

Celebrating Maria Goeppert twice: nuclear shell model and $\beta\beta$ decay

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Celebrating 75 Years of the Nuclear Shell Model
and Maria Goeppert-Mayer

Argonne National Laboratory, 20th July 2024



UNIVERSITAT DE
BARCELONA



Collaborators



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N. Shimizu



R. Weiss



A. Lovato, B. Wiringa



C. Brase, A. Schwenk

Creation of matter in nuclei: $0\nu\beta\beta$ decay

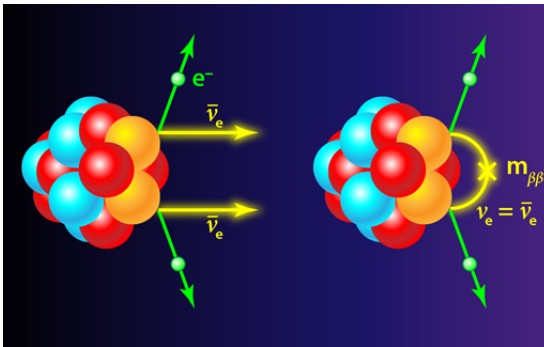
Lepton number conserved
in all processes observed:

single β decay,
 $\beta\beta$ decay with ν emission...

Neutral massive particles (Majorana ν 's)
allow lepton number violation:

neutrinoless $\beta\beta$ decay
creates two matter particles (electrons)

Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

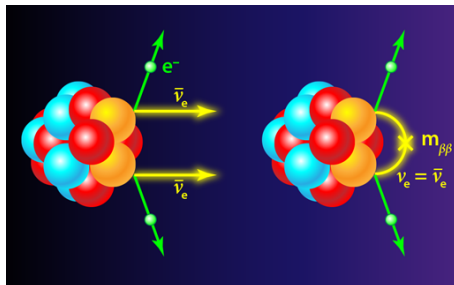


Nuclear matrix elements

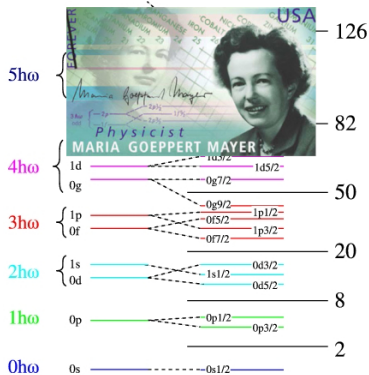
Nuclear matrix elements needed in low-energy new-physics searches

$$\langle \text{Final} | \mathcal{L}_{\text{leptons-nucleons}} | \text{Initial} \rangle = \langle \text{Final} | \int dx j^\mu(x) J_\mu(x) | \text{Initial} \rangle$$

- Nuclear structure calculation of the initial and final states:
Shell model, QRPA, IBM, Energy-density functional
Ab initio many-body theory
QMC, Coupled-cluster, IMSRG...
- Lepton-nucleus interaction:
Hadronic current in nucleus:
phenomenological,
effective theory of QCD



Nuclear shell model



Nuclear shell model configuration space only keep essential degrees of freedom

- High-energy orbitals: always empty
- **Valence space:** where many-body problem is solved
- Inert core: always filled

$$H|\Psi\rangle = E|\Psi\rangle \rightarrow H_{\text{eff}}|\Psi\rangle_{\text{eff}} = E|\Psi\rangle_{\text{eff}}$$

$$|\Psi\rangle_{\text{eff}} = \sum_{\alpha} c_{\alpha} |\phi_{\alpha}\rangle, \quad |\phi_{\alpha}\rangle = a_{i_1}^+ a_{i_2}^+ \dots a_{i_A}^+ |0\rangle$$

Shell model diagonalization:

$\sim 10^{10}$ Slater det. Caurier et al. RMP77 (2005)

$\gtrsim 10^{24}$ Slater det. with Monte Carlo SM

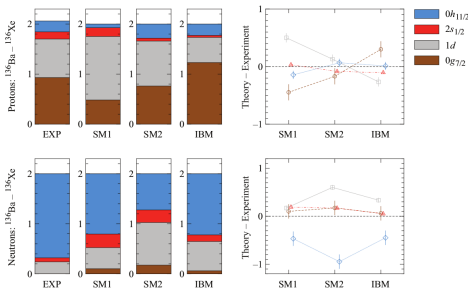
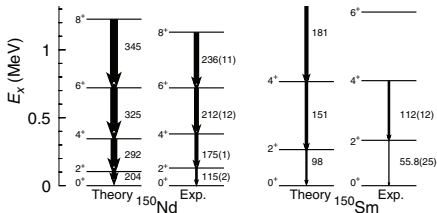
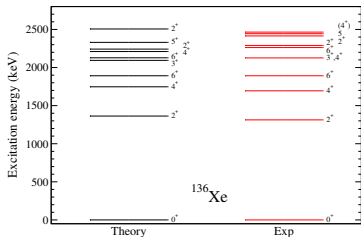
Otsuka, Shimizu, Y.Tsunoda
Phys. Scr. 92 063001 (2017)

H_{eff} includes effects of

- inert core
- high-energy orbitals

Tests of nuclear structure

Spectroscopy well described: masses, spectra, transitions, knockout...



Schiffer et al. PRL100 112501(2009)

Kay et al. PRC79 021301(2009)

...

Szwec et al., PRC94 054314 (2016)

Rodríguez et al. PRL105 252503 (2010)

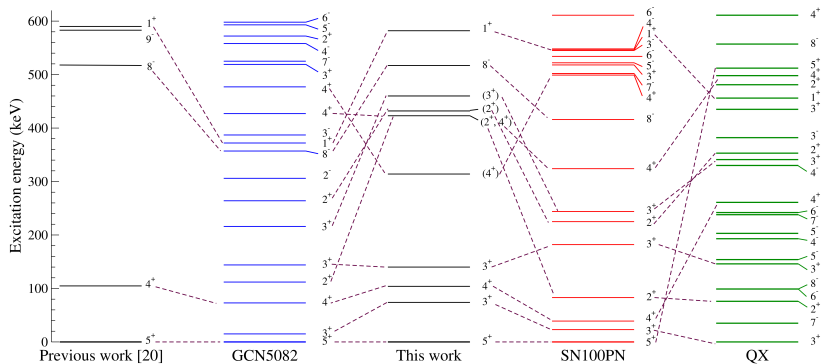
...

Vietze et al. PRD91 043520 (2015)

^{136}Cs experimental spectrum

While all these interactions are well, tested recent data on ^{136}Cs suggests GCN5082 results agree better with experiment than QX

Rebeiro, Triambak et al. PRL131 052501 (2023)



QX systematically smaller ^{136}Xe $0\nu\beta\beta$ -decay nuclear matrix elements

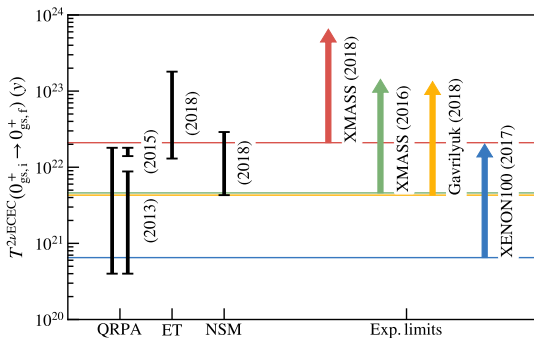
Two-neutrino ECEC of ^{124}Xe

Two-neutrino $\beta\beta$ predicted for ^{48}Ca before measurement

Caurier, Poves, Zuker, PLB 252 13(1990)

Recent predictions for $2\nu\text{ECEC } ^{124}\text{Xe}$ half-life:

shell model error bar largely dominated by “quenching” uncertainty



Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk
PLB 797 134885 (2019)

Shell model, QRPA and Effective theory (ET) predictions suggest experimental detection close to XMASS 2018 limit

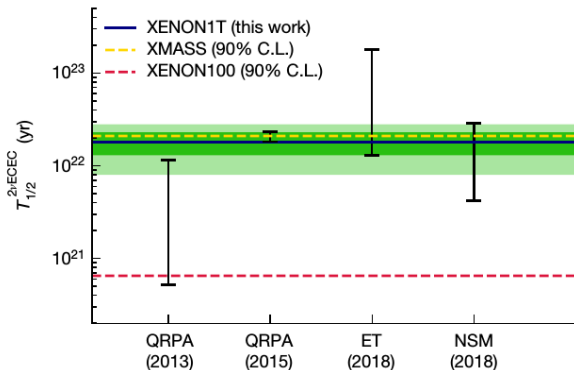
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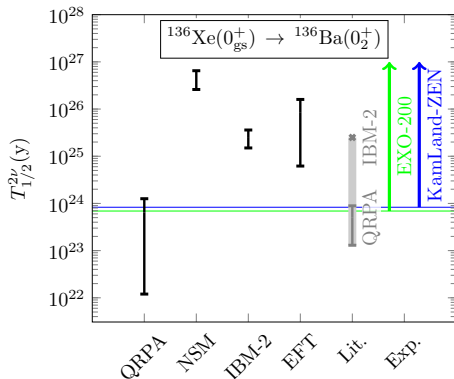
XENON1T
Nature 568 532 (2019)

Shell model, QRPA and Effective theory (ET) predictions suggest experimental detection close to XMASS 2018 limit

$2\nu\beta\beta$ decay of ^{136}Xe to $^{136}\text{Ba } 0_2^+$

Current experiments sensitive to two-neutrino $\beta\beta$ of ^{136}Xe to $^{136}\text{Ba } 0_2^+$

EXO-200, KamLAND-Zen



Nuclear shell model
QRPA, EFT and IBM
very different predictions!

Barea et al.
PRC 91 034304 (2015)

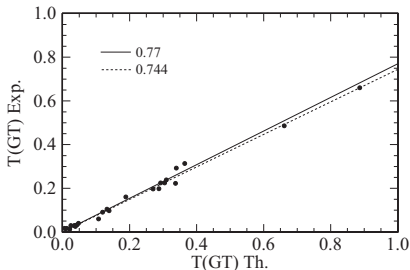
Pirinen, Suhonen
PRC 91, 054309 (2015)

Jokiniemi, Romeo, Brase, Kotila et al.
PLB 838 137689 (2023)

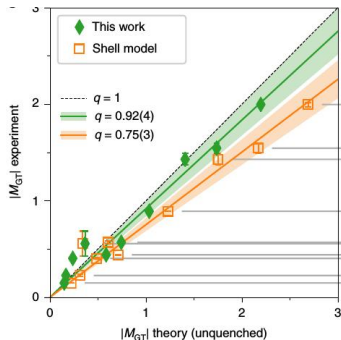
Very good test of theoretical calculations!

Gamow-Teller β decay with IM-SRG

β decays (e^- capture) challenge for nuclear theory



Martinez-Pinedo et al. PRC53 2602(1996)



Gysbers et al. Nature Phys. 15 428 (2019)

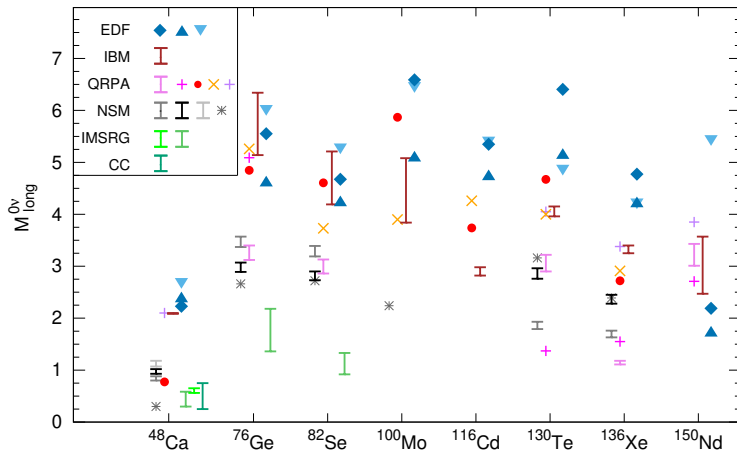
$$\langle F | \sum_i [g_A \sigma_{iT_i}^-]^{\text{eff}} | I \rangle, \quad [\sigma_{iT}]^{\text{eff}} \approx 0.7 \sigma_{iT}$$

Phenomenological models
need σ_{iT} “quenching”

Ab initio calculations
including correlations
and meson-exchange currents
do not need any “quenching”

$0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations: factor ~ 3



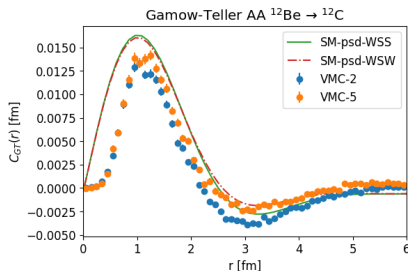
Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

Shell model vs quantum Monte Carlo: correlations

Compare $\beta\beta$ transition densities in nuclear shell model and quantum Monte Carlo calculations in light nuclei

$$4\pi r^2 \rho_{GT}(r) = \langle \Psi_f | \sum_{a < b} \delta(r - r_{ab}) \sigma_{ab} \tau_a^+ \tau_b^+ | \Psi_i \rangle \quad M_{GT}^{0\nu} = \int_0^\infty dr C_{GT}^{0\nu},$$

Agree at long distances, shell model misses short-range correlations



Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Similar findings in Wang et al. PLB 798 134974 (2019)

Generalized contact formalism (GCF)

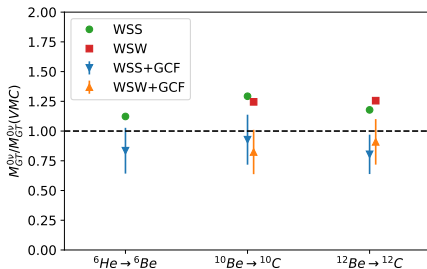
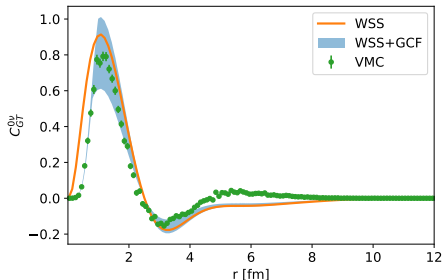
Generalized contact formalism Weiss, Bazak, Barnea, PRL 114 012501 (2015)

Separation of scales: wf, transition density factorize for nearby nucleons

$$\Psi \xrightarrow{r_{ij} \rightarrow 0} \sum_{\alpha} \varphi^{\alpha}(\mathbf{r}_{ij}) A^{\alpha}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j}), \quad \rho_{GT}(r) \xrightarrow{r \rightarrow 0} -3|\varphi^0(r)|^2 C_{pp,nn}^0(f, i)$$

The contact $C^0(f, i) = \frac{A(A-1)}{2} \langle A^{\alpha}(f) | A^{\beta}(i) \rangle$ is model dependent

Replace shell-model by QMC contact: improve transition density



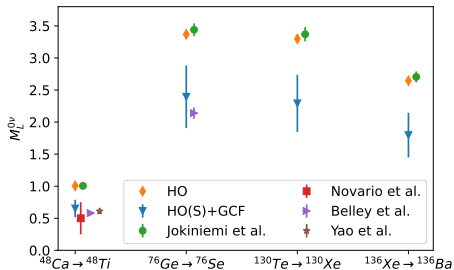
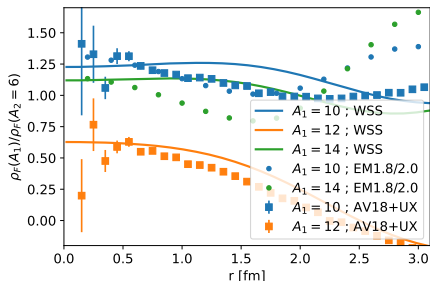
Shell model + Generalized contact formalism

Generalized contact formalism Weiss, Bazak, Barnea PRL 114 012501 (2015)

The contact $C^0(f, i) = \frac{A(A-1)}{2} \langle A^\alpha(f) | A^\beta(i) \rangle$ is model dependent
(shell model, quantum Monte Carlo, no-core shell model...)

but the ratio $C_{pp,nn}^0(X)/C_{pp,nn}^0(Y)$ relatively model independent:

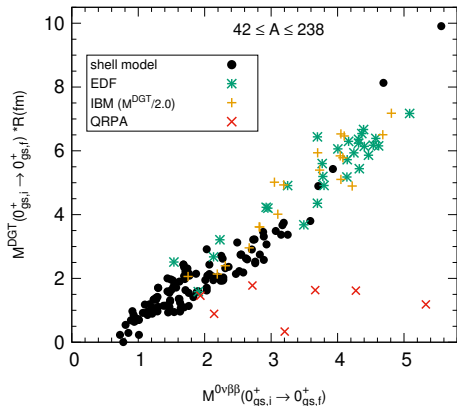
QMC in light nuclei + two shell-model calculations: reach heavy nuclei



Good agreement benchmark in light nuclei with ab initio NMEs

Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

DGT and $0\nu\beta\beta$ decay: heavy nuclei



Shimizu, JM, Yako, PRL120 142502 (2018)

DGT transition to ground state

$$M^{\text{DGT}} = \sqrt{B(\text{DGT}_{-;0}; 0_{\text{gs}}^+ \rightarrow 0_{\text{gs}}^+)}$$

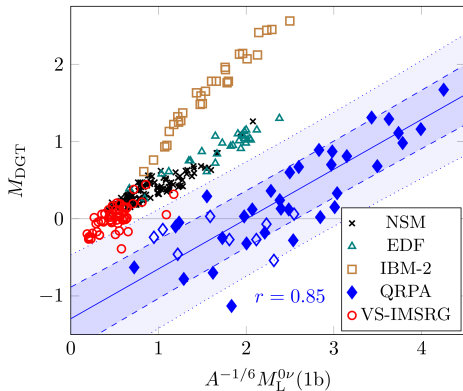
very good linear correlation
with $0\nu\beta\beta$ decay
nuclear matrix elements

Correlation across wide range,
nuclei from Ca to Ge and Xe

Found in:
nuclear shell model
energy-density functional theory
QRPA
ab initio VS-IMSRG (weaker)

Yao et al. PRC106 014315(2022)
Jokiniemi, JM, PRC 107 044316 (2023)
Wang et al. PLB 855 138796 (2024)

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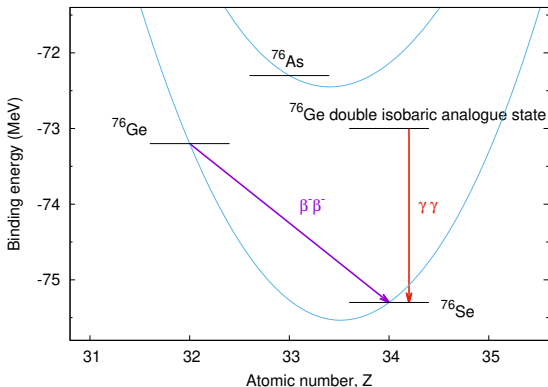
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Yao et al. PRC106 014315(2022)
Jokiniemi, JM, PRC 107 044316 (2023)
Wang et al. PLB 855 138796 (2024)

$\gamma\gamma$ decay of the DIAS of the initial $\beta\beta$ nucleus

Explore correlation between $0\nu\beta\beta$ and $\gamma\gamma$ decays, focused on double-M1 transitions

$$M_{M1 M1}^{\gamma\gamma} = \sum_k \frac{\langle 0_f^+ | \sum_n (g_n^I I_n + g_n^S \sigma_n)^{IV} | 1_k^+ (\text{IAS}) \rangle \langle 1_k^+ (\text{IAS}) | \sum_m (g_m^I I_m + g_m^S \sigma_m)^{IV} | 0_i^+ (\text{DIAS}) \rangle}{E_k - (E_i + E_f)/2}$$



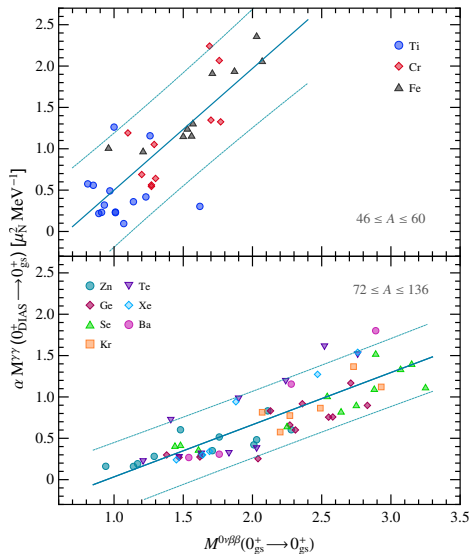
Similar initial and final states but both in same nucleus for electromagnetic transition

M1 and GT operators similar, physics of spin operator
M1 also angular momentum

Different energy denominator

Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Correlation between $M1M1$ and $0\nu\beta\beta$ NMEs



Good correlation between $M1M1$ same-energy photons and shell-model $0\nu\beta\beta$ NMEs

A dependence:
energy denominator
dominant states at higher energy in heavier nuclei

Overall, study ~ 50 transitions
several nuclear interactions
for each of them

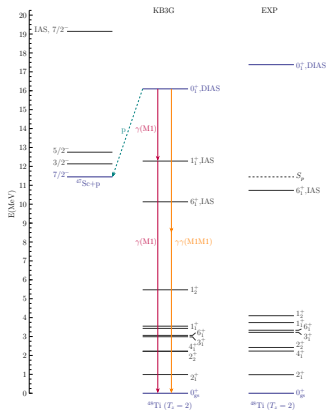
Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Experimental feasibility of $\gamma\gamma$ decay?

$\gamma\gamma$ decays are very suppressed with respect to γ decays
just like $\beta\beta$ decays are much slower than β decays

$\gamma\gamma$ decays have been observed recently
in competition with γ decays

Waltz et al. Nature 526, 406 (2015), Soderstrom et al. Nat. Comm. 11, 3242 (2020)



Competing decay channels:

Study in detail leading
decay channels for $M1M1$ decay
in DIAS of $\beta\beta$ nuclei

Particle emission $M1$, $E1$ decay:
 $BR \sim 10^{-7} - 10^{-8}$

Experimental proposal for ^{48}Ti
by Valiente-Dobón et al.

Romeo, Stramaccioni et al., in prep

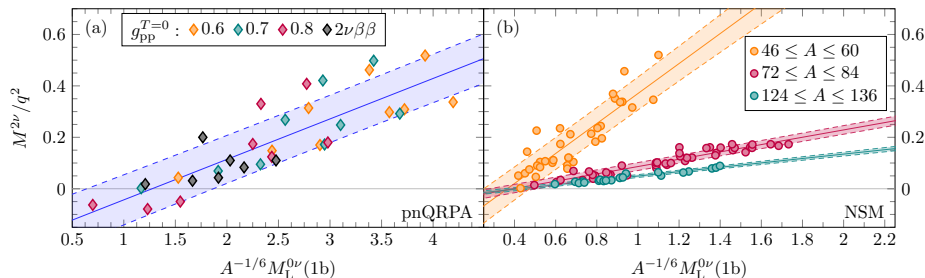
Correlation of $0\nu\beta\beta$ decay and $2\nu\beta\beta$ decay

Good correlation between 2ν and 0ν modes of $\beta\beta$ decay in nuclear shell model (systematic calculations of different nuclei) and QRPA calculations (decays of $\beta\beta$ emitters with different g_{pp} values)

Similar but not common correlation, depends on mass for shell model

$0\nu\beta\beta - 2\nu\beta\beta$ correlation also observed in ^{48}Ca , ^{136}Xe

Horoi et al. PRC106 054302 (2022), PRC107 045501 (2023)

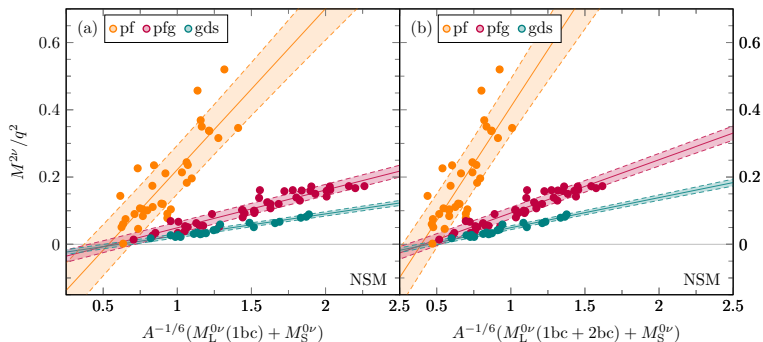


Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)



Correlation of $0\nu\beta\beta$ decay to $2\nu\beta\beta$: general case

A good correlation between $2\nu\beta\beta$ and $0\nu\beta\beta$ also appears when we include to the calculation of $0\nu\beta\beta$ NMEs 2b currents and the short-range nuclear matrix element

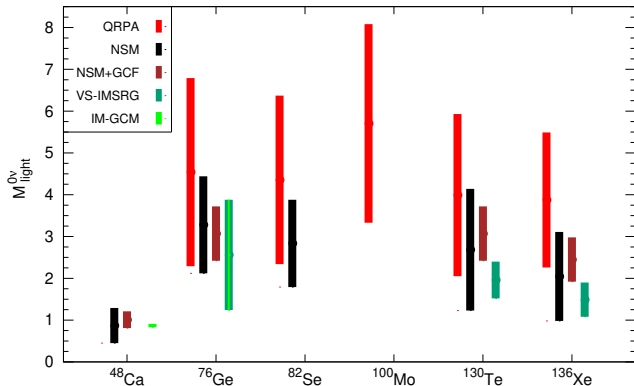


Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs with 2b currents, short-range NME

$0\nu\beta\beta$ decay total (long- and short-range) NMEs

Not-so-large difference in nuclear matrix element calculations!



Wirth et al.
PRL127 242502 (2021)

Belley et al.
arXiv:2307:15156

Belley et al.
PRL132 182502 (2024)

Jokiniemi et al.
PRC 107 044305 (2023)

Weiss et al.
PRC106 065501 (2022)

Gómez-Cadenas, Martín-Albo, JM, Mezzeto, Monrabal, Sorel, Riv. Nuovo Cim. 46, 619 (2023)

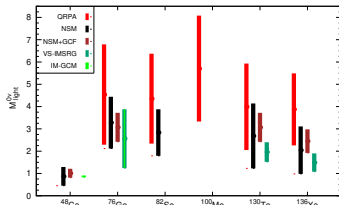
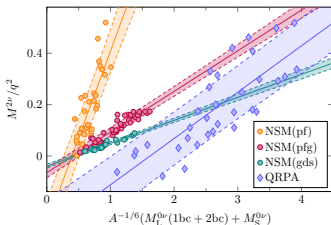
Summary

Calculations of $0\nu\beta\beta$ NMEs challenge nuclear many-body methods, searches demand reliable NMEs

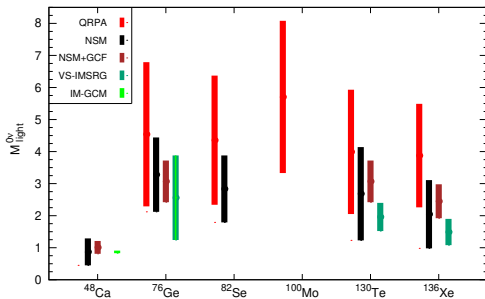
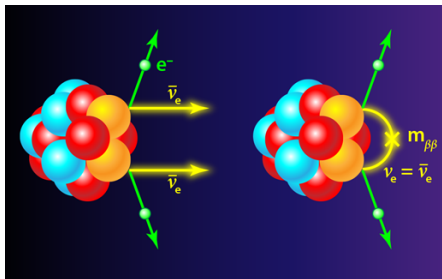
Individual nuclear spectroscopy $2\nu\beta\beta$ measurements test many-body methods used to compute $0\nu\beta\beta$ NMEs

Systematic calculations of double Gamow-Teller transitions, electromagnetic $M1M1$ decay of DIAS good correlation with $0\nu\beta\beta$ NMEs

Good $0\nu\beta\beta - 2\nu\beta\beta$ correlation exploit $2\nu\beta\beta$ data to obtain $0\nu\beta\beta$ NMEs with theoretical uncertainties



Thank you very much for your attention!



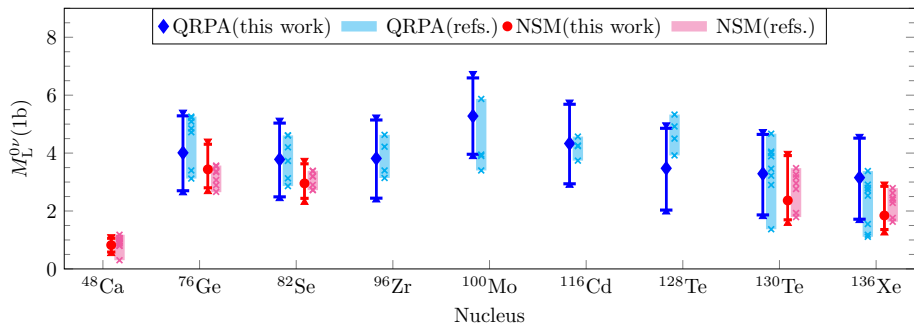
$0\nu\beta\beta$ NMEs from $2\nu\beta\beta - 0\nu\beta\beta$ correlation

NMEs consistent with previous nuclear shell model, QRPA results

Theoretical uncertainty involves

systematic calculations covering dozens of nuclei and interactions
error of each calculation (eg quenching) and experimental $2\nu\beta\beta$ error

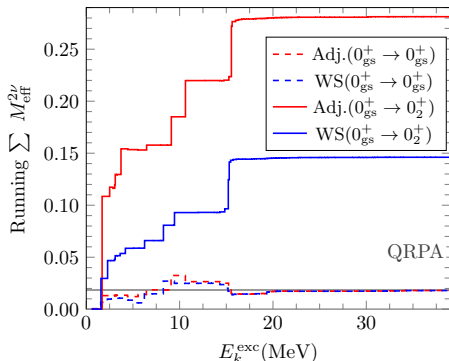
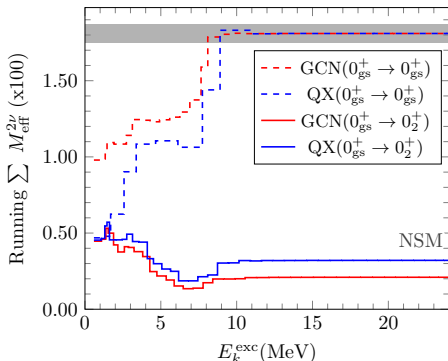
Previous theoretical uncertainty mostly ignored: collection of calculations



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

$^{136}\text{Xe} \rightarrow ^{136}\text{Ba } 0_2^+$ running sums

Subtle cancellation NME running sum, depends on many-body method



Jokiniemi, Romeo, Brase, Kotila et al. PLB 838 137689 (2023)

Shell-model running sum shows cancellations in decay to ground state

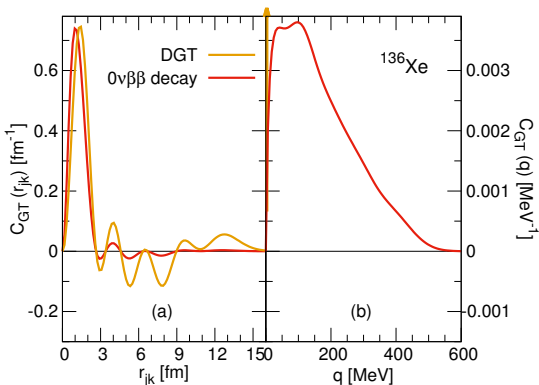
QRPA running sum shows cancellations in decay to excited state

Since ground-state decay fitted to data, very different decay to excited state

Short-range character of DGT, $0\nu\beta\beta$ decay

Correlation between DGT and $0\nu\beta\beta$ decay matrix elements explained by transition involving low-energy states combined with dominance of short distances between exchanged/decaying neutrons

Bogner et al. PRC86 064304 (2012)



$0\nu\beta\beta$ decay matrix element limited to shorter range

Short-range part dominant in double GT matrix element due to partial cancellation of mid- and long-range parts

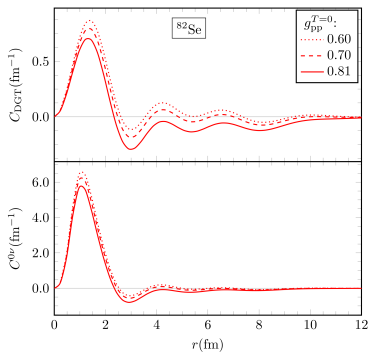
Long-range part dominant in QRPA DGT matrix elements

Shimizu, JM, Yako,
PRL120 142502 (2018)

Short-range character of DGT, $0\nu\beta\beta$ decay

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Bogner et al. PRC86 064304 (2012)



Jokiniemi, JM, PRC 107 044316 (2023)

$0\nu\beta\beta$ decay matrix element limited to shorter range

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Long-range part dominant in QRPA DGT matrix elements

Shimizu, JM, Yako, PRL120 142502 (2018)

Short-range NME: relative impact

Modified decay rate:

$$T_{1/2}^{-1} = G_{01} g_A^4 (M_{\text{long}}^{0\nu} + M_{\text{short}}^{0\nu})^2 \frac{m_{\beta\beta}^2}{m_e^2}$$

Assume $g_{\nu}^{NN} \sim 1 \text{ fm}^2$

Cirigliano et al.

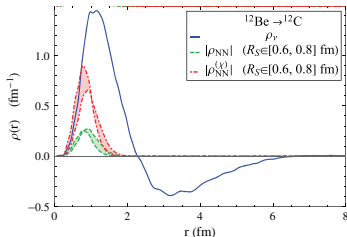
PRC100 055504 (2019)

TABLE II. Values of $C_1 + C_2$ obtained from the CIB contact interactions in various chiral potentials.

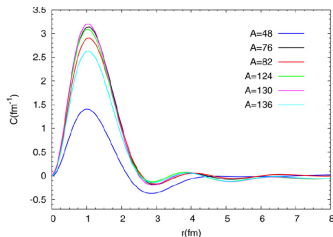
| Model | Ref. | R_S (fm) | C_U^{TT} (fm ²) | $(C_1 + C_2)/2$ (fm ²) | Model | Ref. | Λ (MeV) | $(C_1 + C_2)/2$ (fm ²) |
|---------|------|------------|-------------------------------|------------------------------------|-----------------------|------|-----------------|------------------------------------|
| NV-Ia* | [38] | 0.8 | 0.0158 | -1.03 | Entem-Machleidt | [34] | 500 | -0.47 |
| NV-IIa* | [38] | 0.8 | 0.0219 | -1.44 | Entem-Machleidt | [34] | 600 | -0.14 |
| NV-Ic | [38] | 0.6 | 0.0219 | -1.44 | Reinert <i>et al.</i> | [39] | 450 | -0.67 |
| NV-IIc | [38] | 0.6 | 0.0139 | -0.91 | Reinert <i>et al.</i> | [39] | 550 | -1.01 |
| | | | | | NNLO _{sat} | [37] | 450 | -0.39 |

$\sim 75\%$ correction for QMC ^{12}Be NME

In heavy nuclei, less severe cancellation of dominant $M^{0\nu}$?



Cirigliano et al. PRL120 202001(2018)



JM et al. NPA818 139 (2009)

2b currents in $0\nu\beta\beta$ decay

In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator when including the product of two 2b currents: computational challenge

Approximate 2b current as effective 1b current normal ordering with respect to a Fermi gas

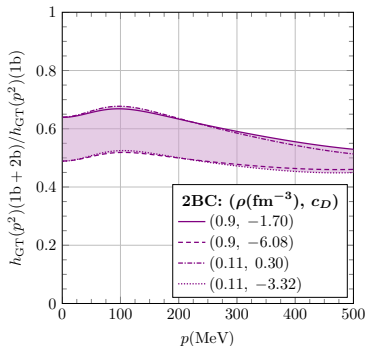
JM, Gazit, Schwenk, PRL107 062501(2011)

Normal-ordering approximation works remarkably well for β decay ($q = 0$)

Gysbers et al. Nature Phys. 15 428 (2019)

Some reduction of quenching due to 2b currents at $p \sim m_\pi$ relevant for $0\nu\beta\beta$ decay

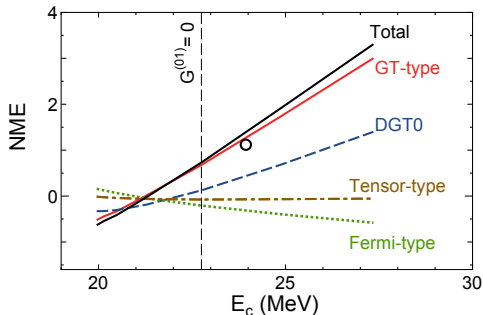
Hoferichter, JM, Schwenk PRD102 074018 (2020)



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

^{48}Ca double GT giant resonance and $0\nu\beta\beta$ decay

Correlation between Double Gamow-Teller resonance in ^{48}Ca and $0\nu\beta\beta$ decay nuclear matrix element



Shimizu, JM, Yako, PRL120 142502 (2018)

Energy of DGT resonance with accuracy to ~ 1 MeV, can give insight on value of $0\nu\beta\beta$ decay matrix element

$$E_c = \frac{\sum_f E_f B(DGT^-, i \rightarrow f)}{\sum_f B(DGT^-, i \rightarrow f)}$$

Good test of nuclear structure calculation

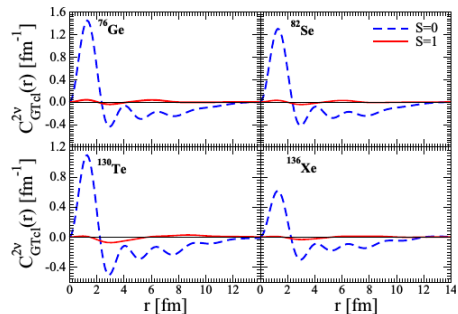
Relatively consistent with *sdpf* calculation (open circle)

Contribution of $S = 1$ pairs to DGT NME

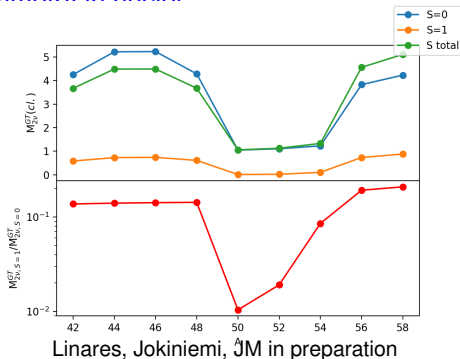
NMEs are dominated by $S = 0$ pairs

In DGT QRPA, $S = 1$ pair contributions extremely suppressed lead to small $M^{DGT} = M_{S=0}^{DGT} + M_{S=1}^{DGT} = 4M_{S=1}^{DGT}$ (isospin conserved)

In contrast, nuclear shell model $M_{S=1}^{DGT}$ small but not negligible, understood in terms of broken $SU(4)$ symmetry in nuclei



Simkovic, Smetana, Vogel PRC98 064325(2019)



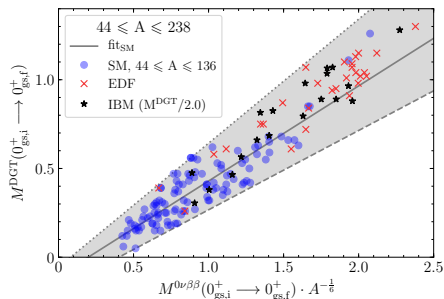
Linares, Jokiniemi, JM in preparation

$0\nu\beta\beta$ decay NMEs in EFT of β decay

Effective field theory of β decay can calculate DGT with uncertainties
(similar to calculation of $2\nu\beta\beta$, no energy denominator)

DGT vs $0\nu\beta\beta$ correlation \Rightarrow predict $0\nu\beta\beta$ NMEs with uncertainties

Because EFT couplings fitted to β decay and GT strengths
shell-model DGT NMEs in correlation need “quenching”: $q = 0.42 - 0.65$



As a result, ET $0\nu\beta\beta$ NMEs

^{76}Ge : $M^{0\nu} = 0.2 - 2.4$

^{82}Se : $M^{0\nu} = 0.2 - 2.7$

small NMEs

large uncertainty:

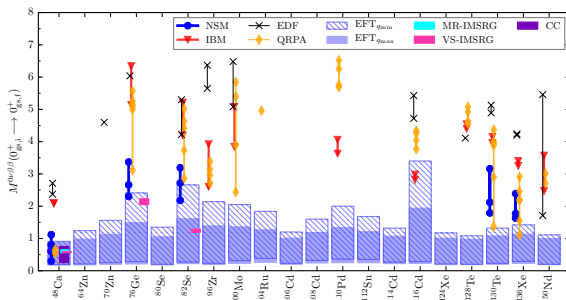
LO in ET, fit, “quenching”

Brase, Coello Pérez, JM, Schwenk
PRC106 034309(2022)

$0\nu\beta\beta$ decay NMEs in EFT of β decay

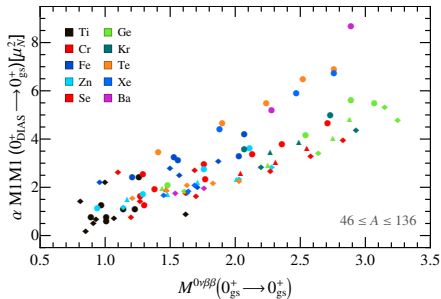
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Brase, Coello Pérez, JM, Schwenk, PRC106 034309 (2022)

Intermediate states of the $M1M1$ transition

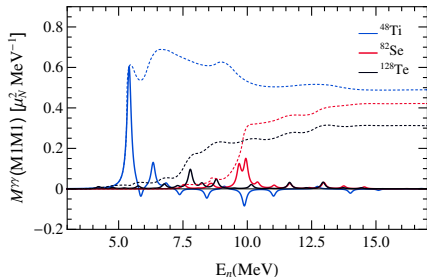


Dominant intermediate states
lower energies for lighter nuclei,
otherwise similar energies

One or few intermediate states
typically dominate the transition

When energy denominators are
(artificially) removed,
same correlation
across the nuclear chart

Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

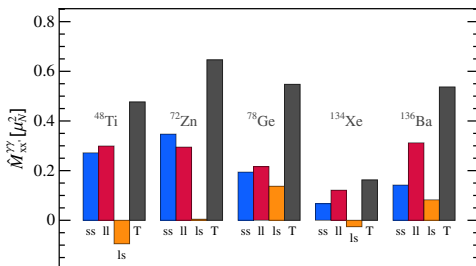


Spin, angular momentum decomposition

The numerator NME can be decomposed into

$$\hat{M}_{\gamma\gamma} = \hat{M}_{ss} + \hat{M}_{ll} + \hat{M}_{ls}$$

spin, angular momentum and interference components



Spin, angular momentum terms
strikingly similar,
always carry same sign

Interference term
can cancel the other two
but always much smaller

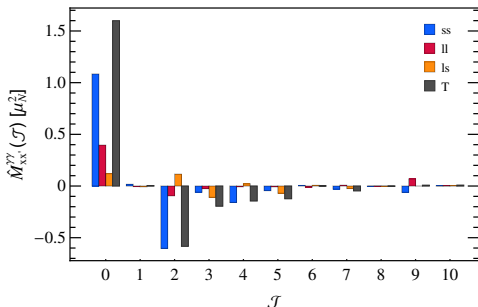
Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Total angular momentum decomposition

The numerator NME can be decomposed into

$$\hat{M}_{\gamma\gamma}(\mathcal{J}) = \hat{M}_{ss}(\mathcal{J}) + \hat{M}_{ll}(\mathcal{J}) + \hat{M}_{ls}(\mathcal{J})$$

spin, angular momentum and interference components
and total angular momentum of the nucleons involved in the transition



Dominance of $\mathcal{J} = 0$ terms
for spin and orbital contributions
just like in $0\nu\beta\beta$ decay

Cancellation from $\mathcal{J} > 0$ terms
less pronounced in orbital part
Explains similar behaviour of spin
and orbital components:

$$s_1 s_2 = S^2 - 3/2 < 0$$

$$l_1 l_2 = \mathcal{L}^2 - l_1^2 - l_2^2 < 0$$

Romeo et al. PLB 827 136965 (2022)

Nuclear matrix elements for new-physics searches

Neutrinos, dark matter studied in experiments using nuclei

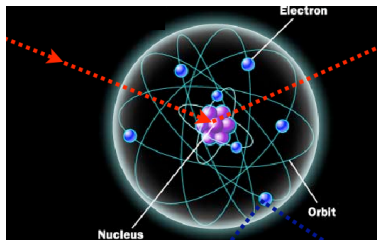
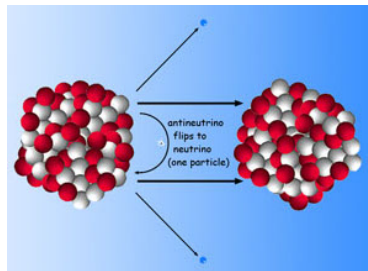
Nuclear structure physics encoded in nuclear matrix elements
key to plan, fully exploit experiments

$$0\nu\beta\beta: \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4 |M^{0\nu\beta\beta}|^2 m_{\beta\beta}^2$$

$$\text{Dark matter: } \frac{d\sigma_{\chi\mathcal{N}}}{dq^2} \propto \left| \sum_i c_i \zeta_i \mathcal{F}_i \right|^2$$

$$\text{CE}\nu\text{NS: } \frac{d\sigma_{\nu\mathcal{N}}}{dq^2} \propto \left| \sum_i c_i \zeta_i \mathcal{F}_i \right|^2$$

$M^{0\nu\beta\beta}$: Nuclear matrix element
 \mathcal{F}_i : Nuclear structure factor



Systematic shell-model calculations

Explore systematic shell-model matrix elements
in configuration spaces relevant for $0\nu\beta\beta$ decay searches

- $^{46-58}\text{Ca}$, $^{50-58}\text{Ti}$, and $^{54-60}\text{Cr}$
in pf-shell with KB3G and GXPF1B interactions
- $^{72-76}\text{Ni}$, $^{74-80}\text{Zn}$, $^{76-82}\text{Ge}$, and $^{82,84}\text{Se}$
in $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$, and $0g_{9/2}$ configuration space
with GCN2850, JUN45, and JJ4BB interactions
- $^{124-132}\text{Sn}$, $^{130-134}\text{Te}$, and $^{134,136}\text{Xe}$
in $1d_{5/2}$, $0g_{7/2}$, $2s_{1/2}$, $1d_{3/2}$, and $0h_{11/2}$ configuration space
with the GCN5082 and QX interactions

Overall, $\sim 20 - 40$ different calculations for each configuration space

Complementary approach to randomly varying nuclear interaction

Horoi et al. PRC106 054302 (2022), PRC107 045501 (2023)

2b currents in $0\nu\beta\beta$ decay

In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator when including the product of two 2b currents: computational challenge

Approximate 2b current as effective 1b current normal ordering with respect to a Fermi gas

JM, Gazit, Schwenk, PRL107 062501(2011)

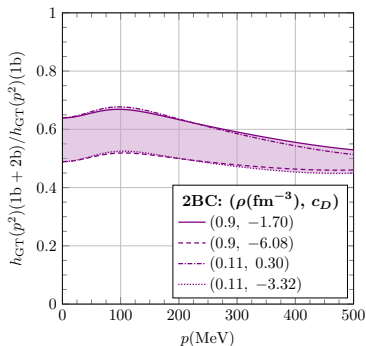
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Hoferichter, JM, Schwenk

PRD102 074018 (2020)

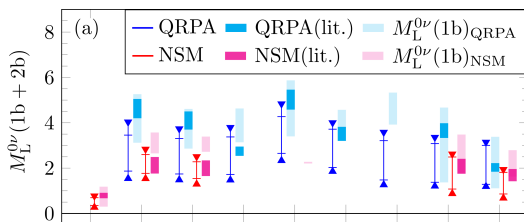
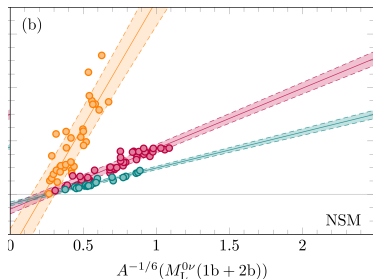


Jokiniemi, Romeo, Soriano, JM

PRC 107 044305 (2023)

Correlation of $0\nu\beta\beta$ decay to $2\nu\beta\beta$: 2b currents

Good correlation between $2\nu\beta\beta$ and $0\nu\beta\beta$
also present when including 2b currents to the $0\nu\beta\beta$ NMEs



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs with 2b currents

Light-neutrino exchange: contact operator

Short-range operator contributes to light-neutrino exchange
for RG invariance of two-nucleon decay amplitude: high-energy ν 's

$$T_{1/2}^{-1} = G_{01} g_A^4 (M_{\text{long}}^{0\nu} + M_{\text{short}}^{0\nu})^2 m_{\beta\beta}^2, \quad \text{Cirigliano et al. PRL120 202001(2018)}$$

$$M_{\text{short}}^{0\nu} \equiv \frac{1.2A^{1/3} \text{ fm}}{g_A^2} \langle 0_f^+ | \sum_{n,m} \tau_m^- \tau_n^- \mathbb{1} \left[\frac{2}{\pi} \int j_0(qr) 2g_\nu^{\text{NN}} g(p/\Lambda) p^2 dp \right] | 0_i^+ \rangle,$$

$$M_{\text{GT}}^{0\nu} \simeq \frac{1.2A^{1/3} \text{ fm}}{g_A^2} \langle 0_f^+ | \sum_{n,m} \tau_m^- \tau_n^- \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \left[\frac{2}{\pi} \int j_0(qr) \frac{1}{p^2} g_A^2 f^2(p/\Lambda) p^2 dp \right] | 0_i^+ \rangle$$

Unknown value (and sign) of the hadronic coupling g_ν^{NN} !

Lattice QCD calculations can obtain value of g_ν^{NN}

Davoudi, Kadam, Phys. Rev. Lett. 126, 152003 (2021), PRD105 094502('22)

match $nn \rightarrow pp + ee$ amplitude calculated with dispersion QCD methods

Cirigliano et al. PRL126 172002 (2021), JHEP 05 289 (2021)

charge-independence breaking of nuclear Hamiltonians

Cirigliano et al. PRC100, 055504 (2019)

Long and short-range NME in heavy nuclei

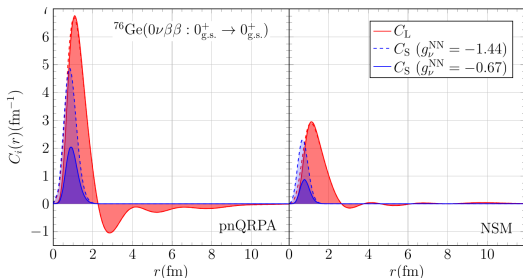
Relatively stable contribution of new term M_S/M_L :

20% – 50% impact of short-range NME in shell model

30% – 70% impact of short-range NME in QRPA

consistent with 43% effect in IM-GCM for ^{48}Ca

using calculated $nn \rightarrow pp + ee$ decay Wirth et al. PRL127 242502 (2021)



Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021)

Uncertainty dominated by coupling g_ν^{NN}

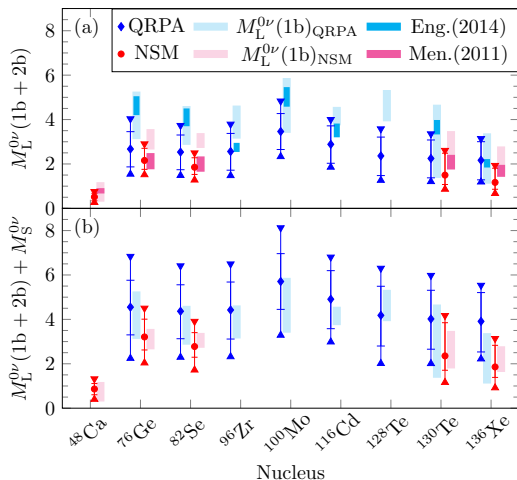
$0\nu\beta\beta$ NMEs from correlation: 2bc, short-range

$0\nu\beta\beta$ NMEs including 2b currents and short-range NME obtained from $0\nu\beta\beta - 2\nu\beta\beta$ correlation and $2\nu\beta\beta$ data

Theoretical uncertainty due to correlation, calculation uncertainties: quenching, 2bc, short-range NME coupling (dominant uncertainty)

First complete estimation of $0\nu\beta\beta$ nuclear matrix elements with theoretical uncertainties

Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)



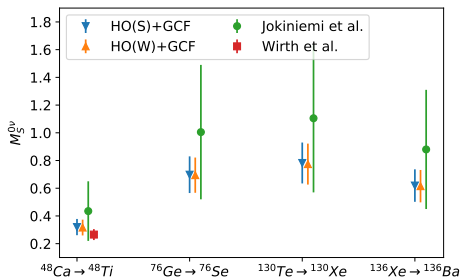
Short-range NME: GCF + shell model

Shell model with short-range correlations from QMC using the GCF
give consistent contribution of new term M_S

~ 25% impact of short-range NME in GCF + shell model
obtained with g_ν^{NN} from AV18 CIB term

consistent with 43% effect in IM-GCM for ^{48}Ca

using calculated $nn \rightarrow pp + ee$ decay Wirth et al. PRL127 242502 (2021)



Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)