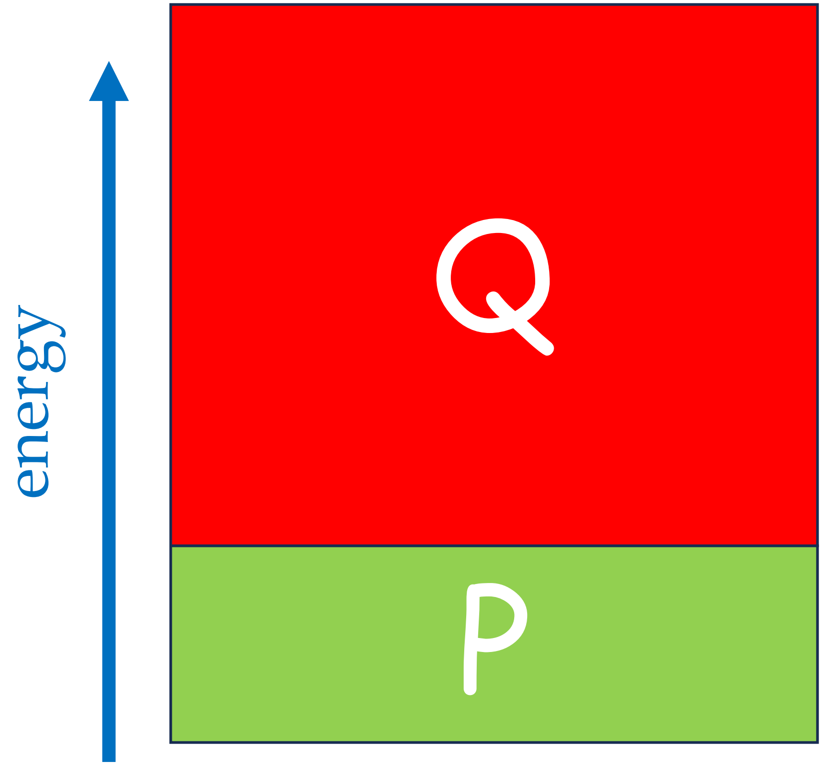
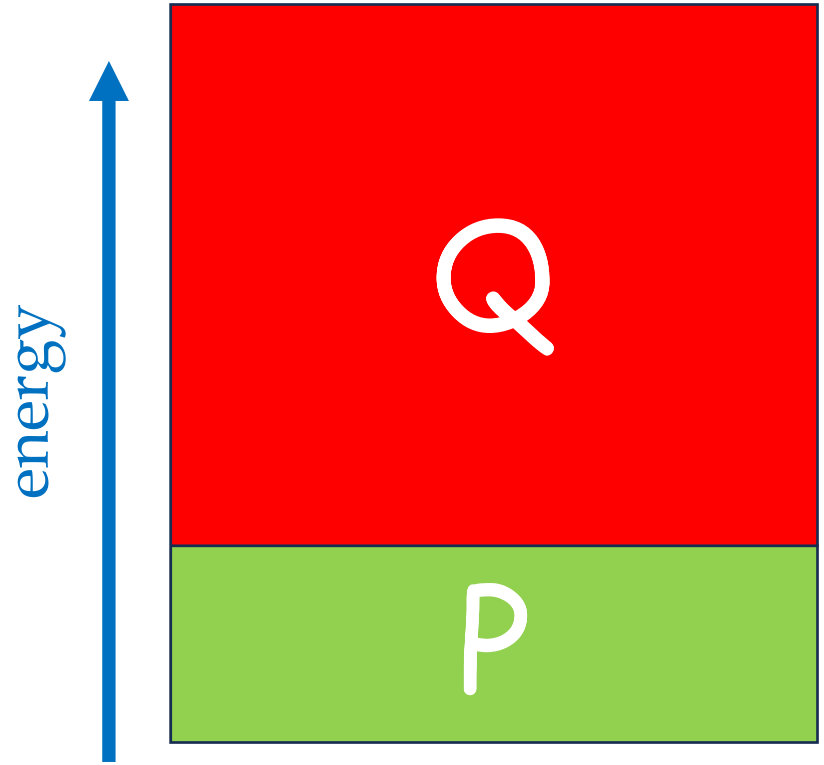


$$P+Q = 1$$

Intruders are states predominantly in Q, but low-lying in energy

The challenge of intruders



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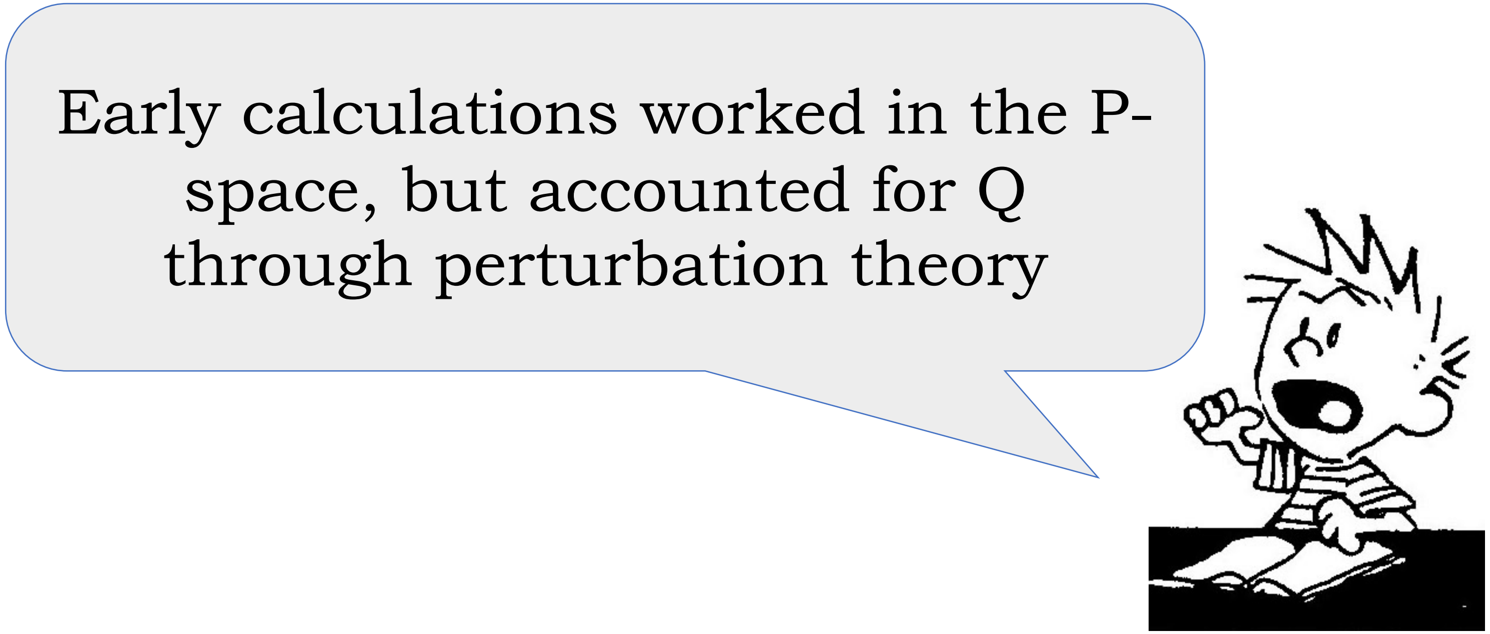
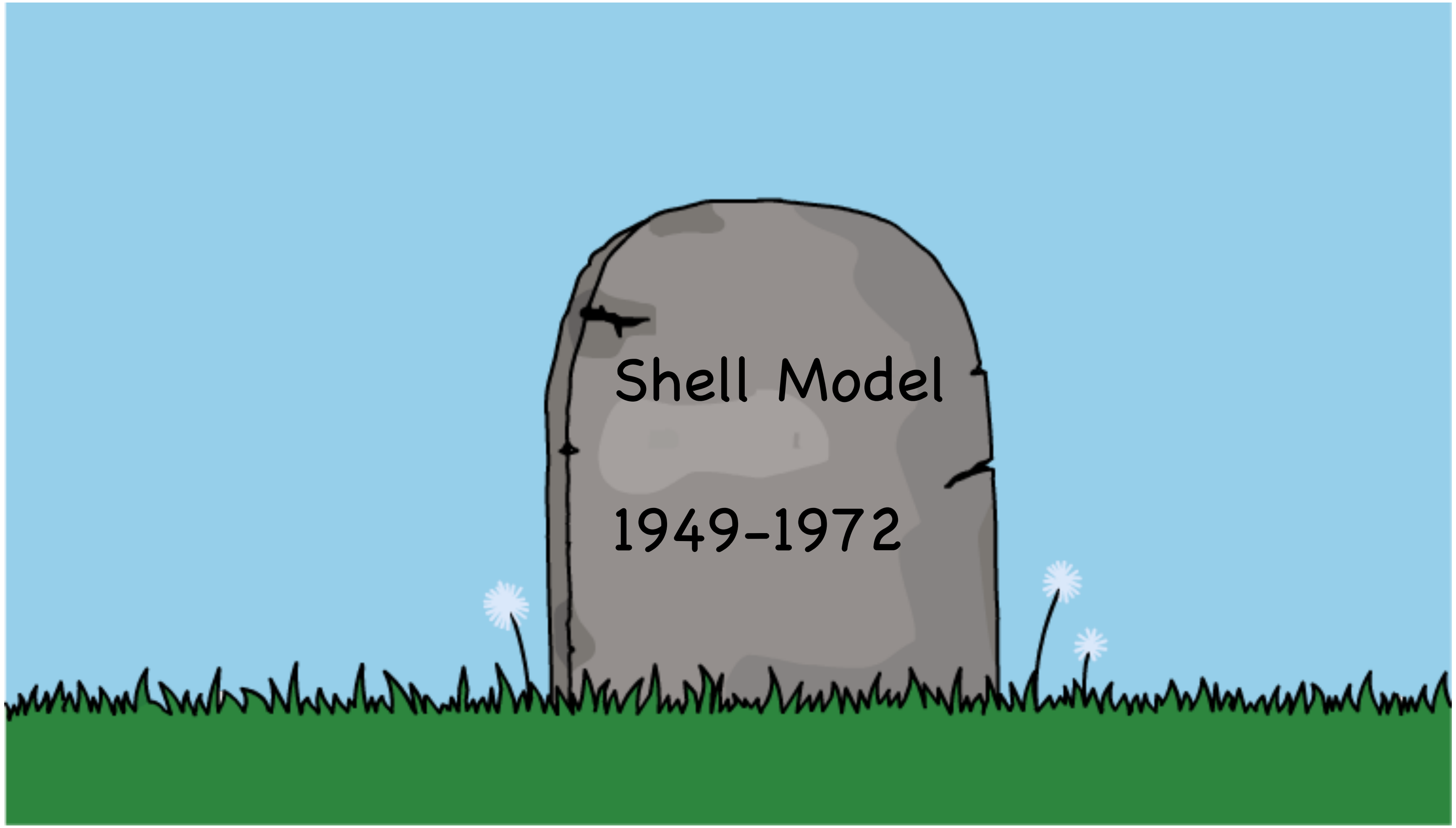


Early calculations worked in the P-space, but accounted for Q through perturbation theory

1970 Barrett and Kirson, 1972 Schucan and Weidenmuller:
intruder states can cause perturbative expansions
to ultimately diverge.

This in particular applies to particle-hole states.

This makes expanding beyond the valence space problematic, and **almost** kills the field (except for a stubborn few) for twenty years.


Shell Model
1949-1972

1991-1993: Vary and Barrett introduced the **no-core shell model**:
Without a core, there is no ”particle-hole” expansion.

Around this same time high-precision phase shift data from NN scattering became available.

Fitted to this data, the **Argonne potential** showed one could reproduce nuclear many-body data.

Then chiral EFT gave a systematic way to characterize nuclear forces

The field lurches back to life!

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The birth of the shell model

Today bases can be a linear combination of simple Slater determinants with fixed total J_z/M ('M-scheme') or configurations with good total J ('J-scheme') or other group-theory label ('symmetry-adapted')

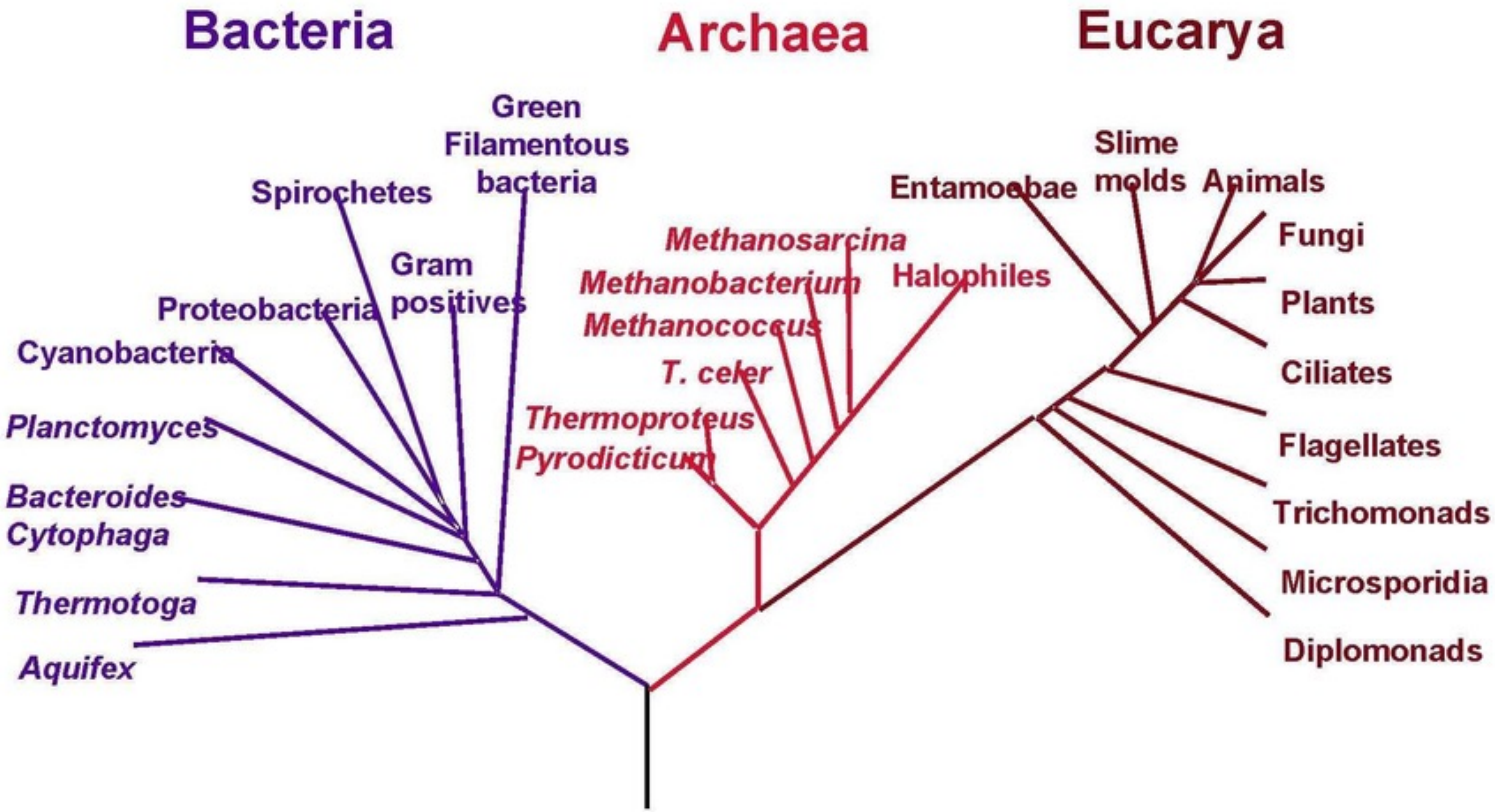
Configuration-interaction:

A superposition of different configurations

$$|\Psi\rangle = c_1|(0s)^4(0p_{3/2})^2\rangle + c_2|(0s)^4(0p_{3/2})^1(0p_{1/2})^1\rangle + \ldots.$$

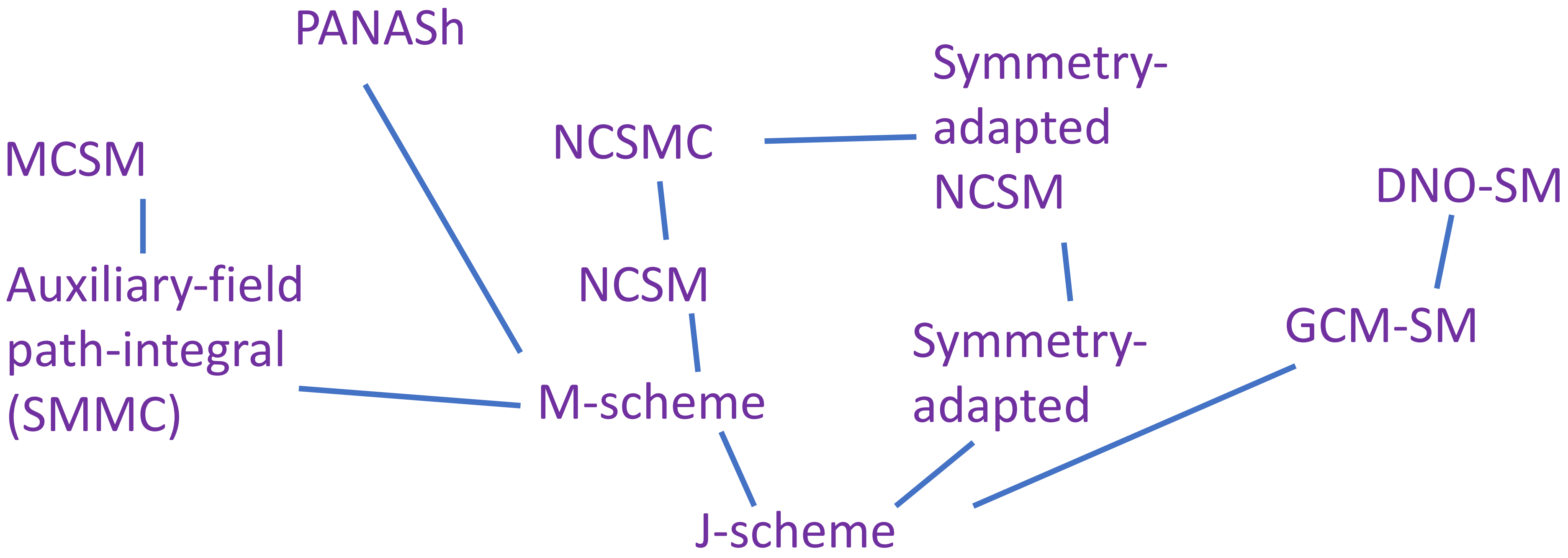
A diversity of approaches

Phylogenetic Tree of Life



A diversity of approaches

Phylogenetic tree of Shell-models



A diversity of approaches

SAN DIEGO STATE UNIVERSITY

Phylogenetic tree of Shell-models

Oliver Gorton

PANASh

Yusuke Tsunoda

xiliary-field
th-integral
MMC)

Yoram Alhassid

NCSMC

James Vary

SI

Petr Navratil

J-scheme

Symmetry-
adapted
NCSM

Sy
ac

Jerry Draayer

Frederic Nowacki

DNO-SM

GCM-SM

Choice of wave function basis

One chooses between *a few, complicated states*
or *many simple states*

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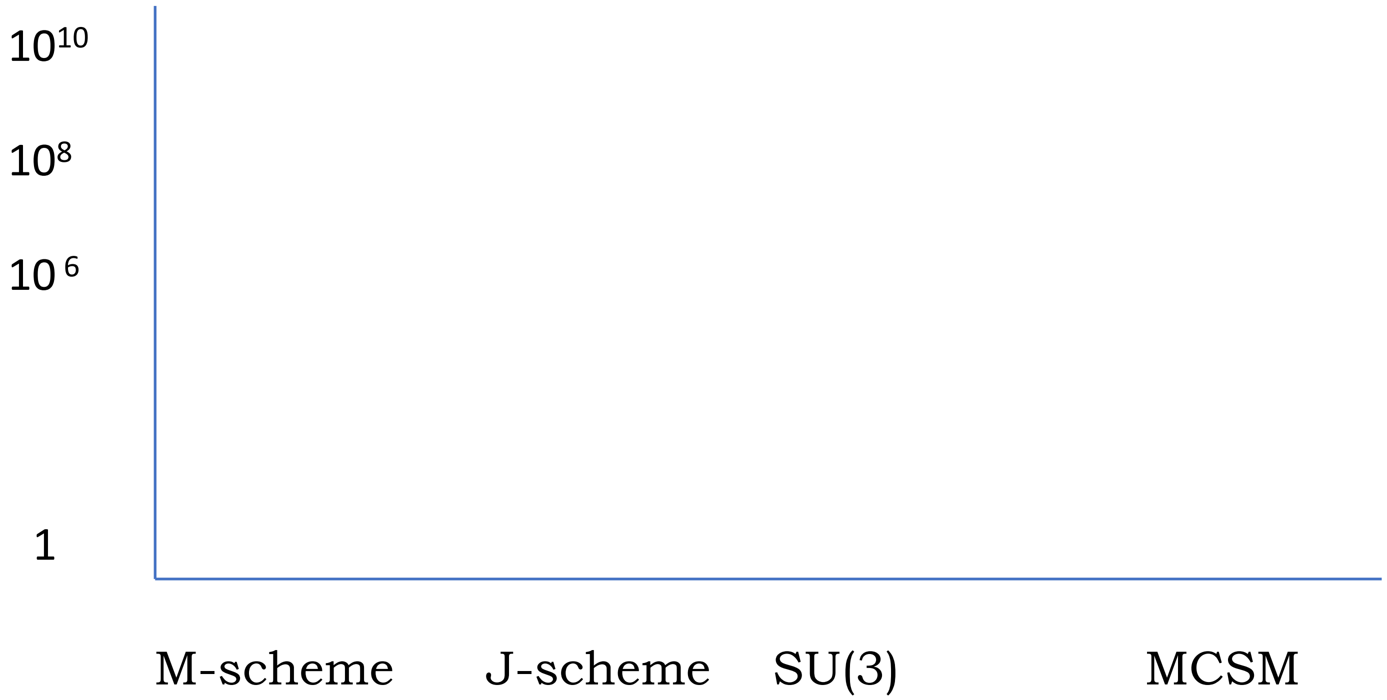

Choice of wave function basis

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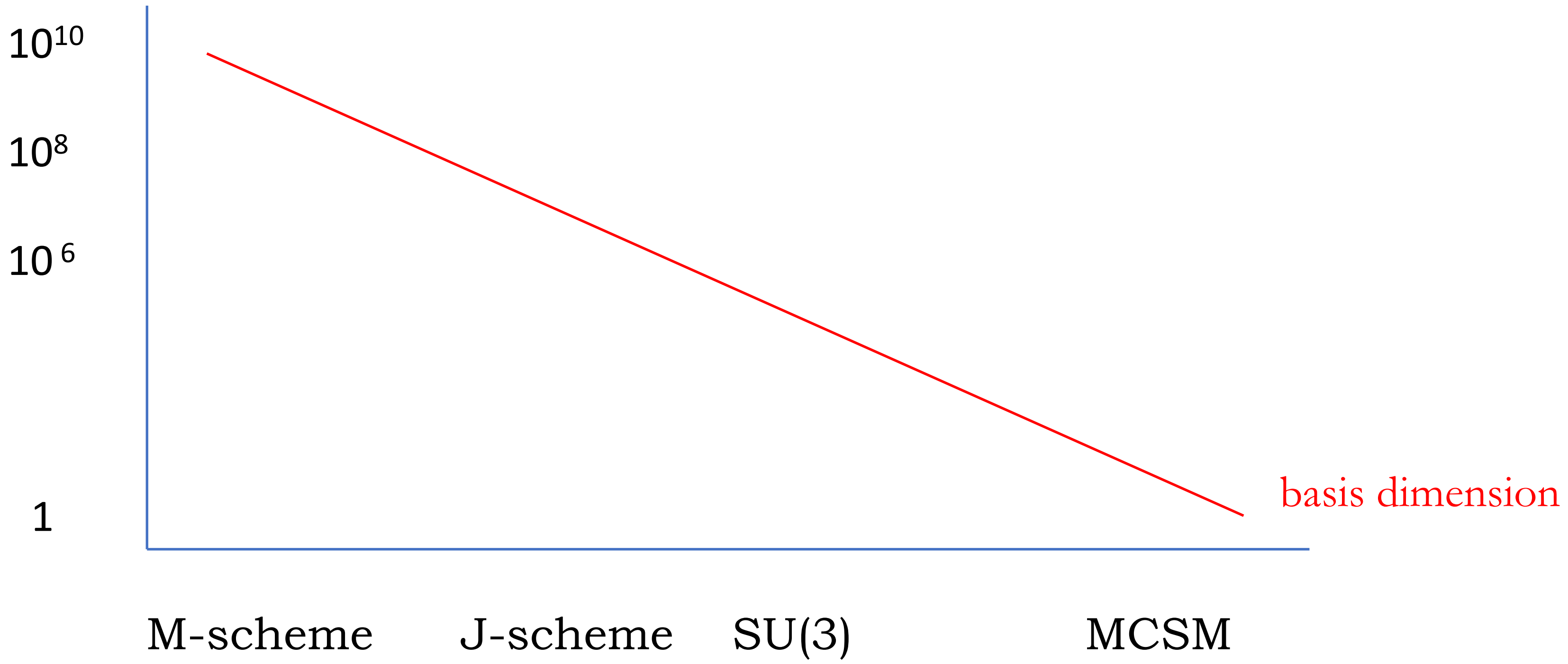

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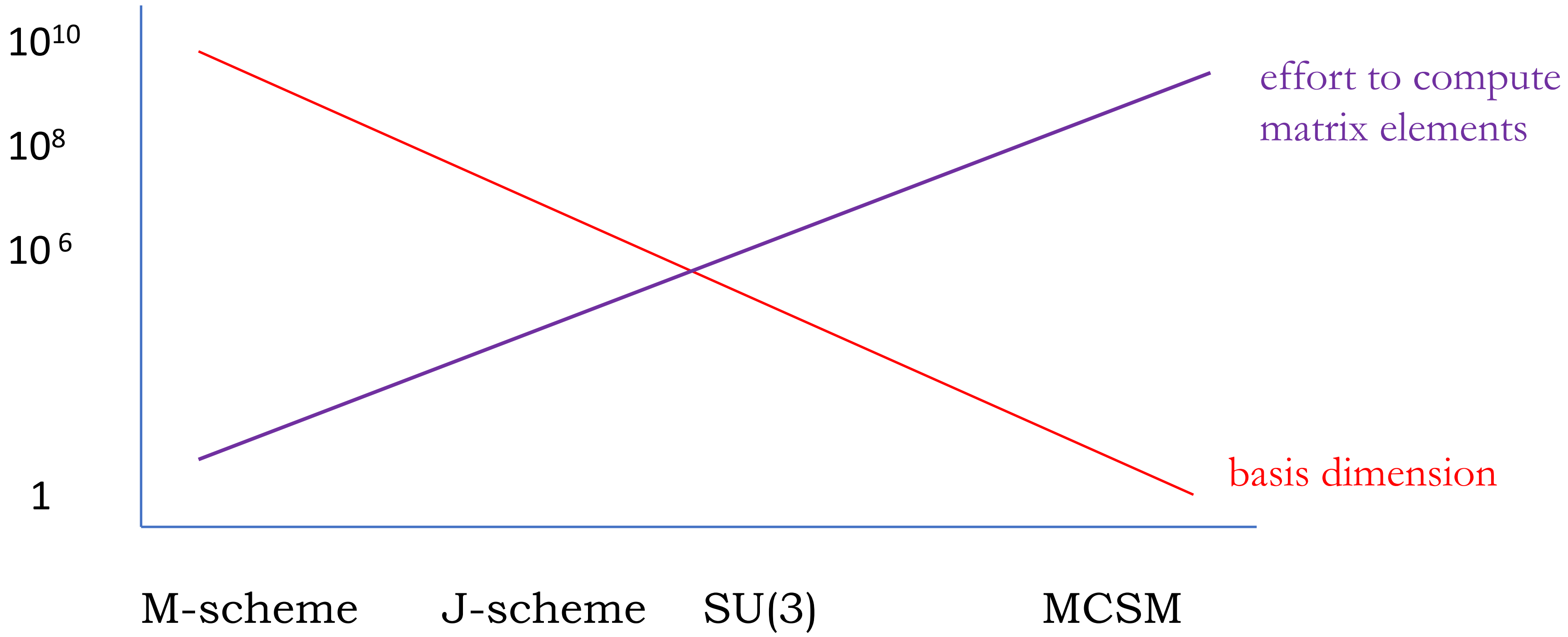

It’s also important to know:

Computational burden is *not* primarily the dimension but is the # of nonzero Hamiltonian matrix elements.

$$\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha}$$

J-scheme matrices are smaller but much denser than M-scheme, and "symmetry-adapted" (i.e. SU(3)) matrices are smaller (and denser) still.

example: ^{12}C $N_{max} = 8$

scheme	basis dim
M	6×10^8
J (J=4)	9×10^7
SU(3) (truncated)	9×10^6

From Dytrych, et al, Comp Phys Comm **207**, 202 (2016)

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scheme	basis dim	# of nonzero matrix elements	
M	6×10^{8}	5×10^{11}	4 Tb of memory!
J (J=4)	9×10^{7}	3×10^{13}	240 Tb of memory!
SU(3) (truncated)	9×10^{6}	2×10^{12}	16 Tb of memory!

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example: ^{12}C $N_{max} = 8$

scheme	basis dim		# of nonzero matrix elements	but least amount of work!
M	6×10^{8}	large dimension	5×10^{11}	4 Tb of memory!
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J- n

M

m

But more ‘complicated’ basis states can give insight

example: ^{12}C $N_{max} = 8$

but least amount of work!

large dimension

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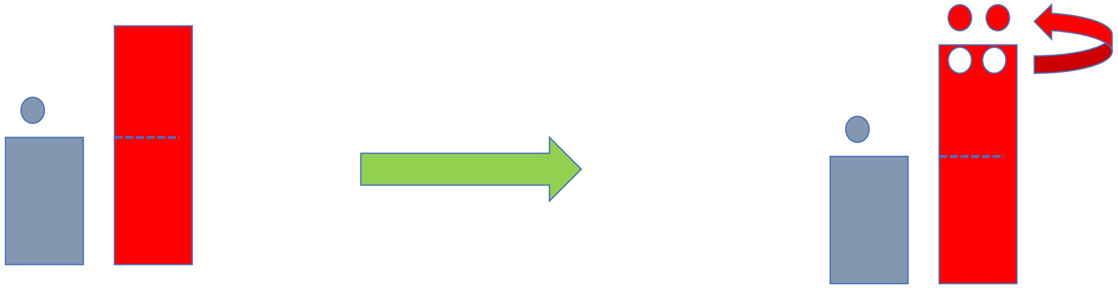
CASE STUDY: ^{11}LI



One proton outside a
filled shell
+ filled neutron shell

One proton outside a
filled shell
+ neutron 2p-2h

"island of inversion"

Frederic Nowacki

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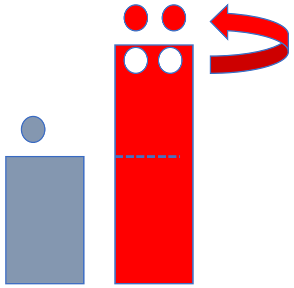
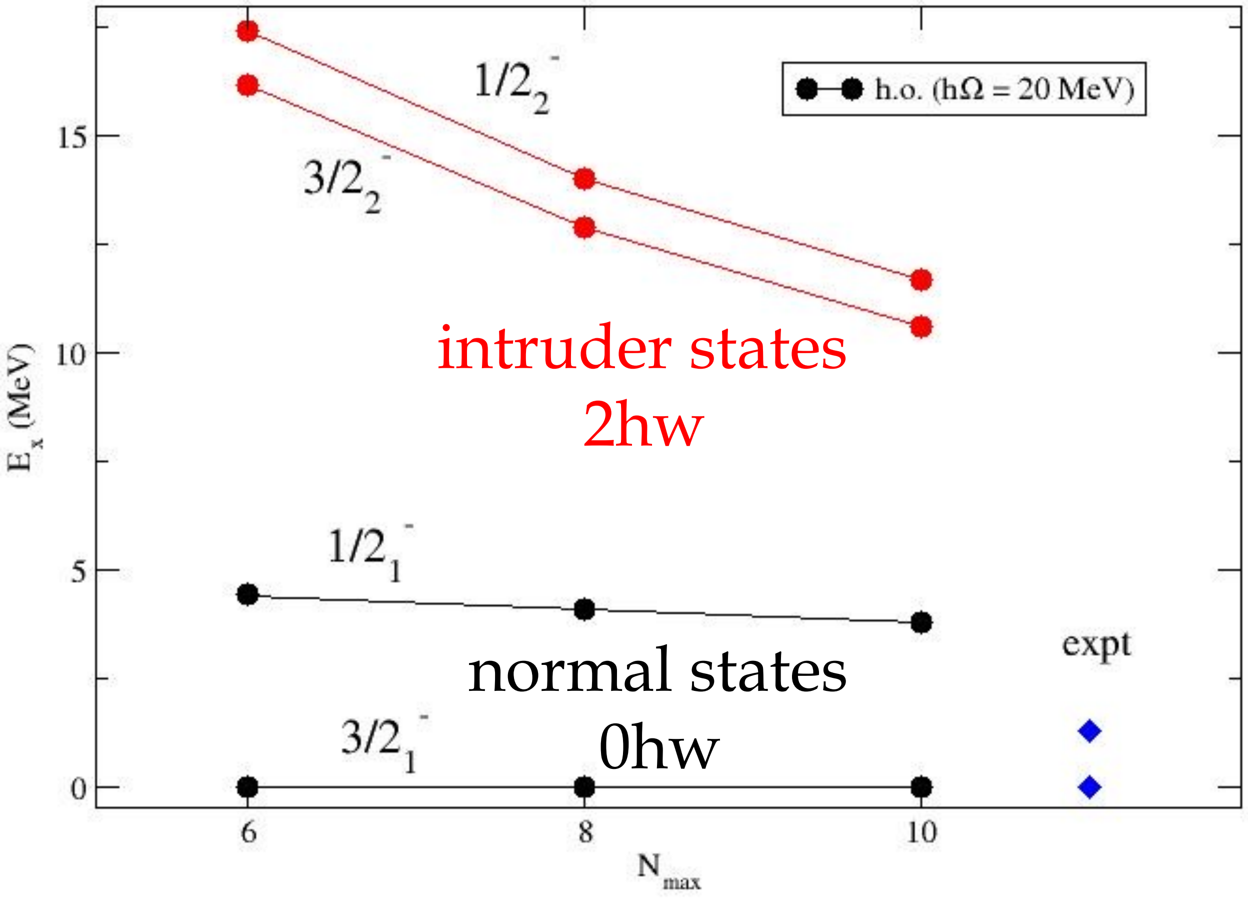
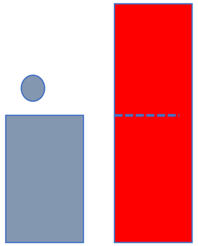




intruder states
2hw

normal states
0hw



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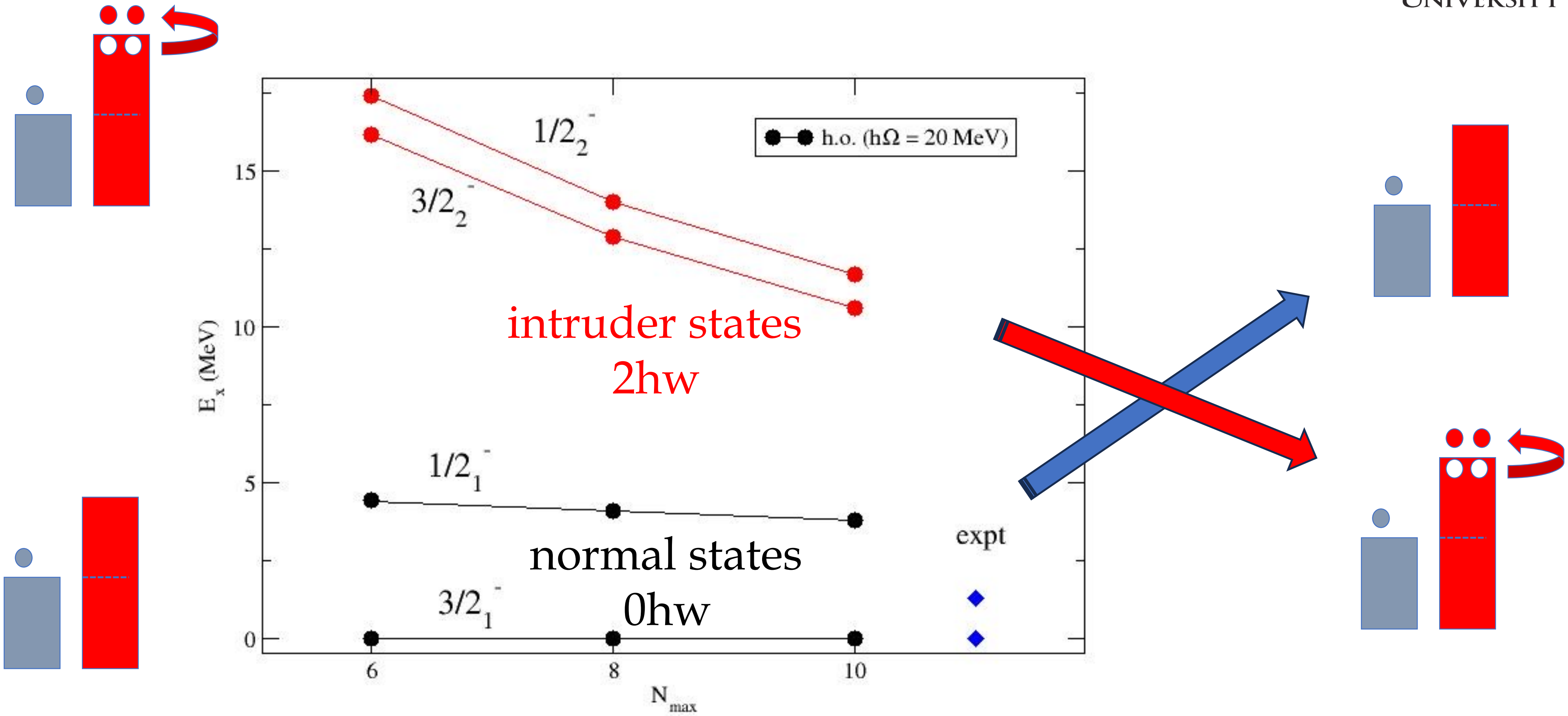

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normal states
0hw

CASE STUDY: ^{11}LI

$$\hat{\mathbf{H}}\left|\Psi\right\rangle = E\left|\Psi\right\rangle$$

$$\left|\Psi\right\rangle = \sum_{\alpha} c_{\alpha}\left|\alpha\right\rangle$$

millions or even billions
of components (in M-scheme)

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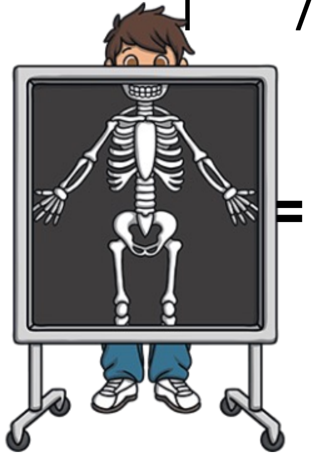


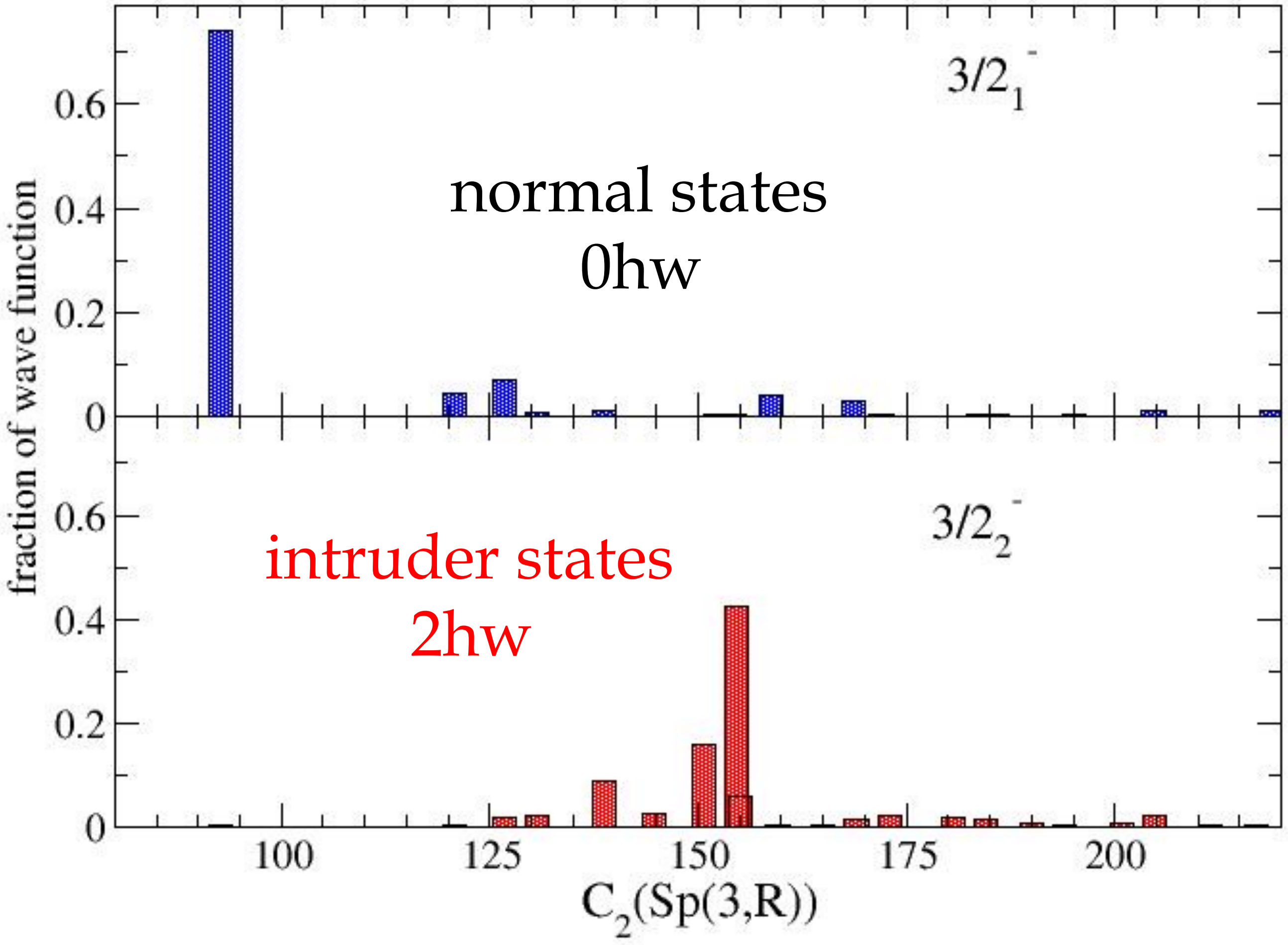
$$= \sum_{\alpha} c_{\alpha}|\alpha\rangle$$

millions or even billions
of components (in M-scheme)

We can ‘x-ray’ the wave function
using group theory

Jerry Draayer

CASE STUDY: ^{11}LI

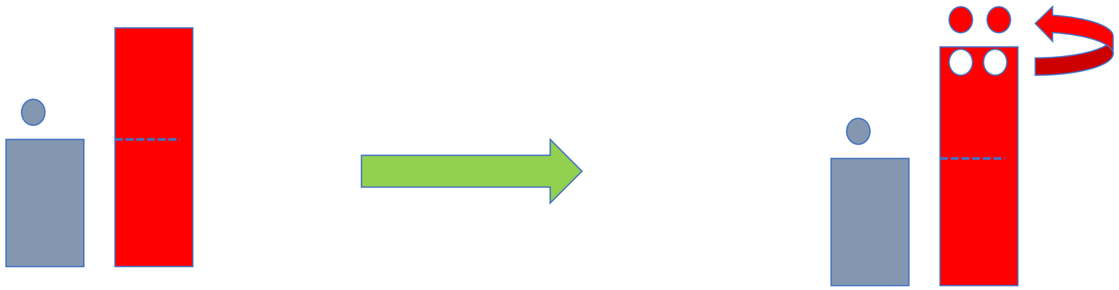


Group-theoretical Decomposition

Symplectic Sp(3,R)

CASE STUDY: ^{29}F

^{29}F is an analog of ^{11}Li



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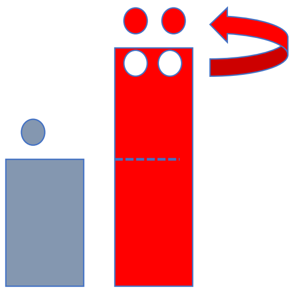

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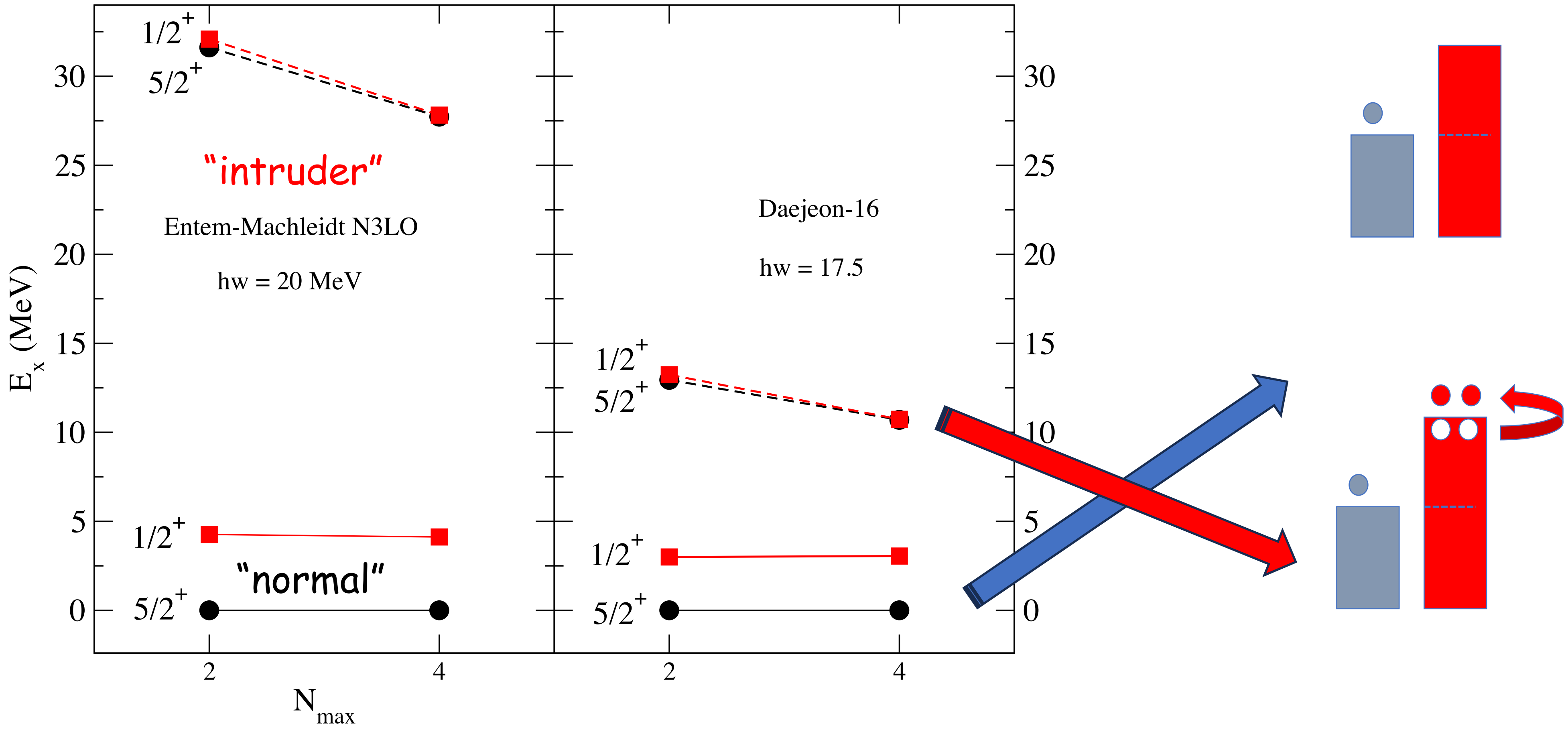
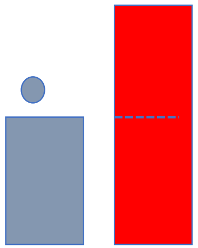
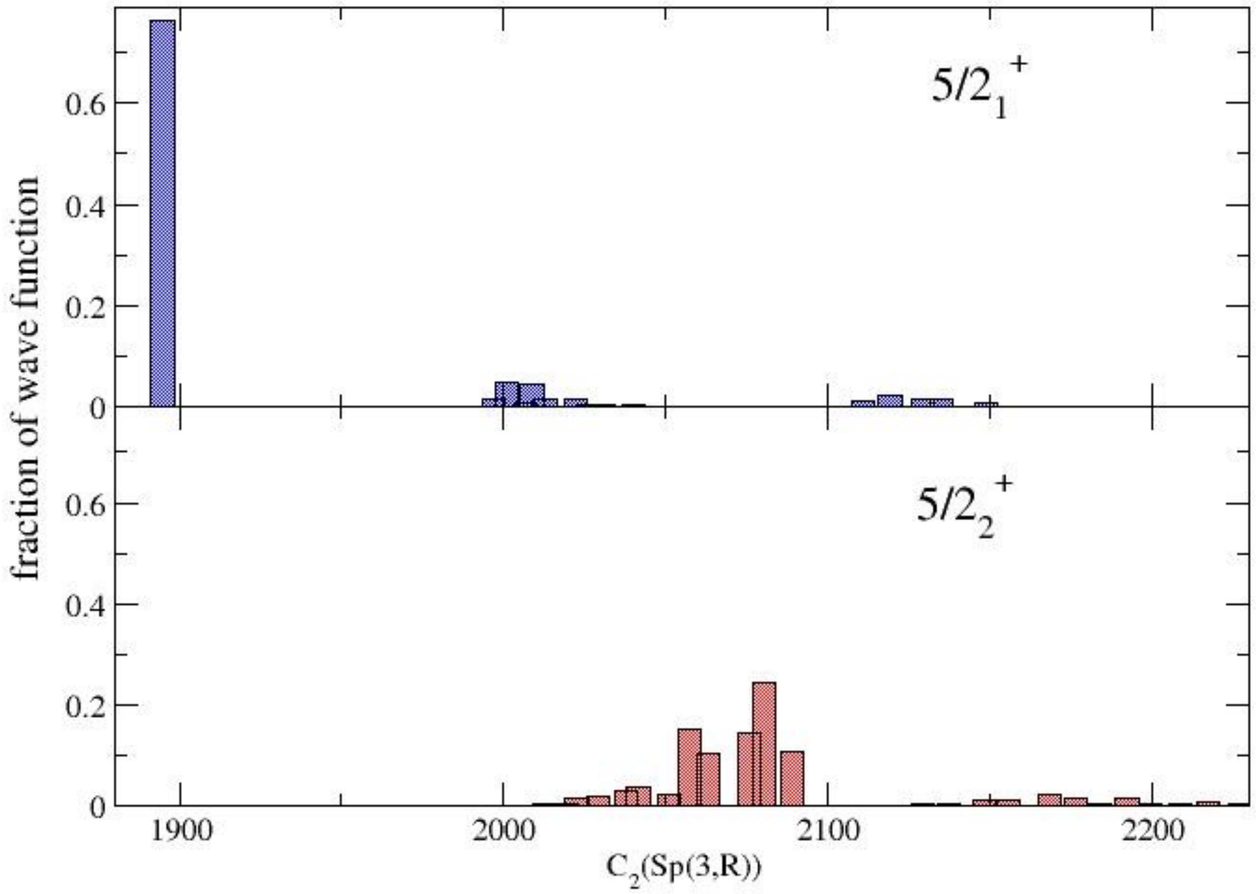




CASE STUDY: ^{29}F



N_{max} = 4, natural orbitals

Group-theoretical Decomposition

Symplectic Sp(3,R)

The future (?) of the shell model

San Diego State University

The future (?) of the shell model

1985

2024

The future (?) of the shell model

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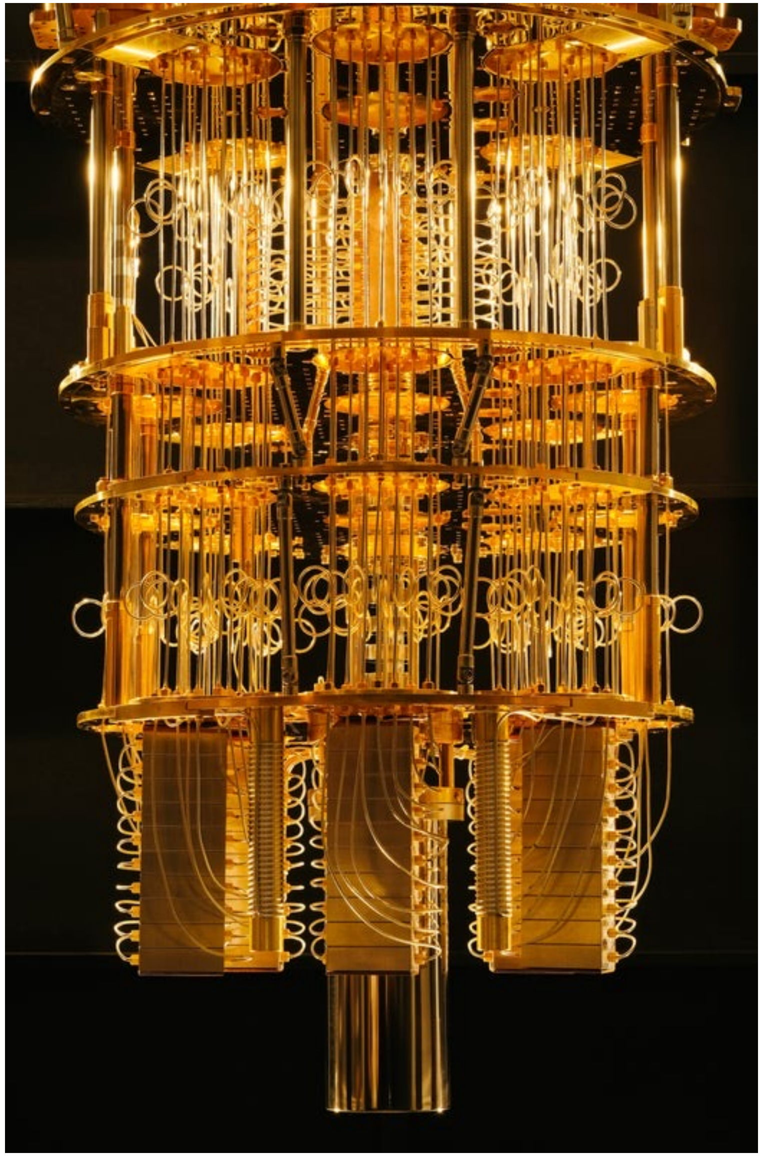
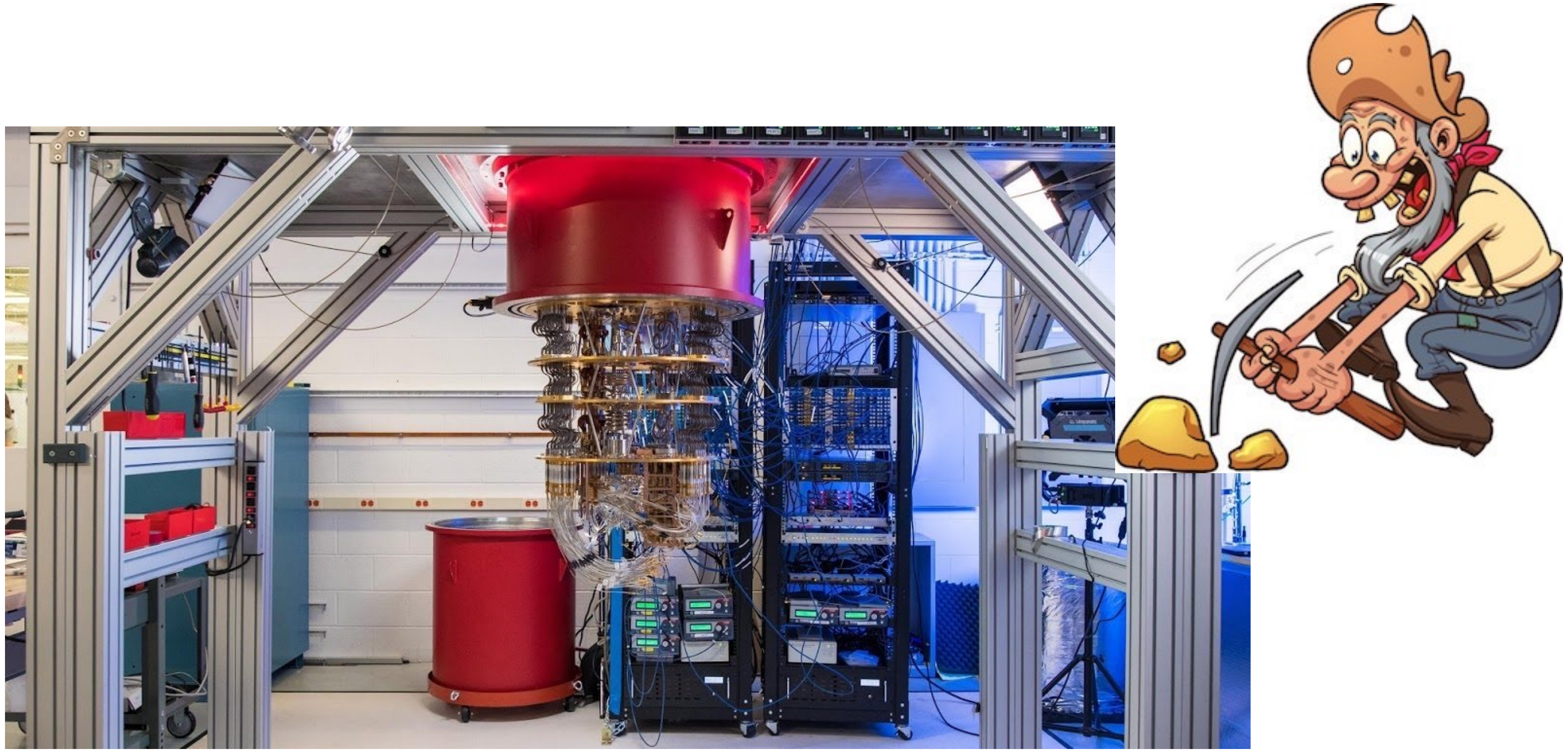


San Diego State University

The quantum computing gold rush….



Jon Engel

Qubits = superposition of bits

$$|\chi\rangle = a\,|0\rangle + b|1\rangle$$

Qubits = superposition of bits

$$|\chi\rangle = a\,|0\rangle + b|1\rangle$$

$$|\chi\rangle|\chi\rangle|\chi\rangle|\chi\rangle|\chi\rangle \ldots = (a\,|0\rangle + b|1\rangle)(a\,|0\rangle + b|1\rangle)\ldots$$
$$= |0000\ldots\rangle + |1000\ldots\rangle + |0100\ldots\rangle \ldots$$

Qubits = superposition of bits

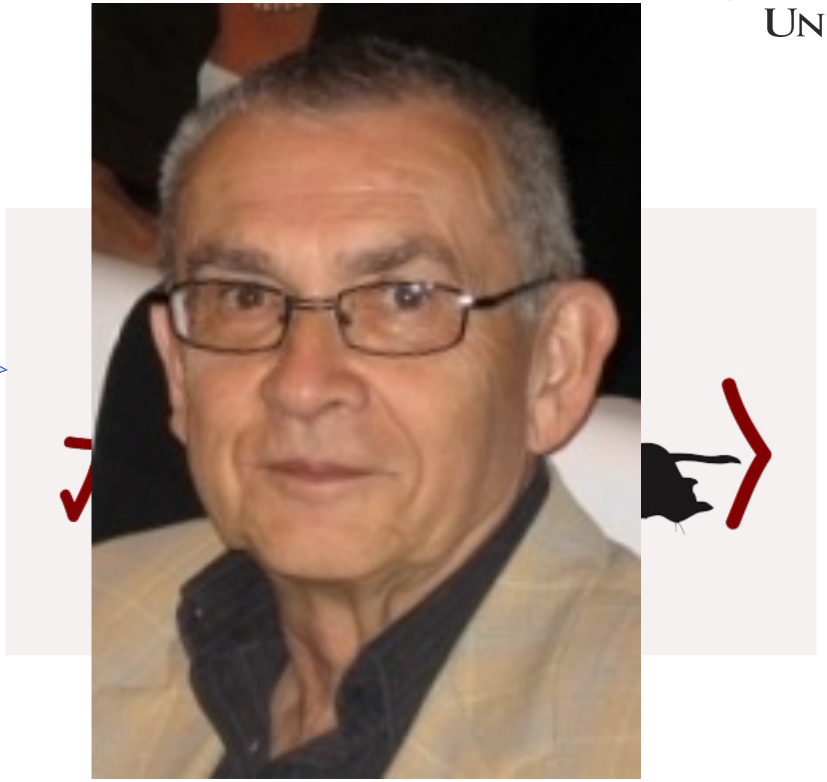
$$|\chi\rangle = a\,|0\rangle + b|1\rangle$$

$$|\chi\rangle|\chi\rangle|\chi\rangle|\chi\rangle|\chi\rangle\ldots$$

1 35 qb ‘word’ = 34.7 *billion* elements in a vector

Qubits = superposition of bits

This solves the problem of **exponential scaling**!



Yuri Manin

$$|\chi\rangle|\chi\rangle|\chi\rangle|\chi\rangle|\chi\rangle \ldots$$

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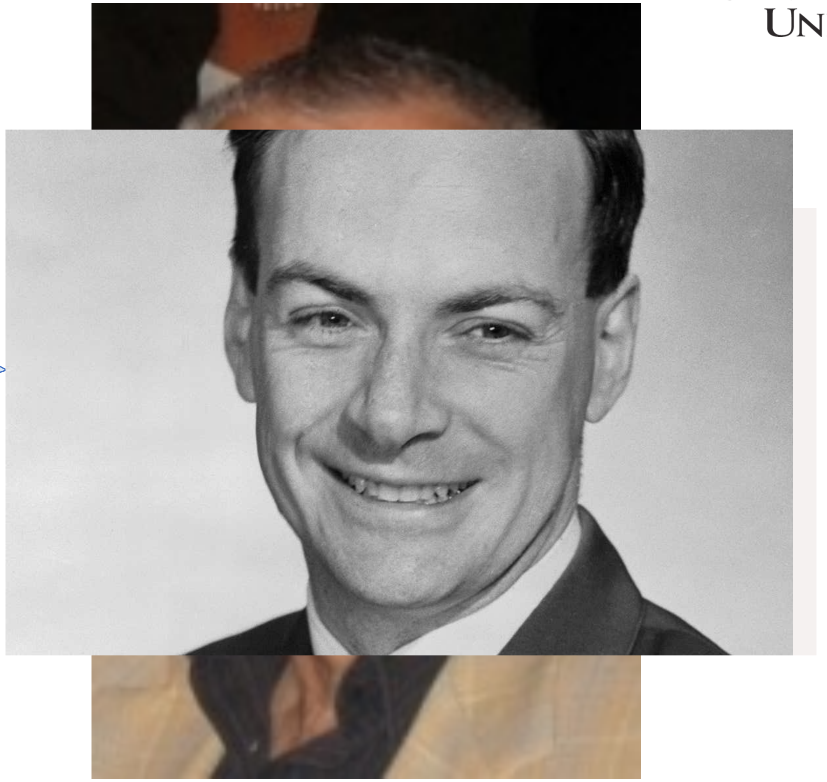
Qubits = superposition of bits

San Diego State
University

So build those quantum computers!



Richard Feynman

$$|\chi\rangle|\chi\rangle|\chi\rangle|\chi\rangle|\chi\rangle \ldots$$

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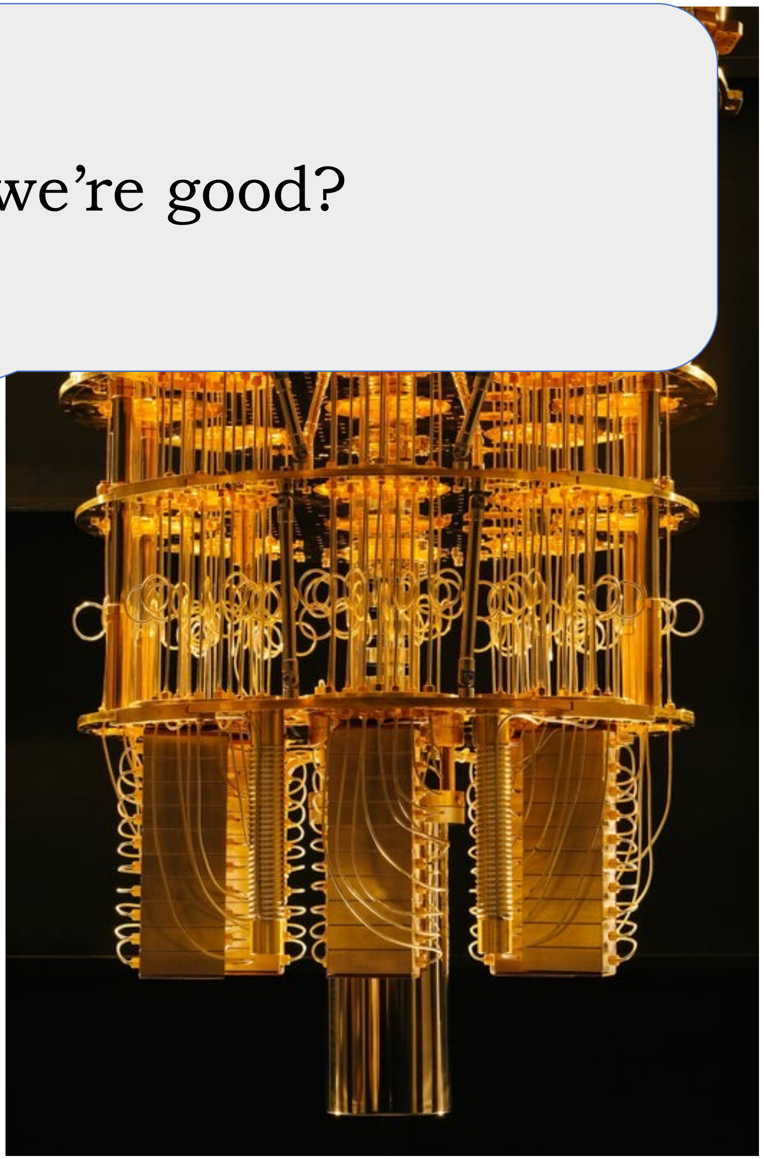
Quantum computing & the shell model

So we’re good?



Quantum computing & the shell model

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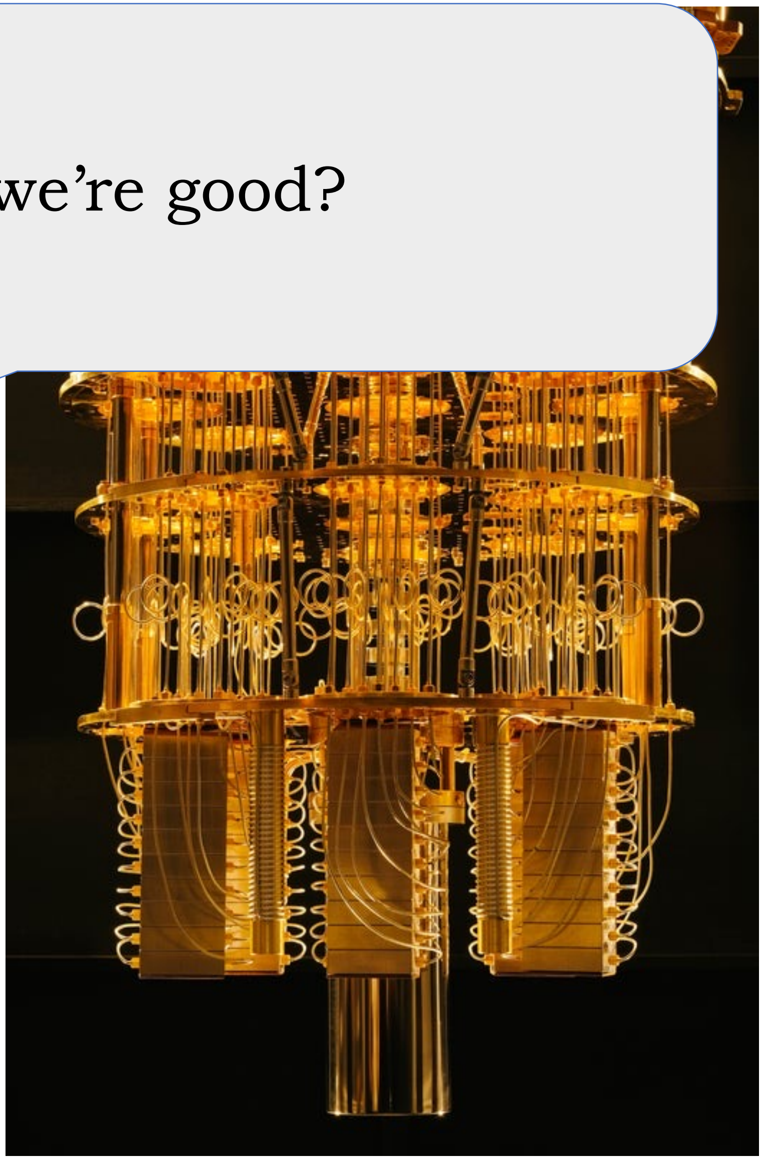

Not quite!

Quantum computing & the shell model

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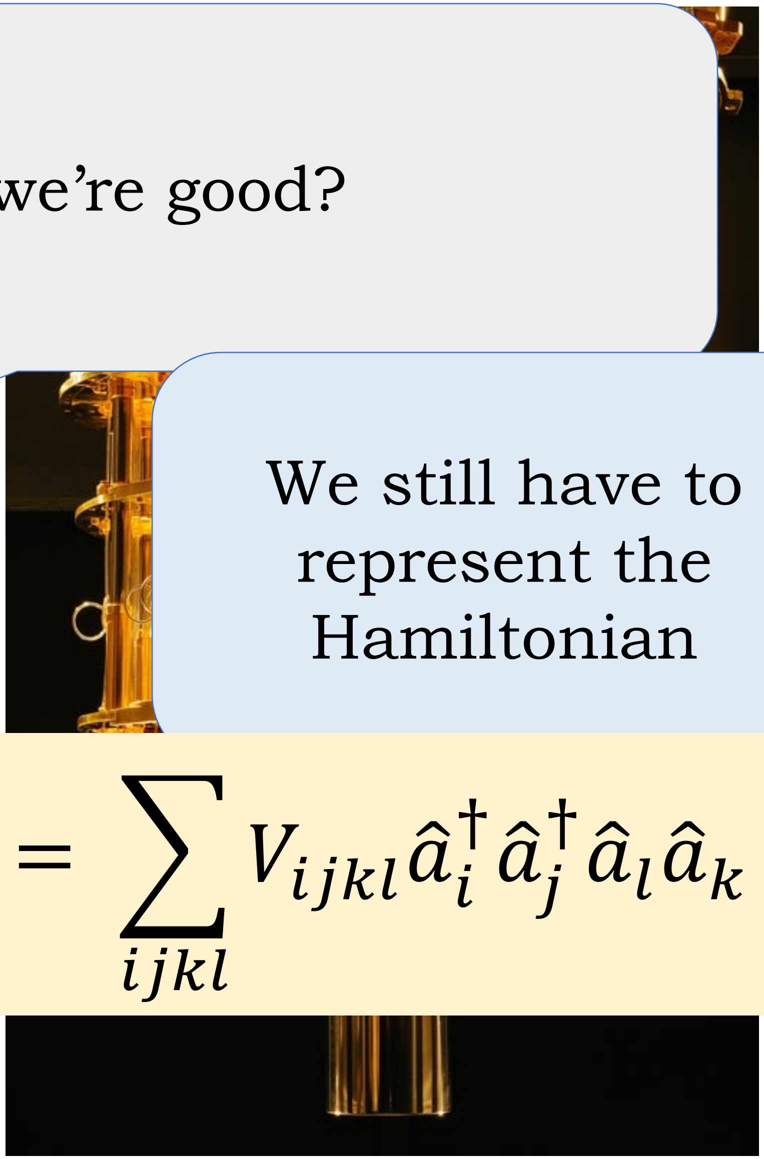

Not quite!

We still have to represent the Hamiltonian

$$\hat{H} = \sum_{ijkl} V_{ijkl} \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_l \hat{a}_k$$

Quantum Lanczos in real time

Parrish and McMahon, arXiv:1909.08925
"Quantum Filter Diagonalization"

Key idea of "Quantum Lanczos": take states at different 'times' to form a non-orthogonal **reduced** basis

$$|\psi_n\rangle = e^{\boxed{-in\Delta t\hat{H}}}|\psi_0\rangle \qquad N_{mn} = \langle\psi_m|\psi_n\rangle \qquad H_{mn} = \langle\psi_m|\hat{H}|\psi_n\rangle$$

In this reduced basis, solve generalized eigenvalue problem:

$$\hat{H}\vec{v} = E\,\hat{N}\vec{v}$$

Quantum Lanczos in real time

SAN DIEGO STATE UNIVERSITY

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Amanda Bowman, SDSU M.S. student

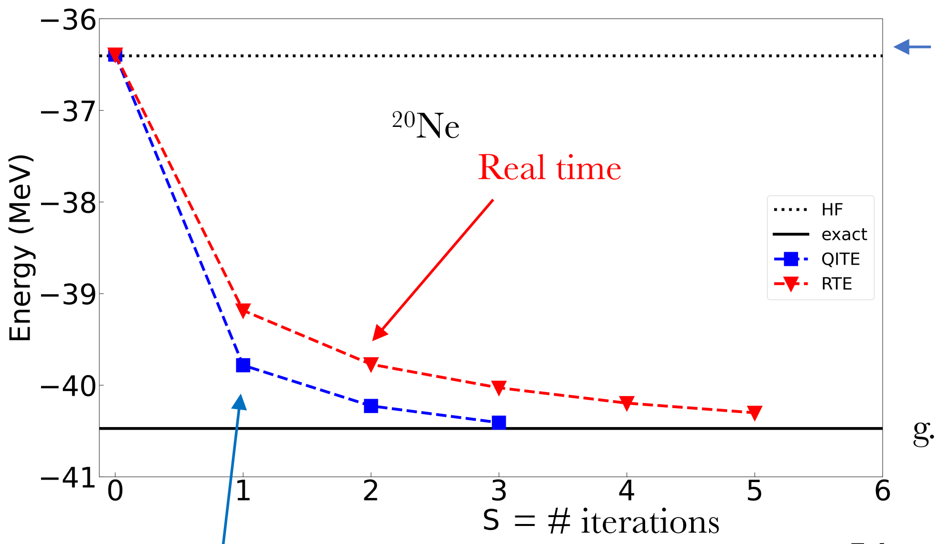
in Computational Science

MS thesis: "*Nuclear Spectra from Quantum Lanczos Algorithm with Real-Time Evolution and Multiple Reference States*"

arXiv:2309.00759

Ionel Stetcu,

Quantum Lanczos in real time

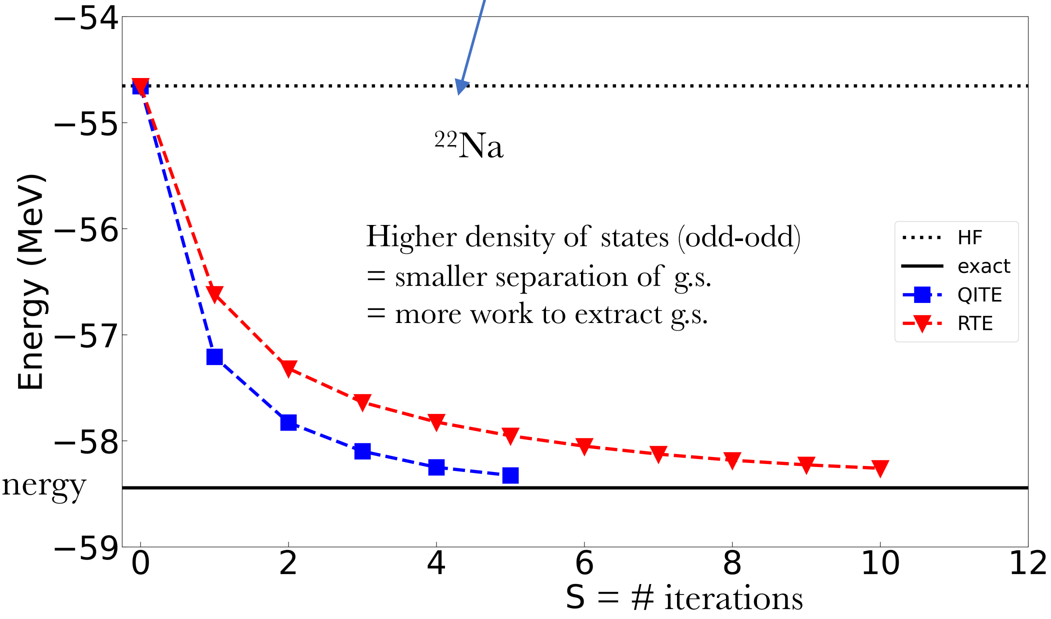


Real time

Imaginary time

Initial state: We evolved in imaginary time a random state until $< H > = E_{HF}$

g.s. energy



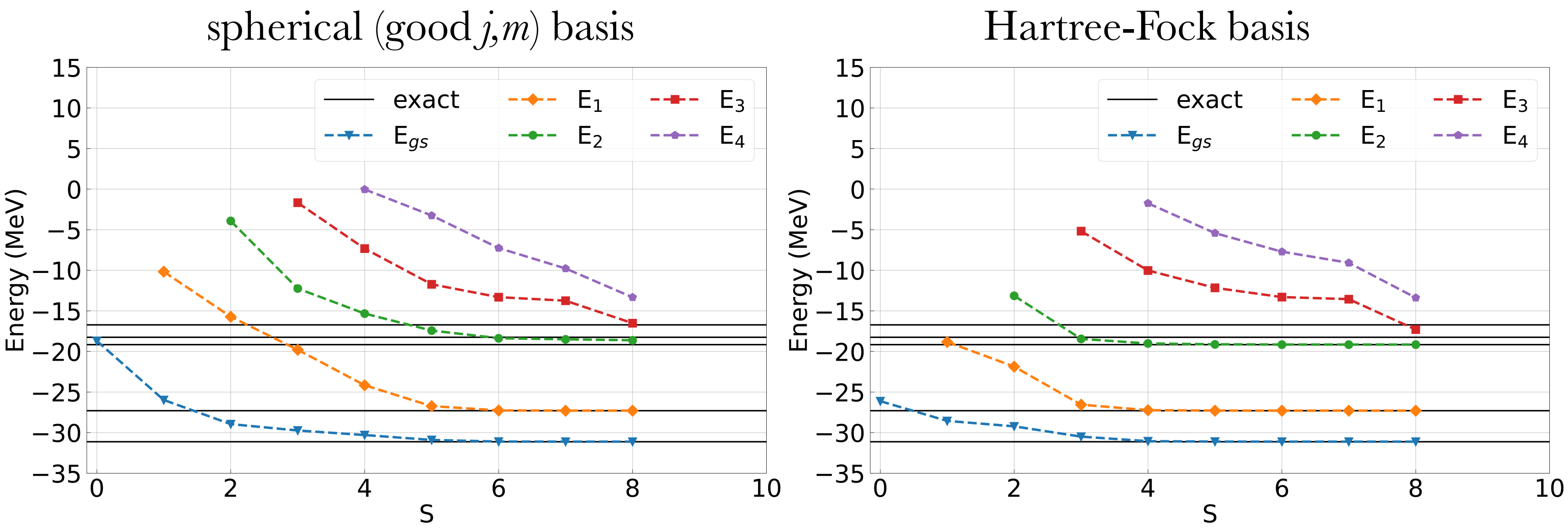
Higher density of states (odd-odd)
= smaller separation of g.s.
= more work to extract g.s.

g.s. energy

Here $\Delta t = \Delta\tau = 0.1$ MeV^{-1}

Quantum Lanczos in real time

^{8}Be in full 0p-shell


spherical (good j,m) basis

Hartree-Fock basis

(a)

(b)

Figure 8.1. Numerical simulations of the QLanczos algorithm with exact real-time evolution to solve for the lowest five energy states of the valence particles of ^{8}Be (two protons and two neutrons in the full p-shell). The simulation was run using a single reference state; (a) the lowest energy configuration in the spherical basis and (b) the Hartree-Fock state. A fixed number of real-time evolution iterations was used ($S = 8$) with a time step size of $\Delta t = 0.1$.

Quantum Lanczos in real time

^{8}Be in full 0p-shell

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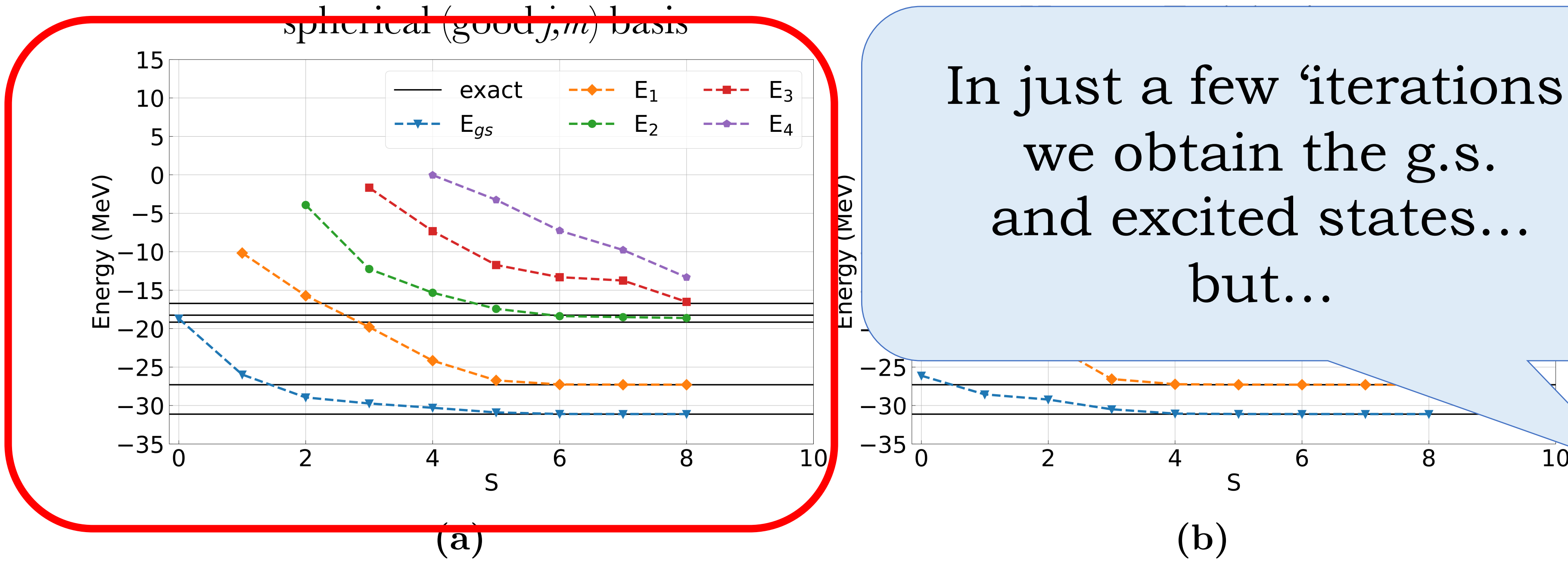

In just a few ‘iterations’ we obtain the g.s. and excited states… but…

Figure 8.1. Numerical simulations of the QLanczos algorithm with exact real-time evolution to solve for the lowest five energy states of the valence particles of ^{8}Be (two protons and two neutrons in the full p-shell). The simulation was run using a single reference state; (a) the lowest energy configuration in the spherical basis and (b) the Hartree-Fock state. A fixed number of real-time evolution iterations was used ($S = 8$) with a time step size of $\Delta t = 0.1$.

Quantum Lanczos in real time

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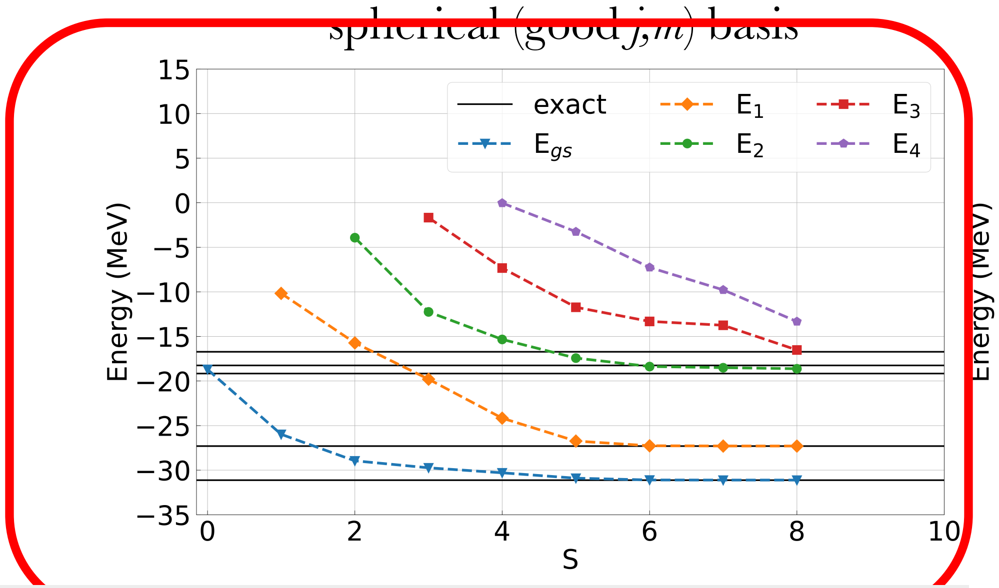
^{8}Be in full 0p-shell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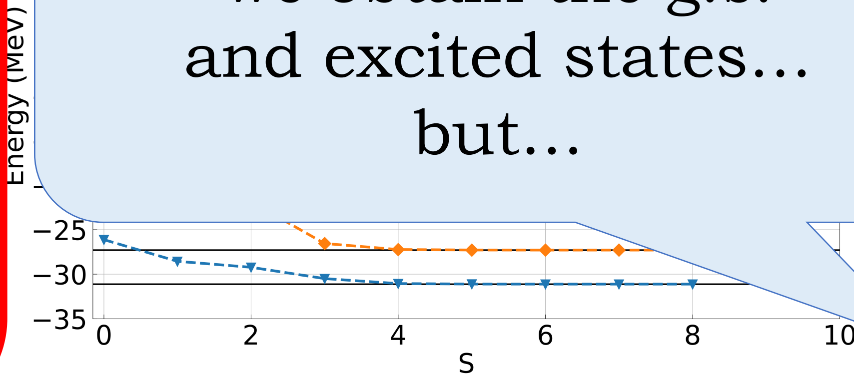

In just a few 'iterations' we obtain the g.s. and excited states... but...

(b)

H requires 975 Pauli strings and ~ 24,000 gates

ne QLanczos algorithm with exact real-
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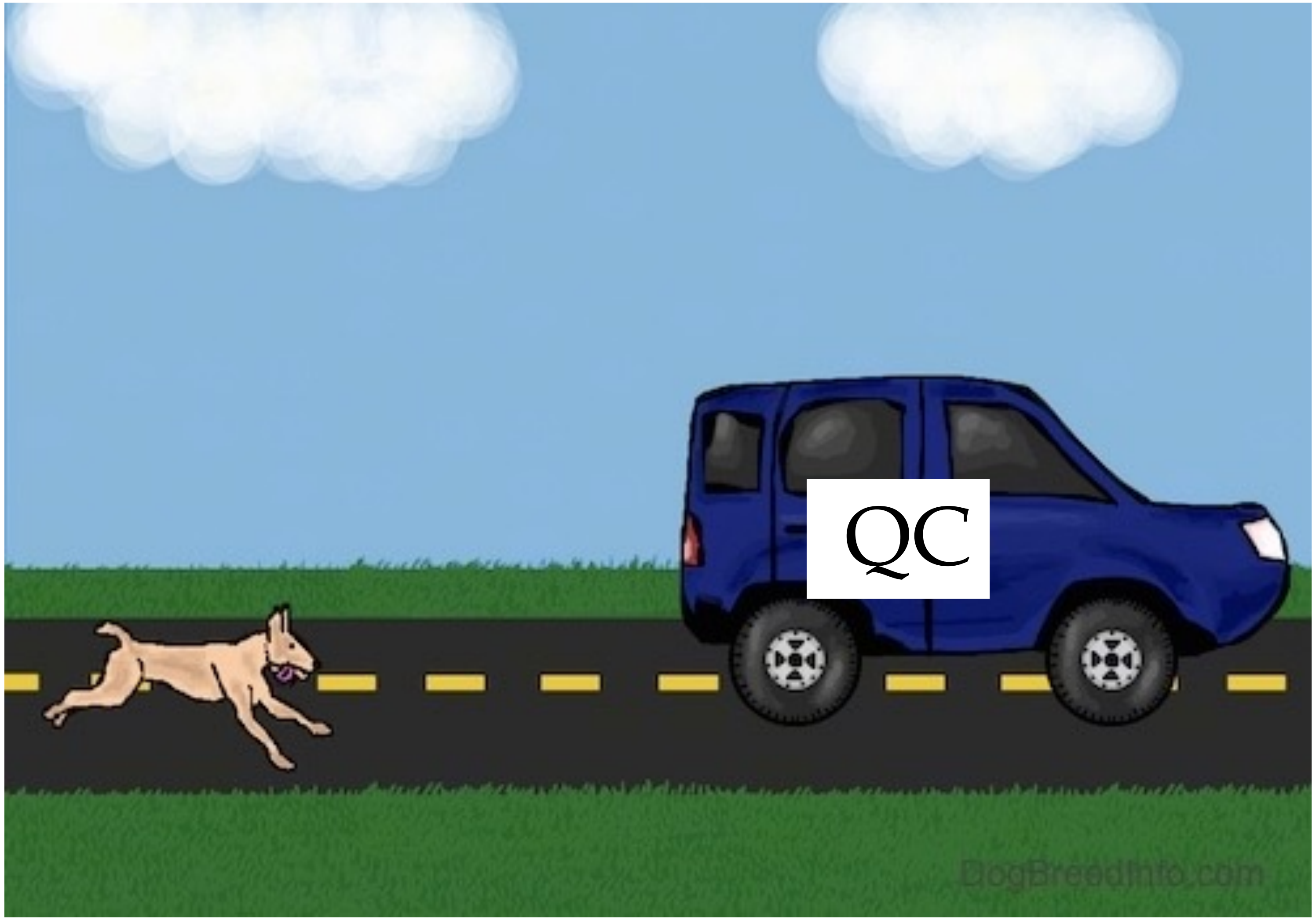
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Let’s look at the data requirements in more detail

Consider ^{12}C, N_{max}=8

M-scheme dimension 0.6 billion

55 single-particle orbitals (n l j)
440 single particle states (n l j m) | 0 1 1 0 0 1 … >

Let’s look at the data requirements in more detail

Consider ^{12}C, N_{max}=8

M-scheme dimension 0.6 billion

55 single-particle orbitals (n l j)

440 single particle states (n l j m) | 0 1 1 0 0 1 … >

= estimate # of qubits needed

Let's look at the data requirements in more detail

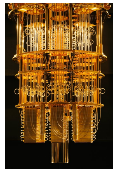
Consider ^{12}C, N_{max}=8

M-scheme dimension 0.6 billion *by superposition*

 # uncoupled 2-body matrix elements
~ 10 million! $V_{ijkl}\ a^+_i\ a^+_j\ a_l\ a_k$ = # 'Pauli strings'

= # of terms to be evaluated in a quantum circuit
(or, # of separate quantum circuits to be evaluated!)

~ 250,000,000 gates
(but **polynomial** scaling)



Quantum computing **useful** for the shell model
is still a ways off!

Summary

Summary

The shell-model has come a long way in 75 years…

and the journey is not yet over!

Enjoy the Symposium!