Elucidating the Ge Nuclei with the Shell Model

Erin E. Peters
University of Kentucky
Interim Director UKAL

Celebrating the 75 years of the shell model and Maria Goeppert Mayer

UKAL 7 MV Model CN VDG (External View) – Located on U. of KY's main campus

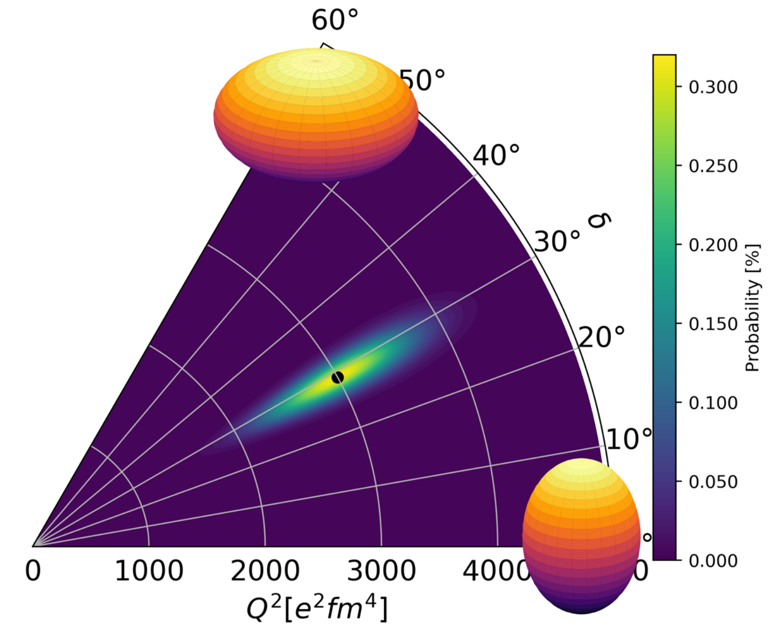
Primary Collaborators:

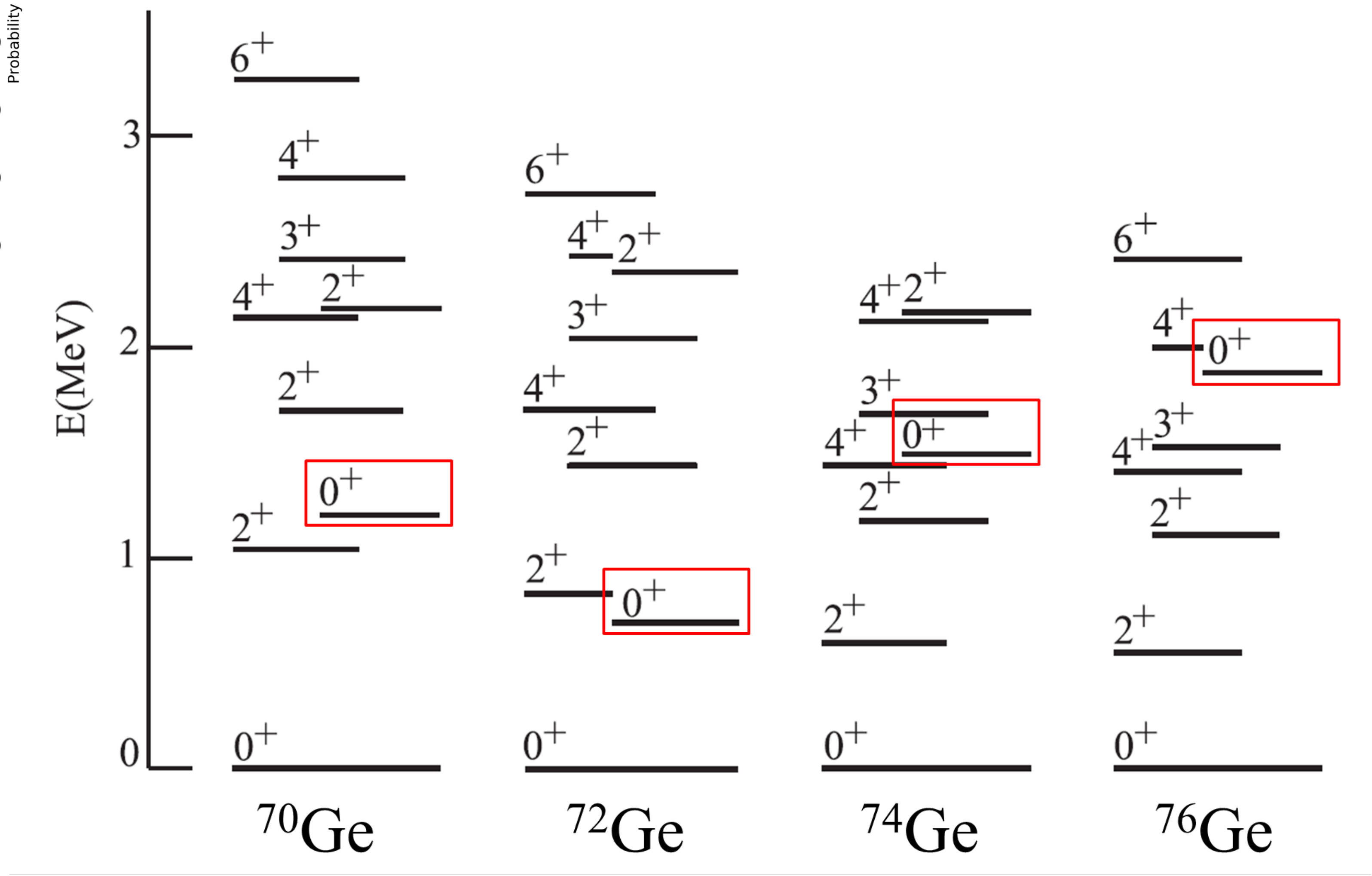
B. A. Brown – FRIB, Michigan State University, retired
S. Mukhopadhyay – former UK postdoc, now Georgia Institute of Technology
S. W. Yates – University of Kentucky, retired

Why study the Ge isotopes?

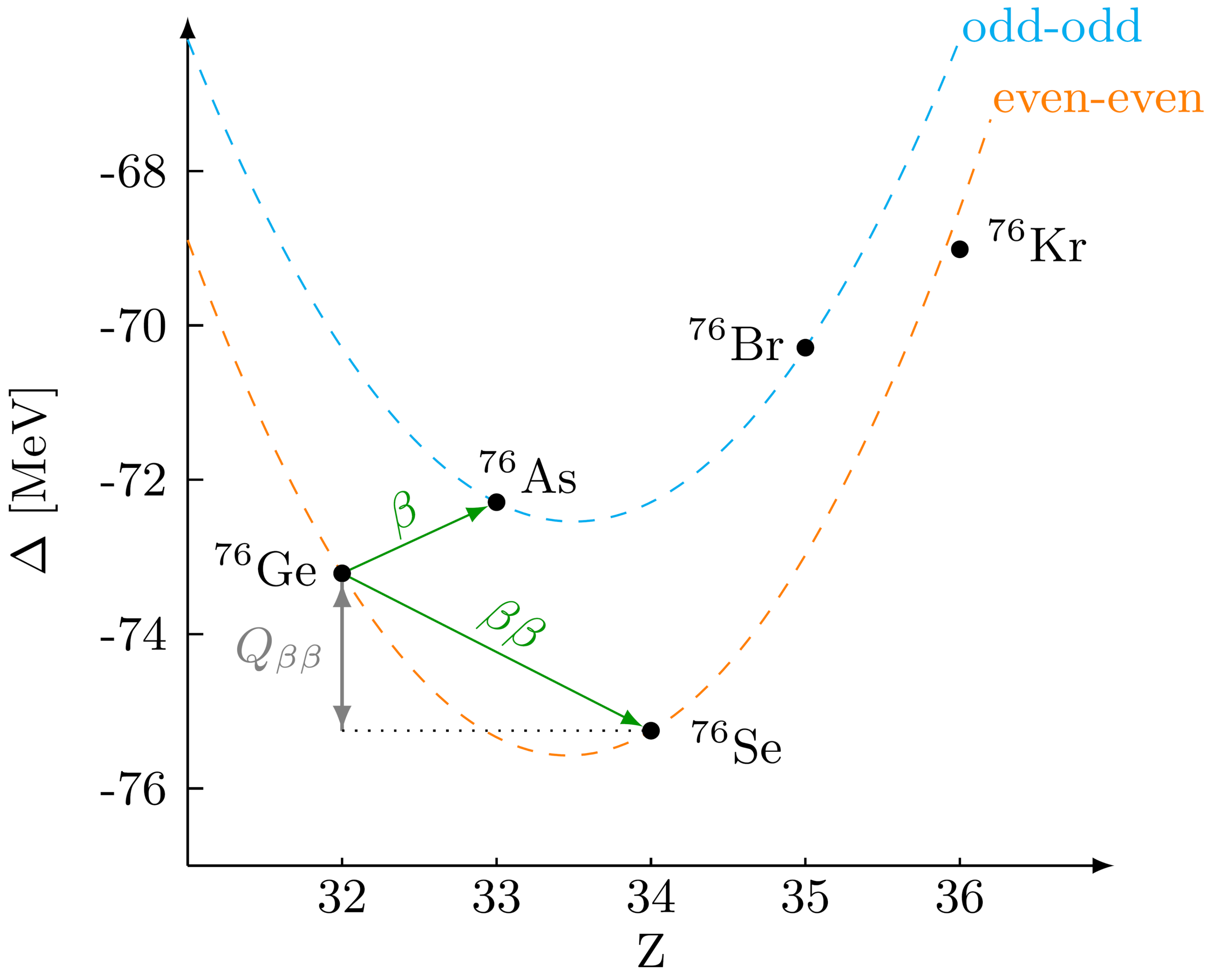
Structurally Interesting!

- Soft/Rigid Triaxiality
- Shape Transition
- Shape Coexistence





Broader Impacts: Neutrinoless Double-beta Decay



Triaxiality in ^{76}Ge

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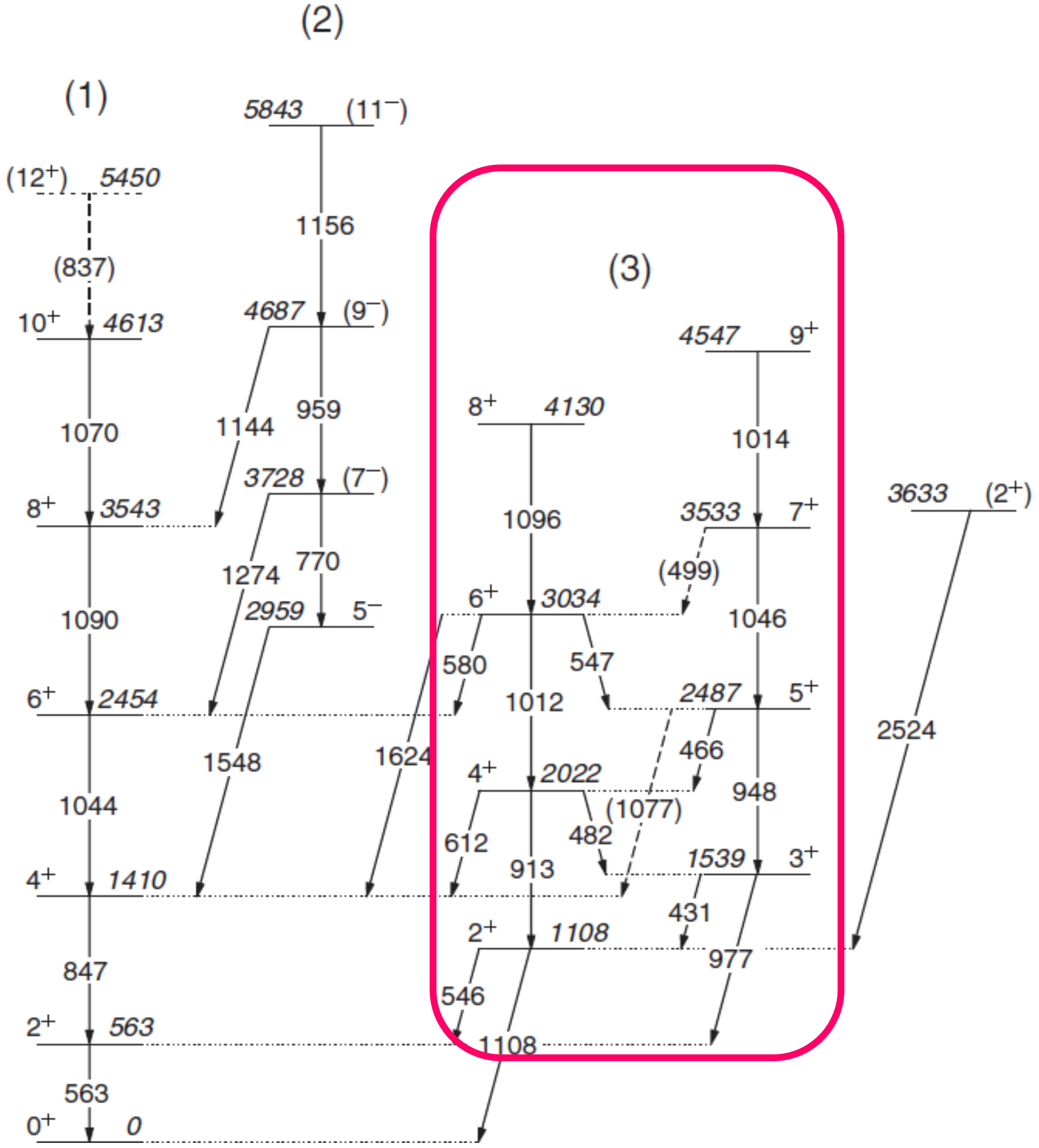
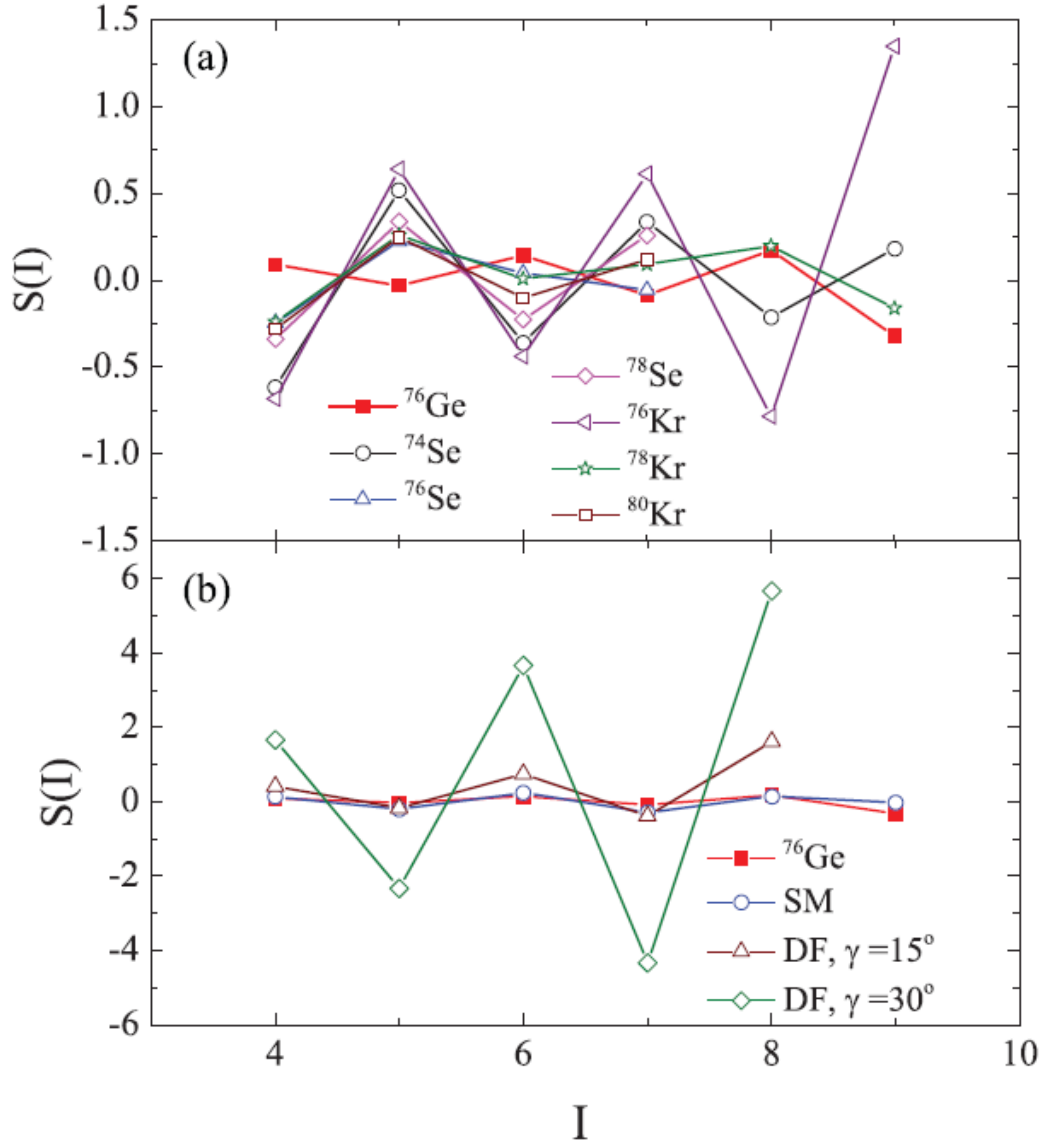
PHYSICAL REVIEW C **87**, 041304(R) (2013)

Evidence for rigid triaxial deformation at low energy in ^{76}Ge

Y. Toh,[1,2] C. J. Chiara,[2,3] E. A. McCutchan,[2,4] W. B. Walters,[3] R. V. F. Janssens,[2] M. P. Carpenter,[2] S. Zhu,[2] R. Broda,[5] B. Fornal,[5] B. P. Kay,[2] F. G. Kondev,[6] W. Królas,[5] T. Lauritsen,[2] C. J. Lister,[2,*] T. Pawłat,[5] D. Seweryniak,[2] I. Stefanescu,[2,3] N. J. Stone,[7,8] J. Wrzesiński,[5] K. Higashiyama,[9] and N. Yoshinaga[10]

“…^{76}Ge may be a rare example of a nucleus exhibiting rigid triaxial deformation in the low-lying states.”



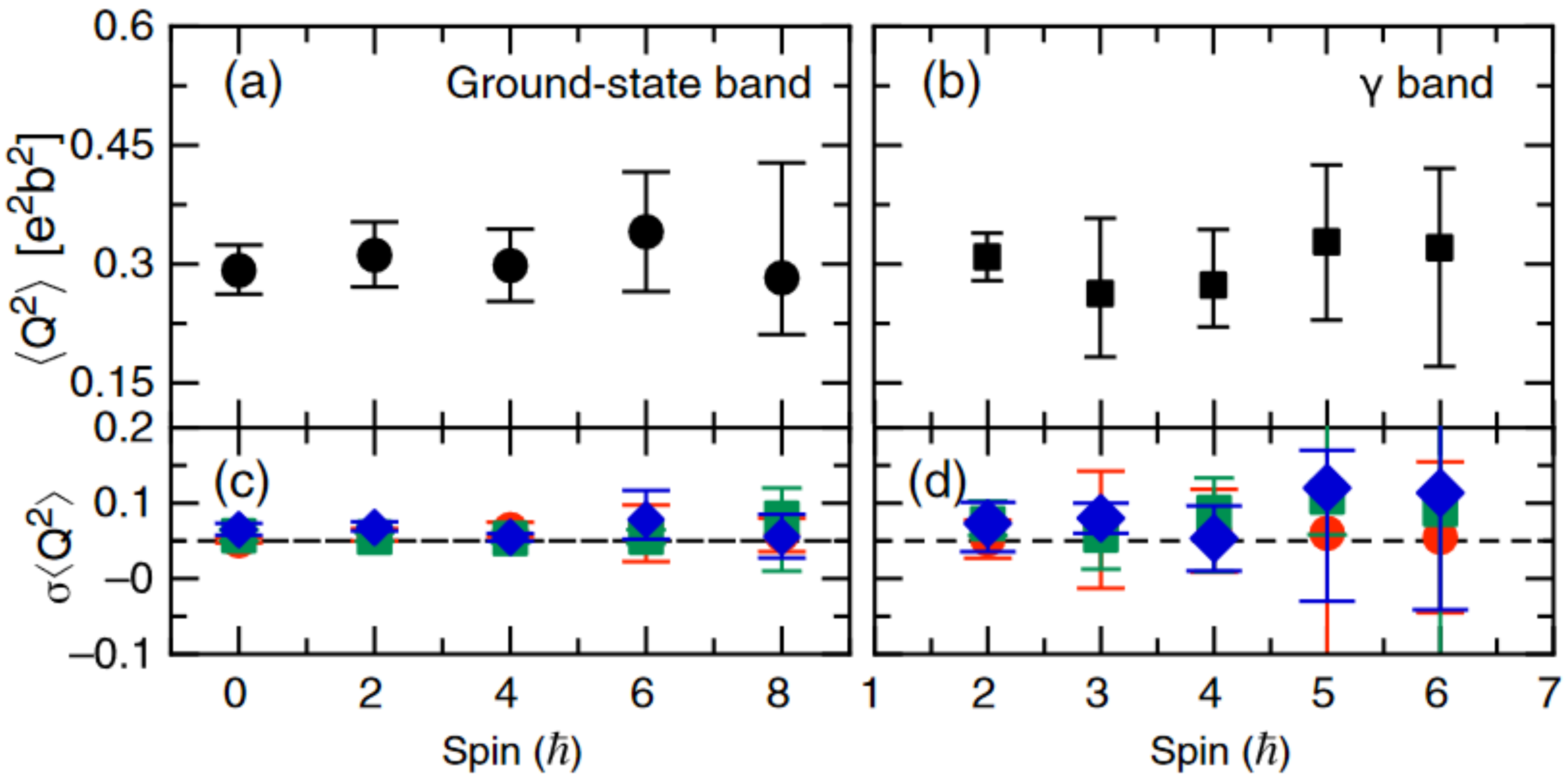


Triaxiality in ^{76}Ge

$\langle Q^2 \rangle$

quadrupole invariant

$\beta \approx 0.28$



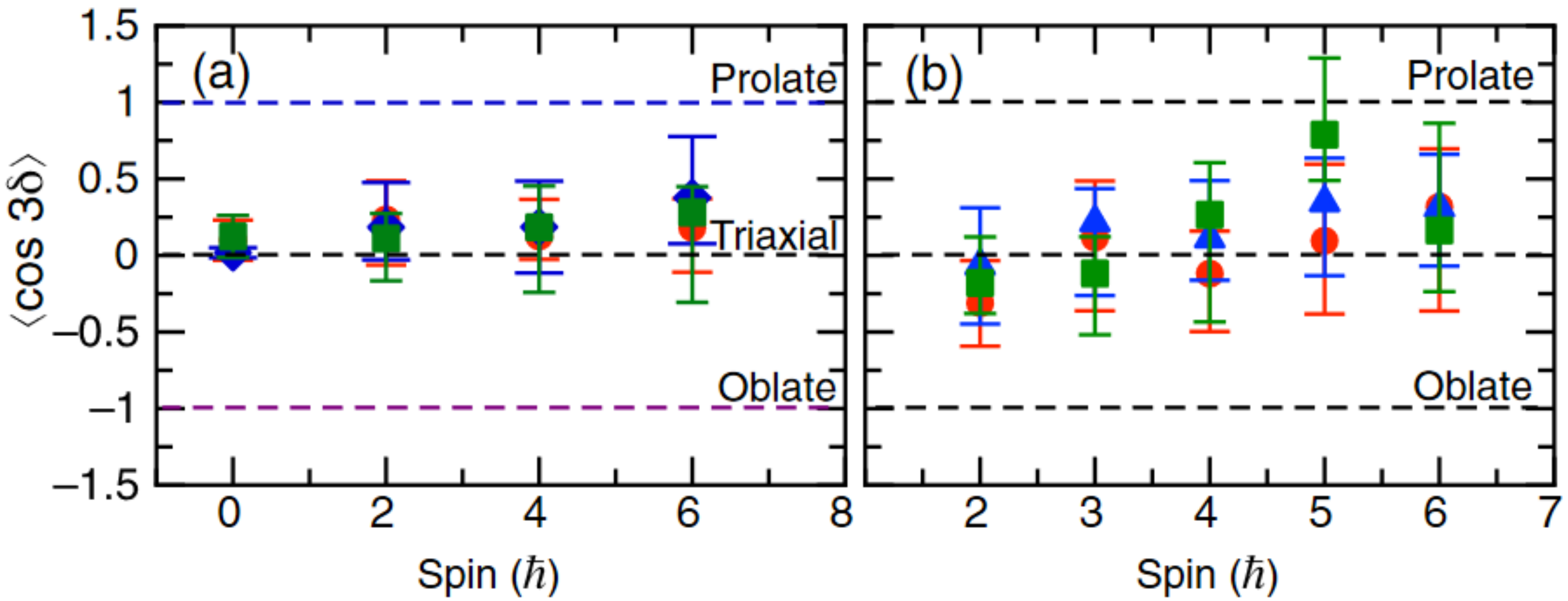
Coulomb excitation at

Argonne NATIONAL LABORATORY

A. D. Ayangeakaa et al., Phys. Rev. Lett. **123**, 102501 (2019).

$\langle \cos 3\delta \rangle$

quadrupole asymmetry



red (circle) J = 0

blue (diamond) J = 2

green (square) J = 4

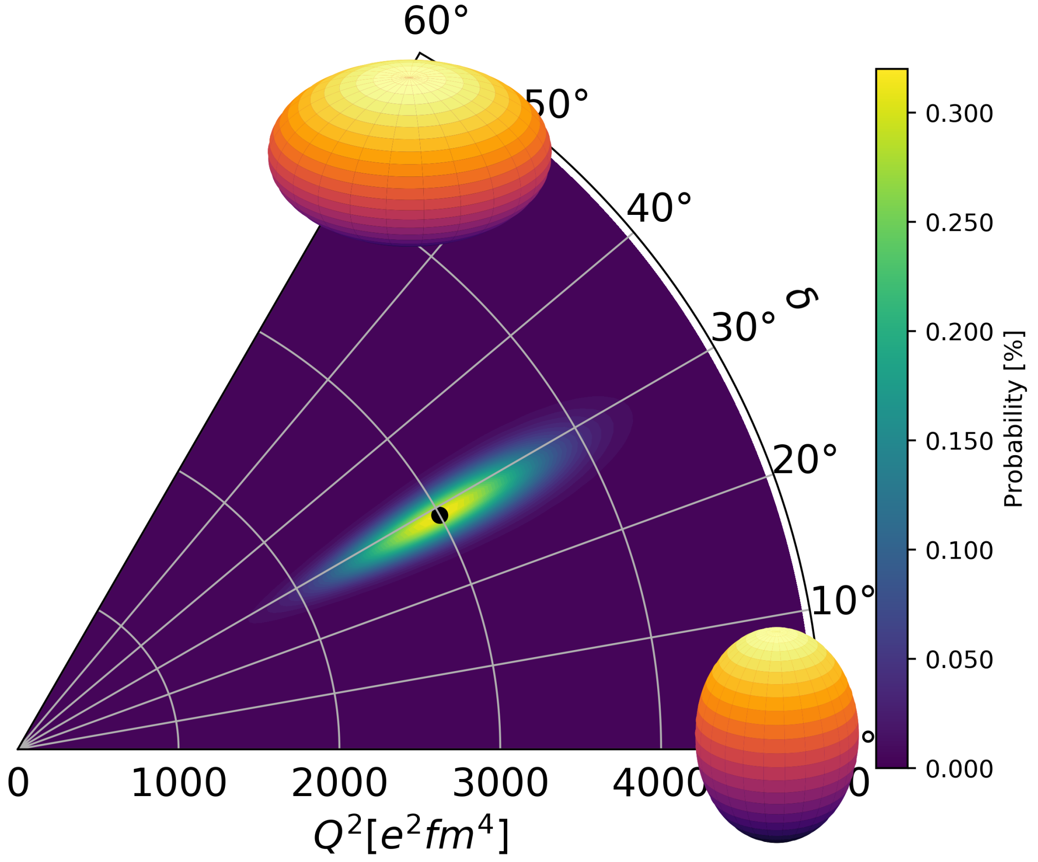
couplings

Triaxiality in ^{76}Ge

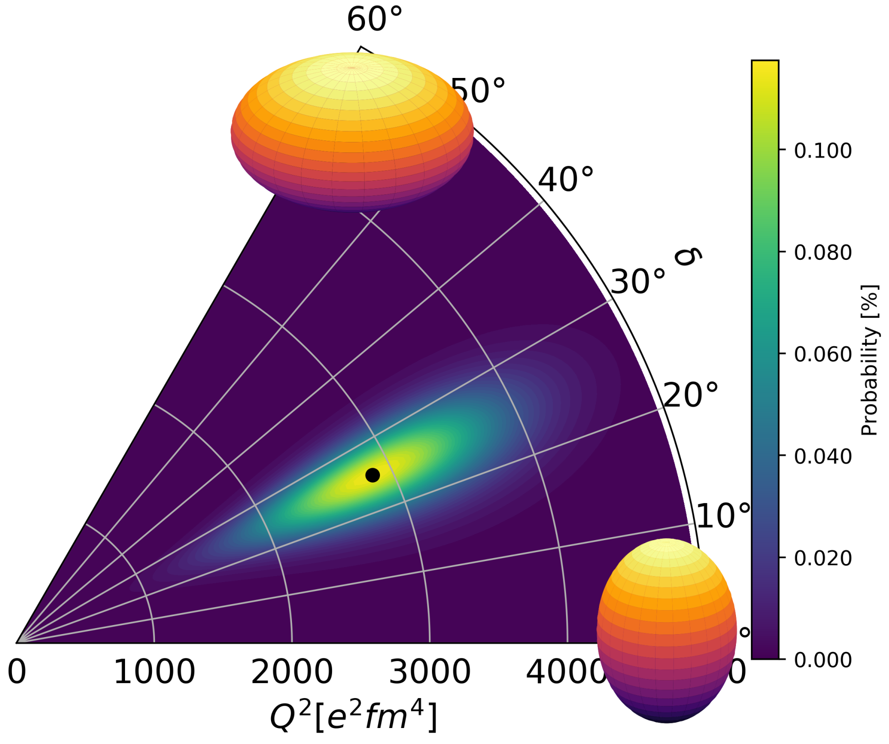
Shell-Model Calculations by **Alex Brown**

Ground-state deformations:

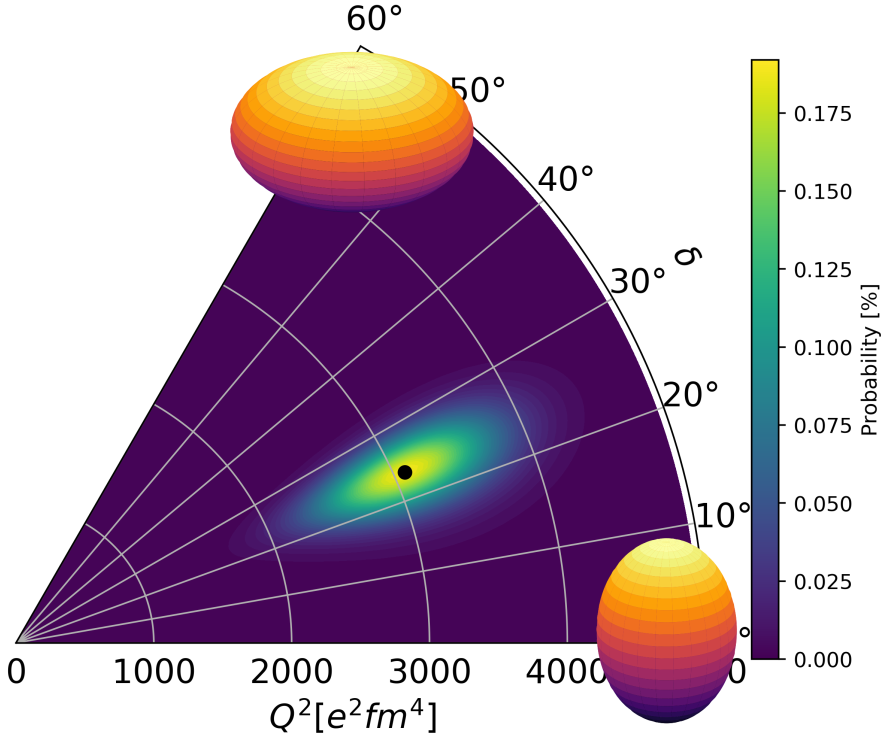
Experimental result



jj44b Hamiltonian



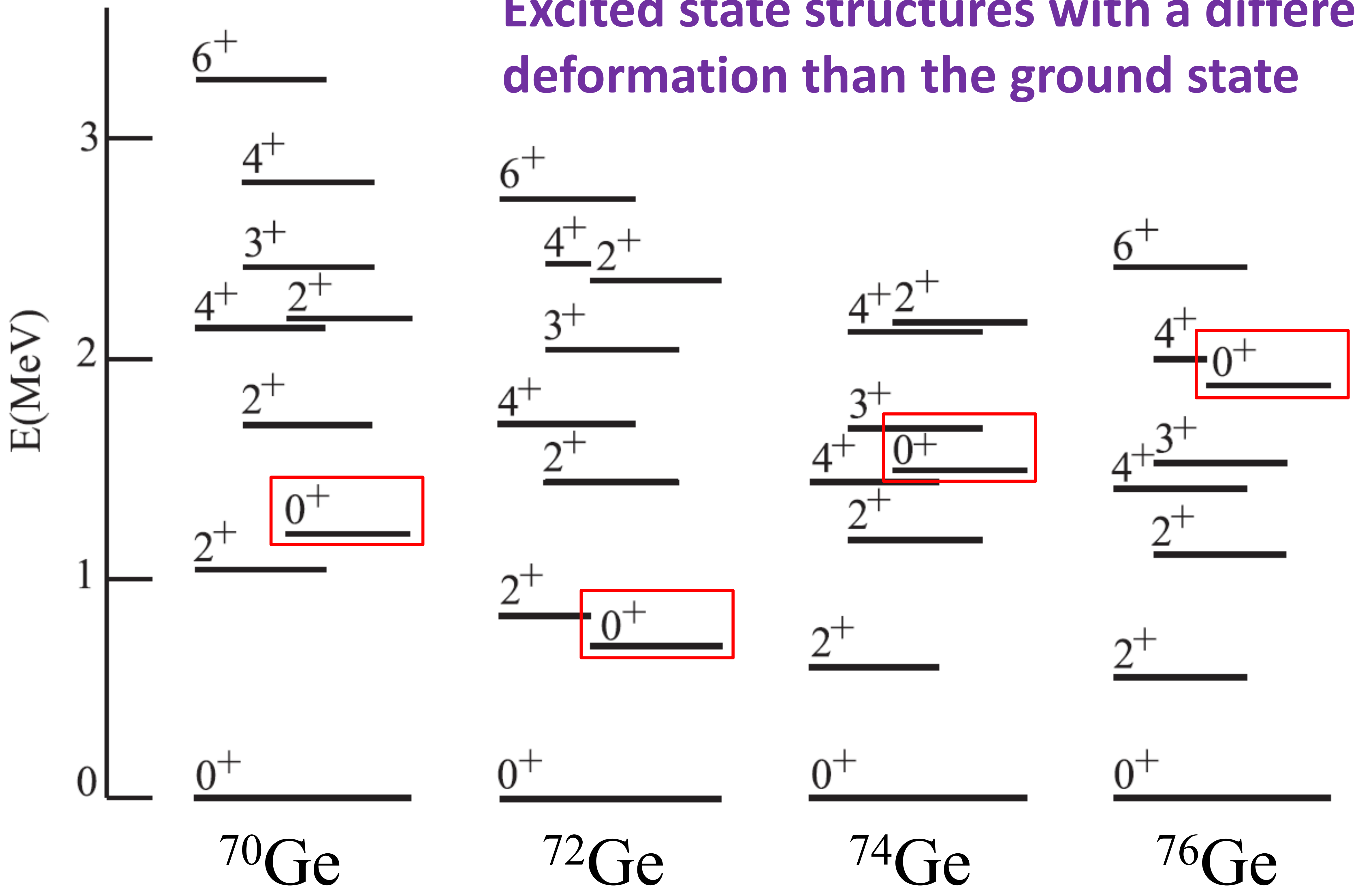
JUN45 Hamiltonian



A.D. Ayangeakaa, et al., Phys. Rev. C 107, 044314 (2023).

Shape Coexistence in Ge Isotopes

Excited state structures with a different deformation than the ground state



Are there bands built on the excited 0^+ states?

Z. Podolyák *et al.*, Int. J. Mod. Phys. E 13, 123 (2004).
K. Heyde and J. L. Wood, Rev. Mod. Phys. 83, 1467 (2011).

^{76}Ge 0νββ candidate

Implications for 0νββ Decay

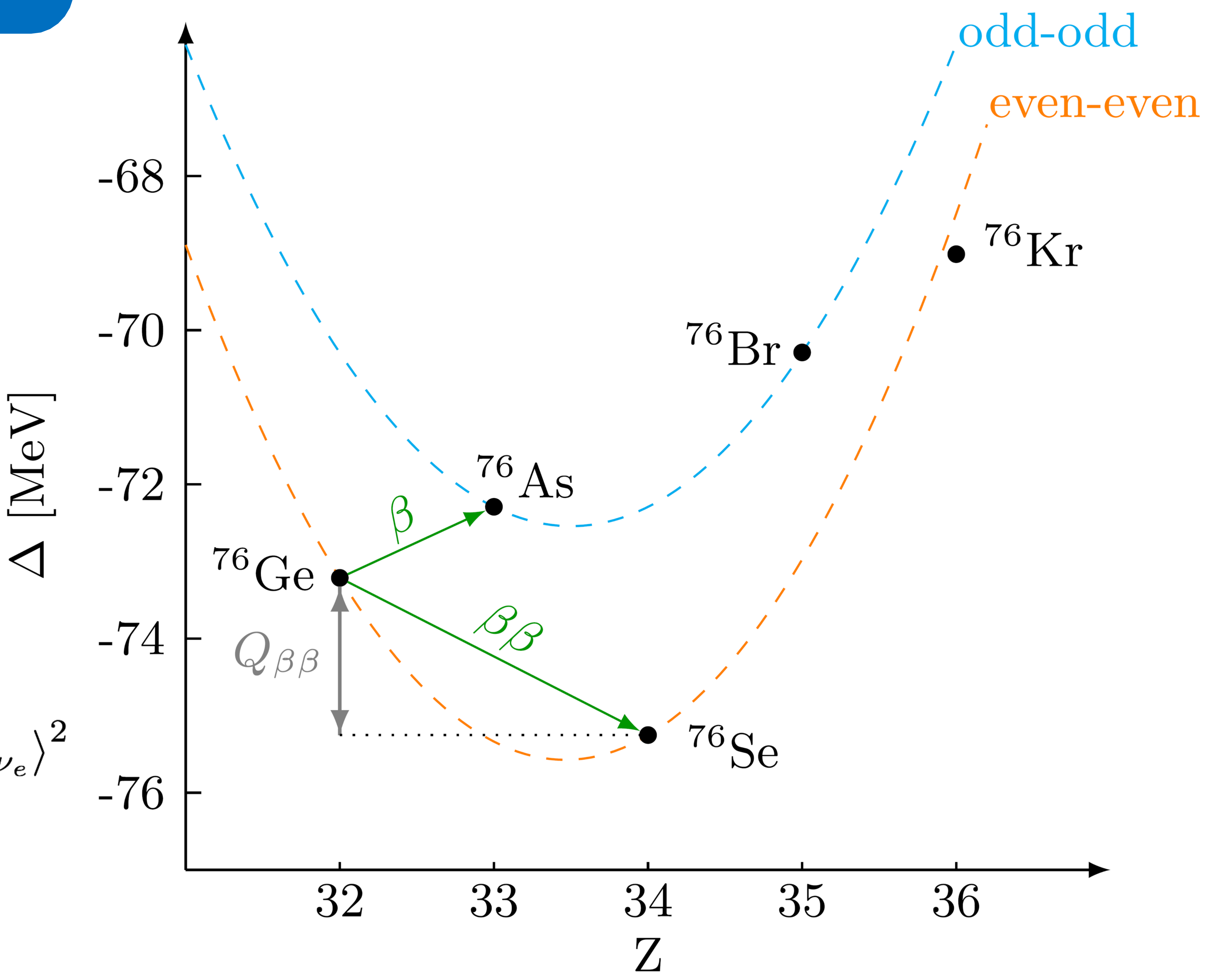
$$^{76}Ge \rightarrow {}^{76}Se + 2\beta^- + 2\bar{\nu}$$

$$^{76}Ge \rightarrow {}^{76}Se + 2\beta^-$$

Is the neutrino its own antiparticle?

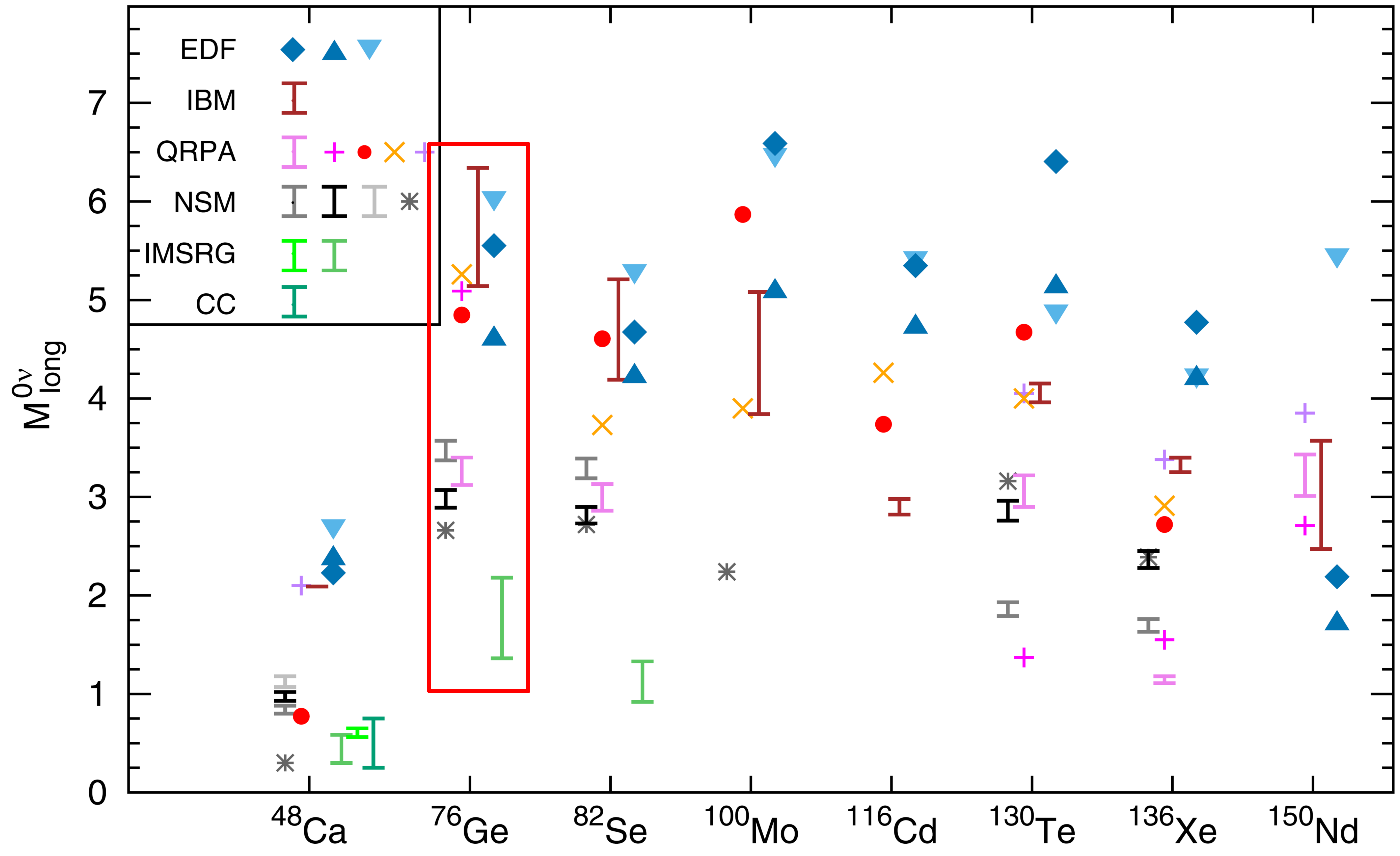
What is the mass of the neutrino?

$$\left[T^{0\nu}_{1/2}\left(0^+ \rightarrow 0^+\right)\right]^{-1} = G^{0\nu}(E_0, Z)\left|M^{0\nu}\right|^2 \langle m_{\nu_e}\rangle^2$$



M. Agostini, et al. Rev. Mod. Phys. 95 025002 (2023).

Calculated Nuclear Matrix Elements



Experimental nuclear structure data are needed to constrain the calculations.

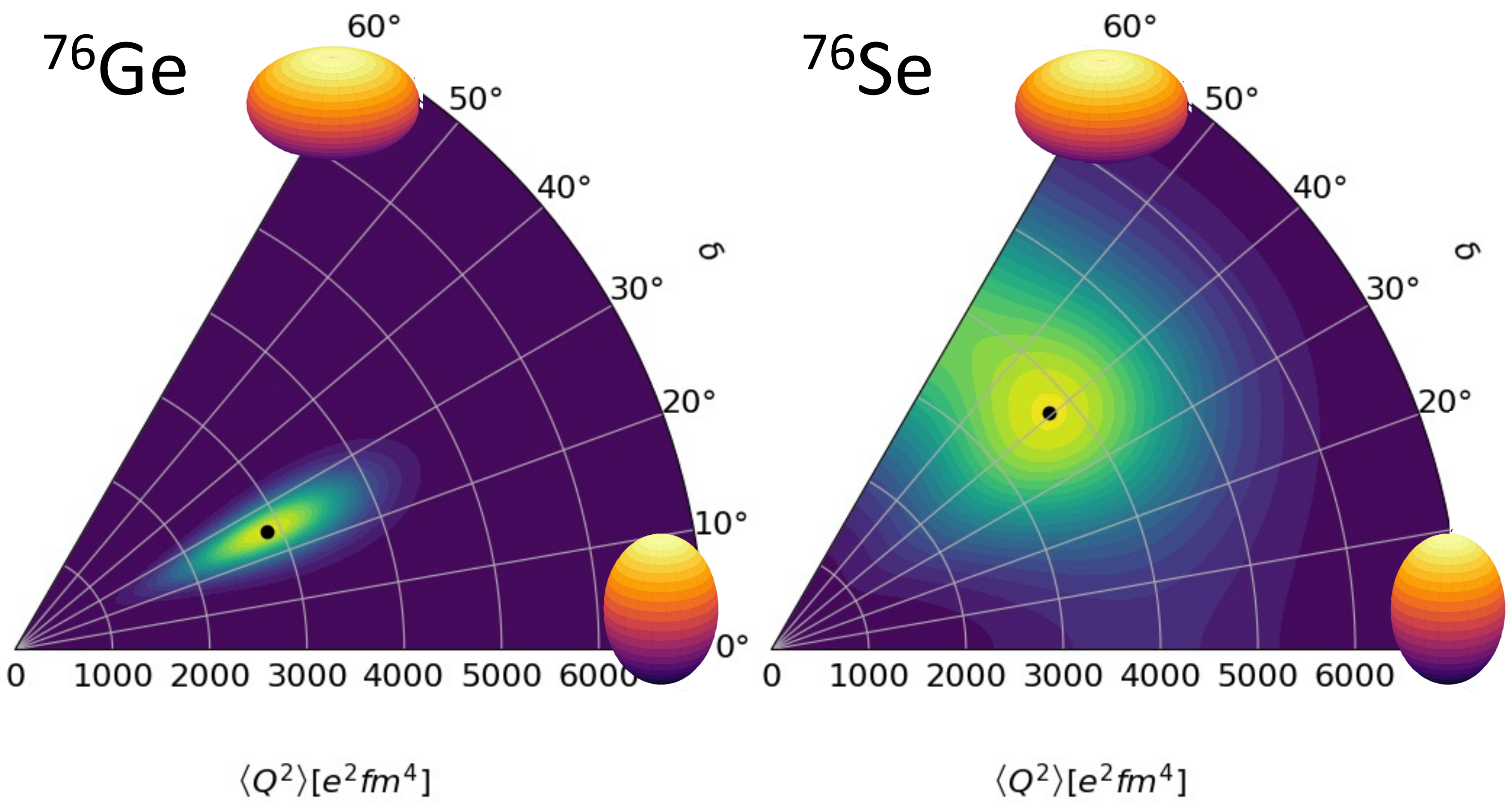
M. Agostini, G. Benato, J. A. Detwiler, J. Menéndez, and F. Vissani, Rev. Mod. Phys. 95, 025002 (2023).

Comparison with ^{76}Se

Ground-state deformations:

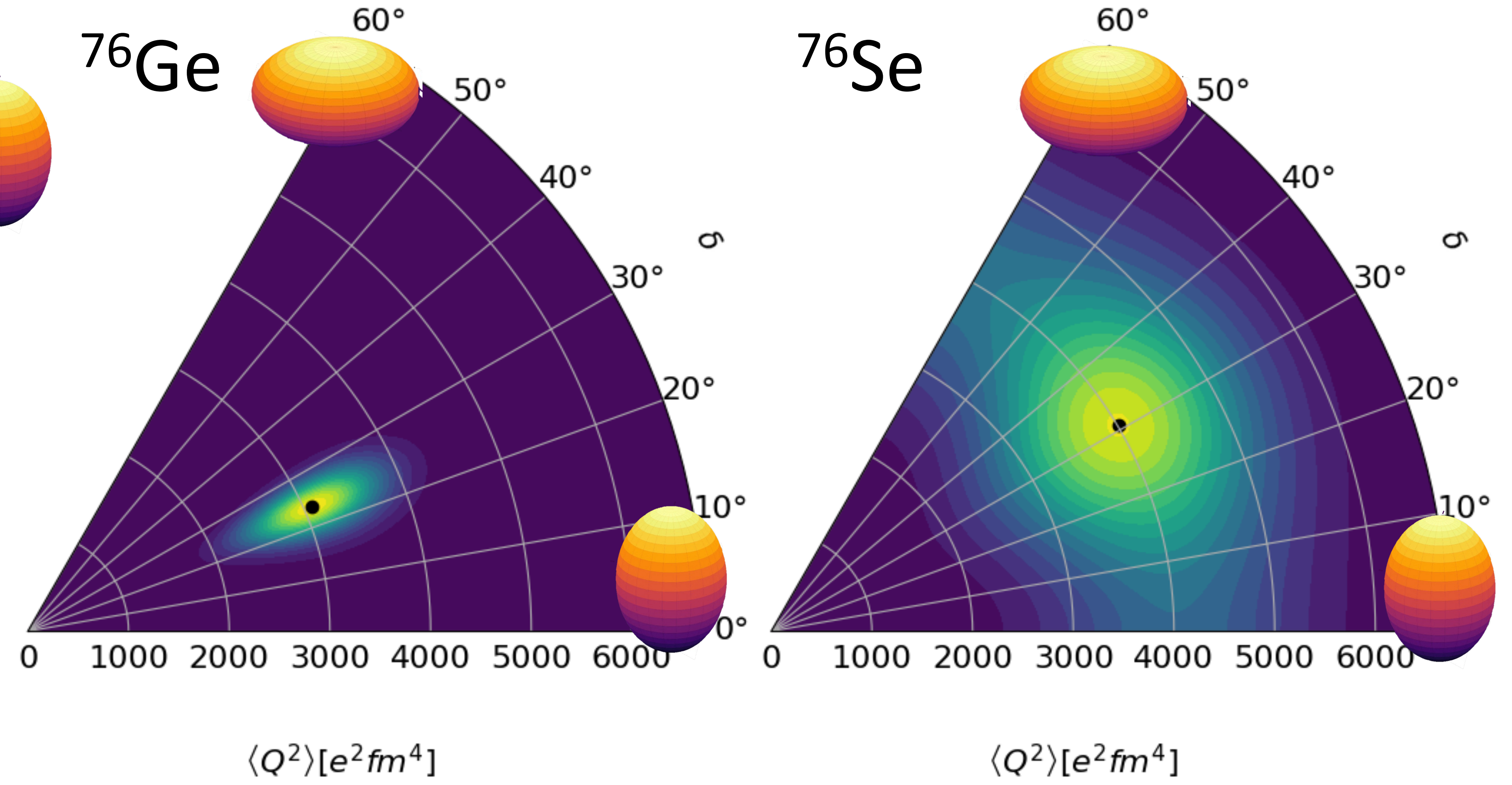
Shell-Model Calculations by **Alex Brown**

jj44b Hamiltonian



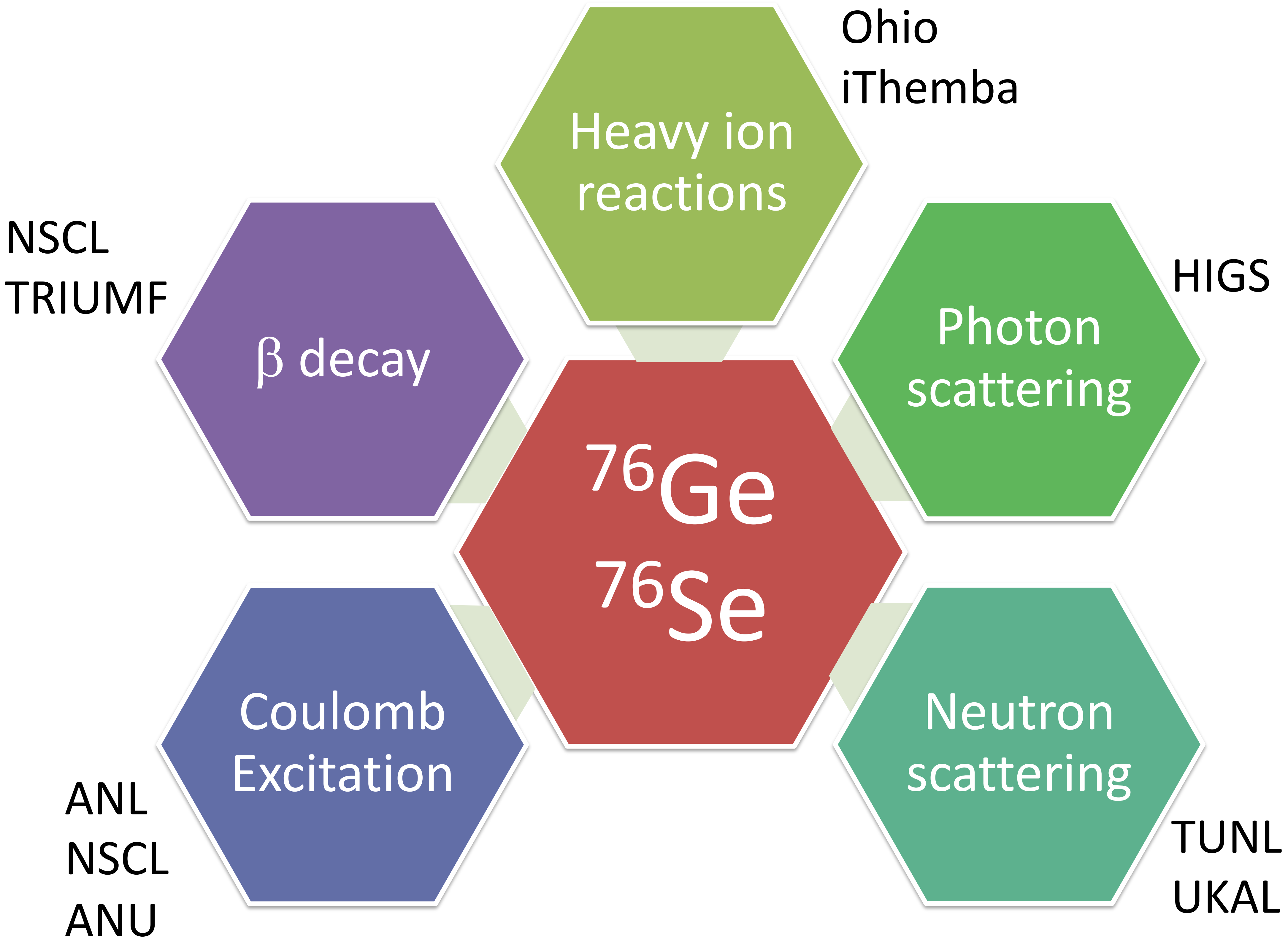
A.D. Ayangeakaa, et al.,
Phys. Rev. C 107, 044314 (2023).

JUN45 Hamiltonian



The difference in deformation quenches the NME.

Experimental Techniques



I. Detailed Nuclear Spectroscopic Data

II. Deformations

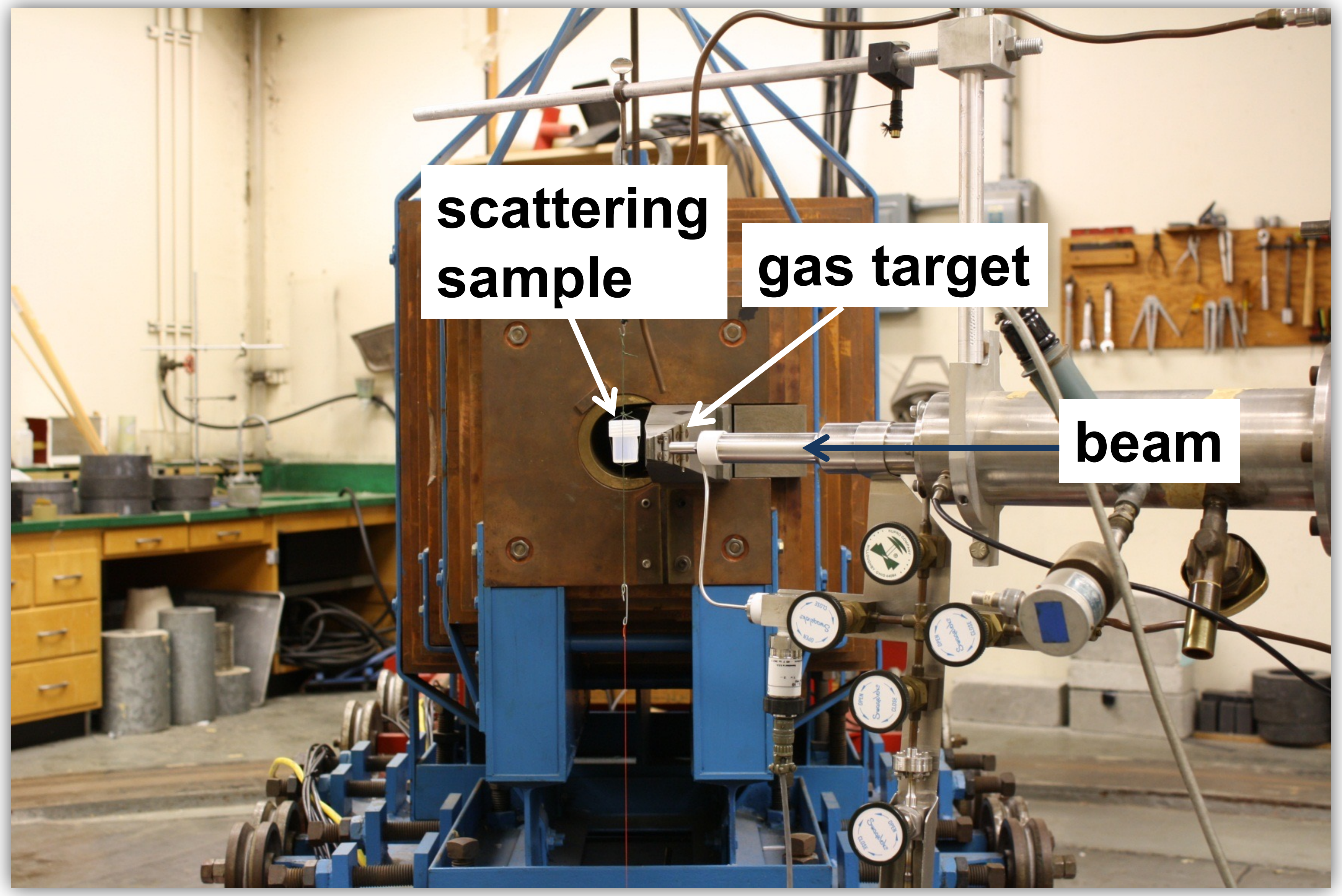
UKAL – Laboratory overview

Primarily perform neutron scattering

- $^{3}H(p,n)^{3}He$, Q = –0.76 MeV, E_n < 5.5 MeV
- $^{2}H(d,n)^{3}He$, Q = 3.3 MeV, E_n = 4 – 9 MeV
- ΔE < 100 keV

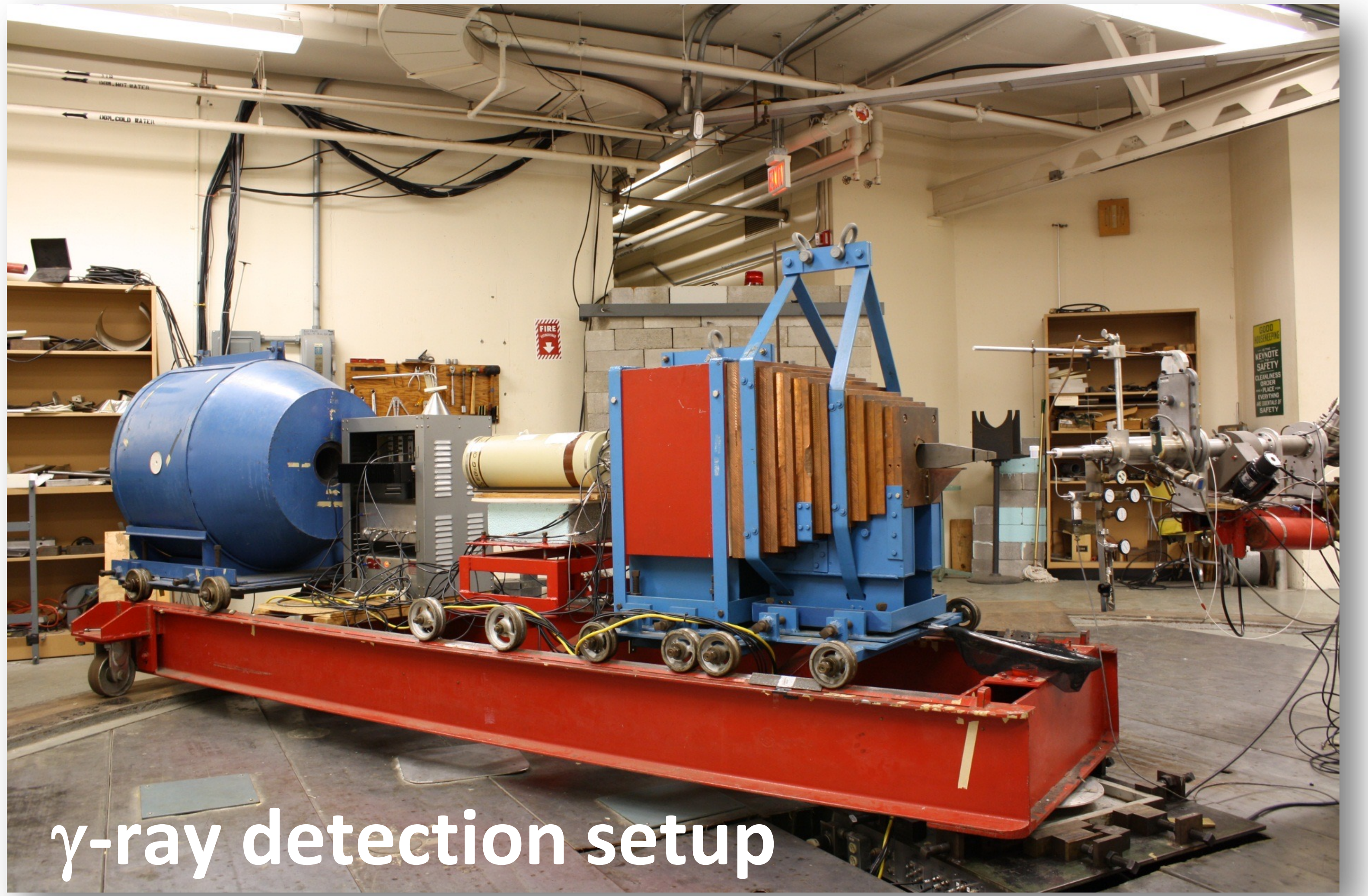
University of Kentucky 60 years 1964-2024 Accelerator Laboratory

- 7 MV Model CN VDG
- p, d, ^{3}He, and α beams
- D.C. (~50 μA)
- Pulsed beams (~5 μA)
- f = 1.875 MHz, Δt ~1 ns

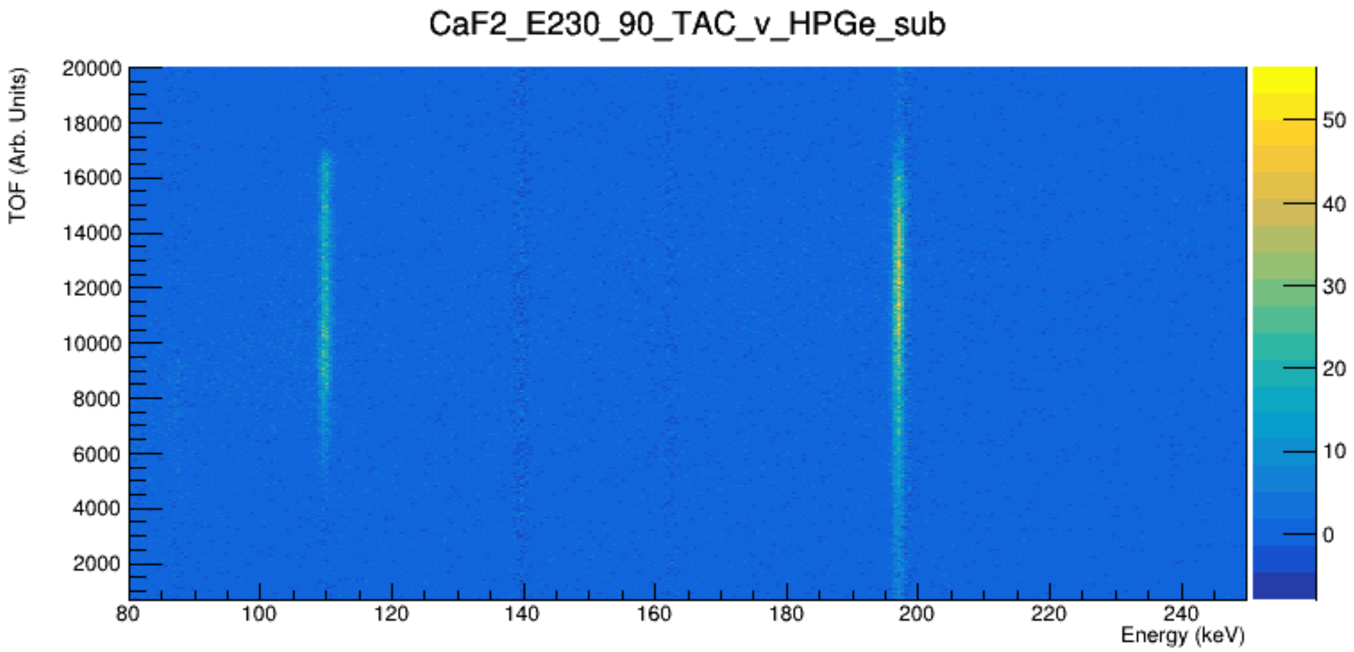


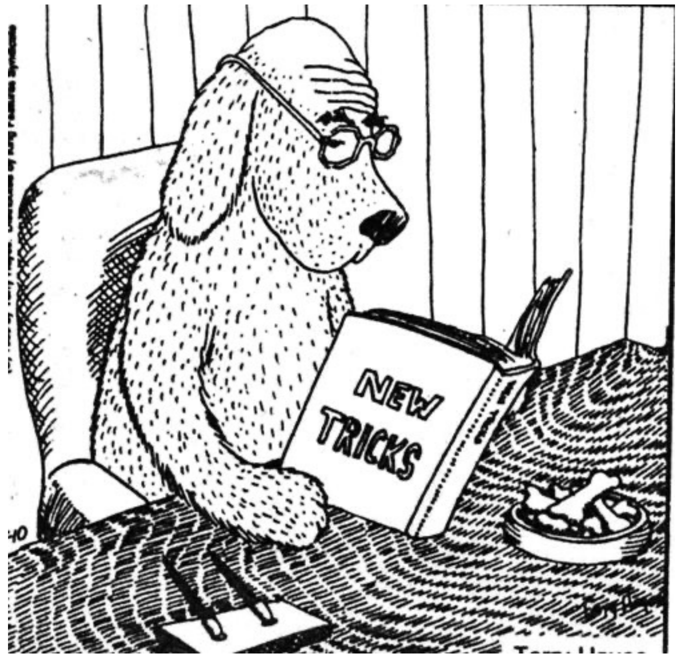
UKAL – Laboratory overview

- Compton-suppressed HPGe
- Flux monitors: long counter, NE213

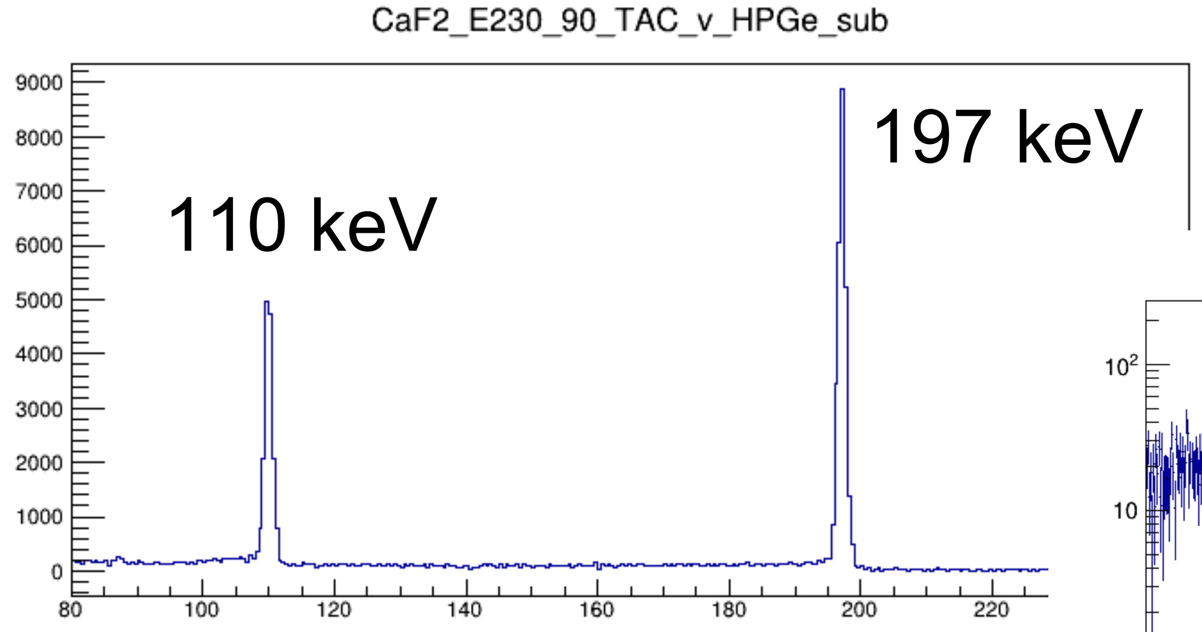


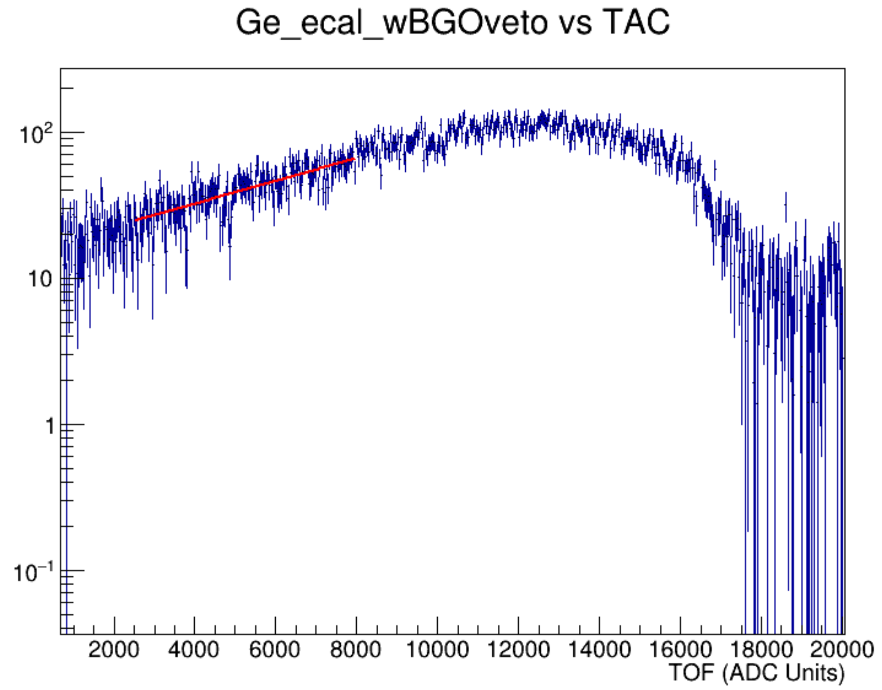
- New CAEN digital data acquisition system
- New γ-ray timing capabilities



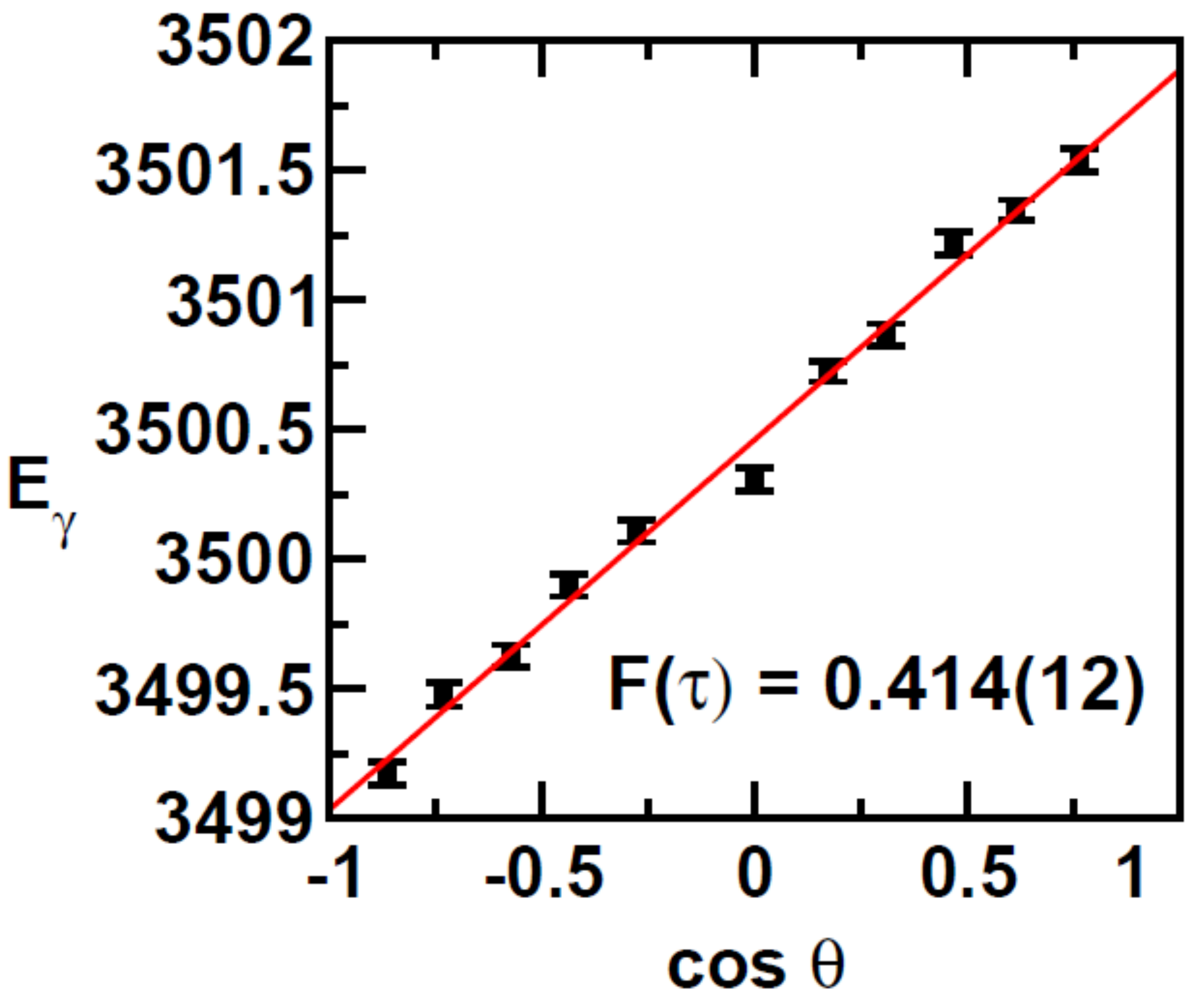


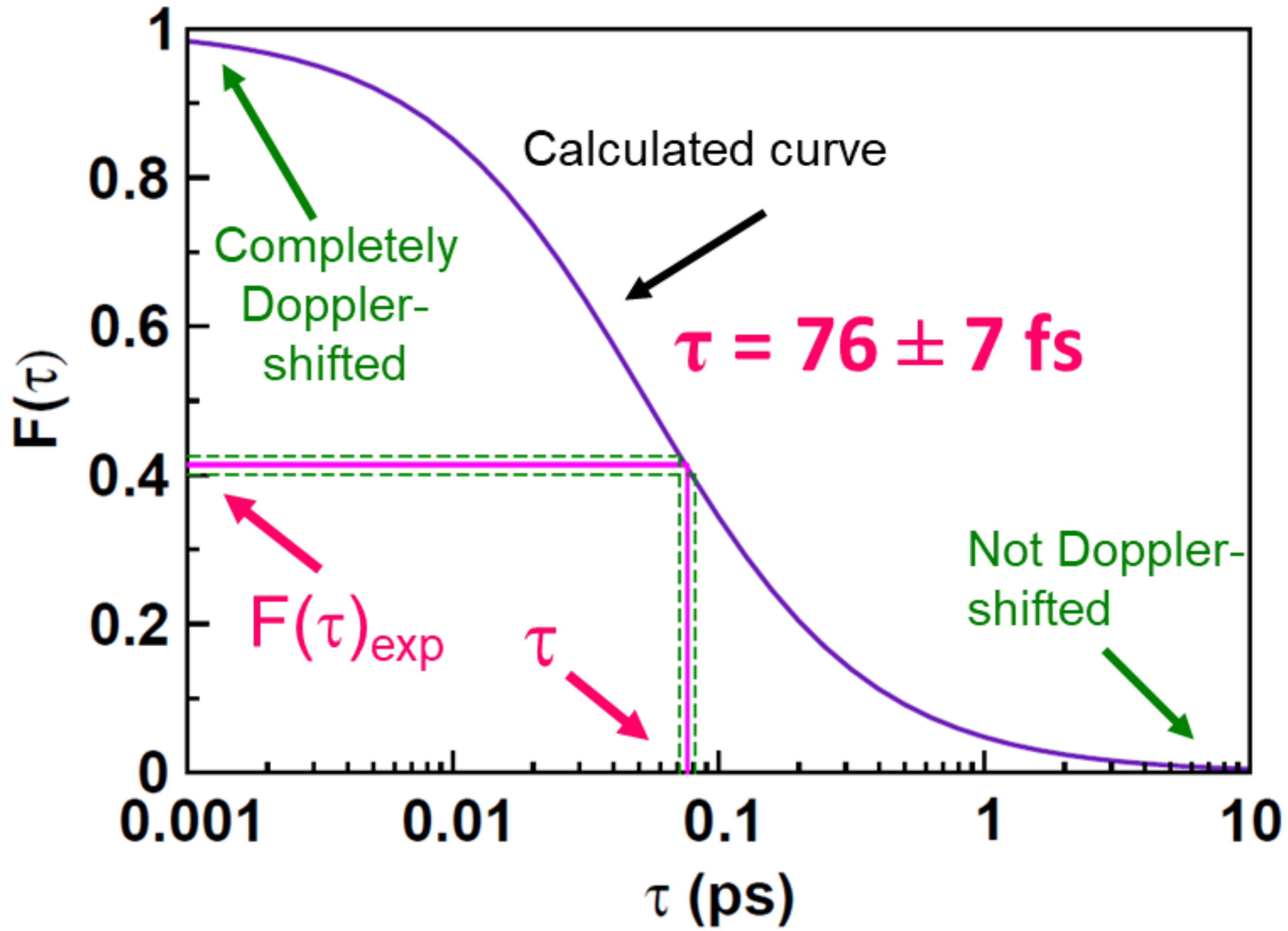
197 keV
$T_{1/2}$ = 89.2(25) ns
NNDC: 89.3(10) ns





INS DSAM Lifetimes



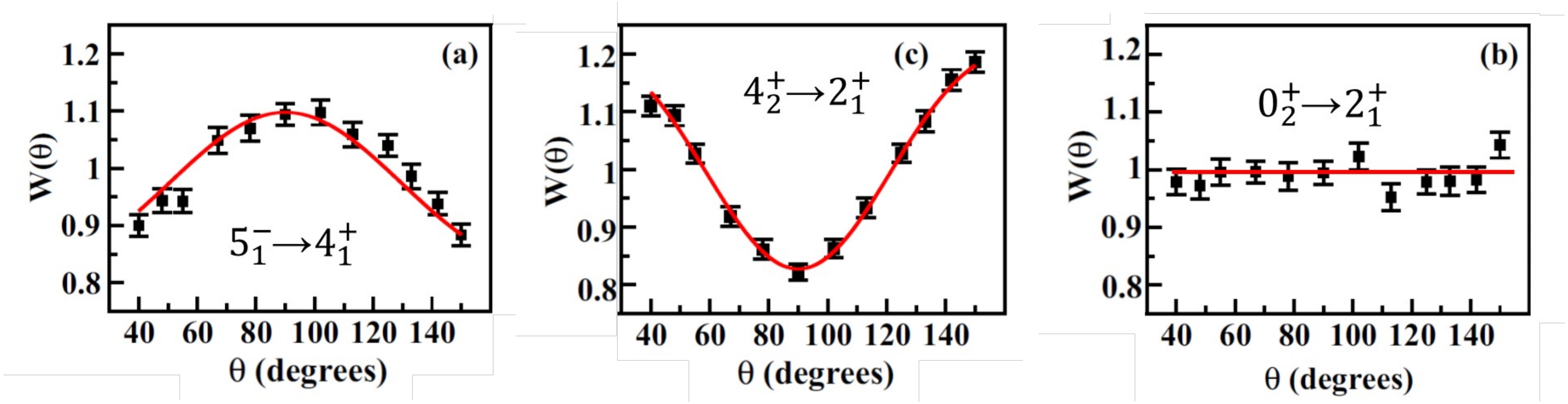


$$E_\gamma(\theta) = E_\gamma\left[1 + F_{exp}(\tau)\frac{v_{cm}}{c}\cos\theta\right]$$

K.B. Winterbon, Nucl. Phys. A246, 293 (1975).

T. Belgya, G. Molnár, and S. W. Yates, Nucl. Phys. A607, 43 (1996).

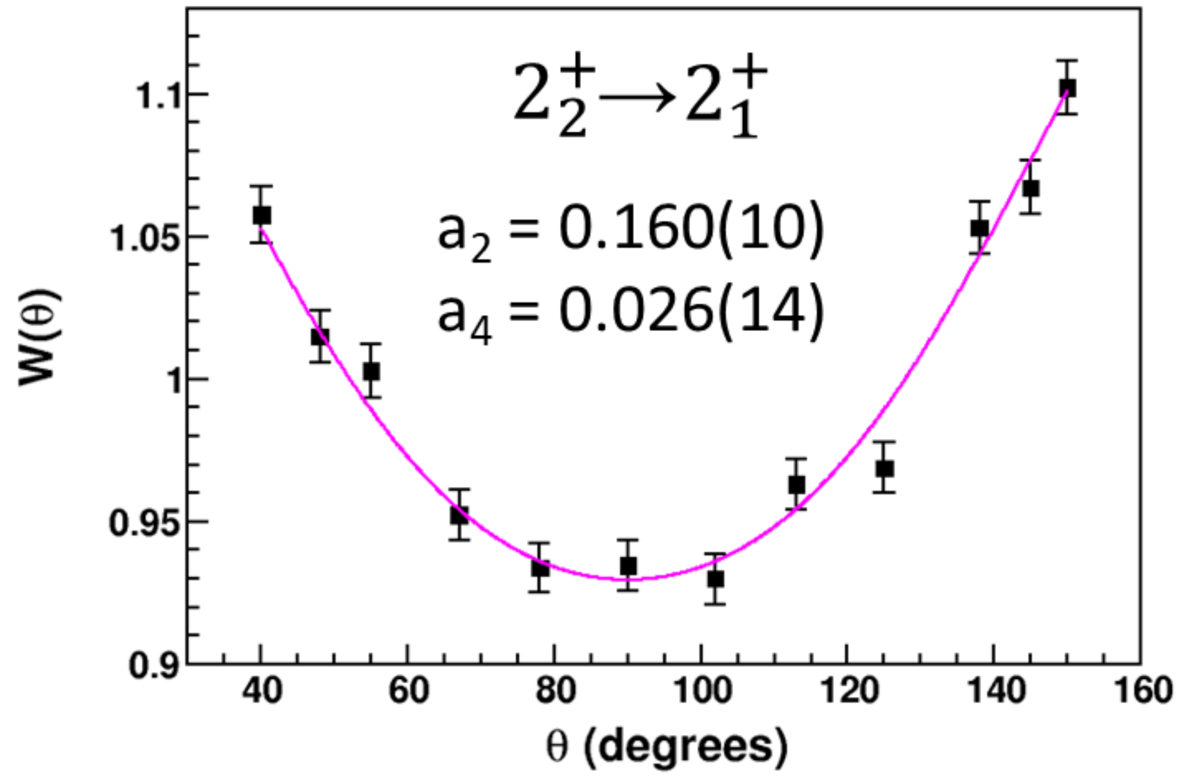
INS Angular Distributions

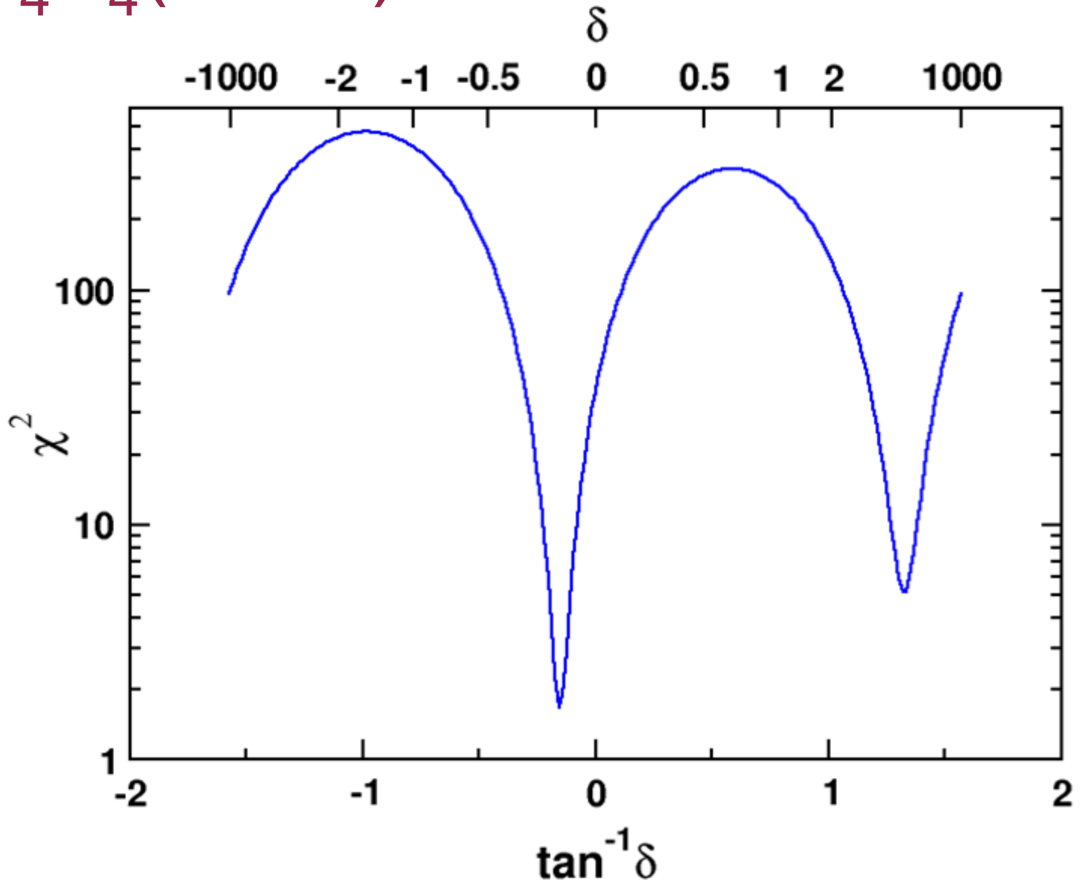


^{134}Xe

$$W(\theta) = 1 + a_2 P_2(\cos\theta) + a_4 P_4(\cos\theta)$$

Comparison with statistical model calculations to extract the E2/M1 mixing ratio



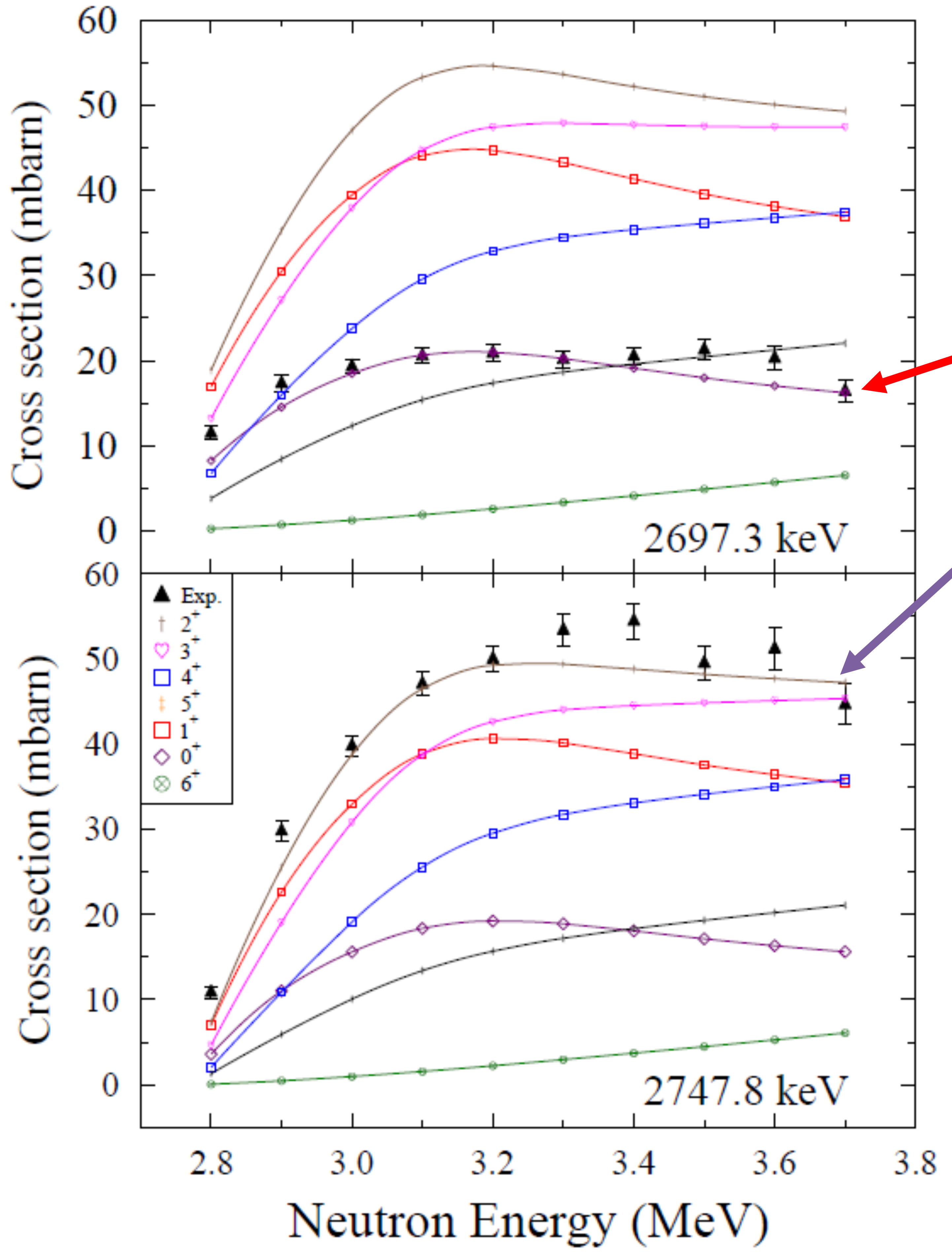


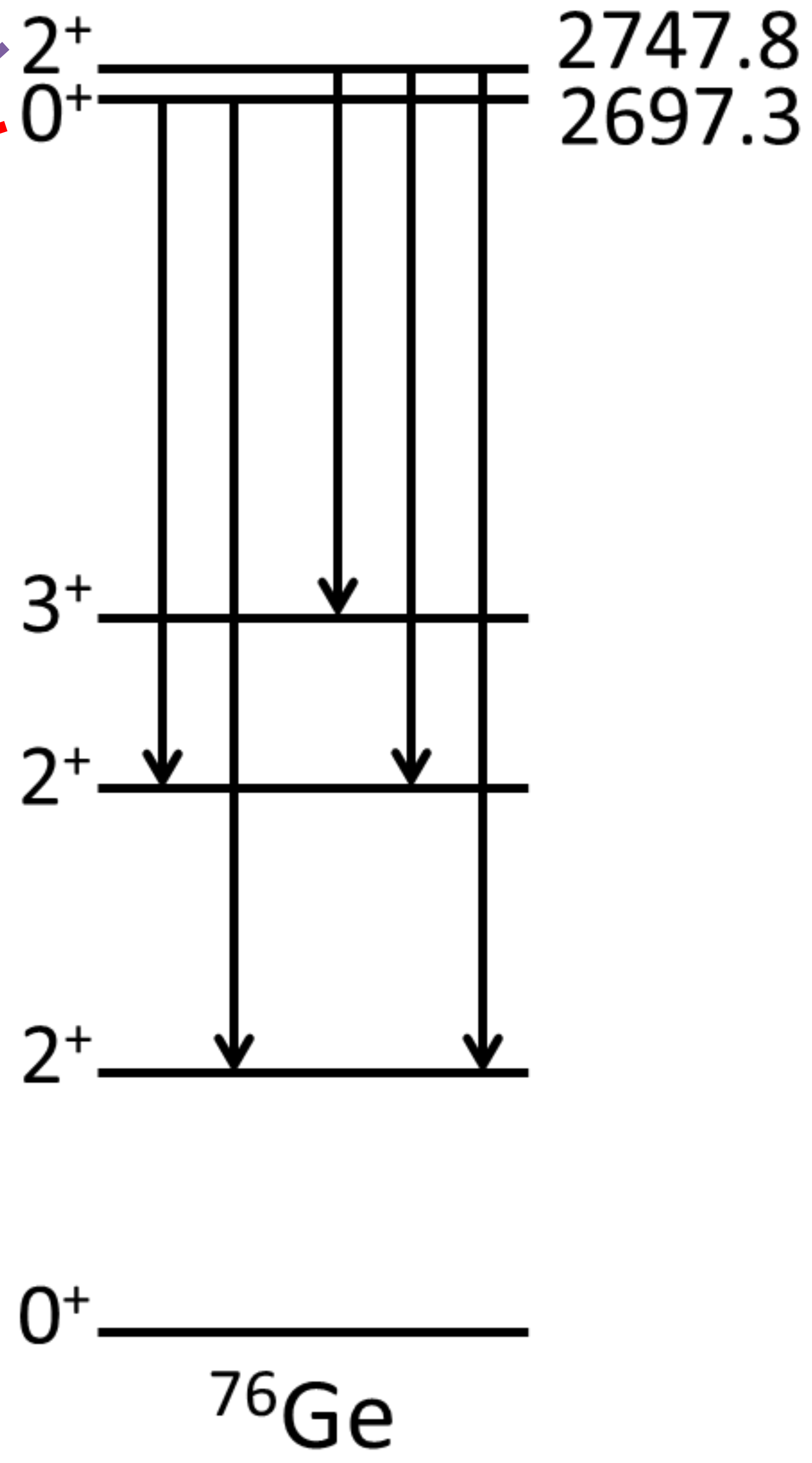
^{56}Fe

$\delta = -0.155^{+13}_{-20}$

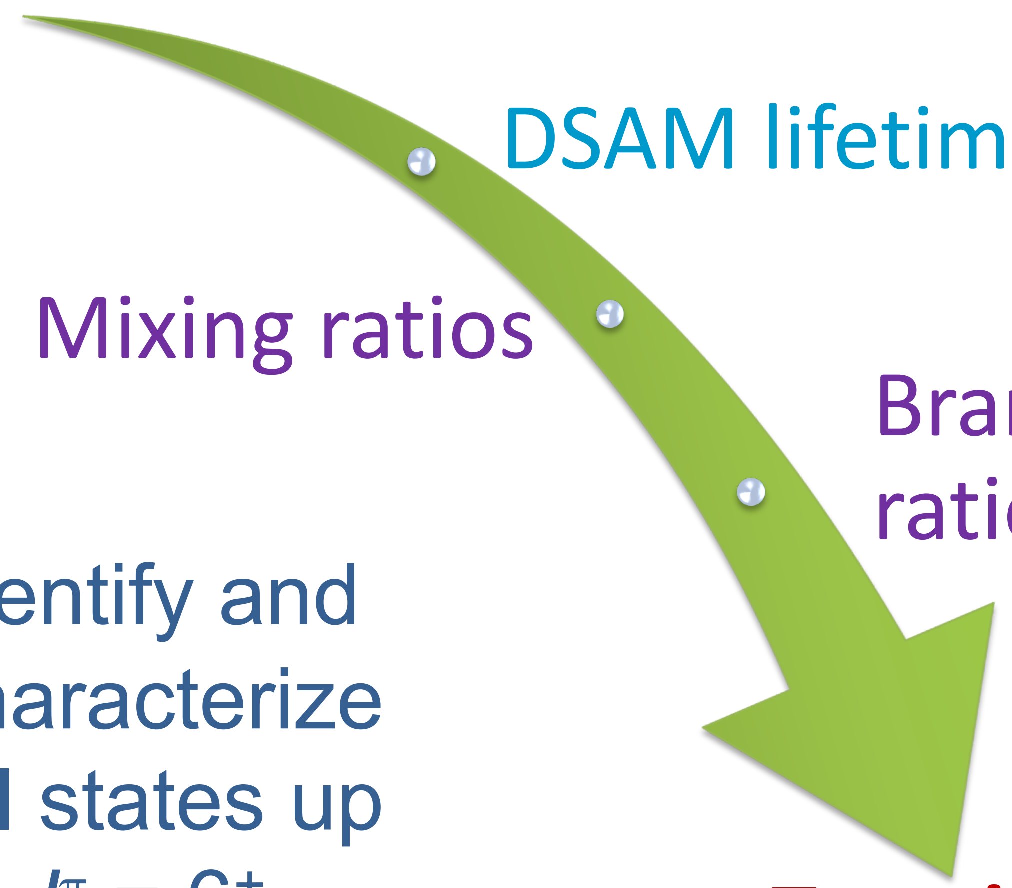
$\delta(NNDC)$
$= -0.18 \pm 0.01$

INS Excitation Functions





UKAL – Inelastic Neutron Scattering



Transition probabilities for:

- Constraining $0\nu\beta\beta$ nuclear matrix element calculations
- Identifying shape coexistence

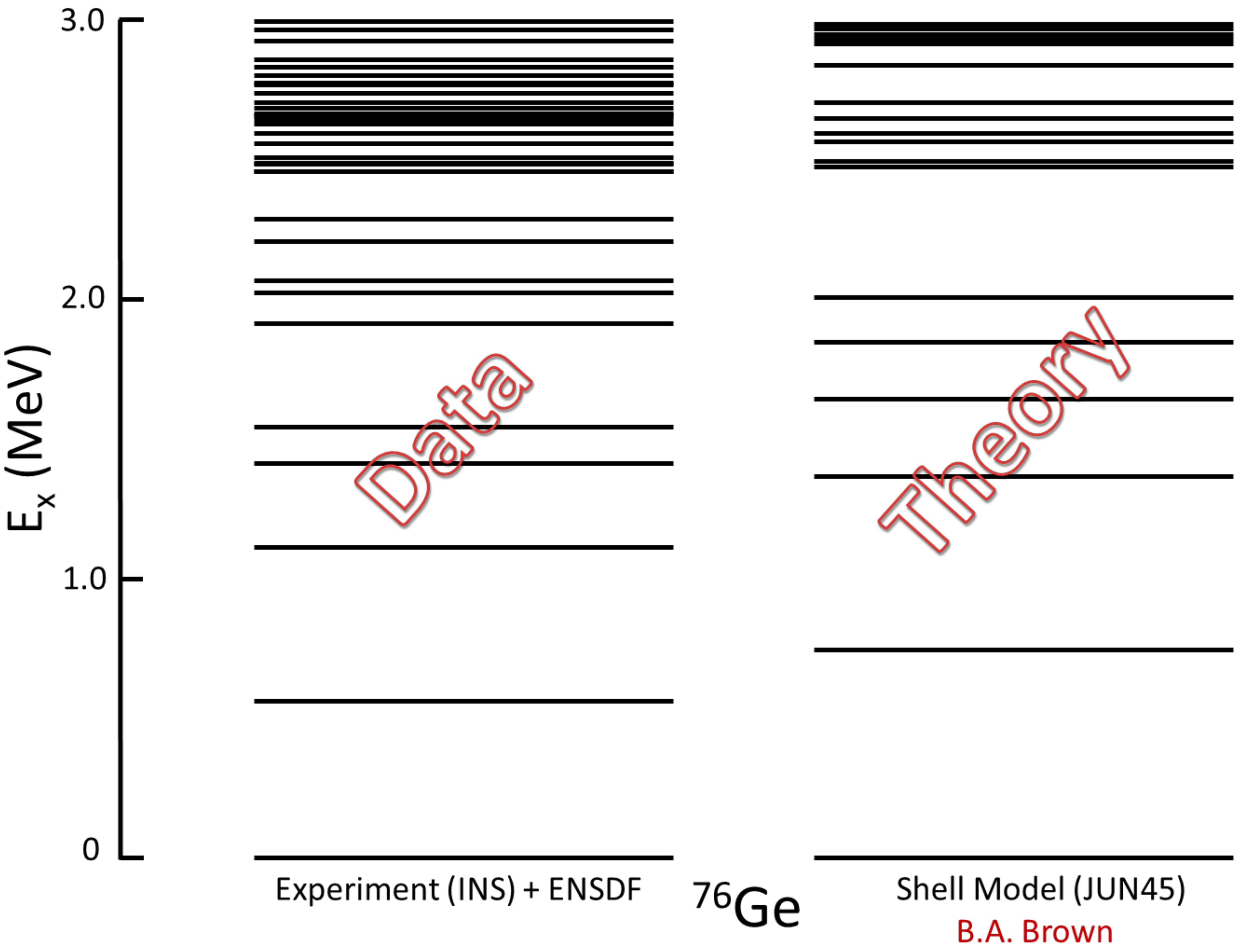
Inelastic Neutron Scattering Advantages:

- No Coulomb barrier
- Statistical population of all states up to ~J^{π} = 6
- Population of non-yrast states
- Level lifetimes (fs-ps) measured using the Doppler-shift attenuation method
- Multipole mixing ratios also measured from γ-ray angular distributions
- Eliminate erroneous states

A Comprehensive Approach

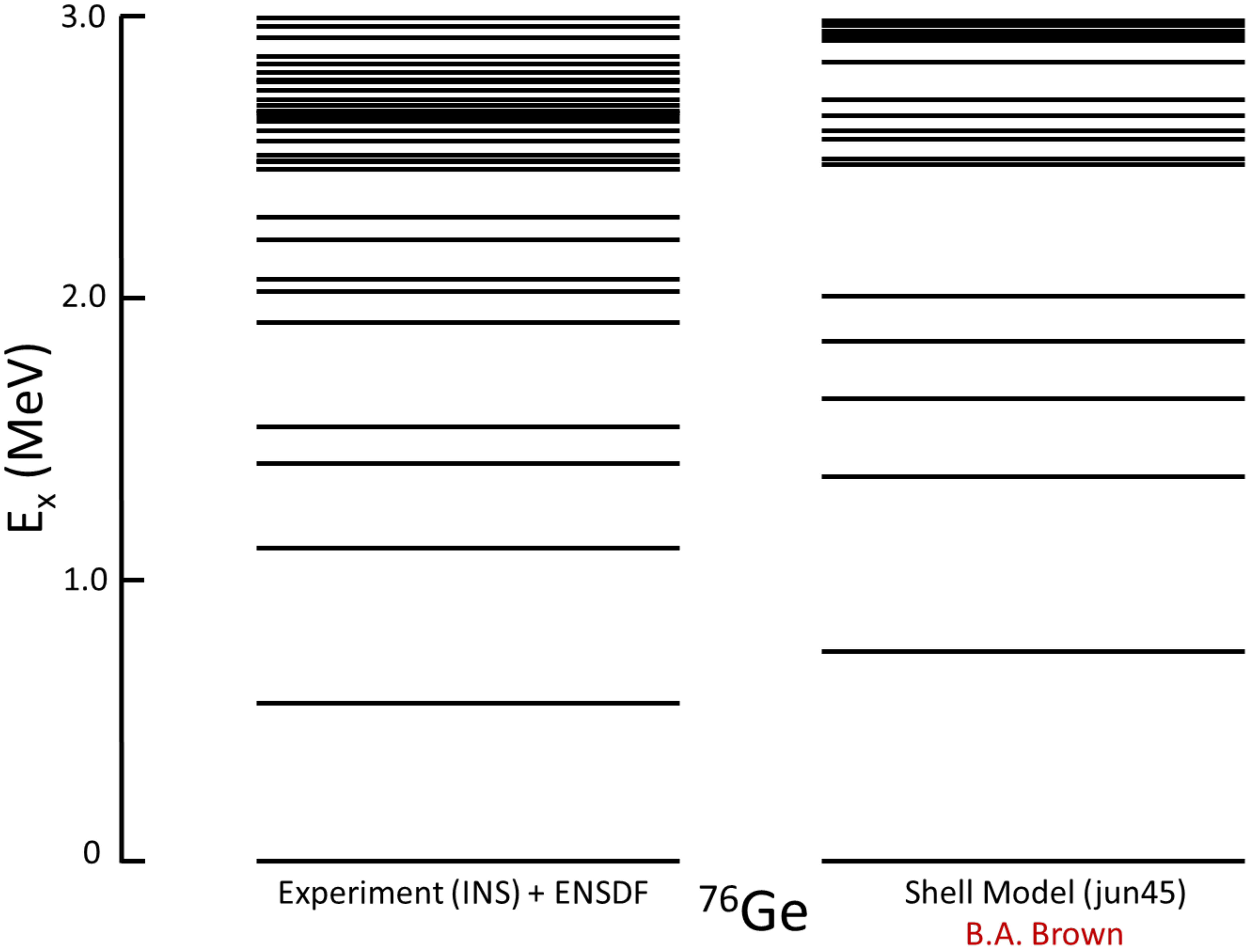
- Eliminate the **erroneous states**.
  - States are populated statistically and non-selectively in INS.
  - Thus, we see population of states with J = 0 – 4 within ~100 keV incident E_n of the level energy and states with J = 5,6 within ~400 keV.
  - If we do not find at least the most intense γ ray(s) purportedly emitted from the state at the appropriate energies, we refute the level, labeling it an "erroneous state".
  - The γ ray is likely misplaced in the level scheme.
  - Coincidence data, while very helpful, are not generally required.
- Identify **all** of the excited states up to some energy (*e.g.*, 3 MeV) in as many nuclei in the region as possible, but certainly those near the nucleus of interest.
- Characterize them as completely as possible.
- Compare these data with theoretical model calculations.

^{76}Ge(n,n`γ)

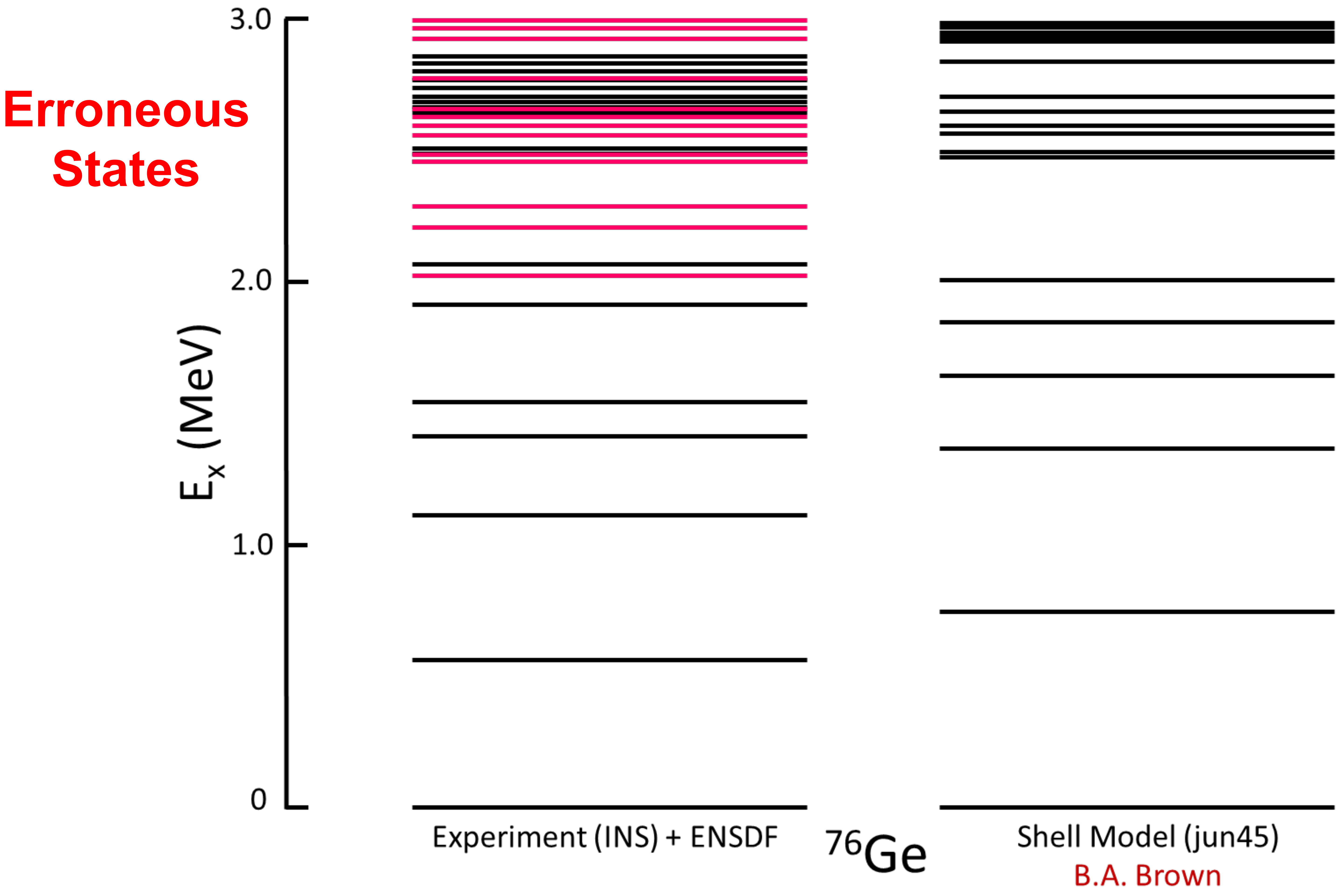


3.0
2.0
1.0
0
Ex (MeV)
Data
Theory
Experiment (INS) + ENSDF
76Ge
Shell Model (JUN45)
B.A. Brown

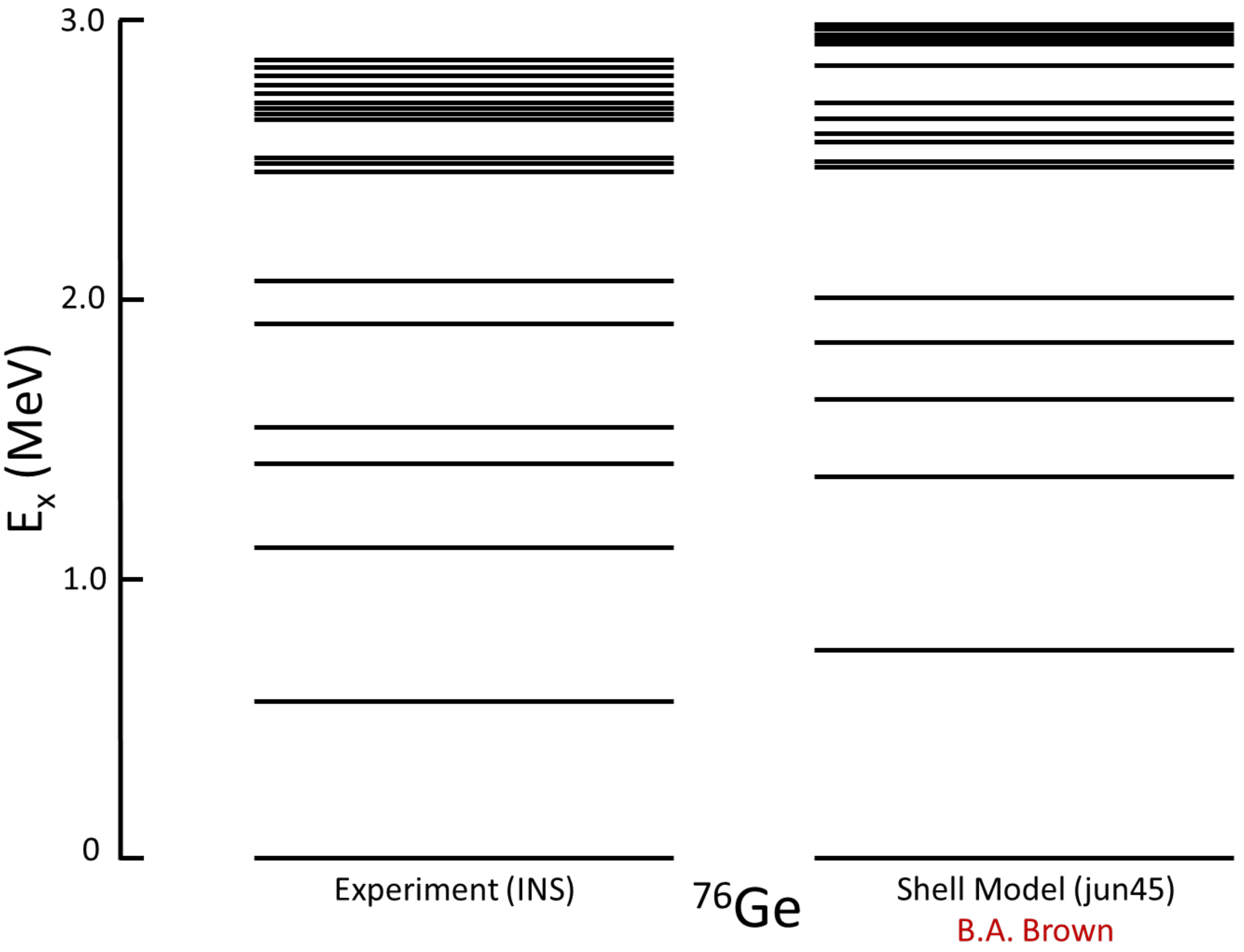
$^{76}Ge(n,n`\gamma)$



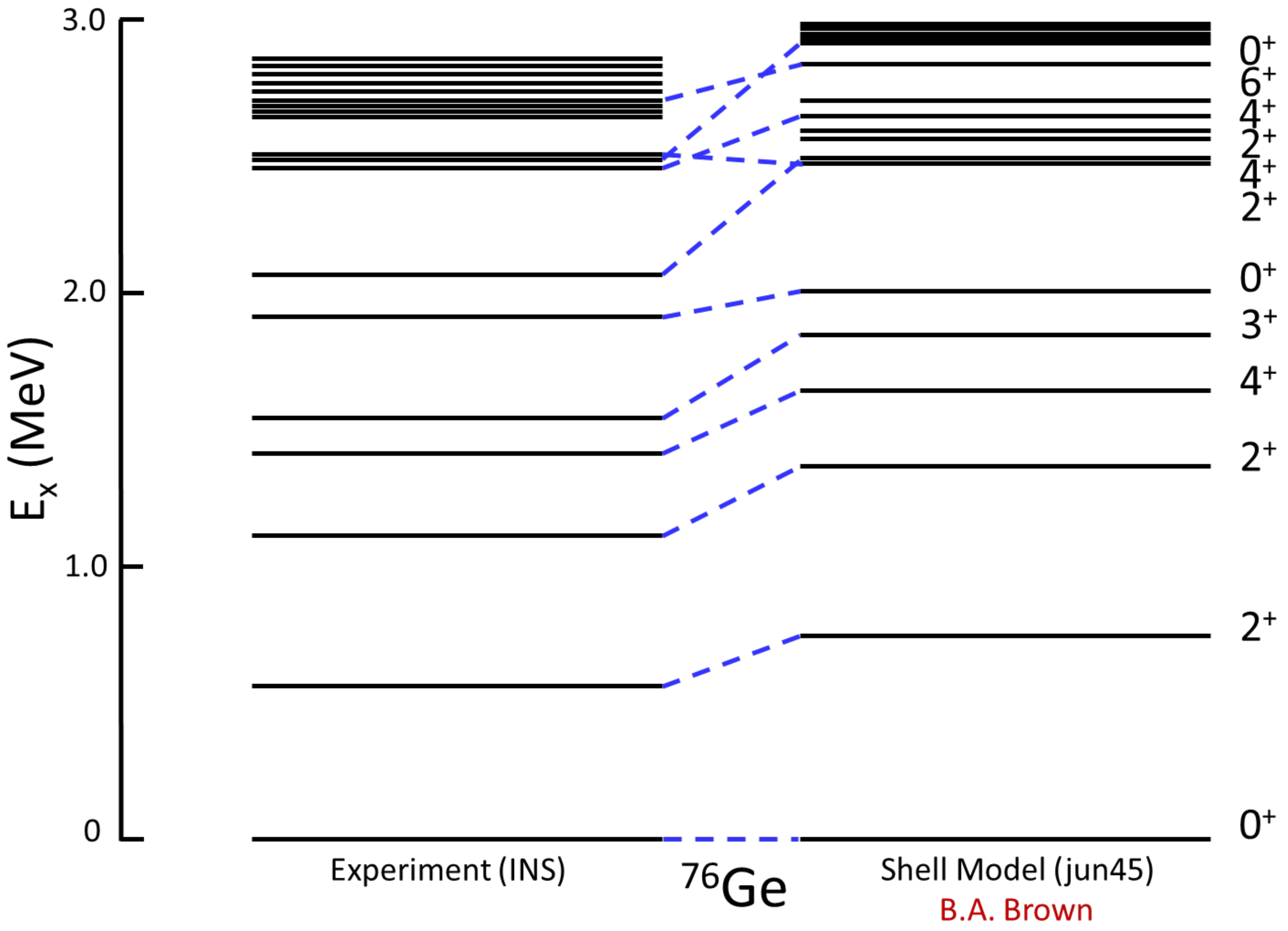
$^{76}Ge(n,n`\gamma)$



$^{76}Ge(n,n`\gamma)$



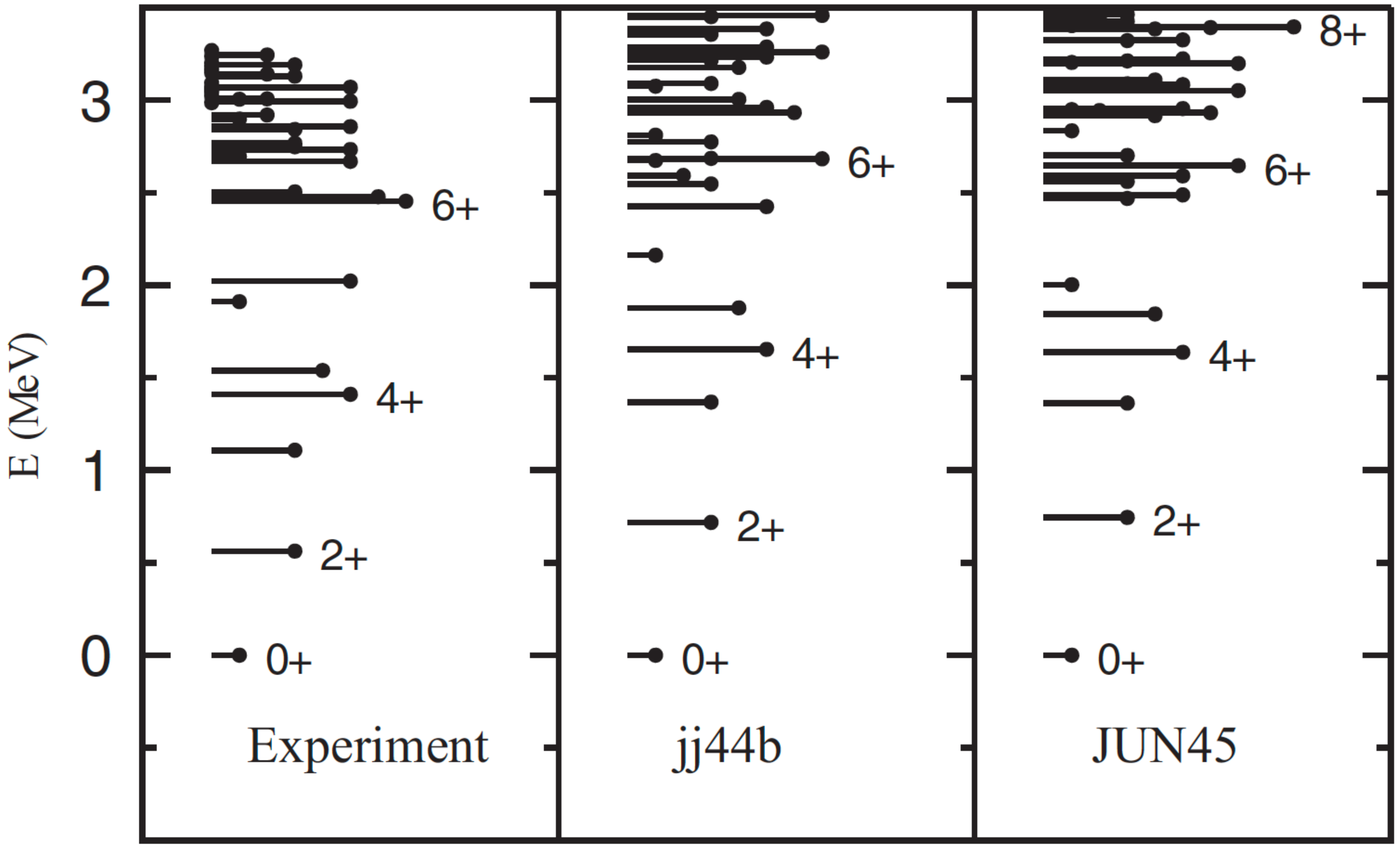
^{76}Ge(n,n`γ)


3.0
2.0
1.0
0
E_x (MeV)
0+
6+
4+
2+
4+
2+
0+
3+
4+
2+
2+
0+
Experiment (INS)
76Ge
Shell Model (jun45)
B.A. Brown

$^{76}Ge(n,n'\gamma)$ and Shell Model

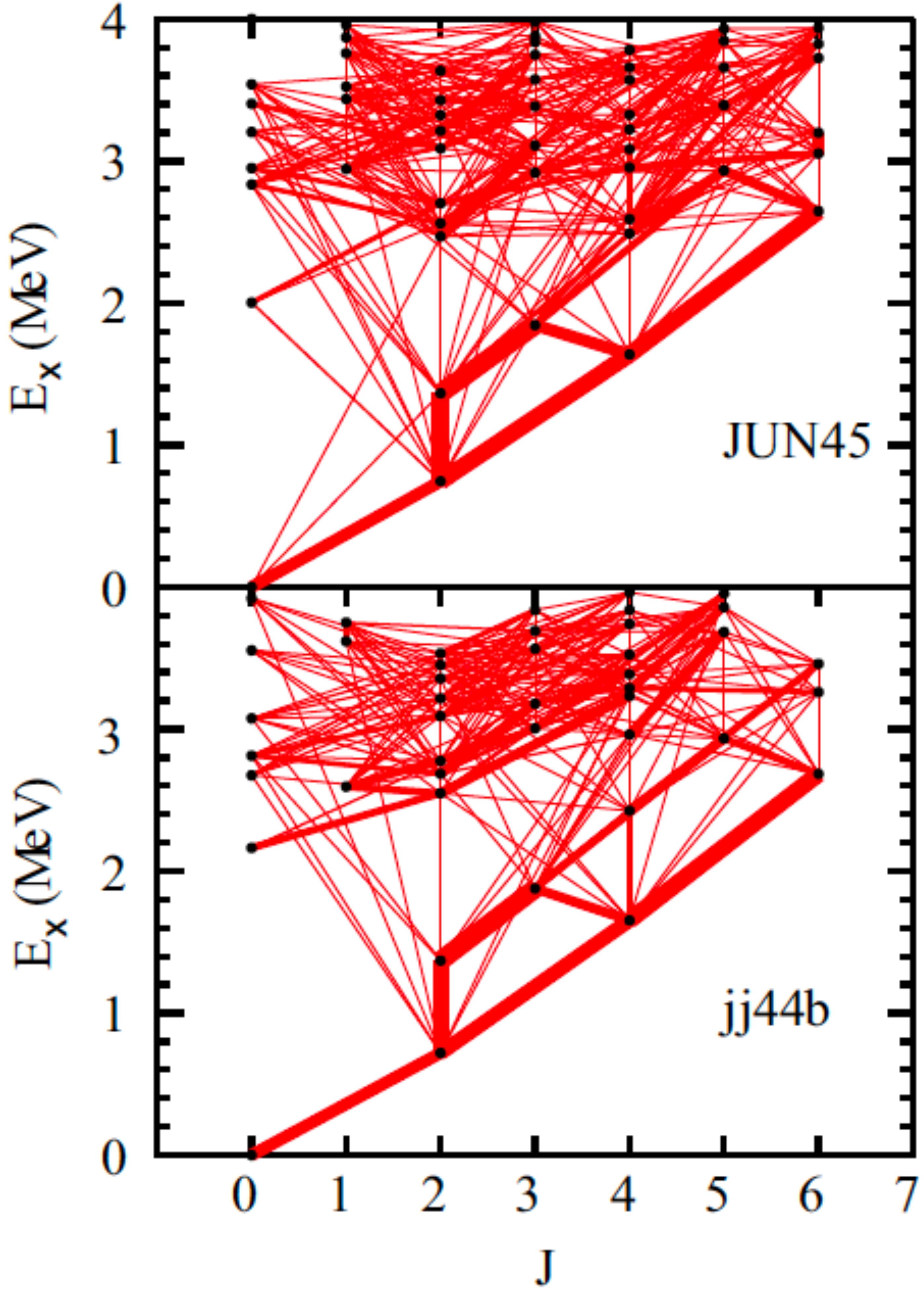
^{56}Ni core

model space jj44: $0f_{7/2}$, $1p_{3/2}$, $1p_{1/2}$, $0g_{9/2}$

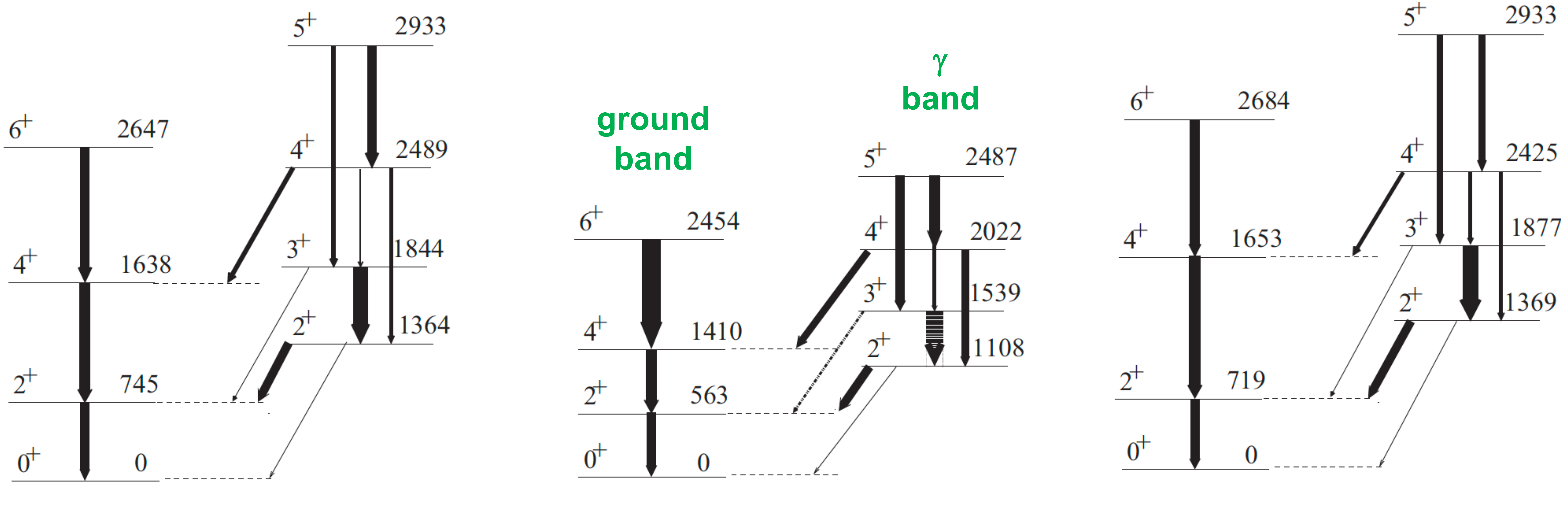


Comparison of the number of states
Both interactions levels ~200 keV > expt.

B(E2) values greater than 1 W.u.



$^{76}Ge(n,n`\gamma)$ and Shell Model



(a) JUN45 (b) Experiment (c) jj44b

B(E2) comparison with experiment shows good agreement.
Reinforces band structure from a microscopic basis

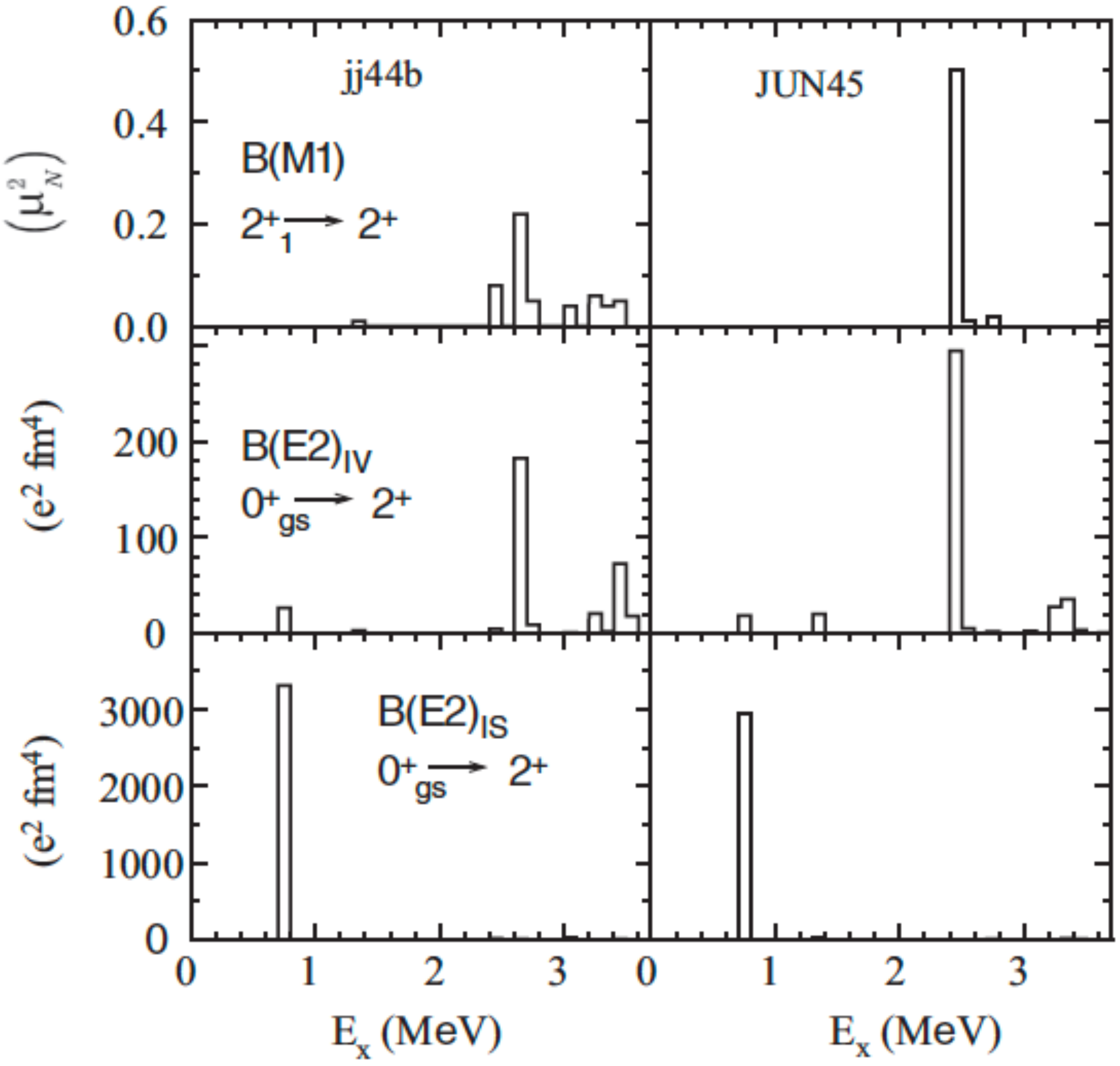
S. Mukhopadhyay, B. P. Crider, B. A. Brown, et al., PRC 95, 014327 (2017).

$^{76}Ge(n,n'\gamma)$ and Shell Model

Experimental mixed-symmetry state is 2.767 MeV 2^+ state

jj44b fragmented M1 strength, dominant component 2.69 MeV

JUN45 single 2.47 MeV level



S. Mukhopadhyay, B. P. Crider, B. A. Brown, et al., PRC 95, 014327 (2017).

$^{74}Ge(n,n`\gamma)$

Erroneous State Example

^{74}Ge: First 10 States 2006 ENSDF Evaluation

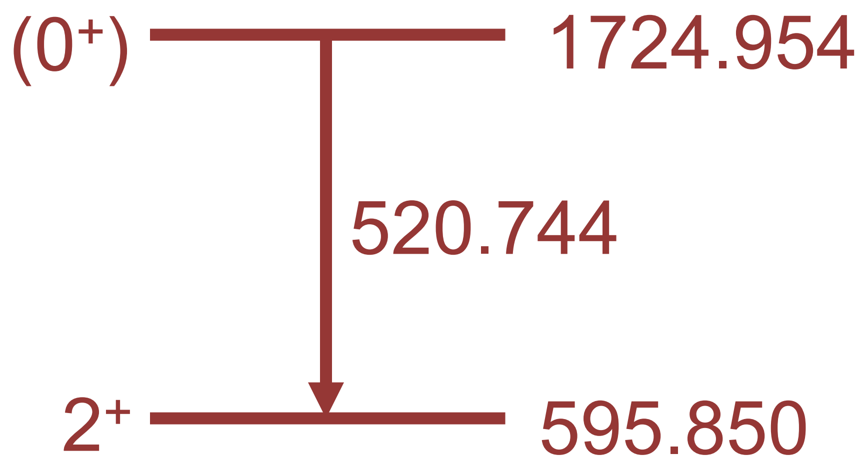
E_i(level)	J_i^π	E_γ	I_γ	E_f	J_f^π	Mult.†	δ	Comments
595.850	2^+	595.847 *6*	100	0.0	0^+	E2		B(E2)(W.u.)=33.0 *4* Mult.: from γ(pol,θ).
1204.205	2^+	608.353 *5*	100 *1*	595.850	2^+	E2+M1	+3.4 *4*	B(M1)(W.u.)=0.00099 *15*; B(E2)(W.u.)=43 *6* δ: from $\gamma\gamma(\theta)$ in ^{74}As ε decay. Other: +2.2 *3* from (n,n′γ).
		1204.208 *12*	46 *3*	0.0	0^+	E2		B(E2)(W.u.)=0.71 *11*
1463.759	4^+	867.898 *6*	100	595.850	2^+	E2		B(E2)(W.u.)=41 *3*
1482.81	0^+	887.19 *7*	100	595.850	2^+	E2		B(E2)(W.u.)=9 *+9−6*
		1482.6		0.0	0^+	E0		From ce data (1983Pa10). $I_{(\gamma+ce)}$: <0.006 from ^{74}As ε decay. q_K^2(E0/E2)<0.12, X(E0/E2)<0.052, ρ^2(E0)>0.032 (2005Ki02, evaluation).
1697.140	$(3)^+$	233.395 *12*	2.1 *2*	1463.759	4^+			
		492.936 *6*	58 *1*	1204.205	2^+	(M1+E2)	+1.3 *4*	δ: from $\gamma(\theta)$ in (n,n′γ) (1970Ch15). Other: 2.0 *+3−6* or 0.75 *+15−6* (1987Do14). Mult.: D+Q from $\gamma(\theta)$. ΔJ^π=no from placement in level scheme.
		1101.267 *12*	100 *1*	595.850	2^+	(M1+E2)	+0.34 *5*	δ: from $\gamma(\theta)$ in (n,n′γ) (1970Ch15). Other: 0.47 *5* (1987Do14). Mult.: D+Q from $\gamma(\theta)$. ΔJ^π=no from placement in level scheme.
1724.954	(0^+)	520.744 *12*	100	1204.205	2^+			
2165.259	$(3,4)^+$	468.11 *3*	6.5 *3*	1697.140	$(3)^+$			
		701.487 *6*	42.7 *3*	1463.759	4^+			
		961.055 *10*	100 *1*	1204.205	2^+	(M1(+E2))	0.01 *1*	δ: from $\gamma(\theta)$ in (n,n′γ) (1987Do14). Mult.: D+Q from $\gamma(\theta)$. ΔJ^π=no from placement in level scheme.
2197.933	2^+	715.17 *3*	35 *2*	1482.81	0^+			
		734.17 *4*	25 *4*	1463.759	4^+			
		993.67 *6*	100 *5*	1204.205	2^+	(E2+M1)	−2.8 *2*	δ: $\gamma\gamma(\theta)$ in ^{74}As ε. Mult from ΔJ^π. Mult.: D+Q from $\gamma(\theta)$. ΔJ^π=no from placement in level scheme.
		1602.0 *2*	45 *4*	595.850	2^+			
		2197.95 *8*	82 *10*	0.0	0^+			
2227.77	0^+	1021.9 *1*	38	1204.205	2^+			
		1631.89 *12*	100	595.850	2^+			
2403.5	1	2403.5 *4*		0.0	0^+			

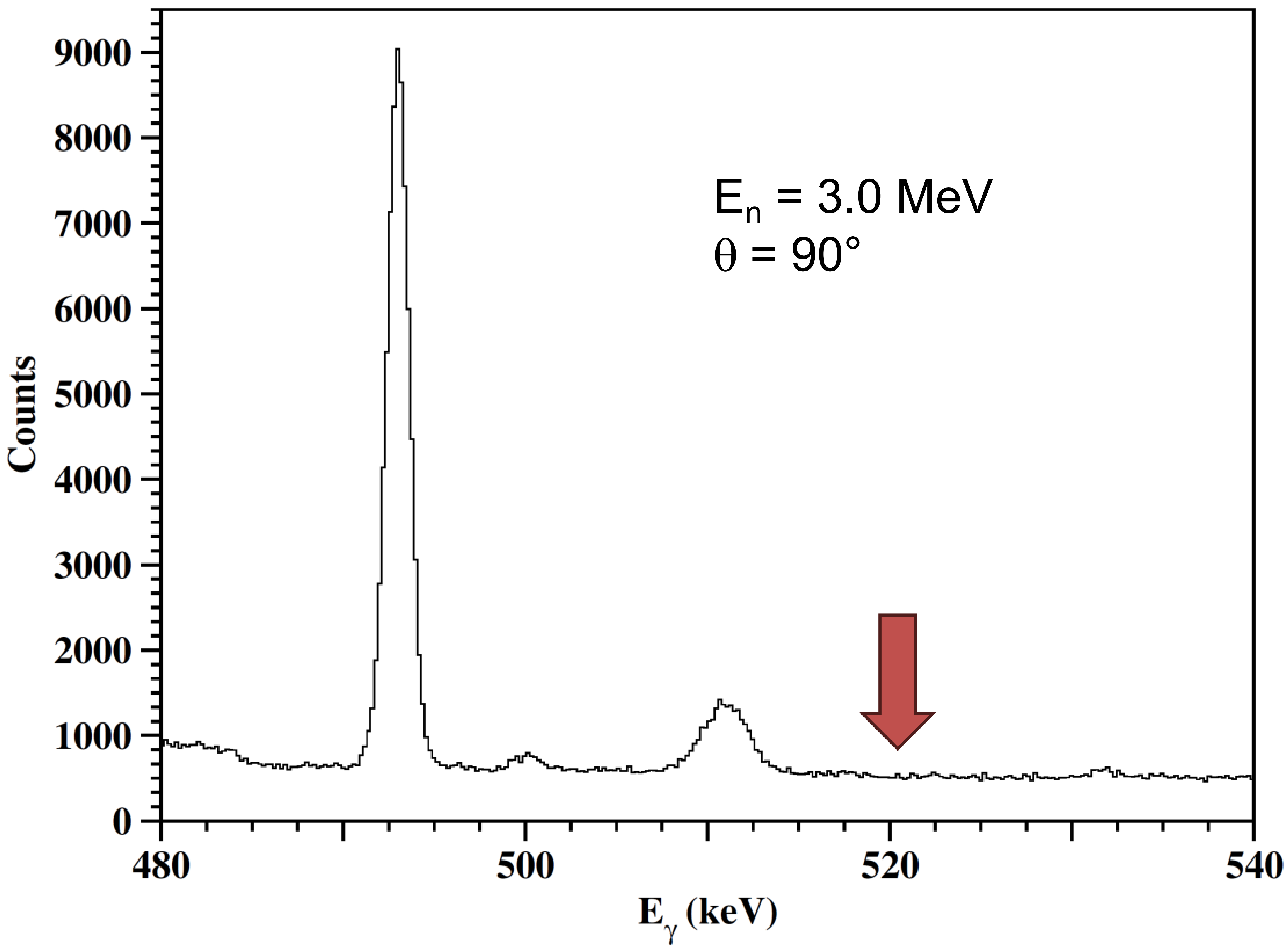
From ENSDF

Erroneous State Example

^{74}Ge

- Reported 520.744(12) keV γ ray from a 1724.954(14) keV (0^+) state





Erroneous State Example

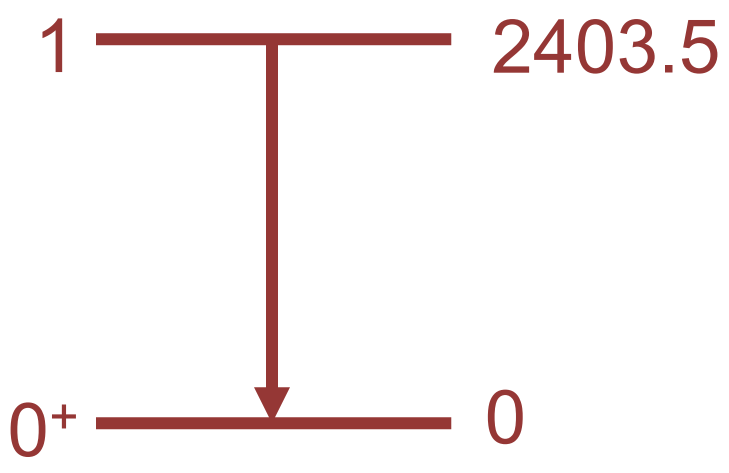
^{74}Ge: First 10 States
2006 ENSDF Evaluation

E_i(level)	J_i^π	E_γ	I_γ	E_f	J_f^π	Mult.†	δ	Comments
595.850	2^+	595.847 6	100	0.0	0^+	E2		B(E2)(W.u.)=33.0 4 Mult.: from γ(pol,θ).
1204.205	2^+	608.353 5	100 1	595.850	2^+	E2+M1	+3.4 4	B(M1)(W.u.)=0.00099 15; B(E2)(W.u.)=43 6 δ: from $\gamma\gamma(\theta)$ in ^{74}As ε decay. Other: +2.2 3 from (n,n′γ).
		1204.208 12	46 3	0.0	0^+	E2		B(E2)(W.u.)=0.71 11
1463.759	4^+	867.898 6	100	595.850	2^+	E2		B(E2)(W.u.)=41 3
1482.81	0^+	887.19 7	100	595.850	2^+	E2		B(E2)(W.u.)=9 +9−6
		1482.6		0.0	0^+	E0		From ce data (1983Pa10). $I_{(\gamma+ce)}$: <0.006 from ^{74}As ε decay. q_K^2(E0/E2)<0.12, X(E0/E2)<0.052, ρ^2(E0)>0.032 (2005Ki02, evaluation).
1697.140	$(3)^+$	233.395 12	2.1 2	1463.759	4^+			
		492.936 6	58 1	1204.205	2^+	(M1+E2)	+1.3 4	δ: from $\gamma(\theta)$ in (n,n′γ) (1970Ch15). Other: 2.0 +3−6 or 0.75 +15−6 (1987Do14). Mult.: D+Q from $\gamma(\theta)$. ΔJ^π=no from placement in level scheme.
		1101.267 12	100 1	595.850	2^+	(M1+E2)	+0.34 5	δ: from $\gamma(\theta)$ in (n,n′γ) (1970Ch15). Other: 0.47 5 (1987Do14). Mult.: D+Q from $\gamma(\theta)$. ΔJ^π=no from placement in level scheme.
~~1724.954~~	~~(0^+)~~	~~520.744 12~~	~~100~~	~~1204.205~~	~~2^+~~			
2165.259	$(3,4)^+$	468.11 3	6.5 3	1697.140	$(3)^+$			
		701.487 6	42.7 3	1463.759	4^+			
		961.055 10	100 1	1204.205	2^+	(M1(+E2))	0.01 1	δ: from $\gamma(\theta)$ in (n,n′γ) (1987Do14). Mult.: D+Q from $\gamma(\theta)$. ΔJ^π=no from placement in level scheme.
2197.933	2^+	715.17 3	35 2	1482.81	0^+			
		734.17 4	25 4	1463.759	4^+			
		993.67 6	100 5	1204.205	2^+	(E2+M1)	−2.8 2	δ: $\gamma\gamma(\theta)$ in ^{74}As ε. Mult from ΔJ^π. Mult.: D+Q from $\gamma(\theta)$. ΔJ^π=no from placement in level scheme.
		1602.0 2	45 4	595.850	2^+			
		2197.95 8	82 10	0.0	0^+			
2227.77	0^+	1021.9 1	38	1204.205	2^+			
		1631.89 12	100	595.850	2^+			
2403.5	1	2403.5 4		0.0	0^+			

From ENSDF

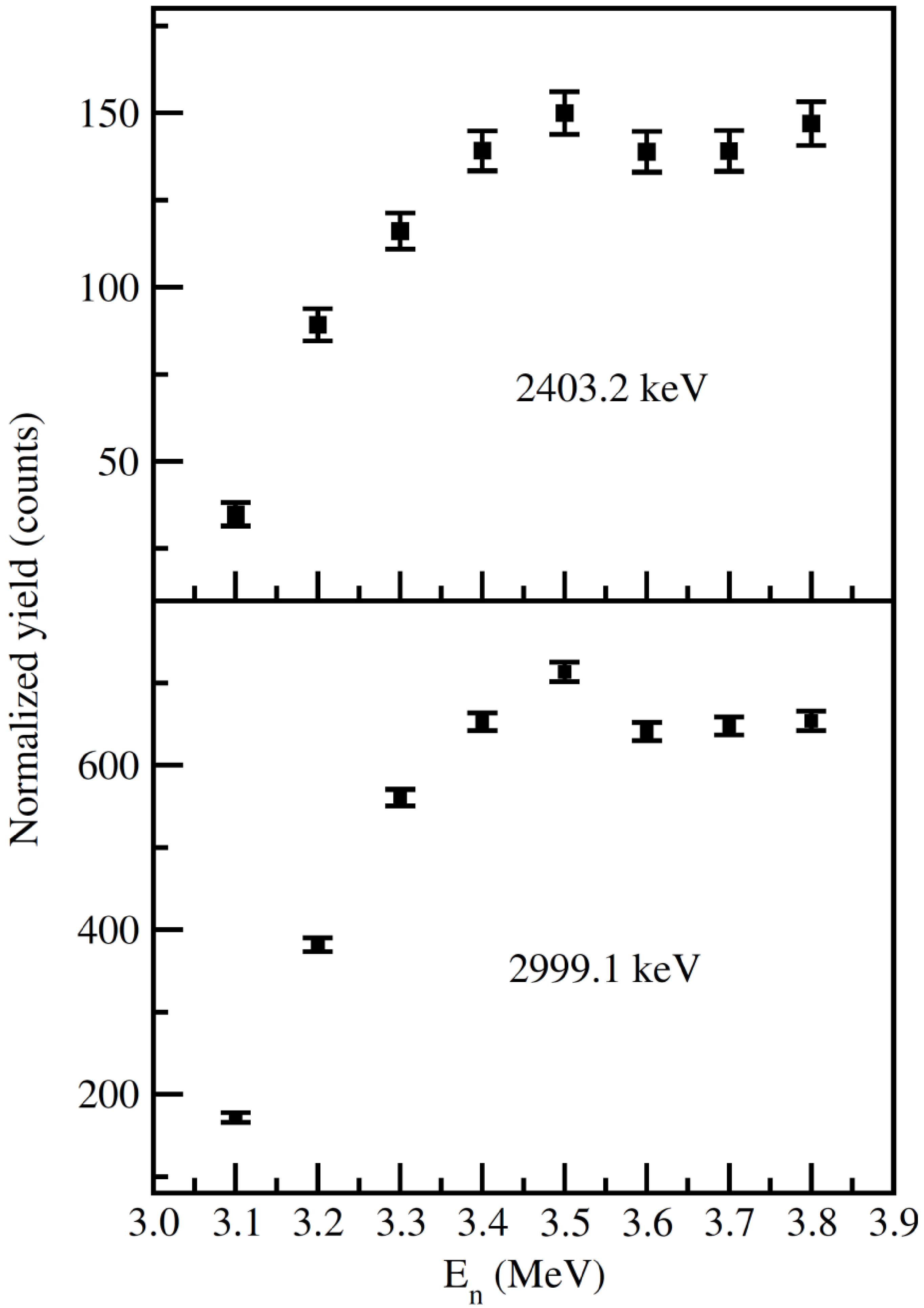
Erroneous State Example

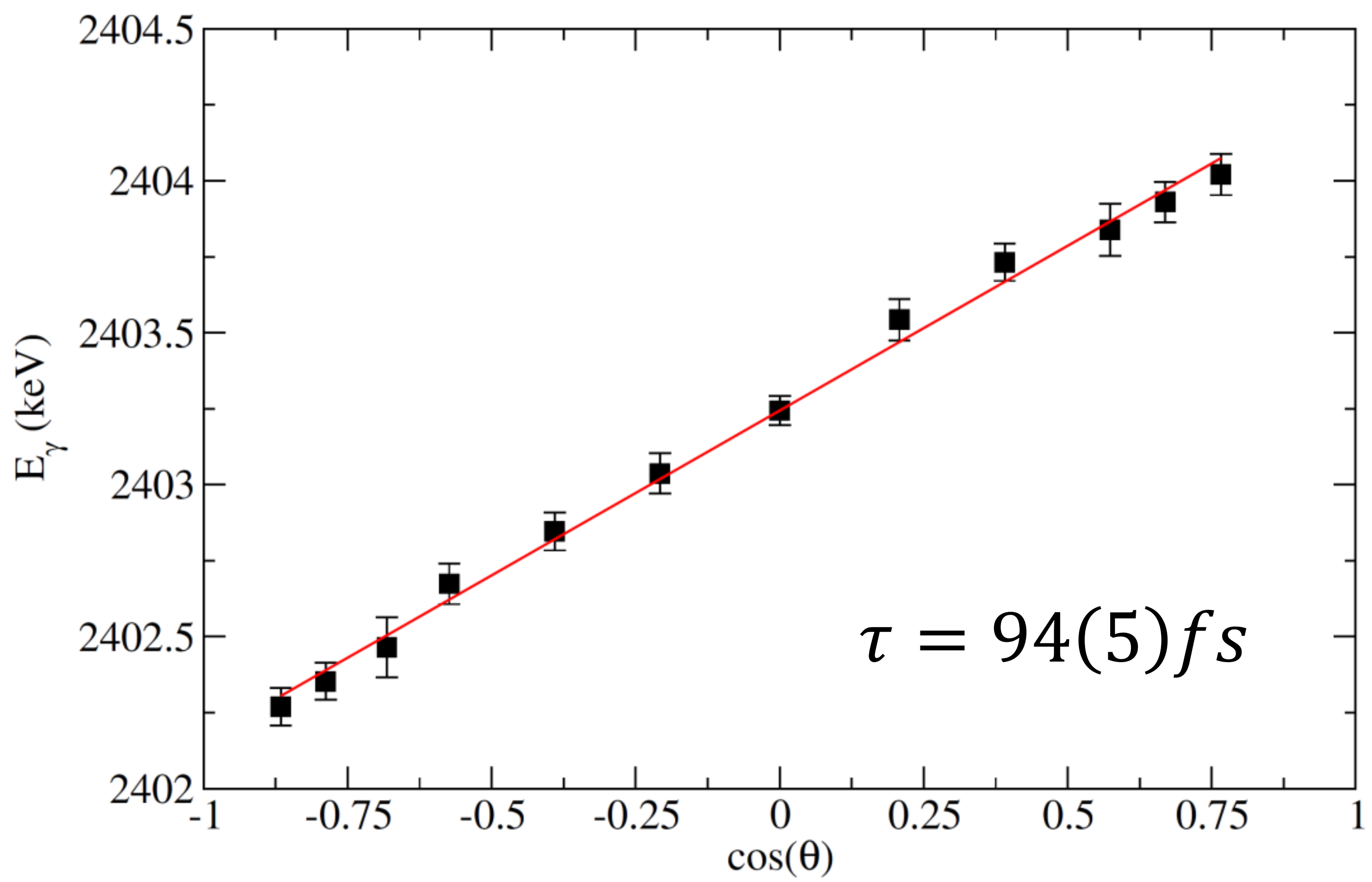
- **2403.5(4) keV γ ray was previously assigned as a ground-state transition from a spin-1 state in ^{74}Ge**

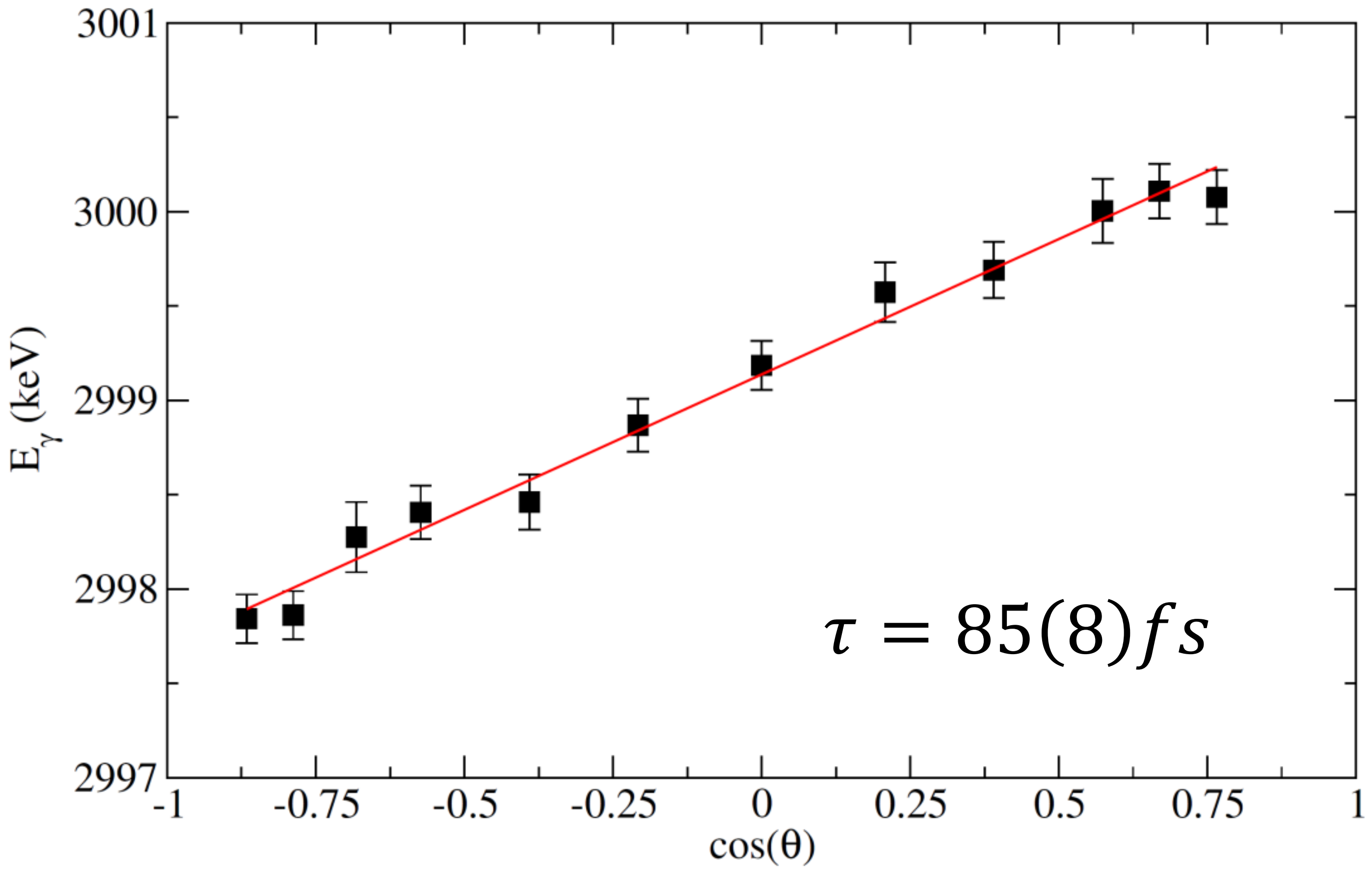


- **Peak should have ~10k cts at 2.5 MeV**

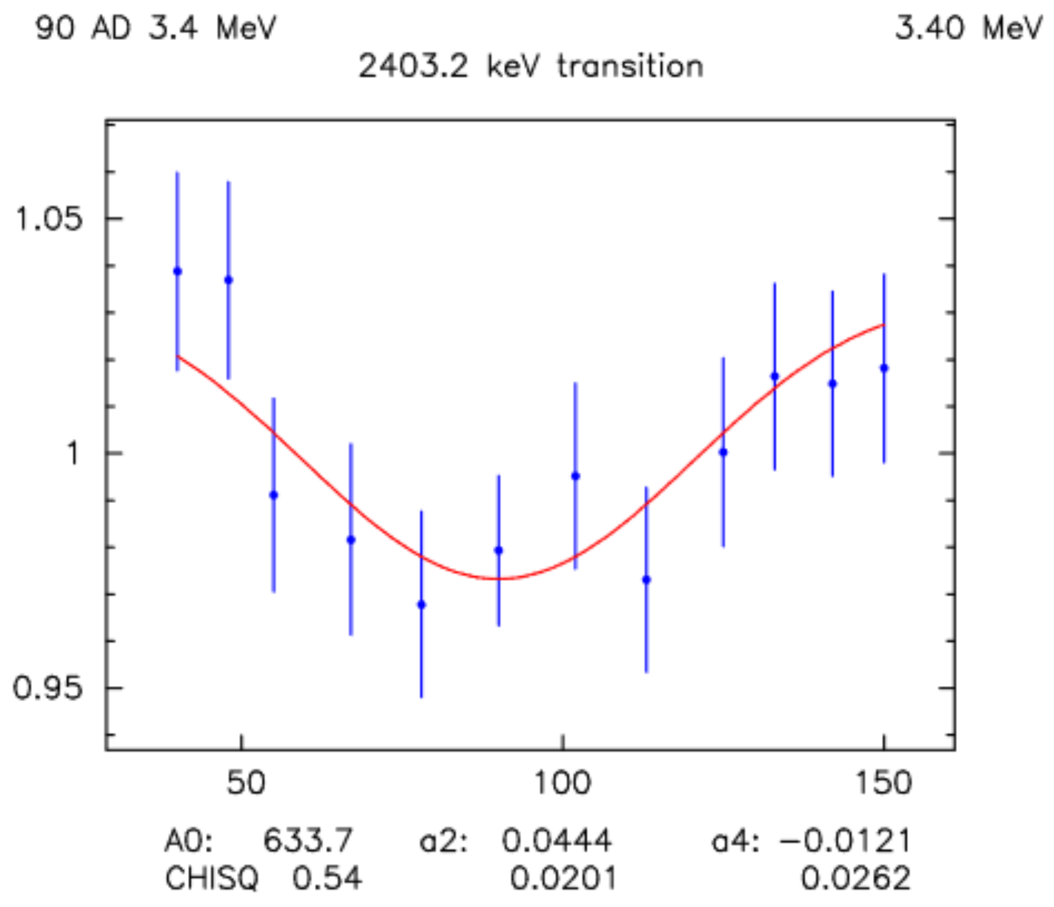
- **2999 keV γ ray is newly observed**

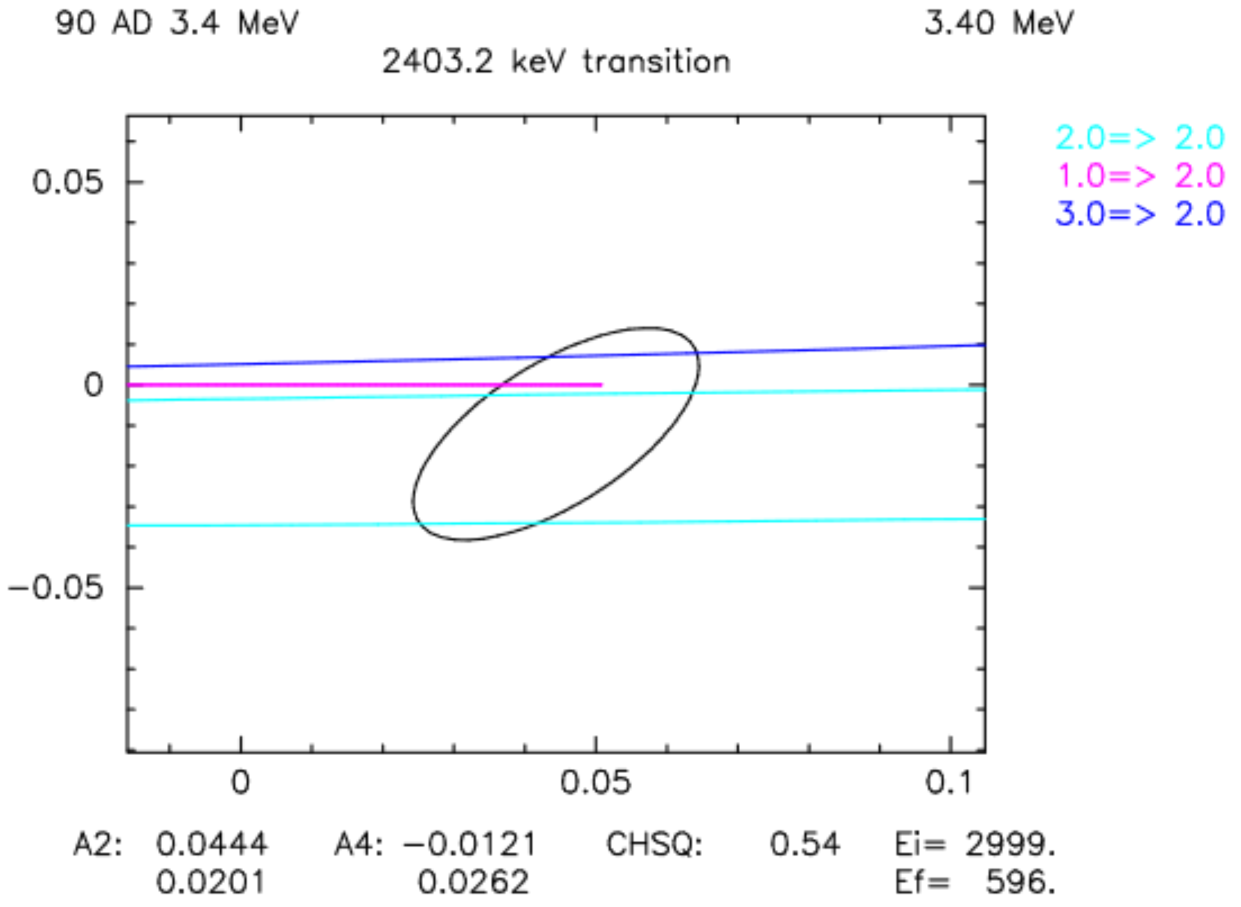


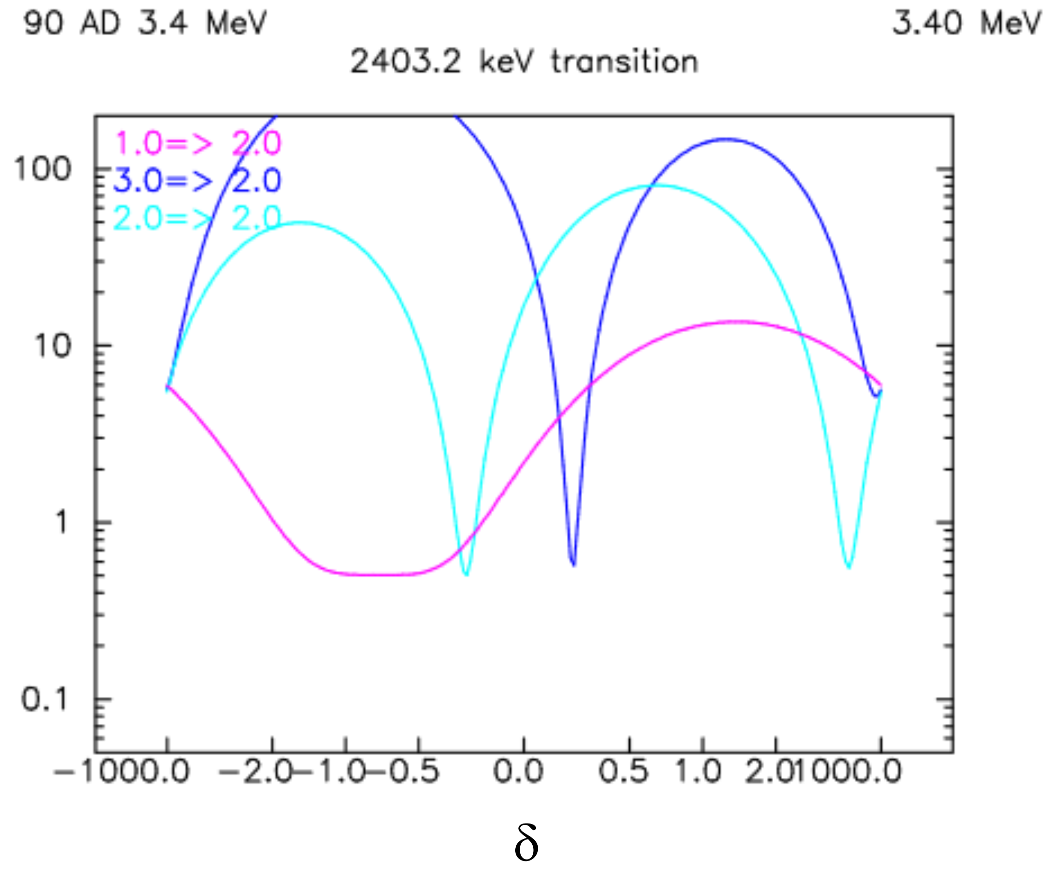


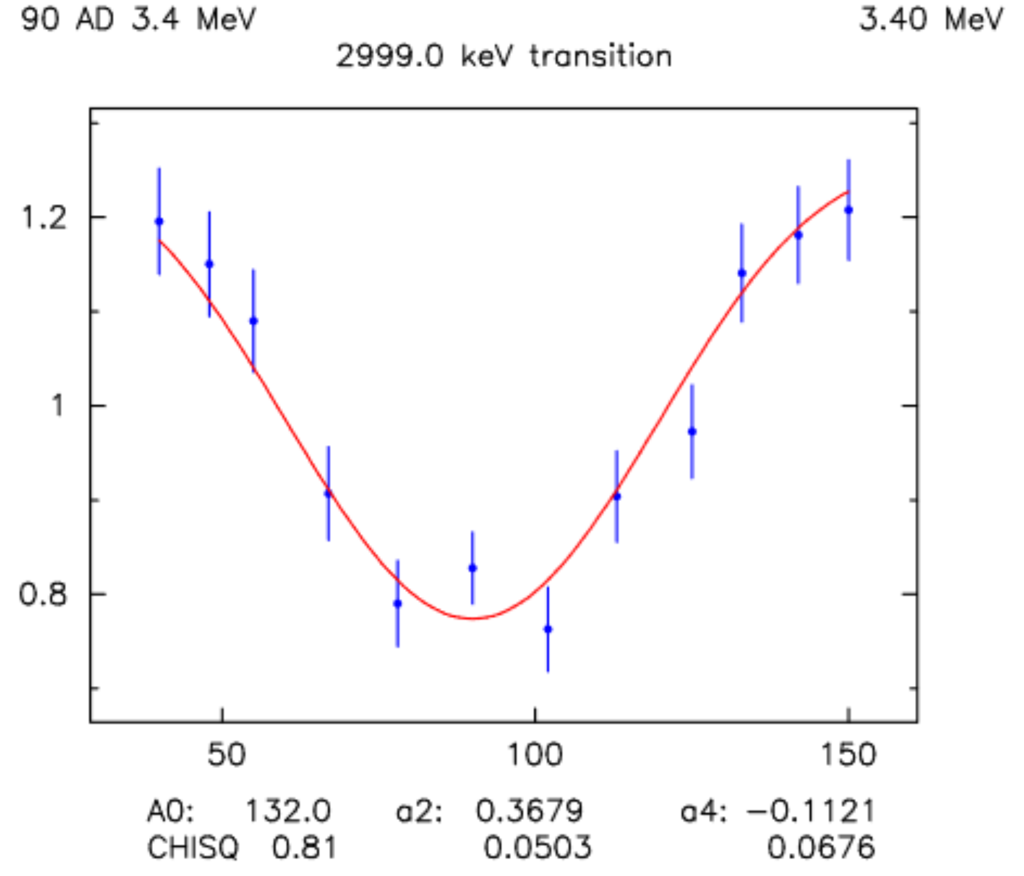


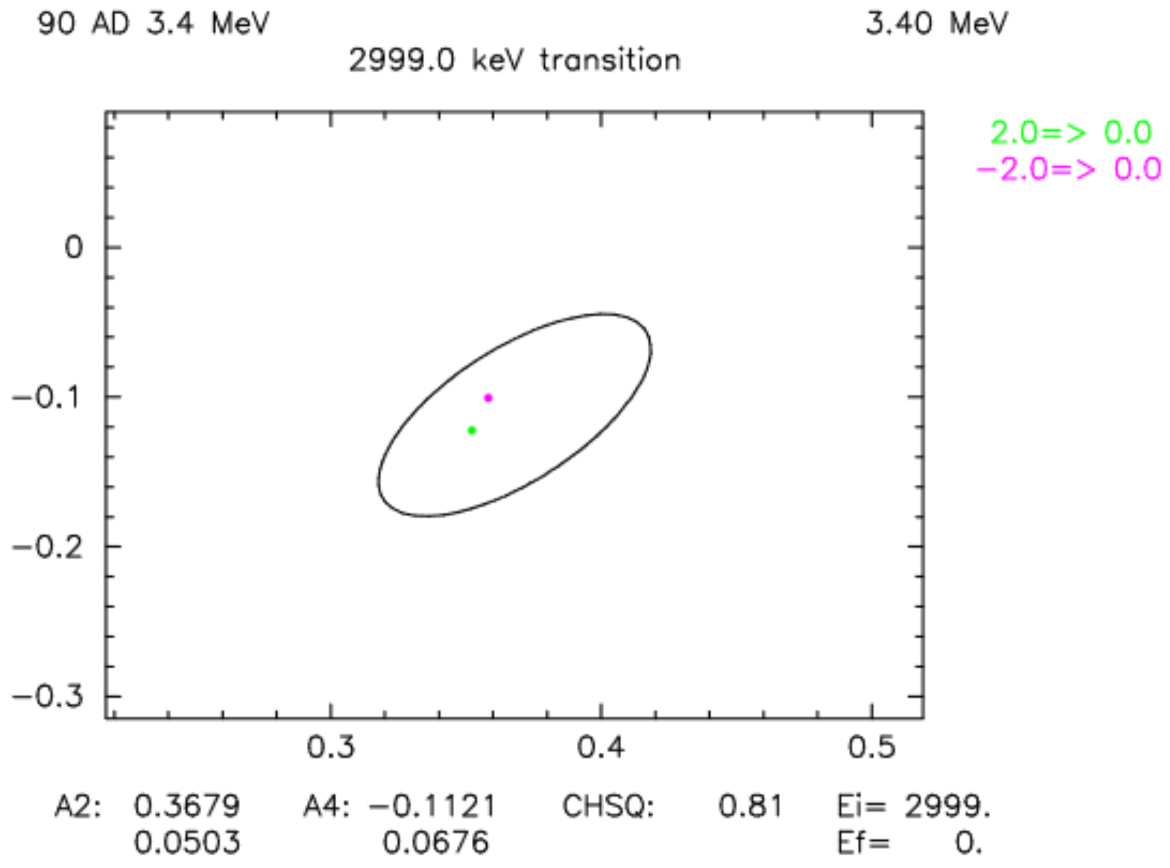
Erroneous State Example

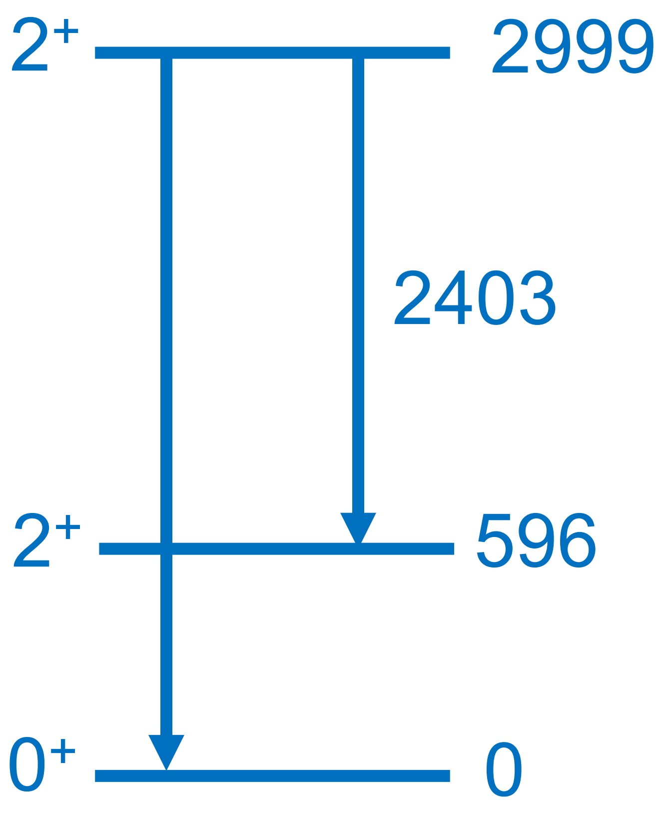








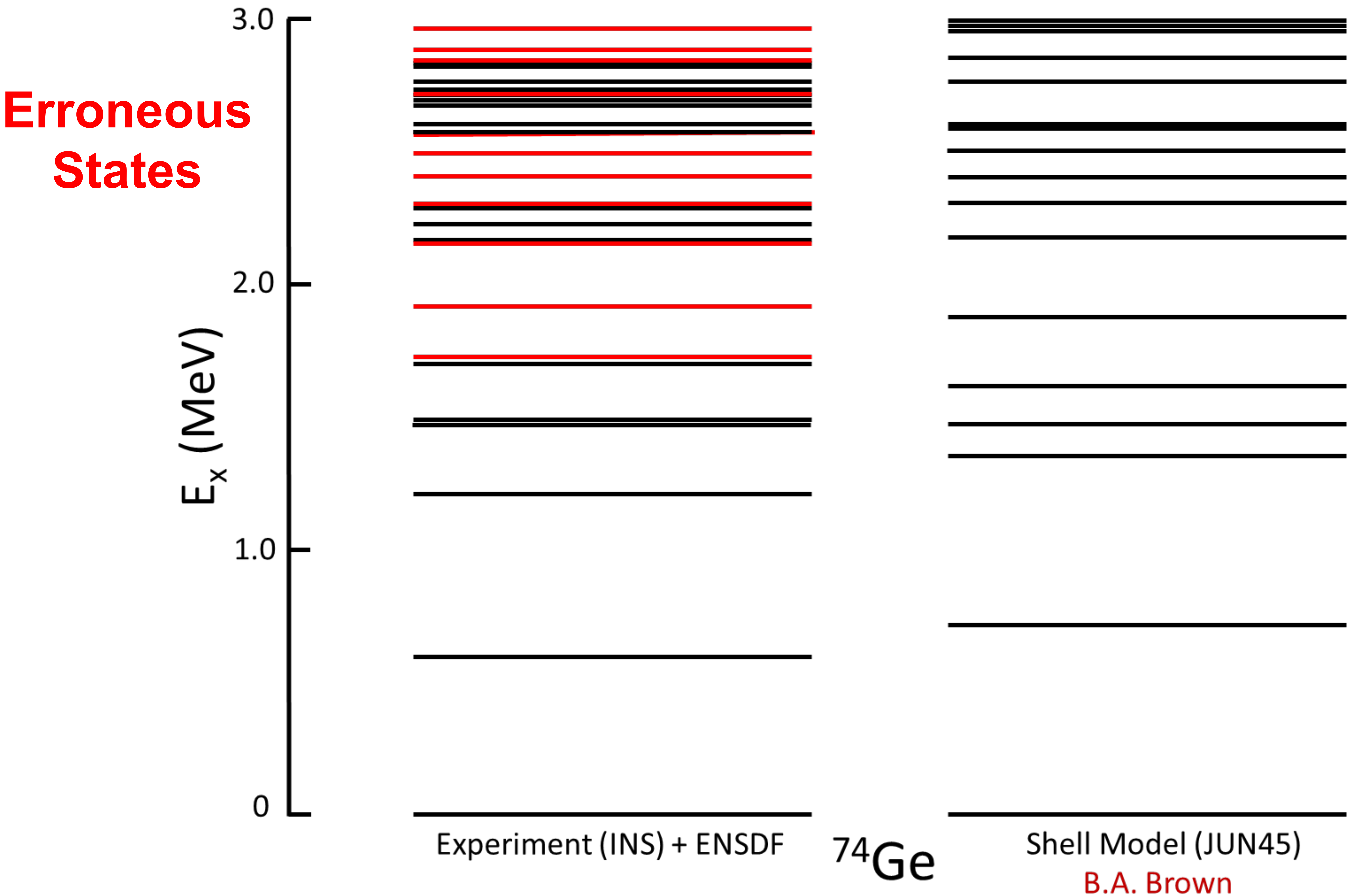




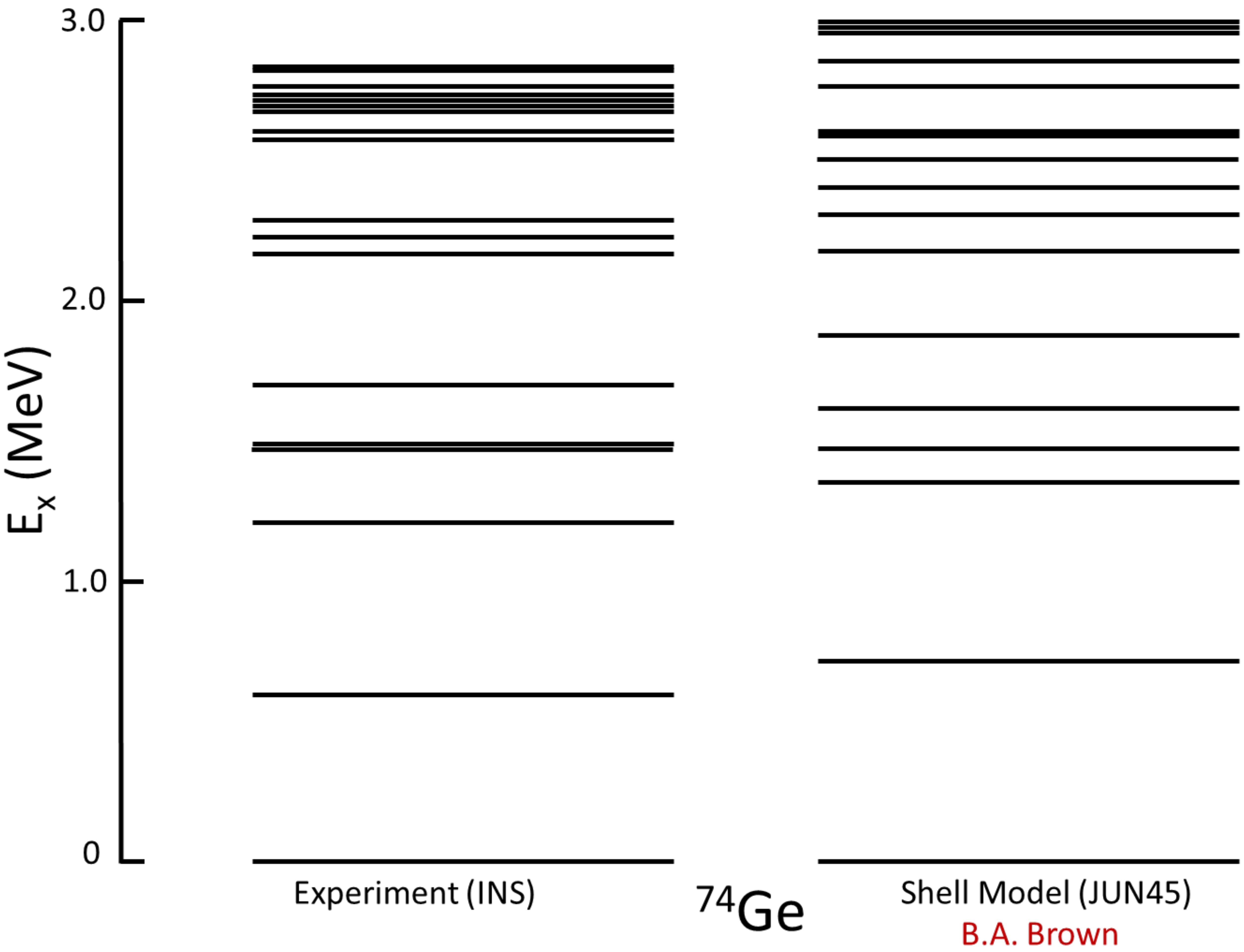
- ✓**Excitation functions & thresholds**
- ✓**Lifetimes**
- ✓**Spins from angular distributions**
- ✓**Energies**

= A consistent picture

$^{74}Ge(n,n`\gamma)$



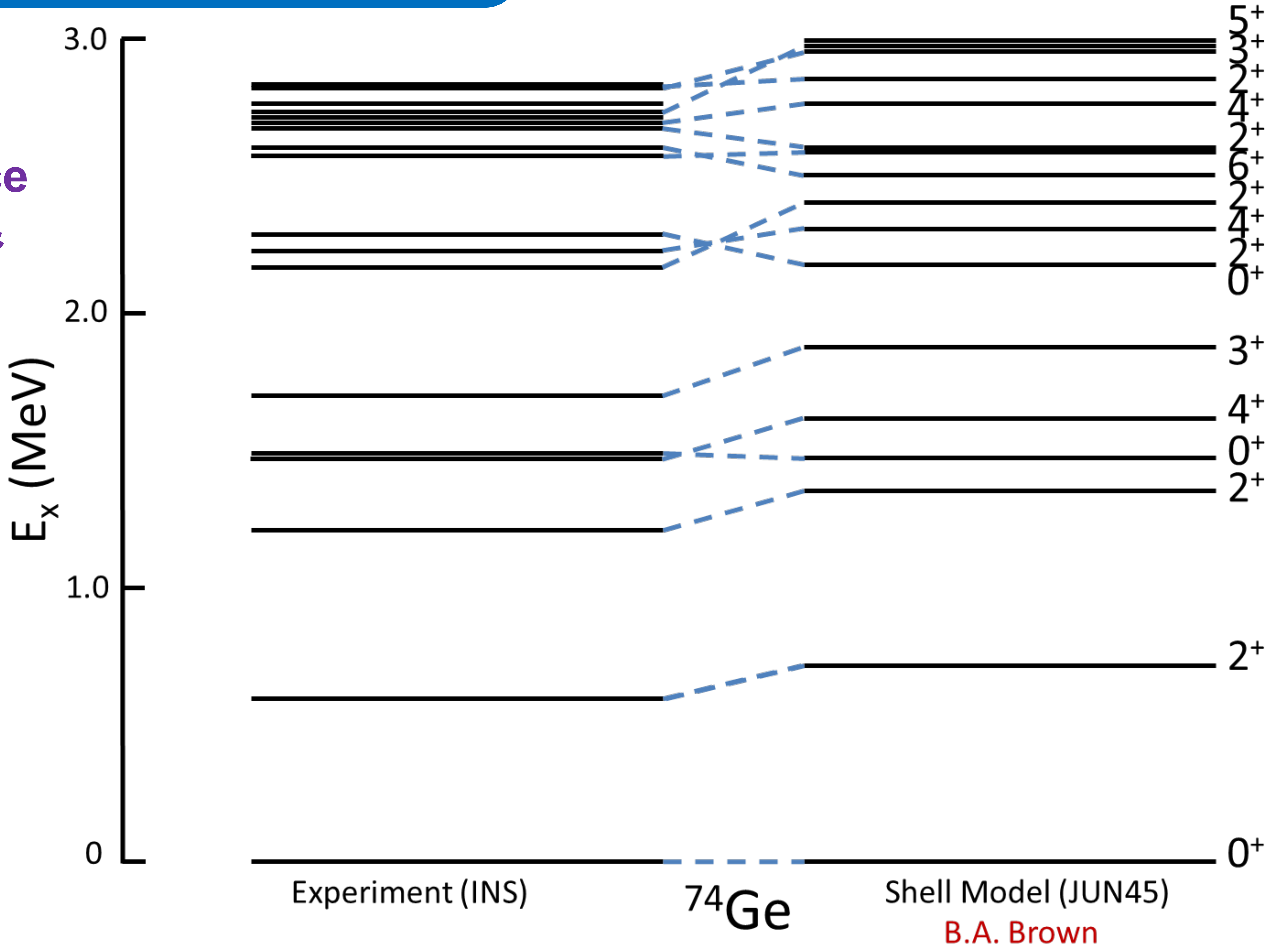
^{74}Ge(n,n`γ)



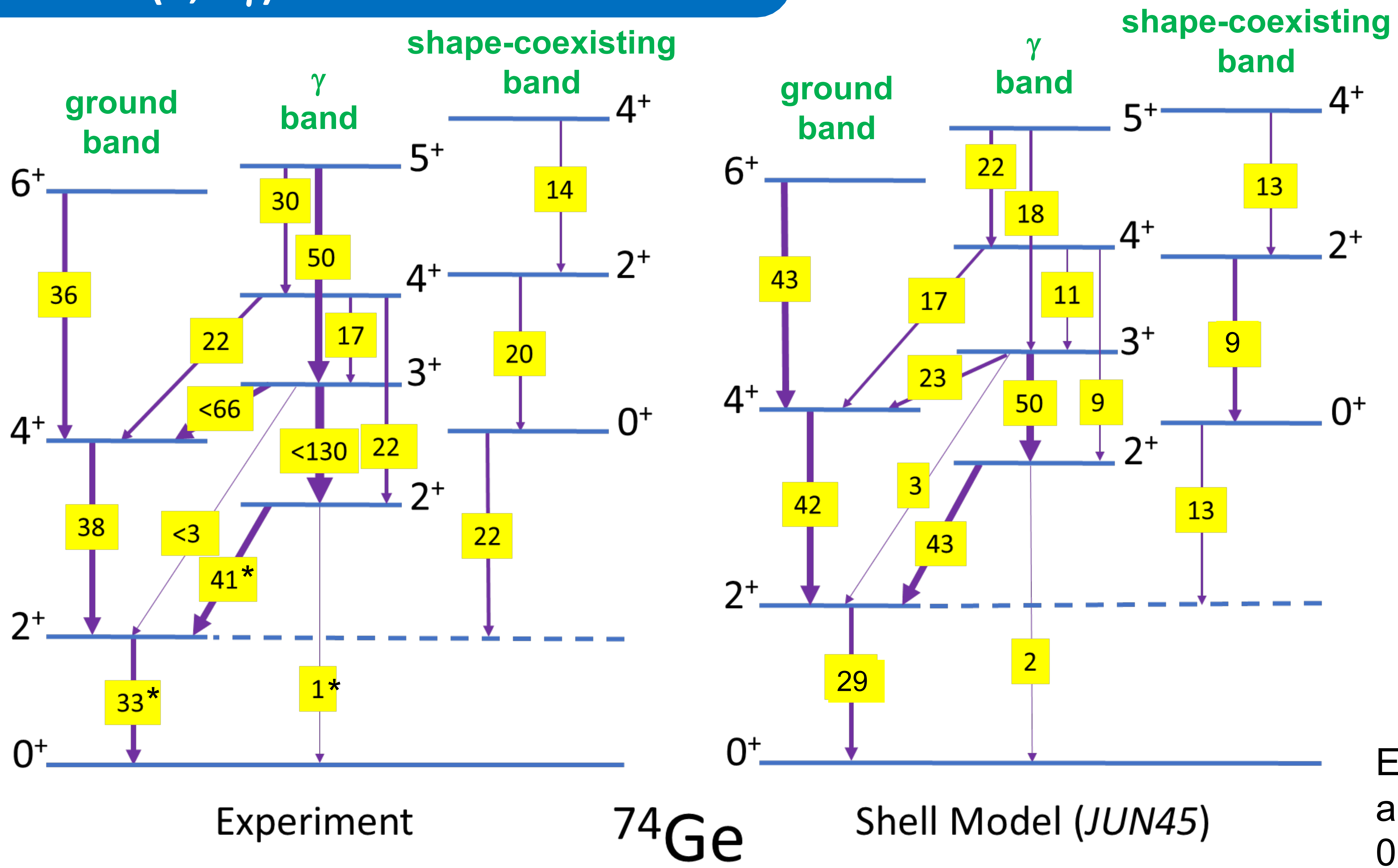
3.0
2.0
1.0
0
E_x (MeV)
Experiment (INS)
^{74}Ge
Shell Model (JUN45)
B.A. Brown

^{74}Ge(n,n`γ) and Shell Model

1-to-1 correspondence of theoretical & experimental levels up to ~2.8 MeV



^{74}Ge(n,n`γ) and Shell Model



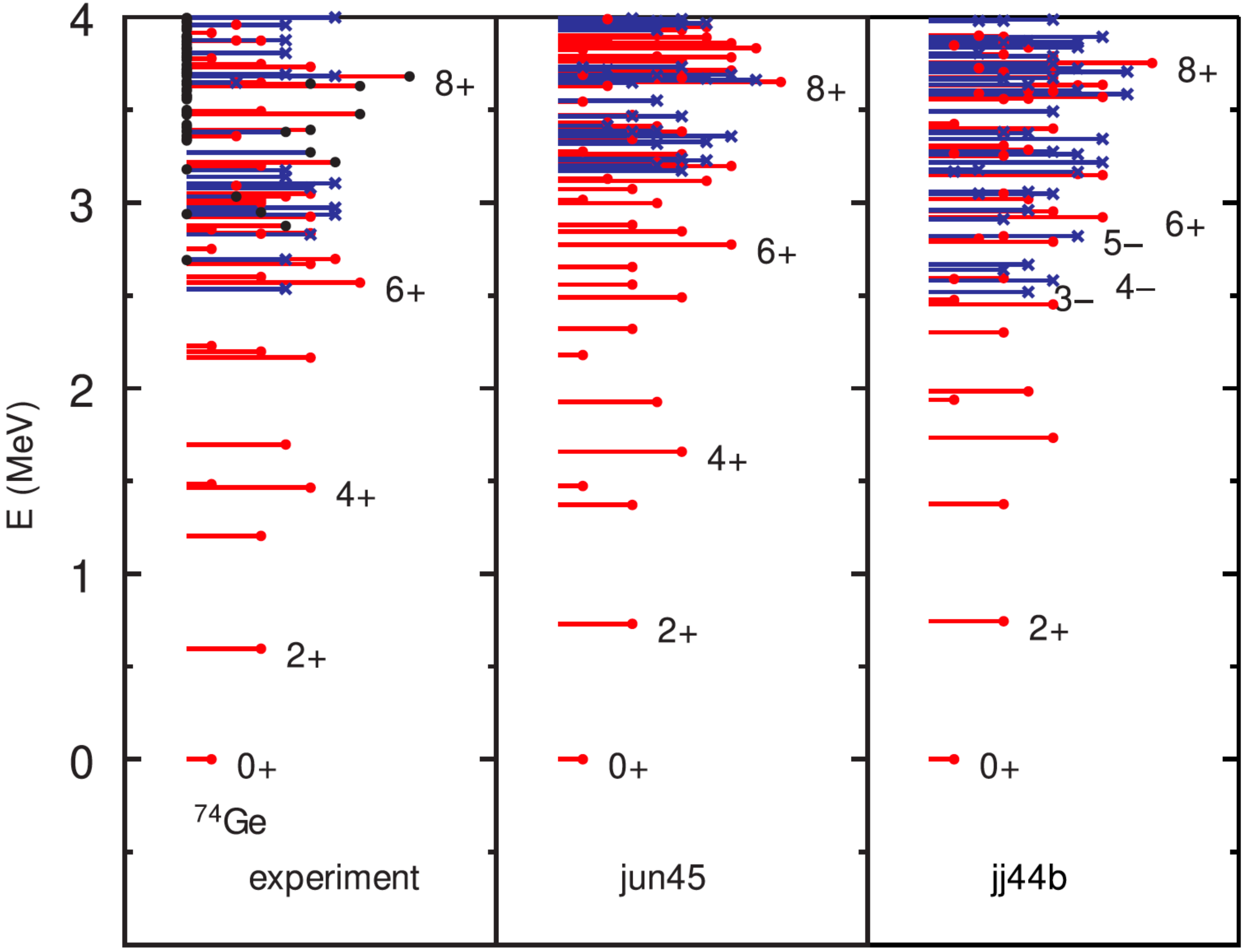
Calculations by Alex Brown with no input from our experiments

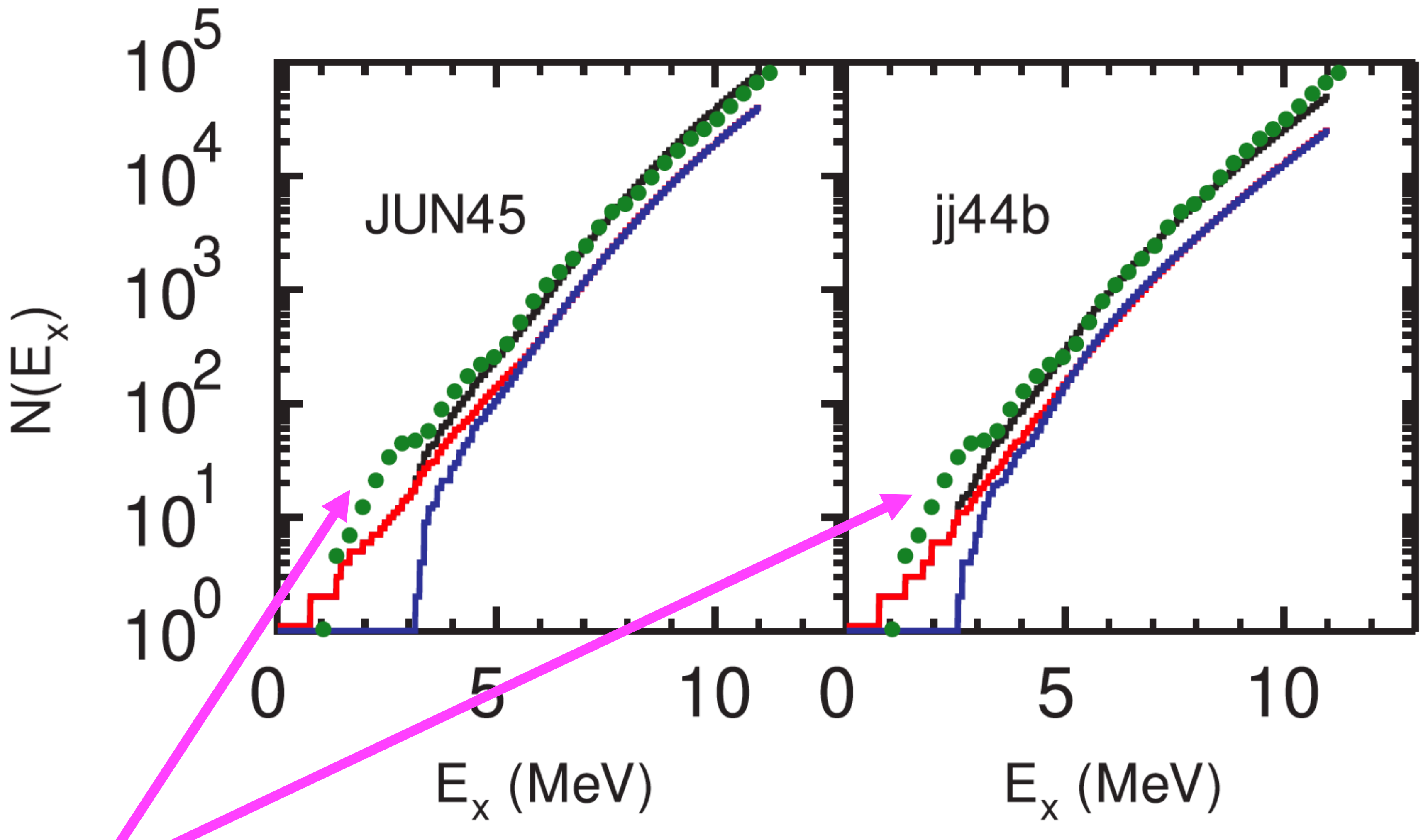
Excellent agreement!

The only ^{74}Ge data used for the SVD fit were the ground-state binding energy and the excitation energies of the lowest two 2^+ states.

EEP, B.A. Brown, et al., Phys. Rev. C 109, 054318 (2024).

^{74}Ge(n,n`γ) Shell Model





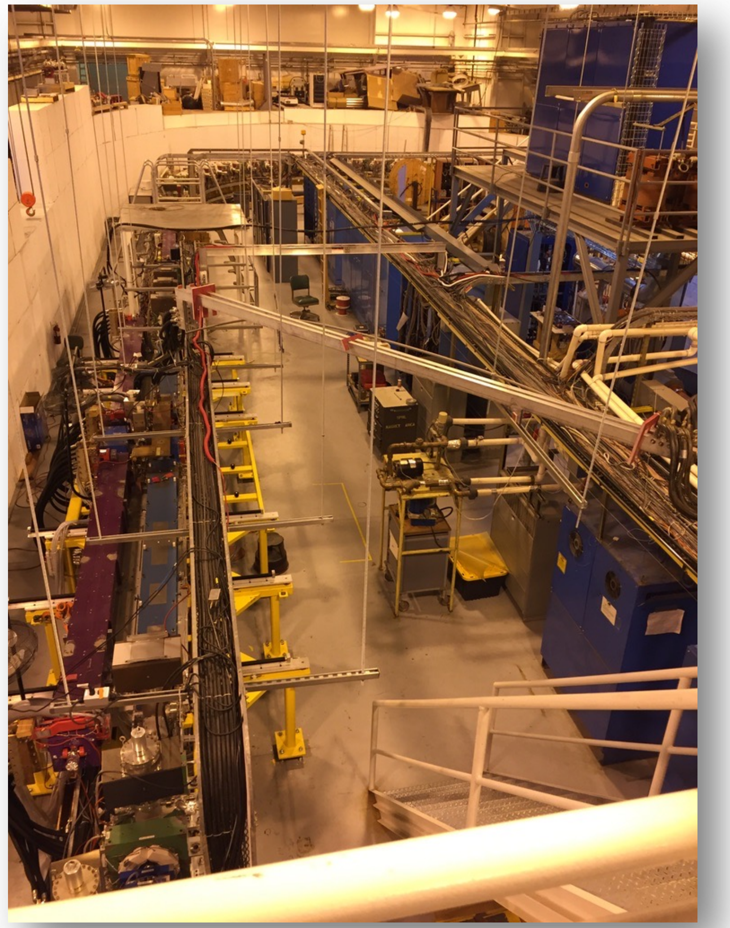
Erroneous states in ENSDF were included.

red = positive-parity states
blue = negative-parity states
black lines = total for both parities
greens points = experimental data
A. V. Voinov, et al.
Phys. Rev. C 99, 054609 (2019).

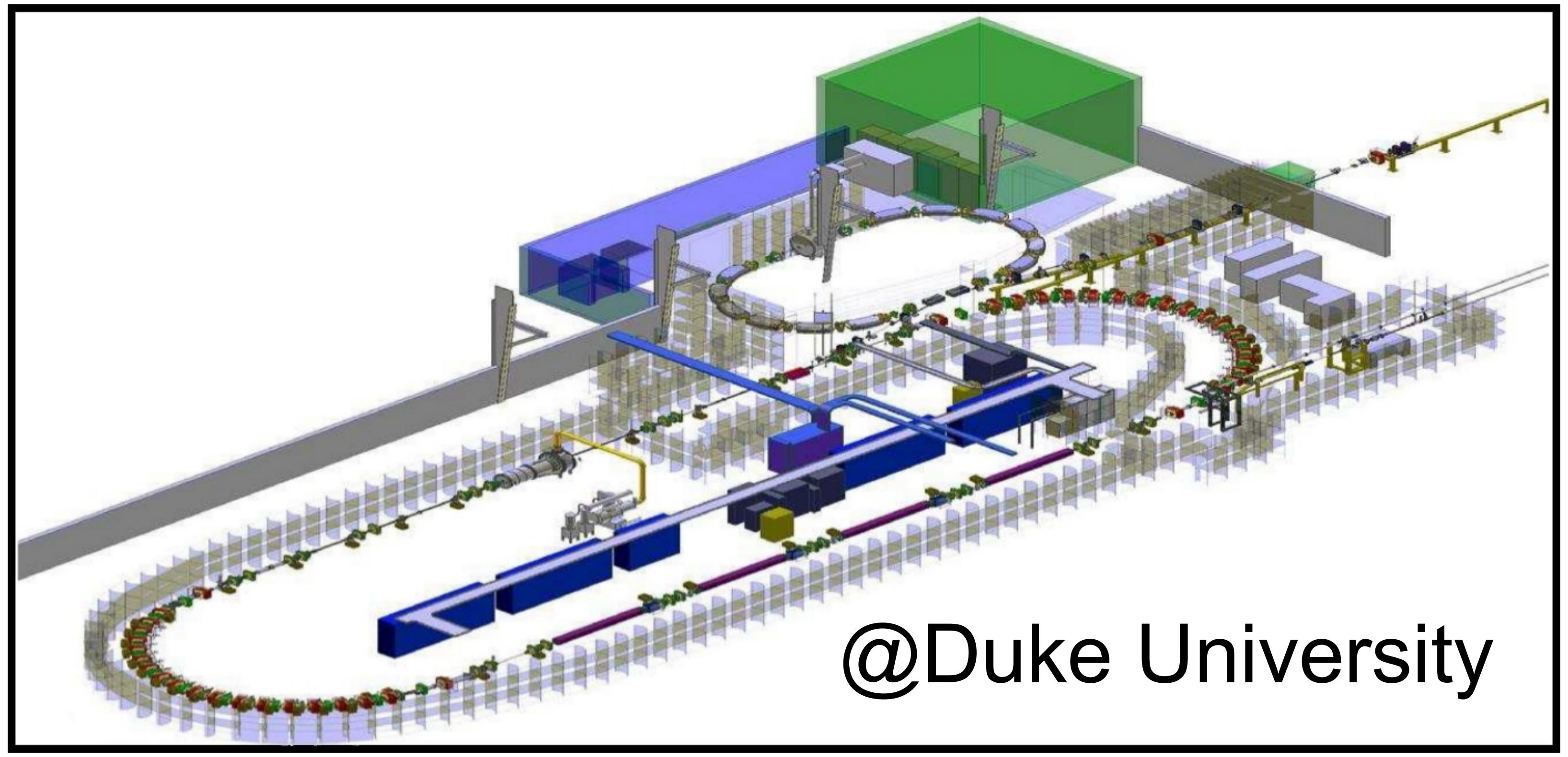
EEP, B.A. Brown, et al., Phys. Rev. C 109, 054318 (2024).

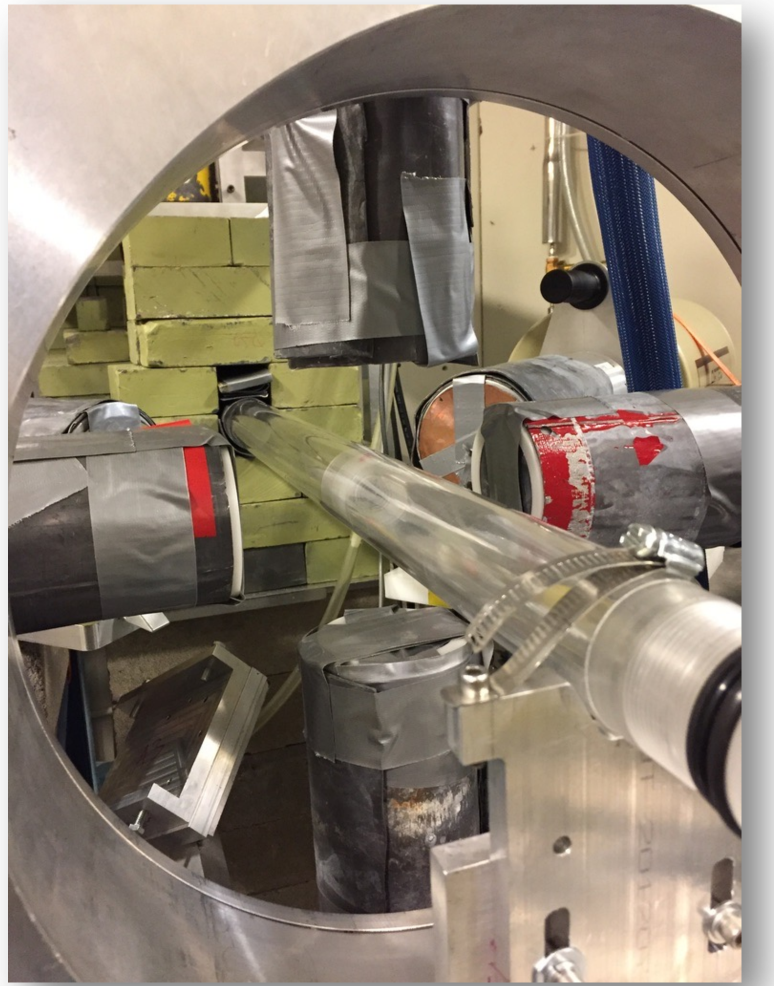
Level densities also in great agreement

^{74}Ge Spin-1 States NRF @ HIγS

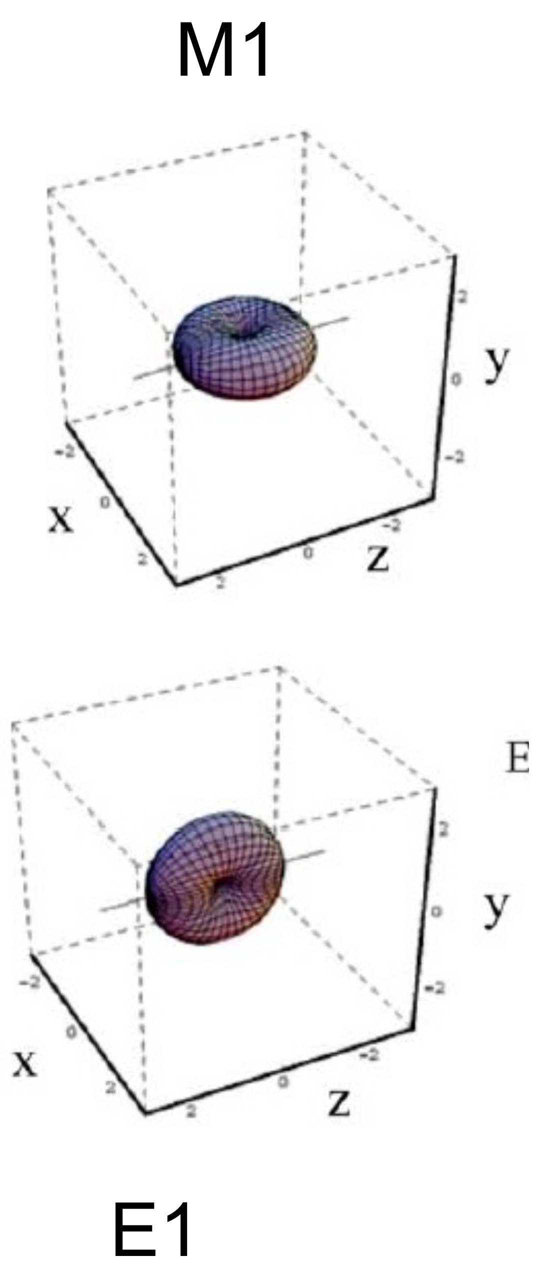


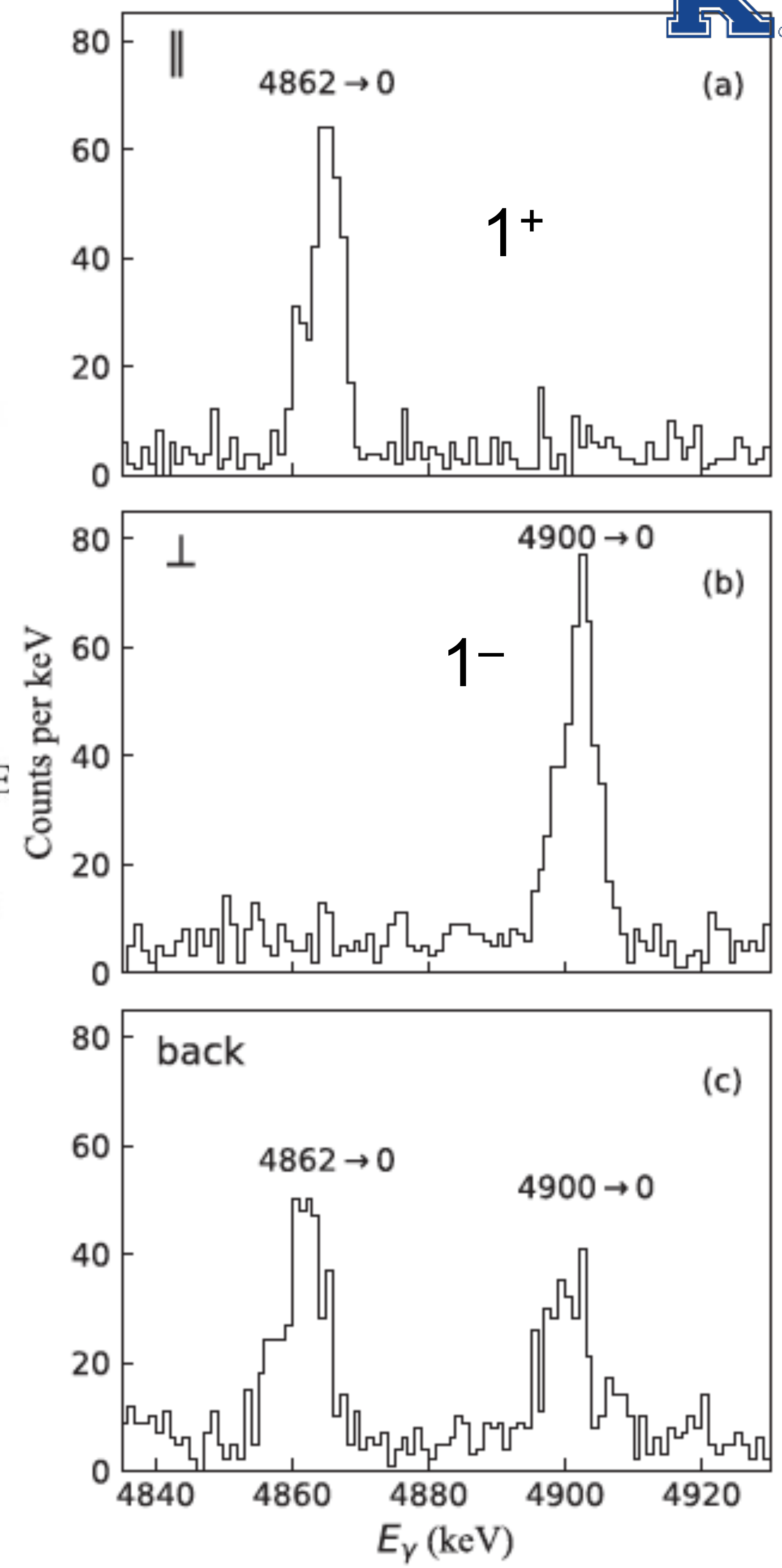
High-Intensity Gamma Source





Polarization plane

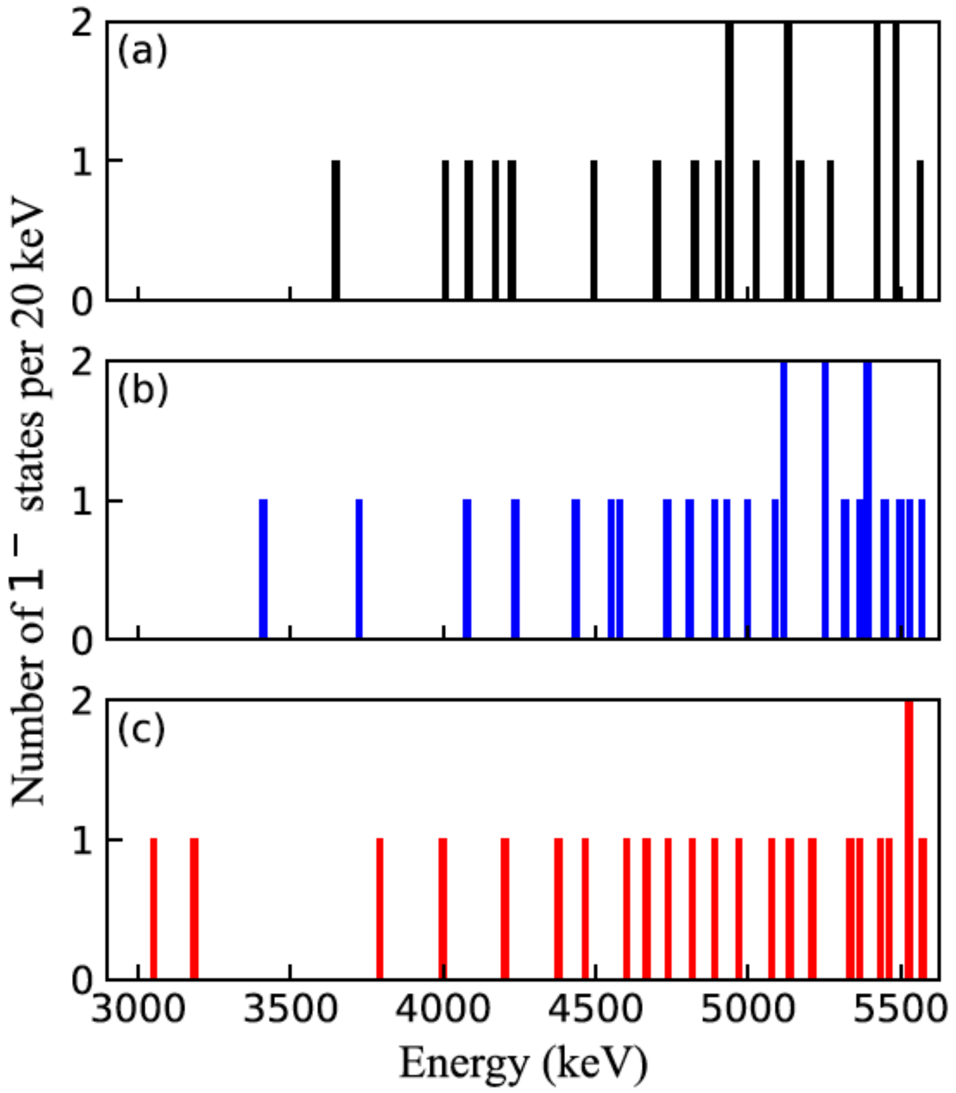




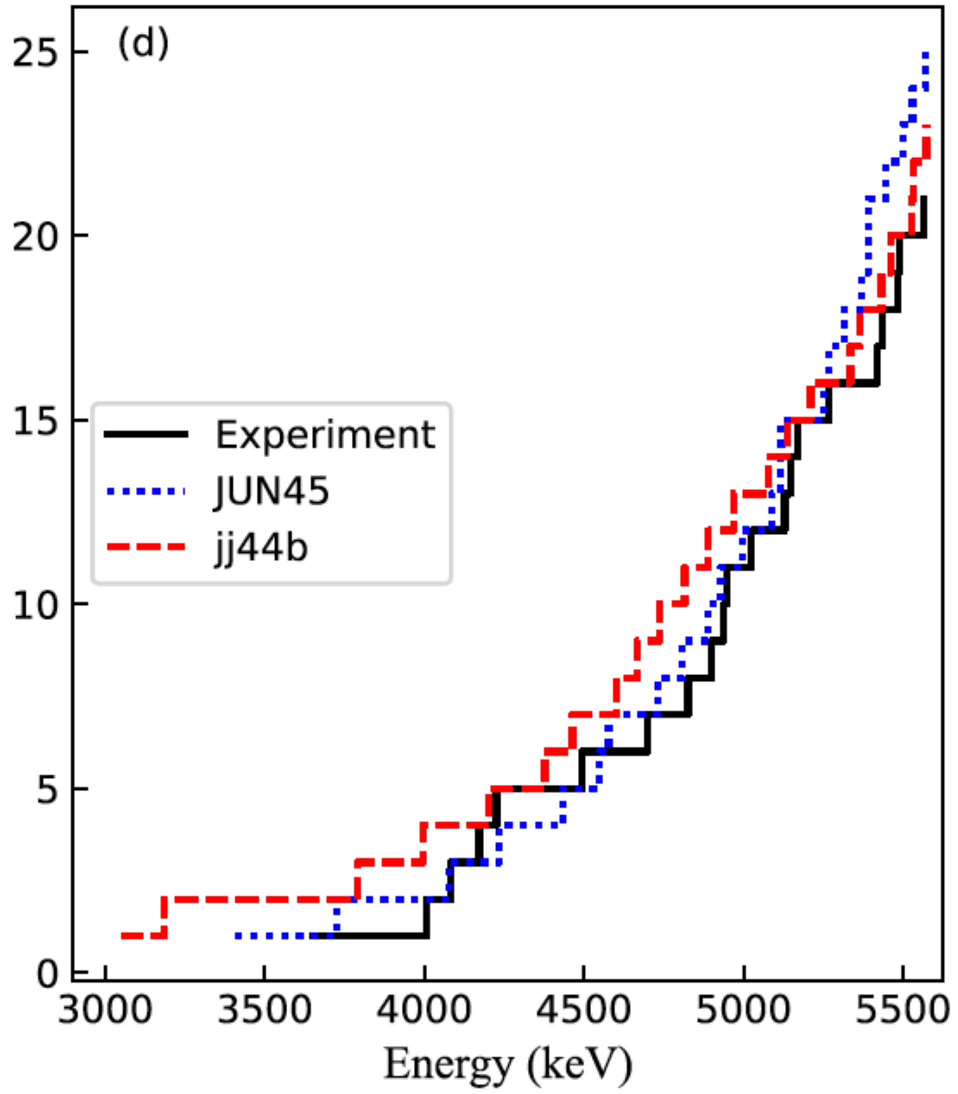
Polarized photon scattering allows the determination of parities of spin-1 states unambiguously

^{74}Ge Spin-1 States NRF & SM

Distribution of 1^- states well reproduced by both interactions.
Cross sections for and 1^+ states slightly better agreement for JUN45.

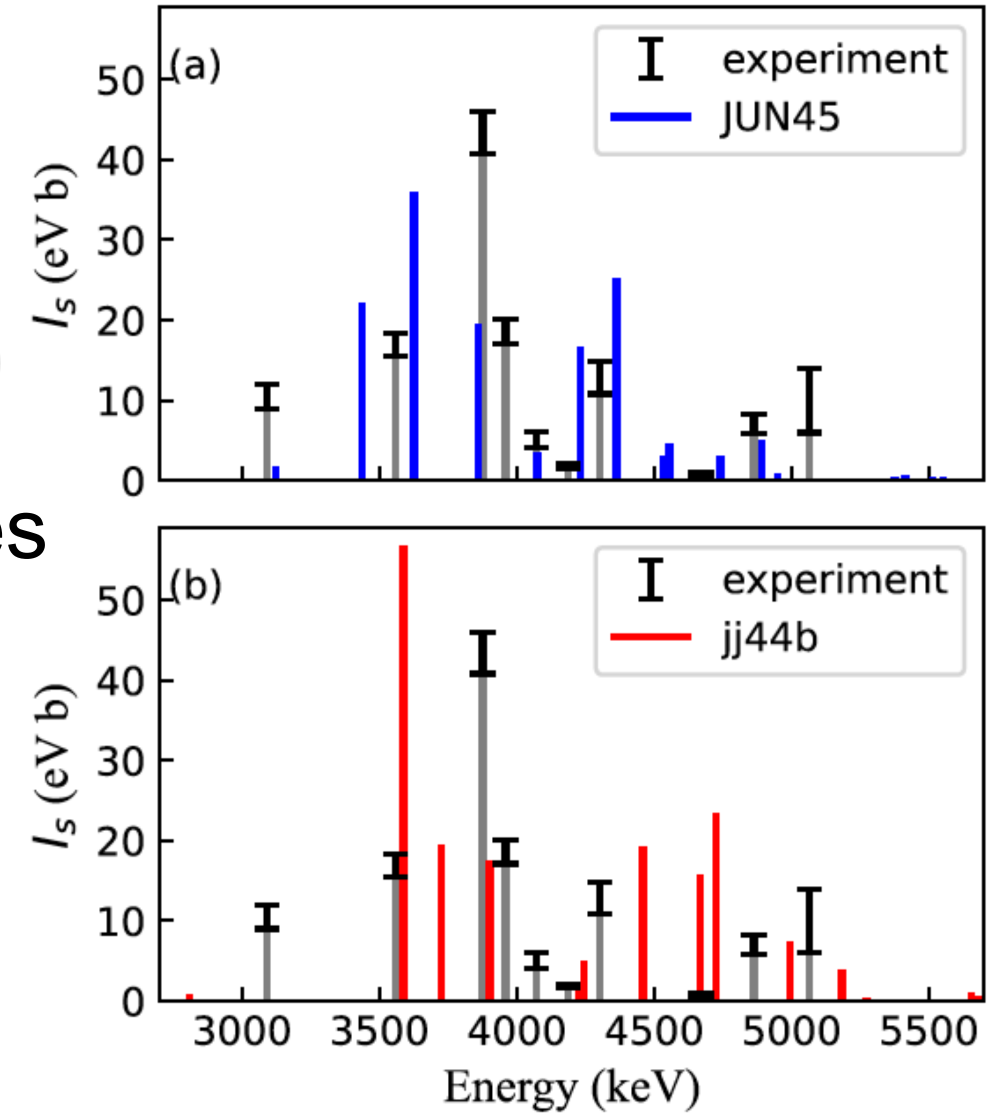


Distribution of 1^- states

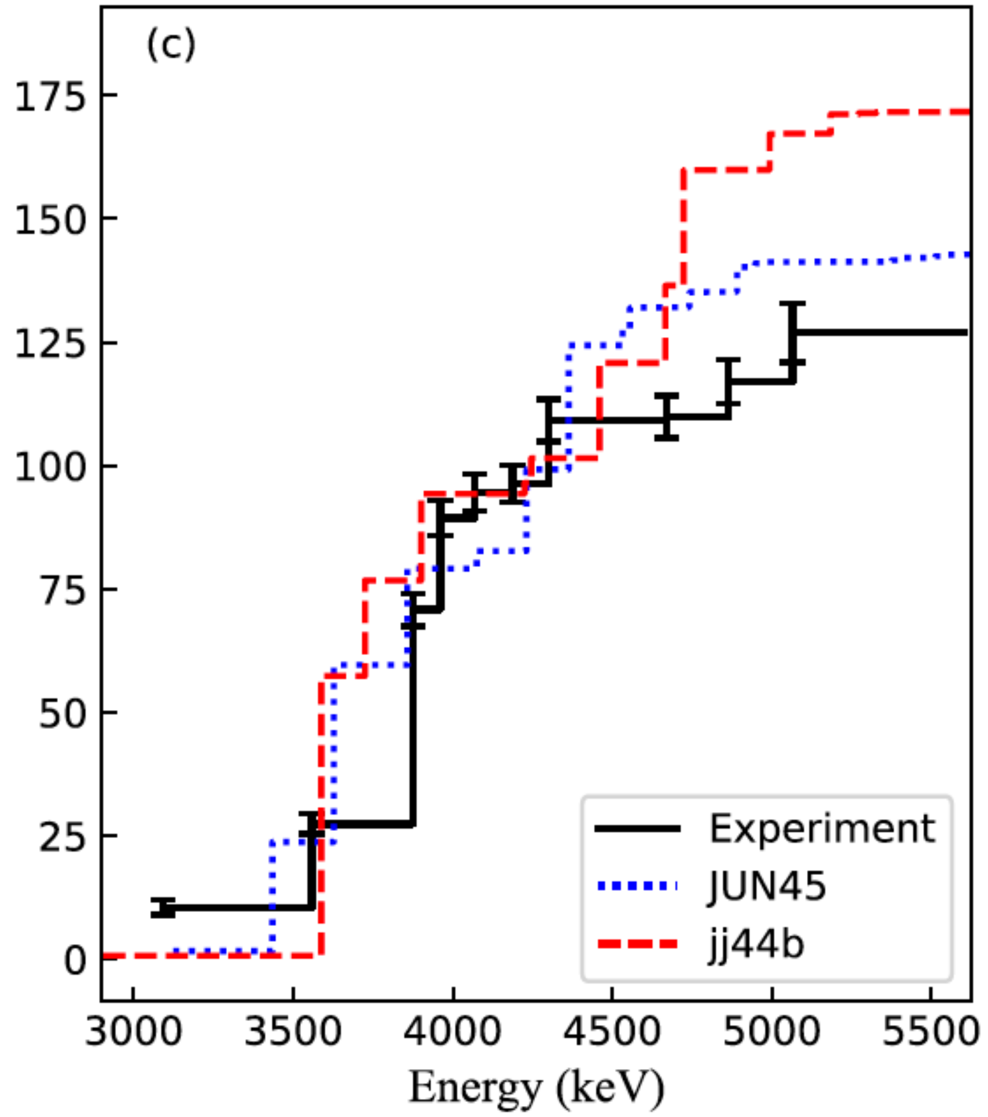


Running sum of 1^- states

Cross sections of 1^+ states



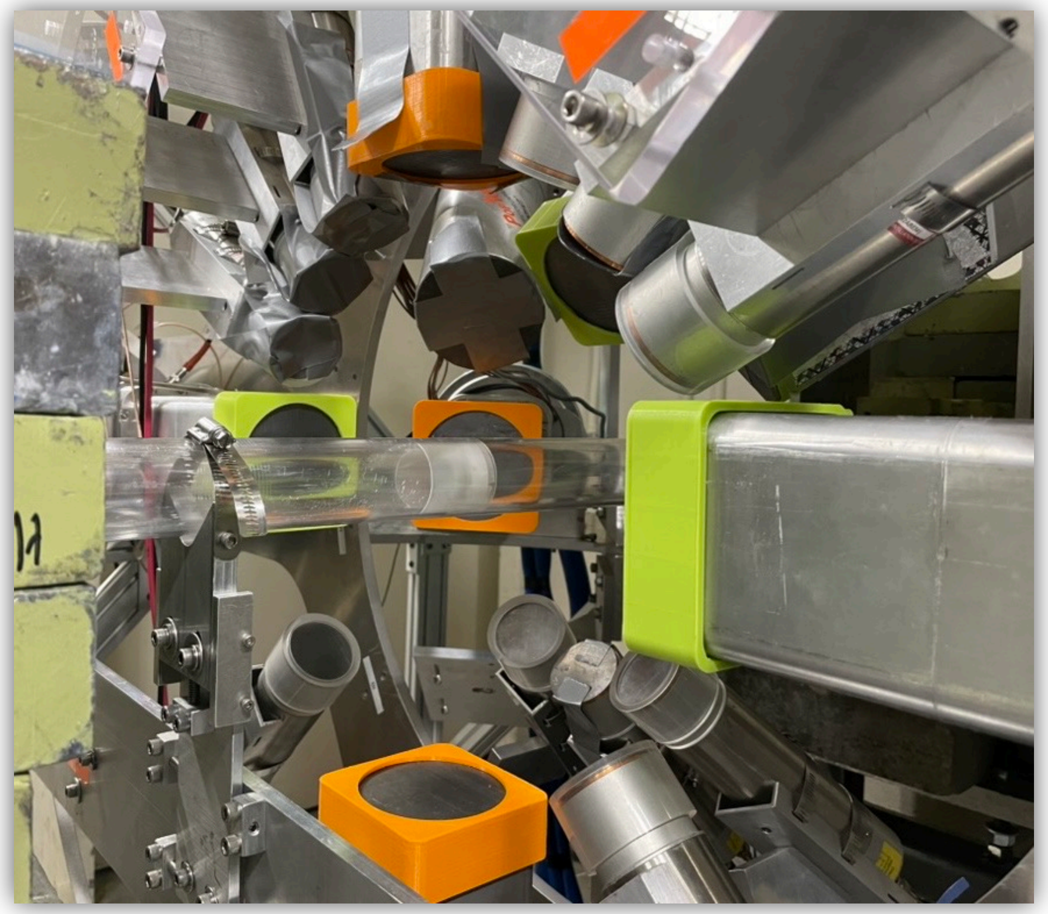
Running sum of cross sections of 1^+ states



S. R. Johnson , R. V. F. Janssens, U. Friman-Gayer, B. A. Brown, et al., PRC 108, 024315 (2023).

^{72}Ge: UKAL, HIγS, & SM Ongoing

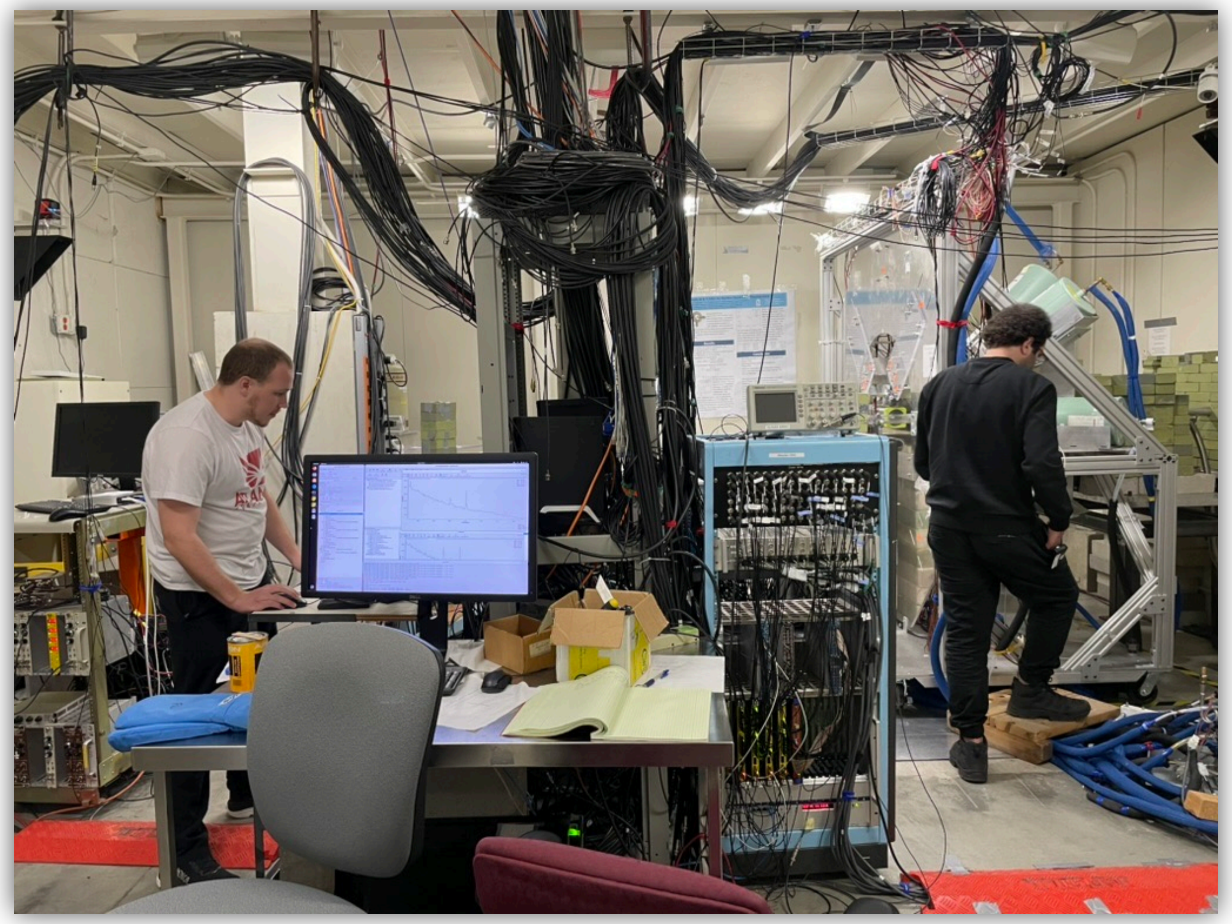
UKAL – INS experiments complete



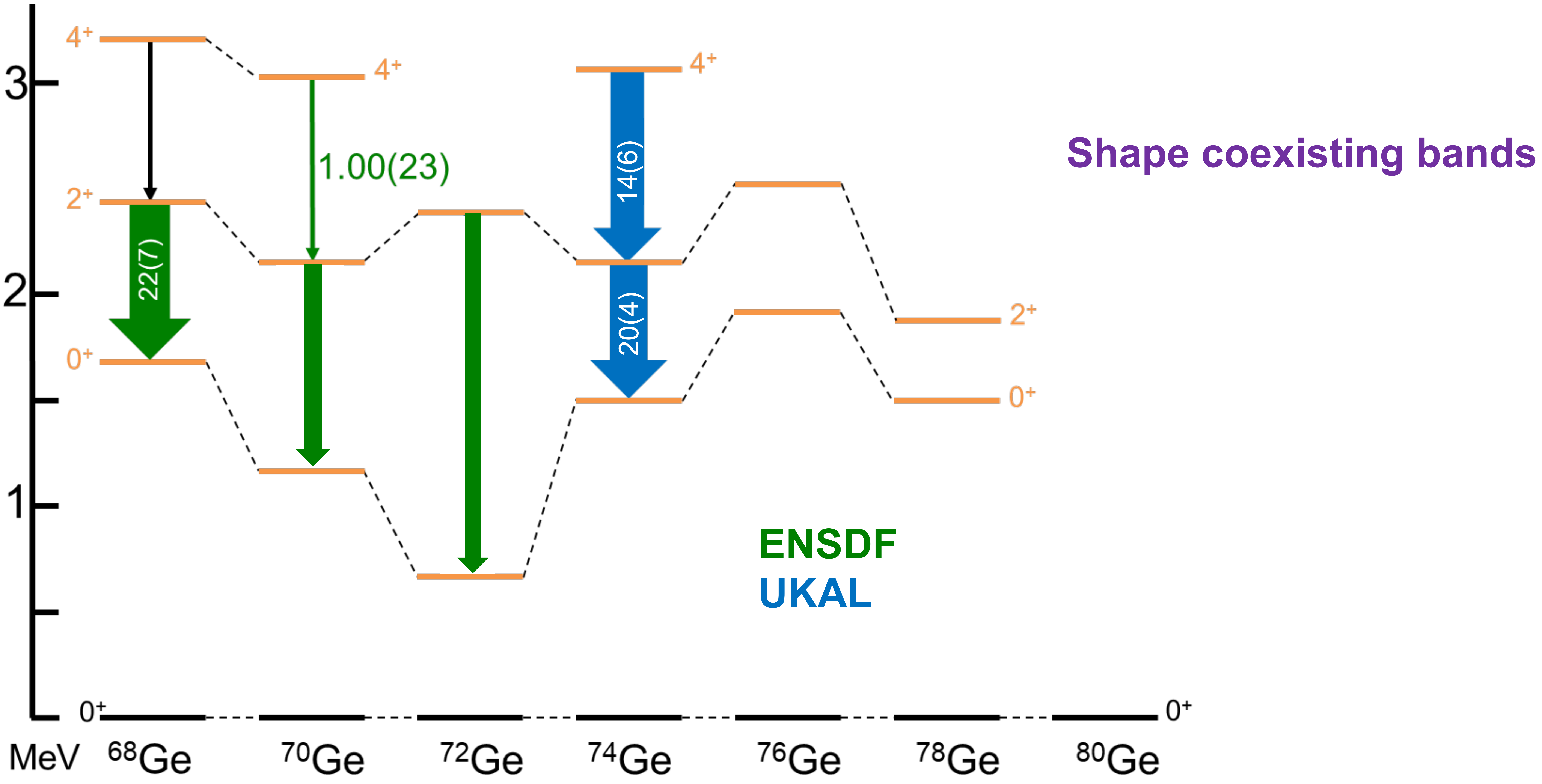
HIγS
February – March 2024
UNC: R.V.F. Janssens, A.D. Ayangeakaa
Miss. St.: B.P. Crider, J.R. Vanhoy
CEA Saclay: W. Korten
UK: S.F. Hicks, E.E. Peters

UK under-graduates
L.D. Martin
B.H. Tomas Lopez

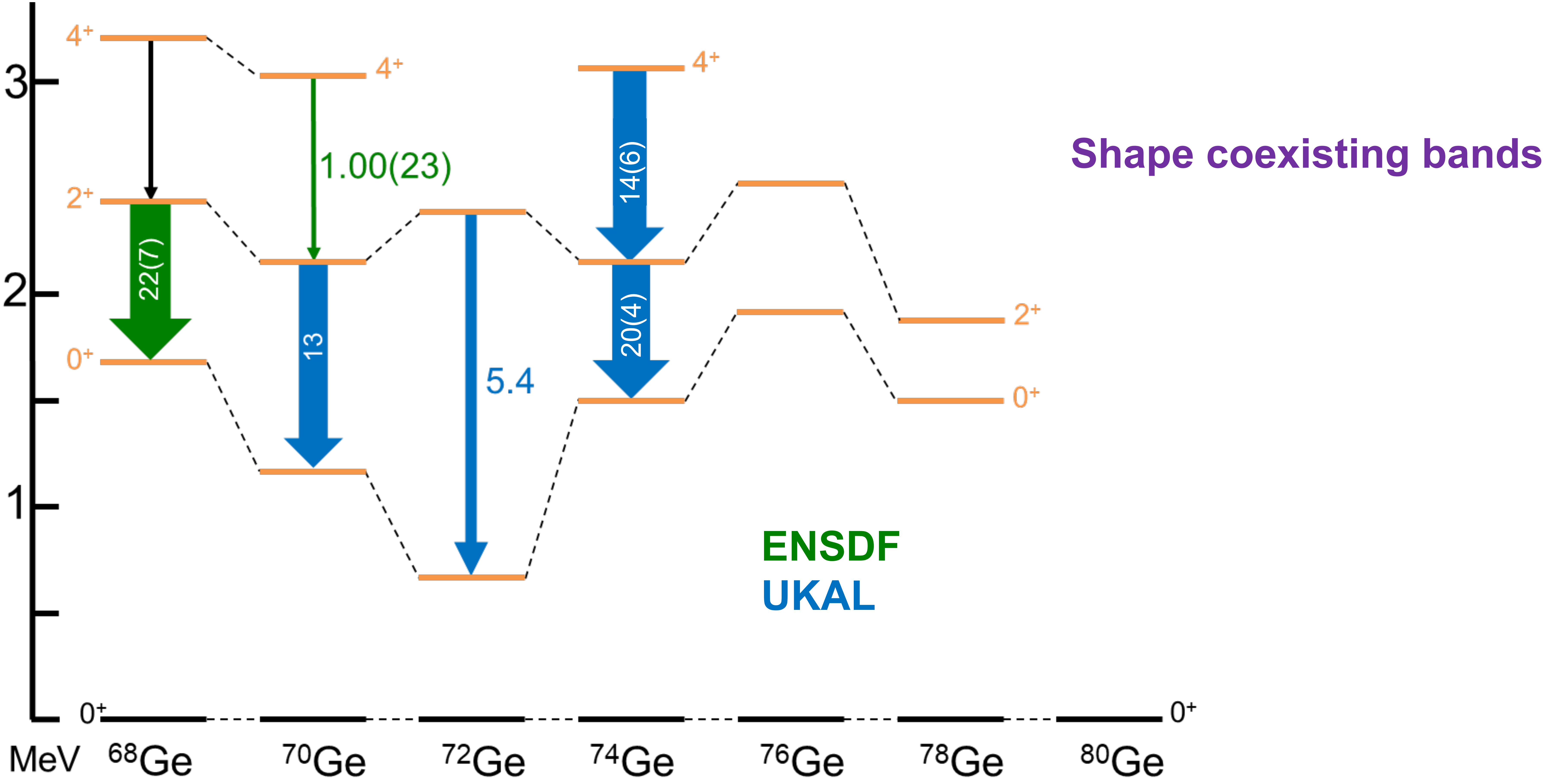
Initial JUN45 calculations complete



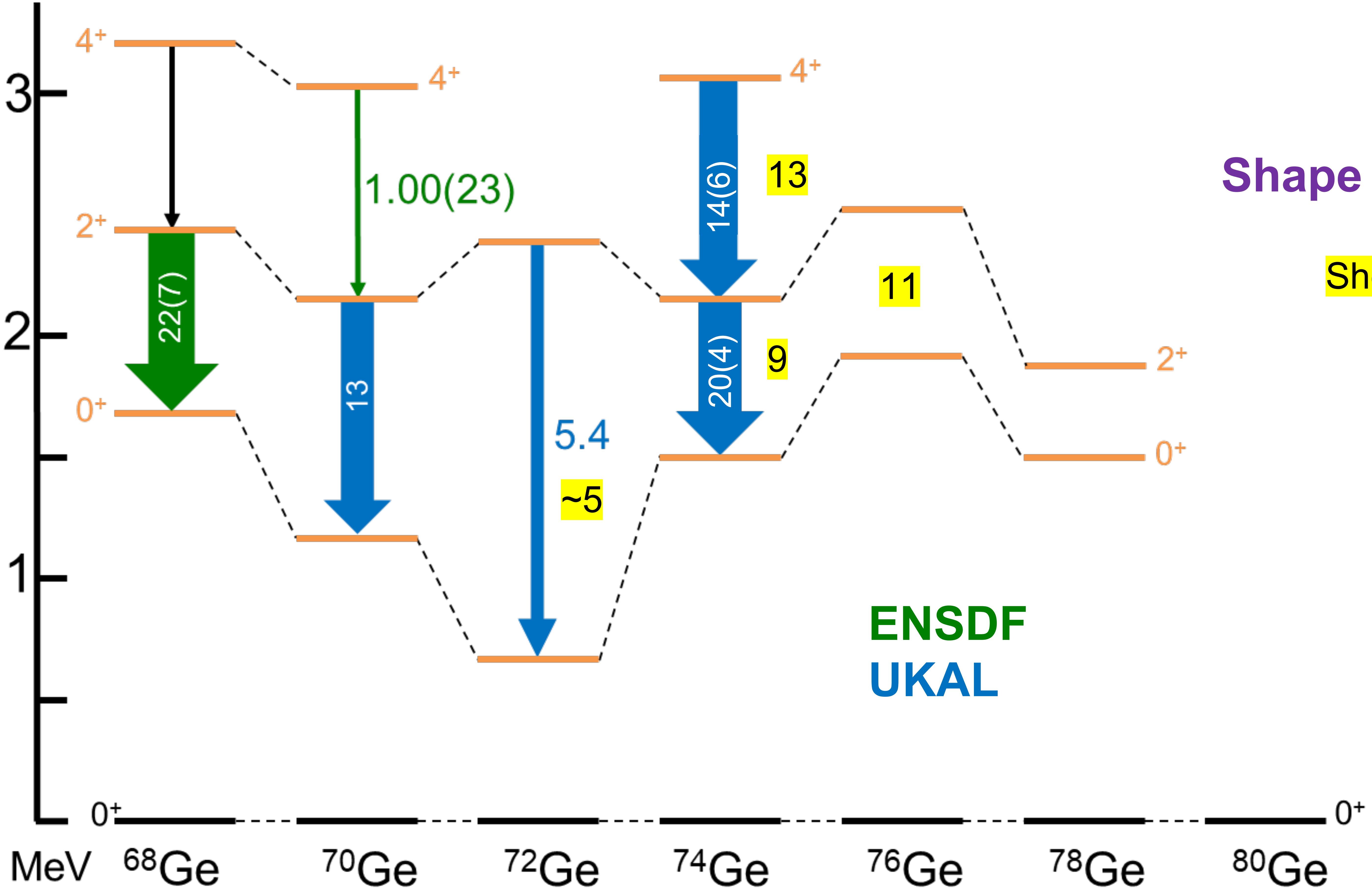
Shape Coexistence in the Ge Isotopes



Shape Coexistence in the Ge Isotopes



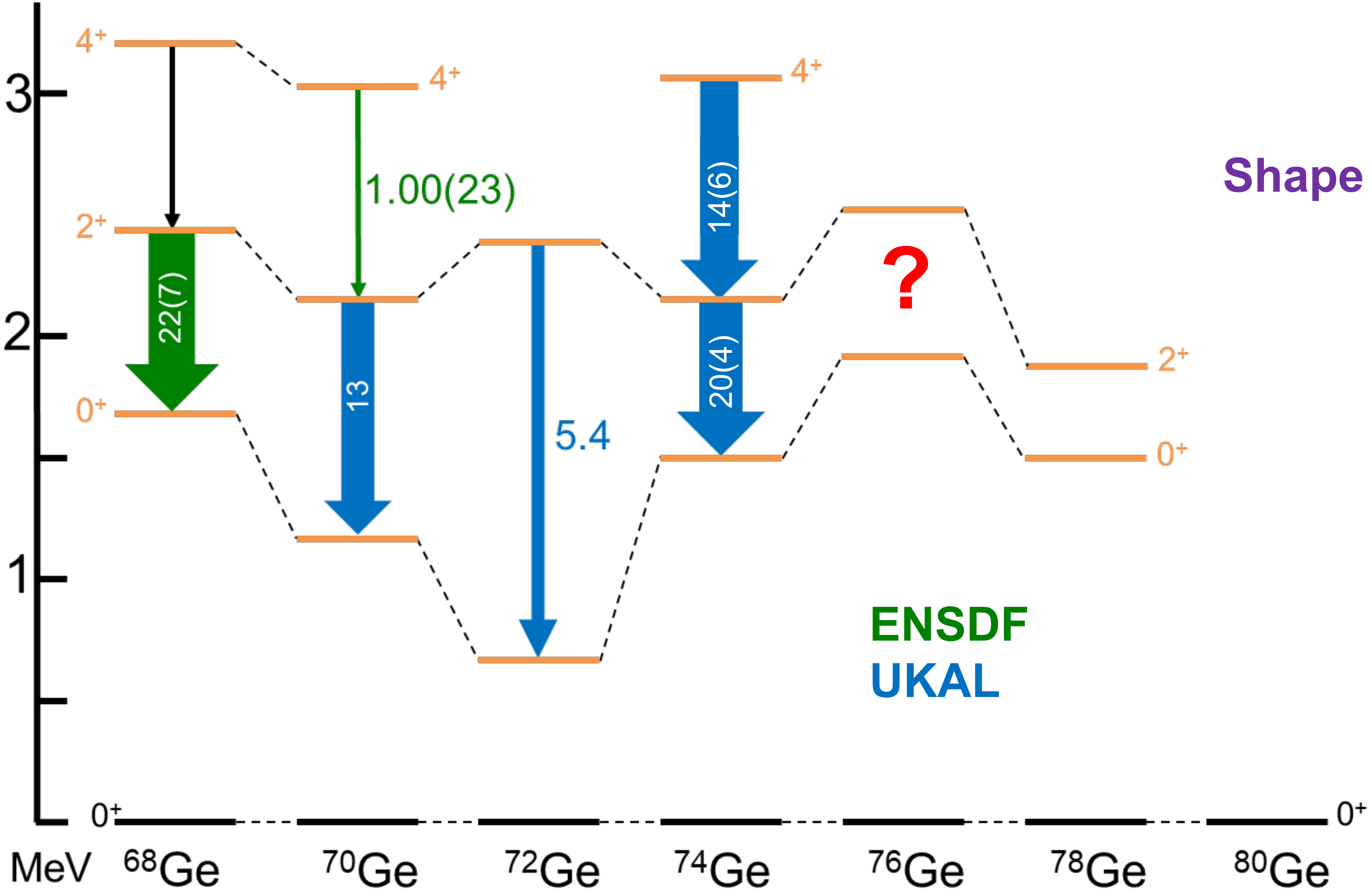
Shape Coexistence in the Ge Isotopes



Shape coexisting bands

Shell-model predictions

Shape Coexistence in the Ge Isotopes

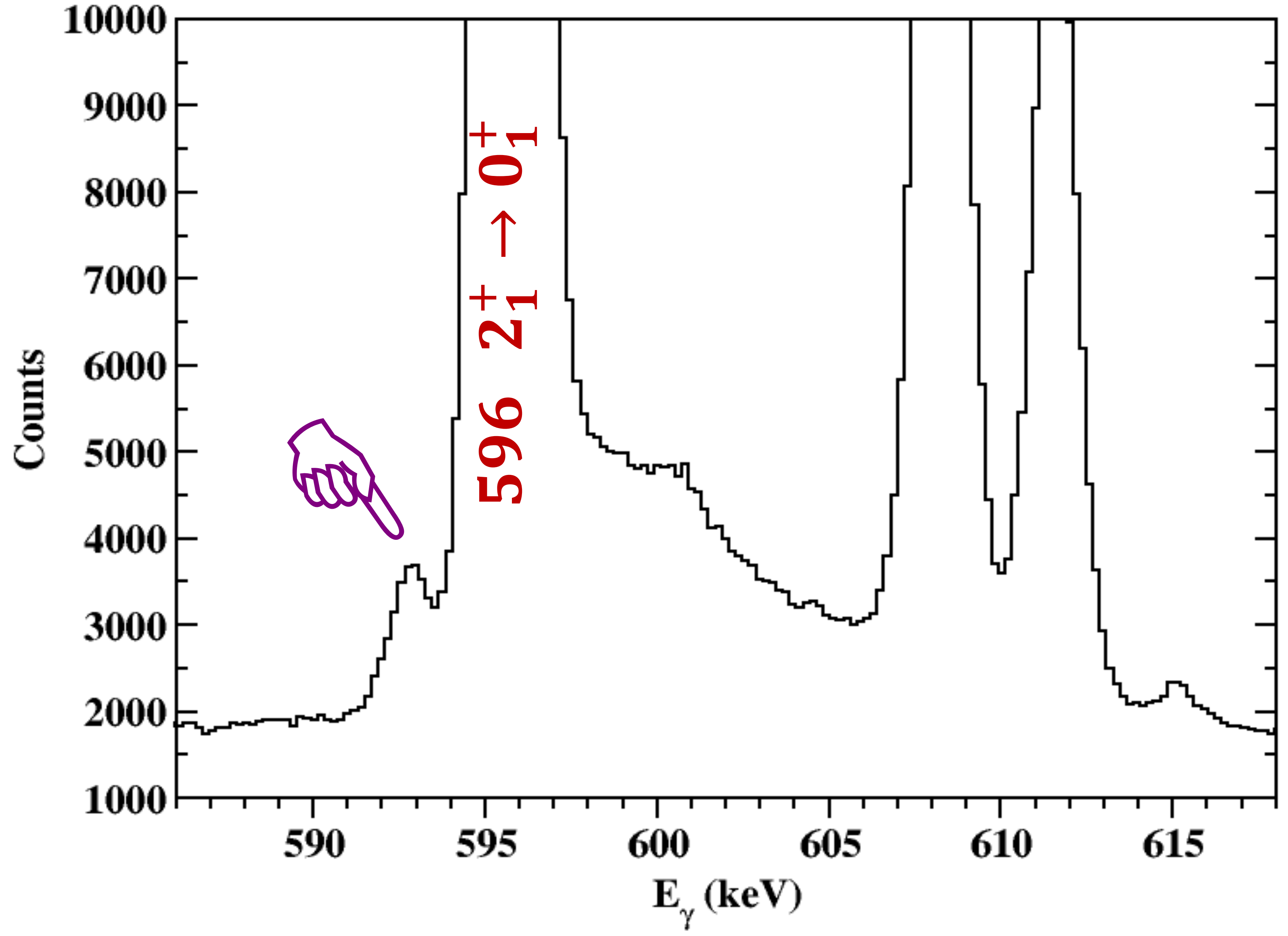


Shape coexisting bands

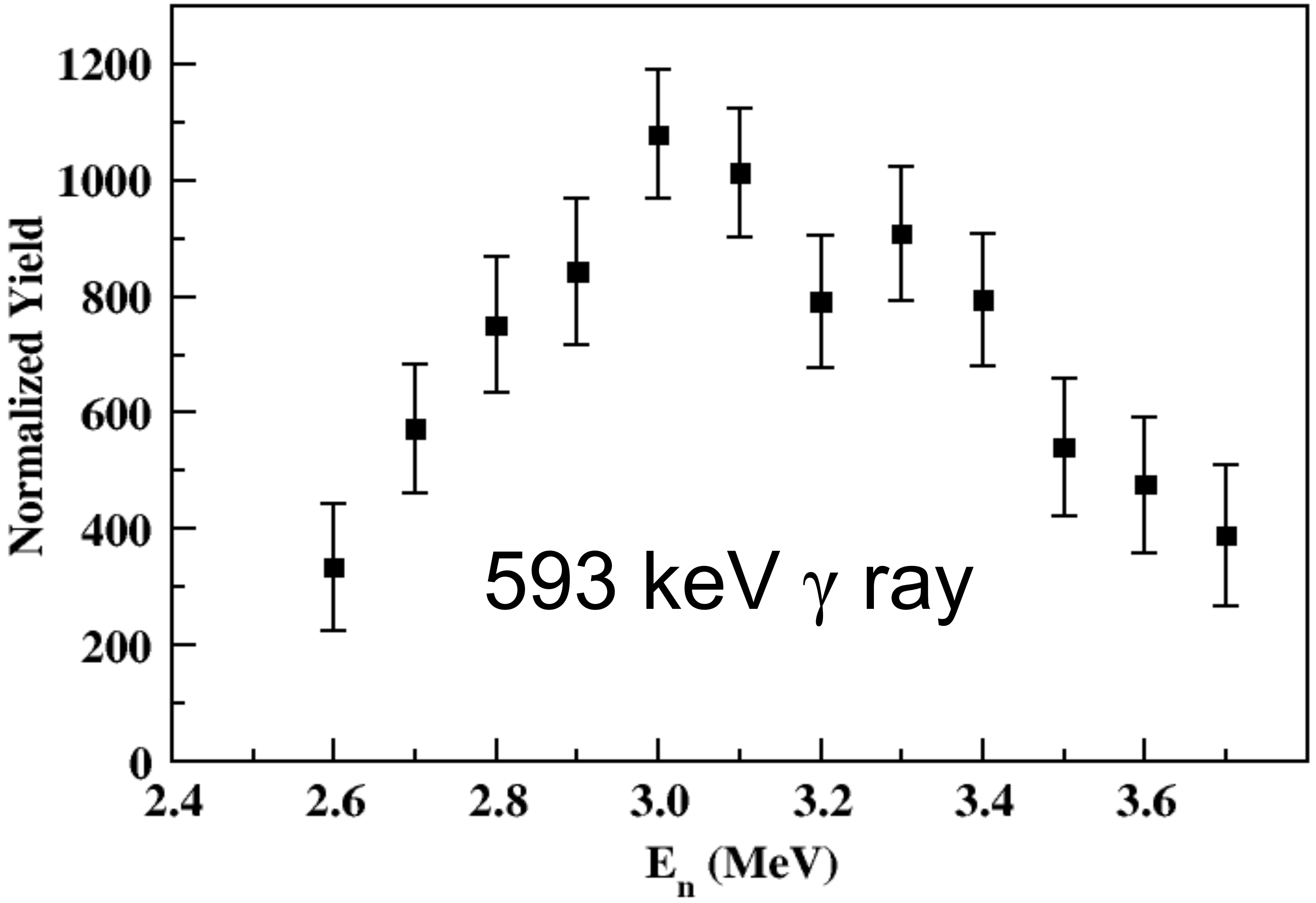
ENSDF

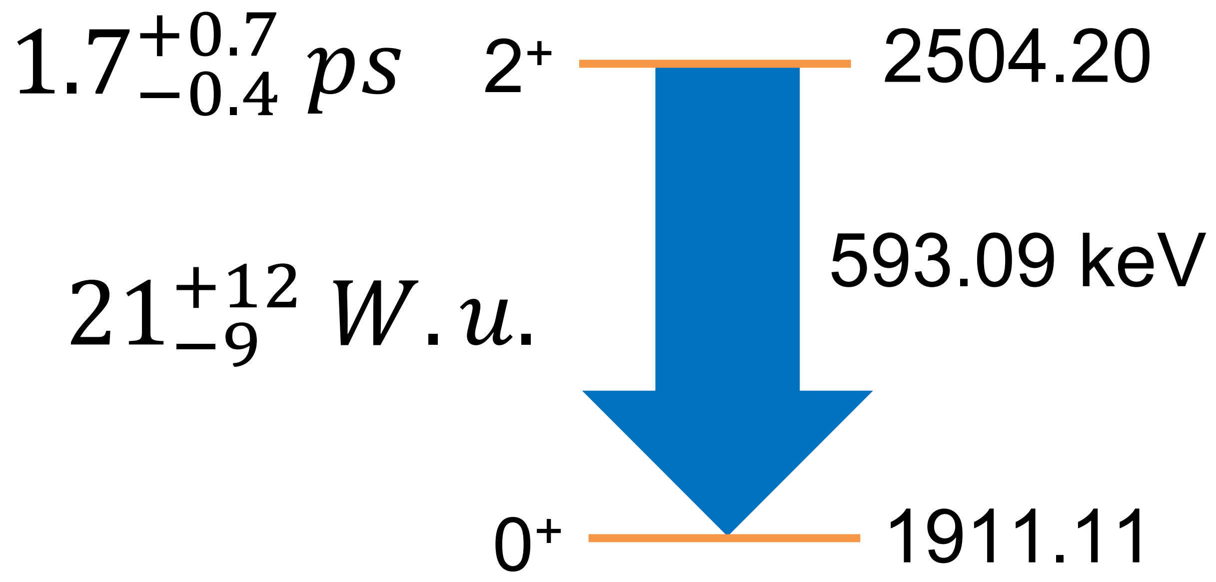
UKAL

$^{76}Ge(n,n`\gamma)$

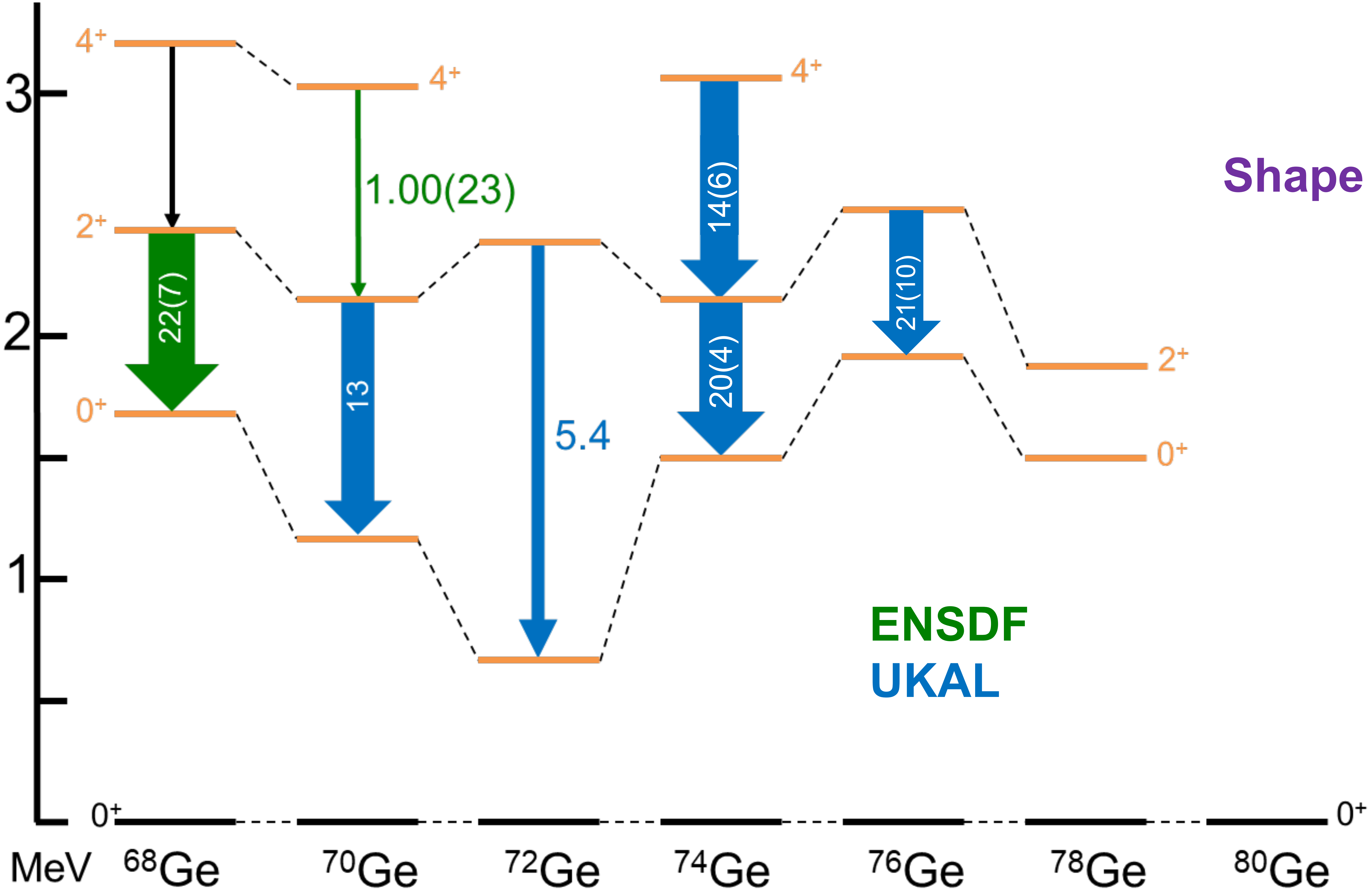


Sum of all angles
at E_n = 3 MeV

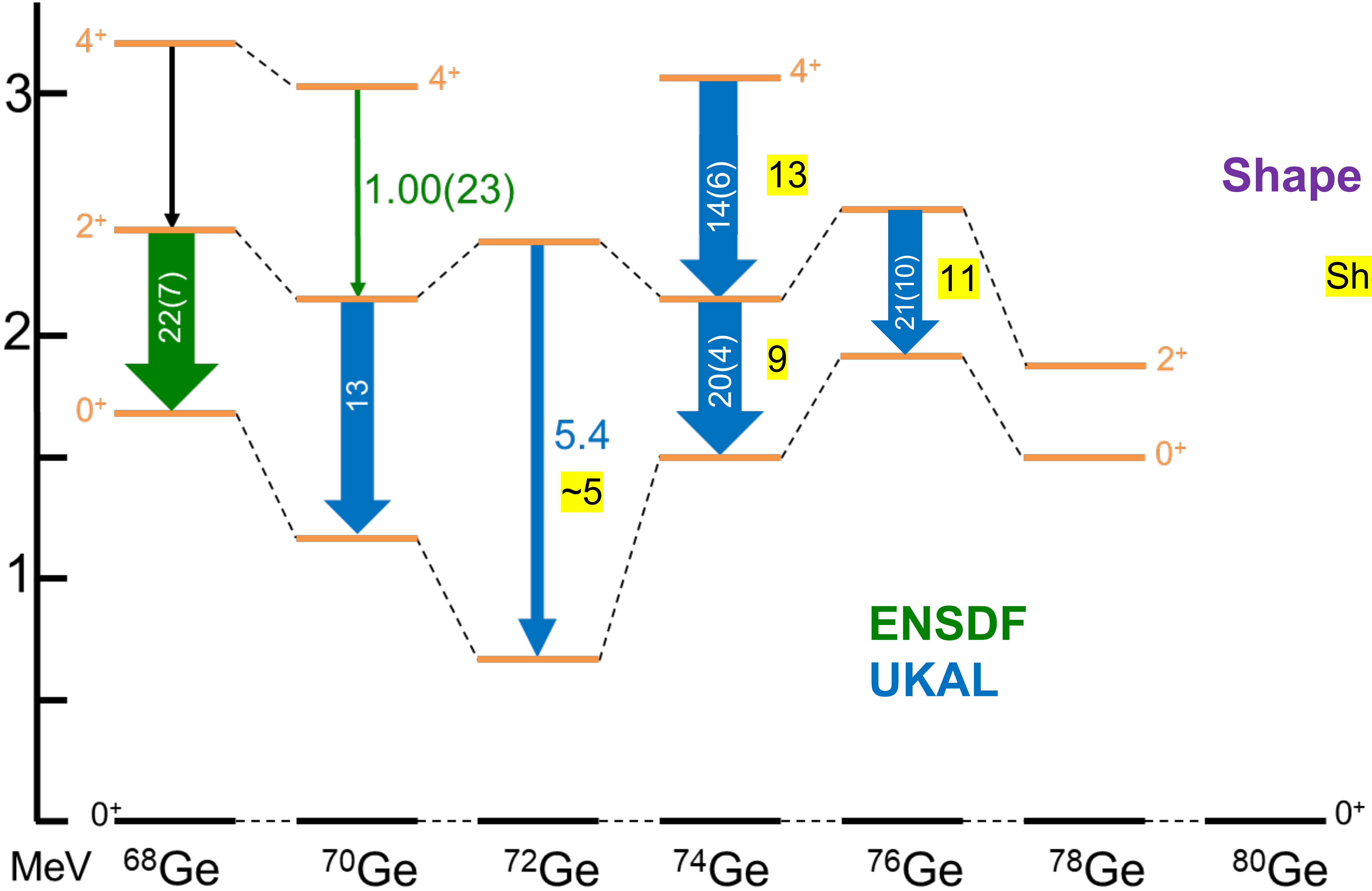




Shape Coexistence in the Ge Isotopes



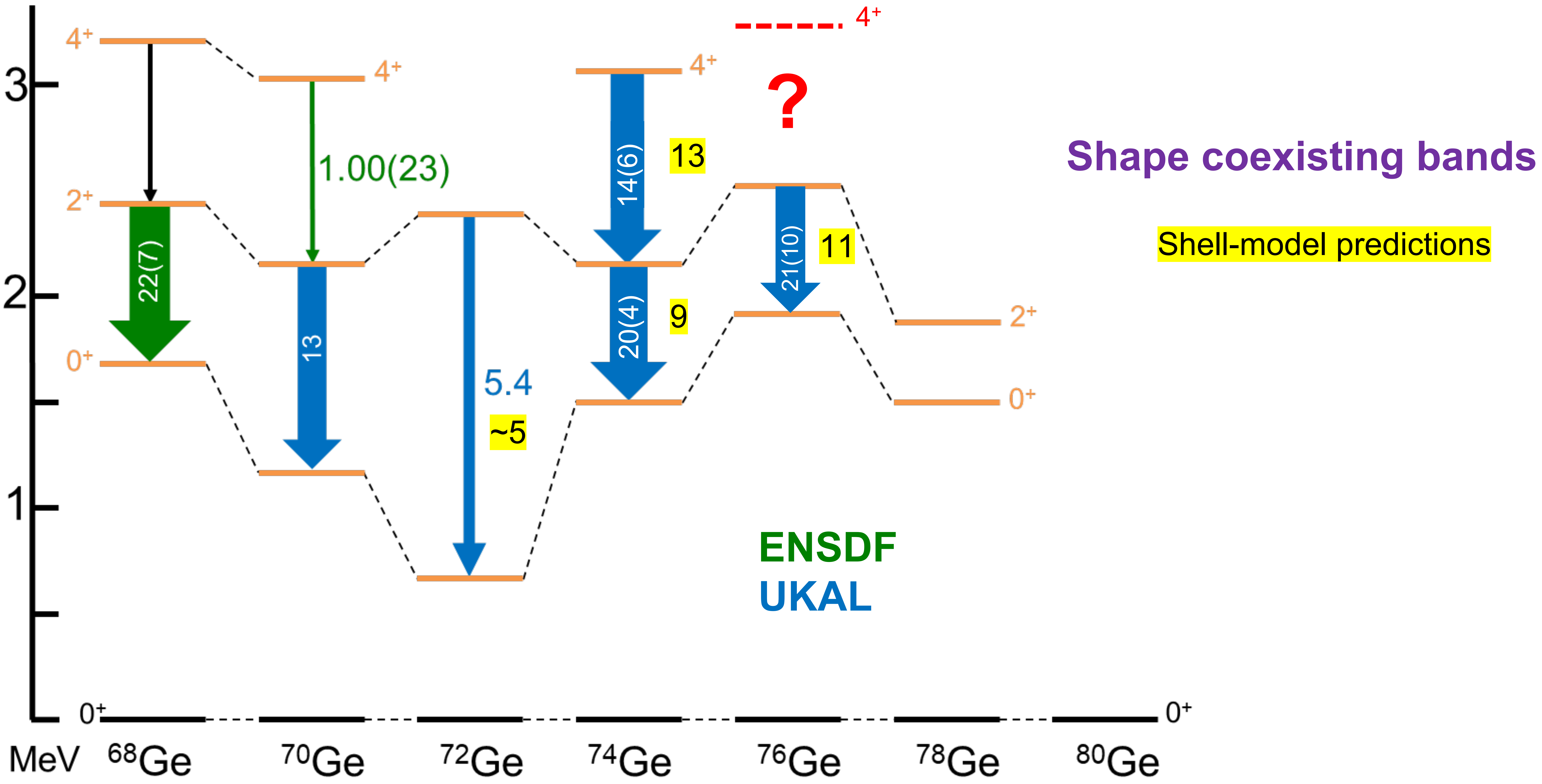
Shape Coexistence in the Ge Isotopes



Shape coexisting bands

Shell-model predictions

Shape Coexistence in the Ge Isotopes



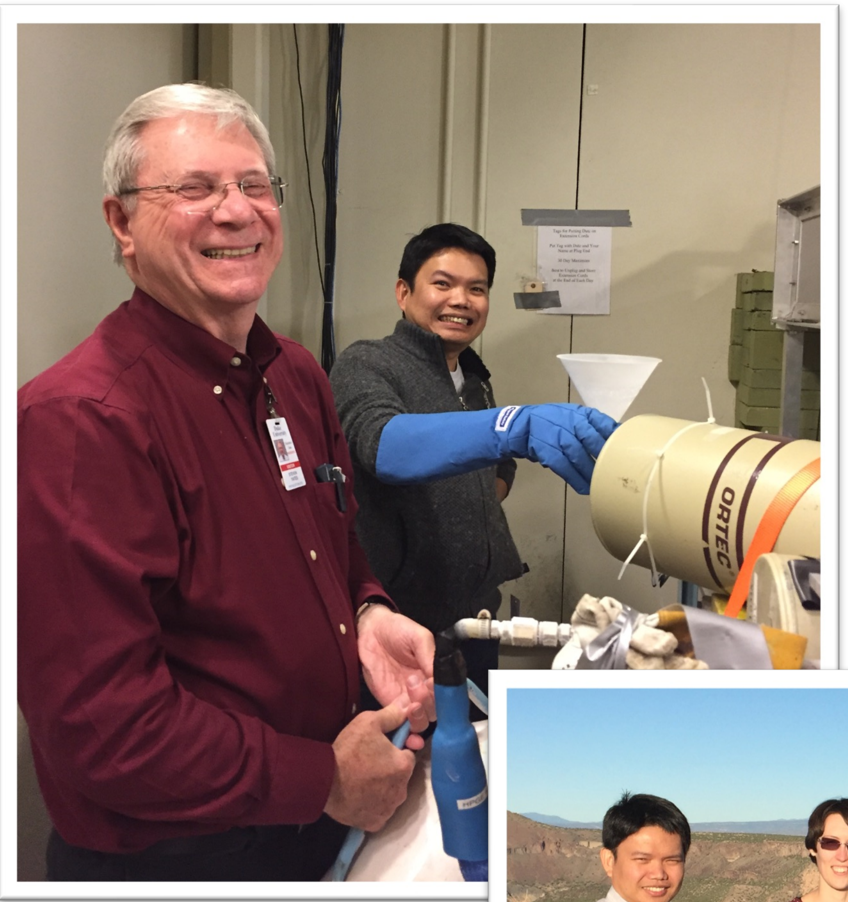
Concluding Remarks

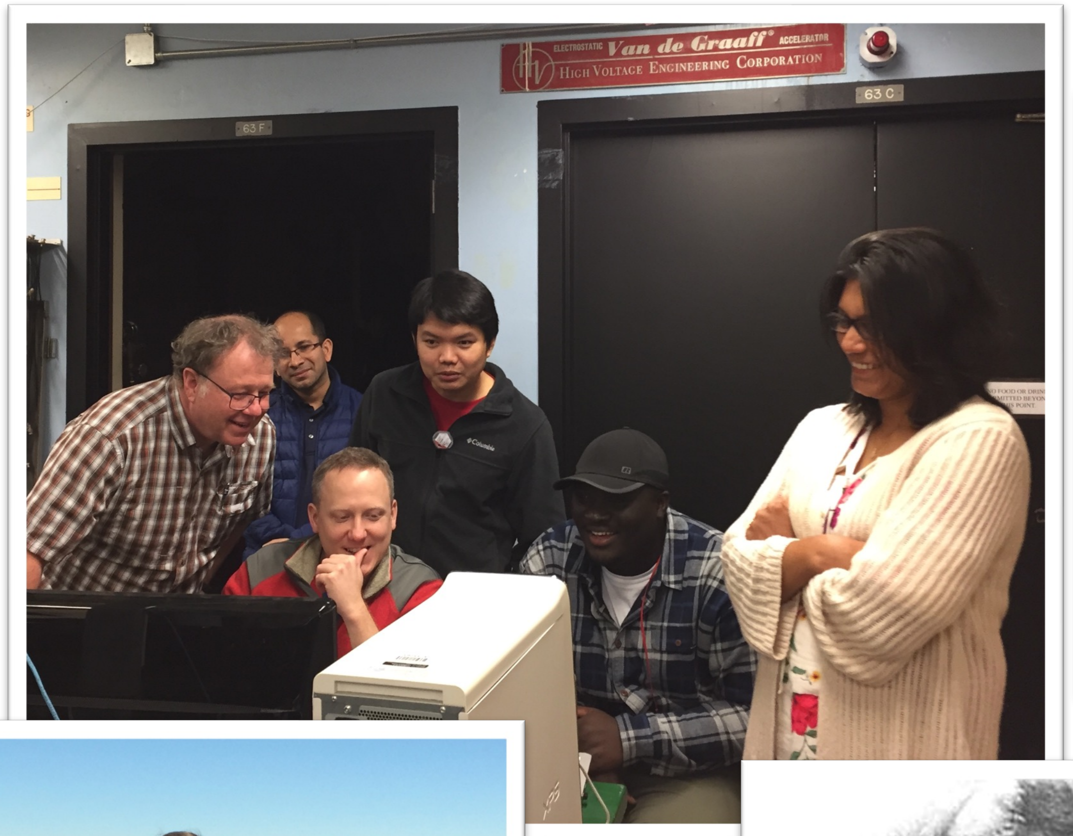
- The nuclear structure is complex (and interesting!) for the Ge nuclei, but
- The shell-model JUN45 and jj44b interactions do an excellent job reproducing experimental data in this isotopic chain.
- This results lends confidence in using these interactions for calculating the $0\nu\beta\beta$ NME.

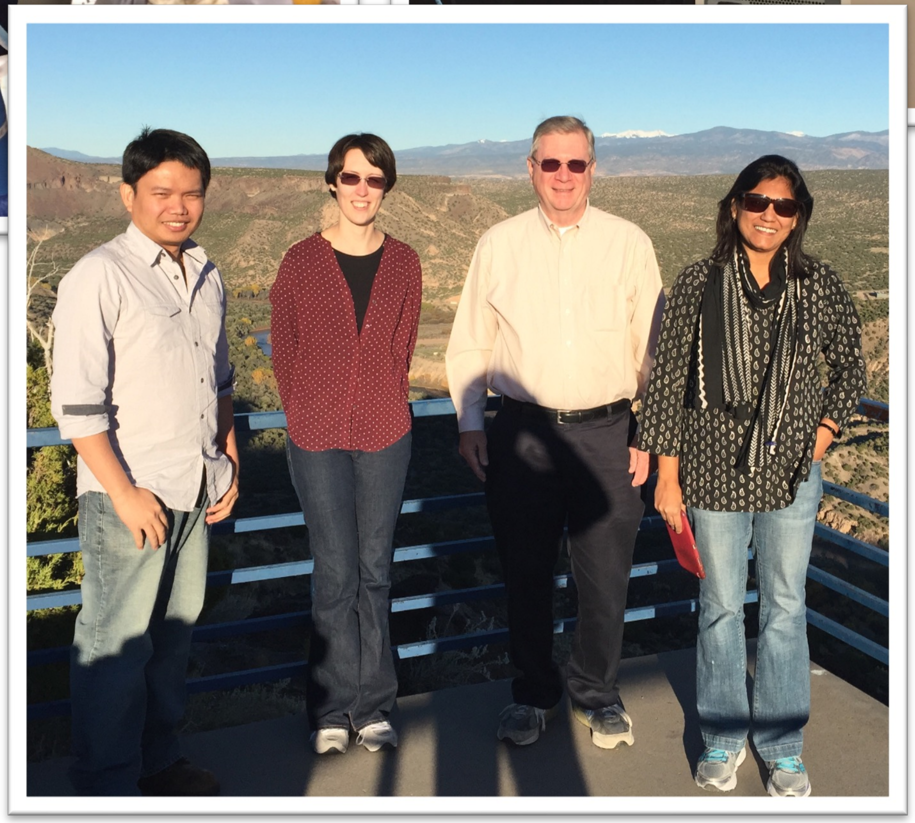
Thank you!!

- Nuclear Structure Studies are funded by the National Science foundation through grants PHY –1913028 and PHY – 2209178

University of Kentucky 60 years 1964-2024 Accelerator Laboratory







Celebrating the 75 years of the shell model and Maria Goeppert Mayer