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The Nilsson Model and Medium-Mass Nuclei

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Overview

- Welcome to the Nilsson Model
- The case of $^{18,19}F$
- Islands of Inversion and the Nilsson Model
  - N=8 and ^{12}Be
  - N=20 and ^{33}Mg, ^{29}F
- Outlook

The Nilsson Model – A Little Bit of History

A Deformed Nuclear Shell Model

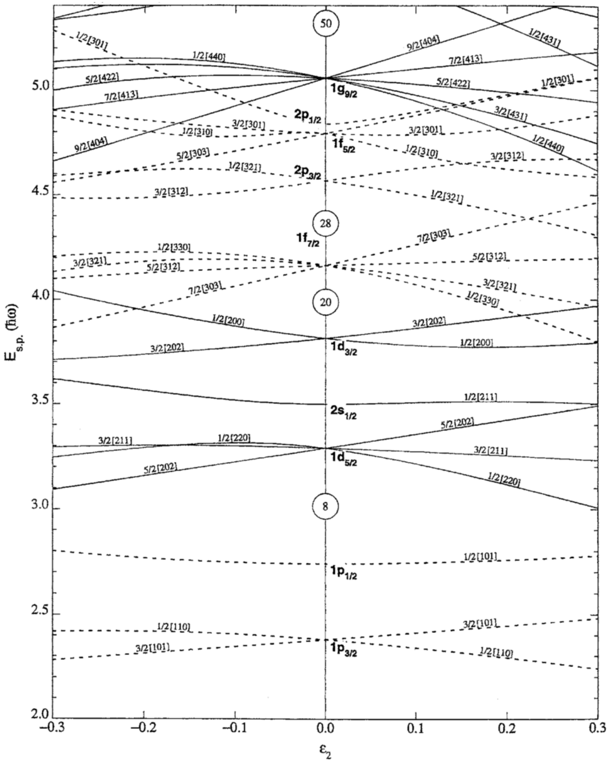
- In the early 1950's the first experimental examples of rotational bands in nuclei showed energy levels following a J(J+1) pattern as for molecular rotational spectra
- This implied a non-spherical nuclear shape – could have been described as coherent superpositions of p-h excitations in a spherical potential, but such a description was intractable at the time
- Models were constructed with the same ingredients as the spherical shell model but in a potential deformed into an ellipsoidal shape – Bohr, Mottelson, Nilsson
- For an axially symmetric potential, have a Hamiltonian of the form:

$$H = \frac{1}{2} m\omega_z^2 z^2 + \frac{1}{2} m\omega_\perp^2 (x^2 + y^2) - c_1 \boldsymbol{\ell} \cdot \boldsymbol{s} - c_2 (\ell^2 - \langle \ell^2 \rangle_N)$$

S. G. Nilsson, Dan. Mat. Fys. Medd. 29, 16 (1955).

The Nilsson Model – A Little Bit of History

A Deformed Nuclear Shell Model

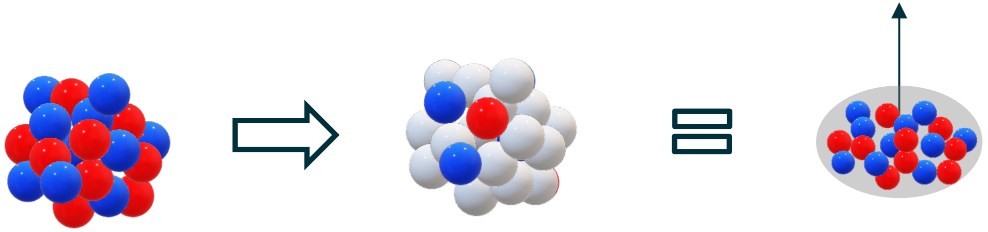


- Nilsson wavefunctions evolve with deformation (β_2/ε_2) and can be expressed as linear combination of spherical orbitals weighted by Nilsson amplitudes
- Still a shell model – nucleons fill Nilsson states in pairs; unpaired valence nucleon(s) determine nuclear state quantum numbers

Single Particle Structure and Collectivity in Light Nuclei

"It was quite a dramatic moment when it was realized that some of the spectra in the light nuclei could be given a very simple interpretation in terms of the rotational coupling scheme."

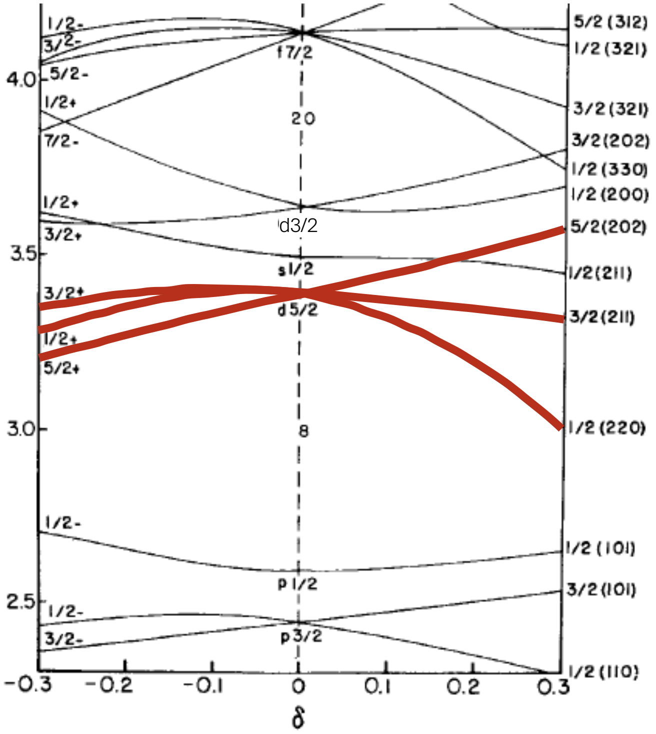
– A. Bohr, 1975 Nobel Lecture



- Duality of single-particle and collective model descriptions for atomic nuclei has been recognized for decades
- ^{19}F was an early example, where shell-model calculations in the sd model space with only 3 valence nucleons outside ^{16}O and a rotational model description both reproduced experimental energy levels

J. P. Elliot and B.H. Flowers, Proc. R. Soc. A 229, 536 (1955).
M. Redlich, Phys. Rev. 99, 1427 (1955).
E. B. Paul, Philosophical Magazine, 311, 2:15 (1957).

^{19}F in the Rotational Model

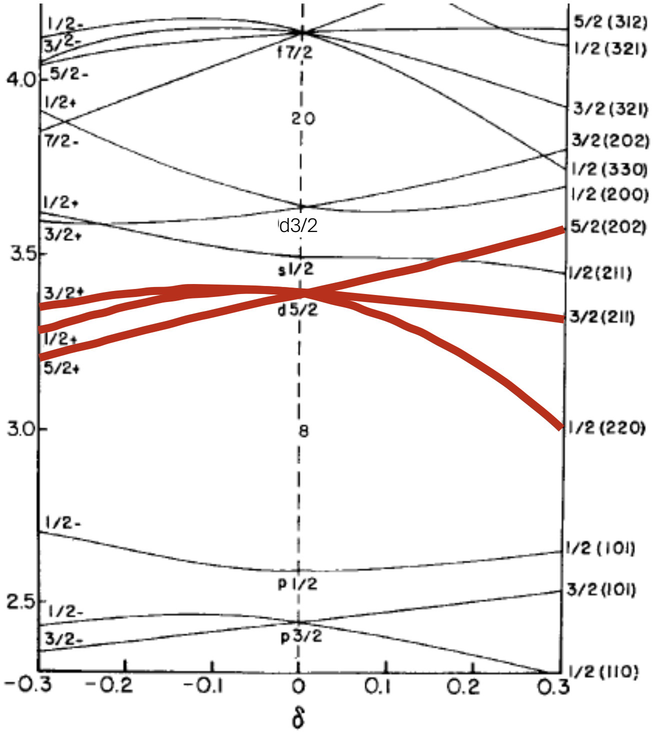


- Within Nilsson (rotational) model, relevant orbitals in the ‘sd’ space are

$$\tfrac{1}{2}[220],\ \tfrac{3}{2}[211],\ \text{and}\ \tfrac{5}{2}[202]$$

E. B. Paul, Philosophical Magazine, 311, 2:15 (1957).
“The interpretation of the levels of ^{19}F in the rotational model”

^{19}F in the Rotational Model



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$$|\tfrac{1}{2}[220]\rangle = C_{1/2,0,1/2}|s_{1/2}\rangle + C_{3/2,2,1/2}|d_{3/2}\rangle + C_{5/2,2,1/2}|d_{5/2}\rangle$$

$$|\tfrac{3}{2}[211]\rangle = C_{3/2,2,3/2}|d_{3/2}\rangle + C_{5/2,2,3/2}|d_{5/2}\rangle$$

$$|\tfrac{5}{2}[202]\rangle = |d_{5/2}\rangle$$

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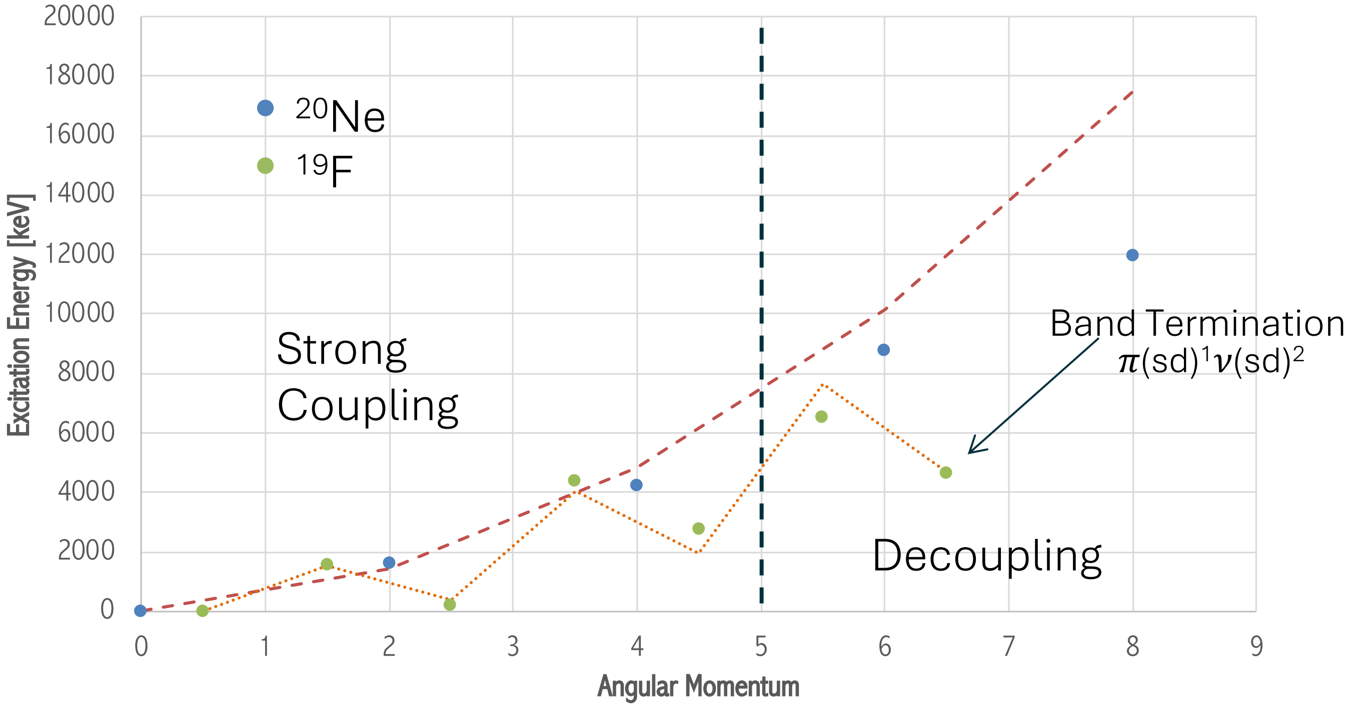
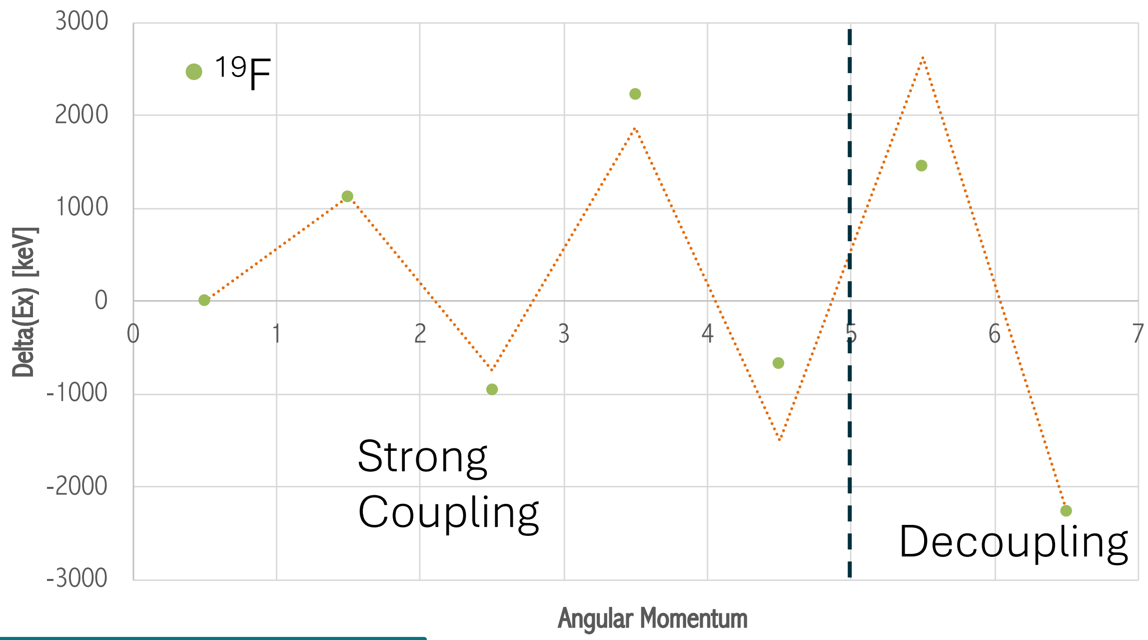
"The interpretation of the levels of ^{19}F in the rotational model"

^{19}F in the Rotational Model



^{19}F in the Rotational Model

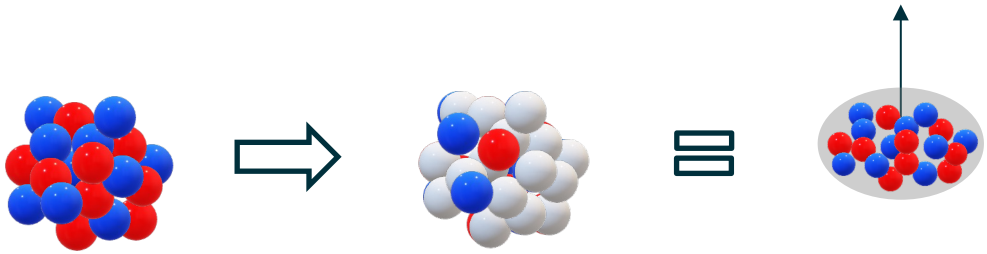


Decoupling parameter

$a_{exp} \approx 2.15$

$a_{Nilsson} = 1.96$

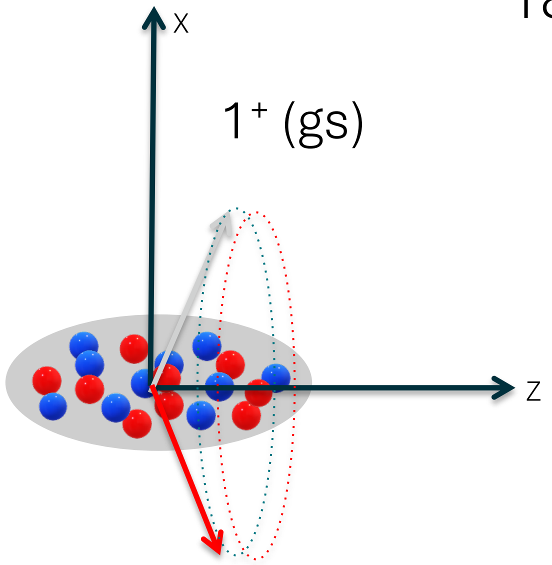
Single Particle Structure and Collectivity in Light Nuclei



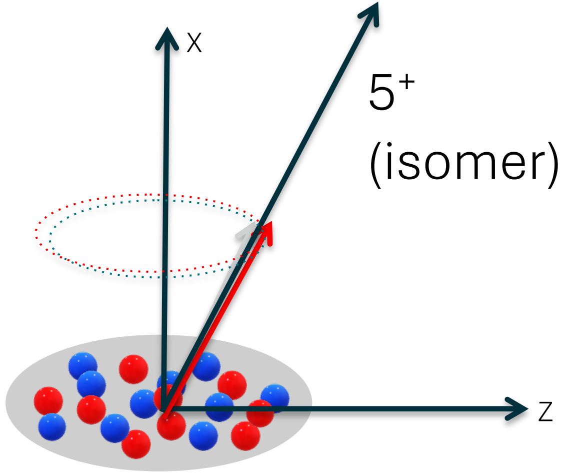
- The success in describing ^{19}F within the rotational model opens the door to considering the description of transfer reactions, i.e. $^{18}F(d, p)^{19}F$
- Consider ^{18}F in the rotational model…

^{18}F in the Rotational Model

^{18}F



Strongly coupled (deformation aligned) band



Doubly-decoupled (rotation aligned) band

A. J. Kreiner, Z. Physik A 288, 373 (1978).

^{18}F(d, p)^{19}F

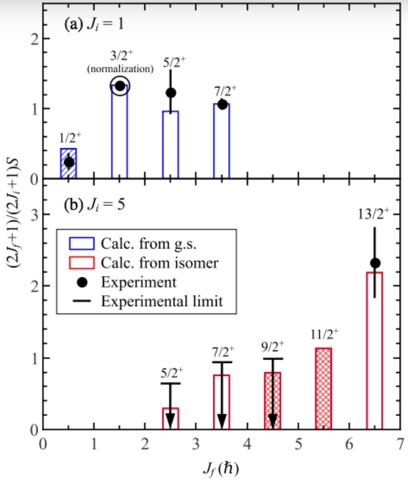
PHYSICAL REVIEW LETTERS **120,** 122503 (2018)

HELIOS Experiment at ANL/ATLAS

Probing the Single-Particle Character of Rotational States in ^{19}F Using a Short-Lived Isomeric Beam

D. Santiago-Gonzalez,[1,2] K. Auranen,[2] M. L. Avila,[2] A. D. Ayangeakaa,[2,*] B. B. Back,[2] S. Bottoni,[2,†] M. P. Carpenter,[2] J. Chen,[2] C. M. Deibel,[1] A. A. Hood,[1] C. R. Hoffman,[2] R. V. F. Janssens,[2,‡] C. L. Jiang,[2] B. P. Kay,[2] S. A. Kuvin,[3] A. Lauer,[1] J. P. Schiffer,[2] J. Sethi,[4,2] R. Talwar,[2] I. Wiedenhöver,[5] J. Winkelbauer,[6] and S. Zhu[2]

- ^{18}F beam with substantial components of both 1^+ ground state and 5^+ isomer (162ns) was used to study (d, p) reactions into rotational band states of ^{19}F
- Relative spectroscopic factors well reproduced by shell-model using the USDB effective interaction



$^{18}F(d, p)^{19}F$

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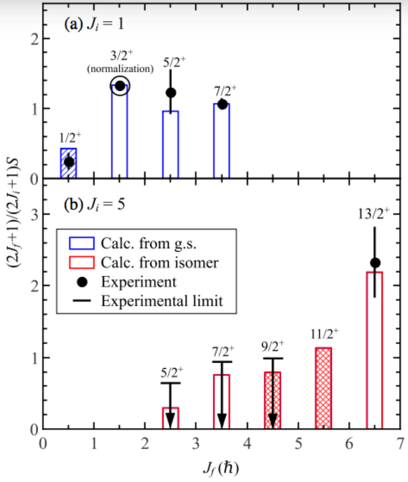
- ^{18}F beam with substantial components of both 1^+ ground state and 5^+ isomer (162ns) was used to study (d, p) reactions into rotational band states of ^{19}F
- Relative spectroscopic factors well reproduced by shell-model using the USDB effective interaction
- Does the rotational model description similarly provide good agreement?



$^{18}F(d,p)^{19}F$ in the Nilsson Formalism

Deformation aligned coupling – strongly coupled band

$$S_{i,f}(j\ell, K) = \frac{(2I_i+1)}{(2I_f+1)} \langle I_i j \Omega_\nu K_i | I_f K_f \rangle^2 C^2_{j,\ell,\nu} U^2_\nu \langle \phi_f | \phi_i \rangle^2$$

Rotation aligned coupling – decoupled band

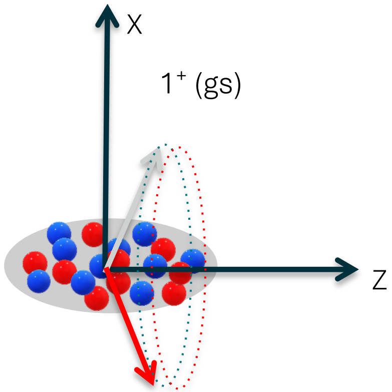
$$S_{i,f} = \left(\sum_K \mathcal{A}_K S^{1/2}_{i,f}(j\ell, K) \right)^2$$

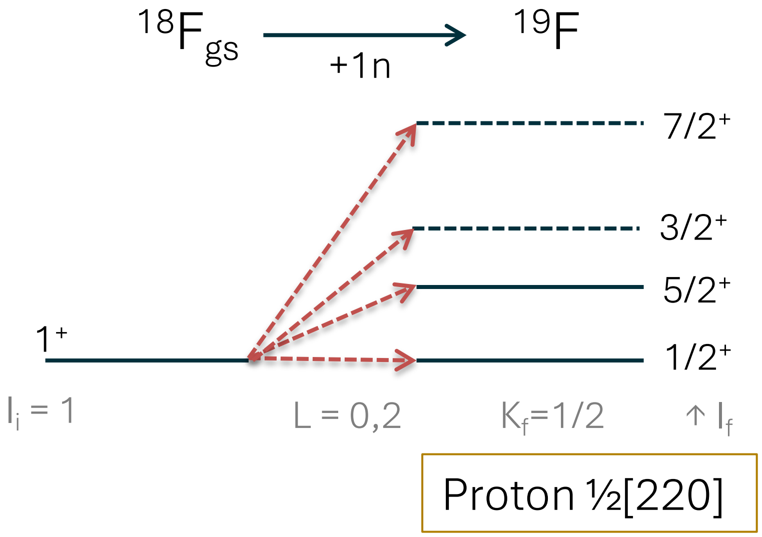
$$\psi_I = \sum_K \mathcal{A}_K | I K \rangle$$

$$\mathcal{A}_K \approx d^{5/2}_{5/2,K}(\pi/2)$$

B. Elbek and P. Tjom, Advances in Nucl. Phys. 3, 259 (1969).
R. G. Lanier, et al., Phys. Rev. 178, 1919 (1969).
F. S. Stephens, R. M. Diamond, and S. G. Nilsson, Phys. Lett. B 44, 429 (1973).

$^{18}F_{gs}(d, p)^{19}F$ in the Nilsson Formalism



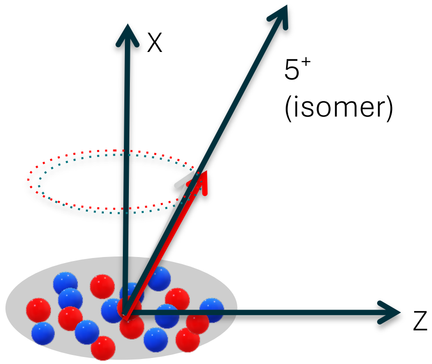


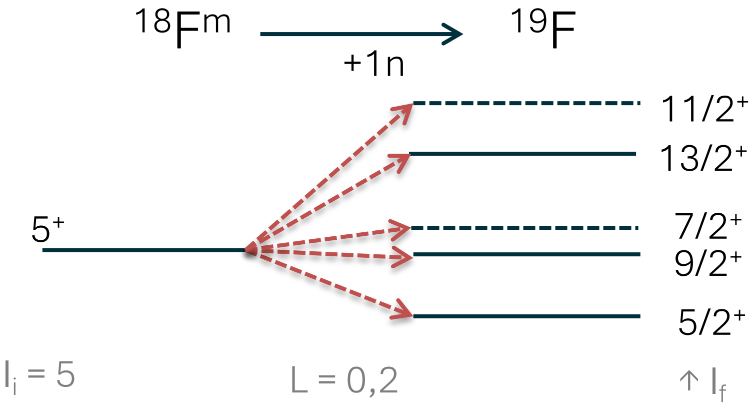
Transfer from the GS $\Longrightarrow$ Deformation-aligned (strong) coupling

$$S_{i,f}(j\ell, K) = \frac{(2I_i+1)}{(2I_f+1)}\langle I_i j \Omega_\nu K_i | I_f K_f \rangle^2 C^2_{j,\ell,\nu} U^2_\nu \langle \phi_f | \phi_i \rangle^2$$

B. Elbek and P. Tjom, Advances in Nucl. Phys. 3, 259 (1969).

$^{18}F^m(d, p)^{19}F$ in the Nilsson Formalism





Transfer from the 5^+ Isomer $\Longrightarrow$ Decoupling

$$\psi_I = \sum_K \mathcal{A}_K |IK\rangle \quad \Longrightarrow \quad S_{i,f} = \left(\sum_K \mathcal{A}_K S_{i,f}^{1/2}(j\ell, K) \right)^2$$

$$\mathcal{A}_K \approx d^{5/2}_{5/2,K}(\pi/2)$$

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$^{18}F(d,p)^{19}F$ in the Nilsson Formalism

Ground state

$$S_{i,f}(j\ell, K) = \frac{(2I_i+1)}{(2I_f+1)}\langle I_i j \Omega_\nu K_i | I_f K_f\rangle^2 C^2_{j,\ell,\nu} U^2_\nu \langle \phi_f | \phi_i \rangle^2$$

Isomer state

$$\psi_I = \sum_K \mathcal{A}_K |IK\rangle$$

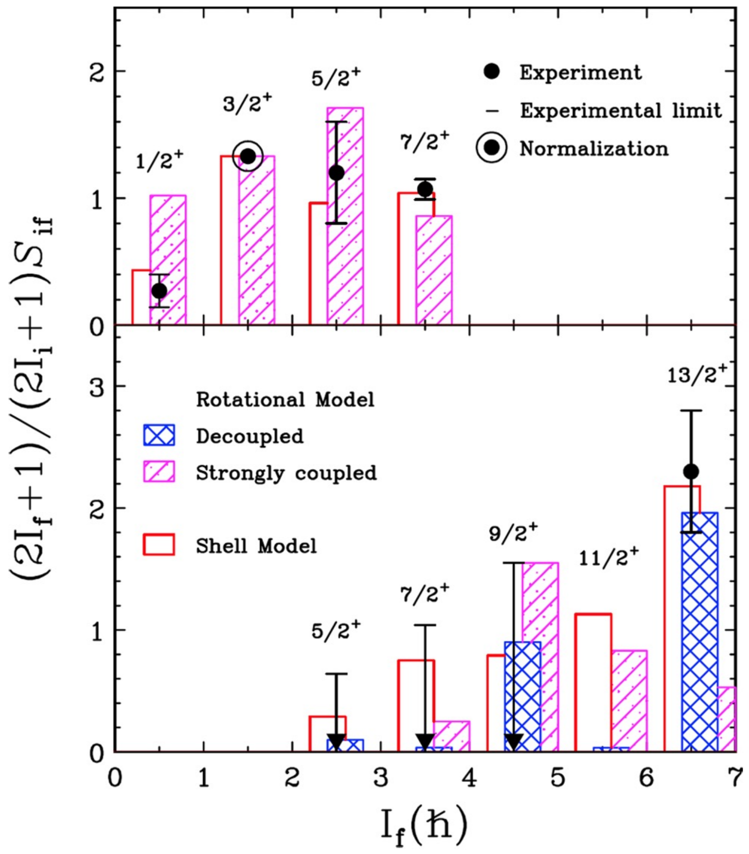
$$S_{i,f} = \left(\sum_K \mathcal{A}_K S^{1/2}_{i,f}(j\ell, K) \right)^2$$

$$\mathcal{A}_K \approx d^{5/2}_{5/2,K}(\pi/2)$$

Fixed

Spectator proton wavefunctions

Results: Spectroscopic Factors



B. E. Chi, Nuclear Physics 83, 97 (1966).

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$$S_{i,f}(j\ell, K) = \frac{(2I_i+1)}{(2I_f+1)} \langle I_i j \Omega_\nu K_i | I_f K_f \rangle^2 C^2_{j,\ell,\nu} U^2_\nu \langle \phi_f | \phi_i \rangle^2$$

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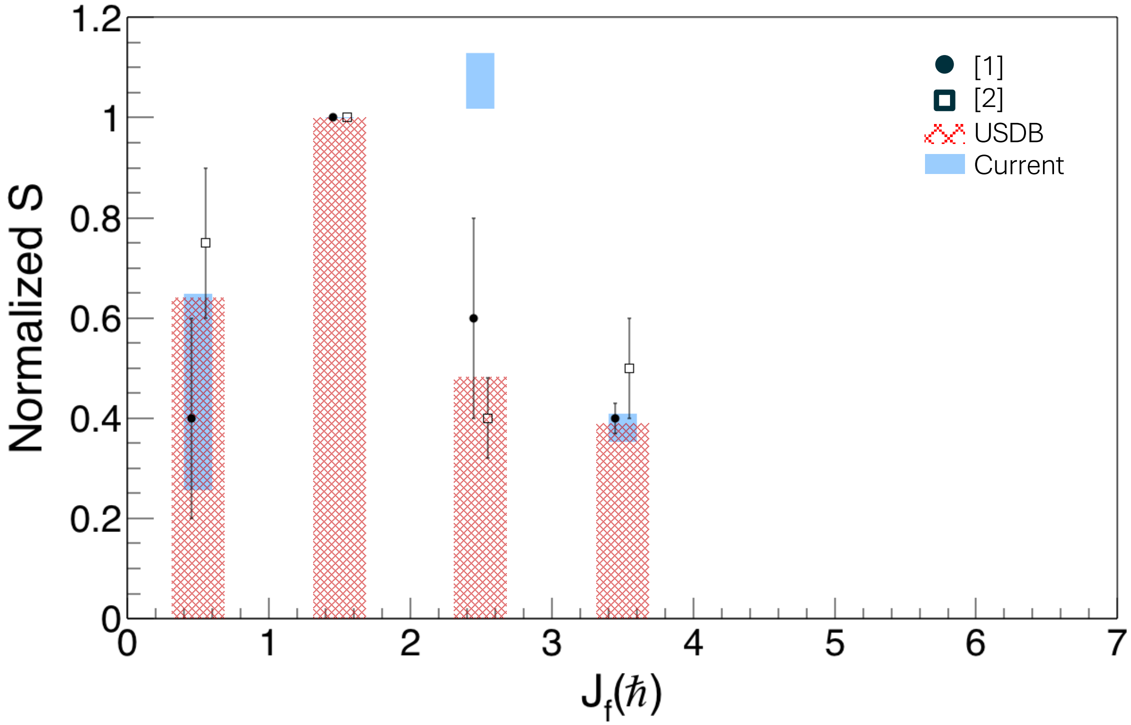
Fixed

Spectator proton wavefunctions

⇒ Minimization on Nilsson amplitudes

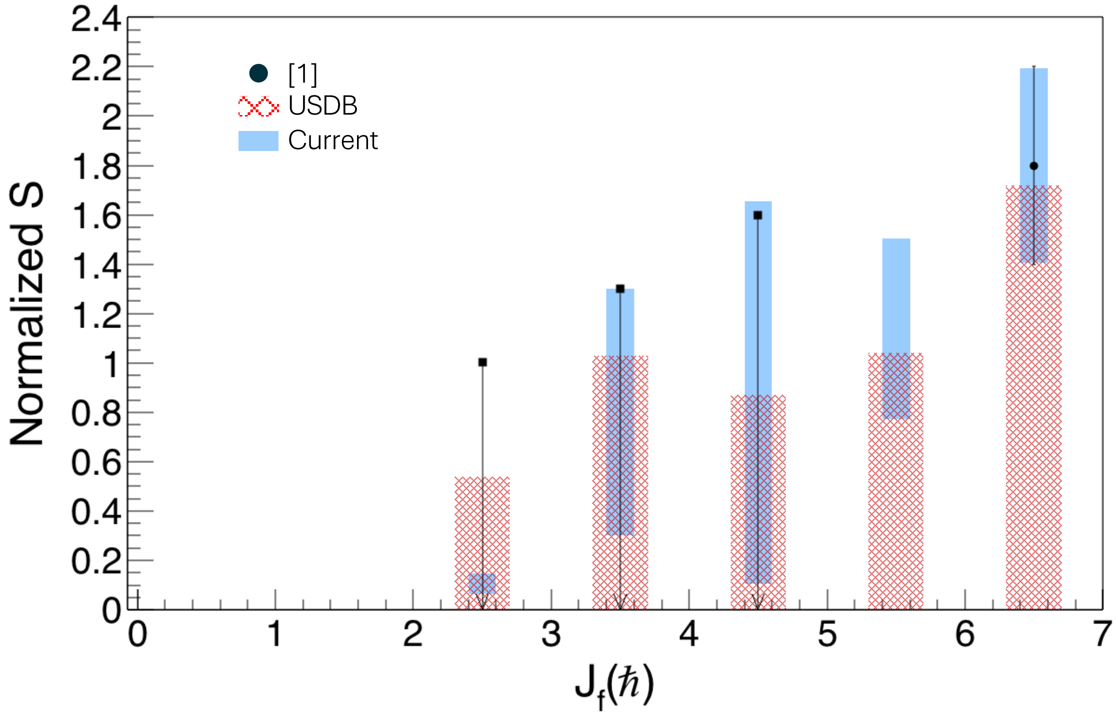
Results: Spectroscopic Factors



Transfer from the GS ⟹ Deformation-aligned (strong) coupling

[1] D. Santiago-Gonzalez et al., Phys. Rev. Lett. 120, 122503 (2018).
[2] R.L. Kozub et al., Phys. Rev. C 73, 044307 (2006).

Results: Spectroscopic Factors

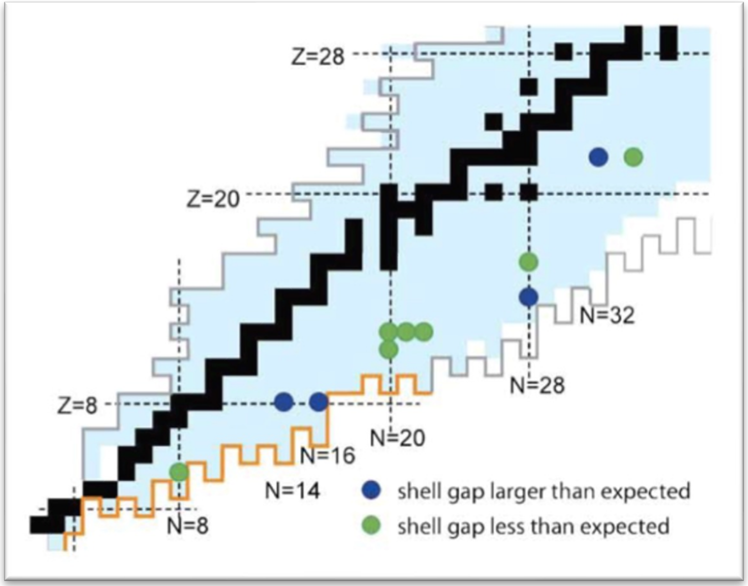


Transfer from the 5^+ Isomer ⟹ Decoupling

[1] D. Santiago-Gonzalez et al., Phys. Rev. Lett. 120, 122503 (2018).

Nilsson Model and Islands of Inversion

- “Classic” magic numbers are generally correct only for stable and near stable isotopes
- Experimental studies of exotic isotopes revealed changes in shell structure and collectivity and provided insight on the important role played by the NN central, tensor, and higher order forces in these changes.



R.V.F Janssens, Nature, 435, 2005.

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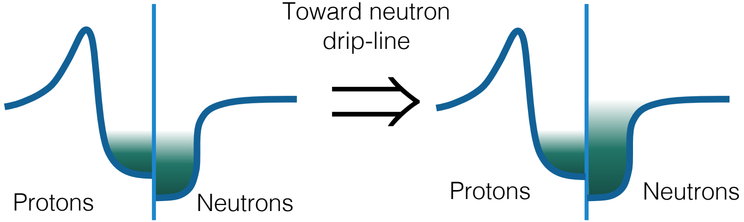

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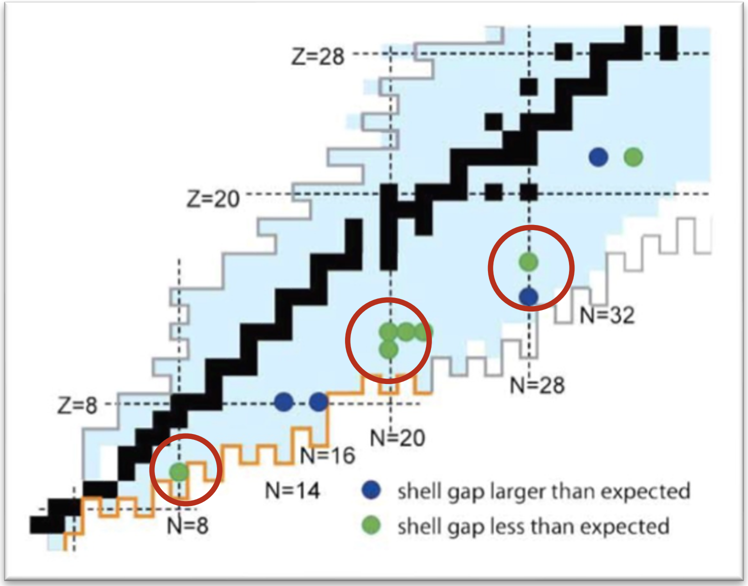

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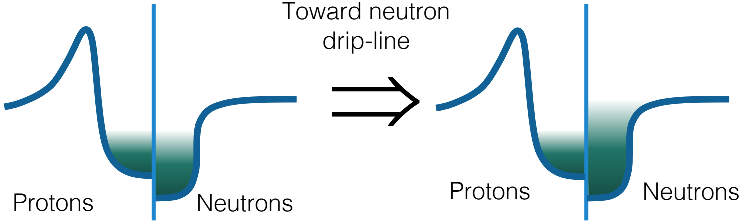
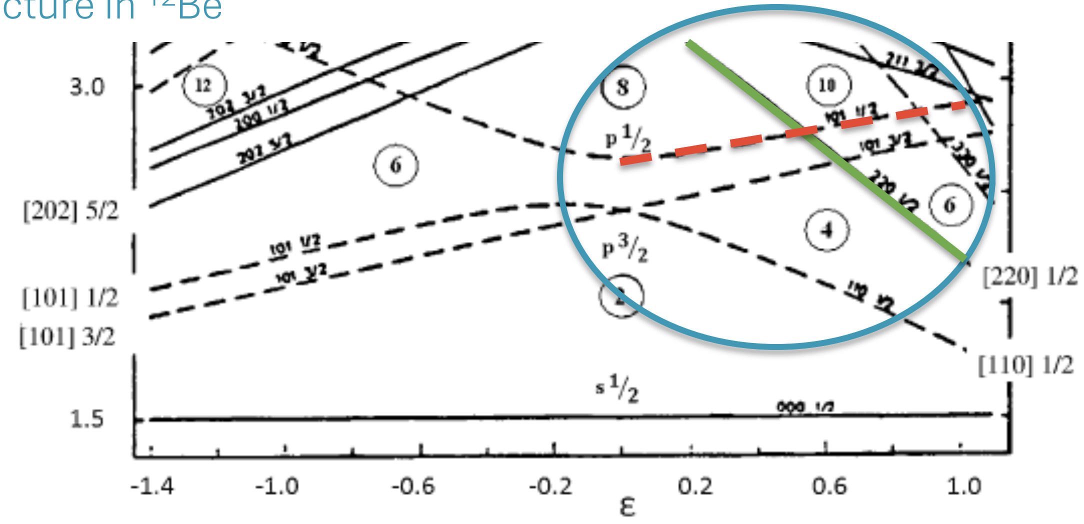


N=8 Island of Inversion

The Nilsson Picture in ^{12}Be

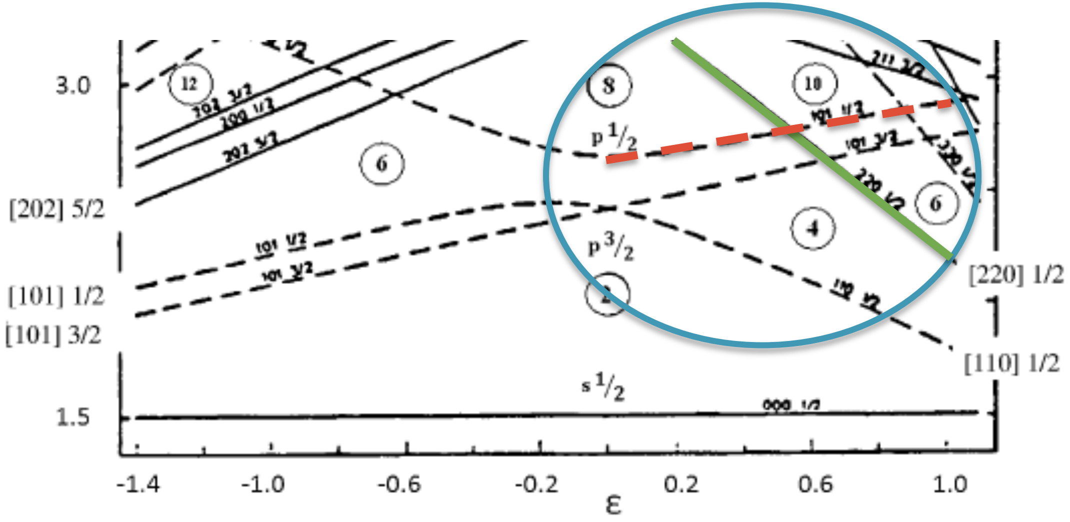


$$|\tfrac{1}{2}[220]\rangle = C_{1/2,0}|s_{1/2}\rangle + C_{3/2,2}|d_{3/2}\rangle + C_{5/2,2}|d_{5/2}\rangle$$

$$|\tfrac{1}{2}[101]\rangle = C_{1/2,1}|p_{1/2}\rangle + C_{3/2,1}|p_{3/2}\rangle$$

A. Bohr and B. R. Mottelson, Nuclear Structure Volume II
W. Von Oertzen, M. Freer, and Y. Kanada-En'yo Physics Reports 432, 43 (2006).
I. Hamamoto and S. Shimoura J. Phys. G: Nucl. Part. Phys. 34, 2715 (2007).

Direct Reaction Studies in the N=8 IoI



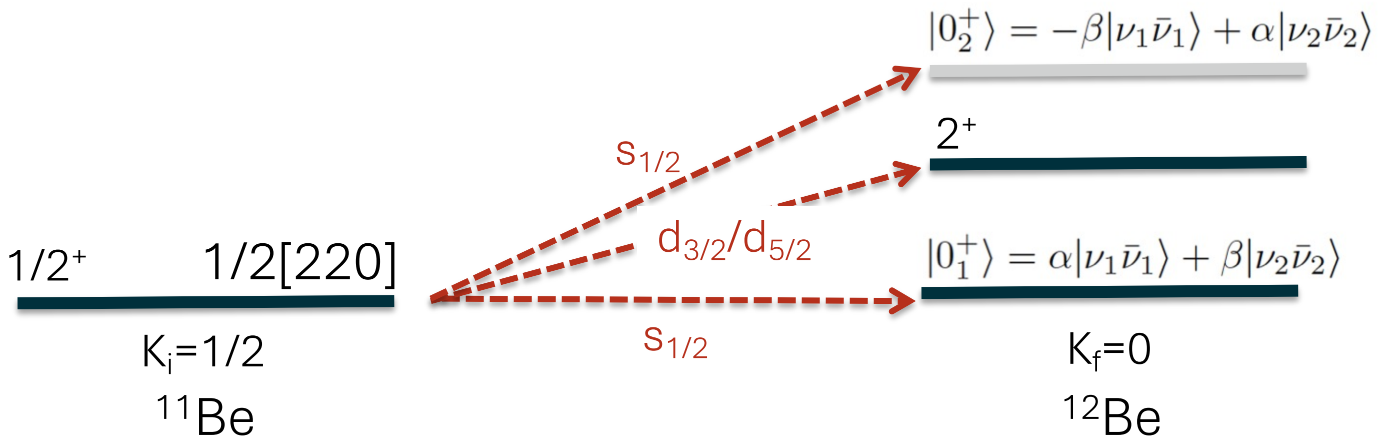
-1n Removal

- A. Navin et al., Phys. Rev. Lett. 85, 266 (2000).
- S. Pain et al., Phys. Rev. Lett. 96, 032502 (2006).

(d,p) – Neutron adding reaction

- R. Kanungo et al., Phys. Lett. B 682, 391 (2010).
- K. T. Schmitt et al., Phys. Rev. Lett. 108, 192701 (2012).
- J. G. Johansen et al., Phys. Rev. C 88, 044619 (2013).

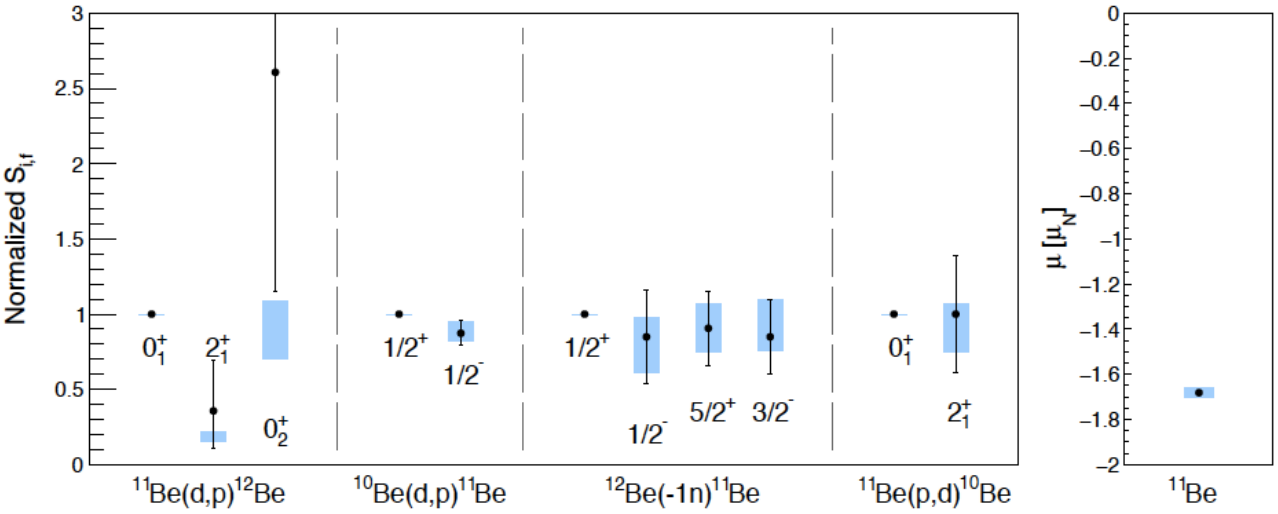
$^{11}Be(d,p)^{12}Be$ à la Nilsson



$$S_{i,f} = \frac{(2I_i+1)}{(2I_f+1)} g^2 \langle I_i j K_i \Delta K | I_f K_f \rangle^2 C_{j,\ell}^2 \langle \phi_f | \phi_i \rangle^2$$

Results in the N=8 Island of Inversion

Total of 12 relations connecting the experimental data to four unknown amplitudes which we determine from a chi^2-minimization procedure.
Weighted fit of the relative spectroscopic factor values with respect to the ground state transition for each of the data sets, and of the absolute value of the ^{11}Be ground-state magnetic moment.



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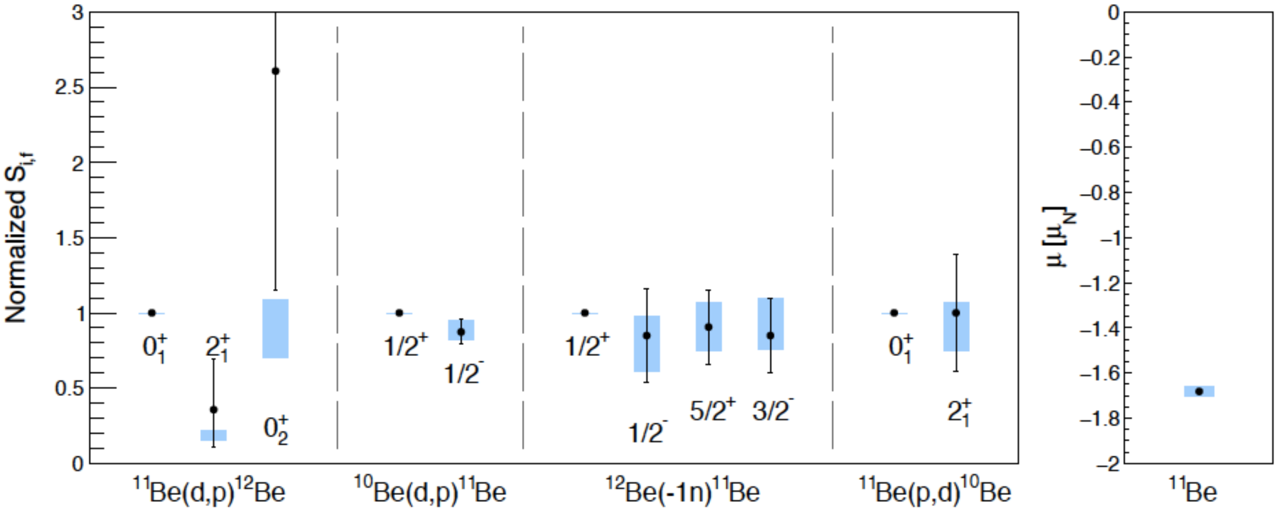
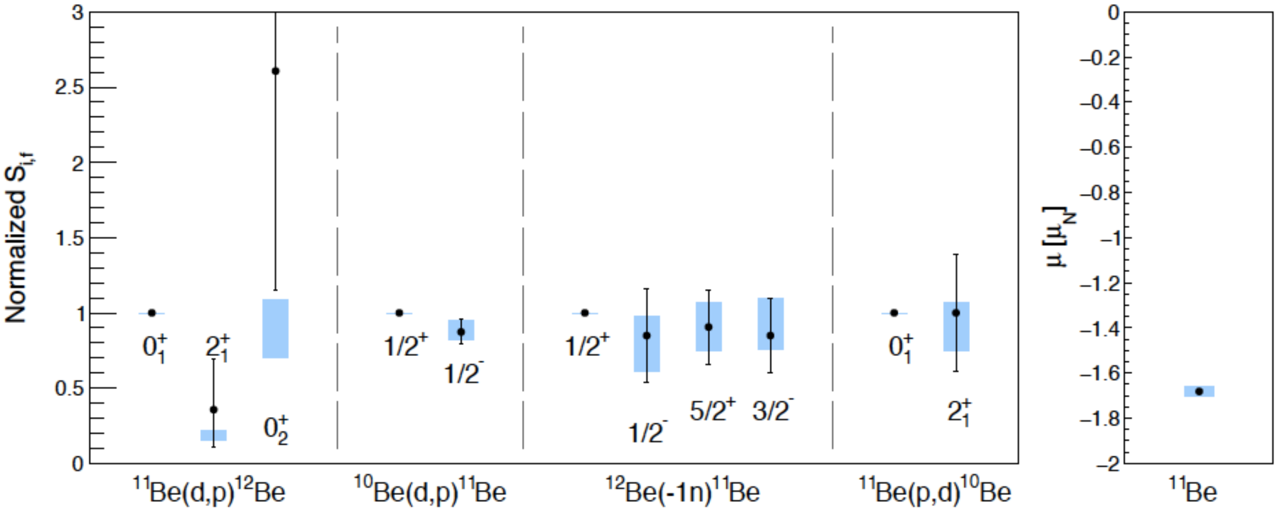


$$\left|\tfrac{1}{2}[220]\right\rangle = -0.72(3)|s_{1/2}\rangle - 0.09(2)|d_{3/2}\rangle + 0.69(2)|d_{5/2}\rangle$$

$$\left|\tfrac{1}{2}[101]\right\rangle = 0.68(4)|p_{1/2}\rangle + 0.73(3)|p_{3/2}\rangle$$

$$\alpha = 0.74(4) \text{ and } \beta = 0.68(4)$$

Results in the N=8 Island of Inversion



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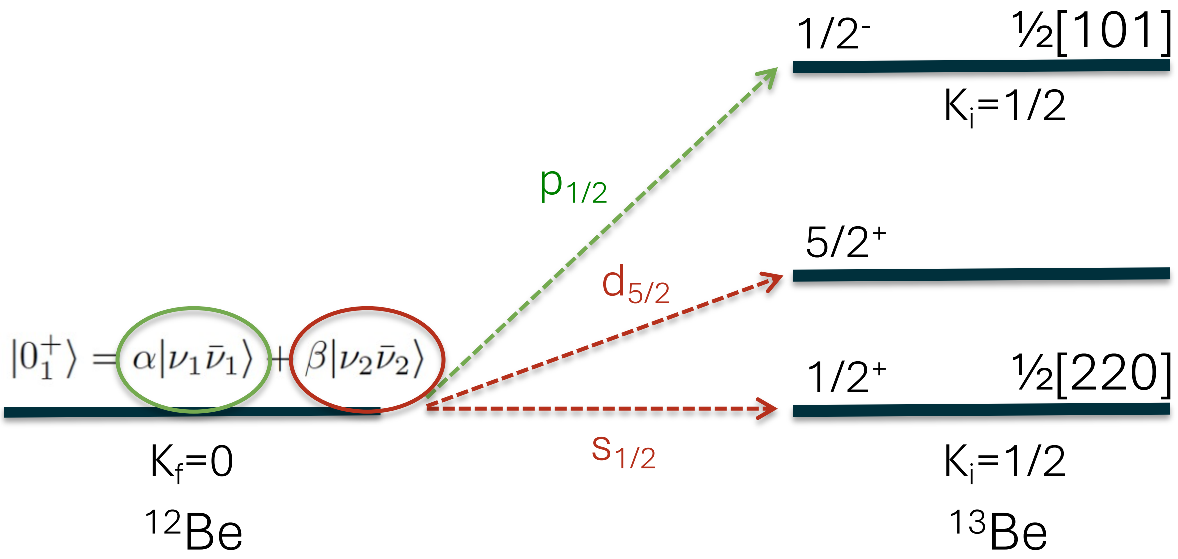
$$\alpha = 0.74(4) \text{ and } \beta = 0.68(4)$$

⇒ Good agreement with literature Nilsson coefficients

Hamamoto and Shimoura, J. Phys. G: Nucl. Part. Phys. **34**, 2715 (2007).

Predictions for $^{12}Be(d,p)^{13}Be$



ν_1 and ν_2

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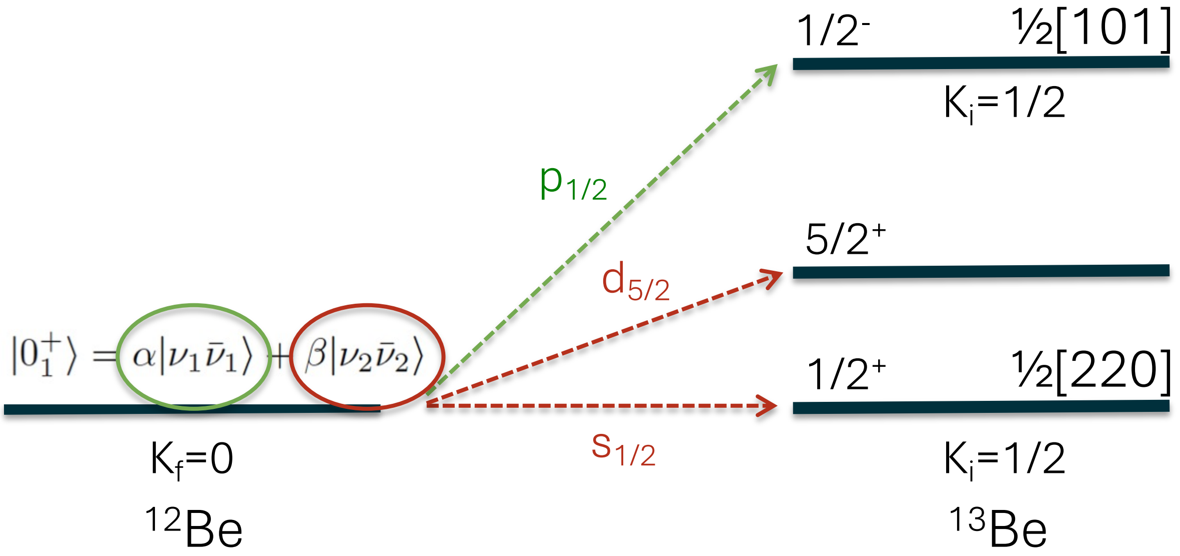
Predictions for ^{12}Be(d,p)^{13}Be



Initial State	Final State	Energy [MeV]	ℓ	Calculated $S_{i,f}$
^{12}Be	^{13}Be			
0_1^+	$\frac{1}{2}^+$	0.00	0	0.24(3)
	$\frac{5}{2}^+$	$\sim$1.8	2	0.07(1)
	$\frac{1}{2}^-$	0+x	1	0.25(4)

$$\left|\tfrac{1}{2}[220]\right\rangle = -0.72(3)|s_{1/2}\rangle - 0.09(2)|d_{3/2}\rangle + 0.69(2)|d_{5/2}\rangle$$

$$\left|\tfrac{1}{2}[101]\right\rangle = 0.68(4)|p_{1/2}\rangle + 0.73(3)|p_{3/2}\rangle$$

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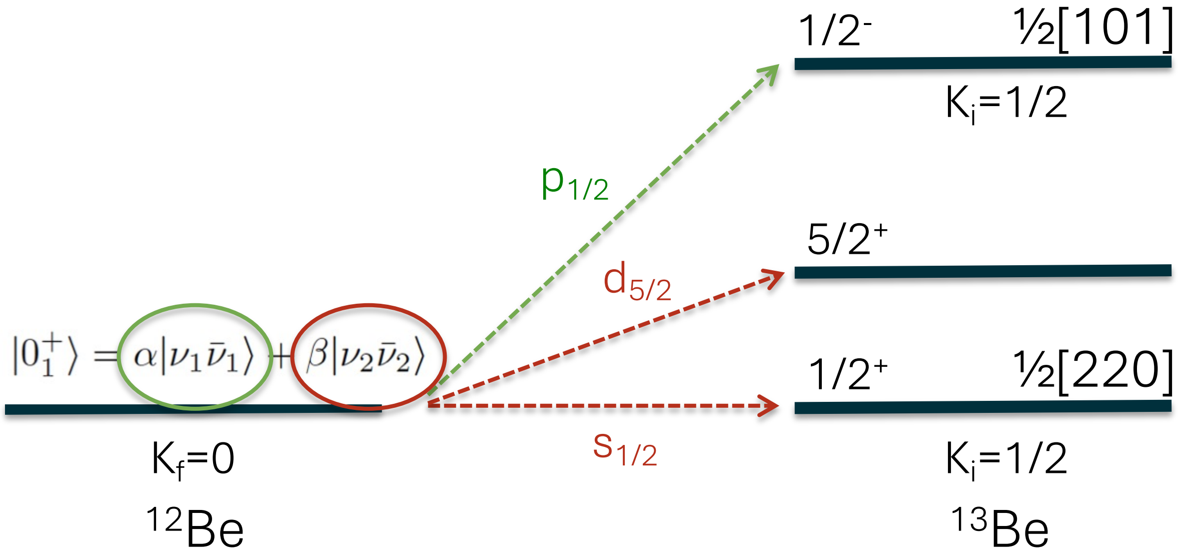


Initial State	Final State	Energy [MeV]	ℓ	Calculated $S_{i,f}$
^{12}Be	^{13}Be			
0_1^+	$\frac{1}{2}^+$	0.00	0	0.24(3)
	$\frac{5}{2}^+$	~1.8	2	0.07(1)
	$\frac{1}{2}^-$	0+x	1	0.25(4)

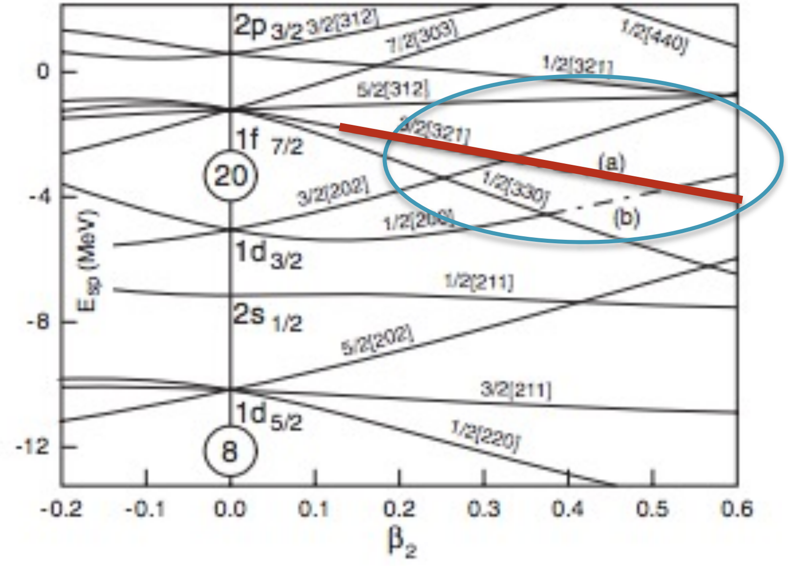
$$|\tfrac{1}{2}[220]\rangle = -0.72(3)|s_{1/2}\rangle - 0.09(2)|d_{3/2}\rangle + 0.69(2)|d_{5/2}\rangle$$

$$|\tfrac{1}{2}[101]\rangle = 0.68(4)|p_{1/2}\rangle + 0.73(3)|p_{3/2}\rangle$$

⇒ Recent results showed two dominant s and p-wave resonances

Kovoor *et al.*Phys. Rev. C **108**, 034601 (2023).

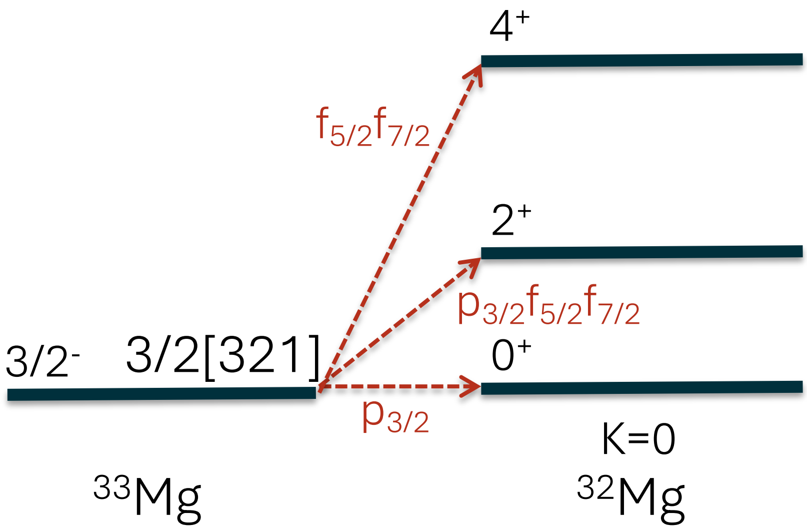
N=20 Island of Inversion: ^{33}Mg -1n removal à la Nilsson



Assume ground state of ^{33}Mg is the 3/2[321] neutron Nilsson level.

$$|\tfrac{3}{2}[321]\rangle = C_{3/2,1}|p_{3/2}\rangle + C_{5/2,3}|f_{5/2}\rangle + C_{7/2,3}|f_{7/2}\rangle$$

N=20 Island of Inversion: ^{33}Mg -1n removal à la Nilsson

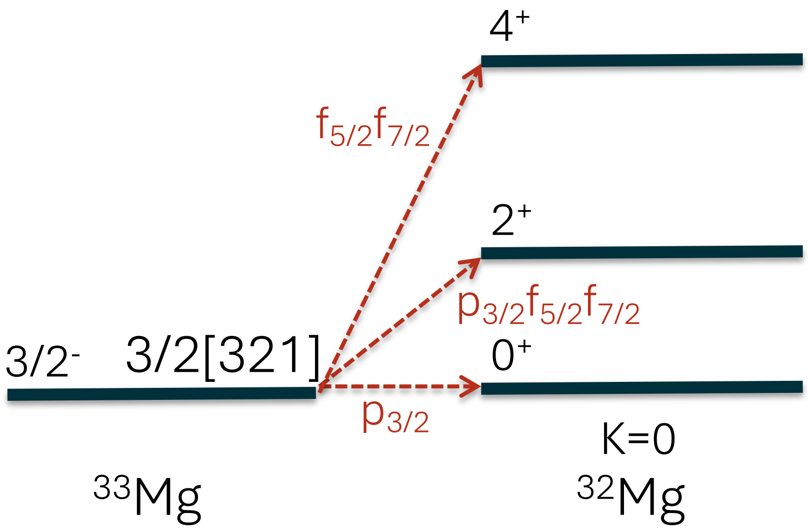


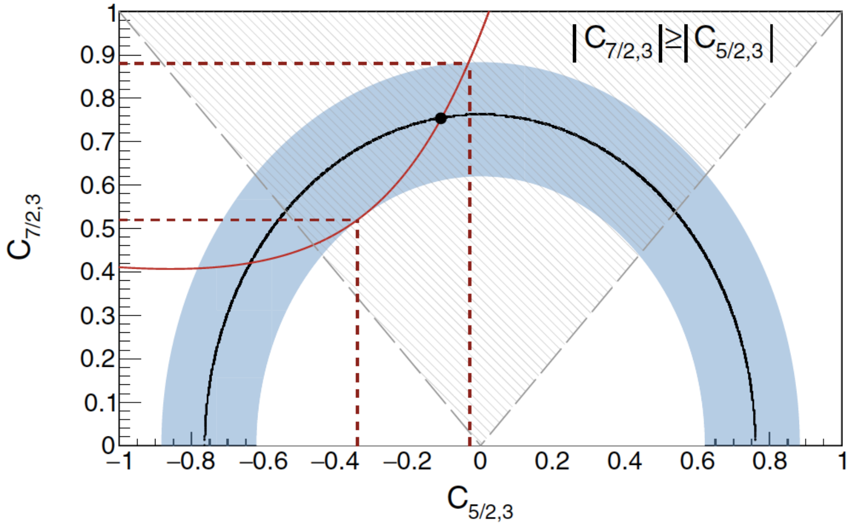
Final state	Energy [MeV]	ℓ	Experimental $S_{j,\ell}$ [21]	Experimental $S_{j,\ell}$ [22]	Calculated $S_{j,\ell}$ Nilsson	Calculated $S_{j,\ell}$ Empirical
0^+	0.00	1	$0.6^{+0.3}_{-0.5}$	0.19 ± 0.1	0.05	0.24
2^+	0.89	1	$0.5^{+0.7}_{-0.3}$		0.05	0.24
		3	$0.5^{+0.2}_{-0.5}$		0.34	0.18
4^+	2.32	3			0.55	0.33

“Standard” Nilsson amplitudes do not reproduce experimental spectroscopic factors – fit amplitudes to the data…

But spectroscopic factors alone cannot uniquely constrain the Nilsson amplitudes – include the ground state magnetic moment of ^{33}Mg.

N=20 Island of Inversion: ^{33}Mg -1n removal à la Nilsson

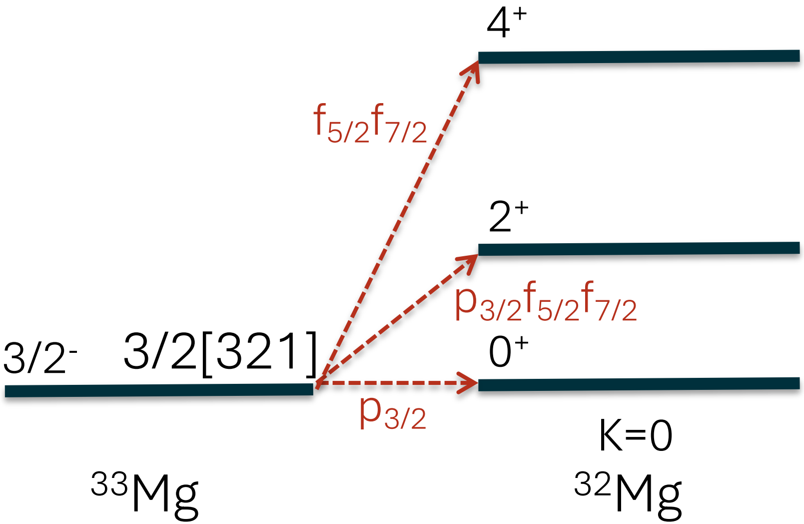


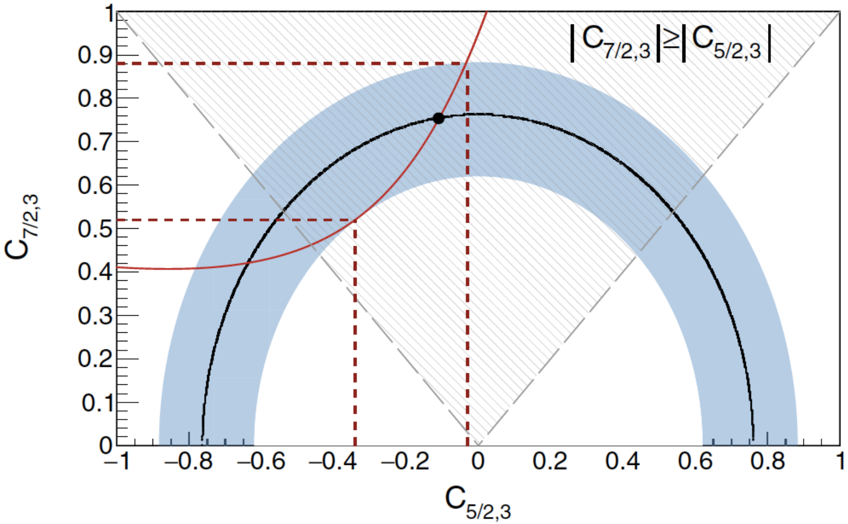


$$\left|\frac{3}{2}[321]\right\rangle = (-0.65 \pm 0.15)|p_{3/2}\rangle + \left(0.75^{+0.13}_{-0.23}\right)|f_{7/2}\rangle + \left(-0.12^{+0.08}_{-0.22}\right)|f_{5/2}\rangle$$

$$\mu = \frac{3}{5}(g_s\langle s_3\rangle + g_R)$$

N=20 Island of Inversion: ^{33}Mg -1n removal à la Nilsson



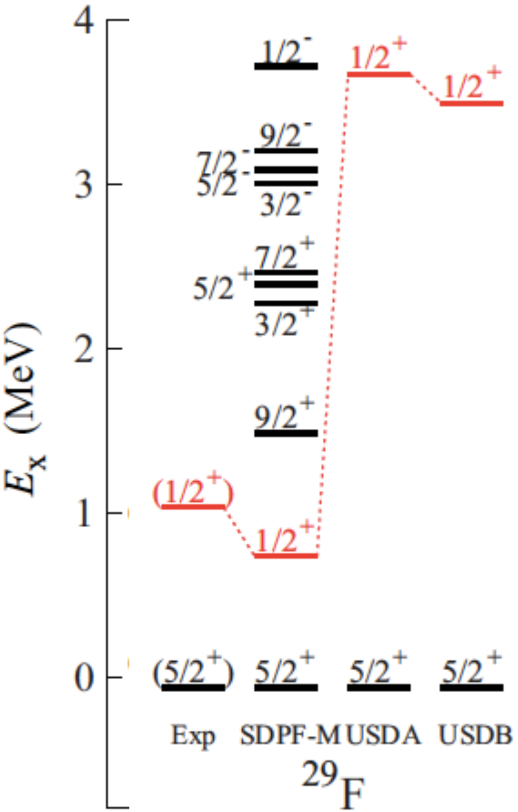


$$\left|\tfrac{3}{2}[321]\right\rangle = (-0.65 \pm 0.15)|p_{3/2}\rangle + (0.75^{+0.13}_{-0.23})|f_{7/2}\rangle + (-0.12^{+0.08}_{-0.22})|f_{5/2}\rangle$$

$$\mu = \frac{3}{5}(g_s\langle s_3\rangle + g_R)$$

⇒ Consistent with narrowed N=20 gap!

The Southern Shore at N=20: ^{29}F



PHYSICAL REVIEW C **95**, 041301(R) (2017)

Low-*Z* shore of the "island of inversion" and the reduced neutron magicity toward ^{28}O

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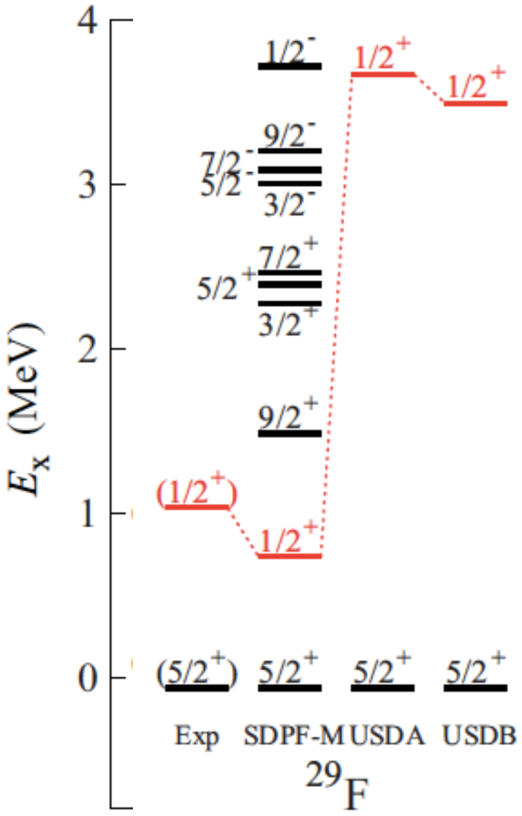
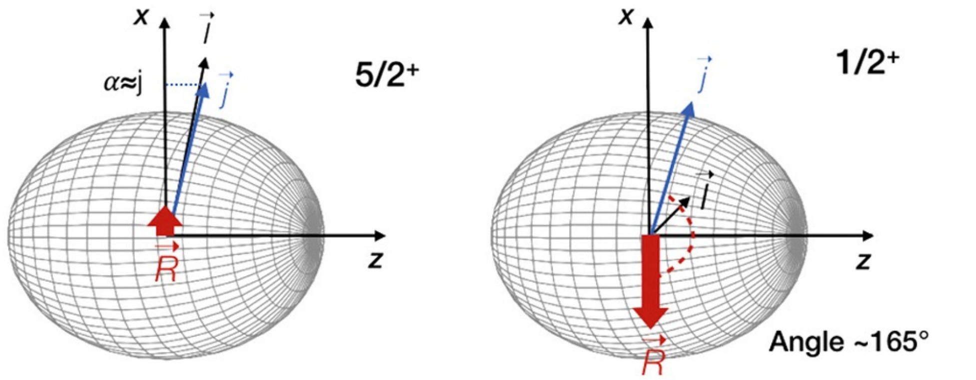
The Southern Shore at N=20: ^{29}F



PRM Hamiltonian

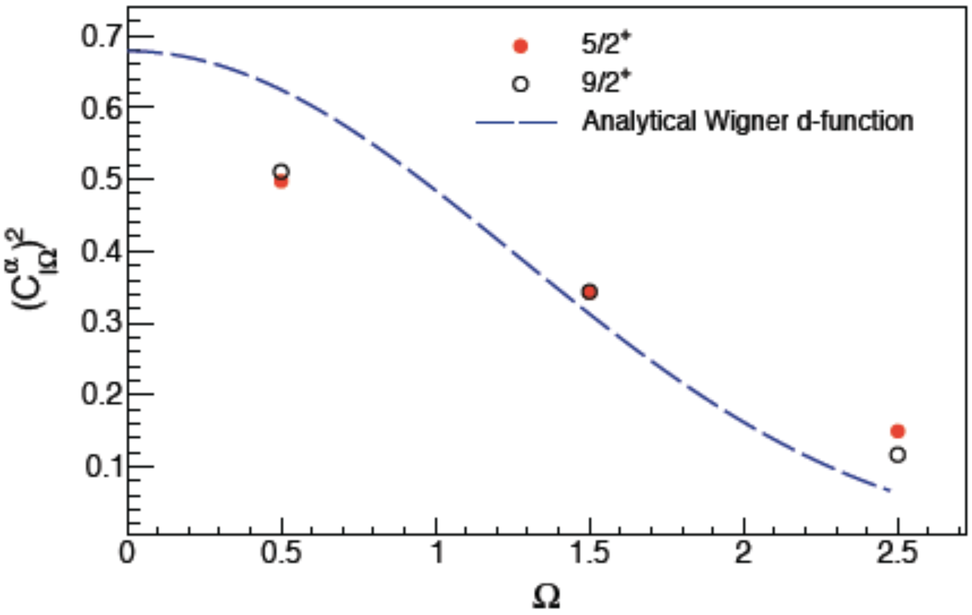
$$H = E_{\Omega} + \frac{\hbar^2}{2\mathscr{I}} I(I+1) + H_C$$

$$H_C = -\frac{\hbar^2}{2\mathscr{I}} (I_+ j_- + I_- j_+)$$



With modest deformation ($\epsilon_2 \sim 0.15$), Coriolis matrix elements dominate over intrinsic level spacings and a rotation-aligned (decoupled band) is expected.

Structure of ^{29}F: PRM Solution

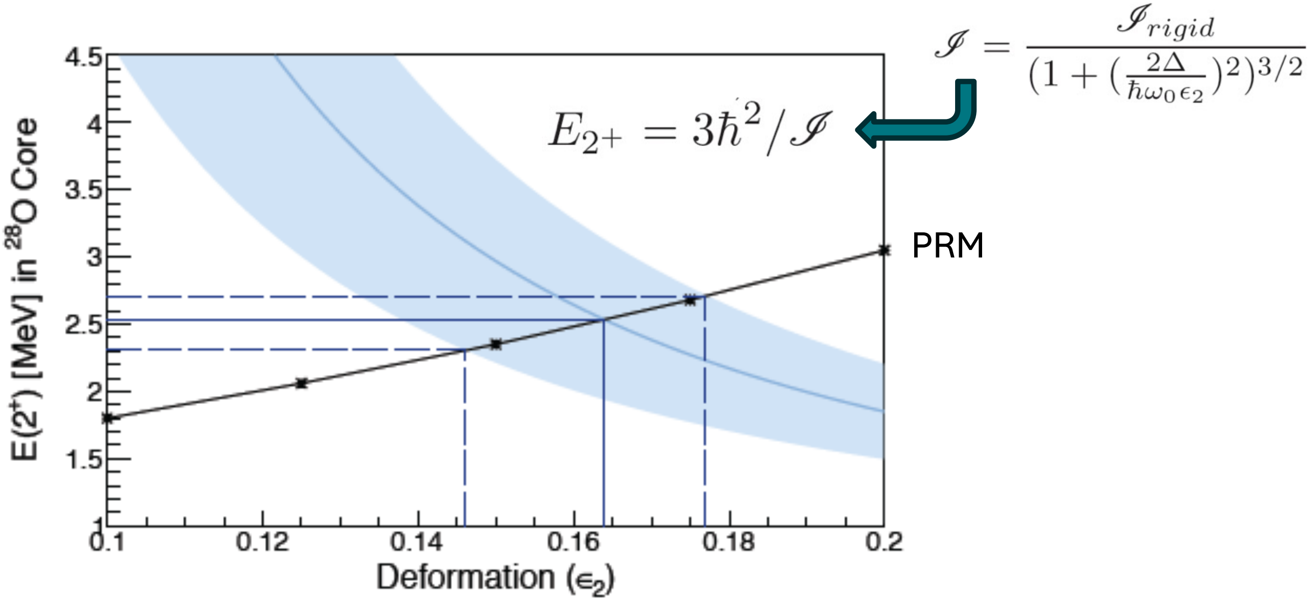


$$|I,\alpha\rangle = \sum_{\Omega_p=1/2}^{5/2} C^{\alpha}_{I\Omega_p} |I,\Omega_p\rangle$$

$$C^{\alpha}_{I\Omega} \approx d^{j}_{\alpha,\Omega}(\pi/2)$$

Decoupled band

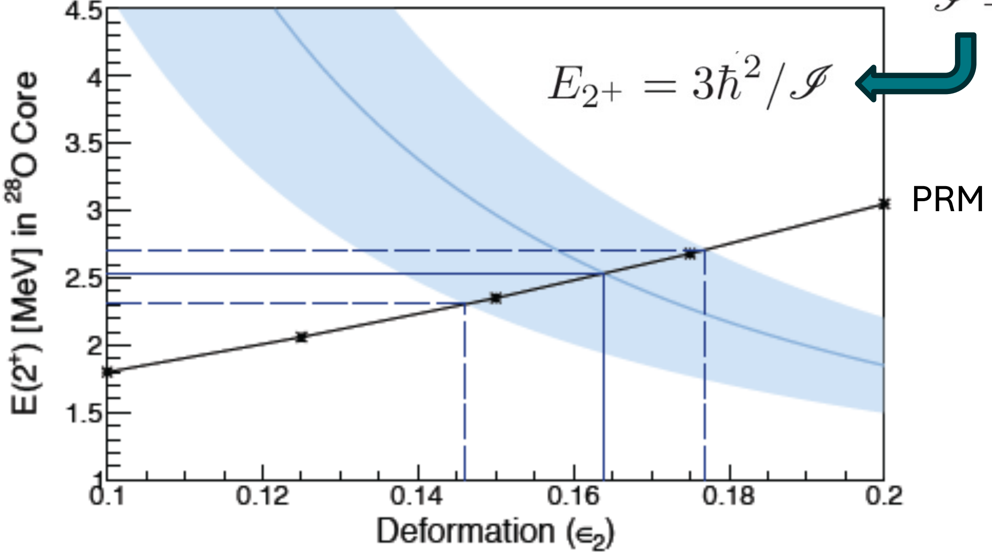
Structure of ^{29}F: PRM Solution



The ½$^+$ excited state in ^{29}F, in the decoupled limit, is proportional to the rotational energy of the core, or E(2^+) in ^{28}O.

Structure of ^{29}F: PRM Solution

$$\mathscr{I} = \frac{\mathscr{I}_{rigid}}{(1 + (\frac{2\Delta}{\hbar\omega_0 \epsilon_2})^2)^{3/2}}$$



$$\frac{\sigma^{\ell=0}_{1/2^+}}{\sigma^{\ell=2}_{5/2^+}} \approx \frac{\sum_{i=1}^{3} C_{1/2,1/2_i} \cdot c_{s_{1/2},1/2_i} \cdot u_{1/2_i}}{\sum_{\Omega_p=1/2}^{5/2} C_{5/2,\Omega_p} \cdot c_{d_{5/2},\Omega_p} \cdot u_{\Omega_p}} \approx 13\%$$

⇒ Measurement = 11(3)%

P. Doornenbal, et al., Phys. Rev. C **95**, 041301(R) (2017).

Summary

- The Nilsson model offers the ”intuition” of the spherical shell model for deformed systems
- The Nilsson model is impressively successful in describing light and medium-mass deformed nuclear systems – in nuclei as light as N=8 this ‘collective’ description captures the physics, reproducing experimental nucleon transfer cross-sections and spectra
- The Islands of Inversion at N=20 and N=28 have also been considered, including ^{29}F and ^{33}Mg – ^{43}P(-1p)^{42}Si was also investigated showing good agreement with data
- Application at N=40 and the He isotopes is underway…stay tuned

Acknowledgments

PHYSICAL REVIEW C **105**, 014309 (2022)

Structure of ^{43}P and ^{42}Si in a two-level shape-coexistence model

A. O. Macchiavelli,[1] H. L. Crawford,[1] C. M. Campbell,[1] R. M. C
P. Fallon,[1] I. Y. Lee,[1] A. Gade,[2] A. Poves,[3] and E.
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Structure of ^{29}F in the rotation-aligned coupling scheme of the particle-rotor model

A.O. Macchiavelli *, H.L. Crawford, P. Fallon, C.M. Campbell, R.M. Clark, M. Cromaz, M.D. Jones, I.Y. Lee, M. Salathe

PHYSICAL REVIEW C **97**, 011302(R) (2018)
Rapid Communications

Analysis of spectroscopic factors in ^{11}Be and ^{12}Be in the Nilsson strong-coupling limit

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PHYSICAL REVIEW C **96**, 054302 (2017)

Spectroscopic factors in the $N = 20$ island of inversion: The Nilsson strong-coupling limit

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[2]*Department of Physic*

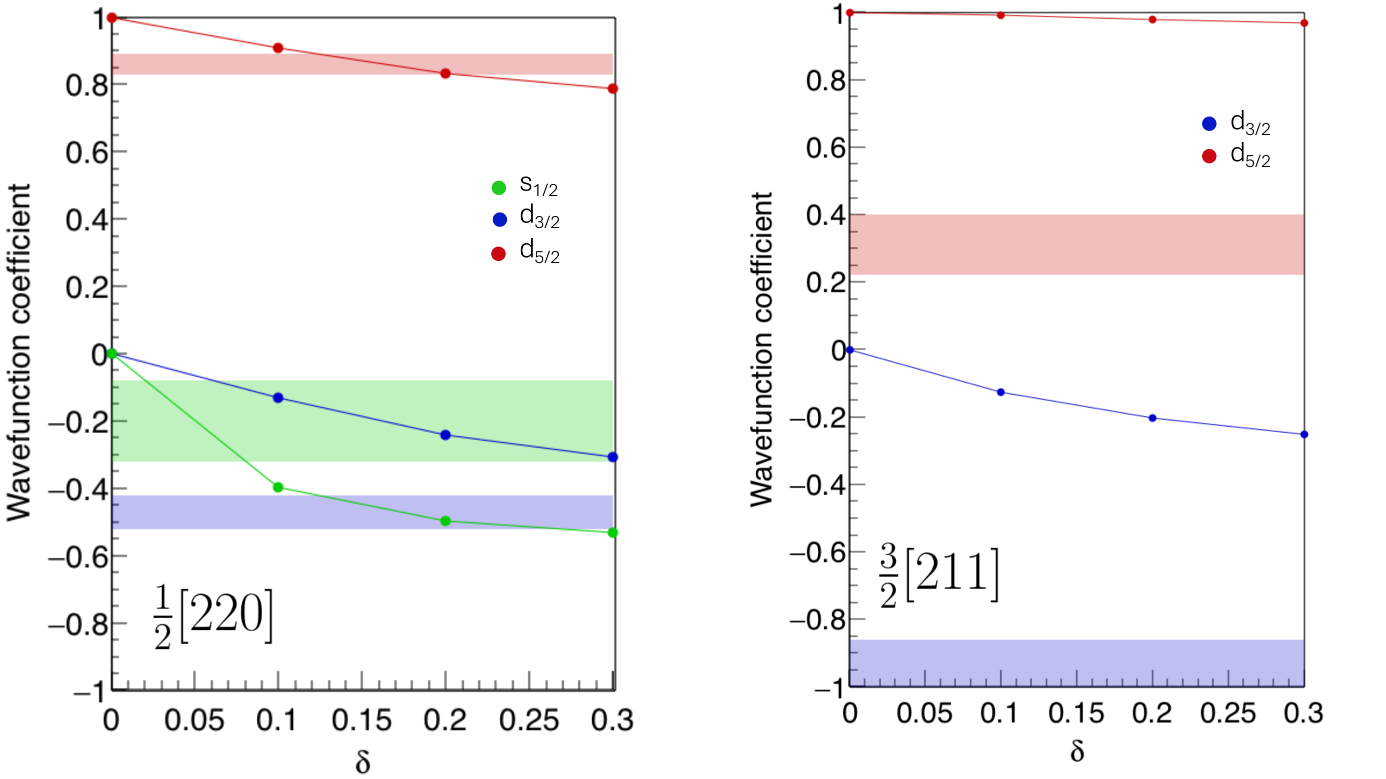
PHYSICAL REVIEW C **101**, 044319 (2020)

Analysis of the $^{18}F^{g,m}(d,p)^{19}F$ reactions in the rotational model

A. O. Macchiavelli, H. L. Crawford, P. Fallon, I. Y. Lee, R. M. Clark, C. M. Campbell, M. Cromaz, C. Morse, and C. Santamaria
Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

Work supported by U.S. DOE, Office of Science under contract number DE-AC02-05CH11231.

Results: Nilsson Amplitudes for ^{18}F



B. E. Chi, Nuclear Physics 83, 97 (1966).

Results for N=8 Island of Inversion

Initial state	Final state	Energy [MeV]	ℓ	Experimental $S_{i,f}$ [17]	[23]	[24,25]	[16][a]	Theoretical $S_{i,f}$ Present work	[17][b]	[18]	[15]	[20]	[24,25][c]	[14]	[13]
^{11}Be	^{12}Be														
$\frac{1}{2}^+$	0_1^+	0.00	0	1				1	1	1					
	2_1^+	2.11	2	0.36(29)				0.19(3)	1.41 [1.41]	0.9					
	0_2^+	2.24	0	2.61(134)				0.84(17)	1.84 [1.55]	0.47					
^{10}Be	^{11}Be														
0^+	$\frac{1}{2}^+$	0.00	0		1			1			1	1			
	$\frac{1}{2}^-$	0.32	1		0.87(8)			0.90(7)			1.24	0.97			
^{12}Be	^{11}Be														
0^+	$\frac{1}{2}^+$	0.00	0			1		1					1		
	$\frac{1}{2}^-$	0.32	1			0.82(22)		0.75(17)					0.84 [0.69]		
	$\frac{5}{2}^+$	1.78	2			0.86(29)		0.92(15)					0.80 [0.8]		
	$\frac{3}{2}^-$	2.69	1			0.71(26)		0.87(18)							
^{11}Be	^{10}Be														
$\frac{1}{2}^+$	0_1^+	0.00	0				1	1			1	1		1	1
	2_1^+	3.4	2				1.0(2)	0.92(15)			0.26	0.24		0.25	0.73