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Scattering Experiment and Lattice QCD'

Deep-inelastic scattering and Drell-Yan process for the 3D structure of the nucleon

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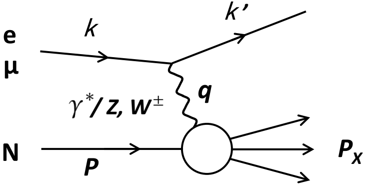
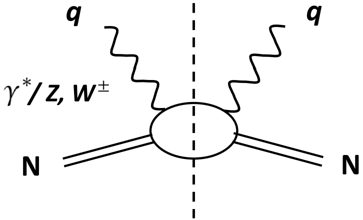
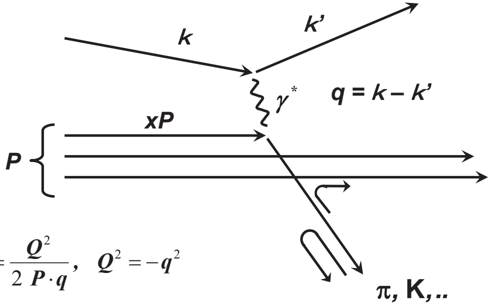
1.1 Deep-inelastic scattering

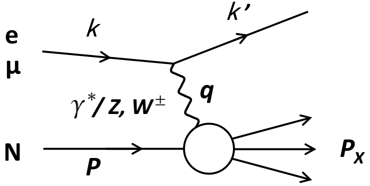




Total absorption cross section of a virtual photon
$\propto$ Imaginary part of forward Compton scattering



$$x = \frac{Q^2}{2\ P \cdot q}, \quad Q^2 = -q^2$$



$$s = (k + P)^2 \quad \text{center of mass energy squared,}$$

$$q = k - k' \qquad \text{momentum transfer}$$

$$t = q^2 = (k - k')^2 \equiv -Q^2,$$

$$\nu = \frac{P\,q}{M},$$

$$x = \frac{Q^2}{2P\,q} = \frac{Q^2}{2M\nu} \quad \text{Bjorken } x \;\; 0 \leq x \leq 1,$$

$$y = \frac{P\,q}{P\,k}$$

$$\begin{aligned} W^2 &= (P+q)^2 = M^2 + 2P\,q + q^2 = M^2 + 2M\nu - Q^2 \\ &= M^2 + 2M\nu\,(1-x), \\ & x \le 1, \\ & x = 1 \quad \leftrightarrow \quad W = M \qquad \text{elastic scattering} \end{aligned}$$

Only 3 out of s, t, ν, x, y, W^2 are independent parameters.

Comparison: Only 2 out of s, t, u are independent parameters in two body scattering when the masses are known.
$s + t + u = \sum_i m_i^2 c^4$.

$$\begin{aligned} k^2 = k'^2 = m_e^2 &\approx 0, \quad E \approx |\vec{k}|, \;\; E' \approx |\vec{k}|, \\ t &= (k-k')^2 = k^2 + k'^2 - 2kk' \approx -2kk', \\ Q^2 &= -t \approx 2kk' = 2\,(EE' - \vec{k}\cdot\vec{k'}) \approx 4EE' \sin^2\frac{\theta}{2}. \end{aligned}$$

In the frame where the initial nucleon is at rest, $P = (M, 0, 0, 0)$.

$$\nu = \frac{P\,q}{M} = E - E' \quad \text{energy transfer}$$

$$y = \frac{P\,q}{P\,k} = \frac{E - E'}{E} \quad \text{fraction of energy transfer}$$

$$\nu > 0 \quad \rightarrow \quad x = \frac{Q^2}{2M\nu} > 0.$$

In the frame where the nucleon is moving very fast, $E \approx |\vec{P}|$, x can be interpreted to be the momentum fraction of the parton in the nucleon.

Cross section and Structure functions

$$\frac{d^2\sigma}{dE'd\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \cdot \left[\frac{F_2(x, Q^2)}{\nu} + 2\frac{F_1(x, Q^2)}{Mc^2}\tan^2\frac{\theta}{2}\right]$$

The differential cross section can be converted by Jacobian determinant:

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{d^2\sigma}{dE'd\cos\theta\, d\phi} = \frac{d^2\sigma}{dx\, dy\, d\phi} \cdot \frac{E'}{M\nu}.$$

$$\frac{d^2\sigma}{dx\, dy\, d\phi} = \frac{d^2\sigma}{dx\, dQ^2\, d\phi} \cdot \frac{Q^2}{y}.$$

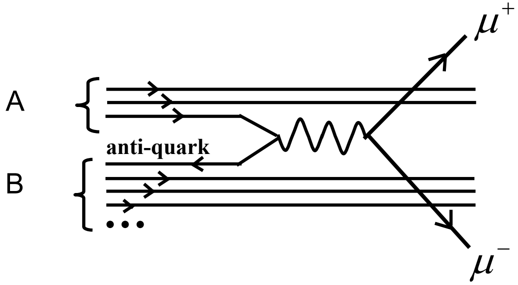
Using the quark-parton model, in the lowest order, parton distribution functions

$$F_2(x, Q^2) = x \cdot \left[e_u^2(u + \bar{u}) + e_d^2(d + \bar{d}) + e_s^2(s + \bar{s}) + ... \right]$$

e_u, e_d, e_s : electric charges of quarks

1.2 Drell-Yan process

$A + B \to \mu^+ + \mu^- + X$



$$\frac{d^2\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9x_1 x_2\, s} \sum_{i=u,d,s} e_i^2 \left[\, q_i^A(x_1)\bar{q}_i^B(x_2) + \bar{q}_i^A(x_1) q_i^B(x_2) \,\right]$$

$$x_1 = \frac{P_2 Q}{P_2 P}, \quad x_2 = \frac{P_1 Q}{P_1 P}$$

$$P = P_1 + P_2, \quad Q = p_{\mu+} + p_{\mu-} = p_{q_i} + p_{\bar{q}_i}.$$

$$x_1 = \frac{P_2 Q}{P_2 P} = \frac{P_2\,(p_{q_i} + p_{\bar{q}_i})}{P_2(P_1 + P_2)}, \qquad x_2 = \ldots$$

■

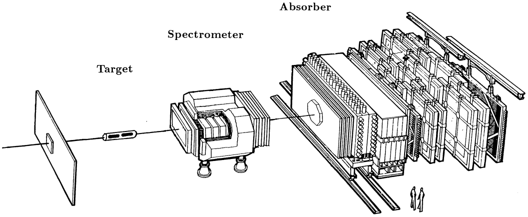
Deep-inelastic scattering | Drell-Yan process

$e_i^2\,[q_i(x) + \bar{q}_i(x)]$ | $e_i^2\left[q_i^A(x_1)\bar{q}_i^B(x_2) + \bar{q}_i^A(x_1)q_i^B(x_2)\right]$

Combined analysis is most effective.

2. Helicity structure of the nucleon

Longitudinally polarized deep-inelastic scattering

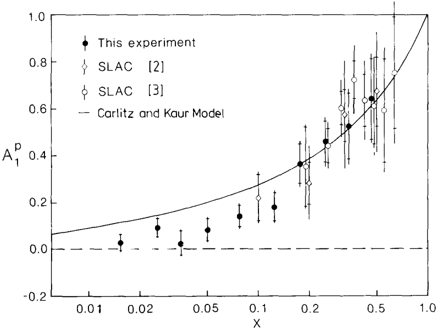


EMC

$$\mu + p, \qquad A_{LL} = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} \approx D\, A_1,$$

$$\gamma^* + p, \qquad A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \approx \frac{g_1(x)}{F_1(x)}$$

$E_\mu = 100,\ 120,\ 200$ GeV



EMC

$$g_1(x) = \frac{1}{2}\sum_i e_i^2 (q_i^{\uparrow}(x) - q_i^{\downarrow}(x))$$

$\Delta u - \Delta d$ neutron beta decay

$\Delta u + \Delta d - 2\Delta s$ hyperon weak decay

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.12 \pm 0.09 \pm 0.14$$

$$\Delta u \equiv \int_0^1 dx (u^{\uparrow}(x) - u^{\downarrow}(x) + \bar{u}^{\uparrow}(x) - \bar{u}^{\downarrow}(x)), \ldots$$

The contibution of spin of quarks and anti-quarks to the proton spin is $(12 \pm 9 \pm 14)\%$. EMC, Nucl. Phys. B 328 (1989) 1.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

The contibution of spin of quarks and anti-quarks to the proton spin is only $(12 \pm 9 \pm 14)\%$.

Contributions of spins of valence quarks are also small.

Then, what are the roles of valence quarks?
Quantum numbers of the proton are determined by the valence quarks !?

Sea quarks may contribute to determine the quantum numbers of the proton.

Search for contributions of orbital angular momenta, L_q, L_G

Search for contributions of gluon spin, ΔG
– deep-inelastic scattering, polarized proton-proton colliders.

Evaluation of $\Delta\Sigma$ from DIS on polarized deuteron.
Integral of $g_1^{\mathrm{d}}(x, Q^2)$

$$\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)_{\mathrm{p}} + \left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)_{\mathrm{n}}$$
$$\longrightarrow \frac{5}{9}\left(\Delta u + \Delta d + \frac{2}{5}\Delta s\right)_{\mathrm{p}}$$

After a correction for Δs, $\Delta\Sigma$ is obtained.
HERMES at DESY, Phys. Rev. D 75 (2007) 012007
0.33 ± 0.039 at $Q^2 = 5$ GeV^2, $0.05 < x < 1$,
COMPASS at CERN,Phys. Lett. B 647 (2007) 8
$0.35 \pm 0.03 \pm 0.05$ at $Q^2 = 3$ GeV^2, $0.004 < x < 0.7$,
to be compared to EMC $0.12 \pm 0.09 \pm 0.14$.

The contribution of spins of quarks and anti-quarks to the proton spin is about $\frac{1}{3}$.

The wave function of the proton in the simplest quark model

No orbital angular momentum, $\ell = 0$
No sea quarks
  No strange quarks
  No anti-quarks

The wave function of the nucleon is expressed as

$$\psi = \xi_{\text{space}} \cdot \eta_{\text{flavor}} \cdot \chi_{\text{spin}} \cdot \phi_{\text{color}}$$

Quarks are Fermi particles. The wave function changes its sign when any two quarks are exchanged.

ϕ_{color} is anti-symmetric.
ξ_{space} is symmetric as only $\ell = 0$ is involved.

As a result, $\eta_{\text{flavor}} \cdot \chi_{\text{spin}}$ is symmetric.

Combination of uud and $\uparrow\uparrow\downarrow$.

$$
\begin{aligned}
|p^{\uparrow} >= \sqrt{\frac{1}{18}}\,(|2u^{\uparrow}u^{\uparrow}d^{\downarrow} + 2u^{\uparrow}d^{\downarrow}u^{\uparrow} + 2d^{\downarrow}u^{\uparrow}u^{\uparrow} \\
- u^{\uparrow}u^{\downarrow}d^{\uparrow} - u^{\uparrow}d^{\uparrow}u^{\downarrow} - d^{\uparrow}u^{\downarrow}u^{\uparrow} \\
- u^{\downarrow}u^{\uparrow}d^{\uparrow} - u^{\downarrow}d^{\uparrow}u^{\uparrow} - d^{\uparrow}u^{\uparrow}u^{\downarrow} >).
\end{aligned}
$$

Expectation values of the spin operators

Expectation values of $\hat{S}_z^u, \ \hat{S}_z^d$:

$$< p^{\uparrow}|\hat{S}_z^u|p^{\uparrow} > = \frac{\hbar}{2} \cdot \frac{4}{3} \tag{1}$$
$$< p^{\uparrow}|\hat{S}_z^d|p^{\uparrow} > = \frac{\hbar}{2} \cdot \left(-\frac{1}{3}\right) \tag{2}$$

The spins of u quarks are parallel to the proton spin on average while the spin of d quark is anti-parallel on average.

The expectation value of $\hat{S}_z = \hat{S}_z^u + \hat{S}_z^d$ is

$$< p^{\uparrow}|\hat{S}_z|p^{\uparrow} >= \frac{\hbar}{2} \cdot \left(\frac{4}{3} - \frac{1}{3}\right) = \frac{\hbar}{2}.$$

The proton spin is 100% carried by the quark spins as is assumed in this model.

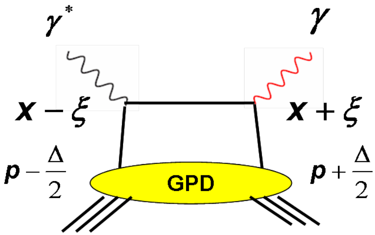
The results of the experiments are very different from this.

3. Generalized parton distributions

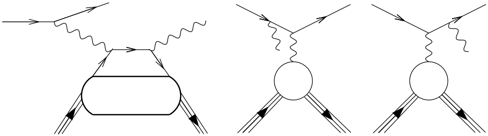
Deeply virtual Compton scattering: $e + N \to e' + \gamma + N$,
Hard exclusive meson production: $e + N \to e' + \text{meson} + N$.
$H(x,\xi,t)$, $E(x,\xi,t)$, $\tilde{H}(x,\xi,t)$, $\tilde{E}(x,\xi,t)$

$$J_{q,G} = \lim_{t\to 0} \int dx\, x \cdot \left[H^{q,G}(x,\xi,t) + E^{q,G}(x,\xi,t)\right]$$

X.D. Ji, Phys. Rev. Lett. 78 610 (1997)

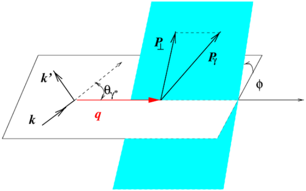


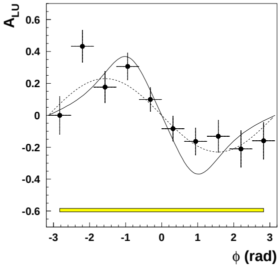
Interference between DVCS and Bethe-Heitler process

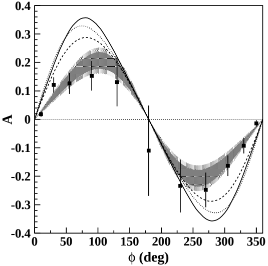


(a) (b)

$$A_{LU}(\phi) = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \mathrm{Im}\,(F \cdot H)\sin\phi$$







Beam-spin asymmetry in DVCS.

HERMES, Phys. Rev. Lett. 87, 182001 (2001)

CLAS, Phys. Rev. Lett. 87, 182002 (2001)

4. Transverse-momentun dependent parton distributions

Sivers function:

$$f_{q/p^\uparrow}(x, k_T) = f_1^q(x, k_T^2) - f_{1T}^{\perp q}(x, k_T^2)\,\frac{(\hat{P} \times k_T) \cdot S}{M}$$

$$\Delta^N f_{q/p^\uparrow}(x, k_T^2) = -\frac{2|k_T|}{M} f_{1T}^{\perp q}(x, k_T^2)$$

Boer-Mulders function:

$$f_{q^\uparrow/p}(x, k_T) = \frac{1}{2}\left(f_1^q(x, k_T^2) - h_1^{\perp q}(x, k_T^2)\,\frac{(\hat{P} \times k_T) \cdot S_q}{M} \right)$$

$$\Delta^N f_{q^\uparrow/p}(x, k_T^2) = -\frac{|k_T|}{M} h_1^{\perp q}(x, k_T^2)$$

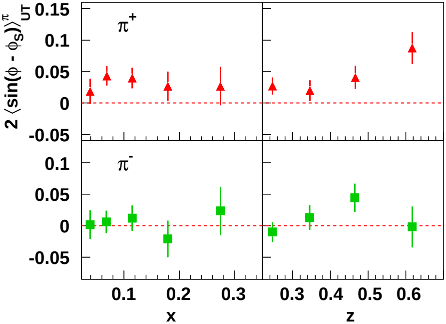
A. Bacchetta et al., Phys. Rev. D70. 117504 (2004)

Single spin asymmetry

$p + p \to \pi^{\pm,0} + X$, The hard scale is determined by p_T.

DIS The hard scale is determined by Q^2. p_T can be low.

Sivers asymmetry:



HERMES, Phys. Rev. Lett. 94 012002 (2005)

5. Summary

- Deep-inelastic scattering and Drell-Yan process are complementary approaches to study the partonic structure of the nucleon.

  $q(x) + \bar{q}(x)$ and $q(x)\bar{q}(x)$.

  Combined analyses are most effective.

- After the pioneering works of electron DIS at SLAC, the polarised muon beam of a few hundreds GeV enabled us to extend the kinematic region of the spin experiments, in particular to the low x region.

- The contribution of spins of quarks and anti-quarks to the proton spin is about $1/3$.

- The large x region was explored by high intensity electron beams.

- The generalized parton distributions were studied by the inteference between DVCS and Bethe-Heitler process.

- DVCS and HEMP require to confirm that the events are exclusive. The experimental methods have been developped.

- Various ways to access TMD's via single spin asymmetry have been studied.

- Future plans such as EIC are much expected as new steps to extend the studies in this field.