# 3D tomography of hadrons with hadron and lepton beams


Shunzo Kumano

Japan Women's University

High Energy Accelerator Research Organization (KEK)

http://research.kek.jp/people/kumanos/


Symposium: 3D Hadron Structure from Next-Generation Scattering Experiment and Lattice QCD I

6th Joint Meeting of the APS Division of Nuclear Physics and the Physical Society of Japan

Waikoloa, Hawaii, USA, November 26 - December 1, 2023

https://indico.frib.msu.edu/event/66/

November 27, 2023

# Contents

- Motivations for studying GPDs (Generalized Parton Distributions)
- Hadron accelerator facilities on GPDs
- Possible studies on GPDs at hadron accelerator facilities
- Comments on GPDs for exotic hadrons

  (transition GPDs could be used for exotic hadrons)
- Comments on GPDs at other facilities:

  Timelike GPDs at KEKB, Neutrino facilities
- Future prospects on GPD project

Talks by Profs. Shibata and Tomida

GPDs ⇔ Energy-momentum tensor
⇔ Gravitational form factors

$$\int_0^1 dz(2z-1)\Phi_q^{\pi^0\pi^0}(z,\zeta,s) = \frac{2}{(P^+)^2}\left\langle \pi^0(p)\pi^0(p') \middle| T_q^{++}(0) \middle| 0 \right\rangle$$

$$\left\langle \pi^0(p)\pi^0(p') \middle| T_q^{\mu\nu}(0) \middle| 0 \right\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu} - P^\mu P^\nu\right)\Theta_{1,q}(s) + \Delta^\mu\Delta^\nu\Theta_{2,q}(s)\right]$$

$\Phi_q^{\pi^0\pi^0}$ : time-like GPD (GDA: generalized distribution amplitude)

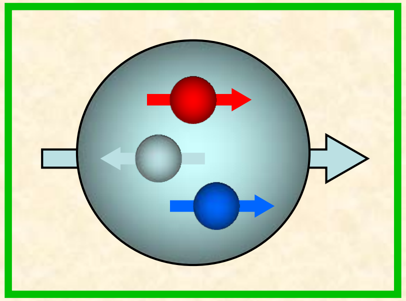
$T_q^{\mu\nu}$ : energy-momentum tensor for quark

$\Theta_{1,q}$, $\Theta_{2,q}$ : gravitational form factos for pion

# Motivations for studying gravitational form factors and GPDs

# Recent progress on origin of nucleon spin

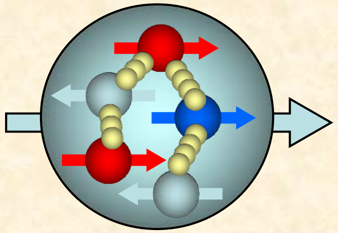
"old" standard model



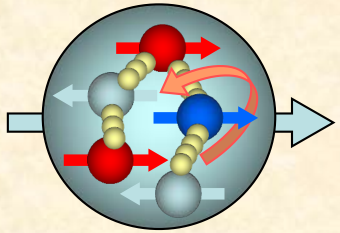
$$p_{\uparrow} = \frac{1}{3\sqrt{2}}\Big(uud\big[2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\big] + \text{permutations}\Big)$$

$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

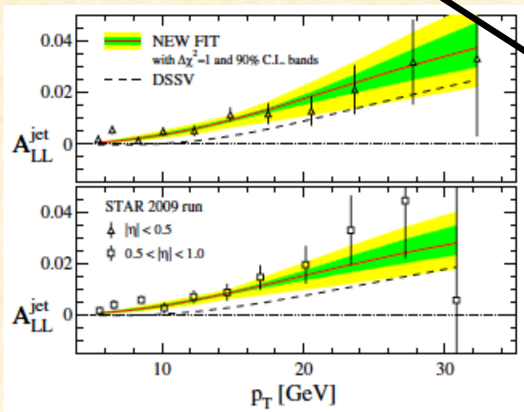
$$\Delta\Sigma = \sum_i \int dx\big[\Delta q_i(x) + \Delta\bar{q}_i(x)\big] \to 1\ (100\%)$$

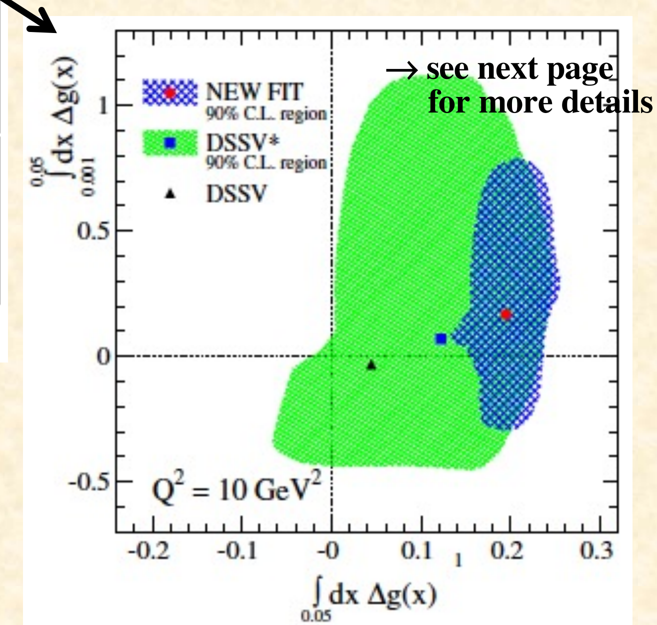


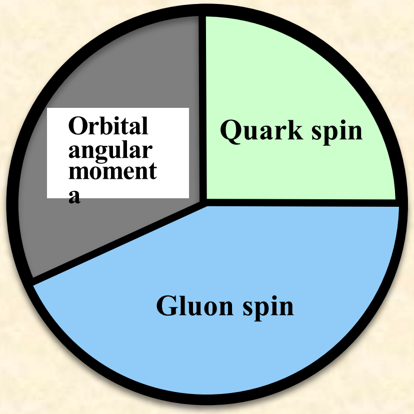
gluon spin



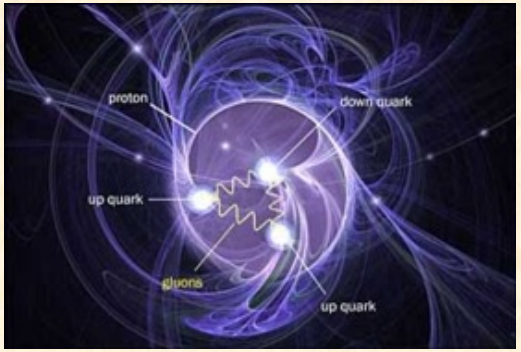
angular momentum



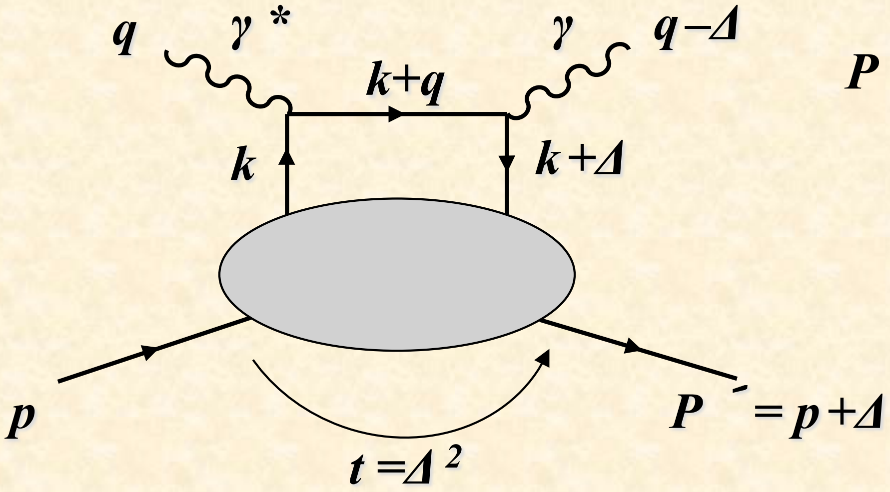

"A possible" spin decmposion




Scientific American (2014)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_{q,g}$$

# Generalized Parton Distributions (GPDs)




$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $\quad x = \frac{Q^2}{2p\cdot q}$

Momentum transfer squared $\quad t = \Delta^2$

Skewdness parameter $\quad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z/2)\gamma^+\psi(z/2)|p\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2)|p\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

**Forward limit: PDFs** $\quad H(x,\xi,t)\big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta f(x),$

**First moments: Form factors**

Dirac and Pauli form factors $F_1, F_2$ $\quad \int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$

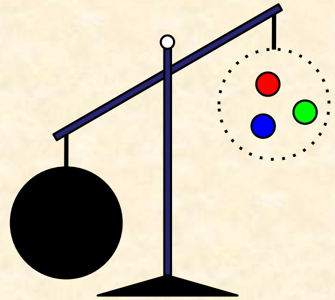
Axial and Pseudoscalar form factors $G_A, G_P$ $\quad \int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$

**Second moments: Angular momenta**

Sum rule: $J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right], \quad J_q = \frac{1}{2}\Delta q + L_q$

$\Rightarrow$ probe $L_q$, key quantity to solve the spin puzzle!

# Origin of hadron masses

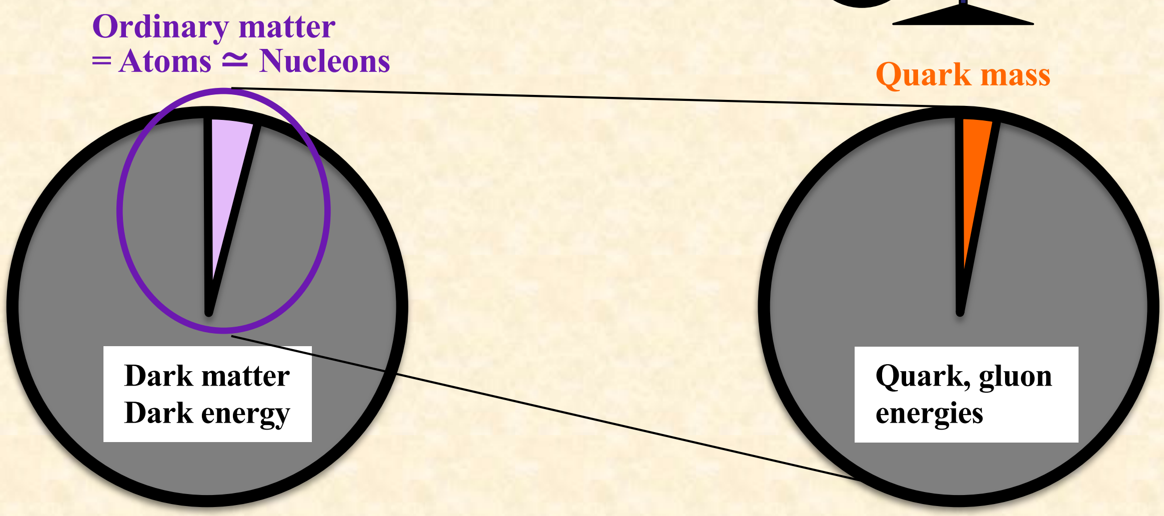


**Mass and spin of the nucleon are two of fundamental quantities in physics.**

**Nucleon mass:** $M = \left\langle p \left| \int d^3x \, T^{00}(x) \right| p \right\rangle$

**Energy-momentum tensor:**

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\ \alpha}(x)$$

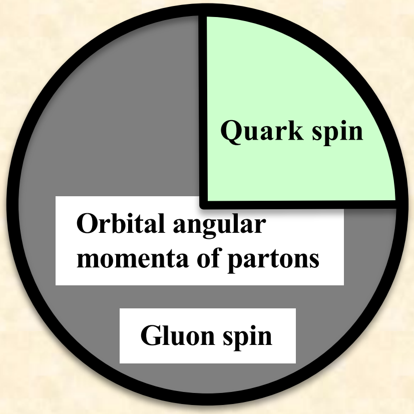



**Dark matter** **Origin of nucleon mass**

**Nucleon spin:** $\frac{1}{2} = \left\langle p \left| J^3 \right| p \right\rangle$

**3rd component of total angular momentum:** $J^3 = \frac{1}{2}\varepsilon^{3jk} \int d^3x \, M^{3jk}(x)$

**Angular-momentum density:** $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x) x^{\mu} - T^{\alpha\mu}(x) x^{\nu}$




**Origin of nucleon spin**

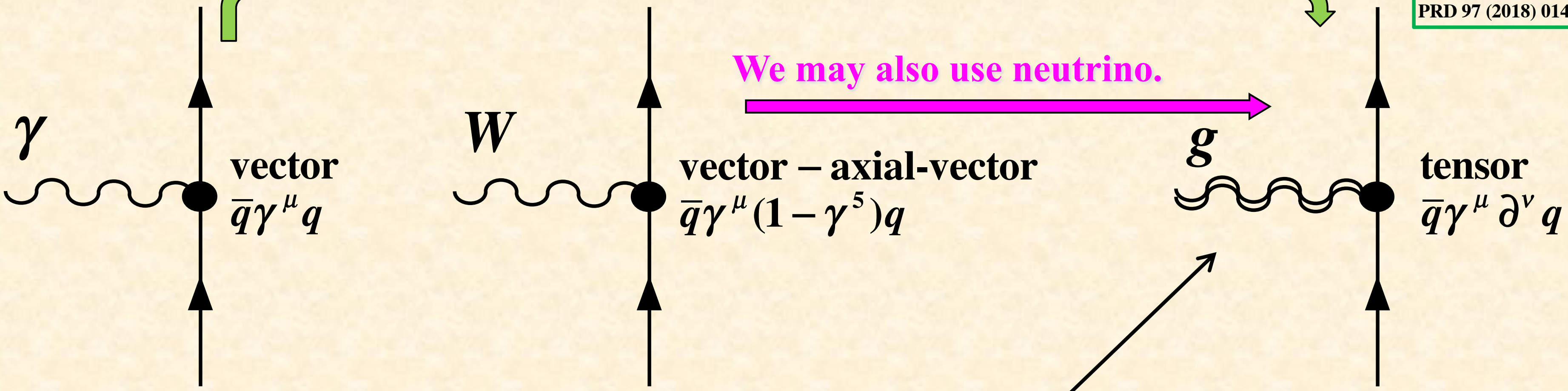
**("Dark spin")**

# Why "gravitational" interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev, PRD 97 (2018) 014020.

We may also use neutrino.




It is possible to probe gravitational sources in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2)\gamma^+ q(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[ H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

Non-local operator of GPDs/GDAs:

$$\left(P^+\right)^n \int dx\, x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2)\gamma^+ q(z/2)\right]_{z^+=0,\vec{z}_\perp=0}$$

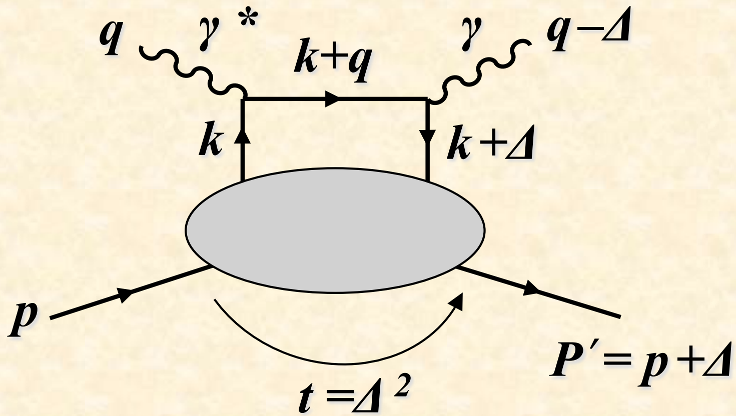
$$= \left(i\frac{\partial}{\partial z^-}\right)^{n-1} \left[\bar{q}(-z/2)\gamma^+ q(z/2)\right]_{z=0}$$

$$= \bar{q}(0)\gamma^+ \left(i\overleftrightarrow{\partial}^+\right)^{n-1} q(0)$$

= energy-momentum tensor of a quark for $n=2$
(electromagnetic for $n=1$)

= source of gravitiy

Virtual Compton
or (timelike) two-photon process



# Nucleon pressure

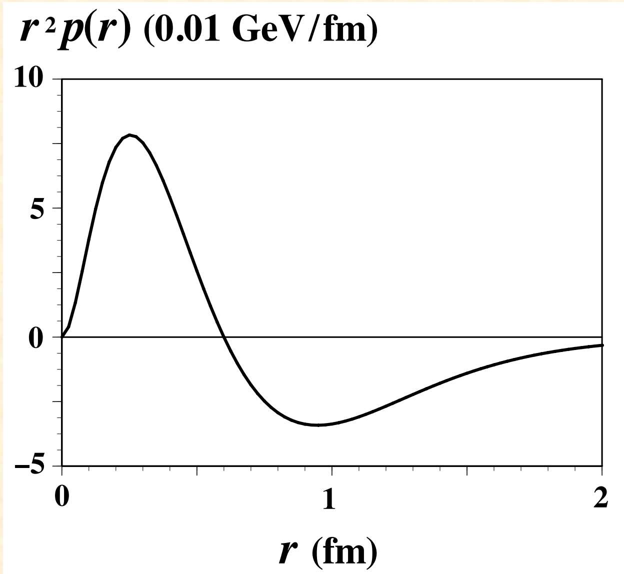
$$\langle N(p')|T_q^{\mu\nu}(0)|N(p)\rangle = \bar{u}(p')\left[A\gamma^{(\mu}\bar{P}^{\nu)} + B\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}Mg^{\mu\nu}\right]u(p)$$

Recent progress

V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557 (2018) 396;

M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) 1830025;

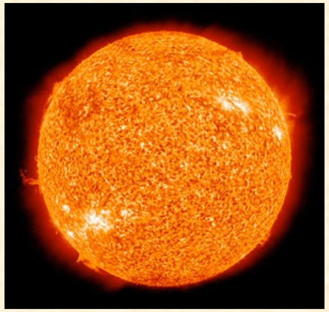
C. Lorce, H. Moutarde, and A. P. Tranwinski, Eur. Phys. J. C 79 (2019) 89.



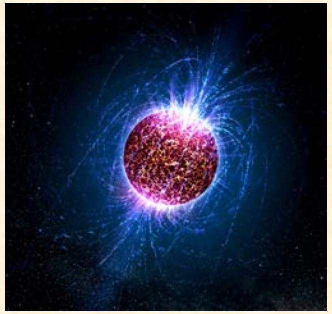

**Highest pressure in nature** 1 Pa (Pascal) = 1 N/m$^2$

Earth atmosphere
$10^5$ Pa =1000 hPa

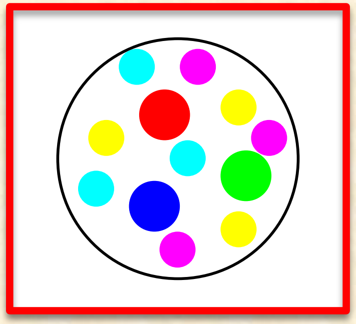
Center of earth
$10^{11}$ Pa = 100GPa



Center of Sun
$10^{16}$ Pa = 10 PPa



Neutron star
$10^{34}$ Pa

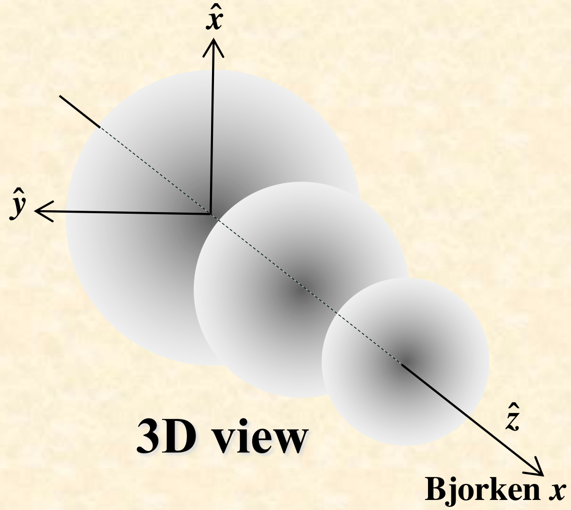


Hadron
$10^{35}$ Pa

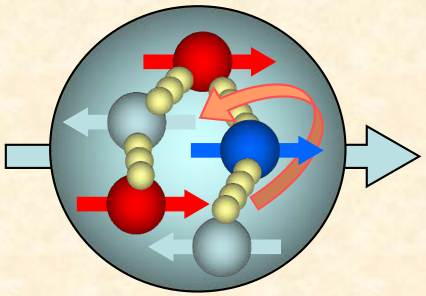
# Proton (hadrons) puzzle studies by hadron tomography

## Hadron tomography

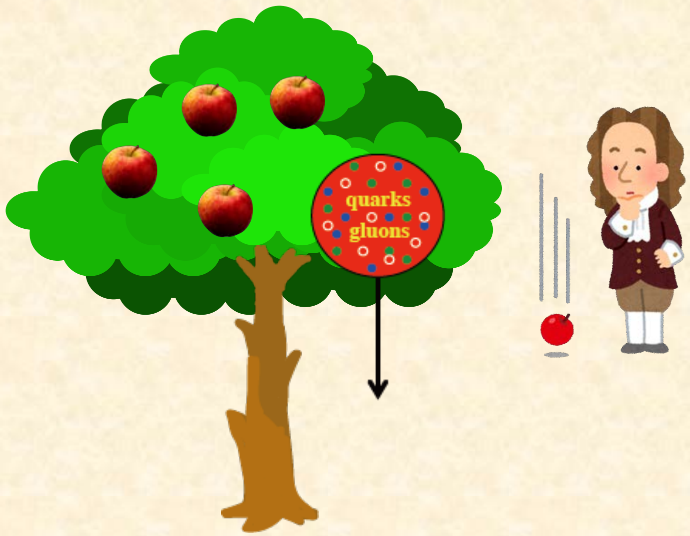

## Proton radius puzzle



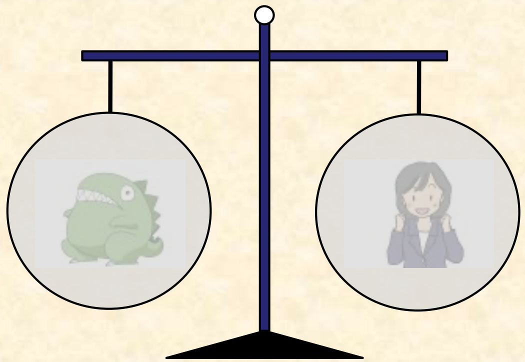

## Origin of nucleon spin



## Source of gravity (mass)



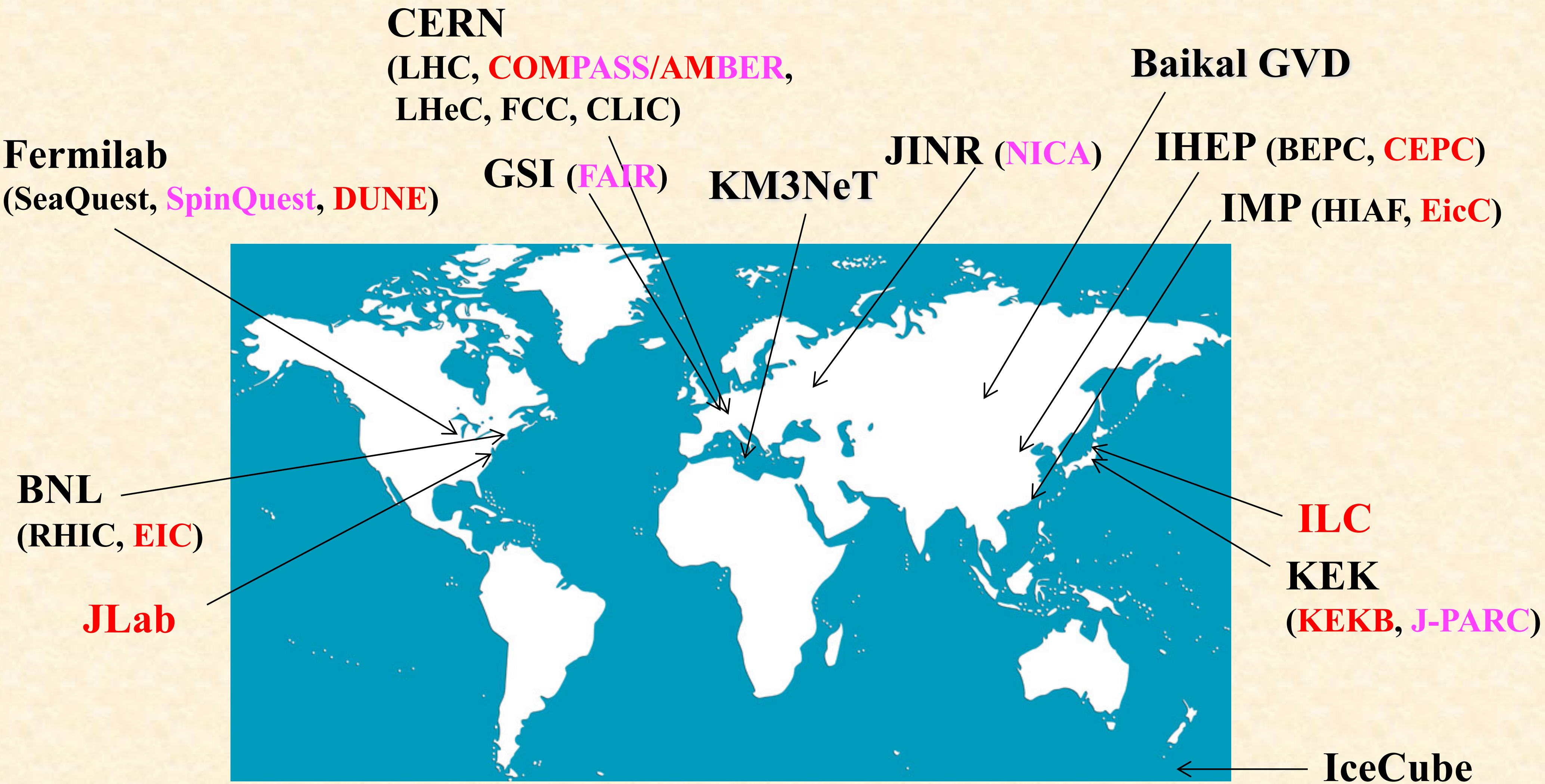

## Exotic hadrons



# Hadron accelerator facilities on GPDs

## (including future possibilities)

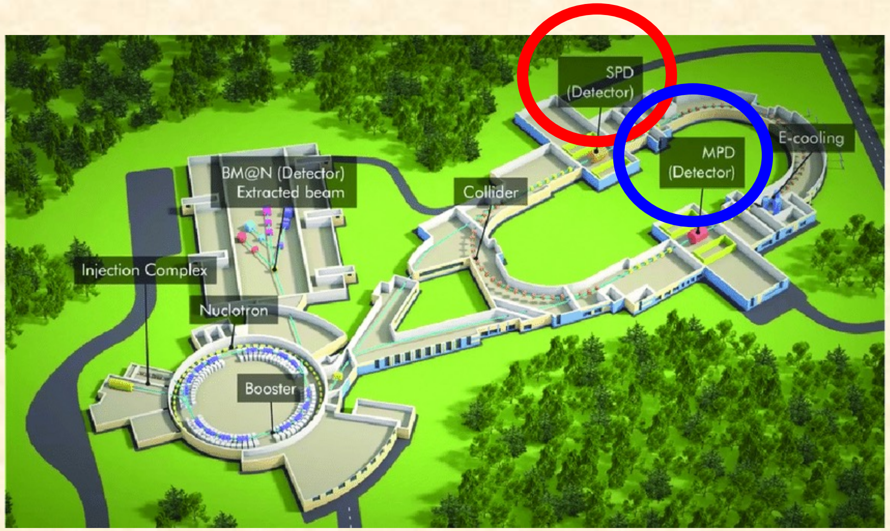
# High-energy hadron physics experiments




**Facilities on hadron structure functions on GPDs including future possibilities.**

**Hadron accelerator facilities. Lepton accelerator facilities.**

# Nuclotron-based Ion Collider fAcility (NICA)





**SPD (Spin Physics Detector for physics with polarized beams)**
**MPD (MultiPurpose Detector for heavy ion physics)**

$$\vec{p}+\vec{p}: \quad \sqrt{s_{pp}} = 12 \sim 27 \text{ GeV}$$

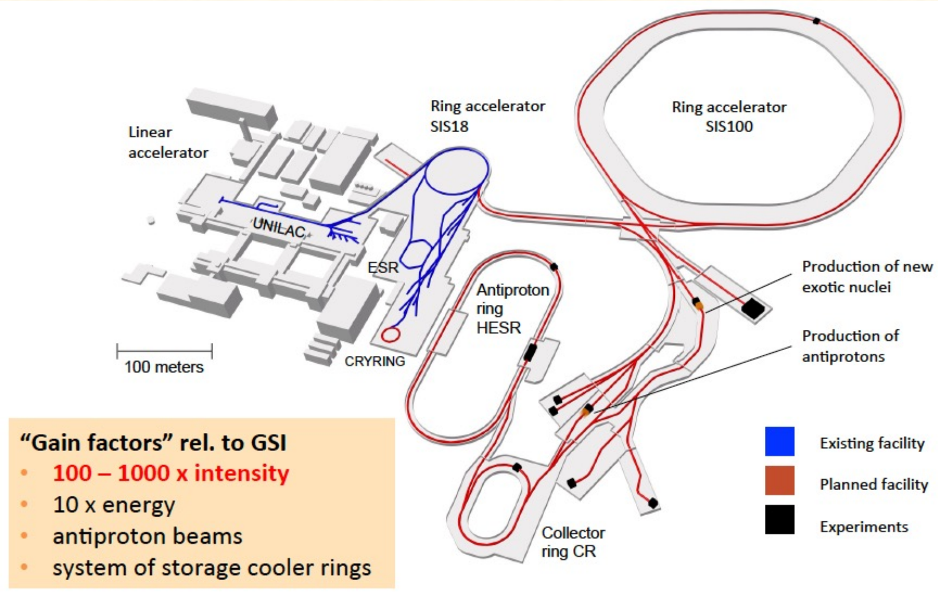
$$\vec{d}+\vec{d}: \quad \sqrt{s_{NN}} = 4 \sim 14 \text{ GeV}$$

$\vec{p}+\vec{d}$ is also possilbe.

**On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al.* (NICA project), arXiv:2011.15005, Progress in Nuclear and Particle Physics in press.**

**Unique opportunity in high-energy spin physics,**
**especially on the deuteron spin physics.**

# GSI-FAIR

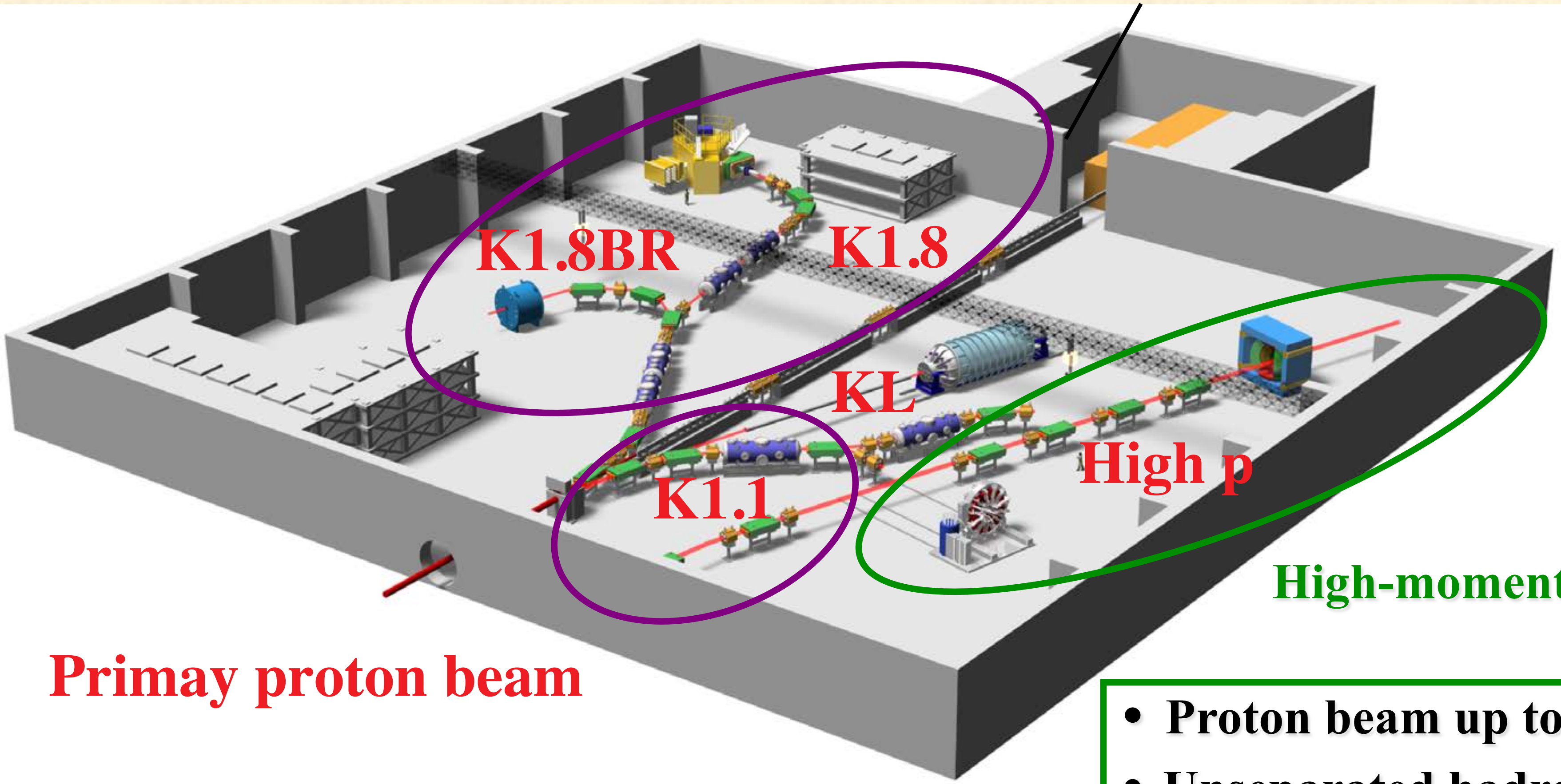



| | |
|---|---|
| **APPA** | Atomic Physics and Fundamental Symmetries,<br>Plasma Physics,<br>Materials Research,<br>Radiation Biology,<br>Cancer Therapy with Ion Beams / Space Research |
| **CBM** | Dense and Hot Nuclear Matter |
| **NUSTAR** | Nuclear Structure and Reaction Studies with nuclei far off stability,<br>Physics of Explosive Nucleosynthesis (r-process) |
| **PANDA** | Hadron Structure & Dynamics with cooled antiproton beams |




# Hadron facility

(Low energy) Kaon and pion experiments are done at these beamlines.



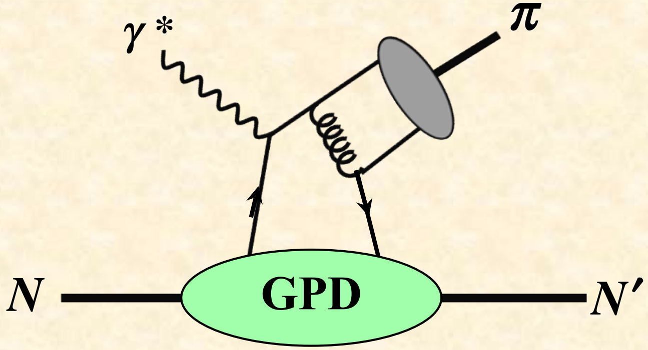

- Proton beam up to 30 GeV
- Unseparated hadron (pion, …) beam up to 15~20 GeV

# Possible studies on GPDs at hadron accelerator facilities

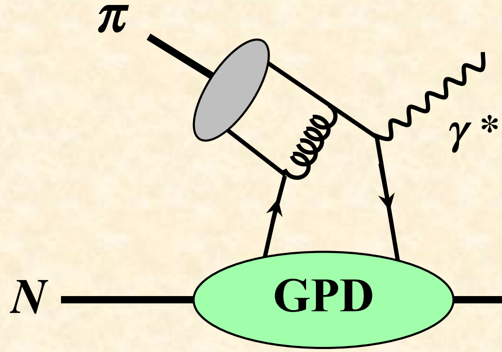
**SK, M. Strikman, K. Sudoh,**
**PRD 80 (2009) 074003;**
**T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka,**
**PRD 93 (2016) 114034.**
**J-PARC LoI 2019-07, J.-K. Ahn *et al*. (2019).**
**J-PARC proposal under preparation (2023),**
**Please get in touch with W.-C. Chang if you are interested in this project.**

# GPD projects at JLab /EIC and J-PARC

**JLab / EIC**

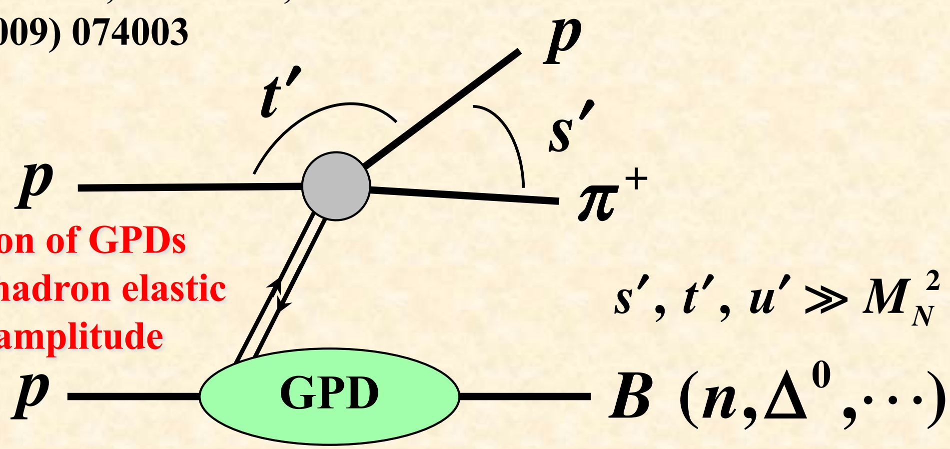



**J-PARC**




$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[ \tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003




Investigation of GPDs
with 2→3 hadron elastic
scattering amplitude

$s', t', u' \gg M_N^2$

J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

$$\pi + N \to \gamma + \gamma + N'$$
$$h + M_B \to h' + \gamma + M_D$$
$$h + M_B \to h' + M_C + M_D$$

# Physics of J-PAC high-momentum beamline

2019 | 2021 | 2023 | 2025

**E16**: Hadron masses in nuclear medium

. . .

GPDs with E16 (proposal?)

**E50**: Charmed baryons

GPDs with E50 (LoI)

There is a possibility for high-energy hadron physics, including nucleon structure, ...

Proposal

Electron pair spectrometer at the J-PARC 50-GeV PS to explore the chiral symmetry in QCD

April 28, 2006
June 07, 2006 rev.1

S. Yokkaichi[1], H. En'yo, M. Naruki, R. Muto, T. Tabaru
*RIKEN*

K. Ozawa, H. Hamagaki
*Center for Nuclear Study, Graduate School of Science, University of Tokyo*

K. Shigaki
*Graduate School of Science, Hiroshima University*

S. Sawada, M. Sekimoto
*High Energy Accelerator Research Organization (KEK)*

F. Sakuma, K. Aoki
*Department of Physics, Kyoto University*

KEK/J-PARC-PAC 2012-19

Charmed Baryon Spectroscopy via the $(\pi, D^{*-})$ reaction

Y. Morino, T. Nakano,* H. Noumi†,* K. Shirotori, Y. Sugaya, and T. Yamaga
*Research Center for Nuclear Physics (RCNP), Osaka University, 10-1, Mihogaoka, Ibaraki, Osaka, 567-0047, Japan*

K. Ozawa‡
*Institute of Particle and Nuclear Studies(IPNS), High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*

T. Ishikawa
*Reserach Center for Electron Photon Science, Tohoku University, 1-2-1, Mikamine, Taihaku-ku, Sendai, Miyagi 982-0826, Japan*

Y. Miyachi
*Physics Department, Yamagata University, 1-4-12, Kojirakawa-machi, Yamagata 990-8560, Japan*

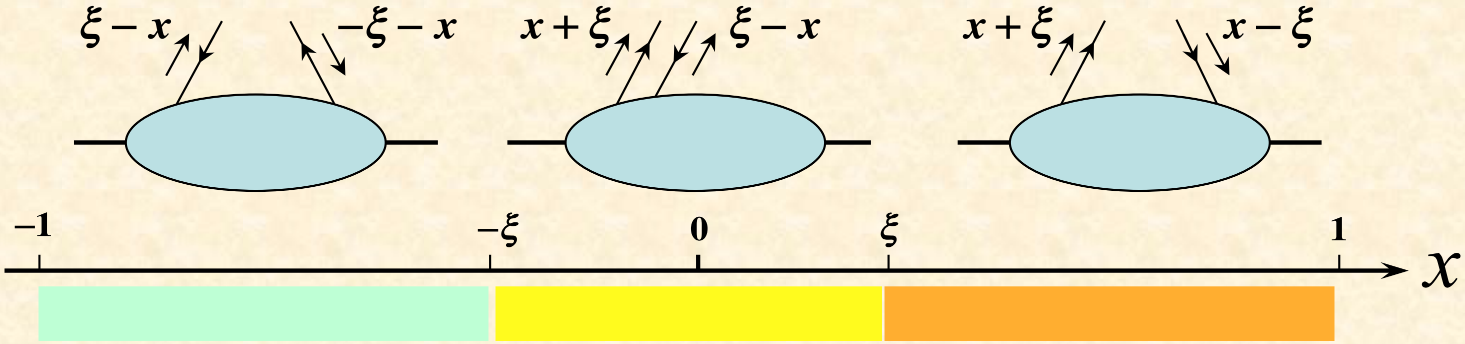
K. Tanida
*Department of Physics and Astronomy, Seoul National University, Seoul 151-747, Korea*

LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J- PARC

JungKeun Ahn,[1] Sakiko Ashikag,[2] Wen-Chen Chang,[3,*] Seonho Choi,[4] Stefan Diehl,[5] Yuji Goto,[6] Kenneth Hicks,[7] Youichi Igarashi,[8] Kyungseon Joo,[5] Shunzo Kumano,[9,10] Yue Ma,[6] Kei Nagai,[3] Kenichi Nakano,[11] Masayuki Niiyama,[12] Hiroyuki Noumi,[13,8,†] Hiroaki Ohnishi,[14] Jen-Chieh Peng,[15] Hiroyuki Sako,[16] Shin'ya Sawada,[8,‡] Takahiro Sawada,[17] Kotaro Shirotori,[13] Kazuhiro Tanaka,[18,10] and Natsuki Tomida[13]

# GPDs in different $x$ regions and GPDs at hadron facilities




$-1 < x < \xi \quad (x+\xi < 0,\, x-\xi < 0)$

$\xi < x < 1 \quad (x+\xi > 0,\, x-\xi > 0)$

$-\xi < x < \xi \quad (x+\xi > 0,\, x-\xi < 0)$

## Quark distribution

Emission of quark with momentum fraction $x+\xi$
Absorption of quark with momentum fraction $x-\xi$

## $q\bar{q}$(meson)-like distribution amplitude

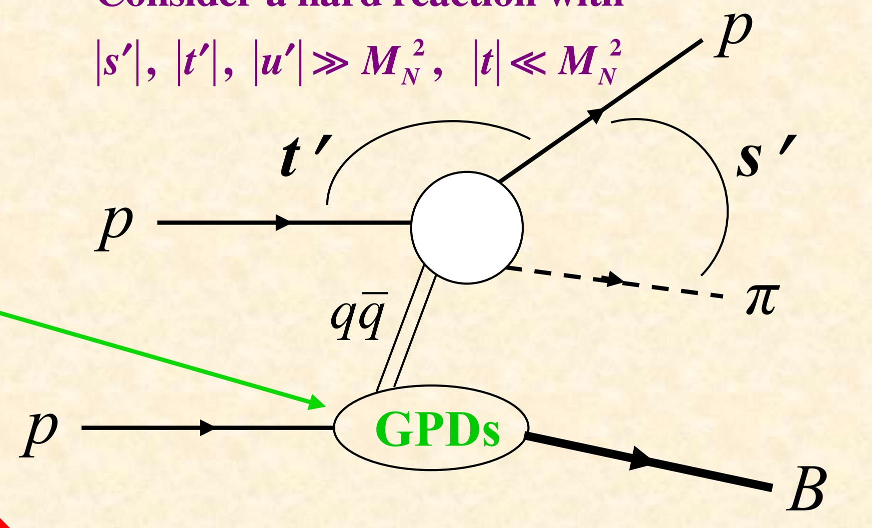
Emission of quark with momentum fraction $x+\xi$
Emission of antiquark with momentum fraction $\xi-x$

## Antiquark distribution

Emission of antiquark with momentum fraction $\xi-x$
Absorption of antiquark with momentum fraction $-\xi-x$

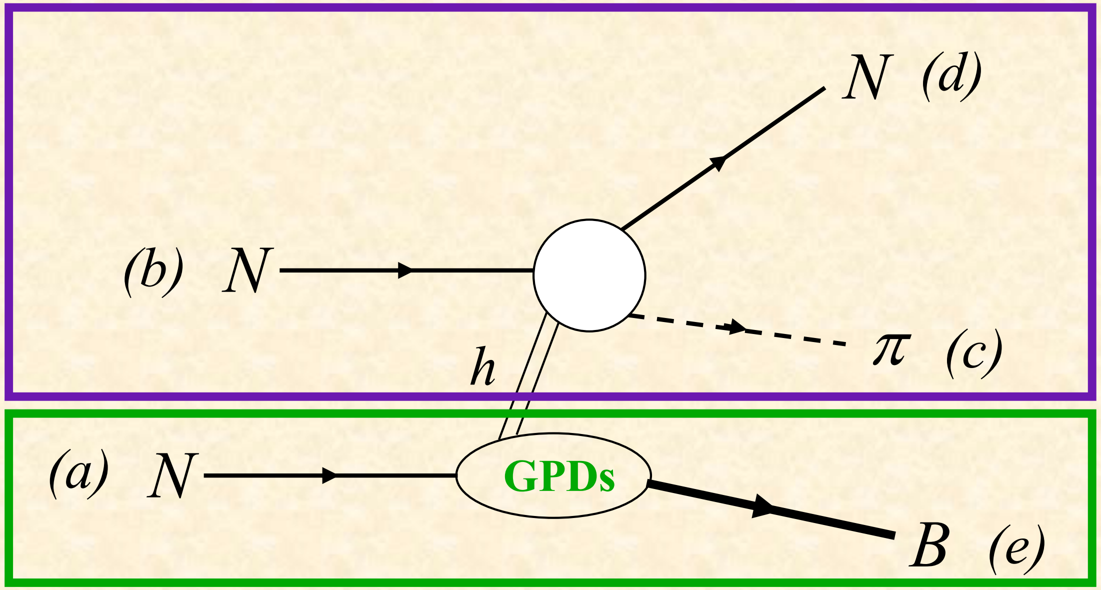
Consider a hard reaction with

$|s'|,\ |t'|,\ |u'| \gg M_N^2,\quad |t| \ll M_N^2$




**Efremov-Radyushkin -Brodsky-Lepage (ERBL) region**

# Cross section estimates for $N + N \to N + \pi + B$




$\dfrac{d\sigma(s',t')}{dt'}$ so as to explain BNL-AGS experimental data on $\pi + p \to \pi + p,\ \pi + p \to \rho + p$

This part is expressed by GPDs.

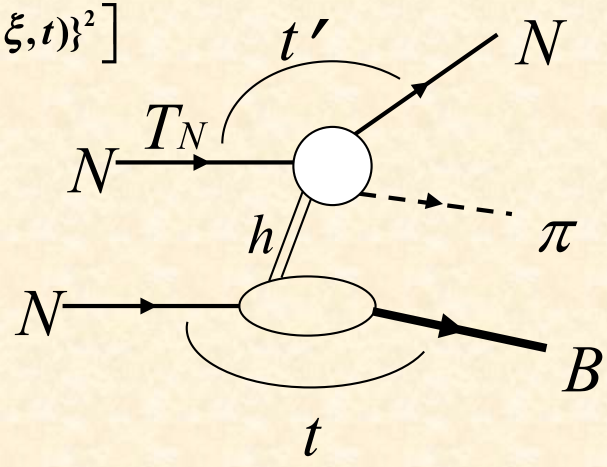
**Purposes of our studies:**

**(1) The ultimate purpose is to extract the GPDs in the ERBL region by measurements at hadron facilities in addition to lepton ones.**

**(2) Since our work is the first one to point out the GPD studies at hadron reactions, we estimate the order of magnitude of cross sections simply by using meson-pole expressions of the GPDs.**
**$\to$ For experimental feasibility studies.**

# Cross section estimate ($\xi$ dependence)

$$\frac{d\sigma_{NN\to N\pi B}}{d\xi\, dt\, dt'} \propto \frac{d\sigma_{MN\to\pi N}}{dt'}\Big[8(1-\xi^2)\{H(x,\xi,t)\}^2+16\xi^2 H(x,\xi,t)E(x,\xi,t)-\frac{t}{m_N^2}(1+\xi)^2\{E(x,\xi,t)\}^2$$
$$+8(1-\xi^2)\{\tilde H(x,\xi,t)\}^2+18\xi^2\tilde H(x,\xi,t)\tilde E(x,\xi,t)-\frac{2t\xi^2}{m_N^2}\{\tilde E(x,\xi,t)\}^2\Big]$$

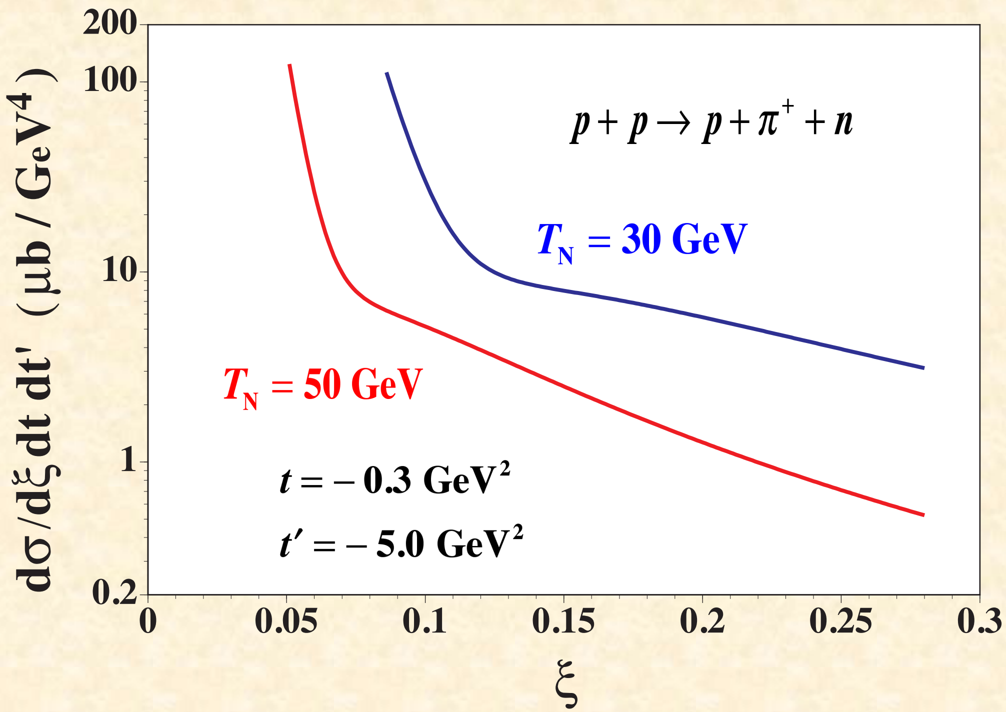



Skewdness parameter: $\xi = \dfrac{p_N^+ - p_B^+}{p_N^+ + p_B^+}$

$\dfrac{d\sigma}{d\xi dt dt'}\left(\dfrac{\mu\mathrm{b}}{\mathrm{GeV}^2}\right)$ as a function of $\xi$

at fixed $T_N = 30\ (50)$ GeV,

$t = -0.3\ \mathrm{GeV}^2,\quad t' = -5\ \mathrm{GeV}^2.$




At this stage, our numerical results are for rough order of magnitude estimates on cross sections by assuming π- and ρ-like intermediate states.

For the details, please look at
SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003.

# Exclusive Drell-Yan $\pi^- + p \to \mu^+\mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[ (1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2\,\mathrm{Re}\left\{\tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t)\right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]$$

$$Q'^2 = q'^2,\ \ t = (p-p')^2,\ \ \tau = \frac{Q'^2}{2p\cdot q_\pi} \simeq \frac{Q'^2}{s-m_\pi^2}$$

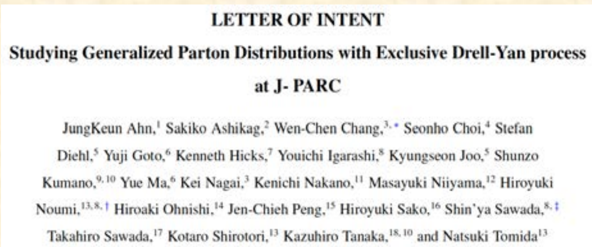
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle p(p')\right|\bar{q}(-z/2)\gamma^+\gamma_5 q(z/2)\left|p(p)\right\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_p^q(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_p^q(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle n(p')\right|\bar{q}_d(-z/2)\gamma^+\gamma_5 q_u(z/2)\left|p(p)\right\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

$$\tilde{H}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s\int_{-1}^{1} dz\frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx'\left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon}\right]\left[\tilde{H}^d(x',\xi,t) - \tilde{H}^u(x',\xi,t)\right]$$

$$\tilde{E}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s\int_{-1}^{1} dz\frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx'\left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon}\right]\left[\tilde{E}^d(x',\xi,t) - \tilde{E}^u(x',\xi,t)\right]$$

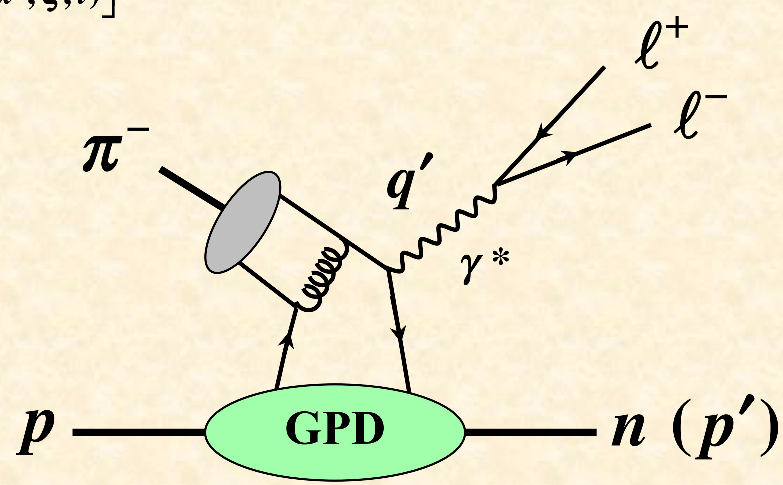
**T. Sawada, W.-C. Chang, SK, J.-C. Peng,
S. Sawada, and K. Tanaka, PRD93 (2016) 114034.**


LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J- PARC

JungKeun Ahn,[1] Sakiko Ashikag,[2] Wen-Chen Chang,[3,*] Seonho Choi,[4] Stefan Diehl,[5] Yuji Goto,[6] Kenneth Hicks,[7] Youichi Igarashi,[8] Kyungseon Joo,[5] Shunzo Kumano,[9,10] Yue Ma,[6] Kei Nagai,[3] Kenichi Nakano,[11] Masayuki Niiyama,[12] Hiroyuki Noumi,[13,8,†] Hiroaki Ohnishi,[14] Jen-Chieh Peng,[15] Hiroyuki Sako,[16] Shin'ya Sawada,[8,‡] Takahiro Sawada,[17] Kotaro Shirotori,[13] Kazuhiro Tanaka,[18,10] and Natsuki Tomida[13]

**LoI for a J-PARC experiment**




$$\pi^-(\bar{u}d) + p(uud) \to n(udd) + \gamma^*(\to \ell^+\ell^-)$$

# GPDs for exotic hadrons

## (If transition GPDs could be studied, this exotic-hadron project becomes realistic. )

**H. Kawamura and SK,**
**Phys. Rev. D 89 (2014) 054007.**

**Constituent counting rule for exotic hadrons:**
**H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;**
**W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006.**

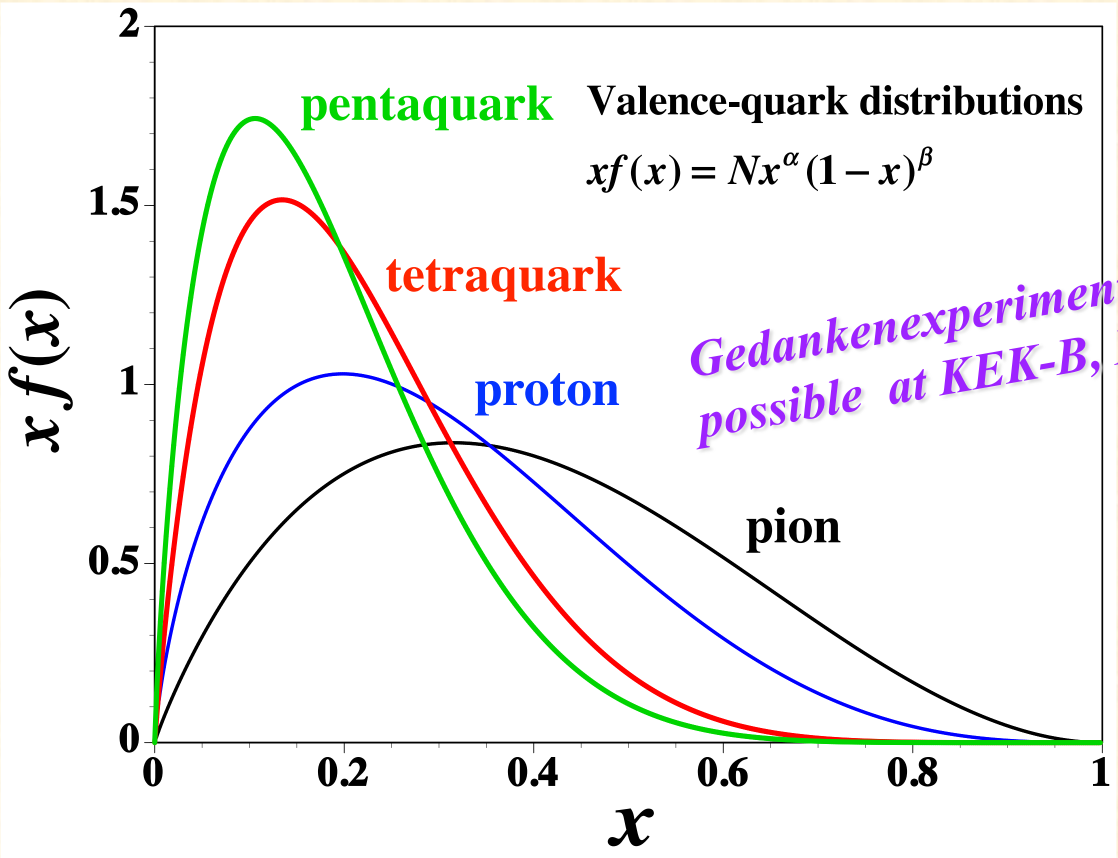
# Simple function of GPDs

$$\boxed{H_q^h(x,t) = f(x)F(t,x)}$$

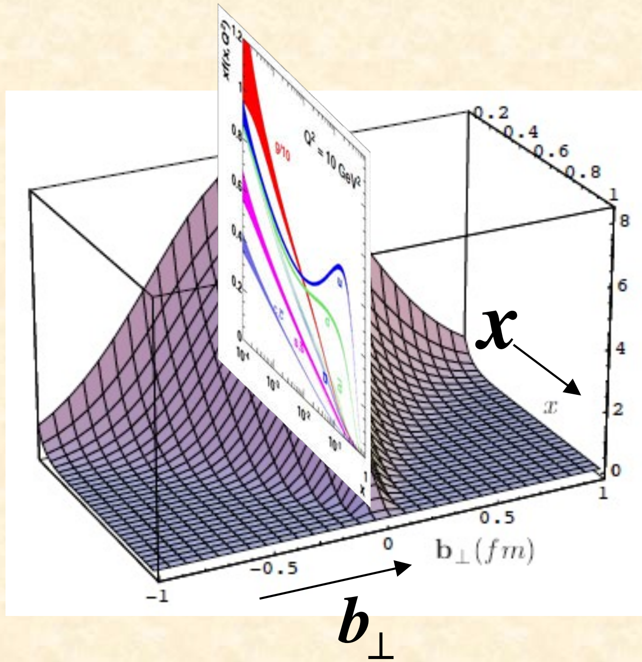
M. Guidal, M.V. Polyakov, A.V. Radyushkin, M. Vanderhaeghen, PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n}(1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers): $\int_0^1 dx f(x) = n$
- Constituent conting rule at $x \to 1$: $\beta_n = 2n - 3 + 2\Delta S$ ($n$ = number of constituents)
- Momentum carried by quarks $\langle x \rangle_q \simeq \int_0^1 dx\, x f(x)$

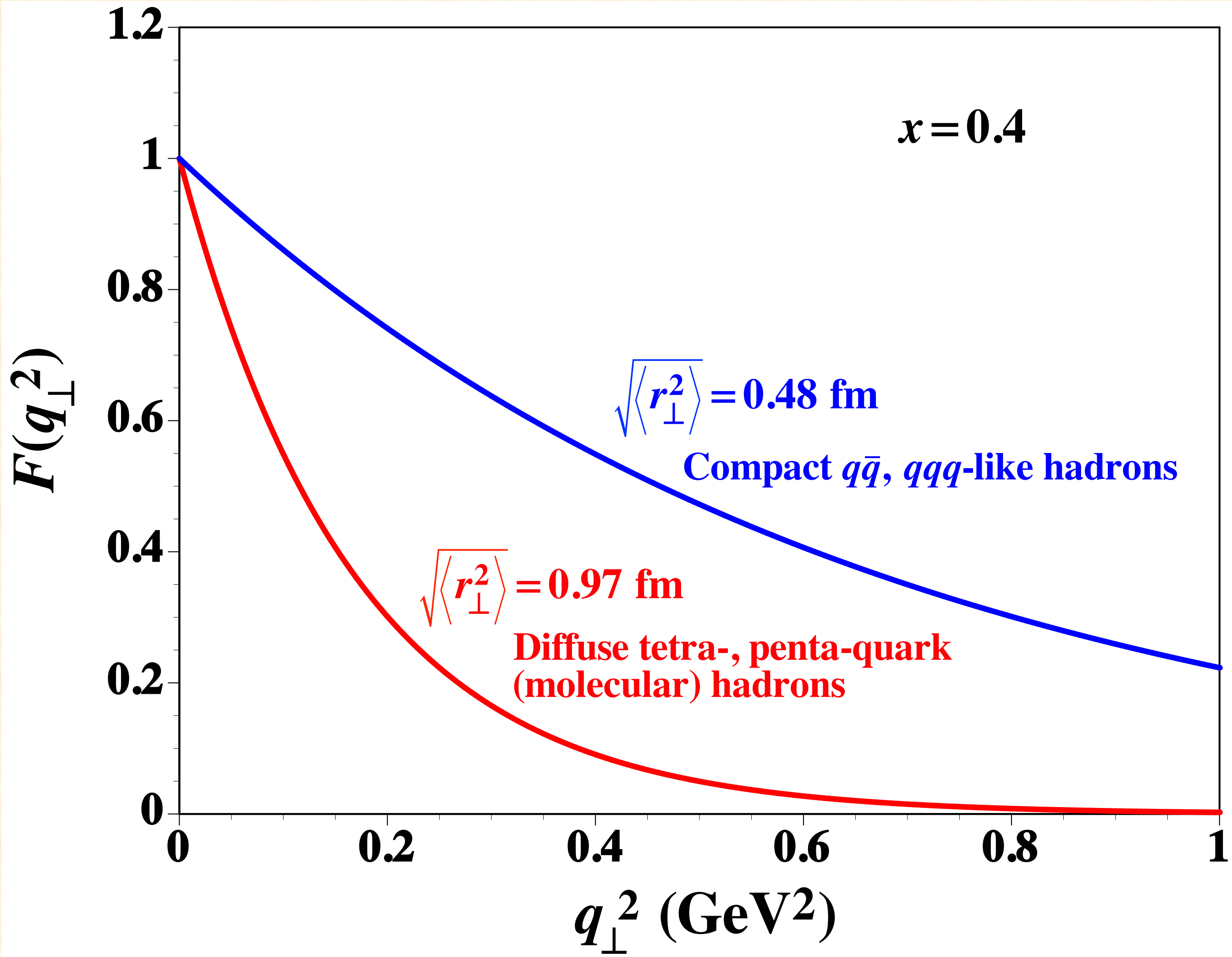


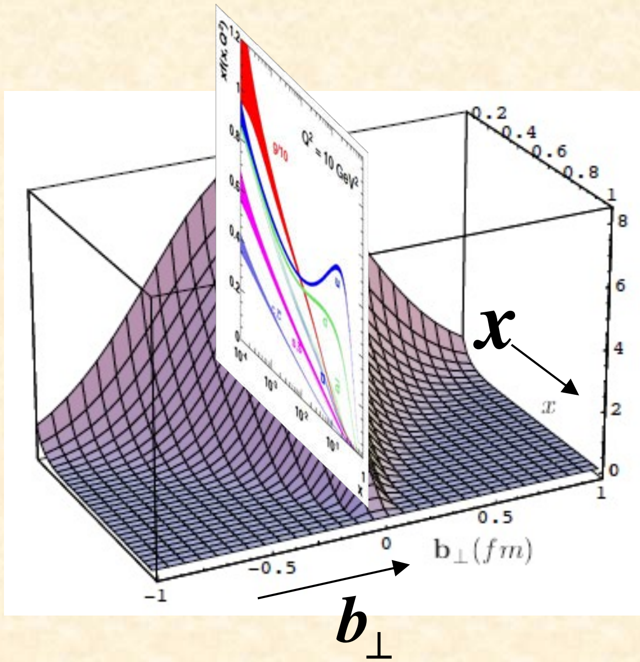

*Gedankenexperiment, but possible at KEK-B, ILC, ...*



# Two-dimensional form factor

$$\boxed{H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}}$$



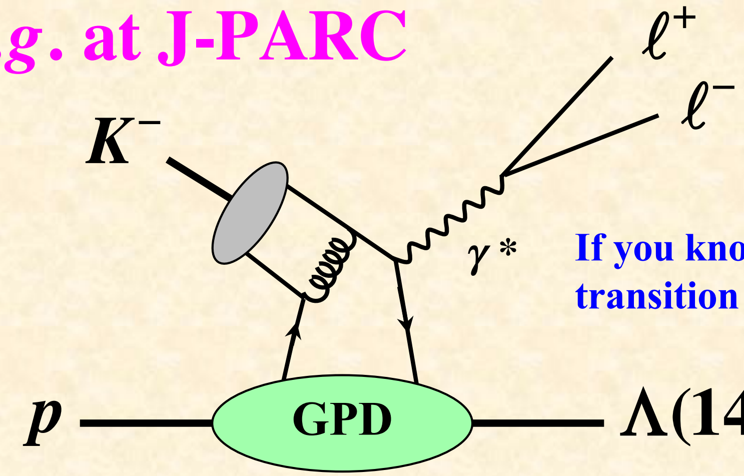
# GPDs for exotic hadrons !?

**Because stable targets do not exist for exotic hadrons, it is not possible to measure their GPDs in a usual way.**

**→ Transition GPDs**

**or → $s \leftrightarrow t$ crossed qunatity = GDAs at KEKB, Linear Collider**

***e.g.* at J-PARC**

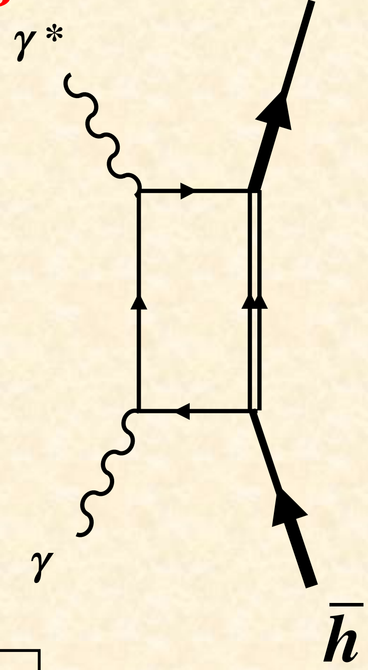



**If you know how to handle this kind of transition GPDs N→Λ, please inform me.**

$$K^-(\bar{u}s) + p(uud) \to \Lambda_{1405}(uud\bar{u}s) + \gamma^*$$

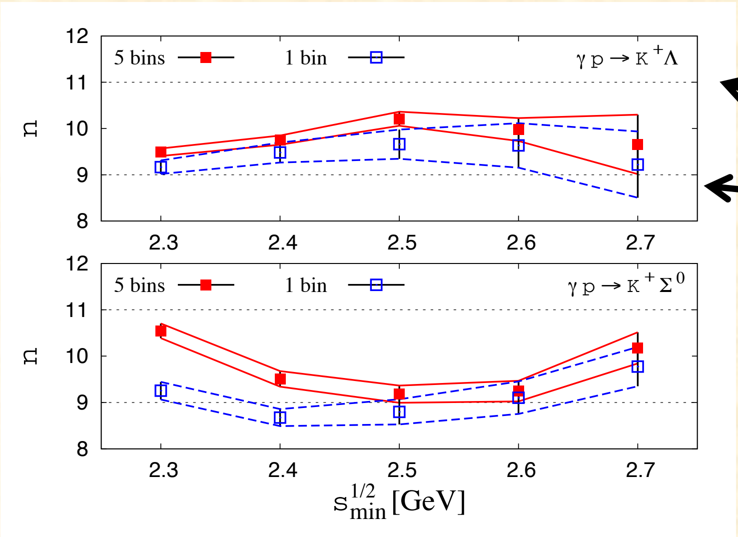
$\Lambda_{1405}$ = pentaquark ($\bar{K}N$ molecule) candidate

***e.g.* KEKB**



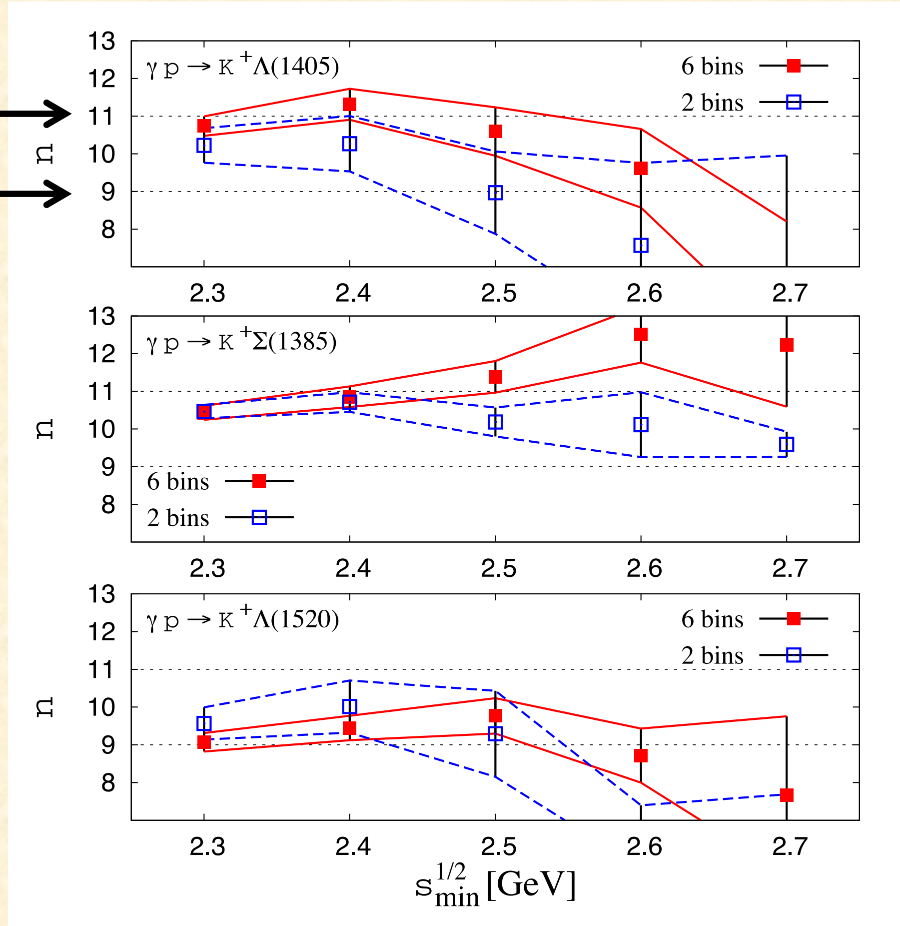

See H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006
for constituent-counting rule for exotic hadron candidates.

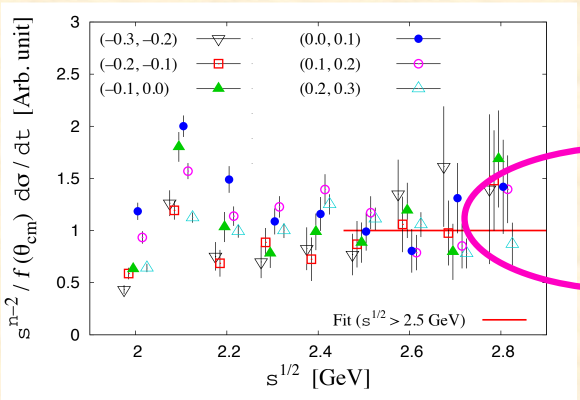
# JLab hyperon productions including Λ(1405)




$n_\Lambda = 5$

$n_\Lambda = 3$



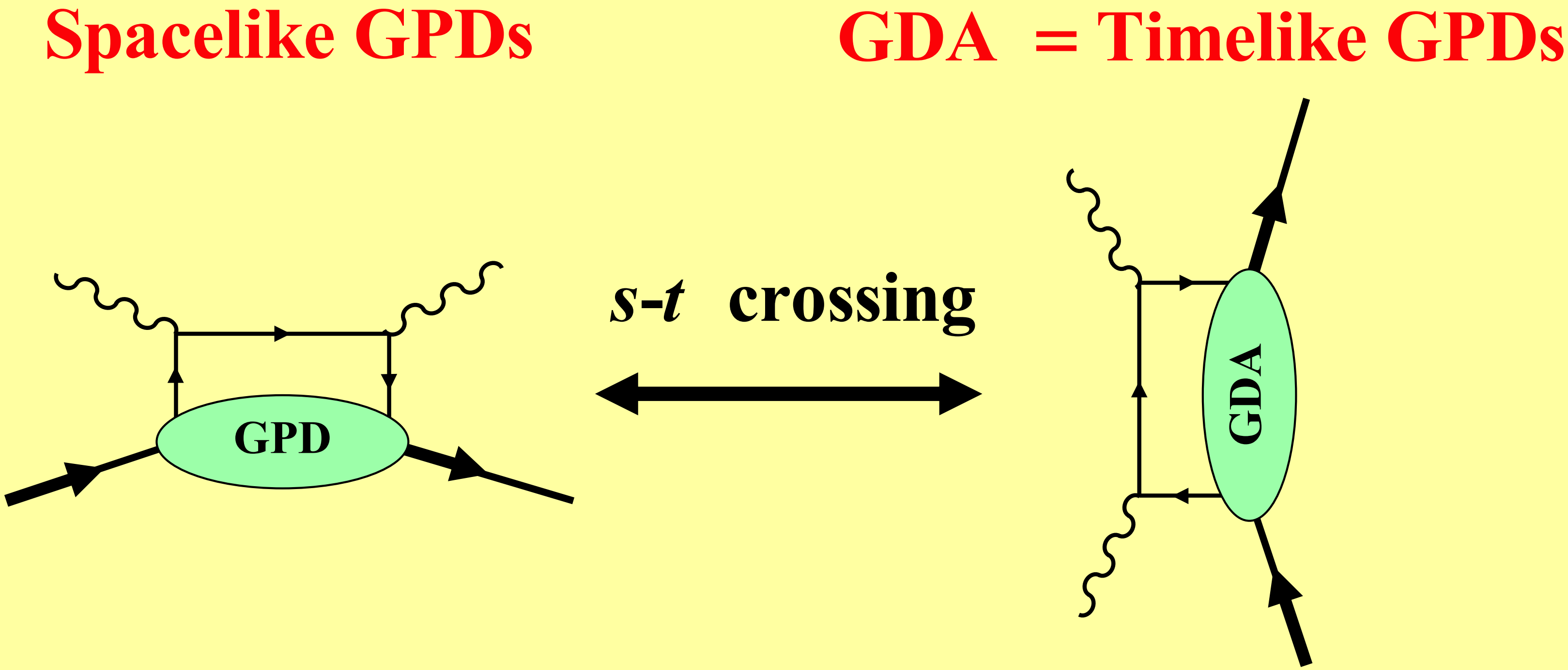

Range of
12 GeV JLab!

- Λ. Λ(1520) and Σ seem to be consistent with ordinary baryons with $n = 3$.
- Λ(1405) looks penta-quark at low energies but $n \sim 3$ at high energies???
- Σ(1385) :   $n = 5$ ???

→ In order to clarify the nature of Λ(1405) $\left[ qqq,\ \bar{K}N,\ qqqq\bar{q} \right]$, the JLab 12-GeV experiment plays an important role!

W.-C. Chang, SK, T. Sekihara, PRD 93 (2016) 034006.

# Generalized Distribution Amplitudes (GDAs) and extraction of gravitational form factors from KEKB data




SK, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020.

# GPD $H_q^h(x,\xi,t)$ and GDA( = timelike GPD) $\Phi_q^{hh}(z,\zeta,W^2)$

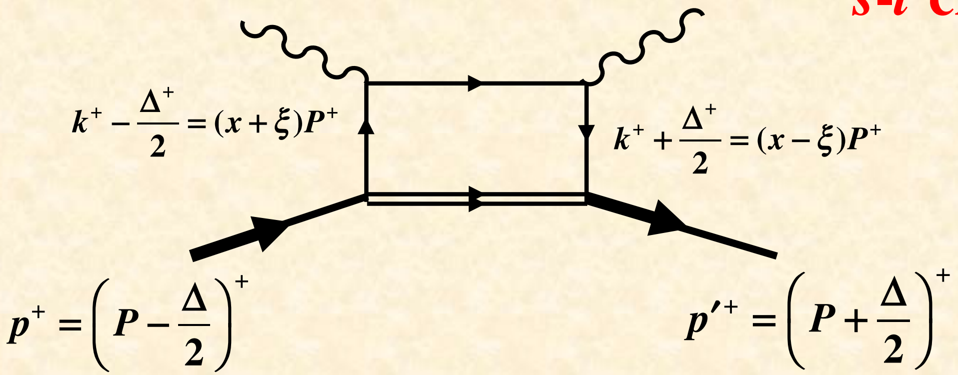
GPD: $$H_q(x,\xi,t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | h(p) \rangle \Big|_{y^+=0,\vec{y}_\perp=0}, \qquad P^+ = \frac{(p+p')^+}{2}$$

GDA: $$\Phi_q(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p)\bar{h}(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | 0 \rangle \Big|_{y^+=0,\vec{y}_\perp=0}$$

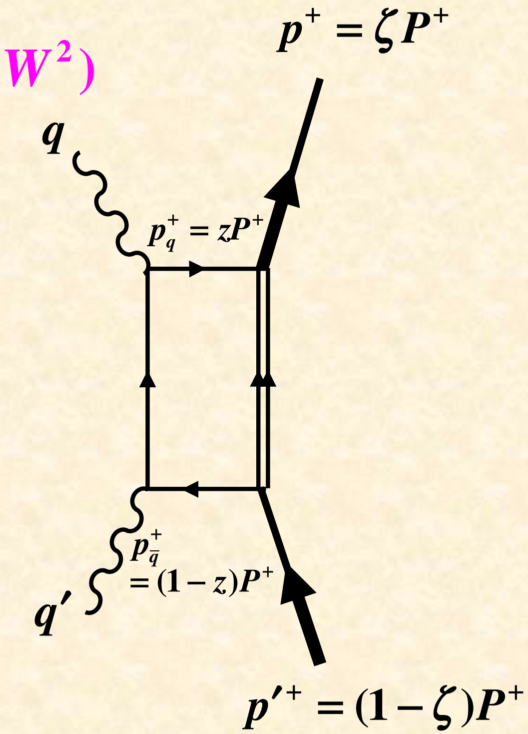
DA: $$\Phi_q^\pi(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2)\gamma^+\gamma_5\psi(y/2) | 0 \rangle \Big|_{y^+=0,\vec{y}_\perp=0}$$

$H_q^h(x,\xi,t)$ $\longleftrightarrow$ $\Phi_q^{hh}(z,\zeta,W^2)$

***s-t* crossing**




$$z \Leftrightarrow \frac{1 - x/\xi}{2}$$
$$\zeta \Leftrightarrow \frac{1 - 1/\xi}{2}$$
$$W^2 \Leftrightarrow t$$




**JLab / COMPASS**

**KEKB**

$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable: $x = \dfrac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

Skewdness parameter: $\xi = \dfrac{p^+ - p'^+}{p^+ + p'^+} = -\dfrac{\Delta^+}{2P^+}$

Bjorken variable for $\gamma\gamma^*$: $z = \dfrac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \dfrac{p^+}{P^+} = \dfrac{1+\beta\cos\theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

# Cross section for $\gamma^* \gamma \to \pi^0 \pi^0$

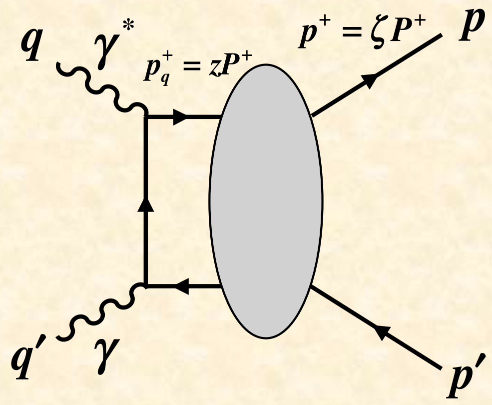
$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}\sum_{\lambda,\lambda'}|\mathcal{M}|^2$$

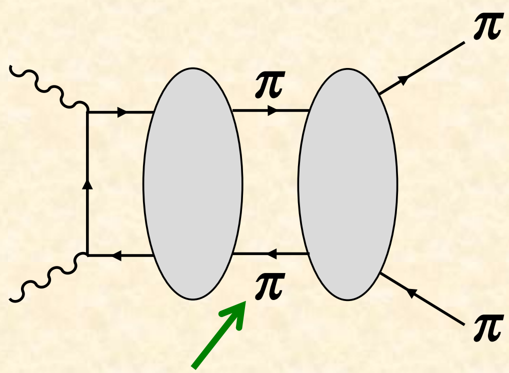
$$\mathcal{M} = \varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i\int d^4\xi\, e^{-i\xi\cdot q}\left\langle \pi(p)\pi(p')\left| T J_{em}^\mu(\xi) J_{em}^\nu(0)\right| 0\right\rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2}\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = -\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')g_T^{\mu\nu}\sum_q \frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

GDA (timelike GPD): $\Phi_q^{\pi\pi}(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-}\langle \pi(p)\pi(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|0\rangle\big|_{y^+=0,\vec{y}_\perp=0}$

$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}|A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$




Including intermediate resonance contributions

- Continuum: GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N_\pi z^\alpha(1-z)^\alpha(2z-1)\zeta(1-\zeta)F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{\left[1+(s-4m_\pi^2)/\Lambda^2\right]^{n-1}},$$ $n=2$ according to constituent counting rule

- Resonances: There exist resonance contributions to the cross section.

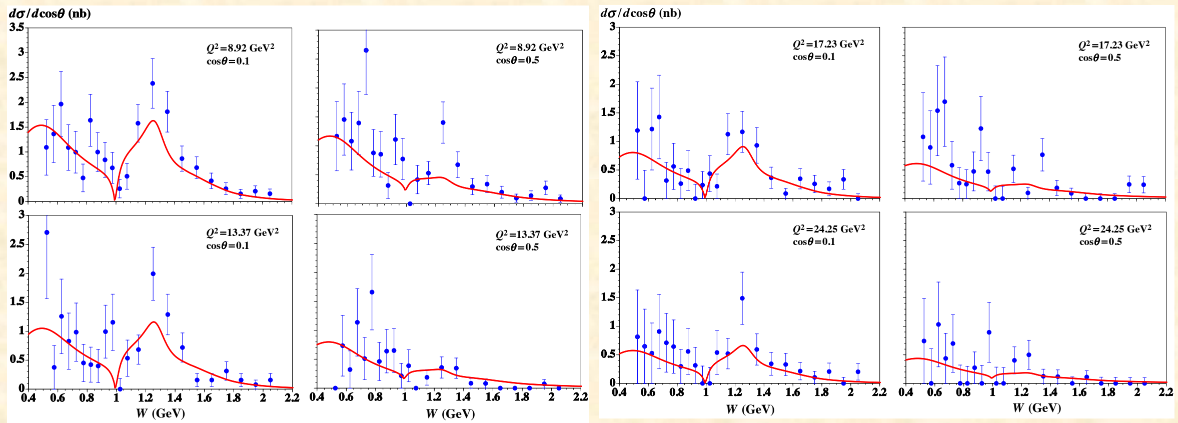
$$\sum_q \Phi_q^{\pi\pi}(z,\zeta,W^2) = 18N_f z^\alpha(1-z)^\alpha(2z-1)\left[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)\right]$$

$$P_2(x) = \frac{1}{2}(3x^2-1)$$

$\tilde{B}_{10}(W) =$ resonance $[f_0(500), f_0(980)]$ + continuum

$\tilde{B}_{12}(W) =$ resonance $[f_2(1270)]$ + continuum

$Q^2 = 8.92,\ 13.37\ \text{GeV}^2$ $\qquad$ $Q^2 = 17.23,\ 24.25\ \text{GeV}^2$




Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.

# Gravitational form factors and radii for pion

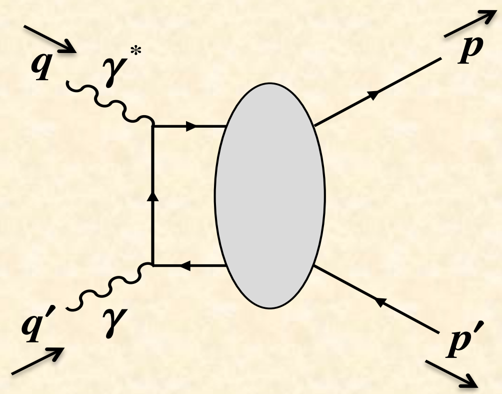
$$\int_0^1 dz(2z-1)\Phi_q^{\pi^0\pi^0}(z,\zeta,s) = \frac{2}{(P^+)^2}\langle \pi^0(p)\pi^0(p')|T_q^{++}(0)|0\rangle$$

$$\langle \pi^0(p)\pi^0(p')|T_q^{\mu\nu}(0)|0\rangle = \frac{1}{2}\Big[\big(sg^{\mu\nu} - P^\mu P^\nu\big)\Theta_{1,q}(s) + \Delta^\mu\Delta^\nu\Theta_{2,q}(s)\Big]$$

$$P = \frac{p+p'}{2}, \qquad \Delta = p' - p$$

$T_q^{\mu\nu}$ : energy-momentum tensor for quark

$\Theta_{1,q}$, $\Theta_{2,q}$ : gravitational form factos for pion




See also Hyeon-Dong Son, Hyun-Chul Kim, PRD90 (2014) 111901.

**Analysiss of $\gamma^*\gamma \to \pi^0\pi^0$ cross section**

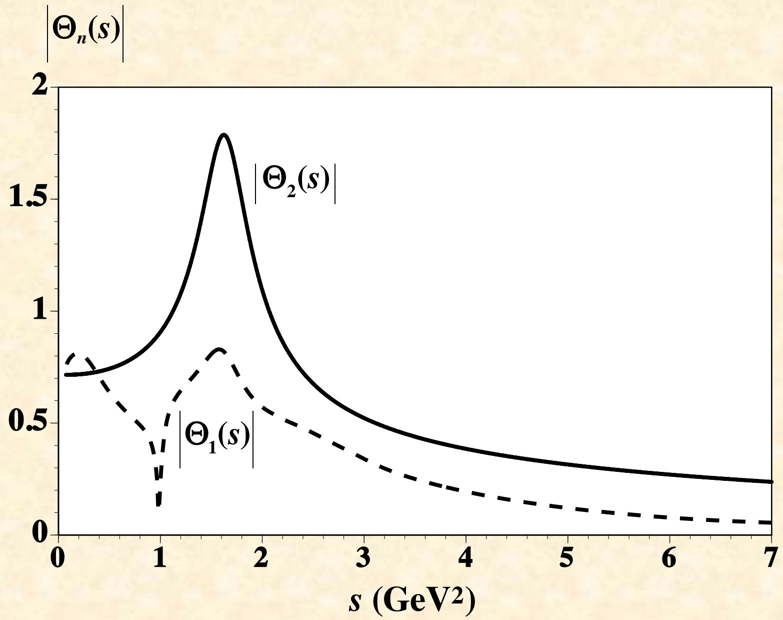
$\Rightarrow$ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z,\zeta,s)$

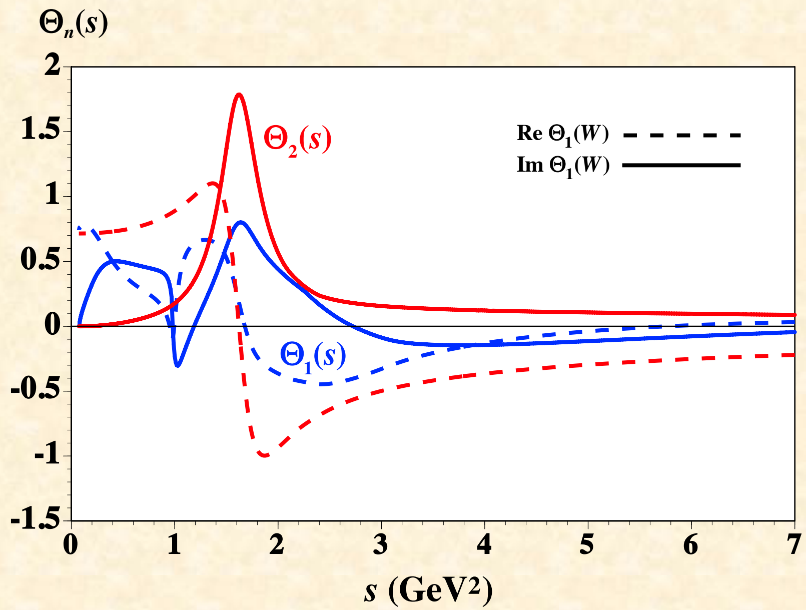
$\Rightarrow$ Timelike gravitational form factors $\Theta_{1,q}(s)$, $\Theta_{2,q}(s)$

$\Rightarrow$ Spacelike gravitational form factors $\Theta_{1,q}(t)$, $\Theta_{2,q}(t)$

$\Rightarrow$ Gravitational radii of pion

Gravitational form factors:
Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.
Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009;
Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12 (2018) 008;
K. Tanaka, JHEP 01 (2019) 120.



# Spacelike gravitational form factors and radii for pion

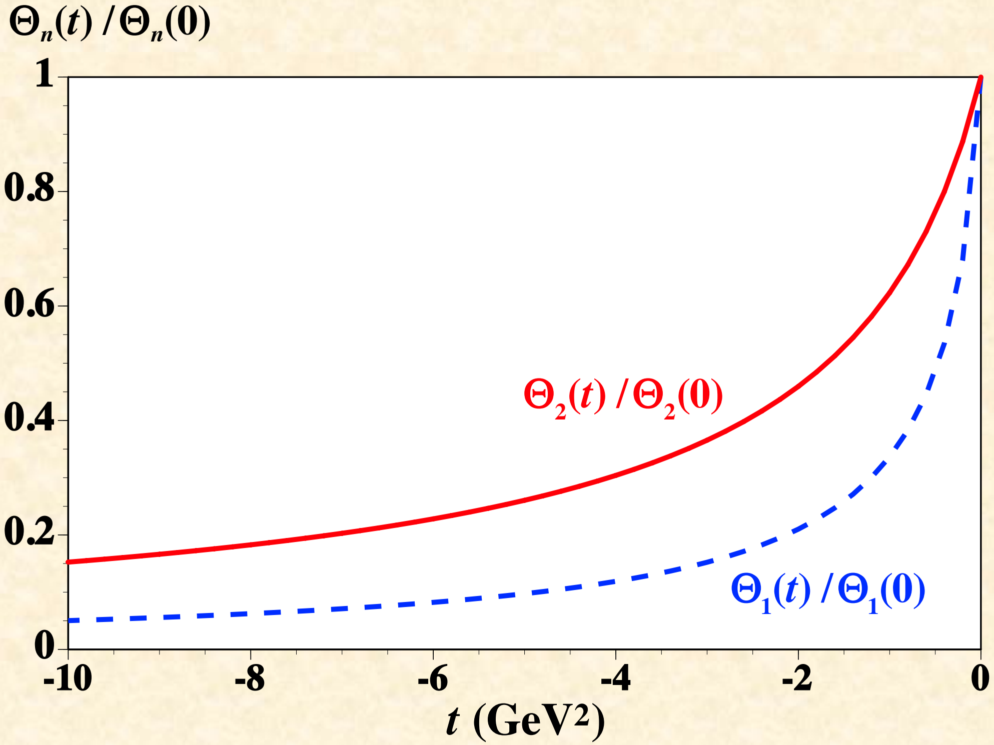
$$F(s) = \Theta_1(s),\ \Theta_1(s),\quad F(t) = \int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r) = \frac{1}{(2\pi)^3}\int d^3q e^{-i\vec{q}\cdot\vec{r}} F(q) = \frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\, e^{-\sqrt{s}r}\, \mathrm{Im}\,F(s)$$

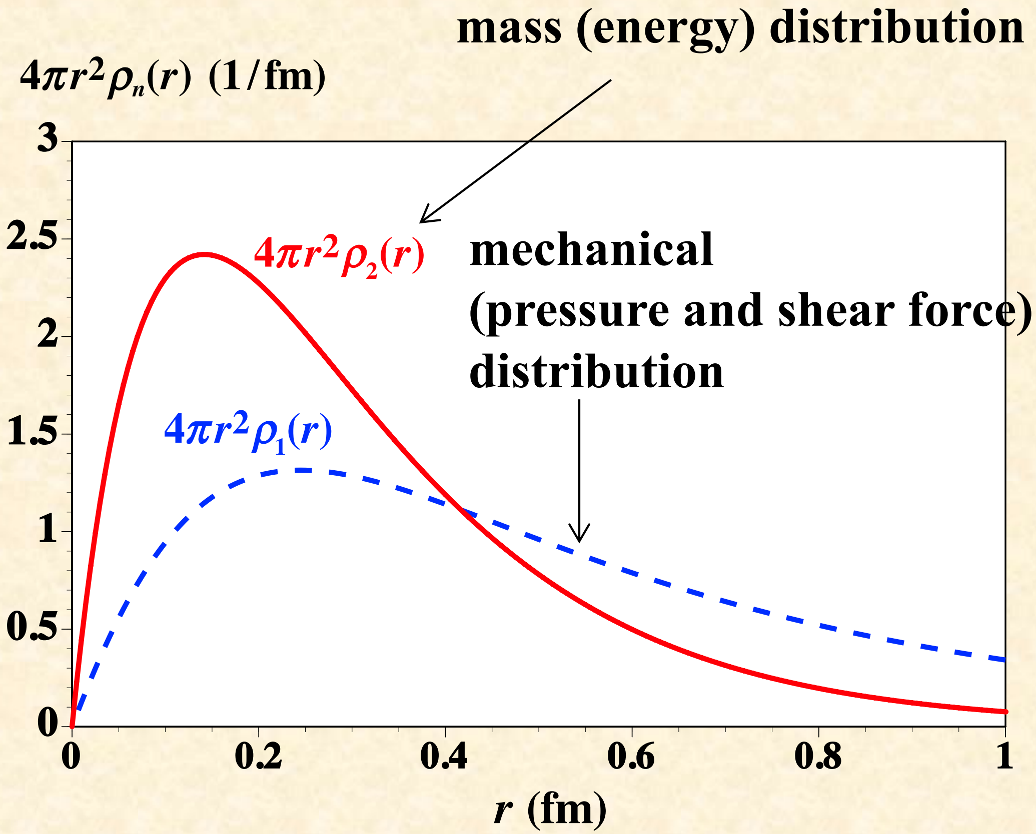
**This is the first report on gravitational radii of hadrons from actual experimental measurements.**

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm},\quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

First finding on gravitational radius from actual experimental measurements

$$\Leftrightarrow\ \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

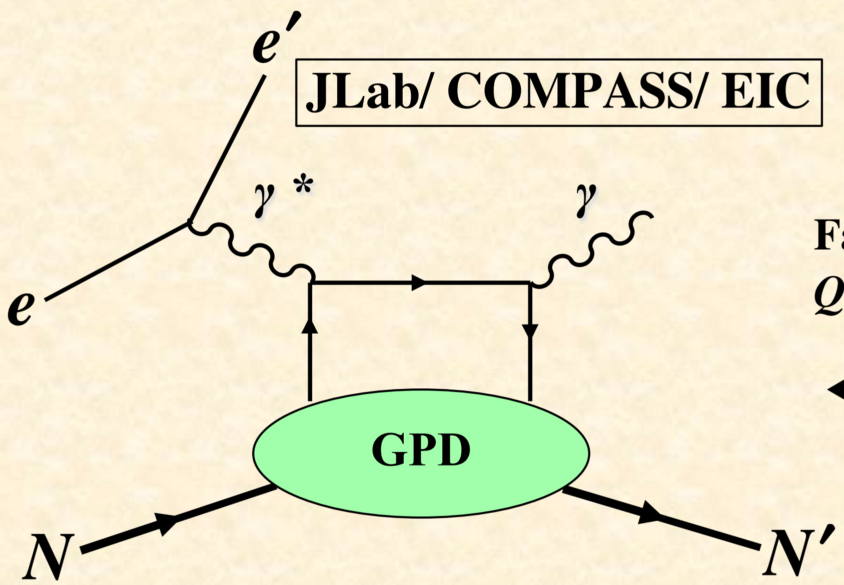


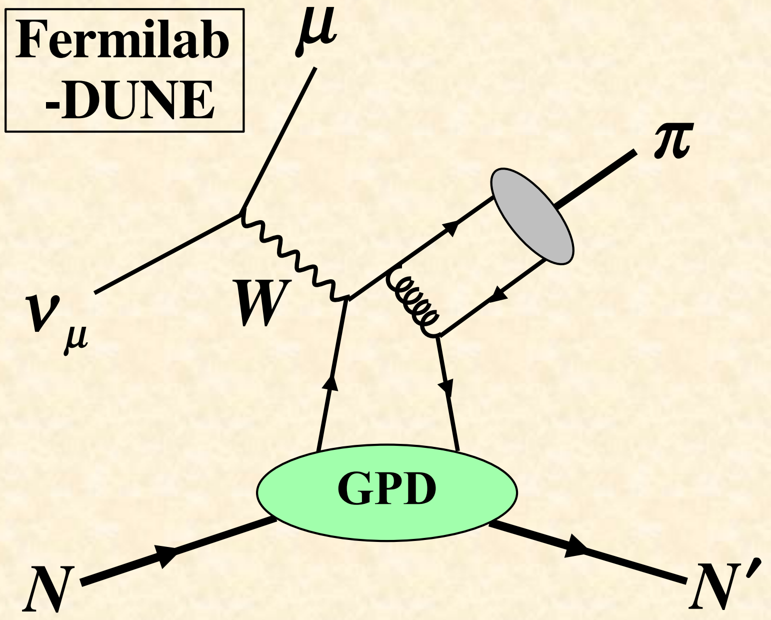
# Possible studies on GPDs at neutrino facilities

- SK,  EPJ Web Conf. 208 (2019) 07003.
- EIC yellow report, R. Abdul Khalek *et al*., arXiv:2103.05419,
  Sec. 7.5.2, Neutrino physics by SK and R. Petti.
- SK and R. Petti, PoS (NuFact2021) 092.
- R. Kunitomo, Bachelor's thesis (2023).
- in progress with X. Chen, S. Wu, Y.-P. Xie

# Neutrino reactions for gravitational form factors @Fermilab-DUNE (Origins of hadron masses and pressures)

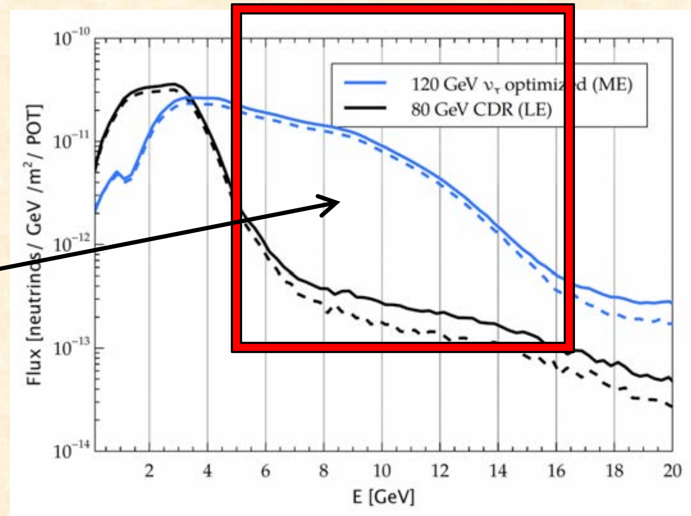



**Factorization condition:**
$Q^2 \gg |t|, \Lambda^2_{QCD}$




**Deep Underground Neutrino Experiment (DUNE) at Long-Baseline Neutrino Facility (LBNF)**

**High-energy part of the LBNF ν beam can be used for the GPD studies.**




**J. Rout *et al.*, PRD 102 (2020) 116018**

# Cross section formalism

B. Pire, L. Szymanowski, J. Wagner, Phys. Rev. D 95, 114029 (2017).

**Cross section**

$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} = \Gamma\, \varepsilon\, \sigma_L, \quad \varepsilon \simeq \frac{1-y}{1-y+y^2/2}, \quad \Gamma = \frac{G_F^2 Q^2}{32\,(2\pi)^4 (s-m_N^2)^2\, y\,(1-\varepsilon)\sqrt{1+4x^2 m_N^2/Q^2}}$$

$$\sigma_L = \varepsilon_L^{*\mu} W_{\mu\nu} \varepsilon_L^{\nu} = \frac{1}{Q^2}\Bigg[(1-\xi^2)\left\{\left|C_q\mathcal{H}_q + C_g\mathcal{H}_g\right|^2 + \left|C_q\widetilde{\mathcal{H}}_q\right|^2\right\} + \frac{\xi^4}{1-\xi^2}\left\{\left|C_q\mathcal{E}_q + C_g\mathcal{E}_g\right|^2 + \left|C_q\widetilde{\mathcal{E}}_q\right|^2\right\}$$
$$-2\xi^2\,\mathrm{Re}\left\{(C_q\mathcal{H}_q + C_g\mathcal{H}_g)(C_q\mathcal{E}_q + C_g\mathcal{E}_g)^*\right\} - 2\xi^2\,\mathrm{Re}\left\{C_q\widetilde{\mathcal{H}}_q(C_q\widetilde{\mathcal{E}}_q)^*\right\}\Bigg]$$

**Quark contributions**

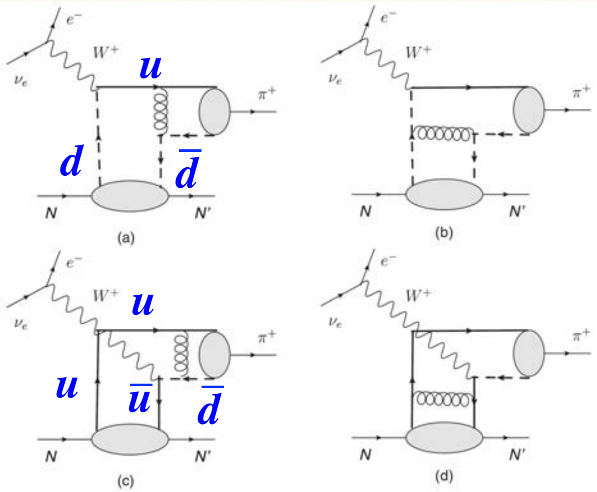
$$T_q = -i\frac{C_q}{2Q} N(p')\left[\mathcal{H}_q \hat{n} + \mathcal{E}_q \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} - \widetilde{\mathcal{H}}_q \hat{n}\gamma_5 - \widetilde{\mathcal{E}}_q \frac{\gamma_5 n\cdot\Delta}{2m_N}\right] N(p)$$

$$\mathcal{F}_q = 2 f_\pi \int \frac{dz\, \phi_\pi(z)}{1-z} \int dx \frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

$$= (\text{pion distribution amplitude}) \cdot (\text{quark GPD})$$

$$F_q(x,\xi,t) \equiv F_d(x,\xi,t) - F_u(-x,\xi,t)$$

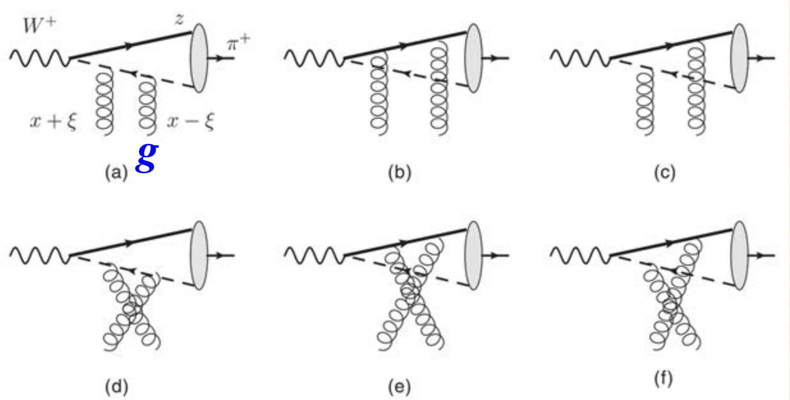
$$F = H,\, E,\, \tilde{H},\, \tilde{E}$$




**Gluon contributions**

$$T_g = -i\frac{C_g}{2Q} N(p')\left[\mathcal{H}^g \hat{n} + \mathcal{E}^g \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N}\right] N(p)$$

$$\mathcal{F}_g = \frac{8 f_\pi}{\xi} \int \frac{dz\, \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$

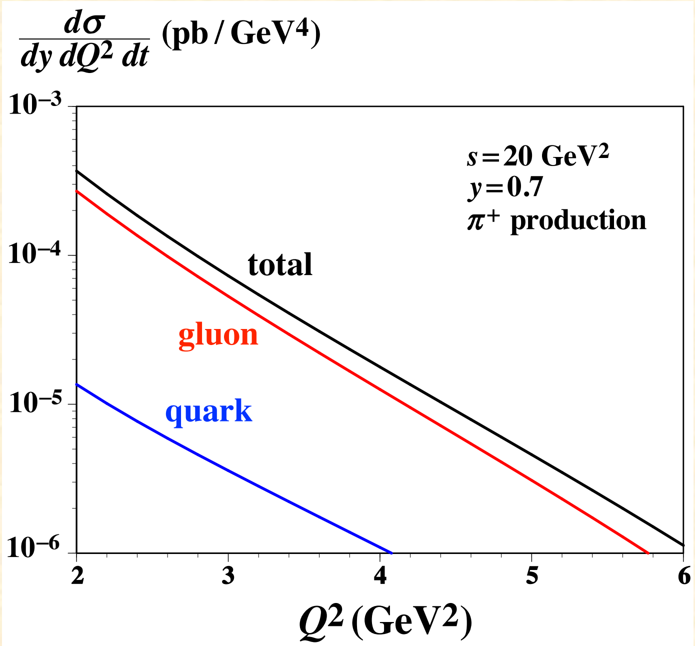


# Cross sections

SK and R. Kunitomo (2023)
in progress with
X. Chen, S. Wu, Y.-P. Xie

## $\pi^+$ production: $\nu p \to \ell^- \pi^+ p$

gluon $\gg$ quark



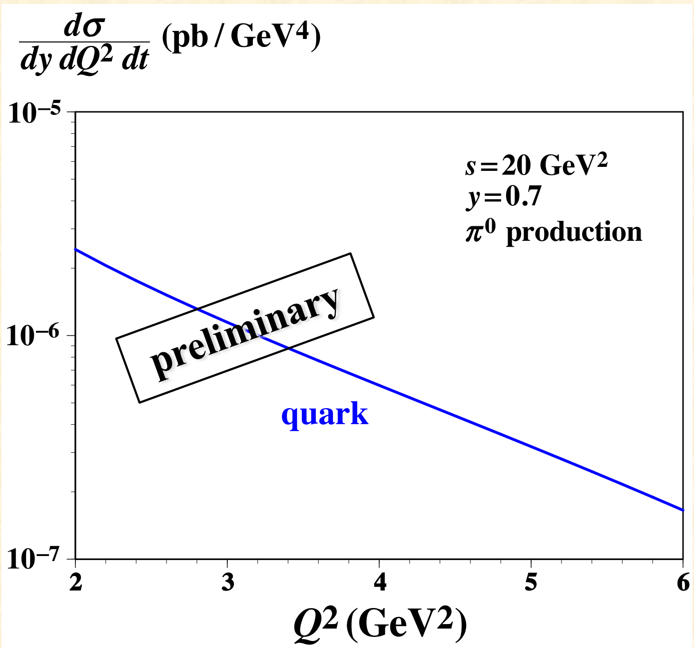

$$\mathcal{F}_q = 2 f_\pi \int \frac{dz\, \phi_\pi(z)}{1-z} \int dx \frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

$$\mathcal{F}_g = \frac{8 f_\pi}{\xi} \int \frac{dz\, \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$

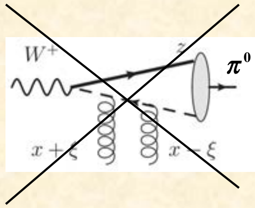
$$\frac{\mathcal{F}_q}{\mathcal{F}_g} \sim \frac{\xi}{8} = \frac{0.1 \sim 0.3}{8} = 0.01 \sim 0.04$$

$$= \text{ a few } \% \ll 1$$

## $\pi^0$ production: $\nu n \to \ell^- \pi^0 p$




no gluon



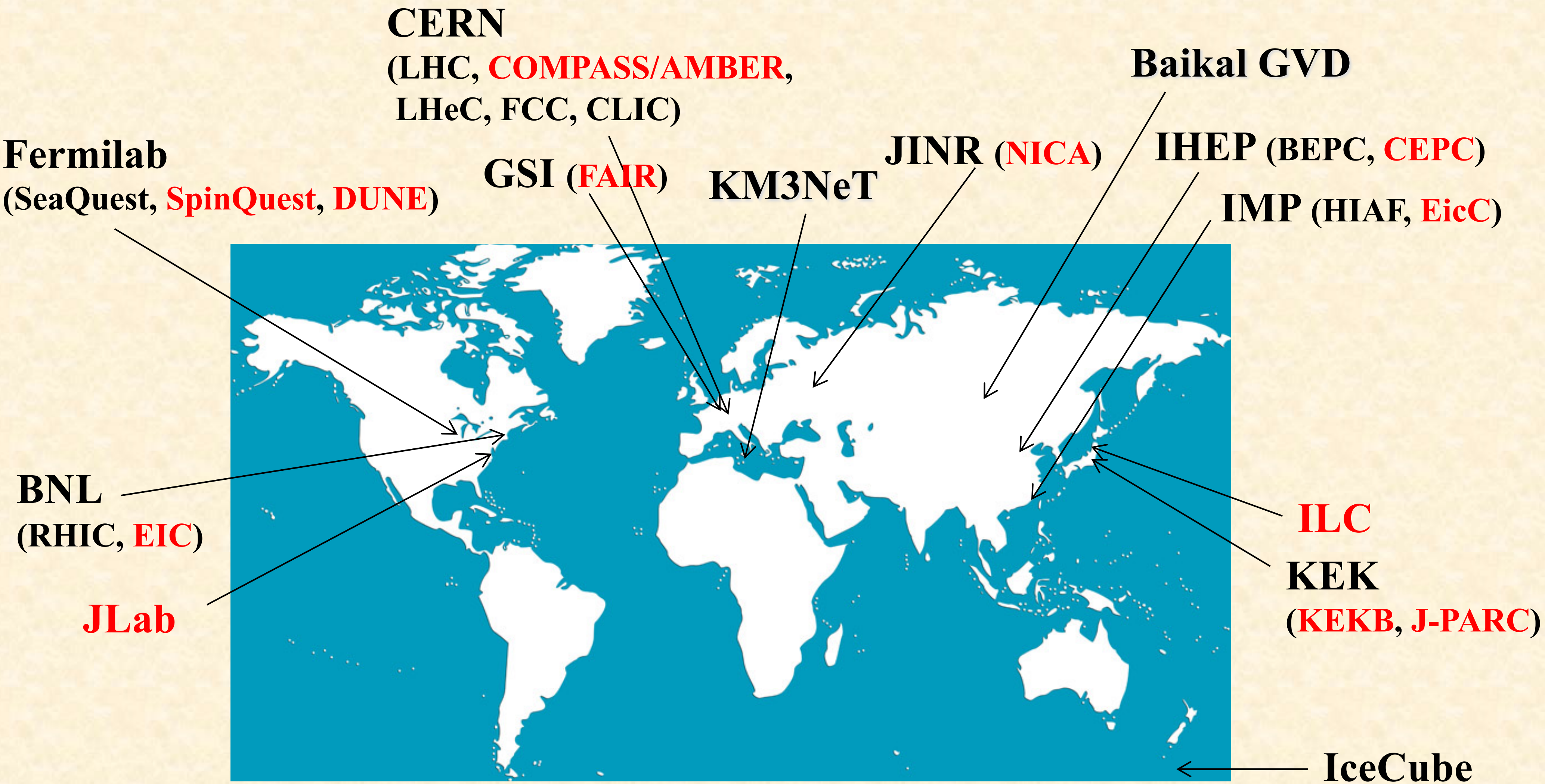

no gluon for $\pi^0$

**Neutrino GPD studies are complementary to the charged-lepton projects.**

- Gluon GPDs could be probed in charged-pion production.
- Quark GPDs could be probed in charged-pion production
- Flavor dependece of quark GPDs could be investigated.

# Future prospects on GPD projects

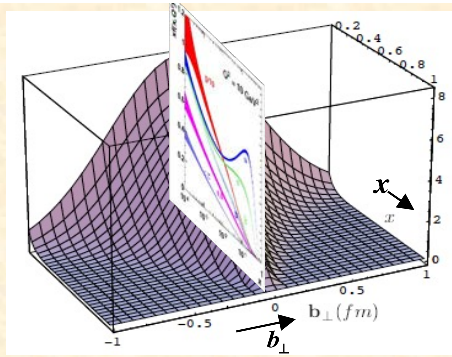
# High-energy hadron physics experiments




Facilities on hadron structure functions on GPDs including future possibilities.

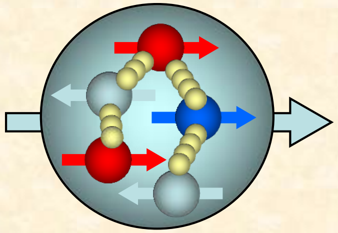
# By hadron tomography


## 3D view of hadrons

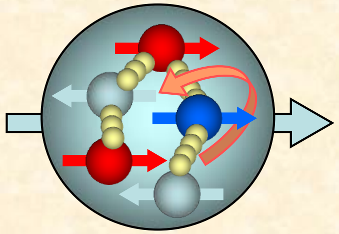


## Origin of nucleon spin

By the tomography, we determine



or



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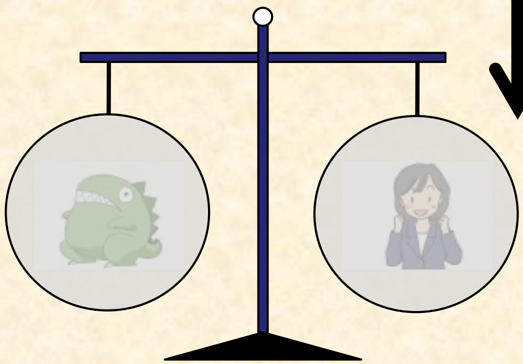
## Exotic hadrons



By tomography, we determine

or

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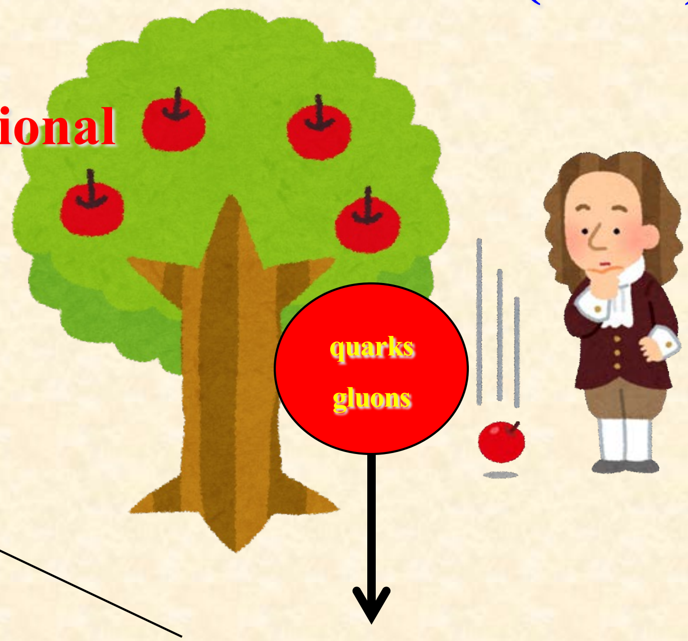
## Origin of gravitational source (mass)

By tomography, we determine gravitational sources in terms of quarks and gluons.



# Summary on GPDs

## Hadron-tomography and gravitational form factors

- Puzzle to find the origin of hadron masses and pressures
  in terms of quark and gluon degrees of freedom

- Puzzle to find the origin of nucleon spin

- Exotic hadron candidates could be studied
  in the same tomography method.

- There are world-wide lepton and hadron accelerator facilities
  which has been used and could be used in future for our studies.

Time has come to understand the gravitational sources
in microscopic (instead of usual macroscopic/cosmic)
world in terms of quark and gluon degrees of freedom.

**The End**

**The End**