Shell Model for Double Beta Decay

Mihai Horoi

Department of Physics, Central Michigan University, Mount Pleasant, Michigan 48859, USA

Support from DOE grant DE-SC0022538 is acknowledged

Two-Neutrino Double Beta Decay

SEPTEMBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Double Beta-Disintegration

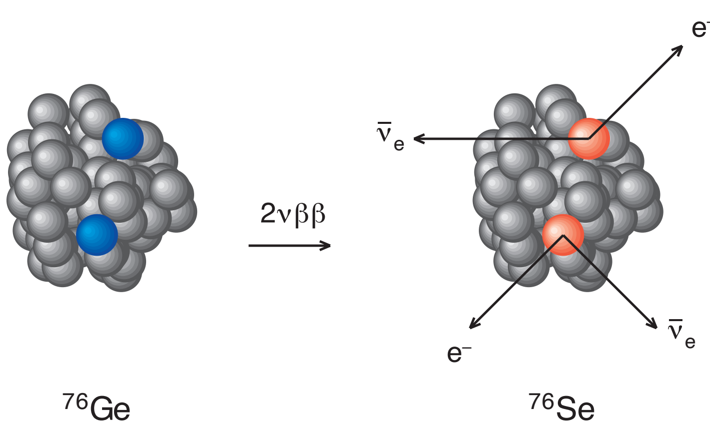
M. Goeppert-Mayer, *The Johns Hopkins University*
(Received May 20, 1935)

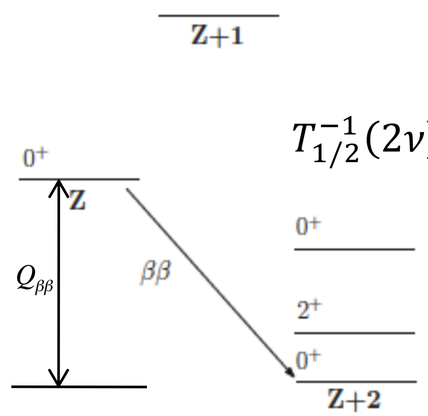
From the Fermi theory of β-disintegration the probability of simultaneous emission of two electrons (and two neutrinos) has been calculated. The result is that this process occurs sufficiently rarely to allow a half-life of over 10^{17} years for a nucleus, even if its isobar of atomic number different by 2 were more stable by 20 times the electron mass.

$$P = 1.15\times10^{-35}F(\epsilon-2)\ \text{sec.}^{-1}$$
$$= 3.6\times10^{-28}F(\epsilon-2)\ \text{year}^{-1}. \qquad (12)$$

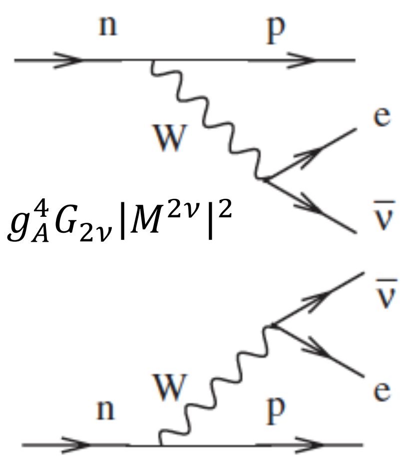
$\epsilon=$	4	6	8	10
$F(\epsilon+2)=$	0.37×10^{2}	9.2×10^{4}	3.4×10^{6}	4.2×10^{7}

$\epsilon=$	12	20
$P(\epsilon-2)=$	3.3×10^{8}	1×10^{11}





$$T_{1/2}^{-1}(2\nu) = g_A^4 G_{2\nu}|M^{2\nu}|^2$$



Neutrinoless Double Beta Decay

DECEMBER 15, 1939 PHYSICAL REVIEW VOLUME 56

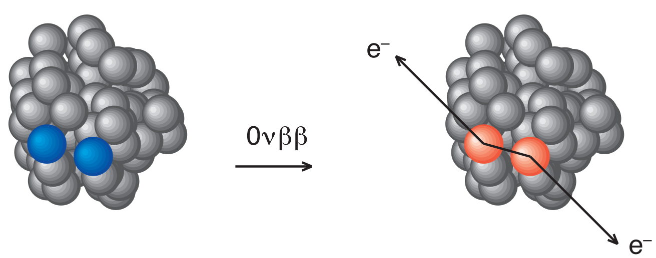
On Transition Probabilities in Double Beta-Disintegration

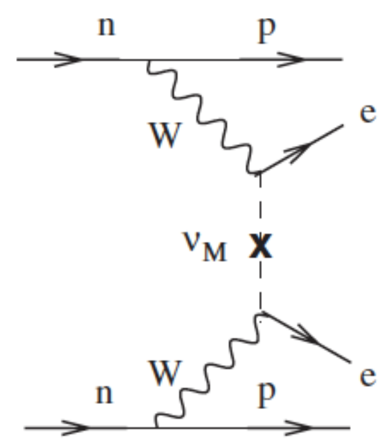
W. H. FURRY
Physics Research Laboratory, Harvard University, Cambridge, Massachusetts
(Received October 16, 1939)

THE probability of double β-disintegration was calculated some years ago by Goeppert-Mayer[1] on the basis of the Fermi theory.[2, 3] The result obtained was extremely small, corresponding to a lifetime of the order of 10^{25} years in the case of two isobars whose masses differ by 0.002

IV. CONCLUSION

We have seen that the phenomenon of double beta-disintegration is one for which there is a decided difference between the results of the Majorana theory and those of the older theory of the neutrino. According to the older theory it seemed certain that double beta-disintegration could never be capable of observation because of its extremely minute probability, but the Majorana theory indicates that this is by no means necessarily the case. Indeed, if the inter-





$$\langle m_{\beta\beta}\rangle = \left|\sum_k m_k U_{ek}^2\right|$$

$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2\left(\frac{< m_{\beta\beta} >}{m_e}\right)^2$$

$$\nu_e(x) = \sum U_{ei}\nu_i(x)$$

$$\left|\nu_e\right\rangle = \sum_i U_{ei}^*\left|\nu_i\right\rangle$$

CENTRAL MICHIGAN UNIVERSITY

First Searches of $0\nu\beta\beta$

Phys. Rev 75, 323 (1949) Letter to the Editor

A Measurement of the Half-Life of Double Beta-Decay from $_{50}Sn^{124}$ *

E. L. FIREMAN
Department of Physics, Princeton University, Princeton, New Jersey
November 29, 1948

TABLE II. $L(A)$ gives the coincidences from specimen A between counters L, and $R(B)$ gives the coincidences from B between counters R. Holder 0° and 180° are the two holder positions. Positions 1 and 2 are positions for the specimens in the holder.

Pos. 1	Holder 0° Coin. counts/hr.	$L(A)$ 16.4±0.3	$R(B)$ 14.3±0.3
	Holder 180° Coin. counts/hr.	$L(B)$ 14.4±0.3	$R(A)$ 15.9±0.3
Pos. 2	Holder 0° Coin. counts/hr.	$L(B)$ 14.6±0.3	$R(A)$ 16.4±0.3
	Holder 180° Coin. counts/hr.	$L(A)$ 16.4±0.3	$R(B)$ 13.9±0.3

A detailed report of this work is being prepared for publication in the Physical Review.

The author is grateful to Professor R. Sherr for accepting the supervision of this research and to Professor E. P. Wigner for many profitable discussions.

* This work is assisted by the Office of Naval Research.
** These isotopes were obtained from Oak Ridge.

[1] Maria Goeppert-Mayer, Phys. Rev. **48**, 512 (1935).
[2] W. H. Furry, Phys. Rev. **56**, 1184 (1939).

PHYSICAL REVIEW VOLUME 86, NUMBER 4 MAY 15, 1952

A Re-Investigation of the Double Beta-Decay from Sn^{124}

E. L. FIREMAN AND D. SCHWARZER
Brookhaven National Laboratory, Upton, New York
(Received February 5, 1952)

Radiations from natural tin and tin enriched with the 124 isotope are examined in a magnetic field with a helium filled cloud chamber that is triggered by internal counters. Only three pictures out of more than four thousand photographs are pictures of two electrons coming out of the same point in the tin and entering the counters, and even these may be pictures of multiply scattered electrons passing through the tin. However, one may set a lower limit to the half-life of double beta-decay from Sn^{124} as 10^{17} years. This is a decay rate less than one-tenth of a value previously reported by one of the authors.

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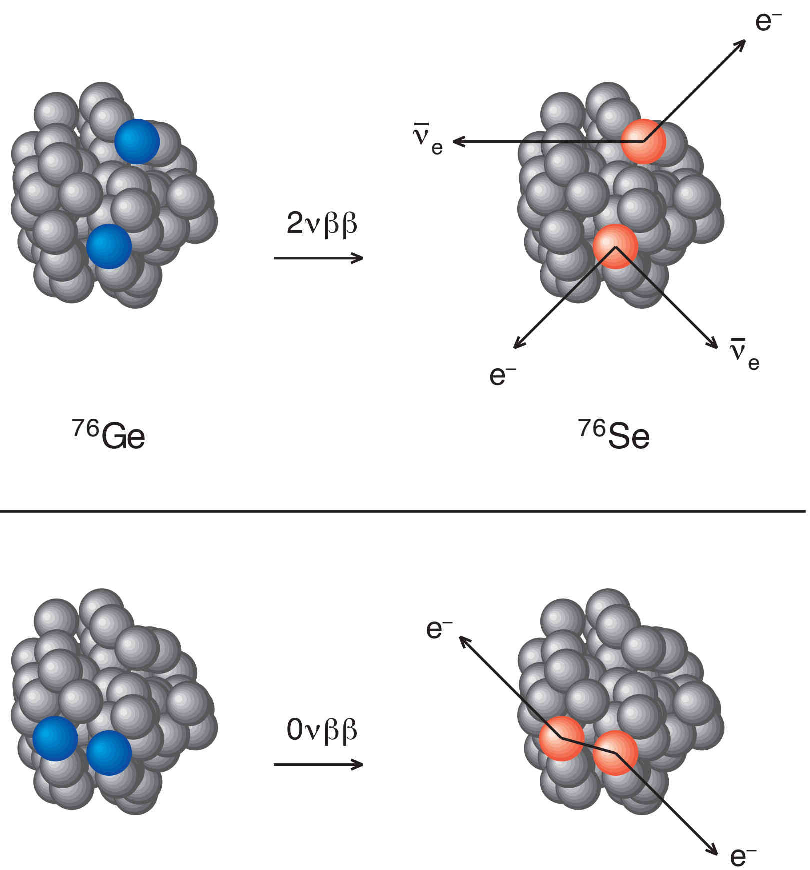
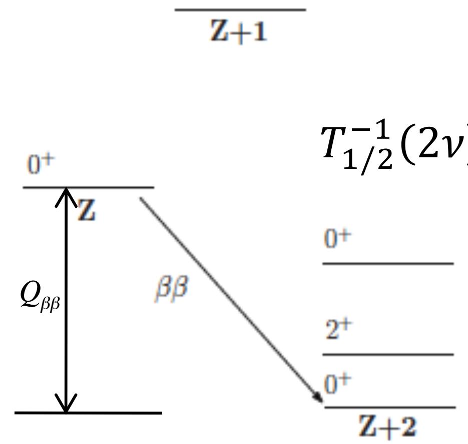
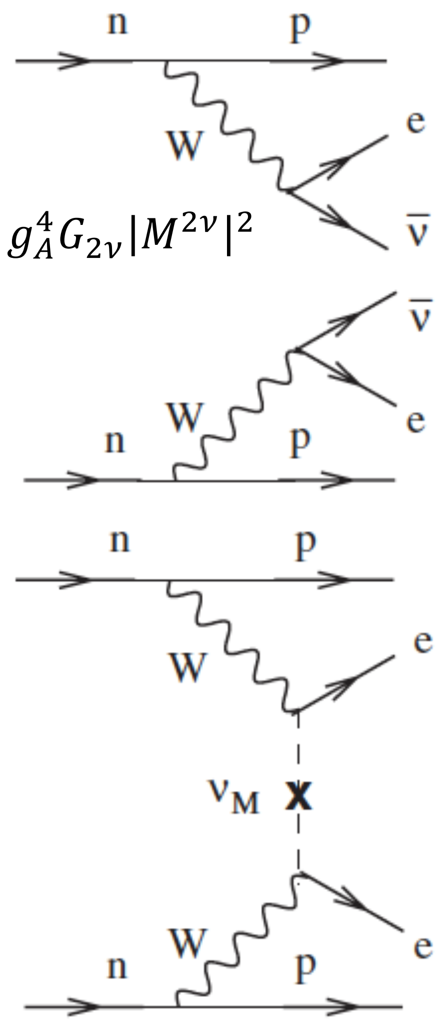
Classical Double Beta Decay Problem





$$T_{1/2}^{-1}(2\nu) = g_A^4 G_{2\nu} |M^{2\nu}|^2$$



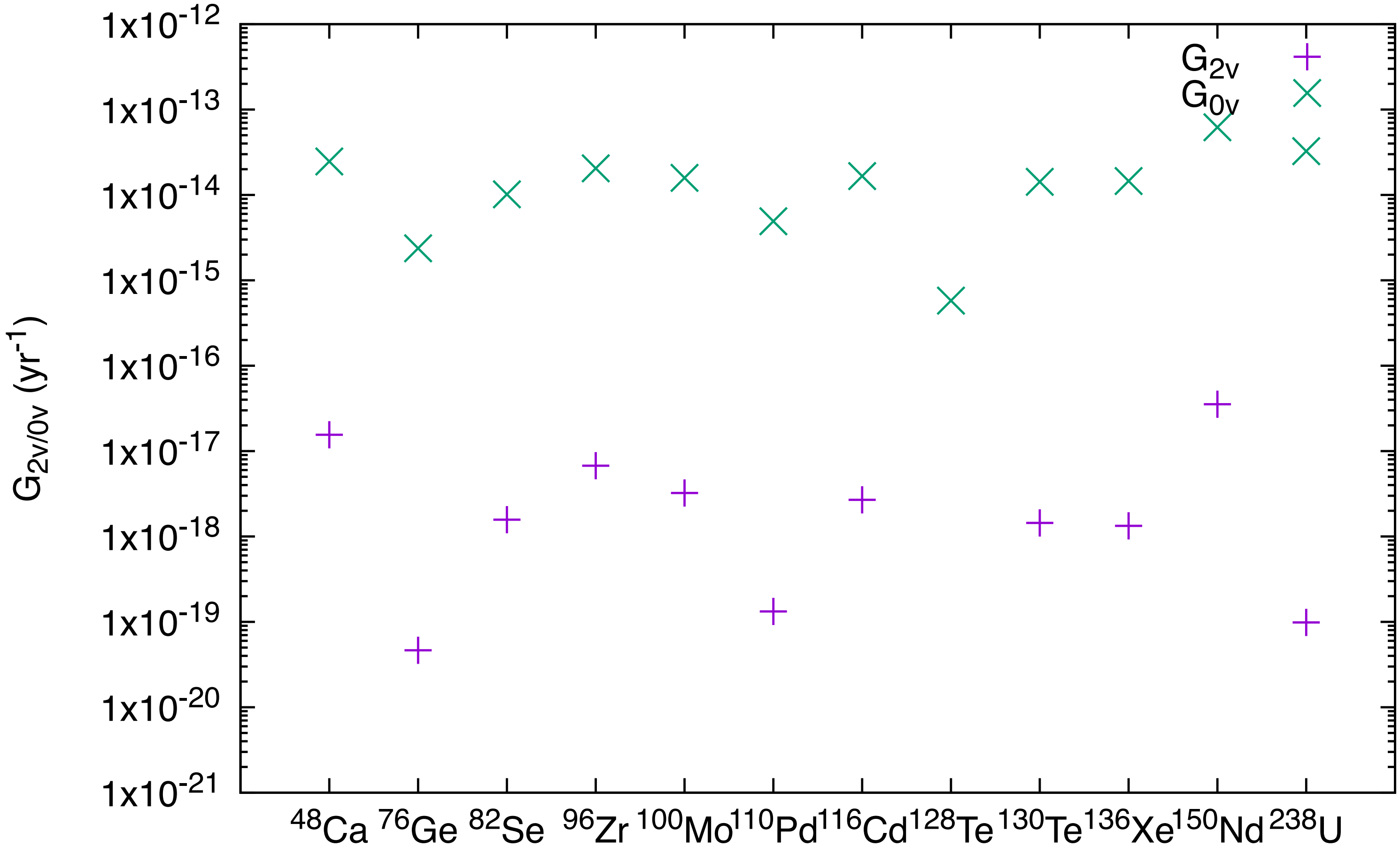
$$\langle m_{\beta\beta} \rangle = \left| \sum_k m_k U_{ek}^2 \right|$$

$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta}) \left[M^{0\nu}(0^+) \right]^2 \left(\frac{< m_{\beta\beta} >}{m_e} \right)^2$$

$$\nu_e(x) = \sum_i U_{ei} \nu_i(x)$$

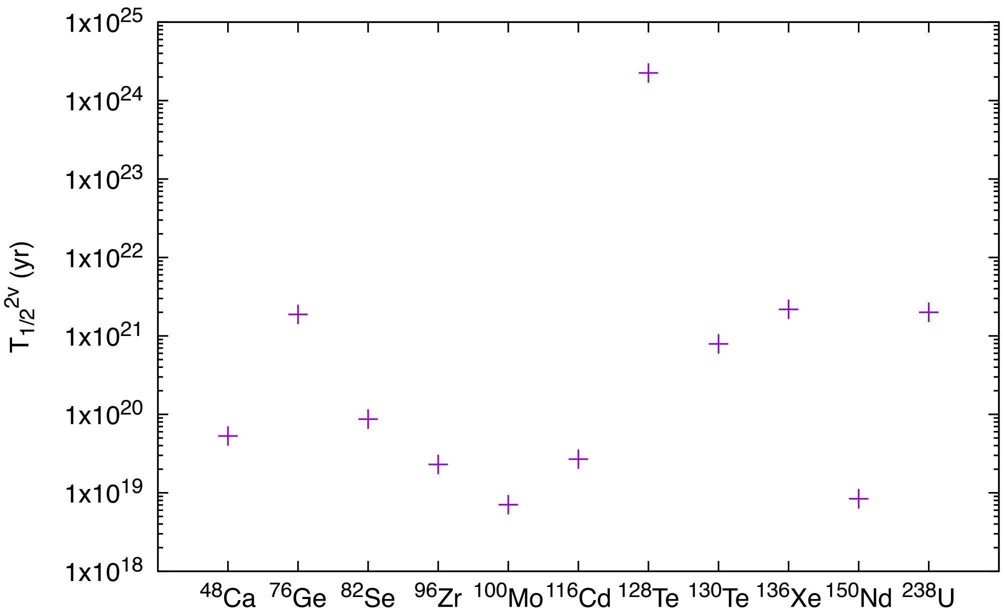
$$|\nu_e\rangle = \sum_i U_{ei}^* |\nu_i\rangle$$

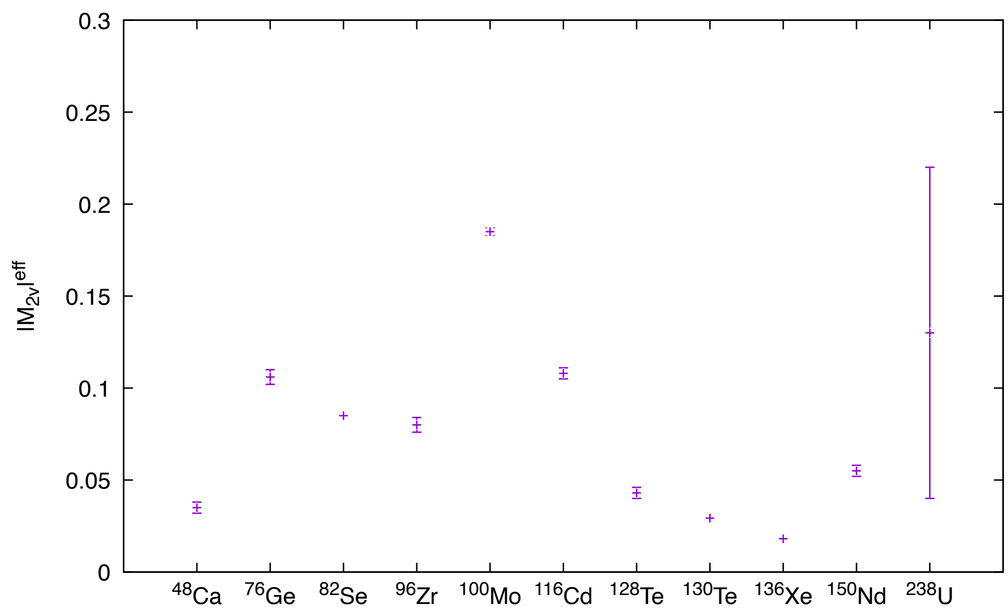
DBD Phase Space Factors



2νββ Half-Lives and NME

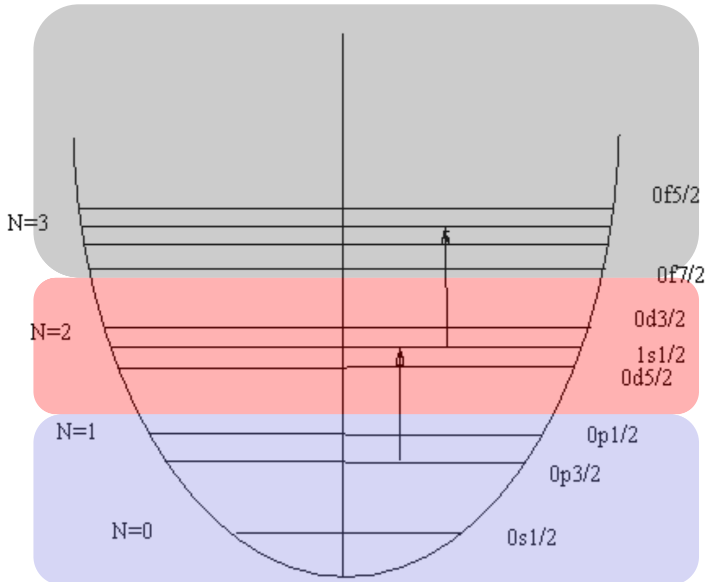
$$\left[T_{1/2}^{2\nu}\right]^{-1} = G_{2\nu} \cdot \left[g_A^2 \left(m_e c^2 \cdot M_{2\nu}\right)\right]^2 \equiv G_{2\nu} \left(M_{2\nu}^{eff}\right)^2$$





A. Barabash, Universe 6, 159 (2020)

Shell Model Effective Hamiltonians



$$g_A\sigma\tau \xrightarrow{quenched} g_A 0.77\sigma\tau$$

q

empty

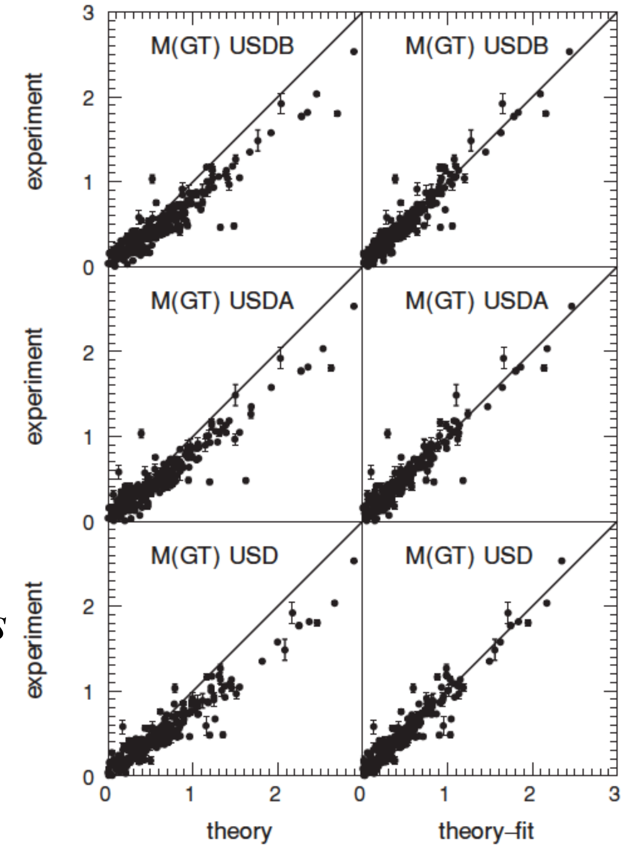
valence

frozen core

$$H_{valence} = H_{2-body}$$

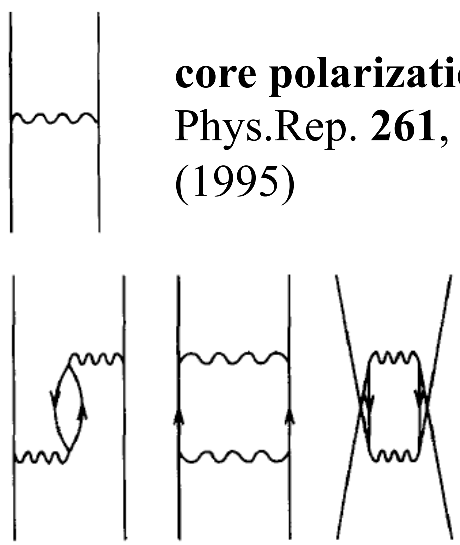
can describe most correlations around the Fermi surface!

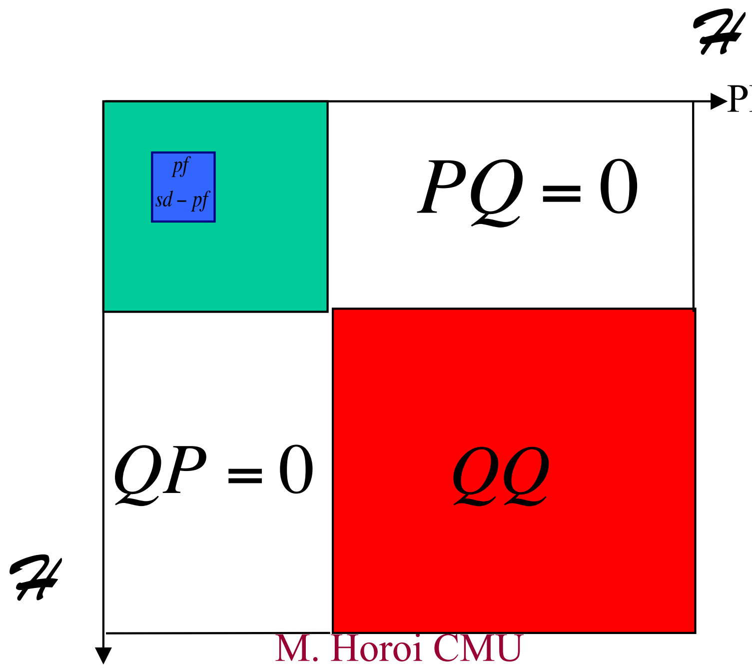
$$H_{valence}\Psi = E_n\Psi$$

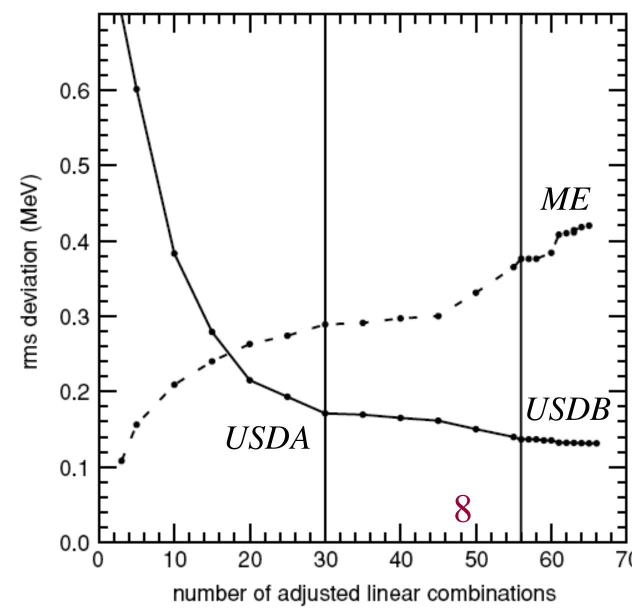


PRC 74, 34315 (2006), 78, 064302 (2008)

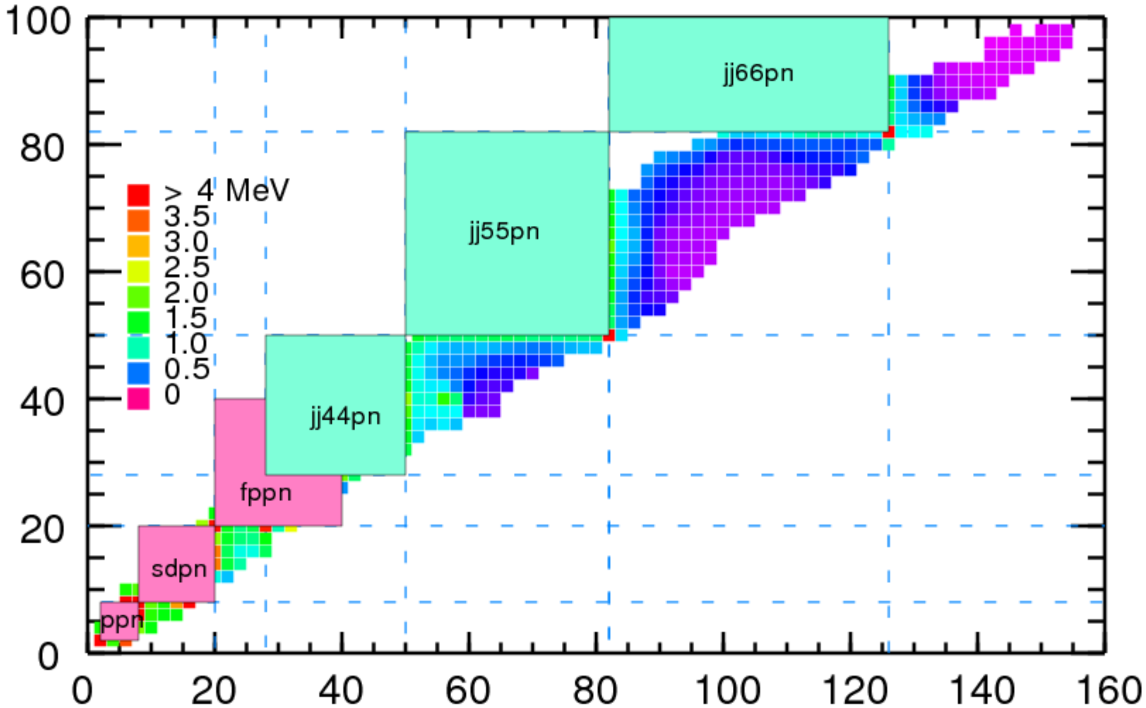
core polarization: Phys.Rep. **261**, 125 (1995)

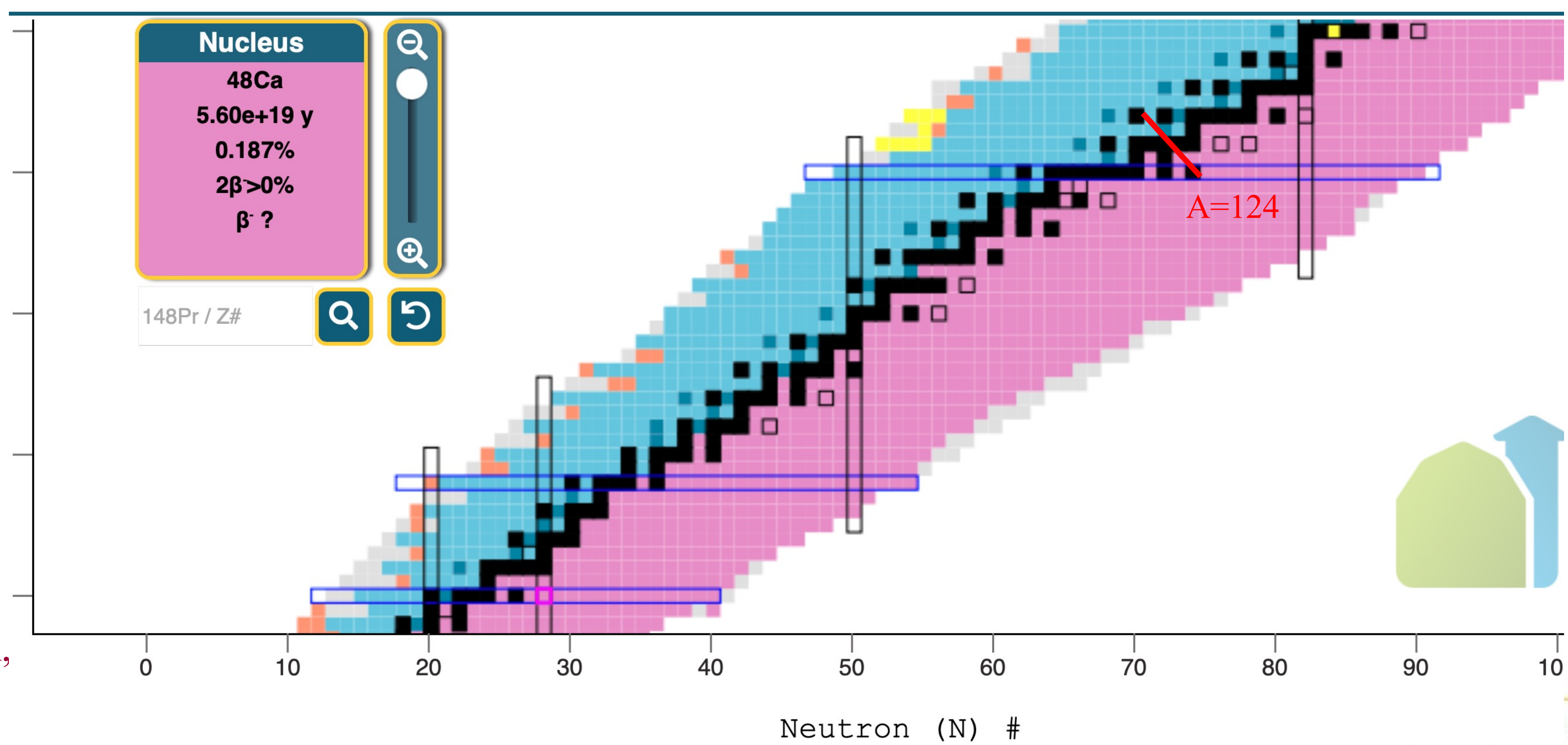






Shell Model Spaces

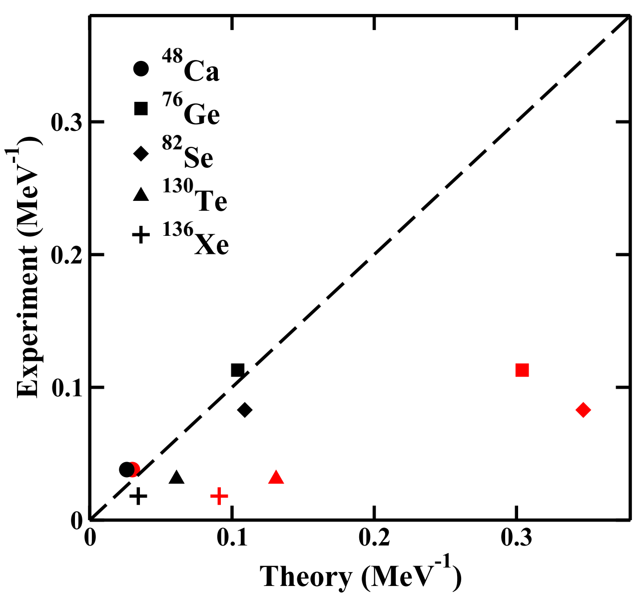




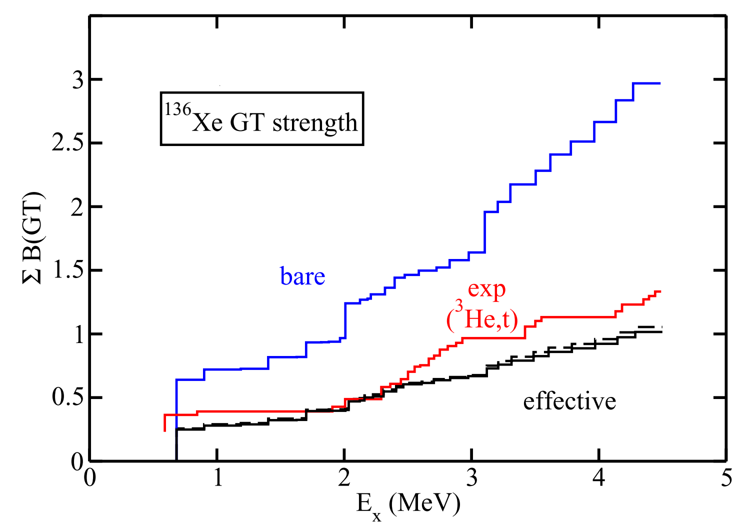
Quenching factor vs Effective Operator

$$M^{2\nu} = \sum_n \frac{\langle \Psi_f | (\sigma\tau^-)^{eff} | 1_n^+ \rangle \langle 1_n^+ | (\sigma\tau^-)^{eff} | \Psi_i \rangle}{E(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta}$$

PHYSICAL REVIEW C **100**, 014316 (2019)

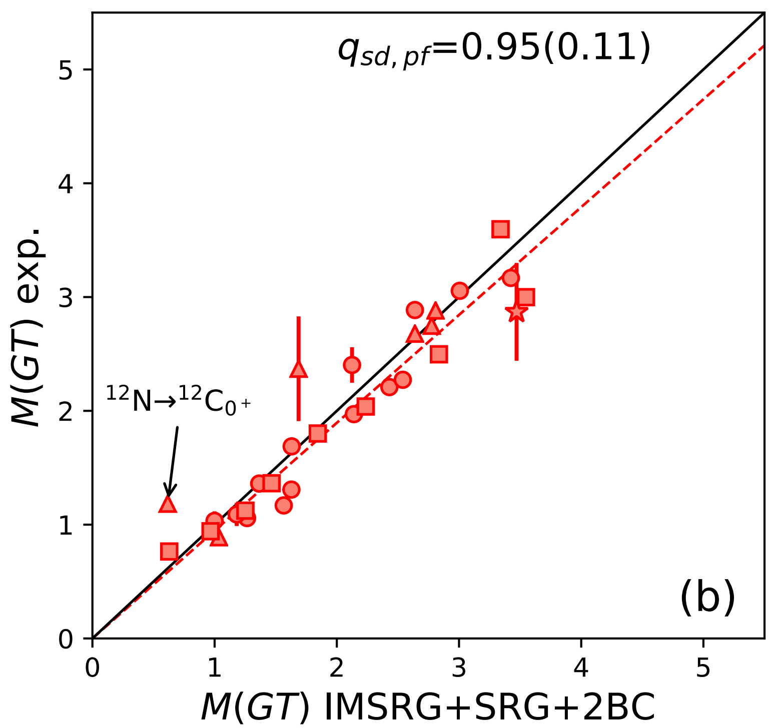


RENORMALIZATION OF THE GAMOW-TELLER OPERATOR ..

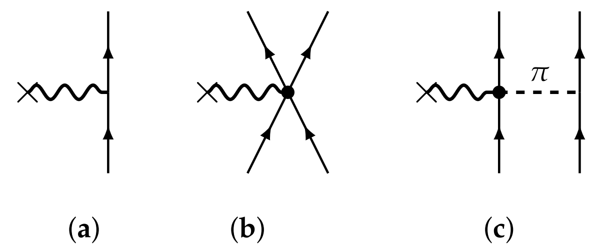


Shell Model: $(\sigma\tau^-)^{eff} = q\tau^-\sigma$

Ab-initio effective Gamow-Teller operator



Citation: Stroberg, S.R. Beta Decay in Medium-Mass Nuclei with the In-Medium Similarity Renormalization Group. *Particles* **2021**, *4*, 521–535. https://doi.org/10.3390/particles4040038



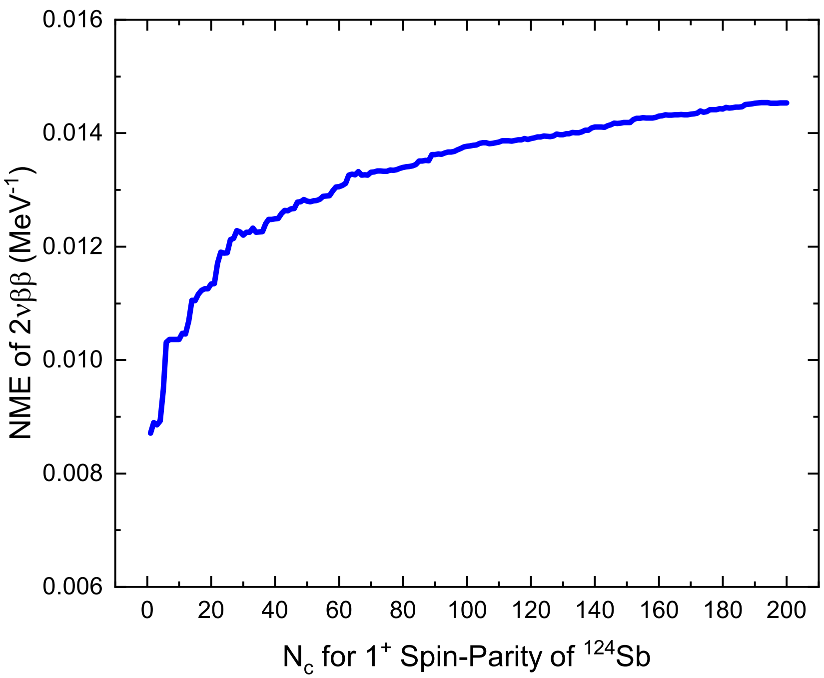
Two-Neutrino NME: Direct Sum ?

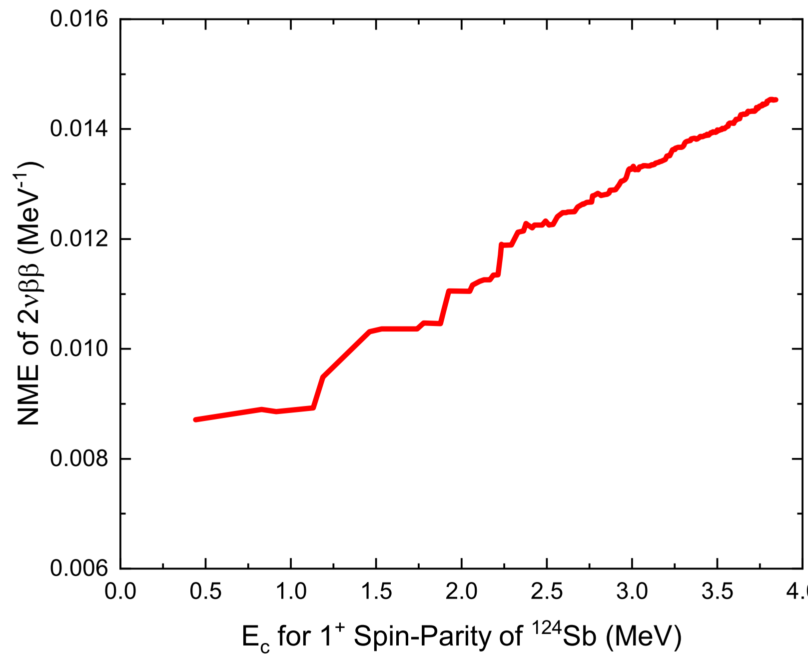
PHYSICAL REVIEW C **109**, 024301 (2024)

Calculation of nuclear matrix elements for $0\nu\beta\beta$ decay of ^{124}Sn using the nonclosure approach in the nuclear shell model

Shahariar Sarkar,[1,*] P. K. Rath,[2] V. Nanal,[3] R. G. Pillay,[1] Pushpendra P. Singh,[1] Y. Iwata,[4]
K. Jha,[1] and P. K. Raina[1,†]

$$M_{GT}^{2\nu} = \sum_{k, E_k^* \leqslant E_c} \frac{\langle f||\sigma\tau_2^-||k\rangle\langle k||\sigma\tau_1^-||i\rangle}{E_k^* + E_0}$$





Strength Function Approach

Create doorway states:

$$|\sigma\tau^{-}0_i^{+} >= c_{-}|dw_{-} >\equiv c_{-}|L_1^{-} >$$
$$|\sigma\tau^{+}0_f^{+} >= c_{+}|dw_{+} >\equiv c_{+}|L_1^{+} >$$

Relation to GT sum rules:

$$3|c_{-}|^2 = B_{sum}(GT; i-) \qquad 3|c_{+}|^2 = B_{sum}(GT; f+)$$

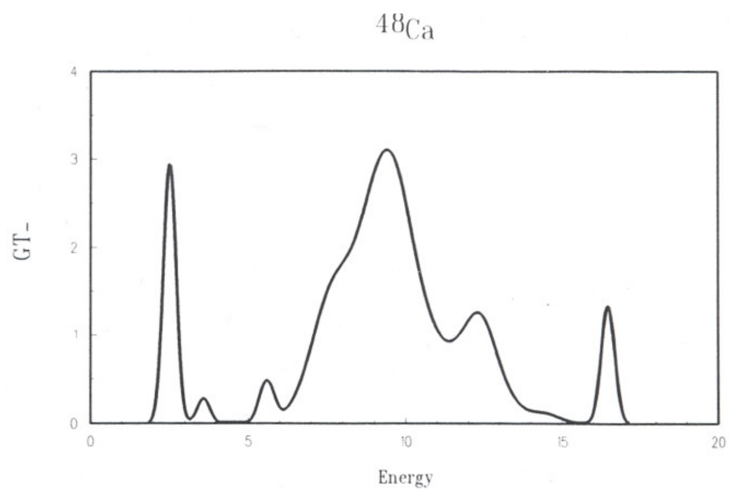
Do a small number of Lanczos iterations starting with $|L_1^{-} >$:

$$M_{2v}(E_k) = \sqrt{B_{sum}(GT; f+)B_{sum}(GT; i-)} \sum_{n=1}^{k<25} \frac{\langle\Psi_f|\tilde{1}_n^{+}\rangle\langle\tilde{1}_n^{+}|\Psi_i\rangle}{\tilde{E}(1_n^{+}) + \frac{Q_{\beta\beta}}{2} - Q_{\beta}}$$

See also M. Horoi, Physics 2022, 4, 1135

Strength Function Approach

Caurier, Poves, Zuker, Phys. Lett. B 252, 13 (1990)



(iii) The convergence with respect to the numbe N of Lanczos iterations on the β^- doorway is ex tremely fast, as the following numbers show:

$M_{GT}^{2\nu}(0^+) = 0.0086, \quad N = 1,$

$M_{GT}^{2\nu}(0^+) = 0.0433, \quad N = 4,$

$M_{GT}^{2\nu}(0^+) = 0.0403, \quad N = 12,$

$M_{GT}^{2\nu}(0^+) = 0.0402, \quad N = 30,$

$M_{GT}^{2\nu}(0^+) = 0.0402, \quad N = 60,$

Engel, Haxton, Vogel, Phys. Rev. C 46, R2153 (1991)

$$M_{GT} = -\frac{1}{2}\left\langle 0_f^+ \left| \sum_{i=1}^{A} \sqrt{3}\sigma_z(i)\tau_-(i)\frac{1}{E_0 - H} \times \sum_{j=1}^{A} \sqrt{3}\sigma_z(j)\tau_-(j) \right| 0_i^+ \right\rangle$$

$$\frac{1}{E_0 - H}|v_1\rangle = g_1(E_0)|v_1\rangle + g_2(E_0)|v_2\rangle + \cdots$$

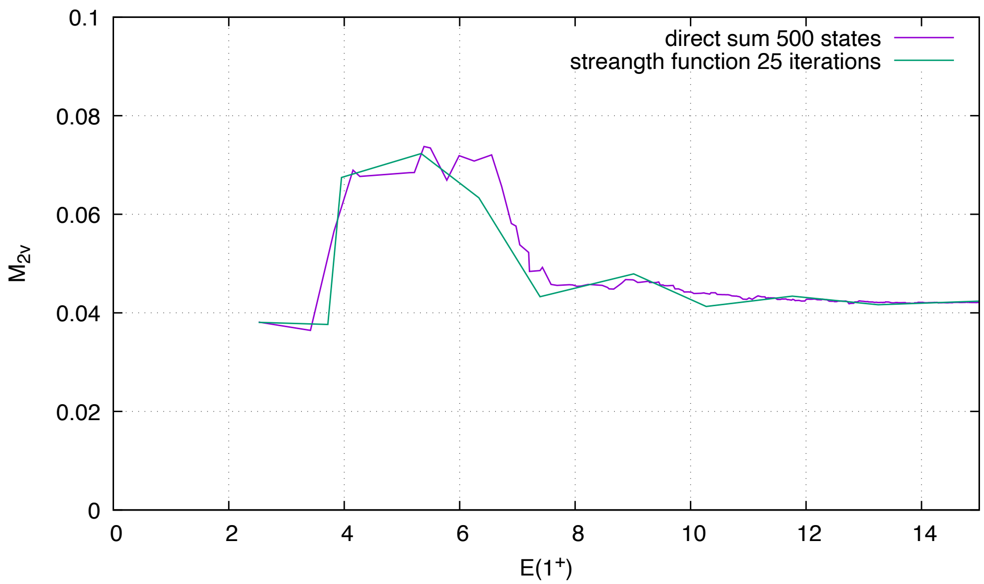
$$g_1(E_0) = \cfrac{1}{E_0 - \alpha_1 - \cfrac{\beta_1^2}{E_0 - \alpha_2 - \cfrac{\beta_2^2}{E_0 - \alpha_3 - \cfrac{\beta_3^2}{\ddots}}}}$$

Two-Neutrino NME: Direct Sum vs Strength Function Approach

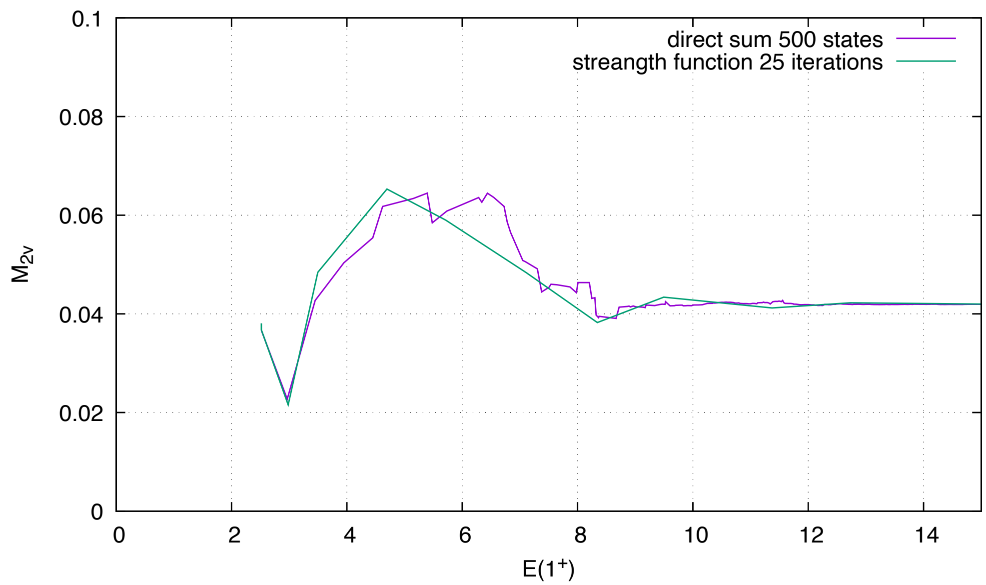
$$M_{2v}(E_k) = \sum_{n=1}^{k} \frac{\langle \Psi_f | q\sigma\tau^- | 1_n^+ \rangle \langle 1_n^+ | q\sigma\tau^- | \Psi_i \rangle}{E(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta}$$

$$M_{2v}(E_k) = \sqrt{B_{sum}(GT; f+) B_{sum}(GT; i-)} \sum_{n=1}^{k<25} \frac{\langle \Psi_f | \tilde{1}_n^+ \rangle \langle \tilde{1}_n^+ | \Psi_i \rangle}{\tilde{E}(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta}$$

^{48}Ca – GXPF1A



^{48}Ca – KB3G



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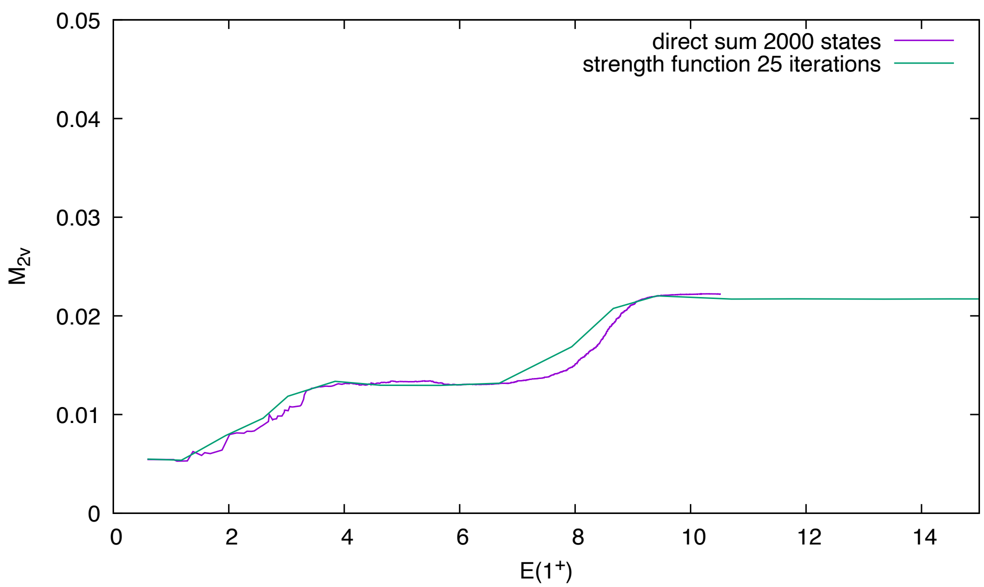
Two-Neutrino NME: Direct Sum vs Strength Function Approach

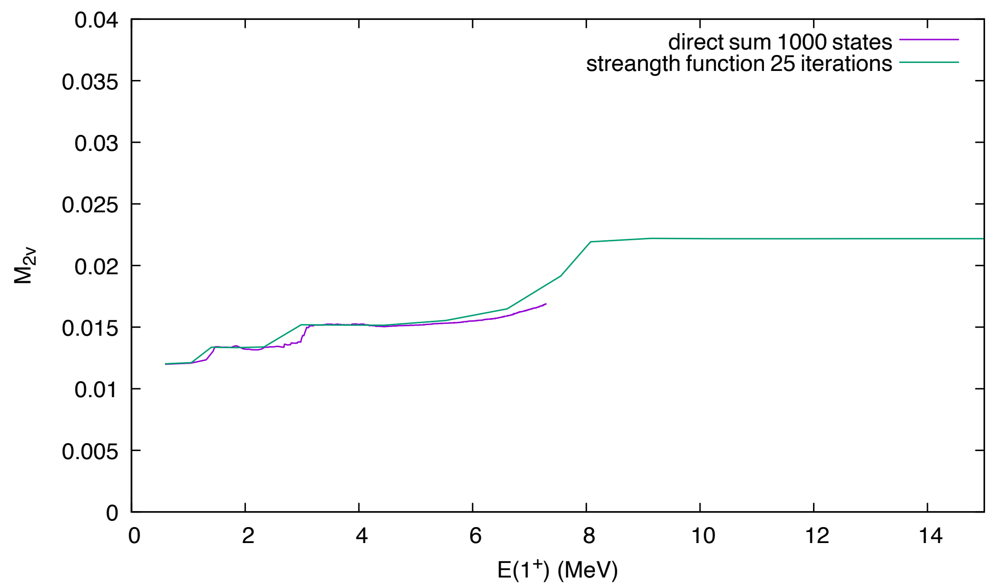
$$M_{2\nu}(E_k) = \sum_{n=1}^{k} \frac{\langle \Psi_f | q\sigma\tau^- | 1_n^+ \rangle \langle 1_n^+ | q\sigma\tau^- | \Psi_i \rangle}{E(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta}$$

$$M_{2\nu}(E_k) = \sqrt{B_{sum}(GT; f+) B_{sum}(GT; i-)} \sum_{n=1}^{k<25} \frac{\langle \Psi_f | \tilde{1}_n^+ \rangle \langle \tilde{1}_n^+ | \Psi_i \rangle}{\tilde{E}(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta}$$

^{136}Xe – SVD



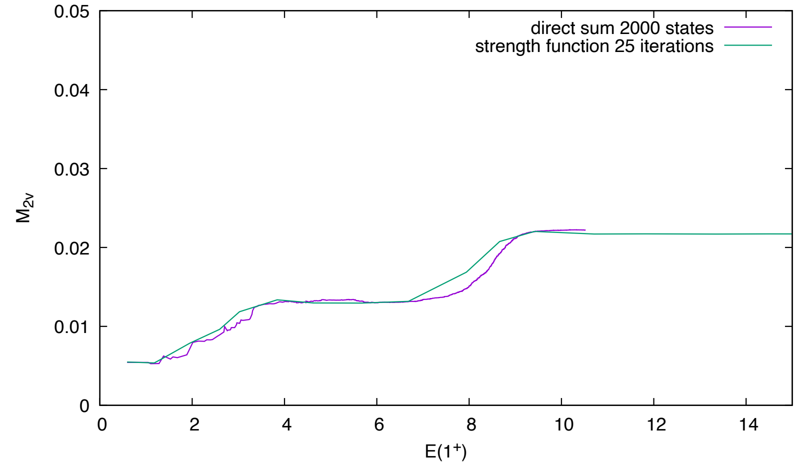
^{136}Xe – GCN5082

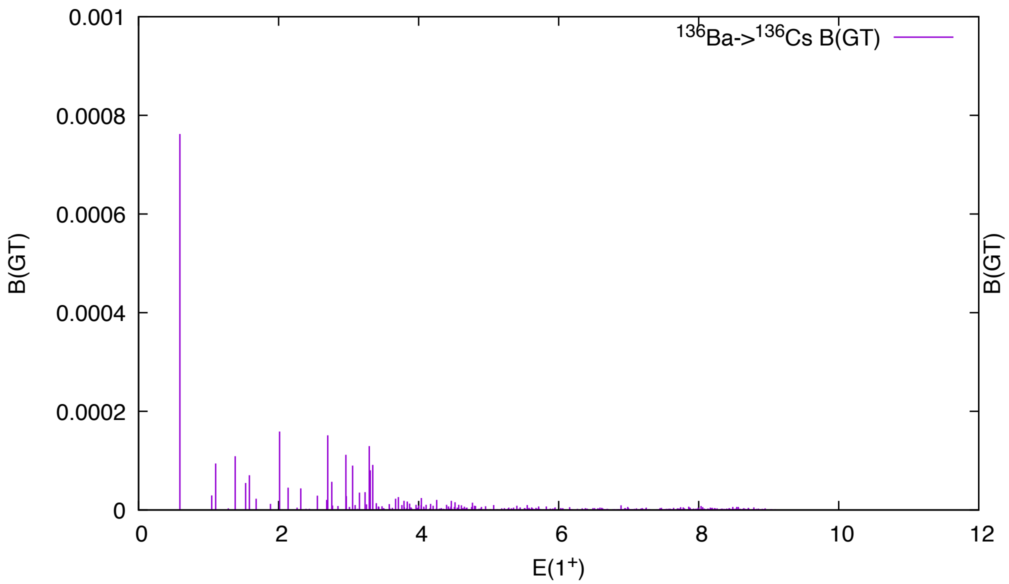


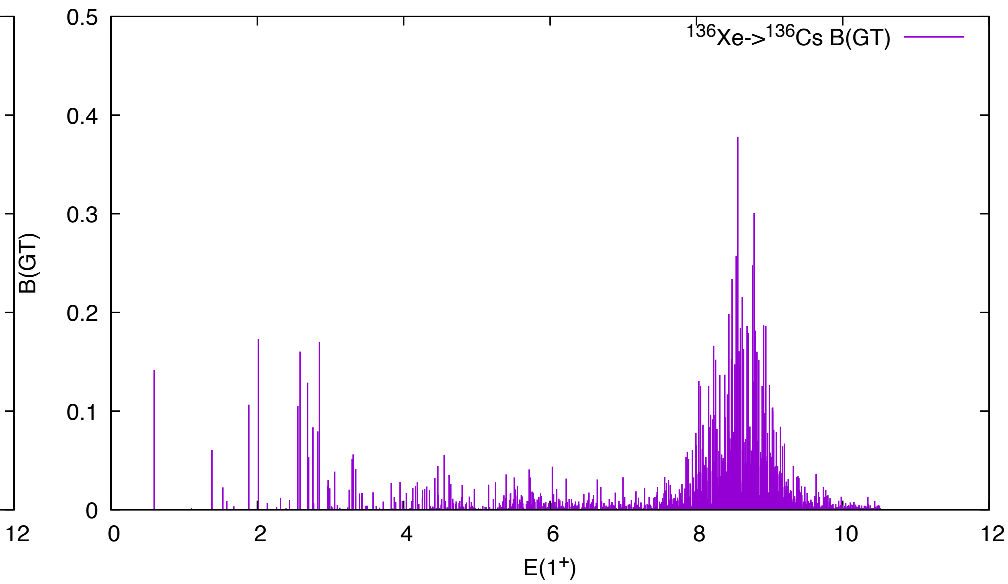
The Late Bump in ^{136}Xe

$$M_{2\nu}(E_k) = \sum_{n=1}^{k} \frac{\langle \Psi_f | q\sigma\tau^- | 1_n^+ \rangle \langle 1_n^+ | q\sigma\tau^- | \Psi_i \rangle}{E(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta}$$

^{136}Xe – SVD

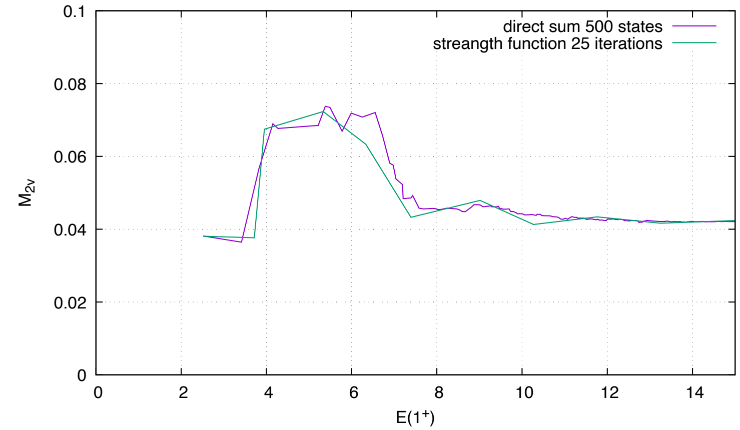




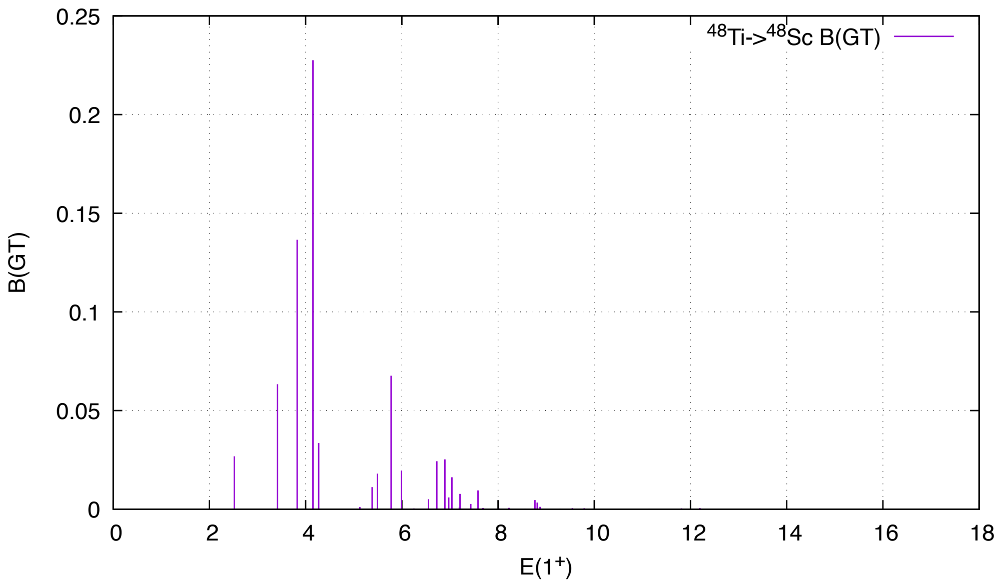


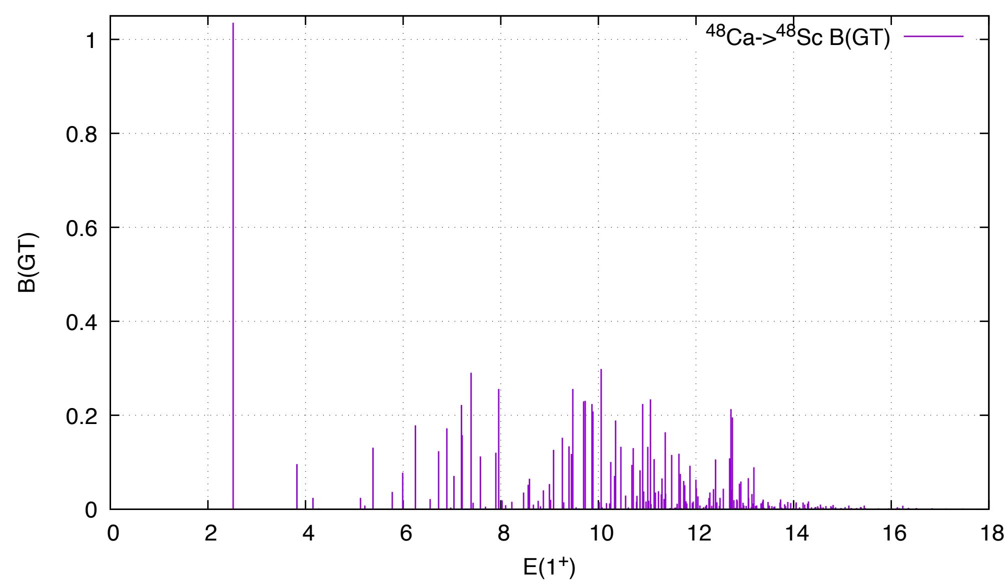
The Late Drop in ^{48}Ca

$$M_{2v}(E_k) = \sum_{n=1}^{k} \frac{\langle \Psi_f | q\sigma\tau^- | 1_n^+ \rangle \langle 1_n^+ | q\sigma\tau^- | \Psi_i \rangle}{E(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta}$$

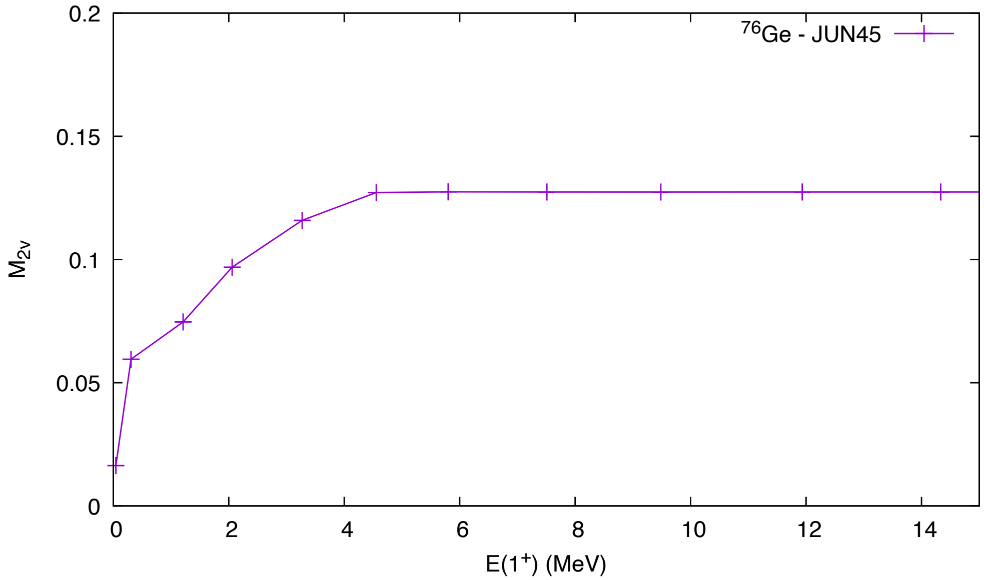


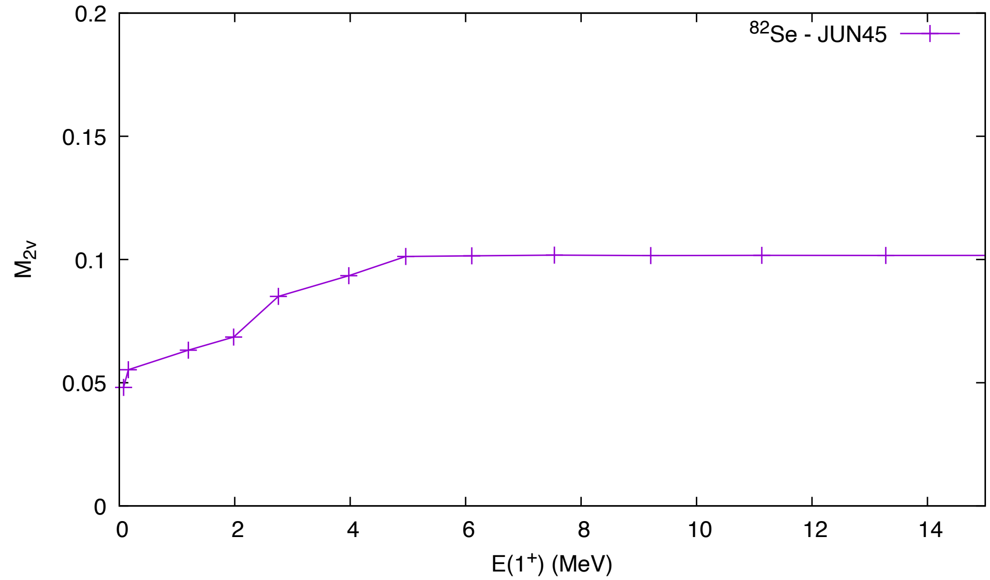
^{48}Ca – GXPF1A

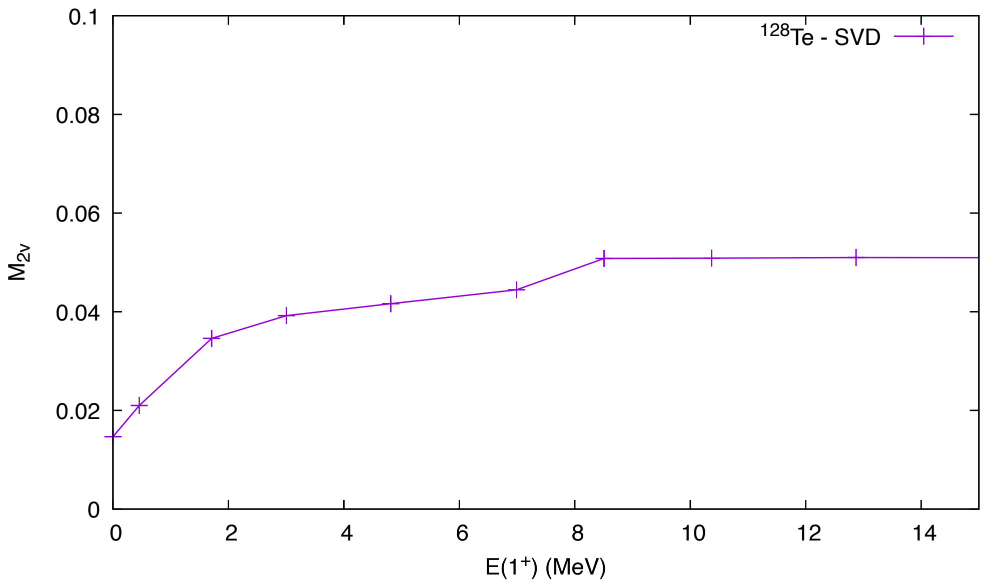


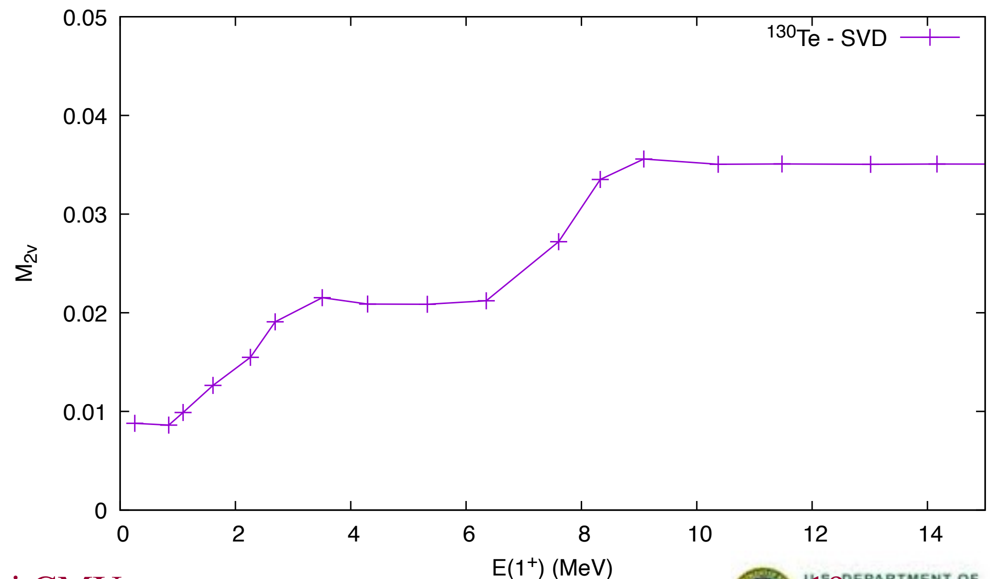


Strength Function: large dimensions









CENTRAL MICHIGAN UNIVERSITY

The 2νECEC decay of ^{124}Xe: Experimental data

532 | NATURE | VOL 568 | 25 APRIL 2019

LETTER

https://doi.org/10.1038/s41586-019-1124-4

Observation of two-neutrino double electron capture in ^{124}Xe with XENON1T

XENON Collaboration*

absolute neutrino mass[15–17]. Here we report the direct observation of 2νECEC in ^{124}Xe with the XENON1T dark-matter detector. The significance of the signal is 4.4 standard deviations and the corresponding half-life of 1.8×10^{22} years (statistical uncertainty, 0.5×10^{22} years; systematic uncertainty, 0.1×10^{22} years) is the

PHYSICAL REVIEW C **106**, 024328 (2022)

Editors' Suggestion

Double-weak decays of ^{124}Xe and ^{136}Xe in the XENON1T and XENONnT experiments

We present results on the search for two-neutrino double-electron capture (2νECEC) of ^{124}Xe and neutrinoless double-β decay ($0\nu\beta\beta$) of ^{136}Xe in XENON1T. We consider captures from the K shell up to the N shell in the 2νECEC signal model and measure a total half-life of $T_{1/2}^{2\nu\mathrm{ECEC}} = (1.1 \pm 0.2_{\mathrm{stat}} \pm 0.1_{\mathrm{sys}}) \times 10^{22}$ yr with a 0.87 kg yr isotope exposure. The statistical significance of the signal is 7.0σ. We use XENON1T data with 36.16 kg yr of ^{136}Xe exposure to search for $0\nu\beta\beta$. We find no evidence of a signal and set a lower limit on the half-life of $T_{1/2}^{0\nu\beta\beta} > 1.2 \times 10^{24}$ yr at 90% CL. This is the best result from a dark matter detector without

arXiv:2009.14451 [pdf, other] nucl-ex

Precise Half-Life Values for Two-Neutrino Double-β Decay: 2020 review

Authors: Alexander Barabash

Nucleus	N	$T_{1/2}(2\nu)$, yr	S/B	Ref., Year
^{130}Ba $ECEC(2\nu)$		$\mathbf{2.1^{+3.0}_{-0.8} \cdot 10^{21}}$ (geochem.)		[87], 1996
		$(\mathbf{2.2 \pm 0.5}) \cdot \mathbf{10^{21}}$ (geochem.)		[11], 2001
		$(\mathbf{0.60 \pm 0.11}) \cdot \mathbf{10^{21}}$ (geochem.)		[88], 2009
		Recommended value: $(\mathbf{2.2 \pm 0.5}) \cdot \mathbf{10^{21}}$		
^{78}Kr $2K(2\nu)$	15	$[\mathbf{1.9^{+1.3}_{-0.7}(stat) \pm 0.3(syst)}] \cdot \mathbf{10^{22}}$	15	[13], 2017
		Recommended value: $(\mathbf{1.9^{+1.3}_{-0.8}}) \cdot \mathbf{10^{22}}$ (?) $^{(a)}$		
^{124}Xe $2K(2\nu)$	126	$[\mathbf{1.8 \pm 0.5(stat) \pm 0.1(syst)}] \cdot \mathbf{10^{22}}$	0.2	[12], 2019
		Recommended value: $(\mathbf{1.8 \pm 0.5}) \cdot \mathbf{10^{22}}$		

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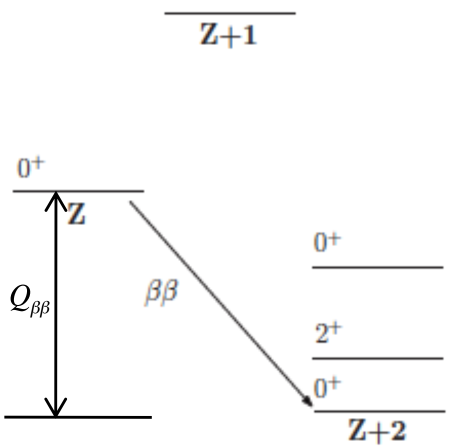
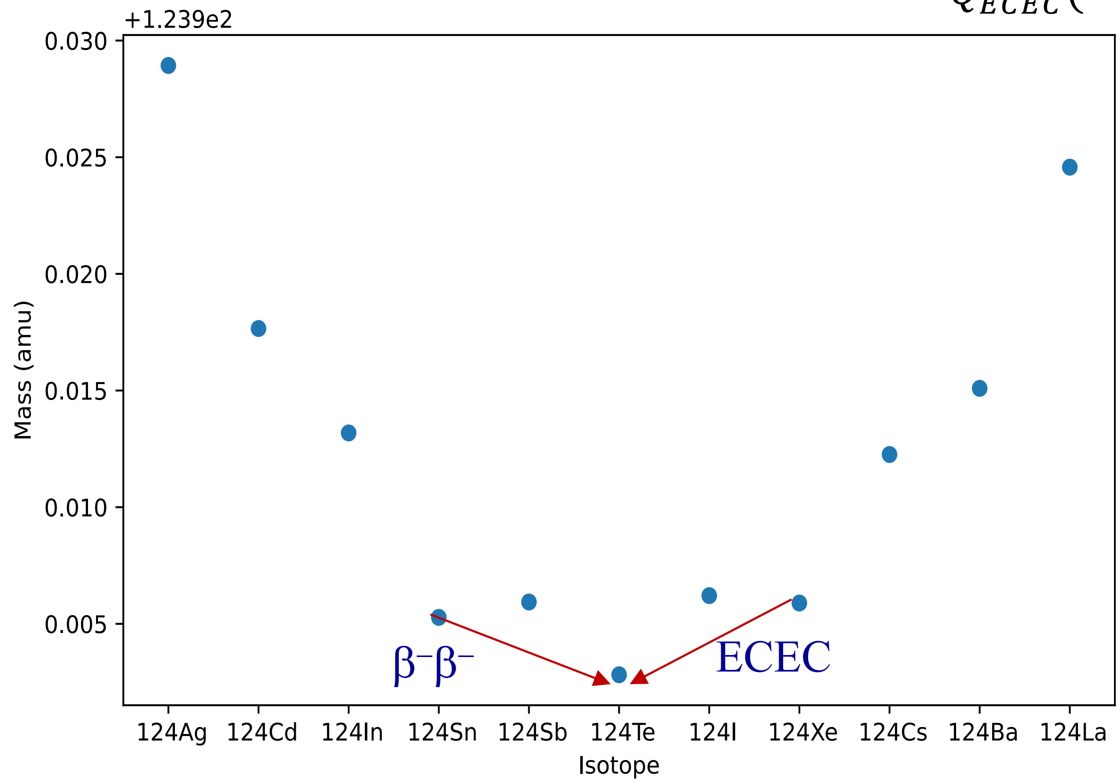
Double Beta /ECEC in A=124

$Q_{ECEC}(^{124}Xe) = 2.857\ MeV$





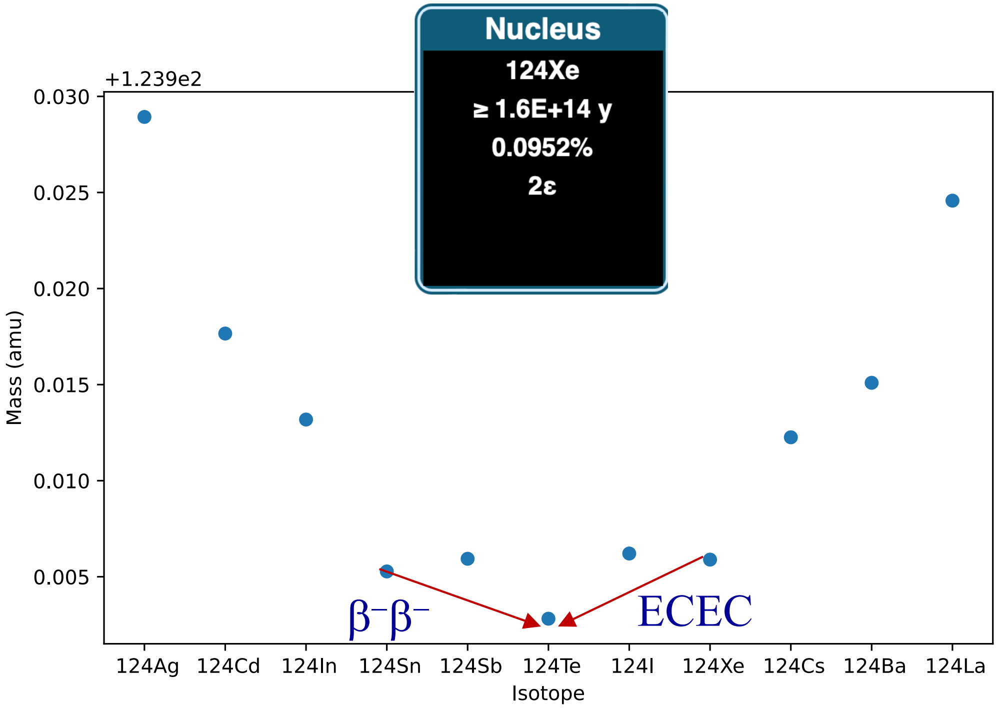
Nucleus
124Xe
≥ 1.6E+14 y
0.0952%
2ε

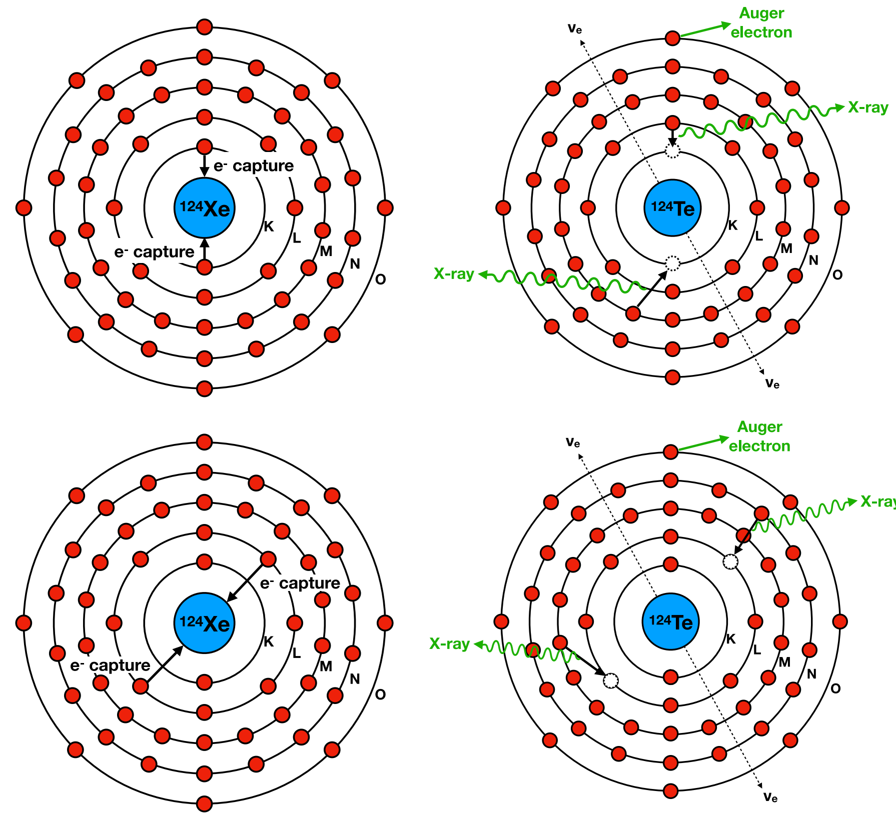
$$T_{1/2}^{-1}(2\nu) = g_A^4 G_{2\nu} |M^{2\nu}|^2$$

$$M^{2\nu} = m_e \sum_n \frac{\langle 0_f^+ | \sigma\tau^- | 1_n^+ \rangle \langle 1_n^+ | \sigma\tau^- | 0_i^+ \rangle}{E_{1_n^+} + E_0}$$

The 2νECEC decay of ^{124}Xe

$$Q_{ECEC}(^{124}Xe) = 2.857\ MeV$$





PHYSICAL REVIEW C **93**, 024308 (2016)

Shell model predictions for ^{124}Sn double-β decay

Mihai Horoi* and Andrei Neacsu†

Theoretical analysis and predictions for the double electron capture of ^{124}Xe

O. Niţescu[a,b,c], S. Ghinescu[d,b,c], V. A. Sevestrean[d,b,c], M. Horoi[e,b], F. Šimkovic[a,f], S. Stoica[b]

arXiv:2402.13784v1

The 2νECEC decay of ^{124}Xe: Shell Model Nuclear Matrix Elements

$$\left[T_{1/2}^{2\nu ECEC}\right]^{-1} = \left(g_A^{\rm eff}\right)^4 \left|M_{GT-1}^{2\nu ECEC}\right|^2 \{G_0^{2\nu ECEC} + \xi_{31}^{2\nu ECEC} G_2^{2\nu ECEC} + \frac{1}{3}\left(\xi_{31}^{2\nu ECEC}\right)^2 G_{22}^{2\nu ECEC} + \left[\frac{1}{3}\left(\xi_{31}^{2\nu ECEC}\right)^2 + \xi_{51}^{2\nu ECEC}\right] G_4^{2\nu ECEC}\},$$

$$g_A^{eff} \rightarrow g_A = 1.276$$

$$\xi_{31}^{2\nu ECEC} = \frac{M_{GT-3}^{2\nu ECEC}}{M_{GT-1}^{2\nu ECEC}}, \qquad \xi_{51}^{2\nu ECEC} = \frac{M_{GT-5}^{2\nu ECEC}}{M_{GT-1}^{2\nu ECEC}}$$

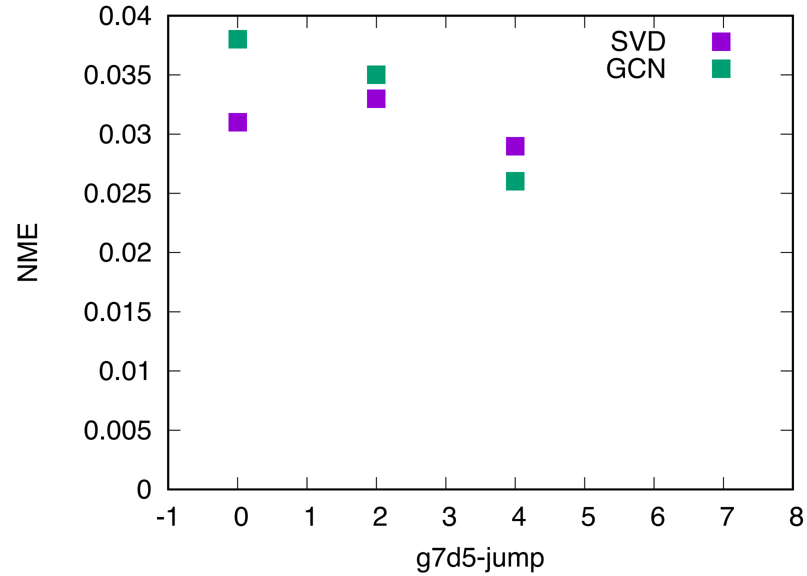
$$M_{GT-1}^{2\nu ECEC} = \sum_n M_{GT}^{2\nu}(n) \frac{m_e}{E_n(1^+) - (E_i + E_f)/2},$$

$$M_{GT-3}^{2\nu ECEC} = \sum_n M_{GT}^{2\nu}(n) \frac{4\, m_e^3}{(E_n(1^+) - (E_i + E_f)/2)^3},$$

$$M_{GT-5}^{2\nu ECEC} = \sum_n M_{GT}^{2\nu}(n) \frac{16\, m_e^5}{(E_n(1^+) - (E_i + E_f)/2)^5}.$$

$$M_{GT}^{2\nu}(n) = \langle 0_f^+ \| \sum_m \tau_m^- \sigma_m \| 1_n^+ \rangle \langle 1_n^+ \| \sum_m \tau_m^- \sigma_m \| 0_i^+ \rangle,$$

$$(\sigma\tau^-)^{eff} \rightarrow q_H\, \sigma\tau^-$$



$q_{SVD} = 0.7$

$q_{GCN} = 0.4$

Model	Channel	jump=0	jump=2	jump=4
SVD	Total	1.94	1.61	2.12
	KK	2.61	2.17	2.86
GCN	Total	1.20	1.40	2.58
	KK	1.62	1.88	3.48

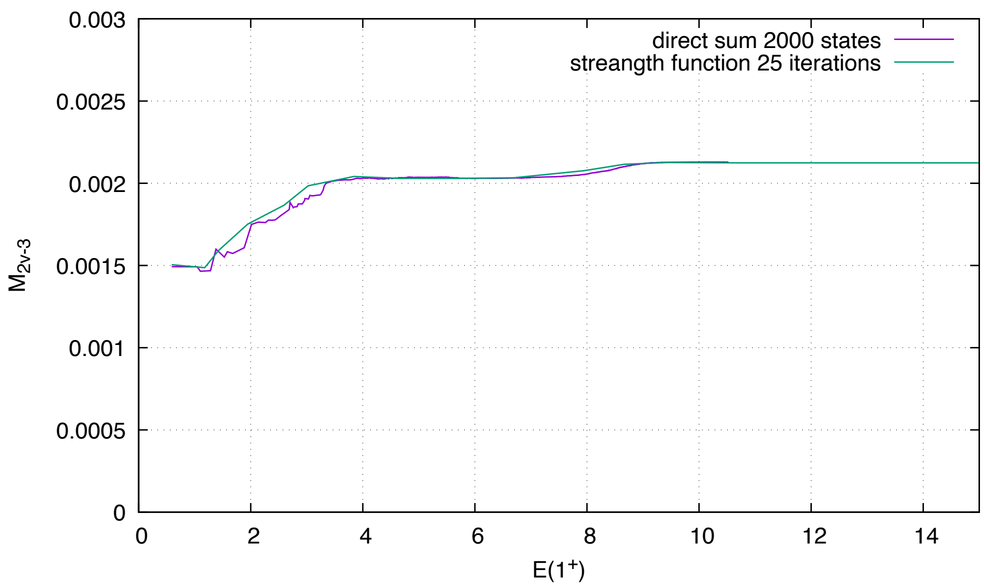
Table 5: The predicted 2νECEC half-lives for ^{124}Xe (in units of 10^{22} yr) from Eqs. (1-2). To be compared with experimental data for the total half-life, $(1.1 \pm 0.2_{stat} \pm 0.1_{sys}) \times 10^{22}$ yr and the inferred data for the KK half-life, $(1.5 \pm 0.3_{stat} \pm 0.1_{sys}) \times 10^{22}$ yr (see section III.F of Ref. [15]).

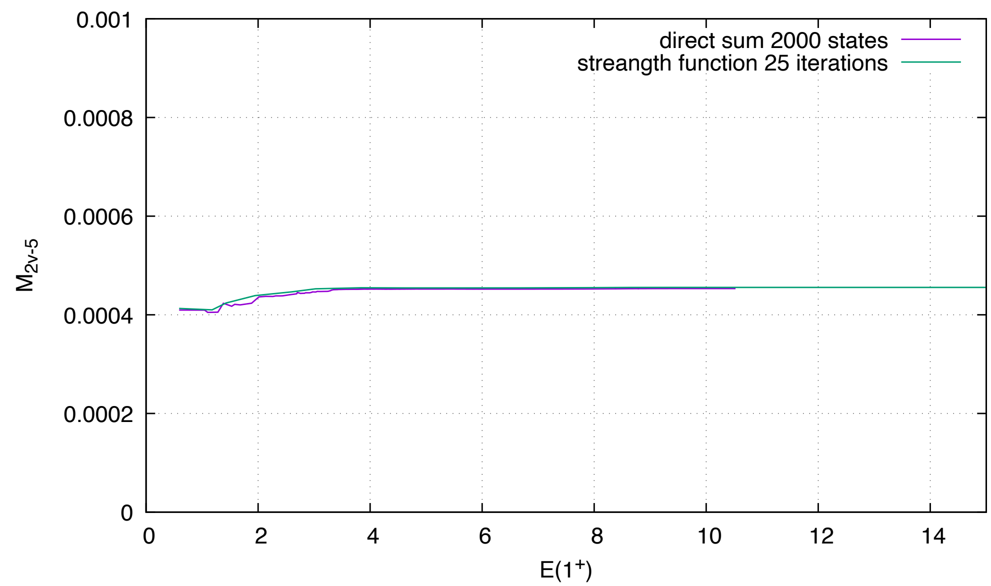
Taylor Expansion NMEs: ^{136}Xe

$$M_{2\nu-3}(E_k) = \sum_{n=1}^{k} \frac{\langle \Psi_f | q\sigma\tau^- | 1_n^+ \rangle \langle 1_n^+ | q\sigma\tau^- | \Psi_i \rangle}{\left(E(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta \right)^3}$$

$$M_{2\nu-5}(E_k) = \sum_{n=1}^{k} \frac{\langle \Psi_f | q\sigma\tau^- | 1_n^+ \rangle \langle 1_n^+ | q\sigma\tau^- | \Psi_i \rangle}{\left(E(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta \right)^5}$$

^{136}Xe – SVD

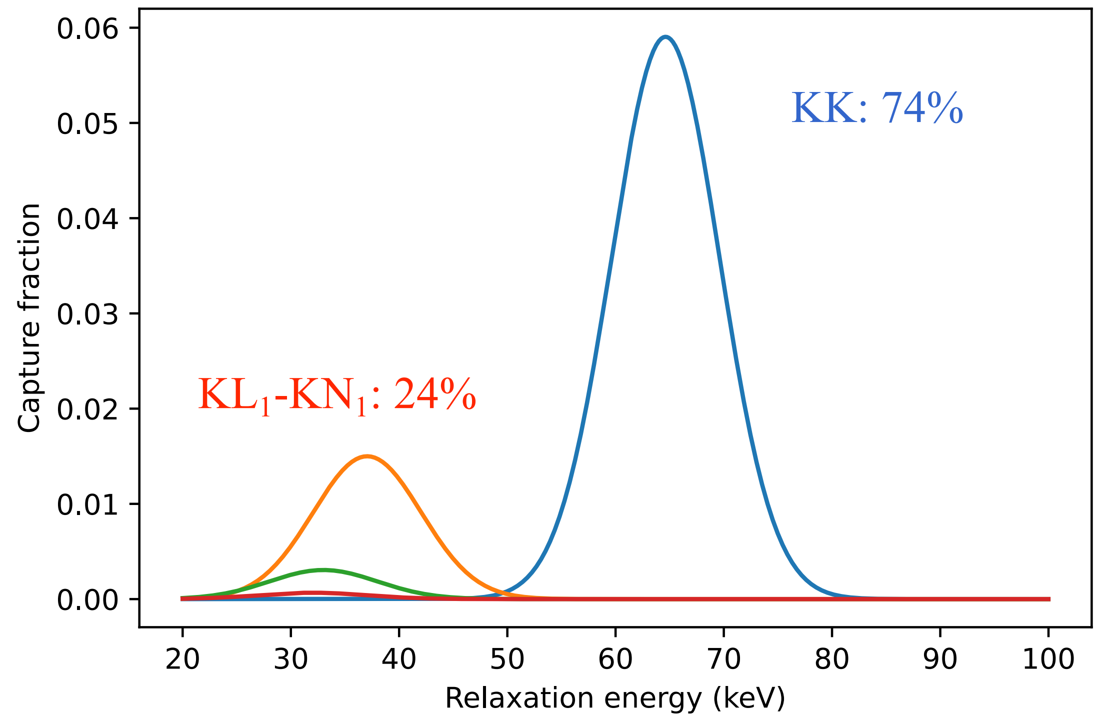




The 2νECEC decay of ^{124}Xe: Predictions for Capture Fractions

Decay Chanel	R_{xy} (keV)	ISM CF (%)
KK	64.62	74.13-74.15
KL_1	37.05	18.76-18.83
KM_1	32.98	3.83-3.84
KN_1	32.11	0.83-0.85
KO_1	31.93	0.13
L_1L_1	10.04	1.22
L_1M_1	6.01	0.49
Other	< 6	0.52-0.55

Table 6: The atomic relaxation energies (Eq. 10) obtained within the DHFS model (second column) and the capture fractions (CF) predicted by ISM (third column). The captures with atomic relaxation energies below 6 keV are subsumed under the label "other". The ranges presented for the KK and KL_1 channels correspond to the minimum and maximum values of the $\xi_{31}^{2\nu ECEC}$ parameter predicted from ISM.



Conclusions

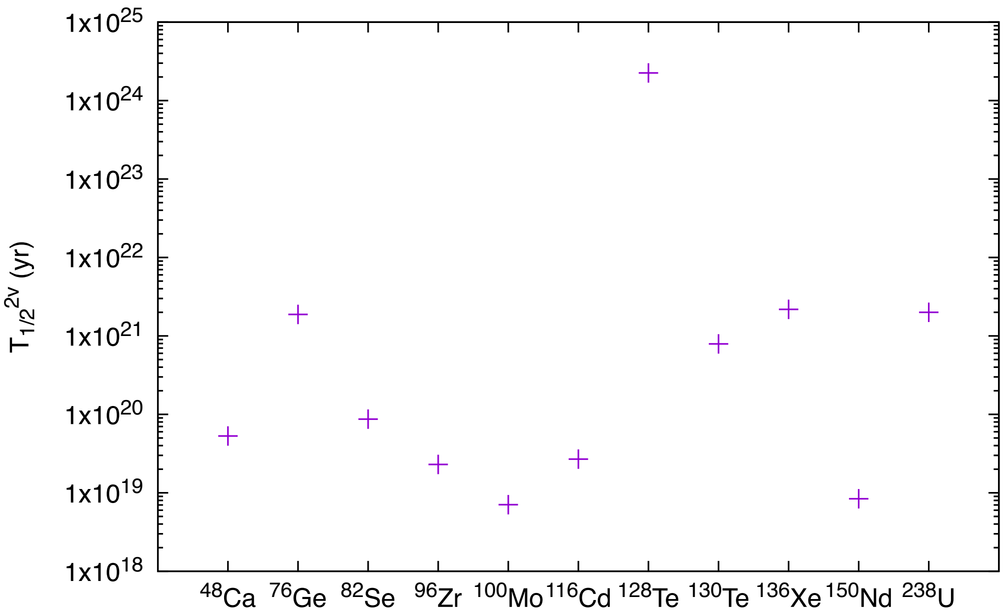
- Shell model calculations of the $2\nu\beta\beta$ nuclear matrix elements can be obtained very efficiently using a strength function approach
- The direct sum is likely to miss convergence in most cases
- The strength function approach is the only option for cases that have very large shell model dimension, such as ^{128}Te
- The proper evaluation of the $2\nu\beta\beta$ nuclear matrix elements is important while they correlate strongly with the $0\nu\beta\beta$ nuclear matrix elements (PRC 106, 05432 (2022), PRC 107, 045501 (2023), Universe 10, 252 (2024))

2νββ Half-Lives and optimal quenching factors

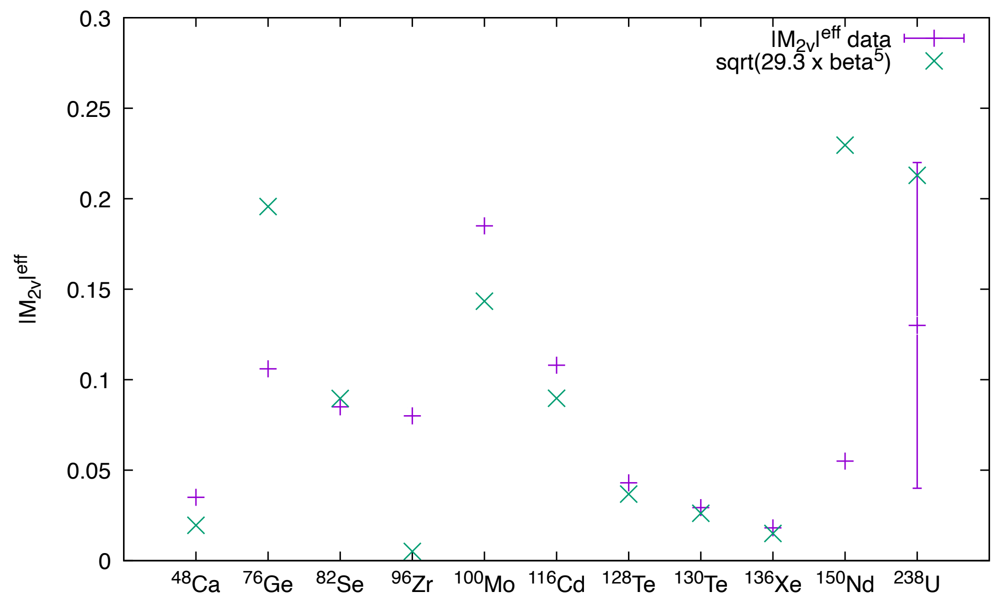
Isotope	Q_{bb}	Q_b	E_0	$E_1(1^+)$	$M_{2\nu}^{eff}$	$M_{2\nu}$	q_{opt}	Hamiltonian
48Ca	4.268	0.280	1.855	2.517	0.0350	0.0421	0.68	GXPF1A
76Ge	2.039	-0.922	1.941	0.044	0.1060	0.1274	0.63	GCN2850
							0.62	JUN45
82Se	2.998	-0.095	1.594	0.075	0.0850	0.1022	0.55	GCN2850
							0.62	JUN45
96Zr	3.356	0.164	1.514	?	0.0800	0.0962		
100Mo	3.034	-0.172	1.689	0.000	0.1850	0.2224		
116Cd	2.813	-0.463	1.869	0.000	0.1080	0.1298		
128Te	0.867	-1.256	1.689	0.000	0.0430	0.0517	0.84	SVD
130Te	2.528	-0.417	1.681	0.255	0.0293	0.0352	0.50	GCN5082
							0.88	SVD
136Xe	2.458	-0.090	1.319	0.590	0.0181	0.0218	0.42	GCN5082
							0.69	SVD
150Nd	3.371	-0.083	1.769	?	0.0550	0.0661		
238U	1.114	-0.147	0.704	0.244	0.1300	0.1563		

2νββ Half-Lives and NME

$$\left[T_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}\cdot\left[g_A^2\left(m_e c^2\cdot M_{2\nu}\right)\right]^2 \equiv G_{2\nu}\left(M_{2\nu}^{eff}\right)^2$$



A. Barabash, Universe 6, 159 (2020)



B. Pritychenko, Nucl. Phys. A 1033, 122628 (2023)