

# Shell Model for Double Beta Decay

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## Double Beta-Disintegration

M. GOEPPERT-MAYER, The Johns Hopkins Universit  $(Received May 20, 1935)$  $\mathcal{Y}$ 

From the Fermi theory of  $\beta$ -disintegration the probability of simultaneous emission of two From the Fermi theory or p-disintegration the probability or simultaneous emission or two<br>electrons (and two neutrinos) has been calculated. The result is that this process occurs sufficiently rarely to allow a half-life of over  $10^{17}$  years for a nucleus, even if its isobar of atomic number different by 2 were more stable by 20 times the electron mass.





July 19-21 2024  $\alpha$ -particle are distributed as  $\alpha$  $Jury 19-21 2024$ 

even more closely, to the simultaneous emission

From the positions of these two bands, together with those







### First Searches of  $0\nu\beta\beta$  $H1TST, N27$  $\frac{1}{2}$  Rev. 3 W. Wahl, Naturwiss. 2008. 3  $T$  counters are shifted by an intersection  $\Omega$  $\bullet$  hes of  $\theta$ ,  $\mathcal{B}$ ides of  $UVDD$ placed between the thin window counters. These specimens

### Phys. Rev 75, 323 (1949) Letter to the Editor

### A Measurement of the Half-Life of Double Beta-Decay from  $_{50}Sn^{124}$  \*

E. L. FIREMAN

Department of Physics, Princeton University, Princeton, New Jersey November 29, 1948

TABLE II.  $L(A)$  gives the coincidences from specimen A between counters Z, and  $R(B)$  gives the coincidences from B between counters R. Holder 0° and 180° are the two holder positions. Positions 1 and 2 are positions for the specimens in the holder.



A detailed report of this work is being prepared for measurement with Al absorbers over the side of each publication in the Physical Review.

The author is grateful to Professor R. Sherr for accepting the supervision of this research and to Professor E. P. Wigner for many profitable discussions.

\* This work is assisted by the Office of Naval Research.<br>\*\* These isotopes were obtained from Oak Ridge.<br><sup>1</sup>Maria Goeppert-Mayer, Phys. Rev. **48**, 512 (1935).<br><sup>2</sup>W. H. Furry, Phys. Rev. **56**, 1184 (1939). <sup>1</sup>Maria Goeppert-Mayer, Phys. Rev. 48, 512 (1935).  $^2$ W. H. Furry, Phys. Rev. 56, 1184 (1939).

### PH YSICAL REVIEW VOLUME 86, NUMBER 4

 $MAY$  15, 1952

### $\mathbf{f}(\mathbf{r}) = \mathbf{f}(\mathbf{r})$  result would indicate that  $\mathbf{f}(\mathbf{r})$ A Re-Investigation of the Double Beta-Decay from Sn<sup>124</sup>

E. L. FIREMAN AND D. SCHWARZER E. L. FIKEMAN AND D. SCHWARZER<br>Brookhaven National Laboratory, Upton, New York (Received February 5, 1952)  $M\Delta$  points pointed out to the set of these results points  $\alpha$  $p_{52}$  $\mathbf{ER}$ 

In the cloud chamber that is triggered by internal counters. Only three pictures out of more than four<br>helium filled cloud chamber that is triggered by internal counters. Only three pictures out of more than four thousand photographs are pictures of two electrons coming out of the same point in the tin and entering thousand photographs are pictures of two electrons coming out of the same point in the tin and entering  $\frac{1}{2}$ However, one may set a lower limit to the half-life of double beta-decay from  $\text{Sn}^{124}$  as  $10^{17}$  years. This is a Radiations from natural tin and tin enriched with the 124 isotope are examined in a magnetic Geld with a the counters, and even these may be pictures of multiply scattered electrons passing through the tin. decay rate less than one-tenth of a value previously reported by one of the authors.

ANL Symposium, July 19-21 2024 INTERNATIONAL ENERGIA ENERGIA<br>Energia energia energi  $\sim$  Theories of the neutrino predict different proper-





## Classical Double Beta Decay Problem



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# DBD Phase Space Factors







## $2\nu\beta\beta$  Half-Lives and NME v pp 11d The 2012 control of the 2012 control of the 2014 control of th

$$
\[T_{1/2}^{2\nu}\]^{-1} = G_{2\nu} \cdot \left[g_A^2 \left(m_e c^2 \cdot M_{2\nu}\right)\right]^2 \equiv G_{2\nu} \left(M_{2\nu}^{eff}\right)^2\]
$$



A. Barabash, Universe 6, 159 (2020)  $\alpha$  is check the convergence. Ref.  $\alpha$  proposed the convergence. Ref.  $\alpha$ A. Barabash, Universe 6, 159 (2020)

ANL Symposium, July 19-21 2024

M. Horoi CMU 7 it was never used in practical calculations. Here, we pro-









# Quenching factor vs Effective Operator



ANL Symposium, July  $19-21$  2024  $\frac{300 \text{ N}}{21212027}$ 



### Ab-initio effective Gamow-Teller operator the listed strength is summed over multiple final states with the same spin and parity.  $\Gamma$  CENTRAL MICHIGAN  $\Delta$  **h**-111110  $\rho$  fter (b) shows *MGT*)th vs *M*(*GT*)exp. The solid line shows *y* = *x* corresponding to the perfect is indicated as a quenching factor at the figure. For this quenching factor,  $\Omega$ include *sd* and *p f* shell nuclei because the *p* shell nuclei have a large scatter due to nuclear check for **updates**  $\mathbf{p}$  $\mathbf{I}$



**Citation:** Stroberg, S.R. Beta Decay in Medium-Mass Nuclei with the In-Medium Similarity Renormalization Group. *Particles* **2021**, *4*, 521–535. https://doi.org/ 10.3390/particles4040038



ANL Symposium, M. Hore July 19-21 2024







### MIGHIGAN Two-Neutrino NME: Direct Sum? technique as opposed to the direct diagonalization of the direct direct diagonalization methods are  $\Gamma$ employed in the present study. Some possible alternatives are 2  $\frac{1}{2}$  $\mathcal{L}$  is the phase-space  $\mathcal{L}$  and  $\mathcal{L}$

written as  $\frac{1}{2}$ 

 $\overline{D}$ tivcio at deview o 100 00/201 PHYSICAL REVIEW C **109**, 024301 (2024)

### In Fig. 6, we demonstrate the dependence of NMEs for **Calculation of nuclear matrix elements for 0**νββ **decay of 124Sn using the nonclosure** 2νββ decay on the cutoff excitation energy (*Ec*) of 1<sup>+</sup> states approach in the nuclear shell model

 $M_{GT}^{2\nu} = \sum$  $k, E^*_k$ ≤ $E_c$  $\langle f||\sigma\tau_2^-||k\rangle\langle k||\sigma\tau_1^-||i\rangle$  $E_k^* + E_0$ 

Shahariar Sarkar $\bigcap_{k=0}^{\infty} \frac{1}{k}$  P K Rath 2 V Nanal  $\bigcap_{k=0}^{\infty} \frac{3}{k}$  R G Pillay  $\bigcap_{k=0}^{\infty} \frac{1}{k}$  Puch Shahariar Sarkar  $\bullet$ , <sup>1,\*</sup> P. K. Rath,<sup>2</sup> V. Nanal  $\bullet$ ,<sup>3</sup> R. G. Pillay  $\bullet$ ,<sup>1</sup> Pushpendra P. Singh  $\bullet$ ,<sup>1</sup> Y. Iwata,<sup>4</sup> K. Jha $\bullet$ ,<sup>1</sup> and P. K. Raina<sup>1,†</sup> K. Jha $\bullet$ ,<sup>1</sup> and P. K. Raina<sup>1,†</sup> where the isospin lowering operator. In this study,  $\mathbf{1}_{\{1,2\}}$ 



ANL Symposium, M. th C<sub>2</sub> Symposium,<br>July 19-21 2024  $\overline{a}$  $JUUY$  19-21 2024

M. Horoi CMU 12 **retical nuclear models in the widely-some of the widely-some of the widely-some of the widely-some of the widely-**





### Strength Function Approach recipes for all its ingredients, and it was never used in practical calculations. Here, I  $P_{\text{univexponent}}$  such to contral michigan street and the expansion proposed to the expansion proposed by the expansion proposed by  $\mathbf{A}$  more all  $\mathbf{A}$ in Ref. [103]. Following Ref. [103], I choose as a starting Lanczos vector *L±* <sup>1</sup> either the initial

## $\alpha$  final states in the decay (only 0+ to 0+ to 0+ to 0+ to 0+ transitions are considered here), on which is decay (only 0+ to 0+ Create doorway states:

$$
|\sigma\tau^{-}0_{i}^{+}\rangle = c_{-}|dw_{-}\rangle \equiv c_{-}|L_{1}^{-}\rangle
$$
  

$$
|\sigma\tau^{+}0_{f}^{+}\rangle = c_{+}|dw_{+}\rangle \equiv c_{+}|L_{1}^{+}\rangle
$$

Relation to GT sum rules:

$$
3|c_{-}|^{2} = B_{sum}(GT; i-) \qquad 3|c_{+}|^{2} = B_{sum}(GT; f+) \qquad
$$

*m MAI MAINOU OI LUITUZOS ITUTUTOIS STAT* <sup>2</sup>*<sup>n</sup>* . (17) Do a small number of Lanczos iterations starting with  $|L_1^->$ :

$$
M_{2\nu}(E_k) = \sqrt{B_{sum}(GT; f+)B_{sum}(GT; i-)} \sum_{n=1}^{k < 25} \frac{\langle \Psi_f | \tilde{1}_n^+ \rangle \langle \tilde{1}_n^+ \rangle |\Psi_i \rangle}{\tilde{E}(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_{\beta}}
$$

See also M. Horoi, Physics 2022, 4, 113 See also M. Horoi, Physics 2022, 4, 1135

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*k* 13 *<sup>L</sup>* (1<sup>+</sup>





#### Strength Function Approach  $\sum$ tranoth  $\sum$ pressed by the phase space factors). In (I) *Em* is the excitation energyof the intermediate I + state and (ii) The double beta matrix elements *Mb'T(J)* ob- $\tau$  and  $\Lambda$  normal *Mb'T(O+)* =0.0402 (MeV)-I,

### Caurier, Poves, Zuker, Phys. Lett. B 252, 13 (1990)



(iii) The convergence with respect to the numbe *N* of Lanczos iterations on the  $\beta$ <sup>-</sup> doorway is ex tremely fast, as the following numbers show:

 $M_{\text{GT}}^{2v}(0^+) = 0.0086, N=1$ ,  $M_{\text{GT}}^{2v}(0^+) = 0.0433, N=4,$  $M_{\text{GT}}^{2v}(0^+)$  = 0.0403,  $N=12$ ,  $M_{\text{GT}}^{2v}(0^+) = 0.0402, \quad N = 30$ ,  $M_{\text{CT}}^{2v}(0^+) = 0.0402, \quad N = 60$ ,

Engel, Haxton, Vogel, Phys. Rev. C 46, R2153 (1991) that even if the *p+* and *P-* doorways are almost or- $\mathcal{L}$  there have been also calculations in heaviele been also calculations in heaviele been also calculated by  $\mathcal{S}_{\mathbf{u}}, \mathbf{u}, \mathbf{v}$  is the standard with the standard  $(1, 1, 1)$ 

$$
M_{GT} = -\frac{1}{2} \left\langle 0_f^+ \left| \sum_{i=1}^A \sqrt{3} \sigma_z(i) \tau_{-}(i) \frac{1}{E_0 - H} \right| \right\rangle
$$
  
 
$$
\times \sum_{j=1}^A \sqrt{3} \sigma_z(j) \tau_{-}(j) \left| 0_i^+ \right\rangle
$$

$$
\frac{1}{E_0 - H} |v_1\rangle = g_1(E_0)|v_1\rangle + g_2(E_0)|v_2\rangle + \cdots
$$



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of the particle-particle-particle-particle-particle-particle-particle-particle interaction and the importance



# Two-Neutrino NME: Direct Sum vs Strength Function Approach

$$
M_{2\nu}(E_k) = \sum_{n=1}^k \frac{\langle \Psi_f | q \sigma \tau^- | 1_n^+ \rangle \langle 1_n^+ | q \sigma \tau^- | \Psi_i \rangle}{E(1_n^+) + \frac{Q_{\beta \beta}}{2} - Q_{\beta}}
$$

 $M_{2\nu}(E_k) = \sqrt{B_{sum}(GT; f+)B_{sum}(GT; i-) }$  $\overline{n=1}$  $\sum_{k \leq 25}^{k \leq 25}$   $\langle \Psi_f | \tilde{1}_n^+ \rangle \langle \tilde{1}_n^+ \rangle | \Psi_i$  $\widetilde{E}(1\frac{+}{n}) + \frac{Q_{\beta\beta}}{2} - Q_{\beta}$ 

 $48Ca - KB3G$ 

 $^{48}Ca - GXPF1A$ 







# Two-Neutrino NME: Direct Sum vs Strength Function Approach

$$
M_{2\nu}(E_k) = \sum_{n=1}^k \frac{\langle \Psi_f | q \sigma \tau^- | 1_n^+ \rangle \langle 1_n^+ | q \sigma \tau^- | \Psi_i \rangle}{E(1_n^+) + \frac{Q_{\beta \beta}}{2} - Q_{\beta}}
$$

$$
M_{2\nu}(E_k) = \sqrt{B_{sum}(GT; f+)B_{sum}(GT; i-)} \sum_{n=1}^{k < 25} \frac{\langle \Psi_f | \tilde{1}_n^+ \rangle \langle \tilde{1}_n^+ \rangle |\Psi_i \rangle}{\tilde{E}(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_{\beta}}
$$

 $136Xe - SVD$   $136Xe - GCN5082$ 







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0 2 4 6 8 10 12

 $E(1^{+})$ 



0 2 4 6 8 10 12

 $E(1^{+})$ 



# The Late Drop in 48Ca

0.1

$$
M_{2\nu}(E_k) = \sum_{n=1}^k \frac{\langle \Psi_f | q \sigma \tau^- | 1_n^+ \rangle \langle 1_n^+ | q \sigma \tau^- | \Psi_i \rangle}{E(1_n^+) + \frac{Q_{\beta \beta}}{2} - Q_{\beta}}
$$



## $48Ca - GXPF1A$





# Strength Function: large dimensions

**CENTRAL MICHIGAN UNIVERSITY** 





#### $T$ he  $2\nu$ ECEC decay of <sup>124</sup>Xe: Experimental data per to 124 and 124 keV. Let us decay o  $124\mathbf{V}_{\mathbf{\rho}}$ .  $\Delta U.$ <sup>17</sup>*Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Rehovot 7610001, Israel* <sup>18</sup>*Department of Physics & Center for High Energy Physics, Tsinghua University, Beijing 100084, China* <sup>19</sup>*LIBPhys, Department of Physics, University of Coimbra, 3004-516 Coimbra, Portugal* <sup>21</sup>*Institute for Astroparticle Physics, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany* <sup>22</sup>*Department of Physics and Chemistry, University of L'Aquila, 67100 L'Aquila, Italy*

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## LETTER **https://doi.org/10.1038/s41586-019-1124-4**

PHYSICAL REVIEW C 106, 024328 (2022)  $R11. 2019$   $\frac{1}{2019}$   $\frac{1}{2019}$   $\frac{1}{201}$   $\frac{1}{201}$   $\frac{1}{201}$   $\frac{1}{201}$ 

**Editors' Suggestion** *University, Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8602, Japan*

**Double-weak decays of 124Xe and 136Xe in the XENON1T and XENONnT experiments** <sub>27</sub> buble-weak decays of <sup>124</sup>Xe and <sup>136</sup>Xe in the XENON1T and XENONnT

### Observation of two-neutrino double electron **capture in <sup>124</sup>Xe with XENON1T U** doub **the instruments with** *X***LTONIII and <b>doub** doub

XENON Collaboration\*

 $\mathbf{0}$  and  $\mathbf{0}$  recoiling nucleus is negative is negative is negative is negative is negative.  $\frac{1}{6}$ energy *Q* are undetected, only the X-rays and Auger electrons are meas $t$ he total energy for double  $t$  is 64.3 keV.3  $T_{\rm eff}$  already been corrected for energy depositions that does not energy depositions that does not expect that do **background in the set of them directly with high set of the set of . 2***<i>y*</del>. **2***<i>S*<sub>1</sub>**EC**</sup>  $\frac{30.14}{\pi}$  the h resent results on the search for two-neutrino double-electron capture (2 $\nu$ ECEC) of <sup>124</sup>Xe and neutrinology  $\beta$  decay (Uv $\beta$ ) of  $\alpha$  also already all the consider captures from the K shell up CCEC signal model and measure a total half-life of  $T_{1/2}^{2 \text{ vECEC}} = (1.1 \pm 0.2_{\text{stat}} \pm 0.1_{\text{sys}}) \times 10^{22} \text{ yr with}$ <br>For the state assumes The statistical significance of the signal is 7.0 – We use YENONUT date with 0.87 kg yr isotope exposure. The statistical significance of the signal is 7.00. We use AENONTT data with<br>36.16 kg yr of <sup>136</sup>Xe exposure to search for  $0\nu\beta\beta$ . We find no evidence of a signal and set a lower limit on g y or  $\alpha$  exposure to search for  $\partial \nu \rho \rho$ , we find no evidence or a signal and set-<br>life of  $T_{\nu}^{\rho \nu \beta \beta} > 1.2 \times 10^{24}$  vr at 90% CL. This is the best result from a dark matter for dark-matter searches  $\frac{1}{2}$  . The current leading lead We present results on the search for two-neutrino double-electron capture ( $2\nu$ ECEC) of <sup>124</sup>Xe and neutrinoless double- $\beta$  decay (0v $\beta\beta$ ) of <sup>136</sup>Xe in XENON1T. We consider captures from the K shell up to the N shell in the 2vECEC signal model and measure a total half-life of  $T_{1/2}^{2 \text{vECEC}} = (1.1 \pm 0.2_{\text{stat}} \pm 0.1_{\text{sys}}) \times 10^{22} \text{ yr with a}$ 0.87 kg yr isotope exposure. The statistical significance of the signal is  $7.0\sigma$ . We use XENON1T data with  $T^{0\nu\beta\beta}$ ,  $12.10^{24}$   $10^{24}$ ,  $10^{26}$   $\sigma$  M<sub>2</sub>. Masson, 15 E. Masson, 16 E. Masson, 15 E. Masson, 45 E. Masson, 15 E. Masson, 15 E. Masson, 15 the half-life of  $T_{1/2}^{0\nu\beta\beta} > 1.2 \times 10^{24}$  yr at 90% CL. This is the best result from a dark matter detector without

**.**  energy *Q* are undetected, only the X-rays and Auger electrons are meas-

#### $T_{1/2}^{2\nu \rm{ECEC}}>2.1\times10^{22}$  yr  $T_{1/2}^{2\nu\text{ECEC}} > 2.1 \times 10^{22} \text{ yr}$  $A.$  Termina,20, $B.$  There is the result,25  $R$  and 21  $R$  Termina,21  $R$  and 21  $R$   $\geq$   $1$  $S_1$  2  $>$  2.1  $\times$  10  $\gamma$ 1

**absolute neutrino mass**<sup>15–17</sup>. Here of  $2\nu$ ECEC in  $^{124}$ Xe with the  $\lambda$ The significance of the signal is  $4.4$  standard deviations and the **corresponding half-life of 1.8**  $\times$  10<sup>22</sup> years (statistical uncertainty,  $0.5 \times 10^{22}$  years; systematic uncertainty,  $0.1 \times 10^{22}$  years) is the The long half-life of double electron capture makes it extremely T<sup>2PECEC</sup>>2.1×10<sup>22</sup> <sub>VT</sub><br>2 We report the direct observation of  $2\nu$ ECEC in <sup>124</sup>Xe with the XENON1T dark-matter detector. 4.4 standard deviations and the  $\mu$  $t_1$  time-projection chamber (TPC): a cylinder with diameter  $\alpha$ 10<sup>44</sup> vears (statistical uncertainty  $\begin{bmatrix} 1 & 1 \end{bmatrix}$  photomultiplier tubes (PMTs). The TPC is used for  $\begin{bmatrix} 2 & 0 \end{bmatrix}$ . ertainty  $0.1 \times 10^{22}$  years) is the  $\sum_{i=1}^{n}$  interactions—the latter by converting ionizations—the latter by converting ionizations—the latter by converting ionizations—the latter by converting ionizations—the latter by converting  $\sum_{i=1}^{n}$  $M^{2\nu}$  MGC and would give a new  $\sim 2$ **absolute neutrino mass15–17. Here we report the direct observation**  dia.<br>10.11

 $\frac{111}{2}$  by 19-21 2024

 $July 17-21 202+$ 

vital experimental constraint for these models, as well as insight into



were selected to have low amounts of radioactive impurities and low amounts of radioactive impurities and low radon emanation rates29. In addition, the anthropogenic β-emitter 85Kr **associated with the Choice of the Choice of the Choice of the Choice of the NME.**  $\overline{O(f)}$  the most  $\overline{O(f)}$  range in  $\overline{13}$ *MICE OF SCIENCE* 



# Double Beta /ECEC in A=124







# The 2*v*ECEC decay of <sup>124</sup>Xe

 $Q_{ECEC}({}^{124}Xe) = 2.857 \text{ MeV}$ 



PHYSICAL REVIEW C **93**, 024308 (2016)

### **Shell model predictions for 124Sn double-**β **decay**

Mihai Horoi\* and Andrei Neacsu*†*

Theoretical analysis and predictions for the double electron capture of  $^{124}Xe$ 

O. Nitescu<sup>a,b,c</sup>, S. Ghinescu<sup>d,b,c</sup>, V. A. Sevestrean<sup>d,b,c</sup>, M. Horoi<sup>e,b</sup>, F. Šimkovic<sup>a,f</sup>, S. Stoica<sup>b</sup>

#### $\delta r$ Vi*n<sup>2</sup>* 2402 12784 $\mu$ <sup>1</sup> *cHoria Hulubei National Institute of Physics and Nuclear Engineering 30 Reactorului POB MG-6 RO-077125 Magurele Romania* arXiv:2402.13784v1

ANL Symposium, a promising between the M. Home July  $19-21$  2024  $JUUY$  19-21  $2024$ 

M. Horoi CMU 23 **for Experimental Applied Physics C***n* 





### The 2*v*ECEC decay of <sup>124</sup>Xe: Shell Model Nuclear Matrix Elements  $\sum_{\text{UNIVERSITY}}$   $\left| \begin{array}{cc} \mathbf{h} & \mathbf{A} \\ \mathbf{v} & \mathbf{A} \end{array} \right|$  $T_{\text{univers}}^{\text{nonivers}}$  and  $T_{\text{no}}$  atomic the power in the lepton energies, we obtain the total inverse halflife of the 2nECEC process in a similar form: electrons are only called  $\mathcal{L}$  and  $\mathcal{L}$ 2n  $\Gamma$  decay of  $124V$ <sub>a</sub>.  $\text{Chol}$ <sup>1</sup>

$$
\begin{aligned}\n\left[T_{1/2}^{2VECEC}\right]^{-1} &= \left(g_A^{\text{eff}}\right)^4 |M_{GT-1}^{2VECEC}|^2 \left\{G_0^{2VECEC} + \frac{1}{3} \left(\xi_{31}^{2VECEC}\right)^2 G_{22}^{2VECEC} \right. \\
&\quad + \left[\frac{1}{3} \left(\xi_{31}^{2VECEC}\right)^2 + \xi_{31}^{2VECEC}\right] G_4^{2VECEC} \right\}, \\
&g_A^{eff} \rightarrow g_A = 1.276 \\
&g_A^{2VECEC} &= \frac{M_{GT-3}^{2VECEC}}{M_{GT-1}^{2VECEC}}, \quad \xi_{51}^{2VECEC} = \frac{M_{GT-5}^{2VECEC}}{M_{GT-1}^{2VECEC}} \\
&M_{GT-1}^{2VECEC} &= \sum_n M_{GT}^{2V} (n) \frac{m_e}{E_n (1^+) - (E_i + E_f)/2}, \\
&M_{GT-3}^{2VECEC} &= \sum_n M_{GT}^{2V} (n) \frac{4 m_e^3}{(E_n (1^+) - (E_i + E_f)/2)^3}, \\
&M_{GT-5}^{2VECEC} &= \sum_m M_{GT}^{2V} (n) \frac{16 m_e^5}{(E_n (1^+) - (E_i + E_f)/2)^3}.\n\end{aligned}
$$

$$
M_{GT-5}^{2vECEC} = \sum_{n} M_{GT}^{2v}(n) \frac{10 m_e}{(E_n(1^+) - (E_i + E_f)/2)^5}.
$$
  

$$
M_{GT}^{2v}(n) = \langle 0_f^+ \|\sum_{m} \tau_m^- \sigma_m \|1_n^+ \rangle \langle 1_n^+ \|\sum_{m} \tau_m^- \sigma_m \|0_i^+ \rangle,
$$

*GT* (*n*) *me*

$$
(\sigma\tau^-)^{eff}\to q_H\;\sigma\tau^-
$$

ANL Symposium, M. *Fig*  $\frac{1}{2}$  +  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{2024}{2024}$ 



represents the principal quantum number and k corresponds to the  $\sigma$ : The predicted  $\Delta$ *v* ECEC in  $1-2$ ). To be compared with experimental data for the total  $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2$  $I_{\text{stat}} \pm 0.1_{\text{sys}} \times 10^{22}$  yr (see section III.F of Ref. [15]). Table 5: The predicted 2vECEC half-lives for  $^{124}$ Xe (in units of  $10^{22}$  yr) from Eqs. (1-2). To be compared with experimental data for the total half-life, (1.1<sup>±</sup>  $(\sigma \tau^{-})^{eff} \rightarrow q_H \sigma \tau^{-}$ <br>0.2<sub>stat</sub>  $\pm$  0.1<sub>sys</sub>) × 10<sup>22</sup> yr and the inferred data for the KK half-life, (1.5  $\pm$  $t_0 \to 0.2$   $t_0 \to 0.1$   $t_0 \to 0.1$   $t_0 \to 0.2$   $t_0 \to 0.2$   $t_0 \to 0.2$ . The ISM point of  $t_0$  and the interior data for the ISIN partition.  $\sigma$  $\sigma$ <sub>stat</sub> =  $\sigma$  $(0.3<sub>stat</sub> \pm 0.1<sub>sys</sub>) \times 10<sup>22</sup>$  yr (see section III.F of Ref. [15]).



## 徶 CENTRAL MIGHIGAN Taylor Expansion NMEs: <sup>136</sup>Xe

$$
M_{2\nu-3}(E_k) = \sum_{n=1}^{k} \frac{\langle \Psi_f | q \sigma \tau^- | 1_n^+ \rangle \langle 1_n^+ | q \sigma \tau^- | \Psi_i \rangle}{\left( E(1_n^+) + \frac{Q_{\beta \beta}}{2} - Q_{\beta} \right)^3}
$$

$$
\frac{|\Psi_i\rangle}{3} \qquad M_{2\nu-5}(E_k) = \sum_{n=1}^k \frac{\langle \Psi_f | q \sigma \tau^- | 1_n^+ \rangle \langle 1_n^+ | q \sigma \tau^- | \Psi_i \rangle}{\left( E(1_n^+) + \frac{Q_{\beta \beta}}{2} - Q_{\beta} \right)^5}
$$

 $136Xe - SVD$ 







### The  $2\nu$ ECEC decay of  $^{124}$ Xe: Predictions for Capture Fractions  $110<sup>2</sup>$  $\Gamma$  1.00054  $\Gamma$  1.00054  $\Gamma$



Table 6: The atomic relaxation energies (Eq. 10) obtained within the DHFS model (second column) and the capture fractions (CF) predicted by ISM (third column). The captures with atomic relaxation energies below 6 keV are subsumed under the label "other". The ranges presented for the KK and  $KL_1$  channels correspond to the minimum and maximum values of the  $\xi_{31}^{2VECEC}$  parameter predicted from ISM.





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# **Conclusions**

- Shell model calculations of the  $2\nu\beta\beta$  nuclear matrix elements can be obtained very efficiently using a strength function approach
- The direct sum is likely to miss convergence in most cases
- The strength function approach is the only option for cases that have very large shell model dimension, such as 128Te
- The proper evaluation of the  $2\nu\beta\beta$  nuclear matrix elements is important while they correlate strongly with the  $0\nu\beta\beta$ nuclear matrix elements (PRC 106, 05432 (2022), PRC 107, 045501 (2023), Universe 10, 252 (2024) )

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# ZVpp Half-Lives and optimal quenching factors







## $2\nu\beta\beta$  Half-Lives and NME v pp 11d The 2012 control of the 2012 control of the 2014 control of th

$$
\[T_{1/2}^{2\nu}\]^{-1} = G_{2\nu} \cdot \left[g_A^2 \left(m_e c^2 \cdot M_{2\nu}\right)\right]^2 \equiv G_{2\nu} \left(M_{2\nu}^{eff}\right)^2\]
$$



A. Barabash, Universe 6, 159 (2020)<br>
122628 (2021)

 $B.$  Pritychenko, Nucl. Phys. A 1033,  $\frac{122020}{2025}$ 122628 (2023)

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M. Horoi CMU 28 it was never used in practical calculations. Here, we provide  $\bigcirc$ 

