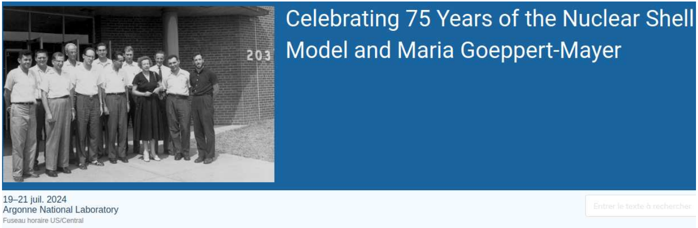
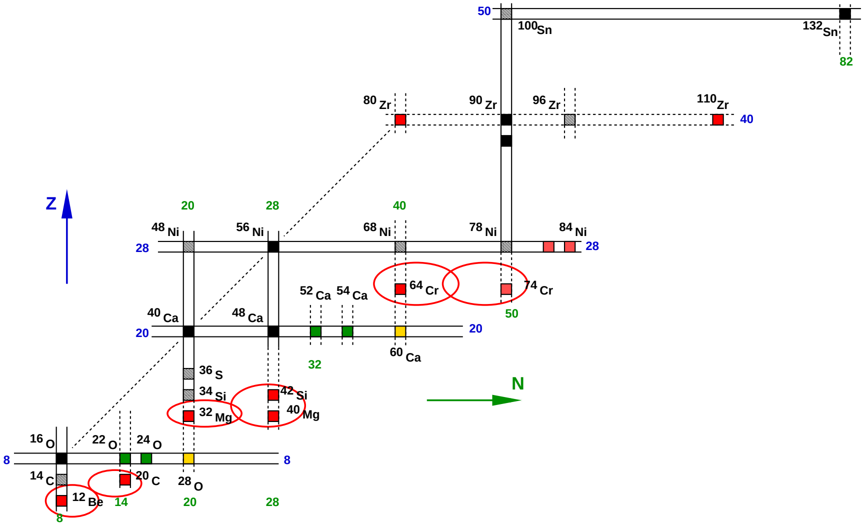
Isospin Symmetric Island of Inversion at the N=Z line

Duy Duc Dao, Frédéric Nowacki



CNRS IN2P3 Les deux infinis
IPHC Institut Pluridisciplinaire Hubert CURIEN STRASBOURG
UNIVERSITÉ DE STRASBOURG

Landscape of medium mass nuclei



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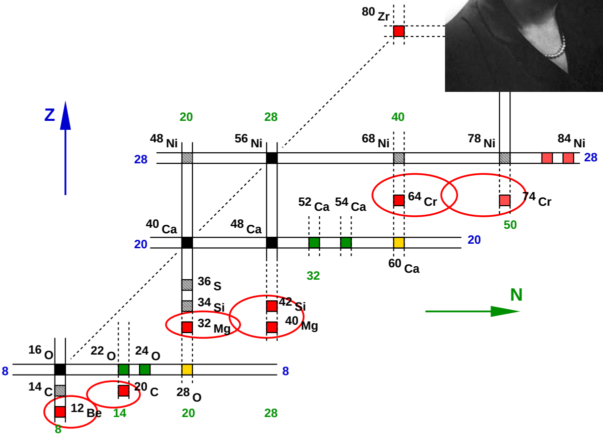

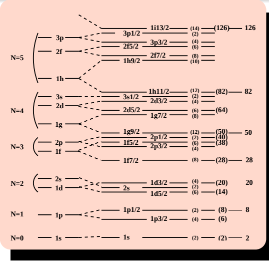

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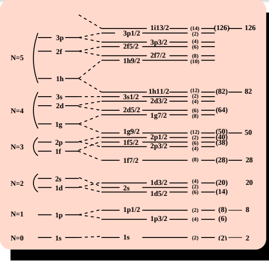

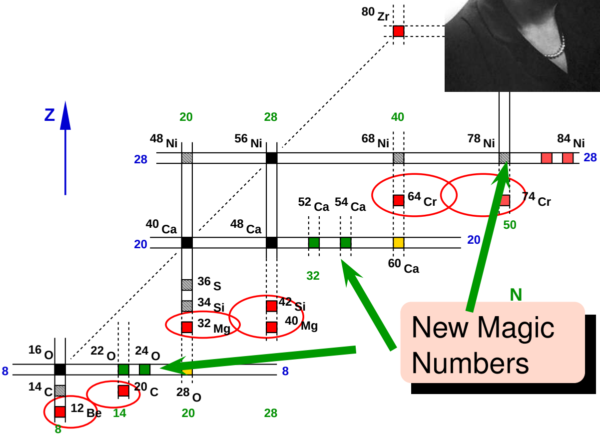

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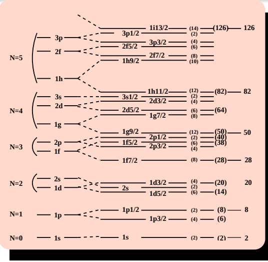
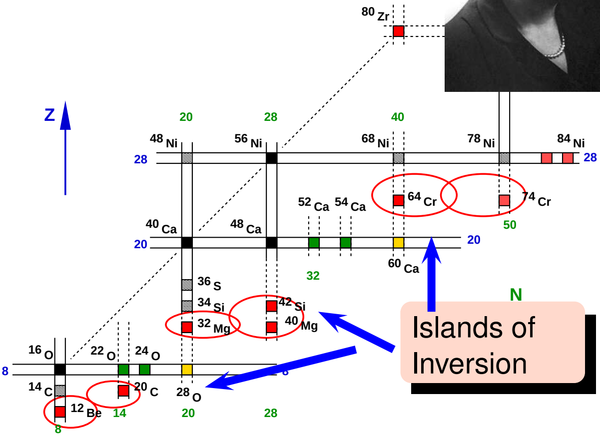


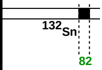


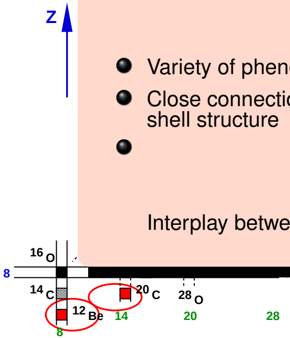
Landscape of medium mass nuclei

UNDERSTANDING REGULARITIES
for both SPHERICAL and DEFORMED systems

- New Magic Numbers: ^{24}O, ^{48}Ni, ^{54}Ca, ^{78}Ni, ^{100}Sn
- Vanishing of shell closures: ^{12}Be, ^{32}Mg, ^{42}Si, ^{64}Cr, ^{80}Zr ...
- Island of deformation around A$\sim$ 32, A$\sim$ 64
- Low-lying dipole excitations in Ne, Ni isotopes
- Variety of phenomena dictated by shell structure
- Close connection between collective behaviour and underlying shell structure
- $$\mathcal{H} = \mathcal{H}_m + \mathcal{H}_\mathcal{M}$$

  Interplay between
  - Monopole field (spherical mean field)
  - Multipole correlations (pairing, Q.Q, ...)



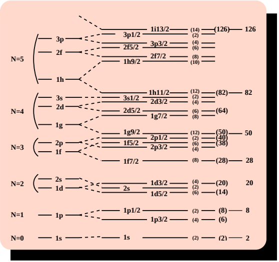


Effective Hamiltonian
Monopole and multipole

Multipole expansion: $\mathcal{H} = \mathcal{H}_{monopole} + \mathcal{H}_{Multipole}$

$\mathcal{H}_{monopole}$:
- *Spherical mean-field*
- *Evolution of the spherical single particle levels*

A. Poves and A. Zuker (Phys. Report 70, 235 (1981))



$\mathcal{H}_{multipole}$:
- *Correlations*
- *Energy gains*

- *Pairing (SU2)* semi-magic (n-n) (p-p)
- *Quadrupole (SU3/pSU3/qSU3)* p-n in H.O. or Δj = 2

M. Dufour and A. Zuker (PRC 54 1996 1641)

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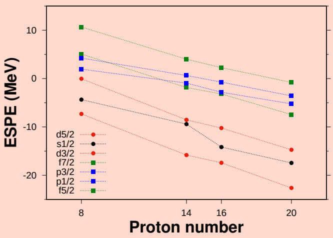

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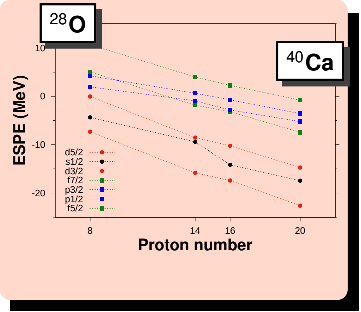

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Effective Hamiltonian
Monopole and multipole

Multipole expansion: $\mathcal{H} = {\color{red}\mathcal{H}_{monopole}} + {\color{blue}\mathcal{H}_{Multipole}}$

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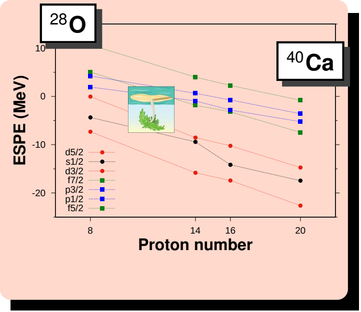

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Effective Hamiltonian

Monopole and multipole

Multipo

$H_{monopole}$:

A. Poves an

$H_{multipole}$:

- *Pai*
- *Qu*

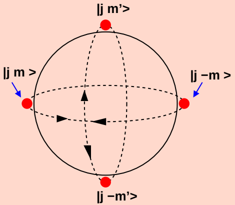
M. Dufour a

- **Pairing regime: spherical nuclei**

  ground state = pairs of like-particles coupled at J=0 (seniority v=0)

  2^+ state (break of pair; v=2) at high energy

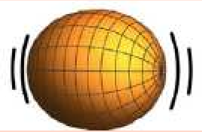
  superfluid nucleus:



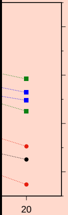
Typical example: **semi-magic Tin isotopes**

- **Quadrupole regime: deformed nuclei**

  prolate nucleus:



Typical example: **open shell N=Z nuclei**



Effective Hamiltonian
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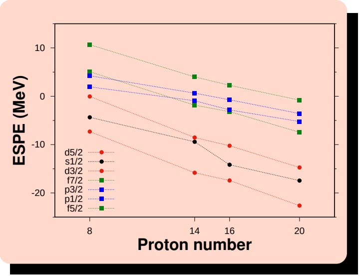

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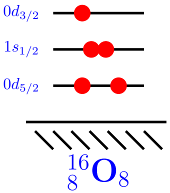
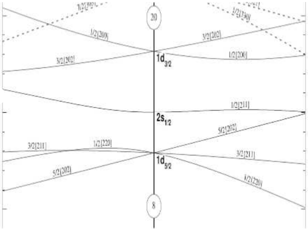
Nilsson-SU3 estimates

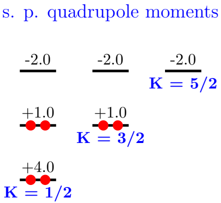
PHYSICAL REVIEW C **92**, 024320 (2015)

Nilsson-SU3 self-consistency in heavy $N = Z$ nuclei

A. P. Zuker,[1] A. Poves,[2,3] F. Nowacki,[1] and S. M. Lenzi[4]







$$Q_0 = 2q^{20} = (2n_z - n_x - n - y)$$

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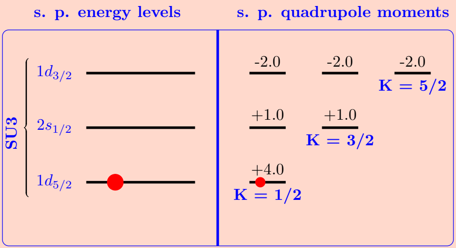

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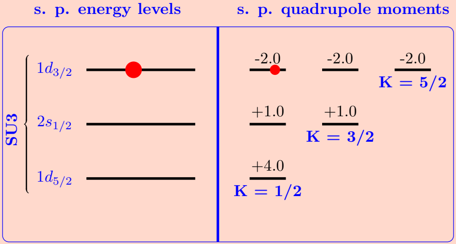

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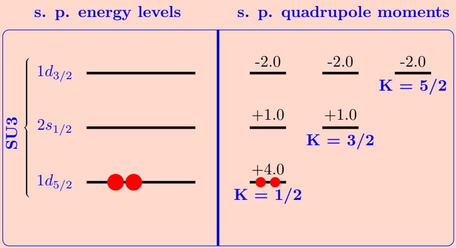

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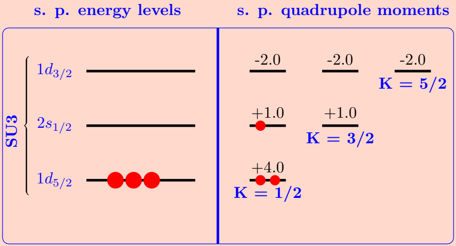

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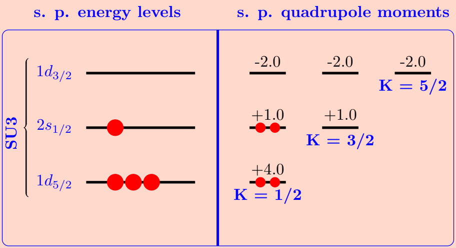

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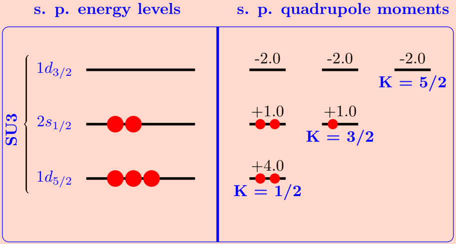

ments
2.0
= 5/2

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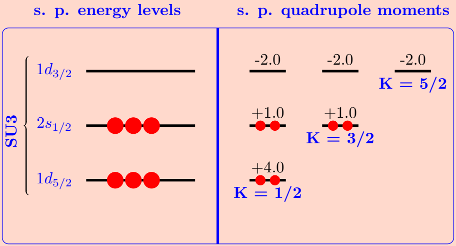

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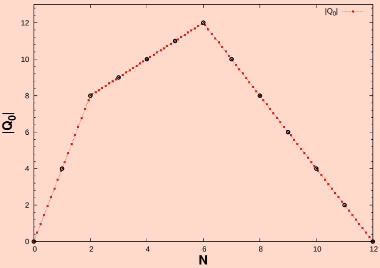

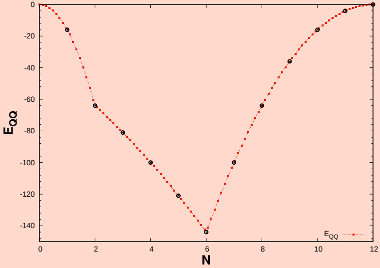

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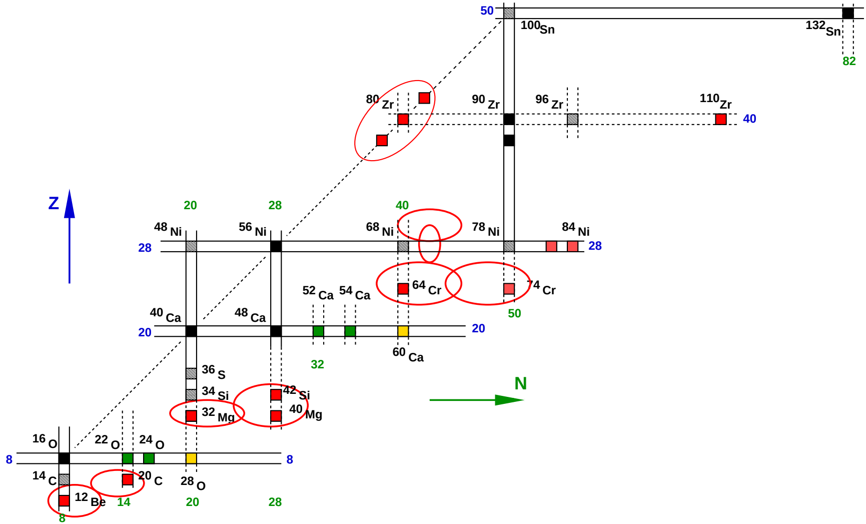
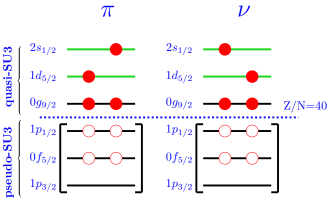
Landscape of medium mass nuclei



◇ Strongly deformed states at $N = Z$:

- Configuration mixing in ^{72}Kr
- Most deformed cases for ^{76}Sr, ^{80}Zr
- New spectroscopy for ^{84}Mo and ^{86}Mo
  NSCL/GRETINA Experiment



R.D.O. Llewellyn *et al.*, PRL **124**, 152501 (2020)

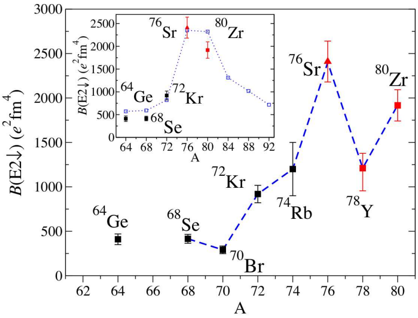


FIG. 3. Schematics of the $B(E2\downarrow)$ values for the $N = Z$ nuclei

- ZBM3 valence space:
  extension of JUN45
  to pseudo-SU3 + Quasi-SU3

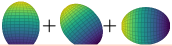
- New effective interactions:
  - Realistic TBME + Monopole “3N” constraints”
  - ab-initio N3LO (2N) interaction
  - ongoing ab-initio N3LO (2N) + 3N (lnl) interaction

- SM + DNO-SM for most deformed cases

Discrete Non-Orthogonal Shell Model

Generator Coordinate Method: $|\Psi_{\text{eff}}\rangle = \sum_i f_i |\Phi_i\rangle$

1) Deformed Hartree-Fock (HF) Slater determinants
2) Restoration of rotational symmetry
3) Mixing of shapes:

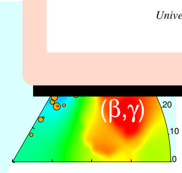
$|\Psi_{\text{eff}}\rangle =$  $\cdots$

PHYSICAL REVIEW C **105**, 054314 (2022)

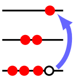
Nuclear structure within a discrete nonorthogonal shell model approach: New frontiers

D. D. Dao and F. Nowacki
Université de Strasbourg, CNRS, IPHC UMR7178, 23 rue du Loess, F-67000 Strasbourg, France

(Received 8 March 2022; accepted 6 May 2022; published 23 May 2022)



- **particle-hole interpretation**:



M-**scheme**

- ***K*–quantum numbers**:

$$P_\alpha^{(J)}(K) = \sum_q \left|M_\alpha^{(J)}(q,K)\right|^2$$

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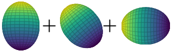
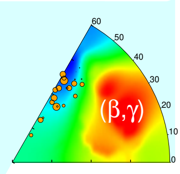
$|\Psi_{\rm eff}\rangle =$  $\cdots$

Intrinsic/Laboratory Description

- **Deformation structure of nuclear states**: $\{J^{\pi}_{\alpha}\}, \;\; q = (\beta, \gamma)$

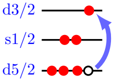
$$M^{(J)}_{\alpha}(q, K) = \sum_{q', K'} [\hat{N}^{1/2}]^{(J)}_{K'K}(q', q)\, f^{(J)}_{\alpha}(q', K')$$



◇ Probability of a configuration (β, γ):

$$\boxed{P^{(J)}_{\alpha}(q) = \sum_{K} |M^{(J)}_{\alpha}(q, K)|^2}$$

- **particle-hole interpretation**:



M**-scheme**

- K**–quantum numbers**:

$$\boxed{P^{(J)}_{\alpha}(K) = \sum_{q} |M^{(J)}_{\alpha}(q, K)|^2}$$

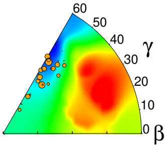
Recent developments of the DNO shell model

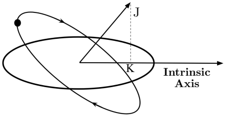
◇ Variation-After-Projection DNO-SM approach

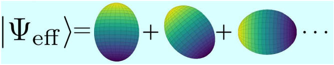
$$\mathcal{H}_{\text{eff}}|\Psi_\alpha^{JM}\rangle = E_\alpha^{(J)}|\Psi_\alpha^{JM}\rangle \Longrightarrow \delta\frac{\langle\Psi_\alpha^{JM}|\mathcal{H}_{\text{eff}}|\Psi_\alpha^{JM}\rangle}{\langle\Psi_\alpha^{JM}|\Psi_\alpha^{JM}\rangle} = 0, \quad |\Psi_\alpha^{JM}\rangle = \sum_{q,K} f_\alpha^{(J)}(q,K)\,\mathcal{P}^J_{MK}\,|\Phi(q)\rangle$$

Double variation AFTER Angular Momentum Projection: Mixing coefficient Slater state

◇ DNO-SM(β,γ)



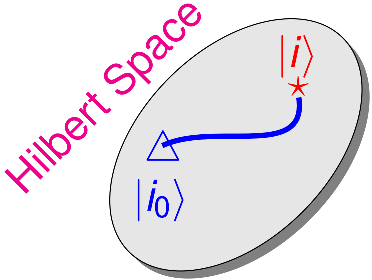




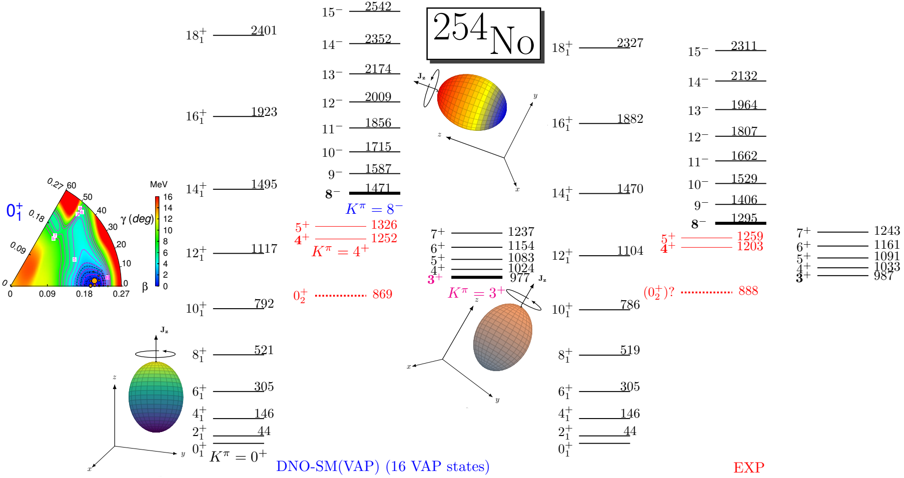
(D.D. Dao and F. Nowacki, PRC 105, 054314 (2022))

◇ DNO-SM(VAP)

- $q = 1, 2, 3, \ldots$
- $J_\alpha^\pi = 0_1^+, \ldots$
- Best energy-favoring Slater states

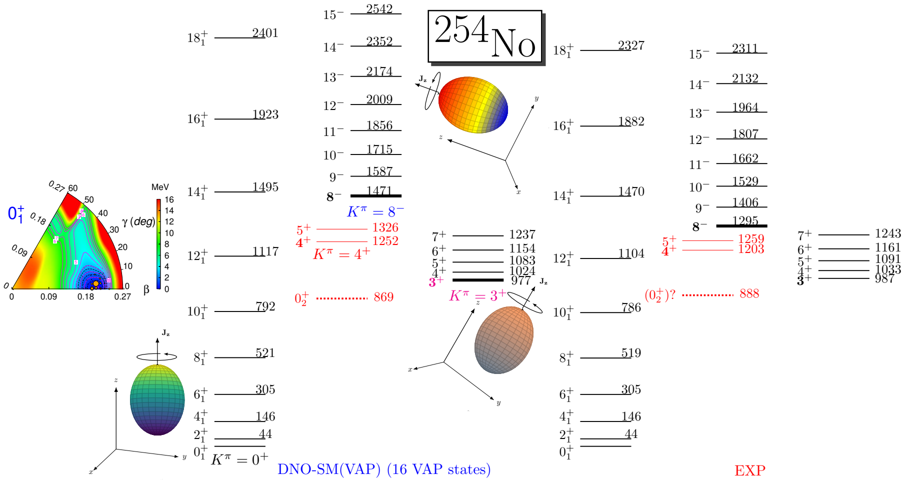


DNO-SM(VAP)



254No
0_1^+
0.27 60
50
40
30
20
10
0
0.18
0.09
0
0 0.09 0.18 0.27
γ (deg)
β
MeV
16 14 12 10 8 6 4 2 0
J_z
z y x
18_1^+ 2401
16_1^+ 1923
14_1^+ 1495
12_1^+ 1117
10_1^+ 792
8_1^+ 521
6_1^+ 305
4_1^+ 146
2_1^+ 44
0_1^+
$K^\pi = 0^+$
15^- 2542
14^- 2352
13^- 2174
12^- 2009
11^- 1856
10^- 1715
9^- 1587
8^- 1471
$K^\pi = 8^-$
5^+ 1326
4^+ 1252
$K^\pi = 4^+$
0_2^+ 869
7^+ 1237
6^+ 1154
5^+ 1083
4^+ 1024
3^+ 977
$K^\pi = 3^+$
DNO-SM(VAP) (16 VAP states)
18_1^+ 2327
16_1^+ 1882
14_1^+ 1470
12_1^+ 1104
10_1^+ 786
8_1^+ 519
6_1^+ 305
4_1^+ 146
2_1^+ 44
0_1^+
15^- 2311
14^- 2132
13^- 1964
12^- 1807
11^- 1662
10^- 1529
9^- 1406
8^- 1295
5^+ 1259
4^+ 1203
$(0_2^+)?$ 888
7^+ 1243
6^+ 1161
5^+ 1091
4^+ 1033
3^+ 987
EXP

DNO-SM(VAP)



First "SM" calculations for superheavies !!!

Island of Inversion at the N=Z line

◇ Strongly deformed states at $N = Z$

- Configuration mixing in ^{72}Kr
- Most deformed cases for ^{76}Sr, ^{80}Zr
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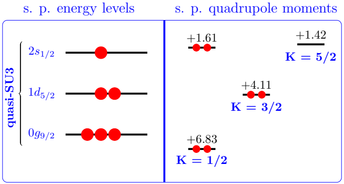

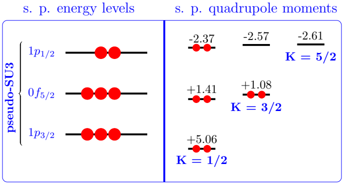

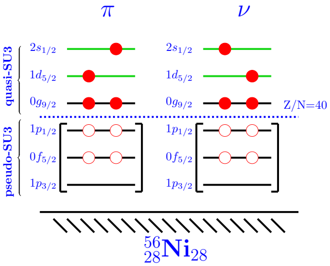

nucleus	Np-Nh*	ZRP	PHF	Exp.	DNO-SM*	SM
		B(E2)(e^2.fm^4)				
^{84}Mo	4p-4h	1104	1193	**1740^{+580}_{-430}**	1765	-
	8p-8h	1891	1732			
^{86}Mo	0p-0h	542	196	**707(71)**	980	731
	2p-2h	1030	871			
	4p-4h	1416	1179			
	6p-6h	1858	1655			

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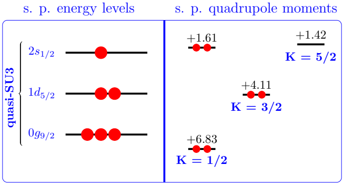

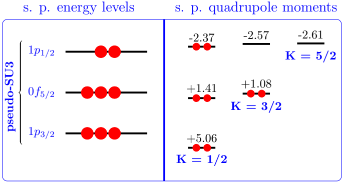

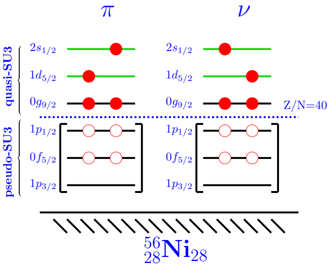

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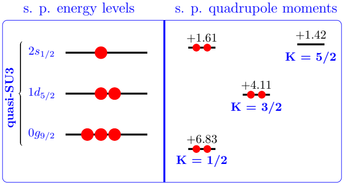

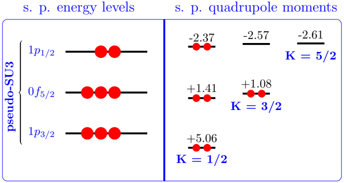

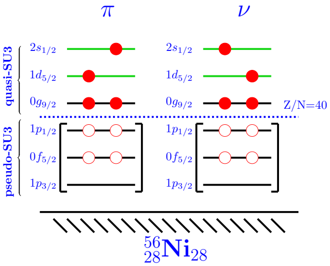

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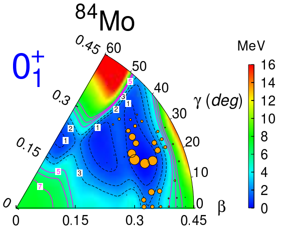

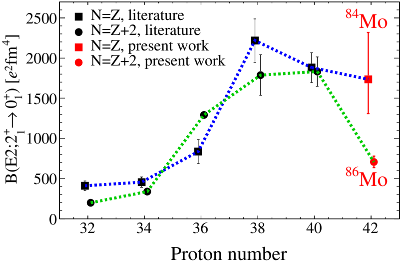

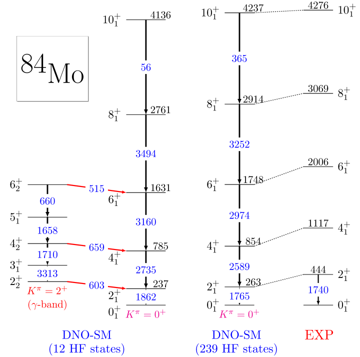

J. Ha, F. Recchia *et al.*, submitted to NATURE

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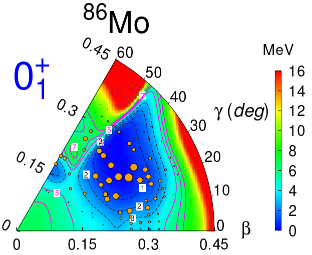

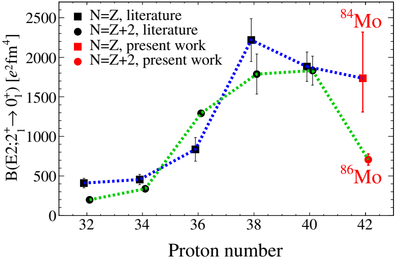

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3N forces in light nuclei

PRL 105, 032501 (2010) PHYSICAL REVIEW LETTERS week ending 16 JULY 2010

Three-Body Forces and the Limit of Oxygen Isotopes

Takaharu Otsuka,[1,2,3] Toshio Suzuki,[4] Jason D. Holt,[5] Achim Schwenk,[5] and Yoshinori Akaishi[6]

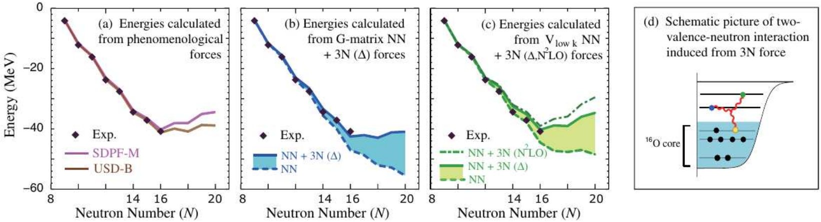


FIG. 4 (color online). Ground-state energies of oxygen isotopes measured from ^{16}O, including experimental values of the bound 16–24 O. Energies obtained from (a) phenomenological forces SDPF-M [13] and USD-B [14], (b) a G matrix and including FM $3N$ forces due to Δ excitations, and (c) from low-momentum interactions $V_{\text{low }k}$ and including chiral EFT $3N$ interactions at N^{2}LO as well as only due to Δ excitations [25]. The changes due to $3N$ forces based on Δ excitations are highlighted by the shaded areas. (d) Schematic illustration of a two-valence-neutron interaction generated by $3N$ forces with a nucleon in the ^{16}O core.

3N forces in light nuclei

PRL **109,** 032502 (2012) PHYSICAL REVIEW LETTERS week ending 20 JULY 2012

Evolution of Shell Structure in Neutron-Rich Calcium Isotopes

G. Hagen,[1,2] M. Hjorth-Jensen,[3,4] G. R. Jansen,[3] R. Machleidt,[5] and T. Papenbrock[1,2]
[1]*Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*
[2]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[3]*Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway*
[4]*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*
[5]*Department of Physics, University of Idaho, Moscow, Idaho 83844, USA*
(Received 16 April 2012; published 17 July 2012)

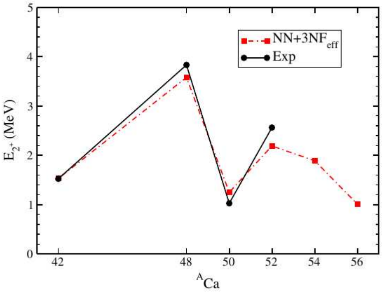


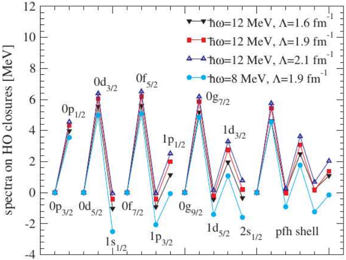
FIG. 2 (color online). (Excitation energies of $J^{\pi} = 2^{+}$ states in the isotopes ^{42,48,50,52,54,56}Ca (experiment: black circles, theory: red squares)

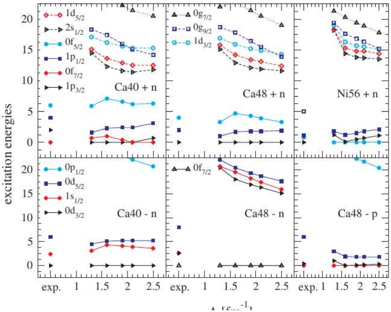
Shell closures and 2N forces only

PHYSICAL REVIEW C **74**, 061302(R) (2006)

Shell-model phenomenology of low-momentum interactions

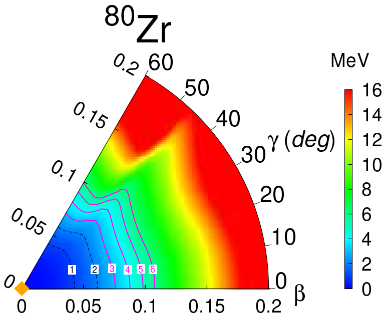
Achim Schwenk[1,*] and Andrés P. Zuker[2,†]
[1]*Nuclear Theory Center, Indiana University, 2401 Milo B. Sampson Lane, Bloomington, Indiana 47408, USA*
[2]*Institut de Recherches Subatomiques, IN2P3-CNRS, Université Louis Pasteur, F-67037 Strasbourg, France*
(Received 14 January 2005; revised manuscript received 20 September 2006; published 12 December 2006)

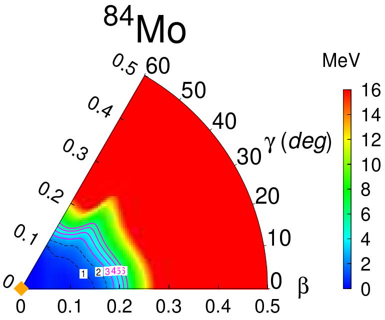




- no Spin-orbite shell closures in ^{12}C, ^{22}O, ^{48}Ca, ^{56}Ni
- too strong H. O. shell closures ^{16}O, ^{40}Ca, ... and ^{80}Zr !!!

N3LO NN calculations





				B(E2)($e^2.fm^4$)		
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^{80}Zr	4p-4h	587	637			
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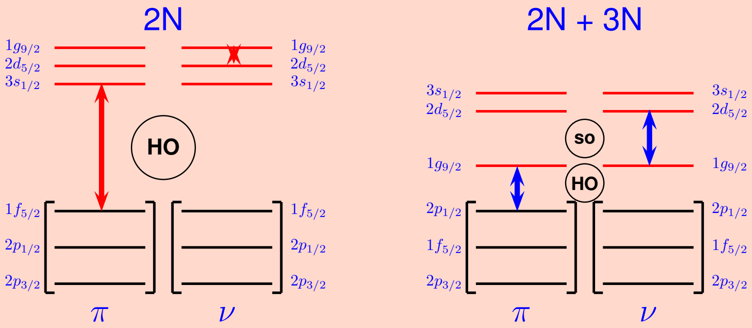

Three body forces and persistence of spin-orbit shell gaps in medium-mass nuclei: Towards the doubly magic ^{78}Ni,

K. Sieja, F. Nowacki

Phys. Rev. **C85**, 051301(R) (2012)

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	12p-12h	2663	2396			
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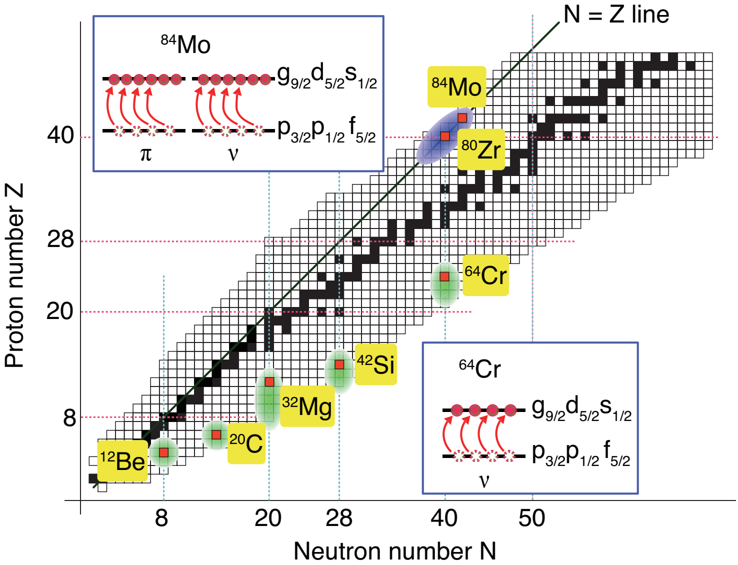
Isospin Symmetric Island of Inversion



Summary

- Monopole drift develops in all regions but the Interplay between correlations (pairing + quadrupole) and spherical mean-field (monopole field) determines the physics.

- New “island of inversion” or “island of deformation” present for neutron-rich systems show up also at N=Z line with very deformed rotors dominated by Many-particles-Many-holes configurations.

- New spectroscopy for ^{84}Mo and ^{86}Mo and first fingerprint of 3N forces in deformed systems

- Around A$\sim$ 80, an “island of enhanced collectivity” show very deformed rotors dominated by Many-particles-Many-holes configurations.

- Ongoing NN + 3N(lnl) ab-initio calculations

Summary

Special thanks to:

- G. Martinez-Pinedo, A. Poves, S. Lenzi
- A. Gade, O. Sorlin, A. Obertelli