

## Applications of the *Ab Initio* No-Core Shell Model

**Celebrating 75 Years of the Nuclear Shell Model and Maria Goeppert-Mayer**

ANL, July 19-21, 2024

Petr Navratil

TRIUMF



# Outline

- Introduction – Shell Model vs. No-Core Shell Model
- *Ab initio* nuclear theory – no-core shell model (NCSM)
- Early NCSM applications - Okubo – Lee – Suzuki (OLS) renormalization
- Recent NCSM applications - Similarity Renormalization Group (SRG) renormalization
- No-Core Shell Model with Continuum (NCSMC) - Unified description of bound and unbound states
- Conclusions

# Shell Model

# No-Core Shell Model (NCSM)

Solving many-nucleon Schroedinger equation

$$H\psi_n = E_n\psi_n$$

Basis expansion method

Harmonic oscillator (HO) or other  
Slater determinant (SD) basis  
Single shell valence space

Relative-coordinate or SD HO  
basis truncated with  $N_{\max}$   
Many HO shells

## Interaction

Effective NN interaction fitted to  
many-nucleon data – CK, USD,  
KB3...

Chiral NN+3N interaction fitted to few-  
body systems ( $NN, A=3,4$ )  
- bare or renormalized by SRG  
(earlier work - Okubo-Lee-Suzuki)

Predicts nuclear structure properties of nuclei

Across nuclear chart

Light nuclei ( $A \leq 20$ )

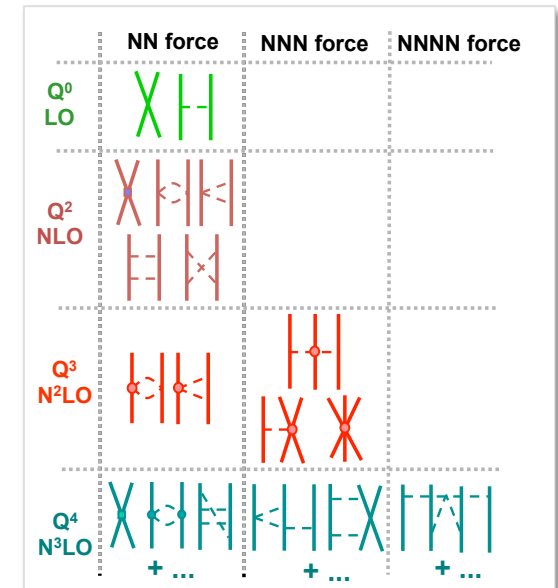
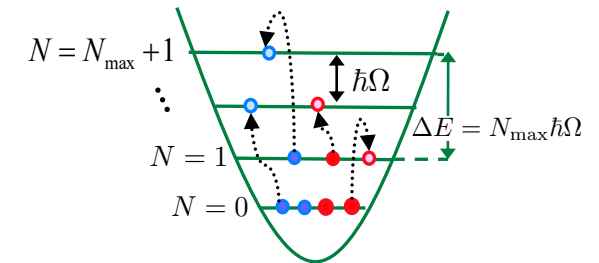
Extendable to describe scattering &  
reactions – NCSM with continuum

Review

*Ab initio* no core shell model

Bruce R. Barrett<sup>a</sup>, Petr Navrátil<sup>b</sup>, James P. Vary<sup>c,\*</sup>

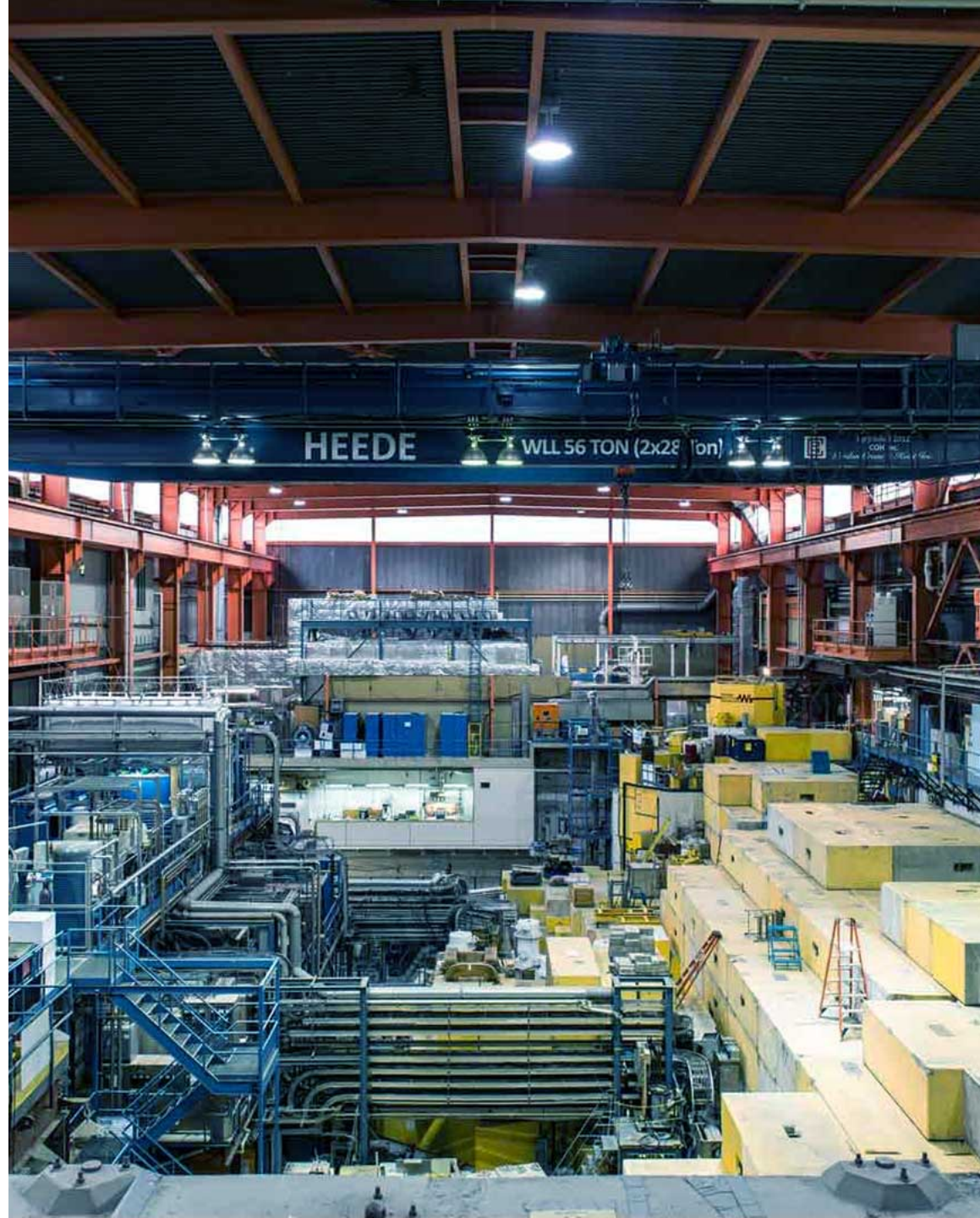
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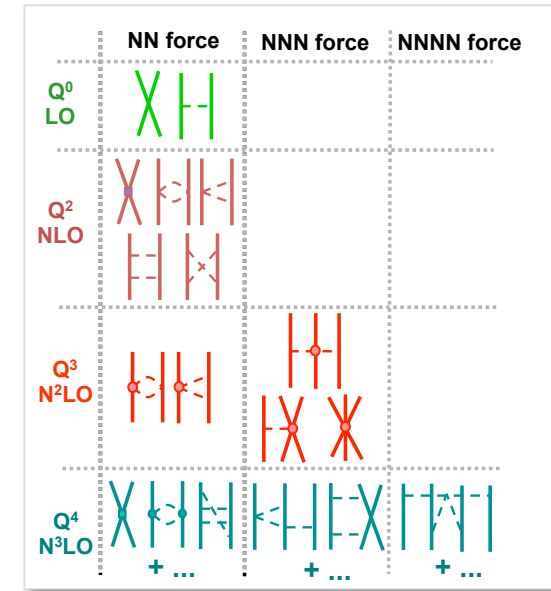
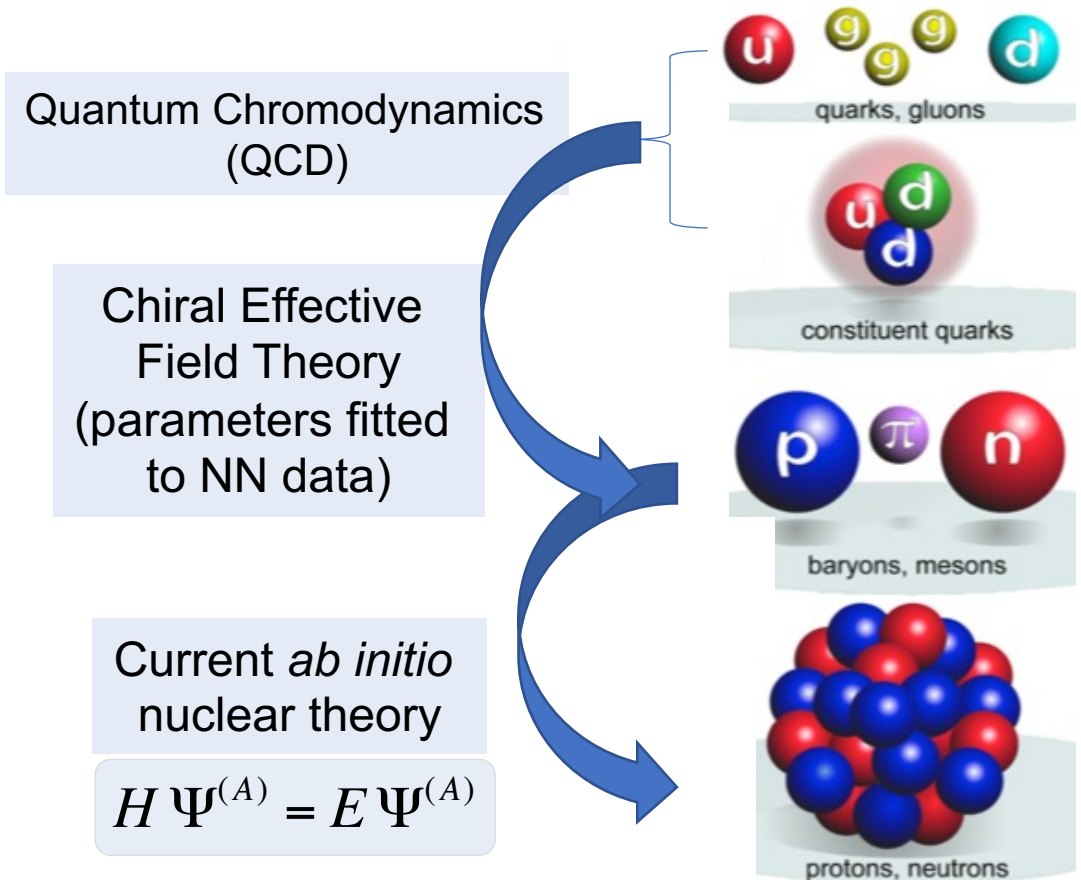
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \lambda \end{matrix} \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (a) \\ \nu \end{matrix} \right\rangle_{(A-a)}$$

*Ab initio* nuclear theory -  
no-core shell model (NCSM)

2024-07-21



# First principles or *ab initio* nuclear theory





Review

*Ab initio* no core shell modelBruce R. Barrett<sup>a</sup>, Petr Navrátil<sup>b</sup>, James P. Vary<sup>c,\*</sup>

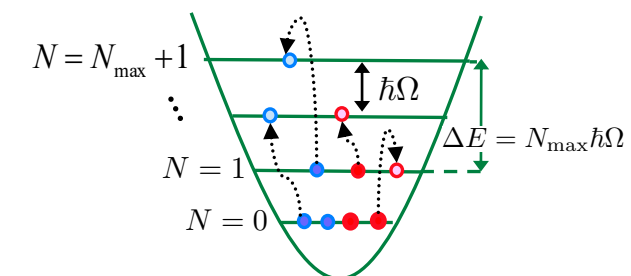
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## Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)

- Basis expansion method
  - Harmonic oscillator (HO) basis truncated in a particular way ( $N_{\max}$ )
  - Why HO basis?
    - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 –  $^4\text{He}$ ,  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ )
    - Equivalent description in relative (Jacobi)-coordinate and Slater determinant basis
- Short- and medium range correlations
- Bound-states, narrow resonances



NCSM



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})$$

$$\Psi_{SD}^A = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj}^{SD} \Phi_{SDNj}^{HO}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$




Review

*Ab initio* no core shell modelBruce R. Barrett<sup>a</sup>, Petr Navrátil<sup>b</sup>, James P. Vary<sup>c,\*</sup>


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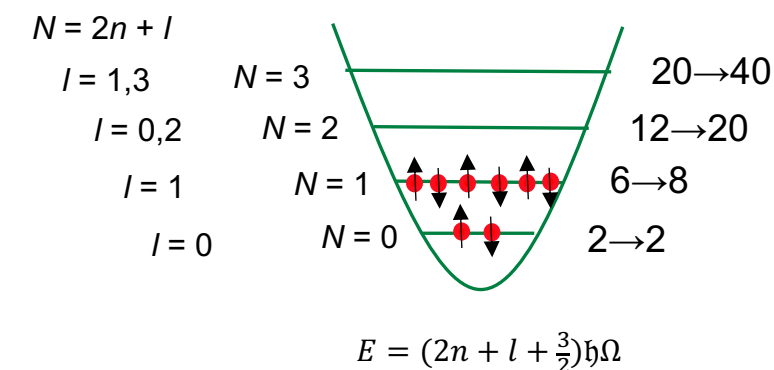
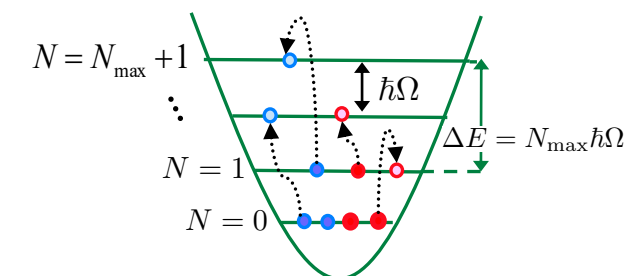
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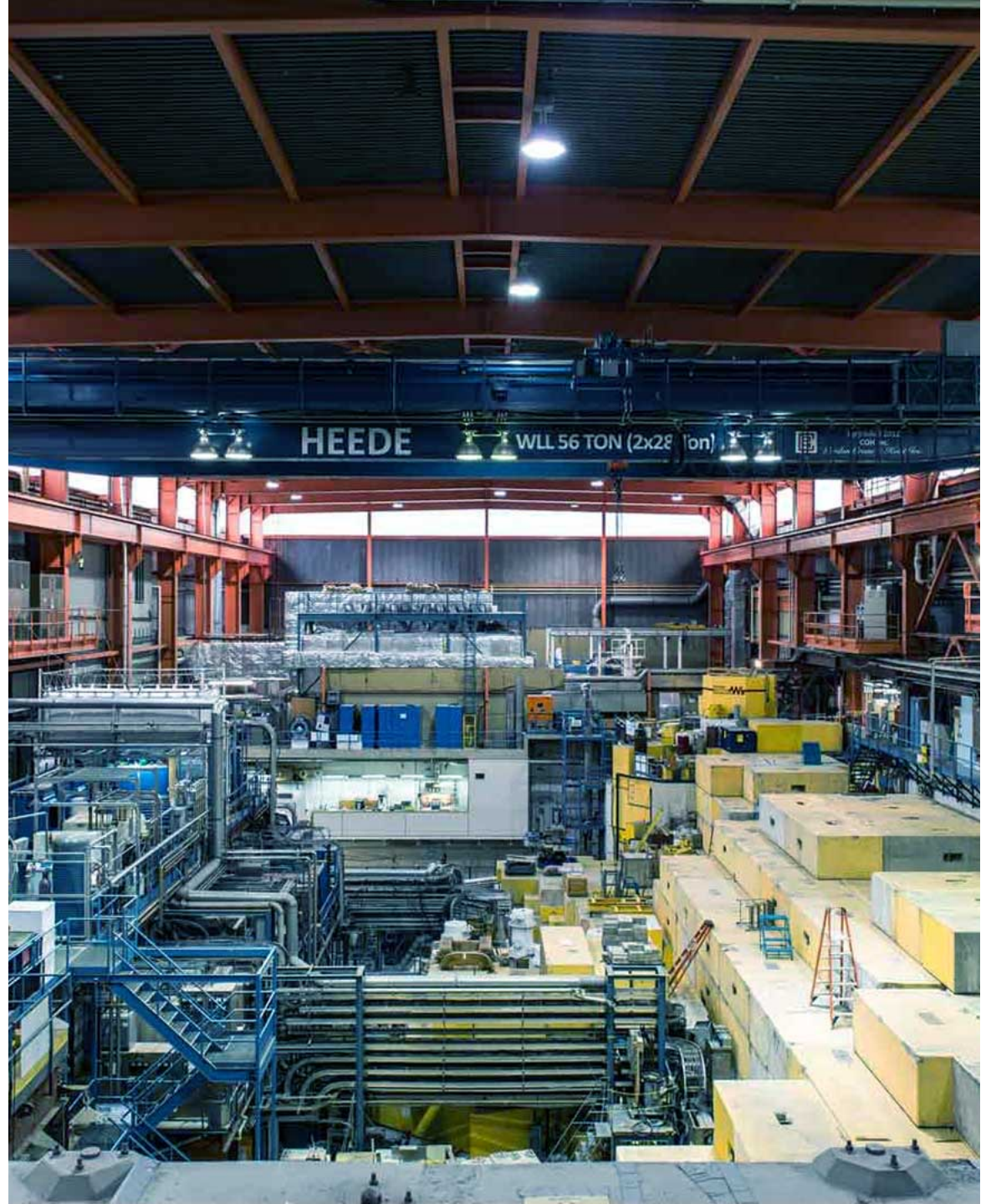


NCSM



Early NCSM applications -  
Okubo – Lee – Suzuki (OLS)  
renormalization  
(calculations not variational)

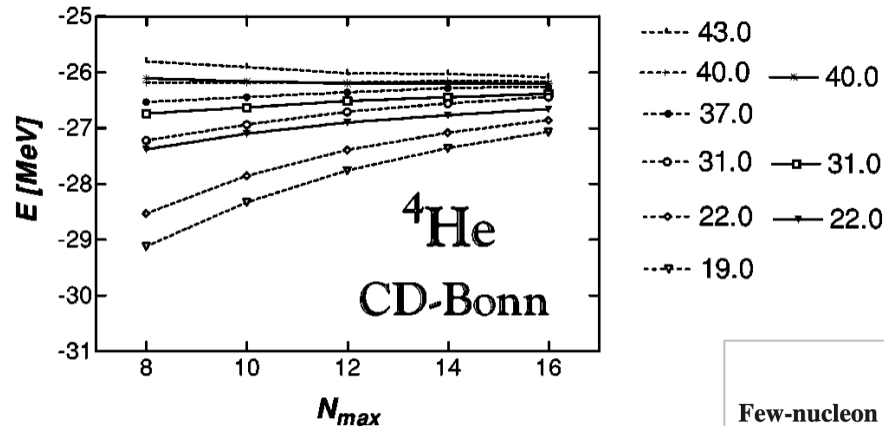
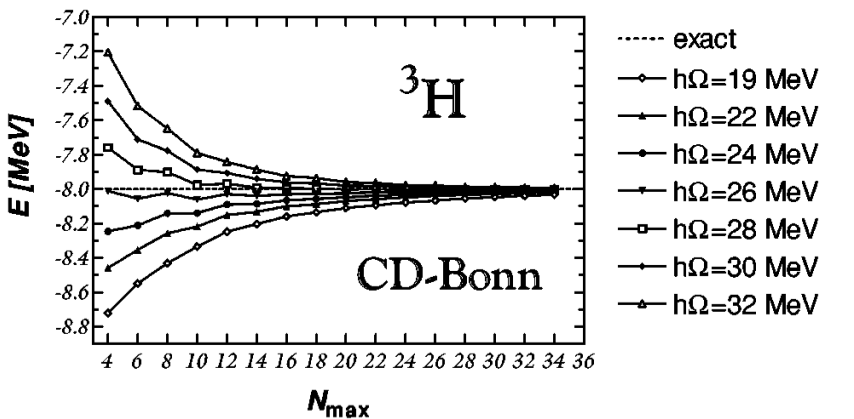
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# NCSM early developments

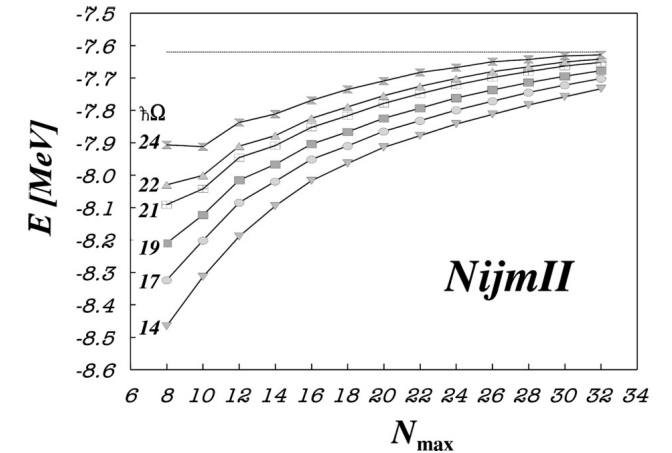
- Confirmation that NCSM calculations of the  $^3\text{H}$  gs energy reproduce Faddeev method results
- Later, the NCSM  $^4\text{He}$  gs energy prediction with the CD-Bonn potential was confirmed by Faddeev-Yakubovsky calculations
  - Jacobi-coordinate HO basis
  - Okubo-Lee-Suzuki effective interaction



PHYSICAL REVIEW C VOLUME 57, NUMBER 2 FEBRUARY 1998

**Shell-model calculations for the three-nucleon system**

P. Navrátil\* and B. R. Barrett  
 Department of Physics, University of Arizona, Tucson, Arizona 85721  
 (Received 21 May 1997)



NCSM is  
an *ab initio*  
method

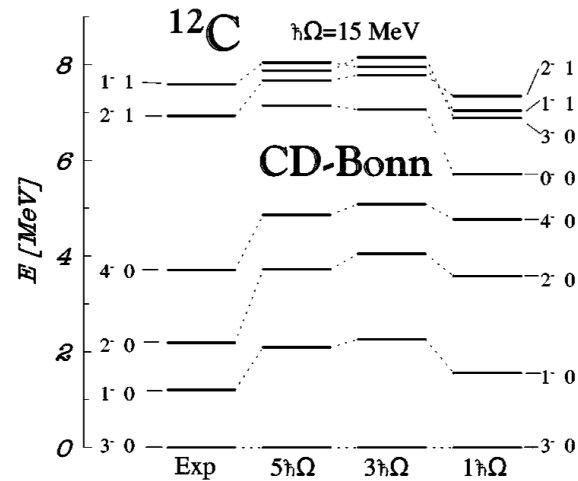
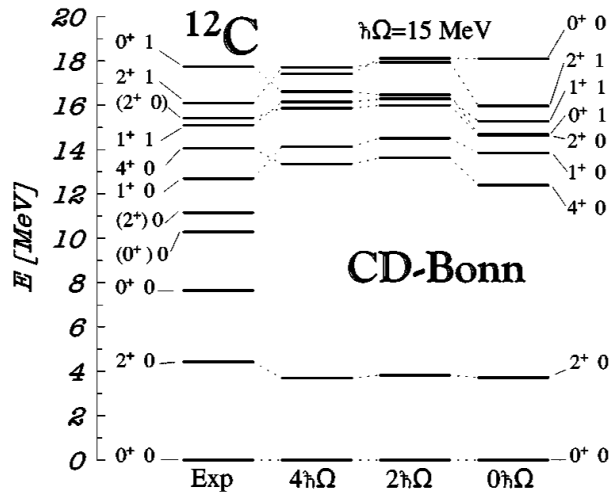
PHYSICAL REVIEW C, VOLUME 61, 044001

**Few-nucleon systems in a translationally invariant harmonic oscillator basis**

P. Navrátil,<sup>1,2</sup> G. P. Kamuntavičius,<sup>1,3,4</sup> and B. R. Barrett<sup>1</sup>

# NCSM early developments

- Structure of  $^{12}\text{C}$ 
  - Energies of states and other properties of a complex nucleus can be predicted from an *ab initio* approach
  - Slater-Determinant HO basis
  - Okubo-Lee-Suzuki effective interaction



VOLUME 84, NUMBER 25      PHYSICAL REVIEW LETTERS      19 JUNE 2000

**Properties of  $^{12}\text{C}$  in the *Ab Initio* Nuclear Shell Model**

P. Navrátil,<sup>1,2</sup> J. P. Vary,<sup>3</sup> and B. R. Barrett<sup>1</sup>

PHYSICAL REVIEW C, VOLUME 62, 054311

**Large-basis *ab initio* no-core shell model and its application to  $^{12}\text{C}$**

P. Navrátil,<sup>1,2,\*</sup> J. P. Vary,<sup>3</sup> and B. R. Barrett<sup>1</sup>

Physical Review C 50<sup>th</sup> Anniversary Milestones

**50 YEARS PHYSICAL REVIEW C**

This year, 2020, is the 50<sup>th</sup> anniversary of *Physical Review C*, which evolved from a section of its parent journal, *The Physical Review*, to one of the most read and trusted journals for nuclear physics. As part of the anniversary celebration, we are putting together a collection of milestone papers that remain central to developments in the field of nuclear physics. These papers announce major discoveries or open up new avenues of research. They would not have come to our journal, had the community not trusted and upheld the top-shelf quality of what PRC has traditionally published and intends to publish in the future.

Large-basis *ab initio* no-core shell model and its application to  $^{12}\text{C}$

Coupled-cluster and configuration-interaction shell model methods originated decades ago. Today, by employing high-precision interactions, new conceptual tools, and powerful computers, these *ab initio* methods have shown the ability to compute energies and other observables, such as electron scattering form factors, for a wide range of atomic nuclei without adjustable parameters. These two papers were early demonstrations of the power of the revitalized methods.

Large-basis *ab initio* no-core shell model and its application to  $^{12}\text{C}$   
 P. Navrátil, J. P. Vary, and B. R. Barrett  
 Phys. Rev. C 62, 054311 (2000)

# Structure of mid-*p*-shell nuclei with chiral NN+3N interactions

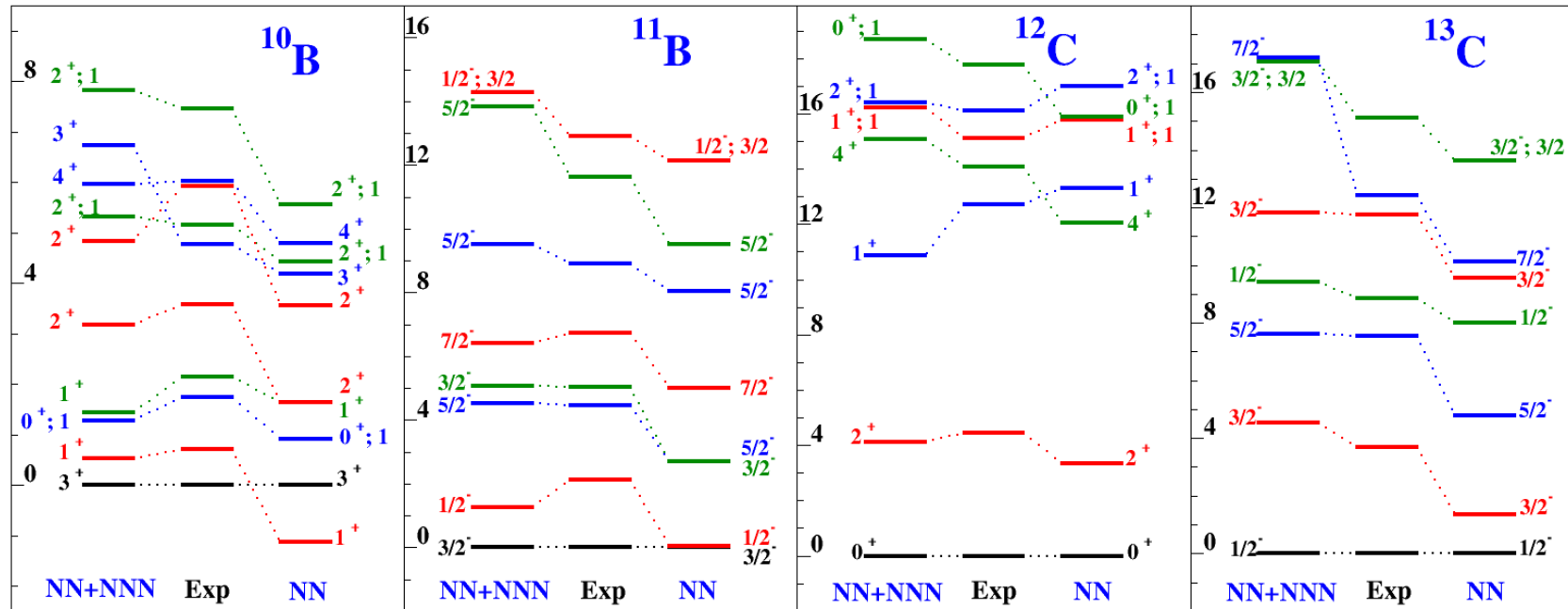
- 3N interaction essential to describe structure of nuclei
  - Both binding energies and excitation levels

PRL 99, 042501 (2007)      PHYSICAL REVIEW LETTERS      week ending 27 JULY 2007

**Structure of  $A = 10-13$  Nuclei with Two- Plus Three-Nucleon Interactions from Chiral Effective Field Theory**

P. Navrátil,<sup>1</sup> V.G. Gueorguiev,<sup>1,\*</sup> J.P. Vary,<sup>1,2</sup> W.E. Ormand,<sup>1</sup> and A. Nogga<sup>3</sup>

<sup>1</sup>Lawrence Livermore National Laboratory, L-414, P.O. Box 808, Livermore, California 94551, USA  
<sup>2</sup>Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA  
<sup>3</sup>Forschungszentrum Jülich, Institut für Kernphysik (Theorie), D-52425 Jülich, Germany  
 (Received 16 January 2007; published 25 July 2007)





# "Anomalous Long Lifetime of Carbon-14"

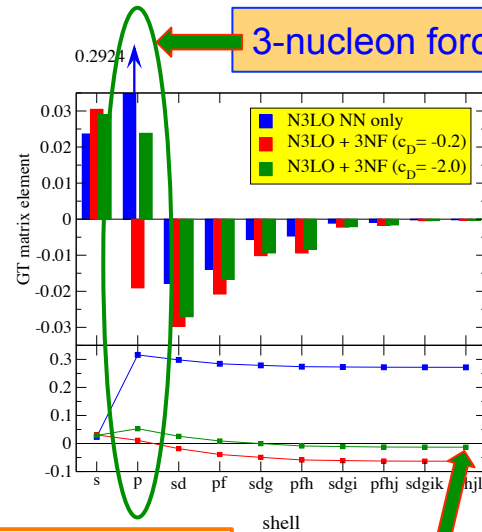
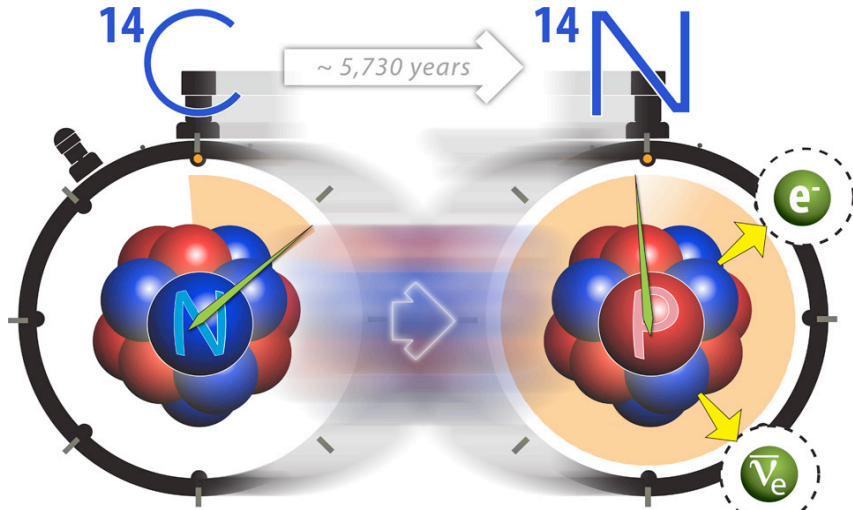


## Objectives

- Solve the puzzle of the long but useful lifetime of  $^{14}\text{C}$
- Determine the microscopic origin of the suppressed  $\beta$ -decay rate

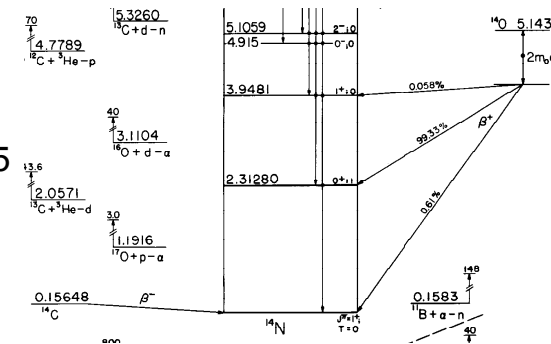
## Impact

- Establishes a major role for strong 3-nucleon forces in nuclei
- Verifies accuracy of *ab initio* microscopic nuclear theory
- Provides foundation for guiding DOE-supported experiments



3-nucleon forces suppress critical component

- Dimension of matrix solved for 8 lowest states  $\sim 1 \times 10^9$
- Solution takes  $\sim 6$  hours on 215,000 cores on Cray XT5 Jaguar at ORNL
- "Scaling of *ab initio* nuclear physics calculations on multicore computer architectures," P. Maris, M. Sosonkina, J. P. Vary, E. G. Ng and C. Yang, 2010 Intern. Conf. on Computer Science, Procedia Computer Science 1, 97 (2010)



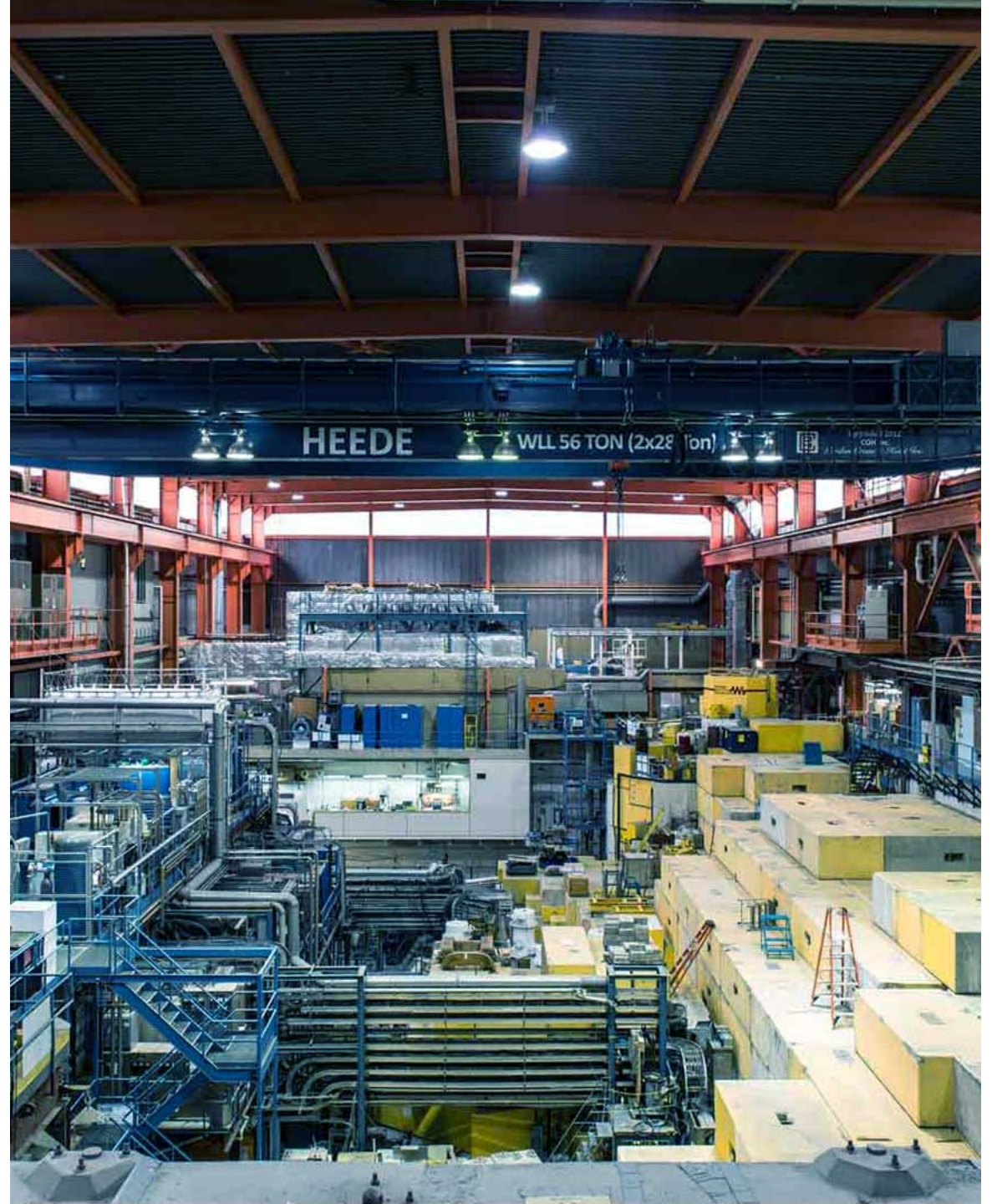
$^{14}\text{N}$  experiment

PRL 106, 202502 (2011) PHYSICAL REVIEW LETTERS week ending 20 MAY 2011  
**Origin of the Anomalous Long Lifetime of  $^{14}\text{C}$**   
 P. Maris,<sup>1</sup> J.P. Vary,<sup>1</sup> P. Navrátil,<sup>2,3</sup> W.E. Ormand,<sup>3,4</sup> H. Nam,<sup>5</sup> and D.J. Dean<sup>5</sup>



# Recent NCSM applications - Similarity Renormalization Group (SRG) renormalization (variational calculations)

2024-07-21

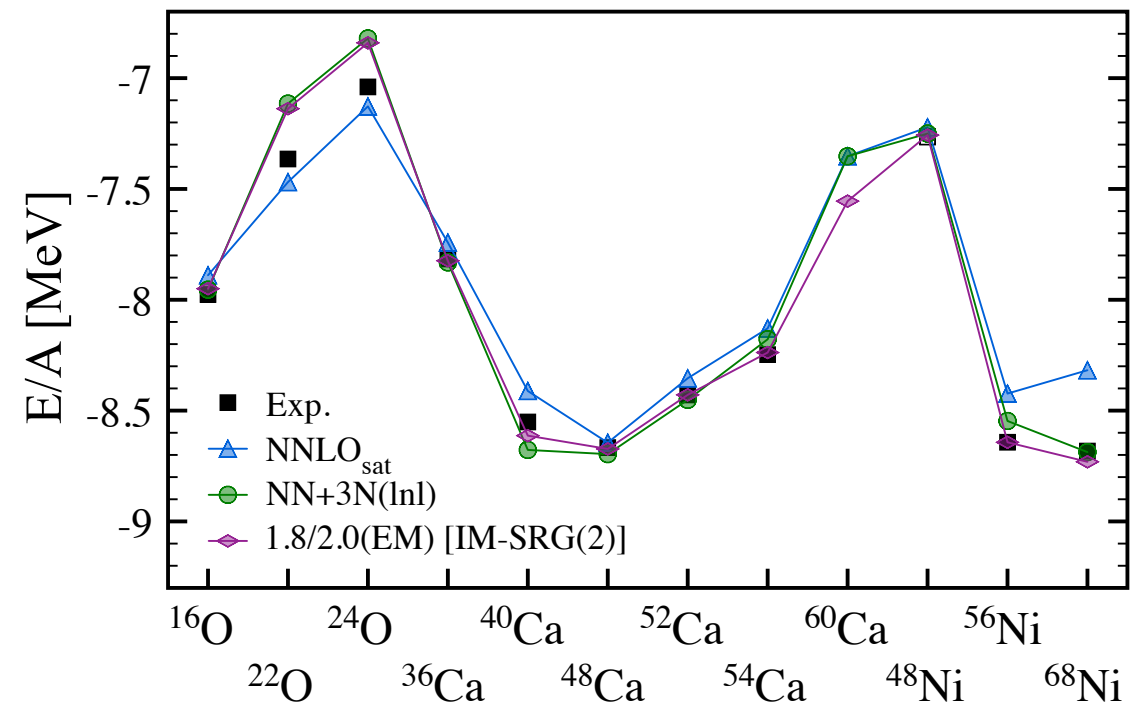
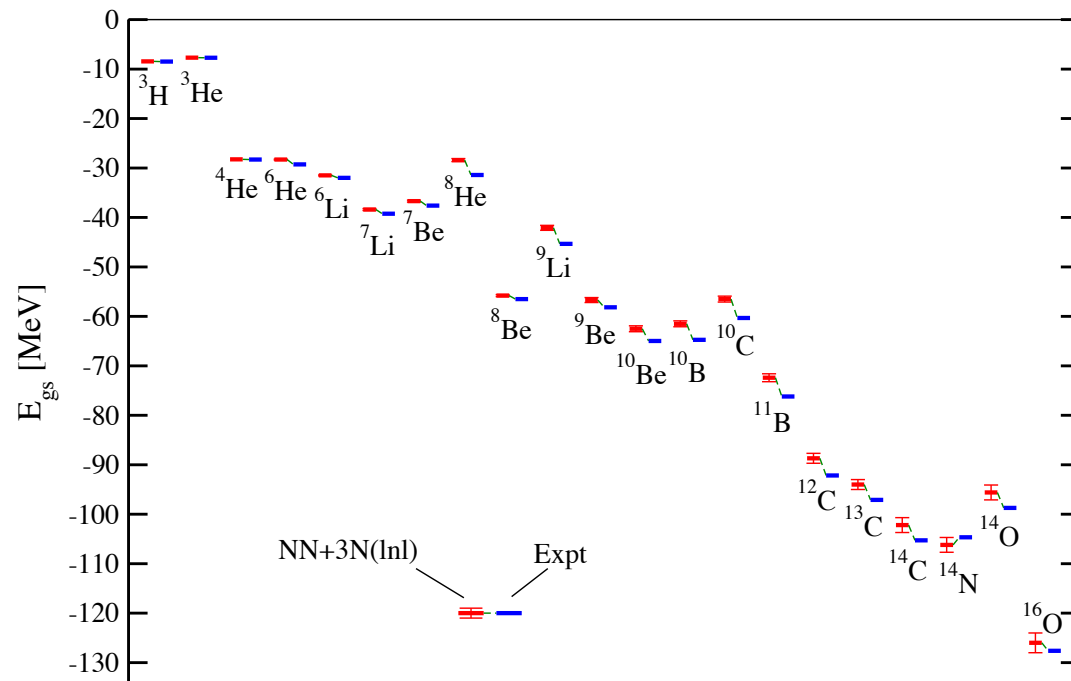


## Binding energies of atomic nuclei with NN+3N forces from chiral Effective Field Theory

14

- Quite reasonable description of binding energies across the nuclear charts becomes feasible
  - The Hamiltonian fully determined in  $A=2$  and  $A=3,4$  systems**
    - Nucleon–nucleon scattering, deuteron properties,  $^3\text{H}$  and  $^4\text{He}$  binding energy,  $^3\text{H}$  half life
  - Light nuclei – NCSM
  - Medium mass nuclei – Self-Consistent Green’s Function method

NN N<sup>3</sup>LO (Entem-Machleidt 2003)  
3N N<sup>2</sup>LO w local/non-local regulator

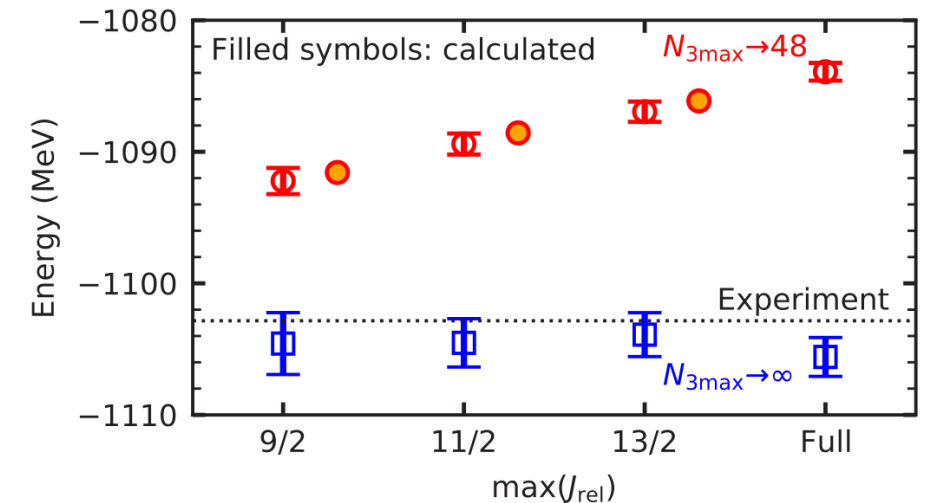
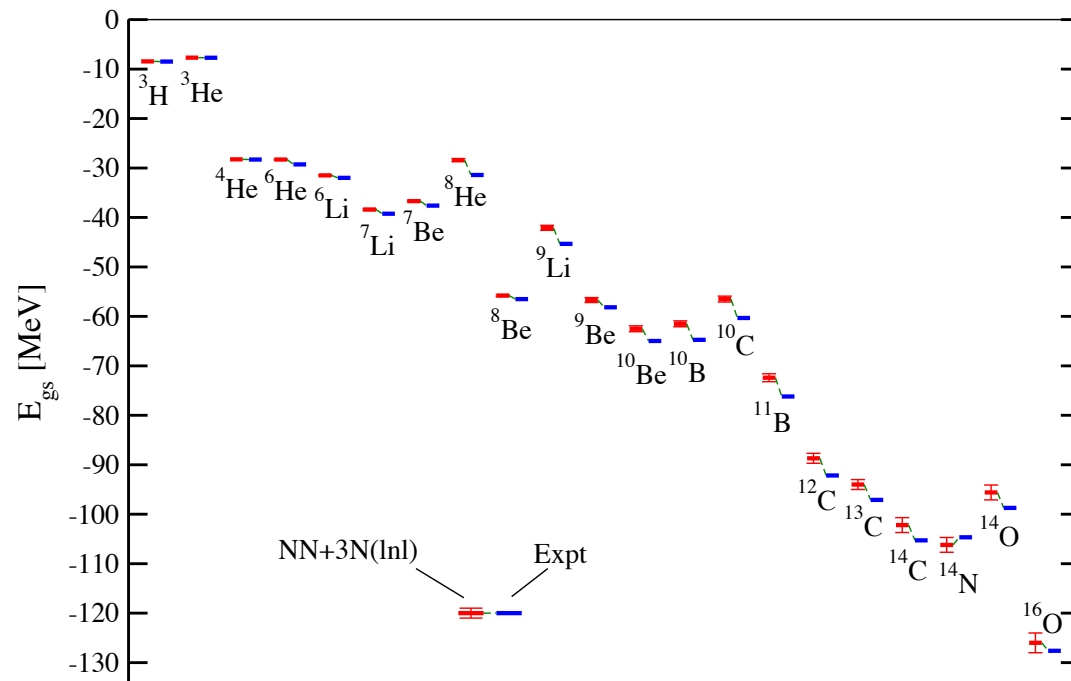


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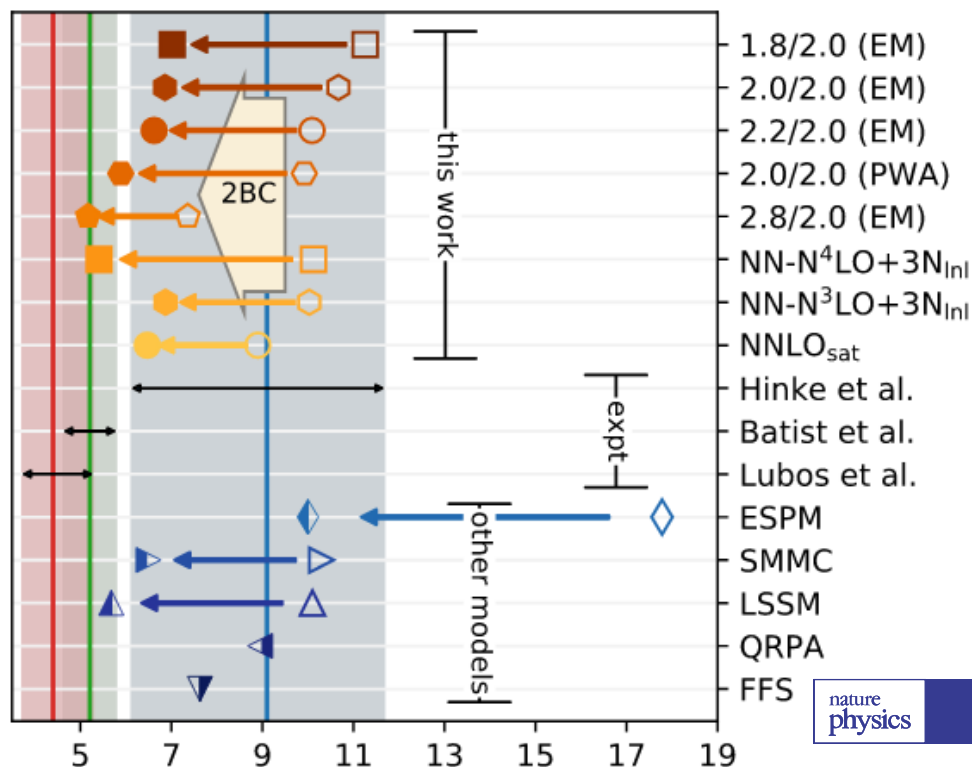
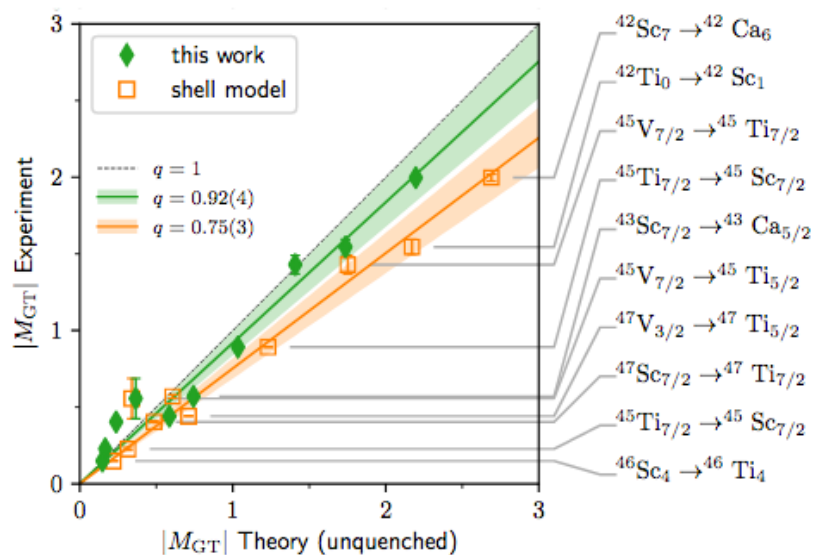
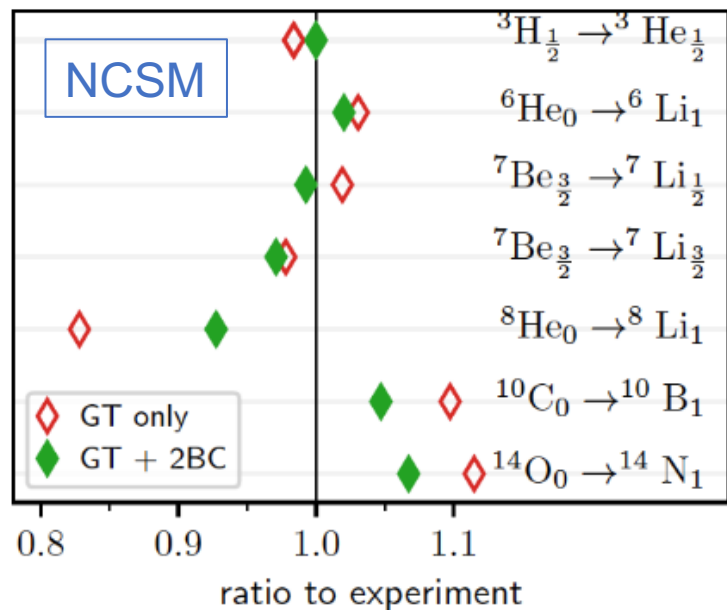
15

- Quite reasonable description of binding energies across the nuclear charts becomes feasible
  - **The Hamiltonian fully determined in  $A=2$  and  $A=3,4$  systems**
    - Nucleon–nucleon scattering, deuteron properties,  $^3\text{H}$  and  $^4\text{He}$  binding energy,  $^3\text{H}$  half life
  - Light nuclei – NCSM
  - Medium mass nuclei – Self-Consistent Green’s Function method

NN N<sup>3</sup>LO (Entem-Machleidt 2003)  
3N N<sup>2</sup>LO w local/non-local regulator



# 50-year-old puzzle of quenched beta decays resolved from first principles



D. Lubos,  
PRL (2019)

**Strong nuclear correlations and two-body currents solve the beta decay quenching problem**

**Discrepancy between experimental and theoretical  $\beta$ -decay rates resolved from first principles**

P.Gysbers<sup>1,2</sup>, G.Hagen<sup>3,4\*</sup>, J.D.Holt<sup>1</sup>, G.R.Jansen<sup>5,6</sup>, T.D.Morris<sup>3,4,6</sup>, P.Navrátil<sup>1</sup>, T.Papenbrock<sup>3,4</sup>, S.Quaglioni<sup>7</sup>, A.Schwenk<sup>8,9,10</sup>, S.R.Stroberg<sup>1,11,12</sup> and K.A.Wendt<sup>7</sup>

nature physics LETTERS  
<https://doi.org/10.1038/s41567-019-0450-7>

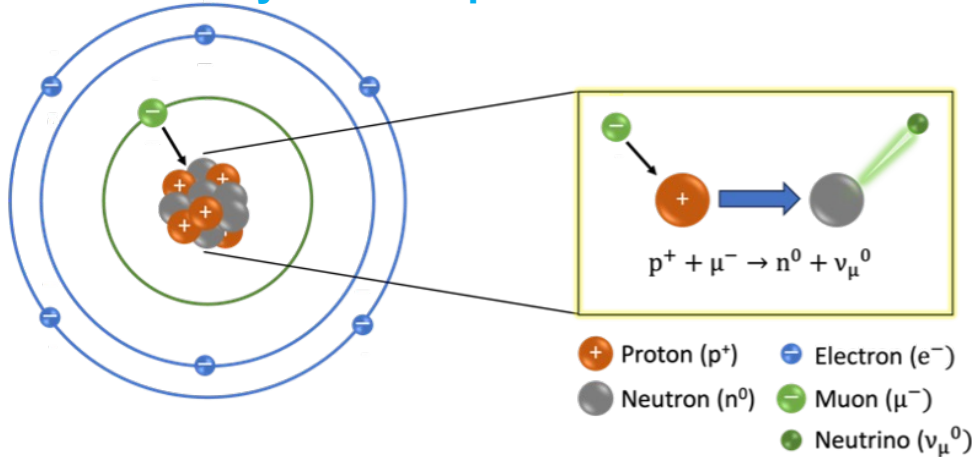
${}^{100}\text{Sn} \rightarrow {}^{100}\text{In}$

The strongest Gamow–Teller strength so far measured in all atomic nuclei



# Muon capture on ${}^6\text{Li}$ , ${}^{12}\text{C}$ , ${}^{16}\text{N}$ from *ab initio* nuclear theory

## Ordinary muon capture on a nucleus



- Momentum exchange  $q = m_{\mu} + E_i - E_f \approx 100 \text{ MeV}$
- Involves vector, axial-vector, magnetic and pseudoscalar nuclear-weak currents  
 → Can be used as a probe of  $0\nu\beta\beta$  decay

PHYSICAL REVIEW C **109**, 065501 (2024)

### Muon capture on ${}^6\text{Li}$ , ${}^{12}\text{C}$ , and ${}^{16}\text{O}$ from *ab initio* nuclear theory

Lotta Jokiniemi ,<sup>1,\*</sup> Petr Navrátil ,<sup>1,2,†</sup> Jenni Kotila ,<sup>3,4,5,‡</sup> and Kostas Kravvaris ,<sup>6,§</sup>

<sup>1</sup>TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

<sup>2</sup>University of Victoria, 3800 Finnerty Road, Victoria, British Columbia V8P 5C2, Canada

<sup>3</sup>Finnish Institute for Educational Research, University of Jyväskylä, P.O. Box 35, Jyväskylä FI-40014, Finland

<sup>4</sup>Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520-8120, USA

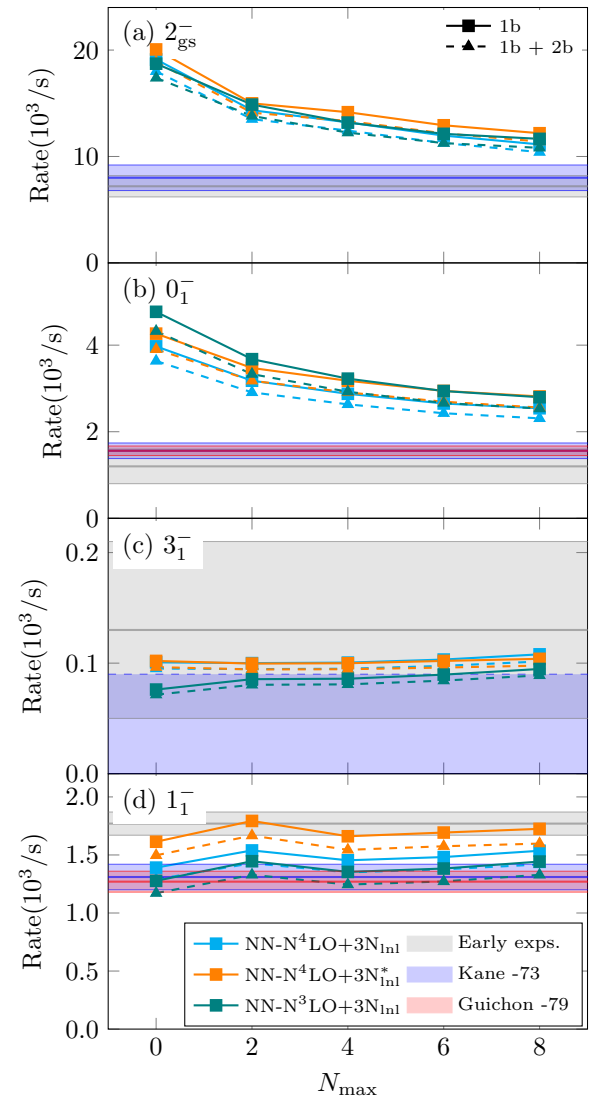
<sup>5</sup>International Center for Advanced Training and Research in Physics (CIFRA), 409, Atomistilor Street, Bucharest-Magurele, 077125, Romania

<sup>6</sup>Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA

(Received 14 March 2024; accepted 21 May 2024; published 10 June 2024)

- *Ab initio* no-core shell-model calculations in good agreement with experiments

$${}^{16}\text{O}(0_{\text{gs}}^+) + \mu^- \rightarrow {}^{16}\text{N}(J_f^{\pi}) + \nu_{\mu}$$



See talk by Lotta Jokiniemi on Saturday

**NCSM applications to parity-violating moments:**  
**How to calculate the sum of intermediate unnatural parity states?**

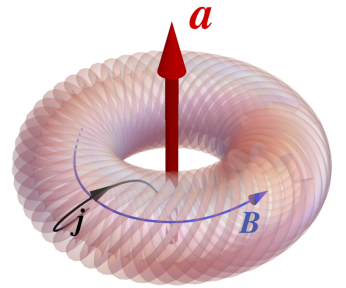
$$a_s = \langle \psi_{\text{gs}} I I_z=I | \hat{a}_{s,0}^{(1)} | \psi_{\text{gs}} I I_z=I \rangle$$

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

- Solving Schroedinger equation with inhomogeneous term

$$(E_{\text{gs}} - H) |\psi_{\text{gs}} I\rangle = V_{\text{NN}}^{\text{PNC}} |\psi_{\text{gs}} I^\pi\rangle$$

- To invert this equation, we apply the Lanczos algorithm



**NCSM applications to parity-violating moments:**  
**How to calculate the sum of intermediate unnatural parity states?**

$$a_s = \langle \psi_{gs} \ I \ I_z=I | \hat{a}_{s,0}^{(1)} | \psi_{gs} \ I \ I_z=I \rangle$$

$$|\psi_{gs} \ I \rangle = |\psi_{gs} \ I^\pi \rangle + \sum_j |\psi_j \ I^{-\pi} \rangle \frac{1}{E_{gs} - E_j} \langle \psi_j \ I^{-\pi} | V_{NN}^{PNC} | \psi_{gs} \ I^\pi \rangle$$

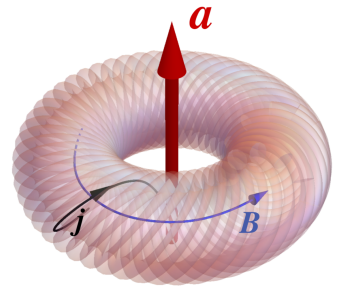
- Solving Schroedinger equation with inhomogeneous term

$$(E_{gs} - H) |\psi_{gs} \ I \rangle = V_{NN}^{PNC} |\psi_{gs} \ I^\pi \rangle$$

- To invert this equation, we apply the Lanczos algorithm
  - Bring matrix to tri-diagonal form ( $\mathbf{v}_1, \mathbf{v}_2 \dots$  orthonormal,  $H$  Hermitian)

$H\mathbf{v}_1 = \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2$
$H\mathbf{v}_2 = \beta_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \beta_2\mathbf{v}_3$
$H\mathbf{v}_3 = \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4$
$H\mathbf{v}_4 = \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5$

- $n^{\text{th}}$  iteration computes  $2n^{\text{th}}$  moment
- Eigenvalues converge to extreme (largest in magnitude) values
- $\sim 150$ - $200$  iterations needed for 10 eigenvalues (even for  $10^9$  states)



**NCSM applications to parity-violating moments:**  
**How to calculate the sum of intermediate unnatural parity states?**

$$a_s = \langle \psi_{gs} \ I \ I_z=I | \hat{a}_{s,0}^{(1)} | \psi_{gs} \ I \ I_z=I \rangle$$

$$|\psi_{gs} \ I \rangle = |\psi_{gs} \ I^\pi \rangle + \sum_j |\psi_j \ I^{-\pi} \rangle \frac{1}{E_{gs} - E_j} \langle \psi_j \ I^{-\pi} | V_{NN}^{PNC} | \psi_{gs} \ I^\pi \rangle$$

- Solving Schroedinger equation with inhomogeneous term

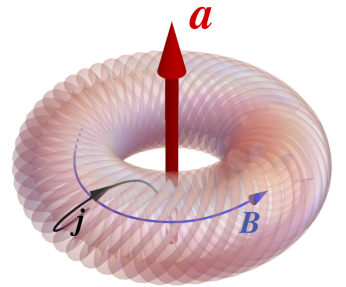
$$(E_{gs} - H) |\psi_{gs} \ I \rangle = V_{NN}^{PNC} |\psi_{gs} \ I^\pi \rangle$$

- To invert this equation, we apply the Lanczos algorithm

$$|\mathbf{v}_1 \rangle = V_{NN}^{PNC} |\psi_{gs} \ I^\pi \rangle$$

$$|\psi_{gs} \ I \rangle \approx \sum_k g_k(E_0) |\mathbf{v}_k \rangle$$

$$\hat{g}_1(\omega) = \frac{1}{\omega - \alpha_1 - \frac{\beta_1^2}{\omega - \alpha_2 - \frac{\beta_2^2}{\omega - \alpha_3 - \frac{\beta_3^2}{\dots}}}}$$



Few-Body Systems 33, 259–276 (2003)  
 DOI 10.1007/s00601-003-0017-z

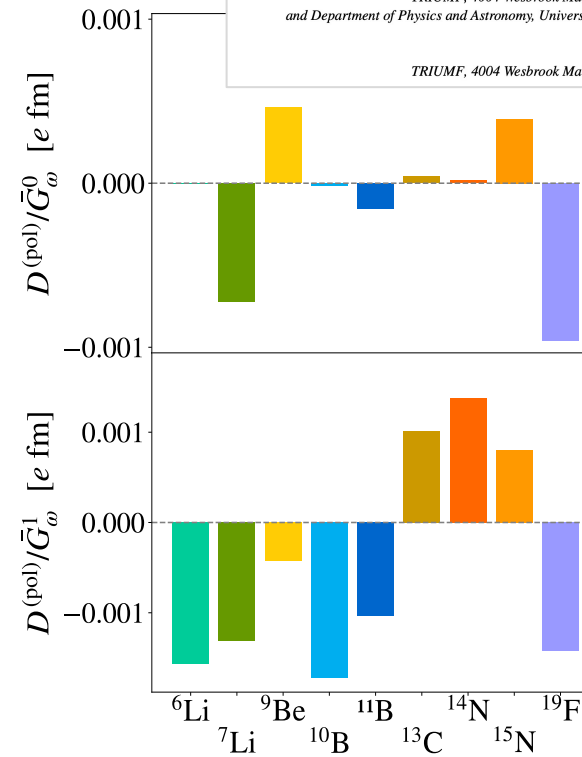
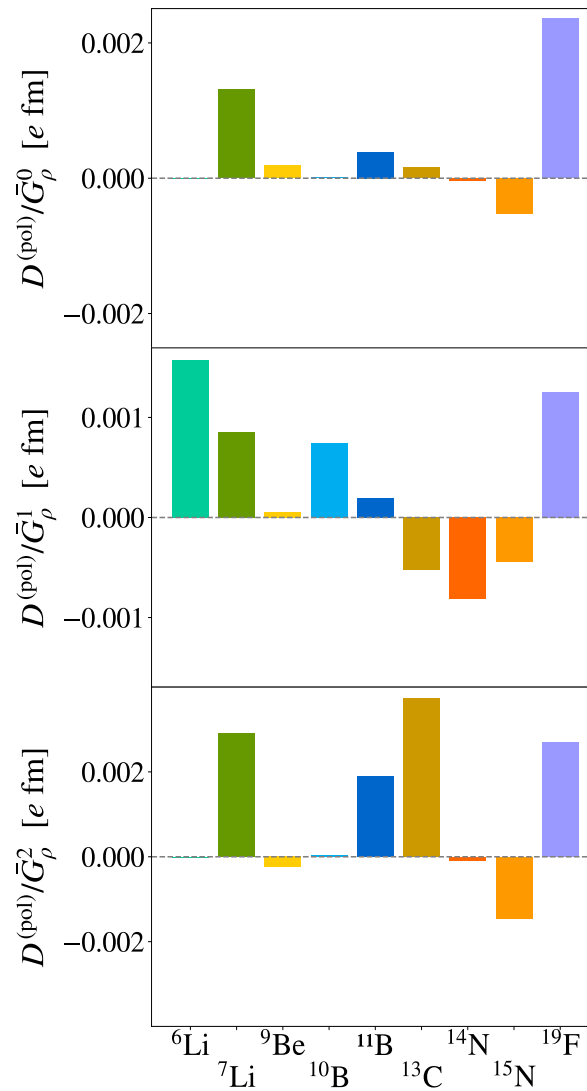
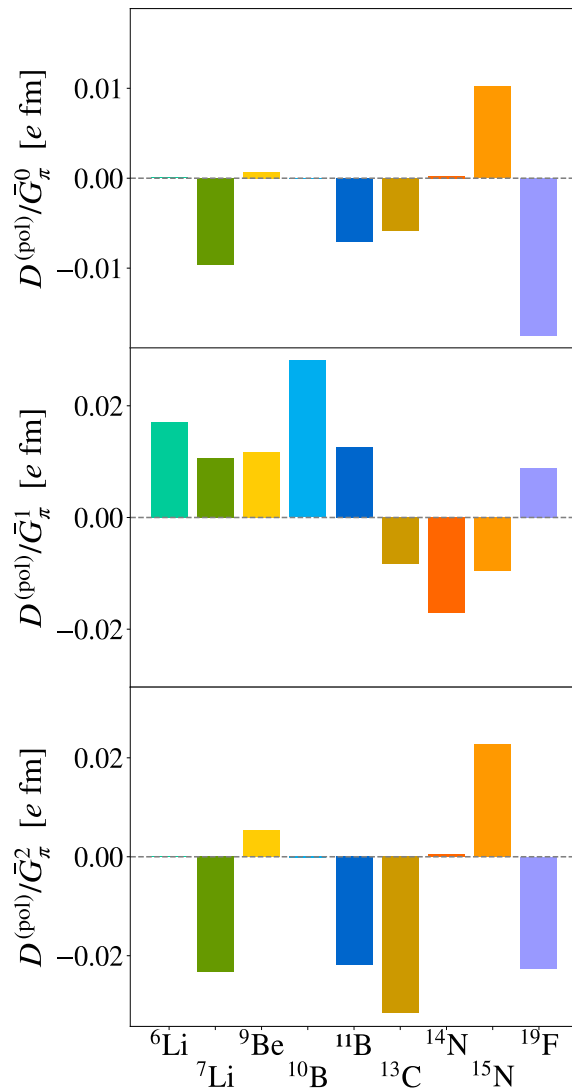
Few-Body  
 Systems  
 Printed in Austria

**Efficient Method for Lorentz Integral  
 Transforms of Reaction Cross Sections**

M. A. Marchisio<sup>1</sup>, N. Barnea<sup>2</sup>, W. Leidemann<sup>1</sup>, and G. Orlandini<sup>1</sup>

Lanczos continued  
 fraction method

# NCSM applications to parity-violating moments: Anapole moments & EDMs of light stable nuclei

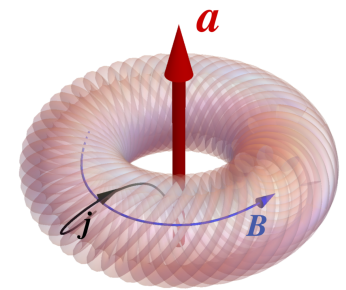


PHYSICAL REVIEW C **104**, 025502 (2021)

*Ab initio* calculations of electric dipole moments of light nuclei

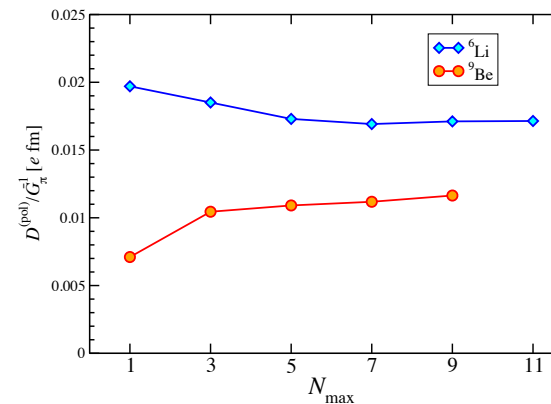
Paul Froese\*  
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada  
and Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada

Petr Navrátil†  
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada



Y. Hao, P. Navratil *et al.*,  
PRA **102**, 052828 (2020)

## Examples of $N_{\max}$ convergence



# Synergy of precision experiments and *ab initio* nuclear theory to test CKM unitarity

## Structure corrections for the extraction of the $V_{ud}$ matrix element from the $^{10}\text{C} \rightarrow ^{10}\text{B}$ Fermi transition

- CKM unitarity sensitive probe of BSM physics
  - $V_{ud}$  element from super-allowed Fermi transitions

$$|V_{ud}|^2 = \frac{\hbar^7}{G_F^2 m_e^5 c^4} \frac{\pi^3 \ln(2)}{\mathcal{F}t} \quad \mathcal{F}t = \frac{K}{G_V^2 |M_{F0}|^2 (1 + \Delta_R^V)}$$

$$\mathcal{F}t(1 + \Delta_R^V) = ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS})$$

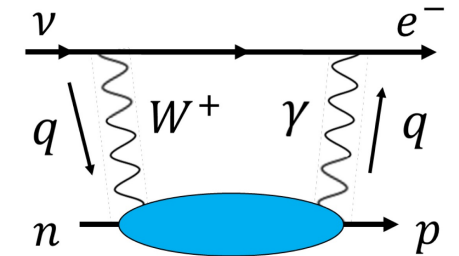
- $\delta_{NS}$  parametrizes correction to free  $\gamma W$  box
- Ab initio* no-core shell model (NCSM)
  - A very good convergence – consistent with what used in latest evaluation with a substantially reduced theoretical uncertainties

$$\delta_{NS} = 2[\Box_{\gamma W}^{VA, \text{nuc.}} - \Box_{\gamma W}^{VA, \text{free n}}]$$

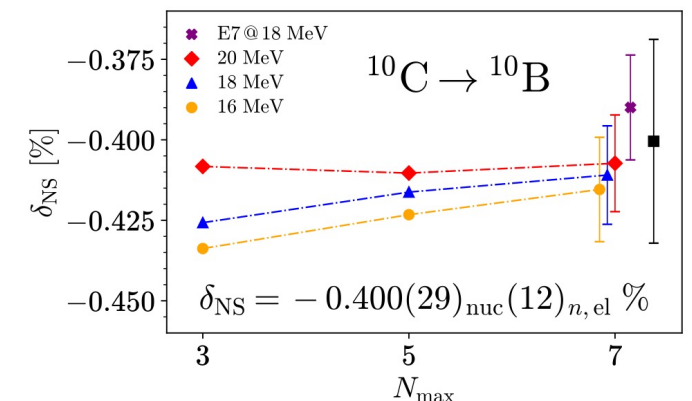
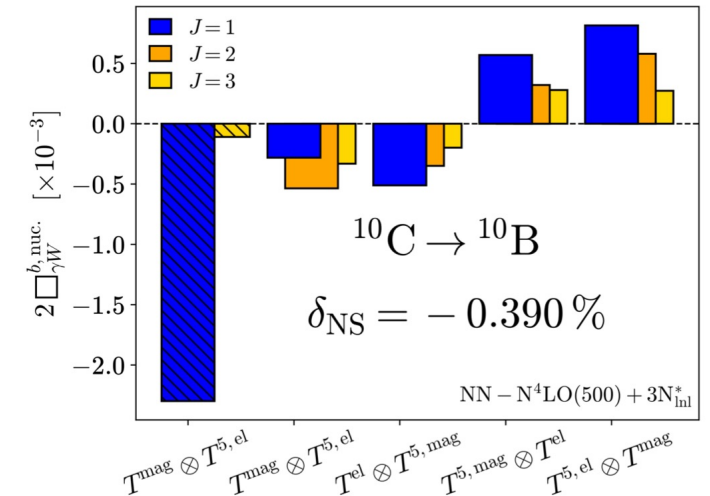
An *ab initio* strategy for taming the nuclear-structure dependence of  $V_{ud}$  extractions:  
the  $^{10}\text{C} \rightarrow ^{10}\text{B}$  superallowed transition arXiv: 2405.19281

Michael Gennari<sup>1,2</sup>, Mehdi Drissi<sup>1</sup>, Mikhail Gorchtein<sup>3,4</sup>, Petr Navrátil<sup>1,2</sup>, and Chien-Yeah Seng<sup>5,6</sup>

NCSM applicable also to  $^{14}\text{O} \rightarrow ^{14}\text{N}$  and possibly  $^{18}\text{Ne} \rightarrow ^{18}\text{F}$ ,  $^{22}\text{Mg} \rightarrow ^{22}\text{Na}$

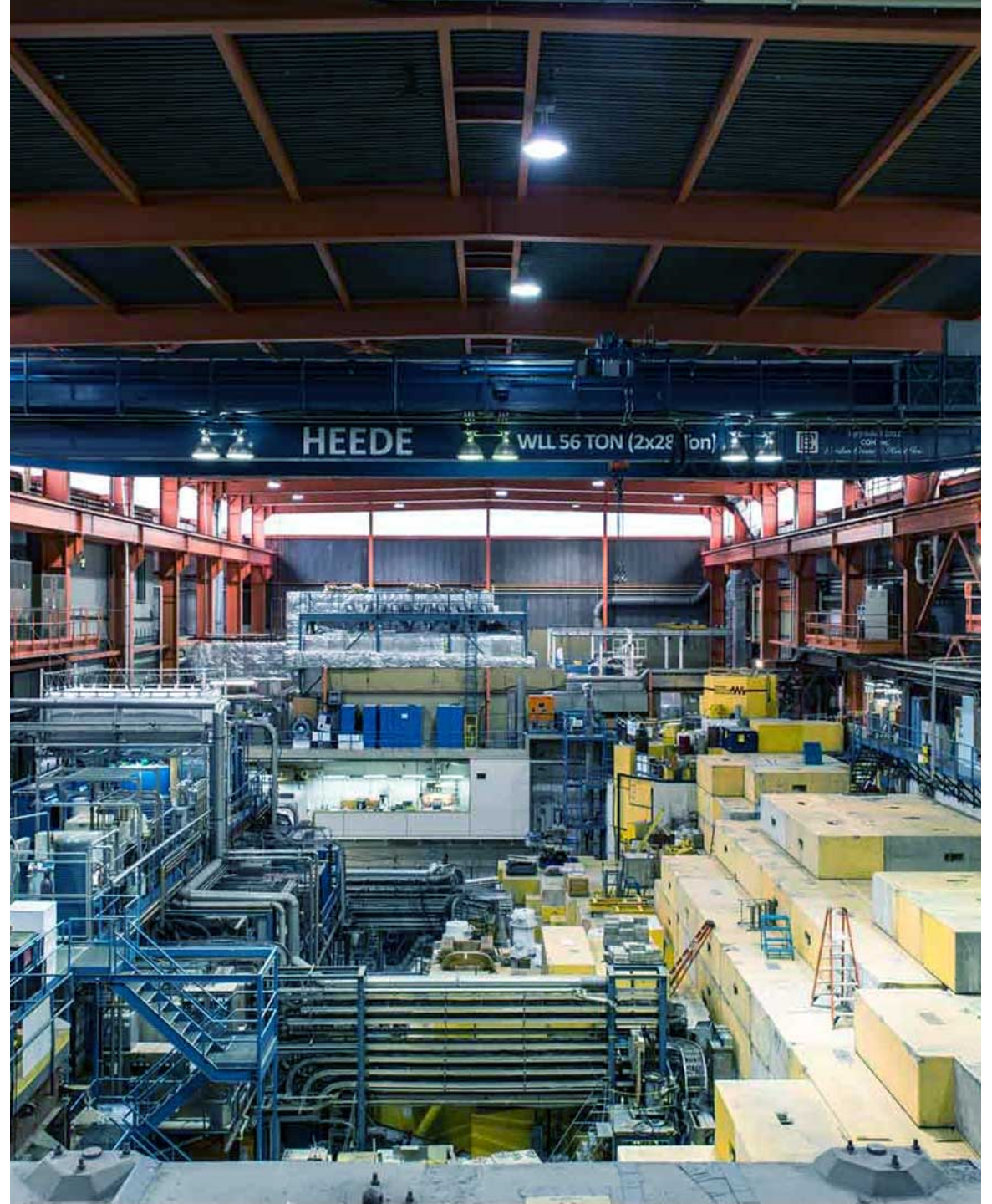


22



# No-Core Shell Model with Continuum (NCSMC) - Unified description of bound and unbound states

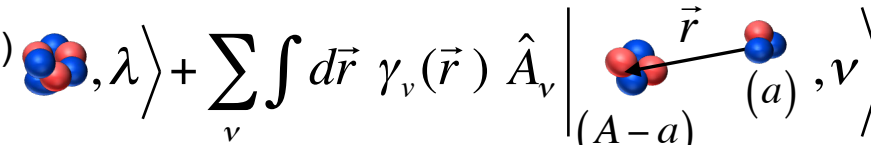
2024-07-21



# Ab Initio Calculations of Structure, Scattering, Reactions

Unified approach to bound & continuum states

## No-Core Shell Model with Continuum (NCSMC)

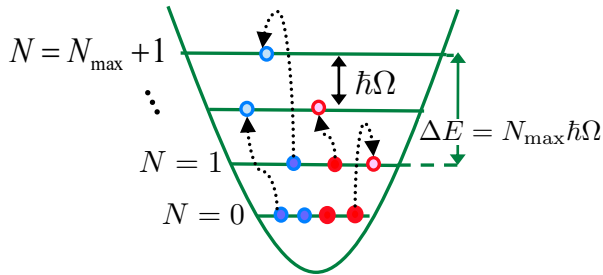
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} (A) \\ \text{Nucleus} \end{array}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} (A-a) \\ \text{Nucleus} \end{array}, \nu \right\rangle$$
The equation shows the wave function for a nucleus with mass number A. The first term is a sum over discrete states labeled by lambda, represented by a cluster of red and blue spheres. The second term is an integral over a continuum state labeled by nu, where the integrand is the product of a weight function gamma\_nu(r) and an operator A\_nu acting on a state with mass number A-a and quantum number nu. A diagram shows a cluster of red and blue spheres with a vector r pointing to a single nucleon (red and blue spheres).



# Ab Initio Calculations of Structure, Scattering, Reactions

## Unified approach to bound & continuum states

### No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \underbrace{\sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \\ \lambda \end{matrix} \right\rangle}_{\text{bound states}} + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & \vec{r} & (a) \\ \nu & & \nu \end{matrix} \right\rangle$$


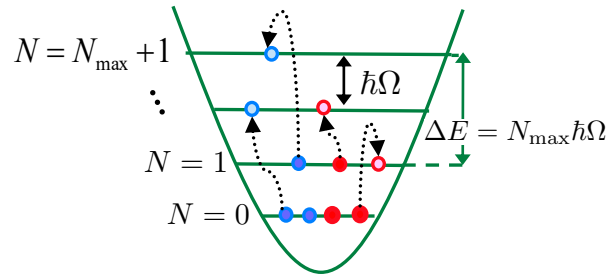
Static solutions for aggregate system,  
describe all nucleons close together

# Ab Initio Calculations of Structure, Scattering, Reactions

## Unified approach to bound & continuum states

### No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \underbrace{\sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \\ \lambda \end{matrix} \right\rangle}_{\text{Static solutions for aggregate system}} + \underbrace{\sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & \vec{r} & (a) \\ \nu & & \nu \end{matrix} \right\rangle}_{\text{Continuous microscopic cluster states}}$$



Continuous microscopic cluster states, describe long-range projectile-target

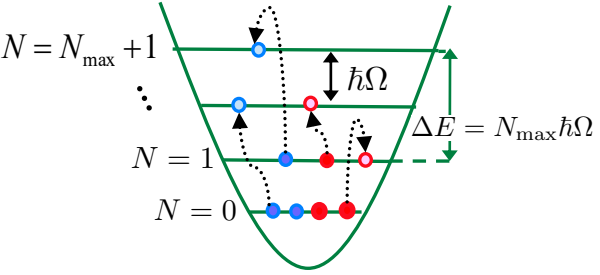
Static solutions for aggregate system, describe all nucleons close together

# Ab Initio Calculations of Structure, Scattering, Reactions

Unified approach to bound & continuum states

## No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \underbrace{\sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \\ \lambda \end{matrix} \right\rangle}_{\text{Unknowns}} + \underbrace{\sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & \vec{r} & (a) \\ \nu & & \nu \end{matrix} \right\rangle}_{\text{Unknowns}}$$



Continuous microscopic cluster states, describe long-range projectile-target

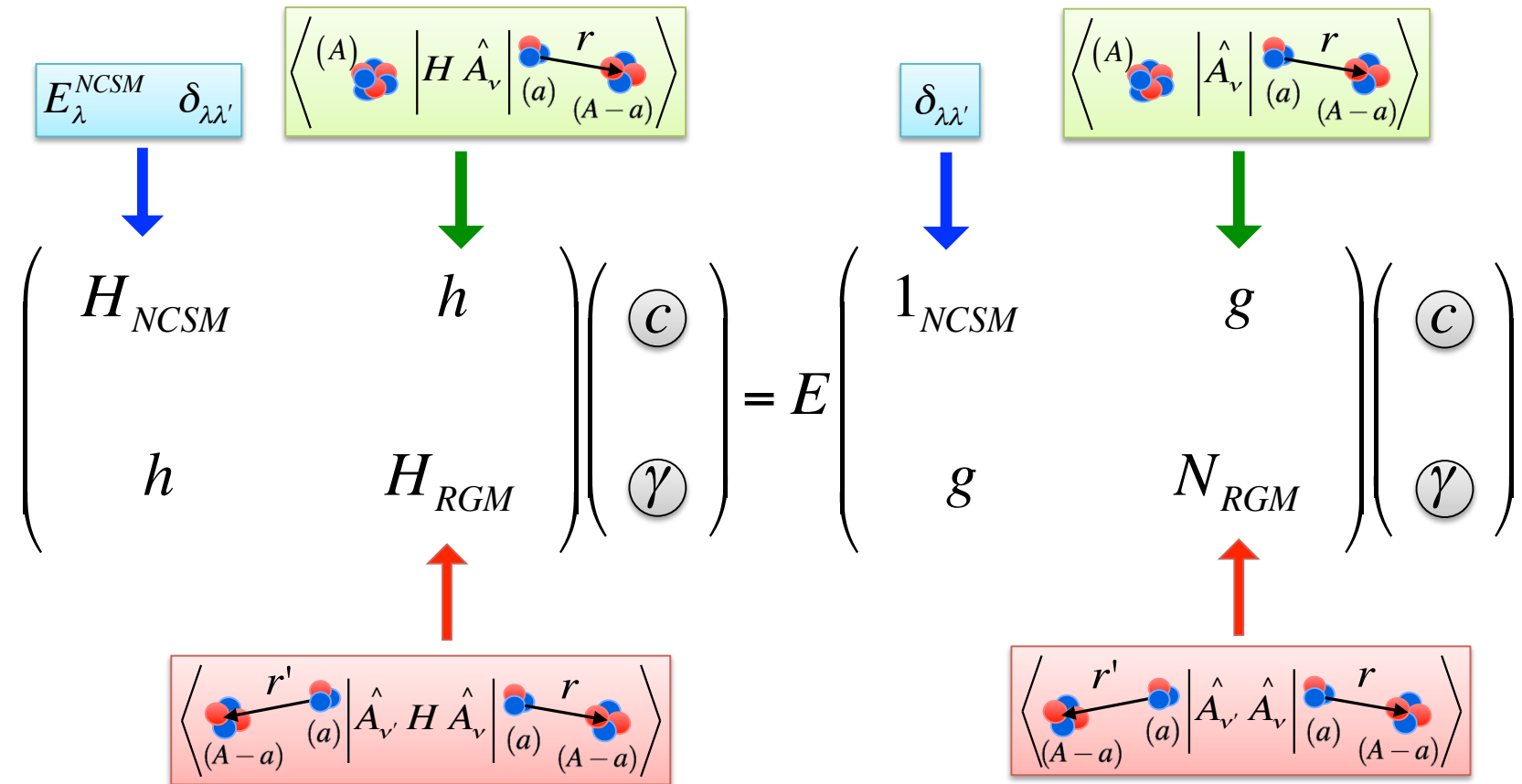
Static solutions for aggregate system, describe all nucleons close together

S. Baroni, P. Navratil, and S. Quaglioni, PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

# Coupled NCSMC equations

$$H \Psi^{(A)} = E \Psi^{(A)}$$

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \end{matrix}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & (a) \\ \text{cluster} & \text{cluster} \end{matrix}, \nu \right\rangle$$

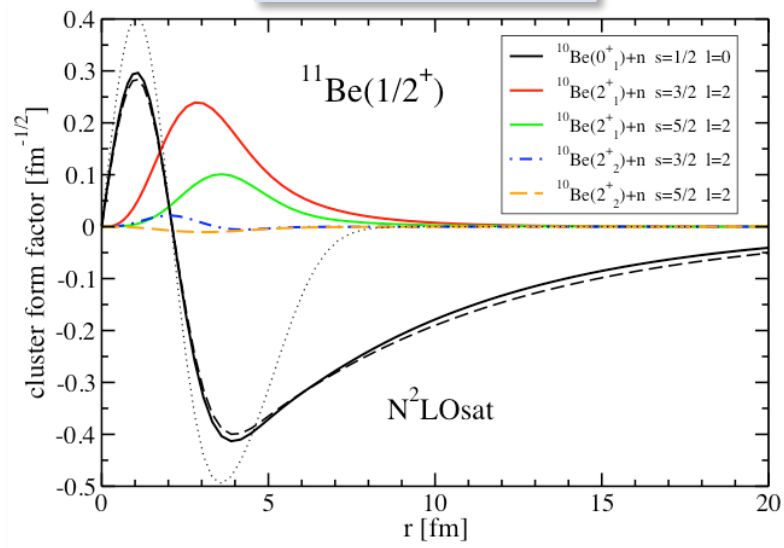


# Studies of exotic nuclei – continuum effects

## Photo-disassociation of $^{11}\text{Be}$

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \lambda \end{matrix} \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) \\ \nu \end{matrix} \right\rangle$$

### Halo structure

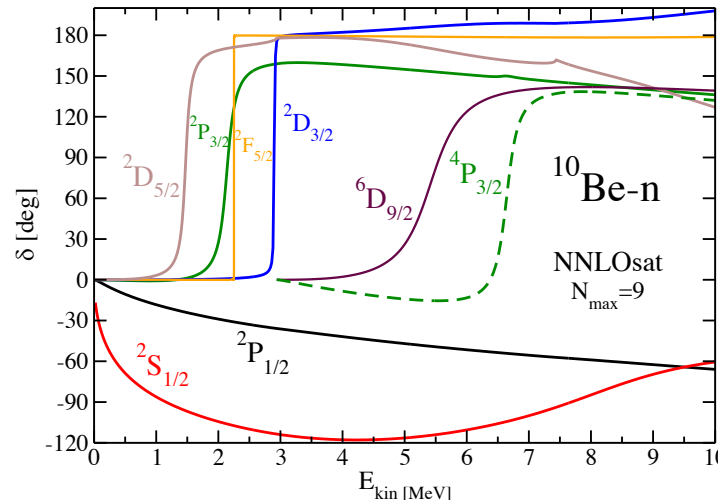
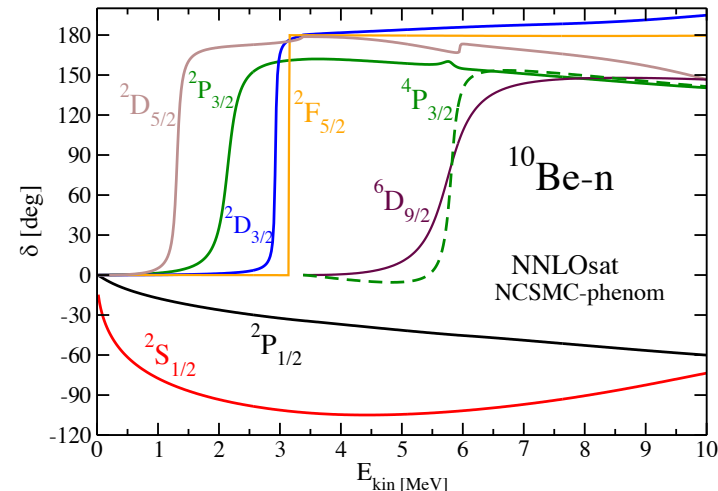


### cluster form factor

$$= r \langle \Phi_{vr}^{J^{\pi T}} | \hat{A}_{\nu} | \psi^{J^{\pi T}} \rangle$$

$$| \Phi_{vr}^{J^{\pi T}} \rangle = \left[ \left( \left| ^{10}\text{Be } \alpha_1 I_1^{\pi_1 T_1} \right| \left| n \frac{1}{2}^+ \frac{1}{2} \right\rangle \right)^{(sT)} Y_{\ell}(\hat{r}_{10,1}) \right]^{(J^{\pi T})} \frac{\delta(r - r_{10,1})}{r r_{10,1}}$$

Bound to bound	NCSM	NCSMC-phenom	Expt.
B(E1; $1/2^+ \rightarrow 1/2^-$ ) [e <sup>2</sup> fm <sup>2</sup> ]	0.0005	0.117	0.102(2)

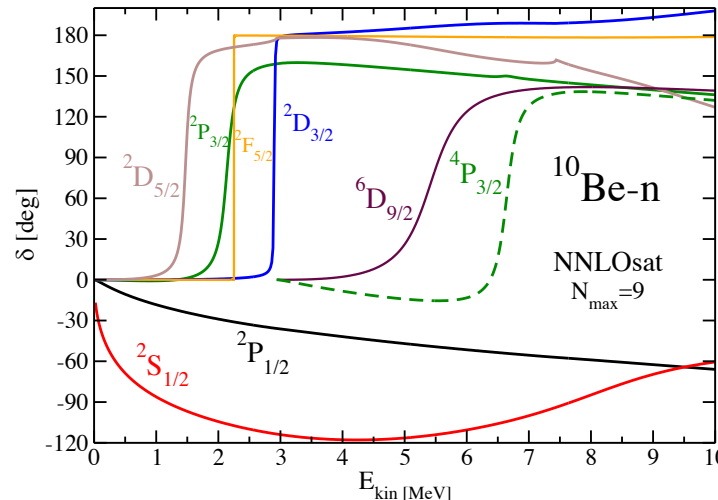
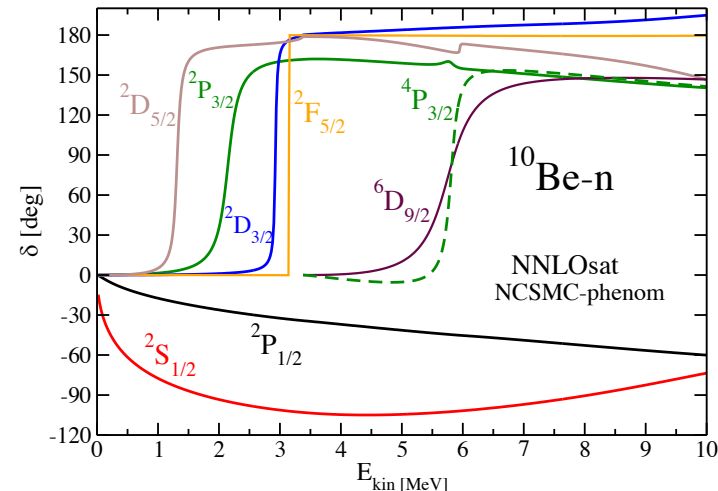
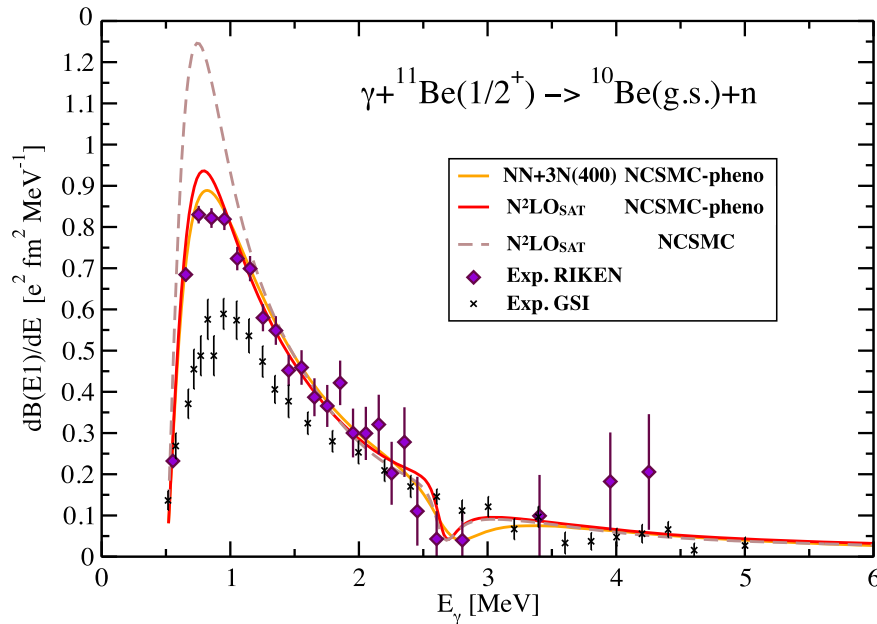


# Studies of exotic nuclei – continuum effects

## Photo-disassociation of $^{11}\text{Be}$

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \lambda \end{matrix} \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) \\ \nu \end{matrix} \right\rangle$$

### Bound to continuum



Bound to bound	NCSM	NCSMC-pheno	Expt.
B(E1; $1/2^+ \rightarrow 1/2^-$ ) [ $e^2 \text{fm}^2$ ]	0.0005	0.117	0.102(2)

Responsible for  ${}^6\text{Li}$  production in BBN –  $10^3$  discrepancy between theory/observation

Deficiency in observation, theory, or new physics?

**NCSMC** calculation w/ chiral forces

## Radiative capture S-factor

Dominated by  $E2$

$M1$  significant at low energy

$E1$  negligible (isospin suppressed)

## Thermonuclear reaction rate

Smaller than NACRE II evaluation

Agrees w/ LUNA; reduced

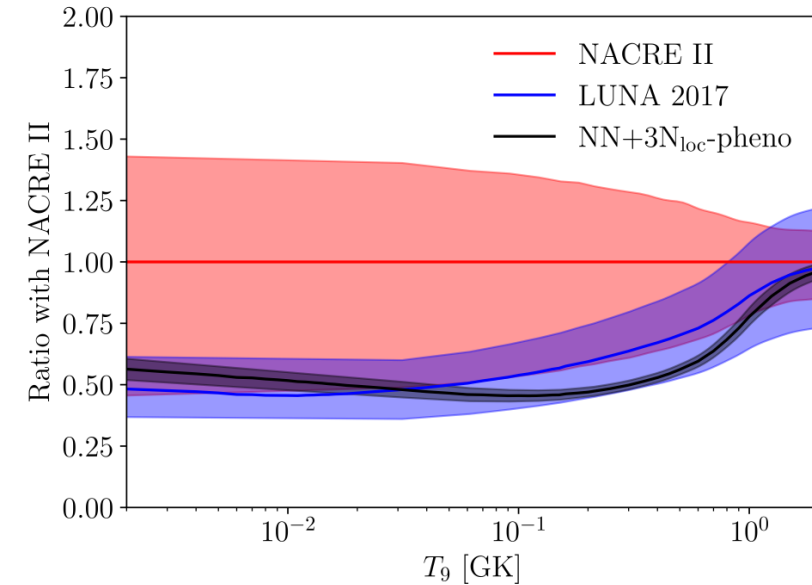
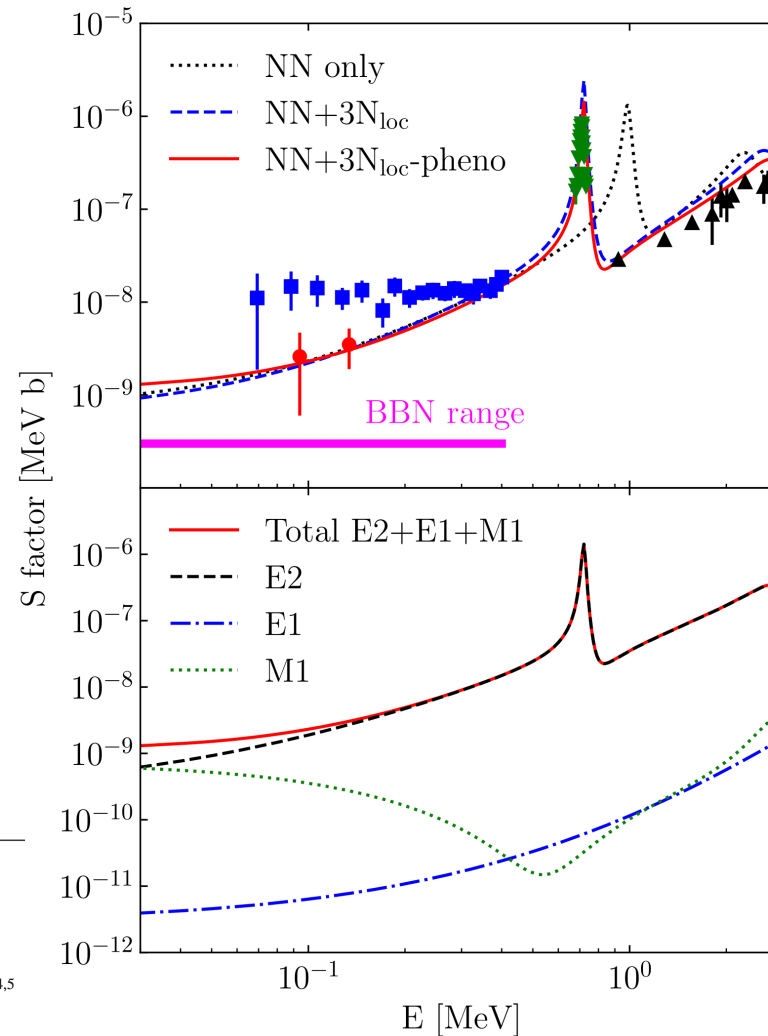
uncertainty

PHYSICAL REVIEW LETTERS **129**, 042503 (2022)

*Ab Initio* Prediction of the  ${}^4\text{He}(d,\gamma){}^6\text{Li}$  Big Bang Radiative Capture

C. Hebborn<sup>1,2,\*</sup>, G. Hupin<sup>3</sup>, K. Kravvaris<sup>2</sup>, S. Quaglioni<sup>2</sup>, P. Navrátil<sup>4</sup>, and P. Gysbers<sup>4,5</sup>

2024-07-21

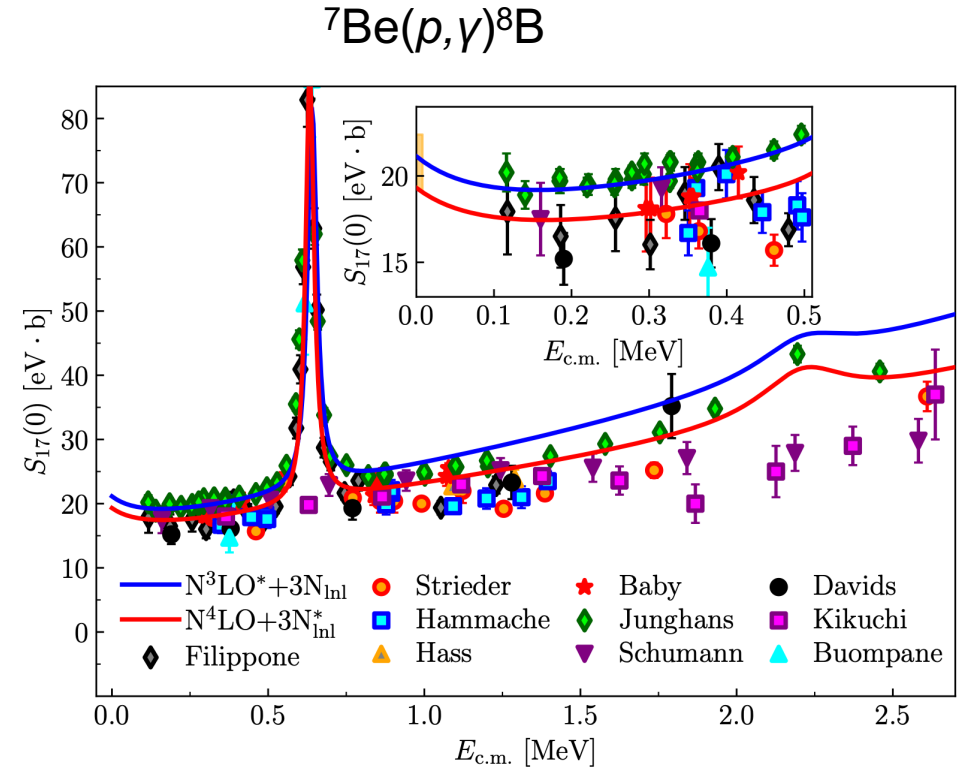


# Radiative capture of protons on ${}^7\text{Be}$

- Solar pp chain reaction, solar  ${}^8\text{B}$  neutrinos
- NCSMC calculations with a set of chiral NN+3N interactions as input
  - Radiative capture S-factor
    - Dominated by  $E1$  non-resonant
    - $M1/E2$  significant at  $1^+$  and  $3^+$  resonances
  - Correlations between results obtained by different chiral interactions and experimental data → evaluation of the S-factor at  $E=0$  energy relevant for the solar physics

Recommended value  $S_{17}(0) \sim 19.8(3) \text{ eV} \cdot \text{b}$

Latest evaluation in *Rev. Mod. Phys.* **83**,195–245 (2011):  
 $S_{17}(0) = 20.8 \pm 0.7(\text{expt}) \pm 1.4(\text{theory}) \text{ eV} \cdot \text{b}$



Physics Letters B 845 (2023) 138156



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*Ab initio* informed evaluation of the radiative capture of protons on  ${}^7\text{Be}$

K. Kravvaris<sup>a,\*</sup>, P. Navrátil<sup>b</sup>, S. Quaglioni<sup>a</sup>, C. Hebborn<sup>c,a</sup>, G. Hupin<sup>d</sup>

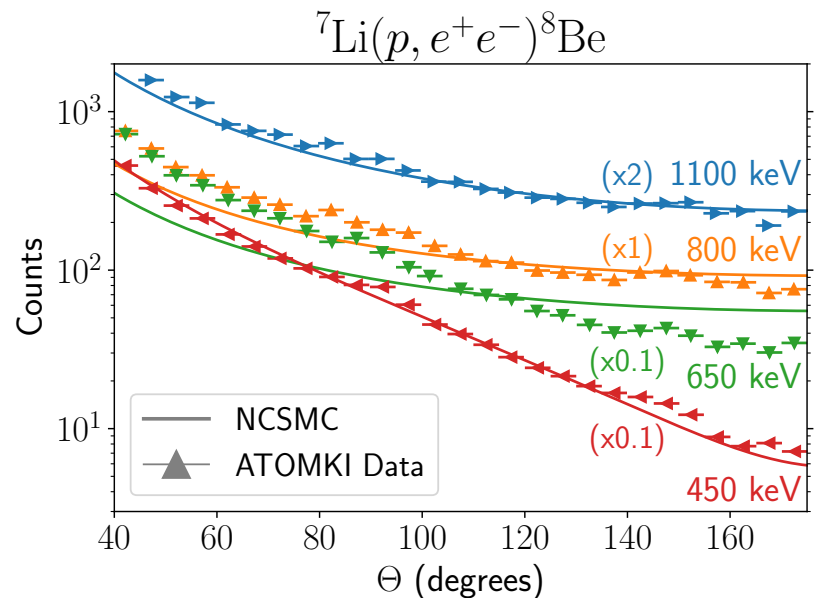




# Recently developed NCSMC capability – charge-exchange reaction calculations

- The first published application -  ${}^7\text{Li}+p$  scattering and radiative capture
  - Wave function ansatz

$$\Psi_{\text{NCSMC}}^{(8)} = \sum_{\lambda} c_{\lambda} |{}^8\text{Be}, \lambda\rangle + \sum_{\nu} \int dr \gamma_{\nu}(r) \hat{A}_{\nu} |{}^7\text{Li} + p, \nu\rangle + \sum_{\mu} \int dr \gamma_{\mu}(r) \hat{A}_{\mu} |{}^7\text{Be} + n, \mu\rangle$$

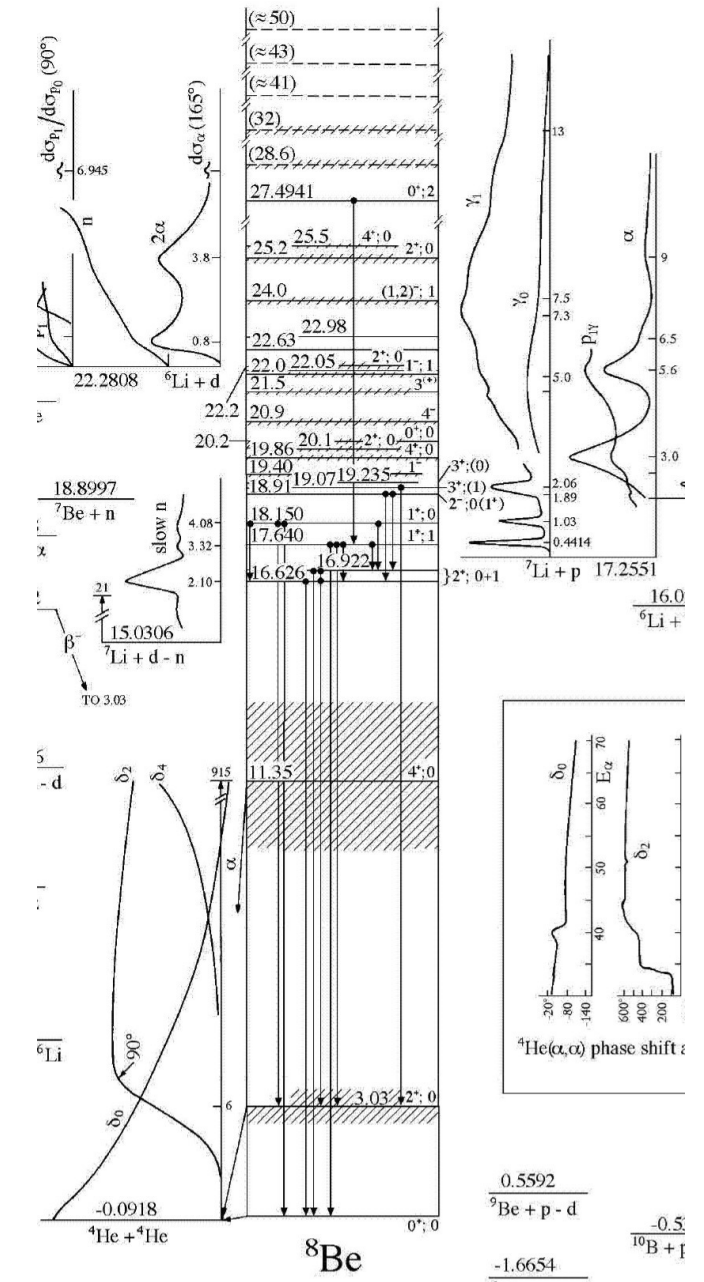


PHYSICAL REVIEW C **110**, 015503 (2024)

Editors' Suggestion

## Ab initio investigation of the ${}^7\text{Li}(p, e^+e^-){}^8\text{Be}$ process and the X17 boson

P. Gysbers<sup>1,2,3</sup>, P. Navrátil<sup>1,4</sup>, K. Kravvaris<sup>5</sup>, G. Hupin<sup>6</sup> and S. Quaglioni<sup>5</sup>



## Conclusions

- *Ab initio* nuclear theory
  - Makes connections between the low-energy QCD and many-nucleon systems
- No-core shell model is an *ab initio* extension of the original nuclear shell model
  - Applicable to nuclear structure, reactions including those relevant for astrophysics, electroweak processes, tests of fundamental symmetries

Thank you!  
Merci!

Thanks to all my collaborators  
over the years!

