# Single Diffractive Hard Exclusive Scattering for studying Generalized Parton Distributions (GPDs)

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In collaboration with Jianwei Qiu

JHEP 08 (2022) 103, PRD 107 (2023) 014007, PRL 131 (2023) 161902

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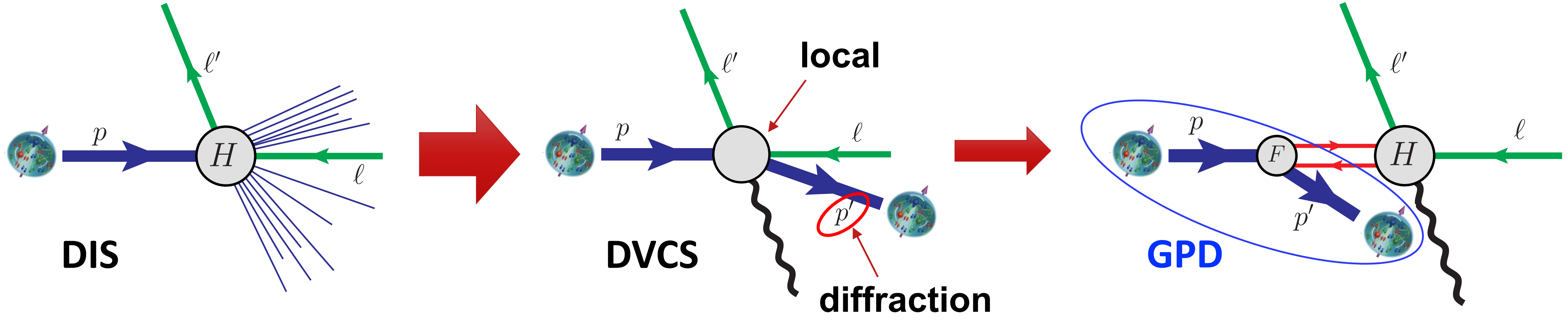
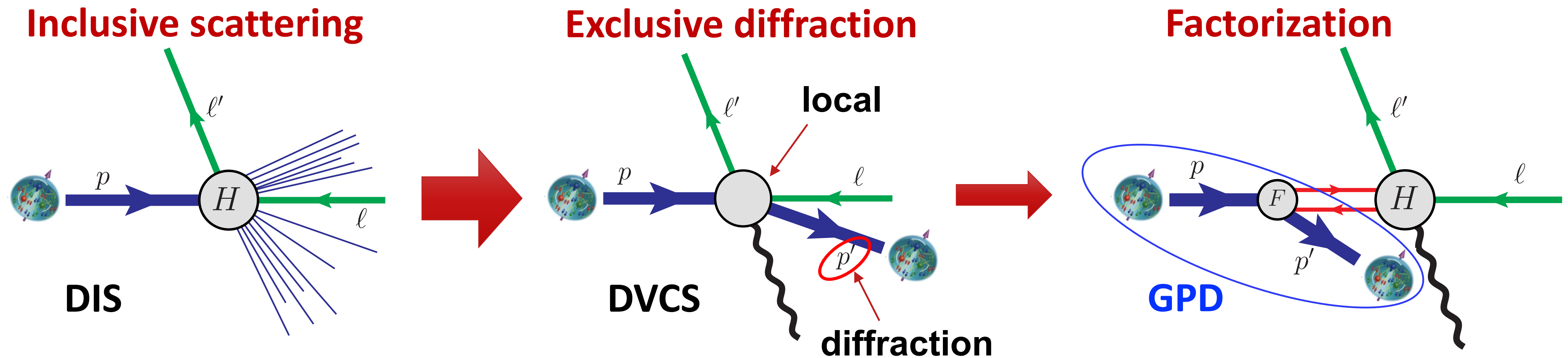
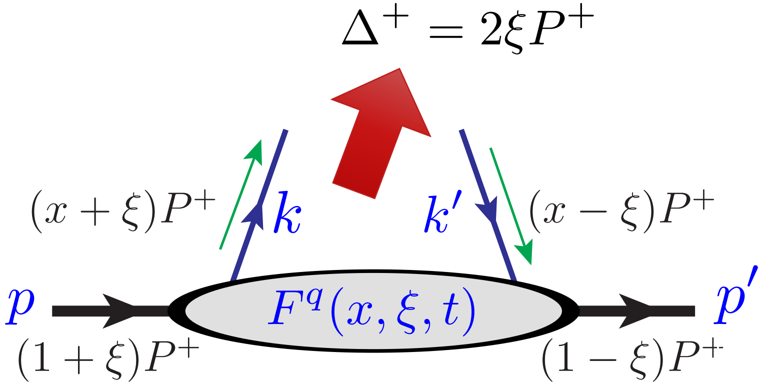
# Exclusive Processes and GPDs

**Inclusive scattering** | **Exclusive diffraction** | **Factorization**



# Exclusive Processes and GPDs




$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+ q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+}\left[ H^q(x,\xi,t)\, \bar{u}(p')\, \gamma^+ u(p) - E^q(x,\xi,t)\, \bar{u}(p')\, \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \right]$$

$$P = \frac{p+p'}{2}$$

$$\Delta = p - p'$$

$$t = \Delta^2$$

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

**Hadron diffraction** $p \to p'$

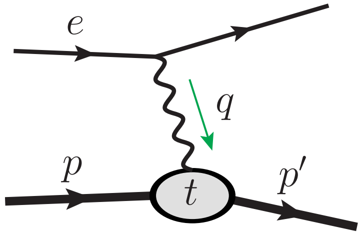
$$x = \frac{(k+k')^+}{(p+p')^+}$$

← **parton momentum**

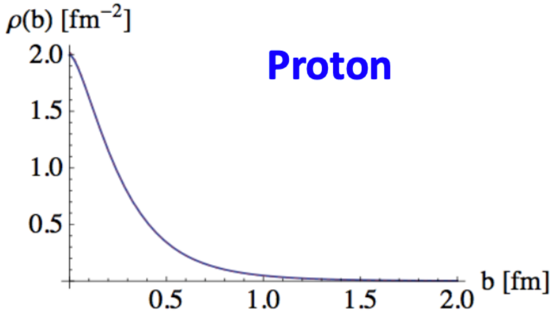
# GPD and QCD Tomography

## ❑ Controllable soft scale $t$

**Elastic EM form factor**



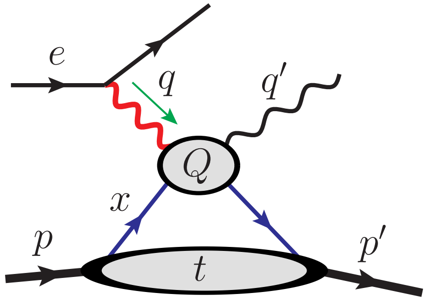

**F. T.** → $F_{1,2}(t)$




- **Flavor, color blind**
- **EM charge radius**
- **No color elastic form factor**

→ **no color radius**

**DVCS**



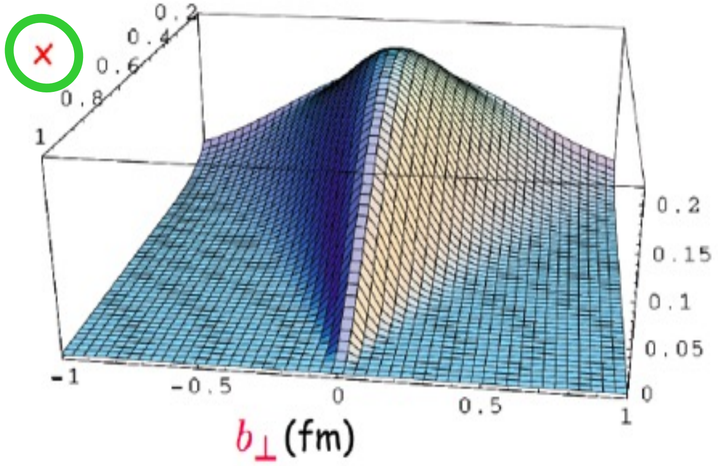

**Two distinct scales at the same time**

- **Hard $Q$: see partons (with $x$)**
- **Low $t$: probe the confined motion ($b_T$)**

$$f_i(x, \boldsymbol{b}_T) = \int \mathrm{d}^2\boldsymbol{\Delta}_T \, e^{i\boldsymbol{\Delta}_T\cdot \boldsymbol{b}_T} F_i(x, 0, -\Delta_T^2)$$

**Parton density in** $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$

**3D image** →




**"Color" density**

↓

**confinement;**
**nuclear force;**
**color radius…**



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# GPD and Hadronic Property

## ❑ QCD energy-momentum tensor

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \textbf{with} \quad T_q^{\mu\nu} = \bar{\psi}_q \, i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi_q - g^{\mu\nu} \bar{\psi}_q \left( i\gamma \cdot \overleftrightarrow{D} - m_q \right) \psi_q \quad \textbf{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} \left( F^a_{\rho\eta} \right)^2$$

## ❑ Gravitational form factor

$$\langle p' | T_i^{\mu\nu} | p \rangle = \bar{u}(p') \left[ A_i(t) \frac{P^\mu P^\nu}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} + m \, \bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

## ❑ Connection to GPD moments

$$\int_{-1}^{1} dx \, x \, F_i(x,\xi,t) \propto \langle p' | T_i^{++} | p \rangle \quad \propto \quad \bar{u}(p') \left[ \underbrace{\left( A_i + \xi^2 D_i \right)}_{\int_{-1}^{1} dx \, x \, H_i(x,\xi,t)} \gamma^+ + \underbrace{\left( B_i - \xi^2 D_i \right)}_{\int_{-1}^{1} dx \, x \, E_i(x,\xi,t)} \frac{i\sigma^{+\Delta}}{2m} \right] u(p)$$

**Need $\xi$-dependence to extract the $D$-term**

## ❑ Angular momentum sum rule

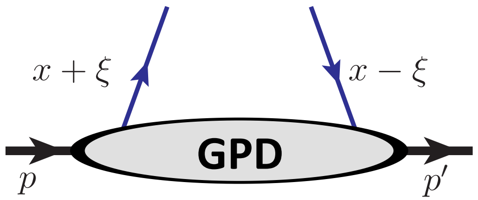
$$J_i = \lim_{t \to 0} \int_{-1}^{1} dx \, x \left[ H_i(x,\xi,t) + E_i(x,\xi,t) \right]$$

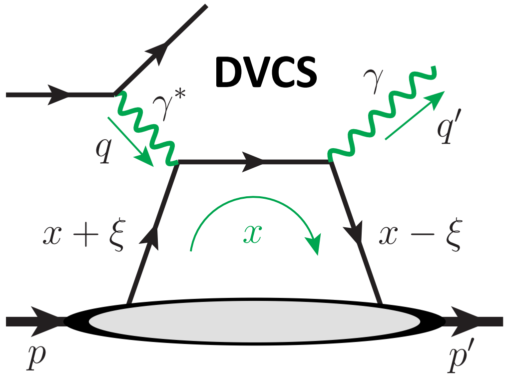
$$i = q, g$$

- **3D tomography**
- **relations to GFF** ➡ ***$x$*-dependence!**
- **angular momentum**
- **…**

# Why is the GPD *x*-dependence so *difficult* to measure?

## ❑ Amplitude nature: exclusive processes




***x* ~ loop momentum**

$$i\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t) \cdot C(x,\xi;Q/\mu)$$

**never pin down to some *x***



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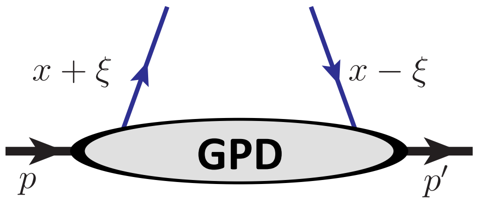


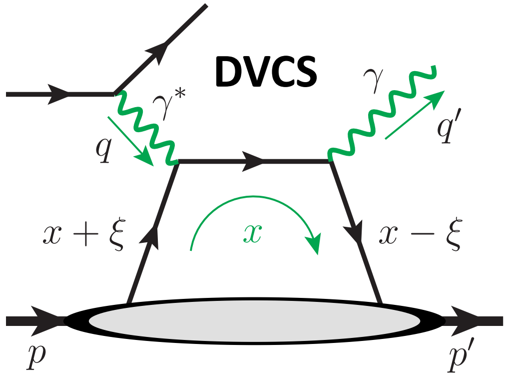


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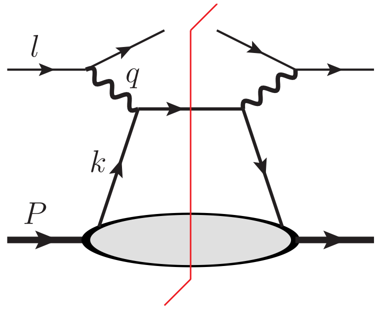
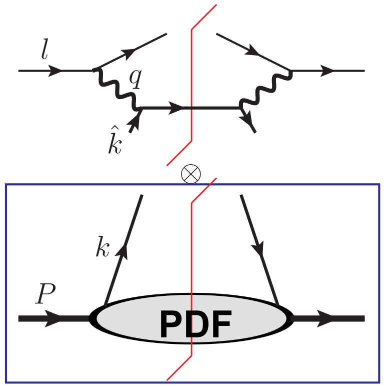
**Compare with DIS**




***cross section*: cut diagram**

$$\sigma_{\mathrm{DIS}} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\, \hat{\sigma}(x/x_B)$$

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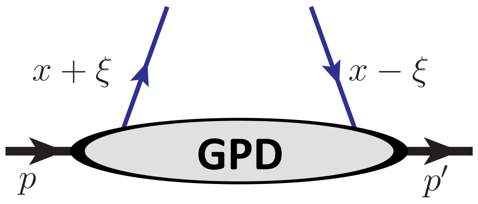
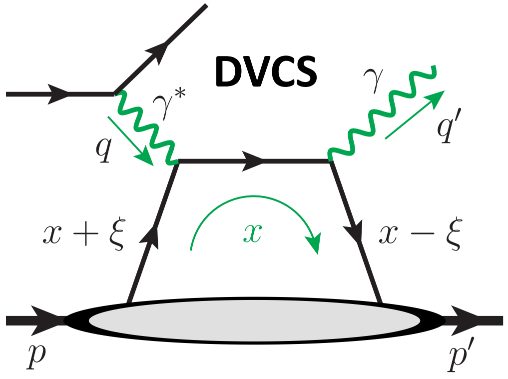



$x \sim$ loop momentum

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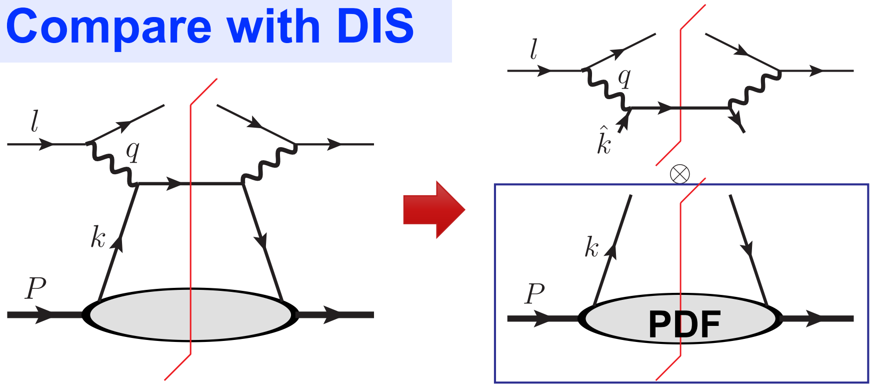



***cross section***: cut diagram

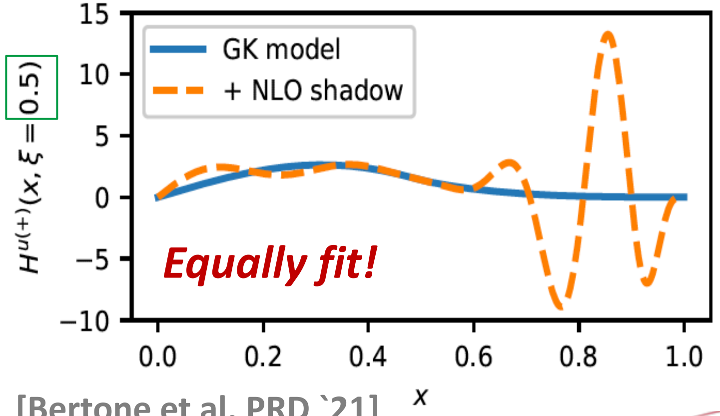
$$\sigma_{\rm DIS} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\,\hat{\sigma}(x/x_B)$$

## ❑ Sensitivity to $x$: comes from $C(x,\xi;Q/\mu)$

$$C(x,\xi;Q/\mu) = T(Q/\mu)\cdot G(x,\xi) \propto \frac{1}{x-\xi+i\epsilon}\cdots$$

$$\Rightarrow\ i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x\, \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$

**“moment”**

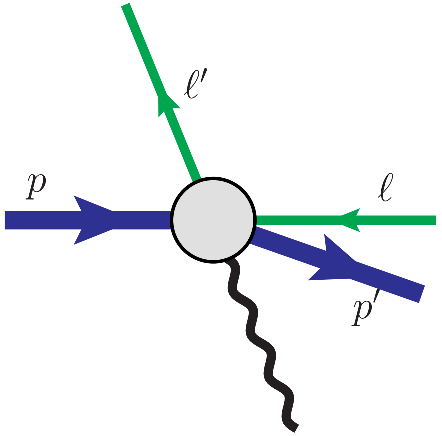



[Bertone et al. PRD `21]

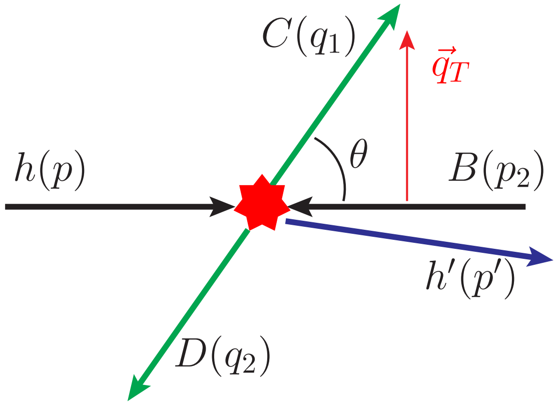
# How to *think* about the GPD processes?

- **Single diffractive hard exclusive process (SDHEP)** [Qiu & Yu, PRD 107 (2023) 014007]

**DVCS in lab frame**




$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$




**2 → 3: *minimal***
**kinematic configuration!**

# How to *think* about the GPD processes?

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[Qiu & Yu, PRD 107 (2023) 014007]

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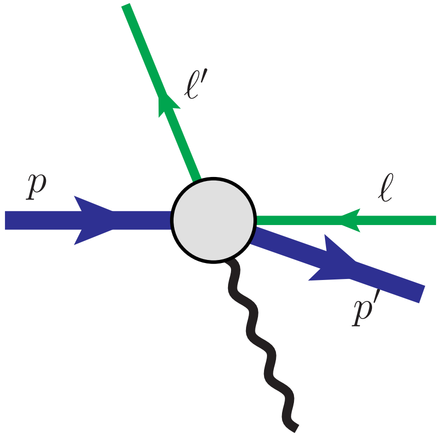
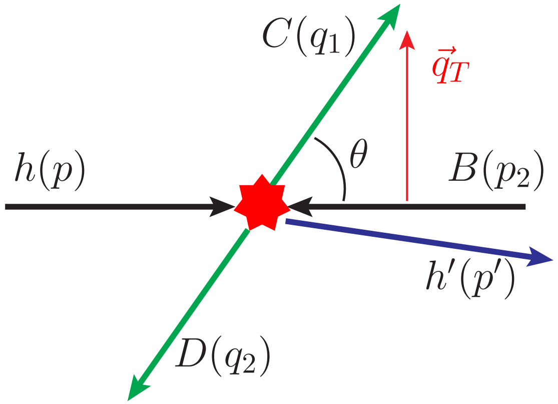


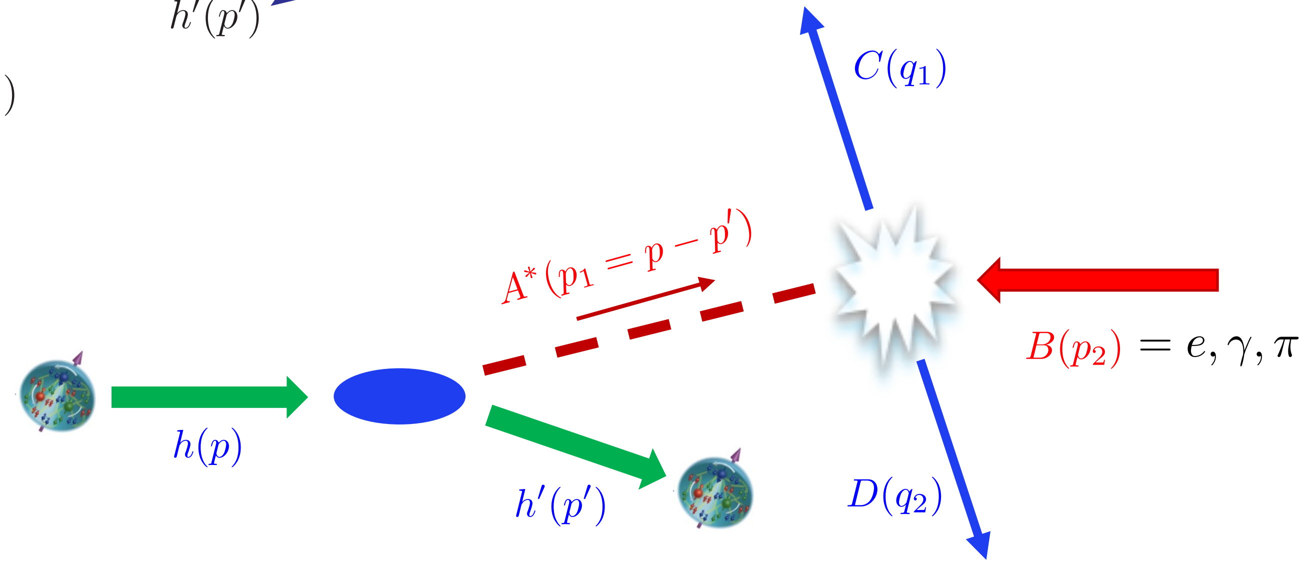

**2 → 3: *minimal* kinematic configuration!**

## ❑ Two-stage process paradigm

**Single diffractive:** $h(p) \to h'(p') + A^*(p_1 = p - p')$

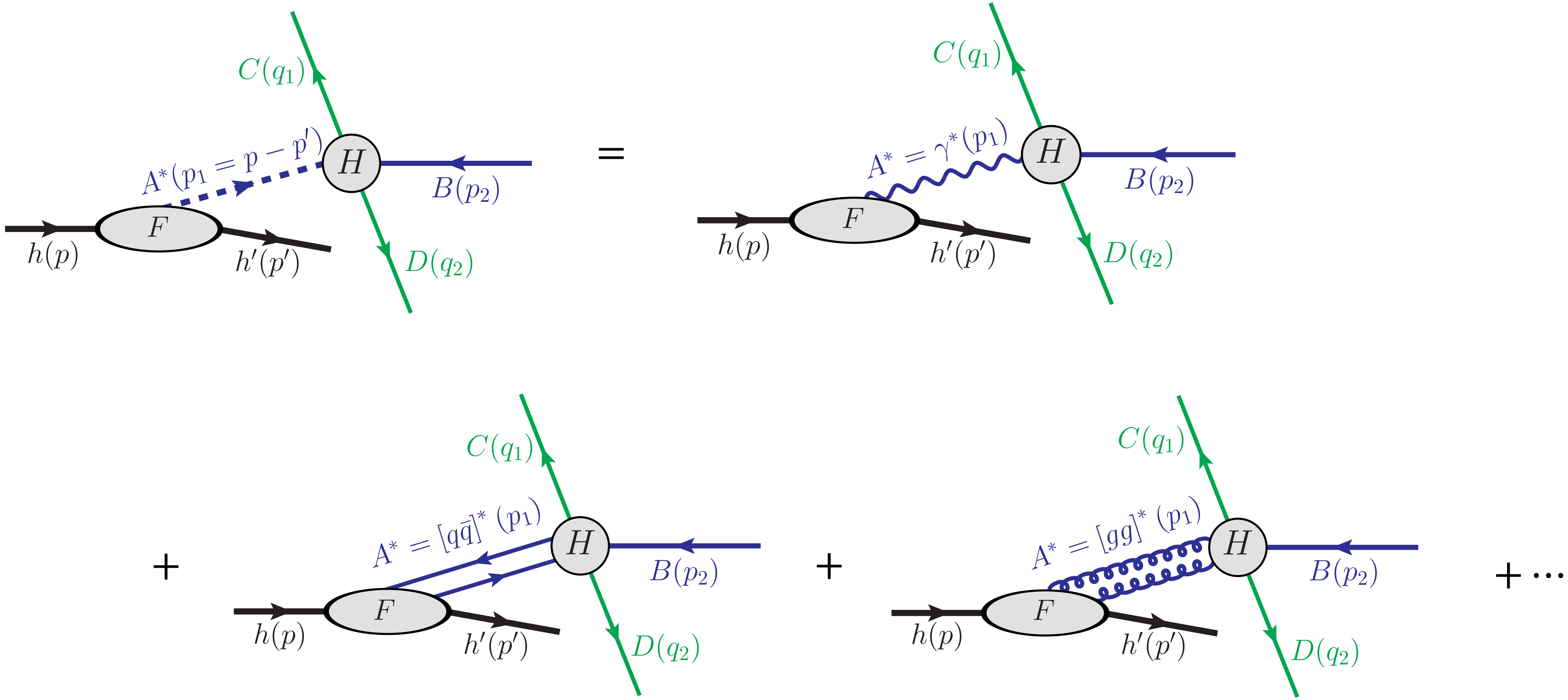
**factorize**

**Hard exclusive:** $A^*(p_1) + B(p_2) \to C(p_3) + D(p_4)$




***Necessary*** **condition for factorization:** $q_T \gg \sqrt{-t} \simeq \Lambda_{\rm QCD}$ $\quad t = (p - p')^2$

# SDHEP: Two-stage paradigm and channel expansion



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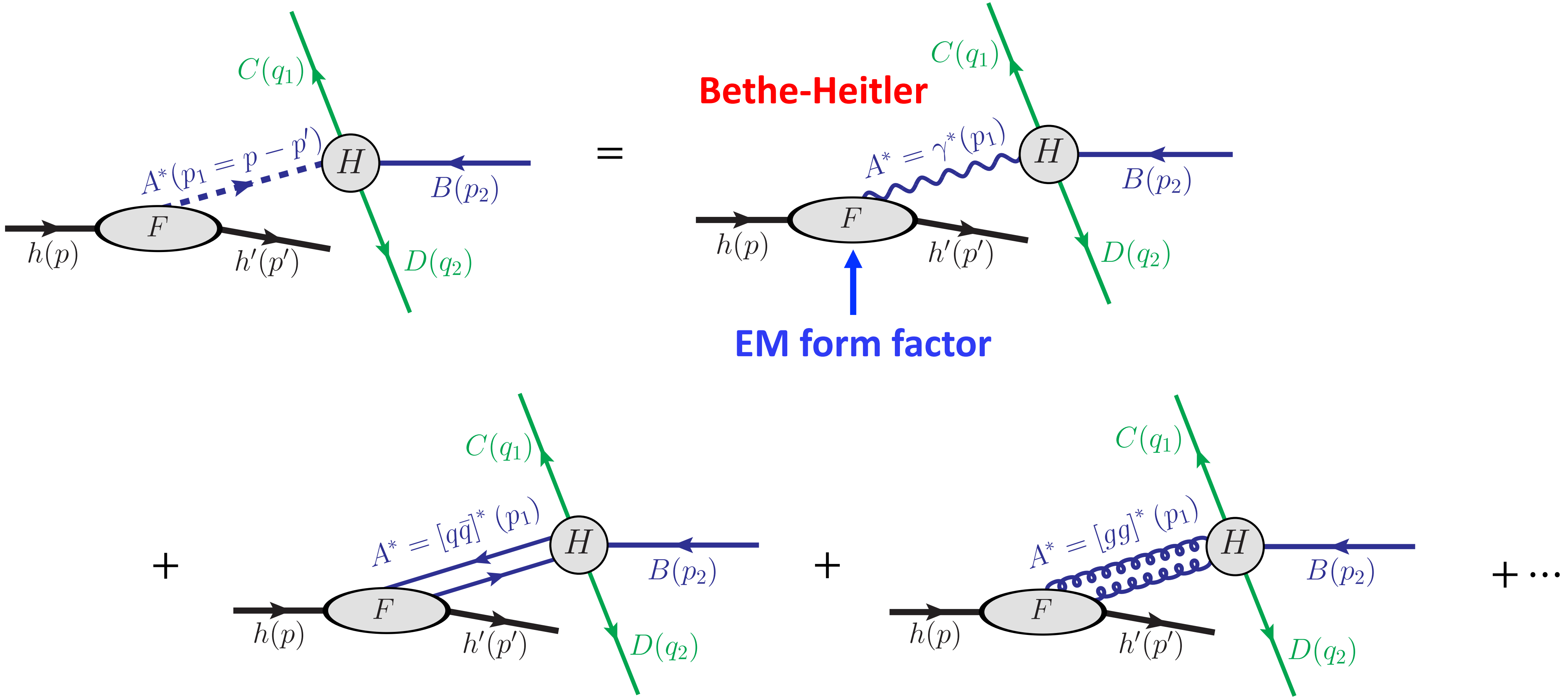

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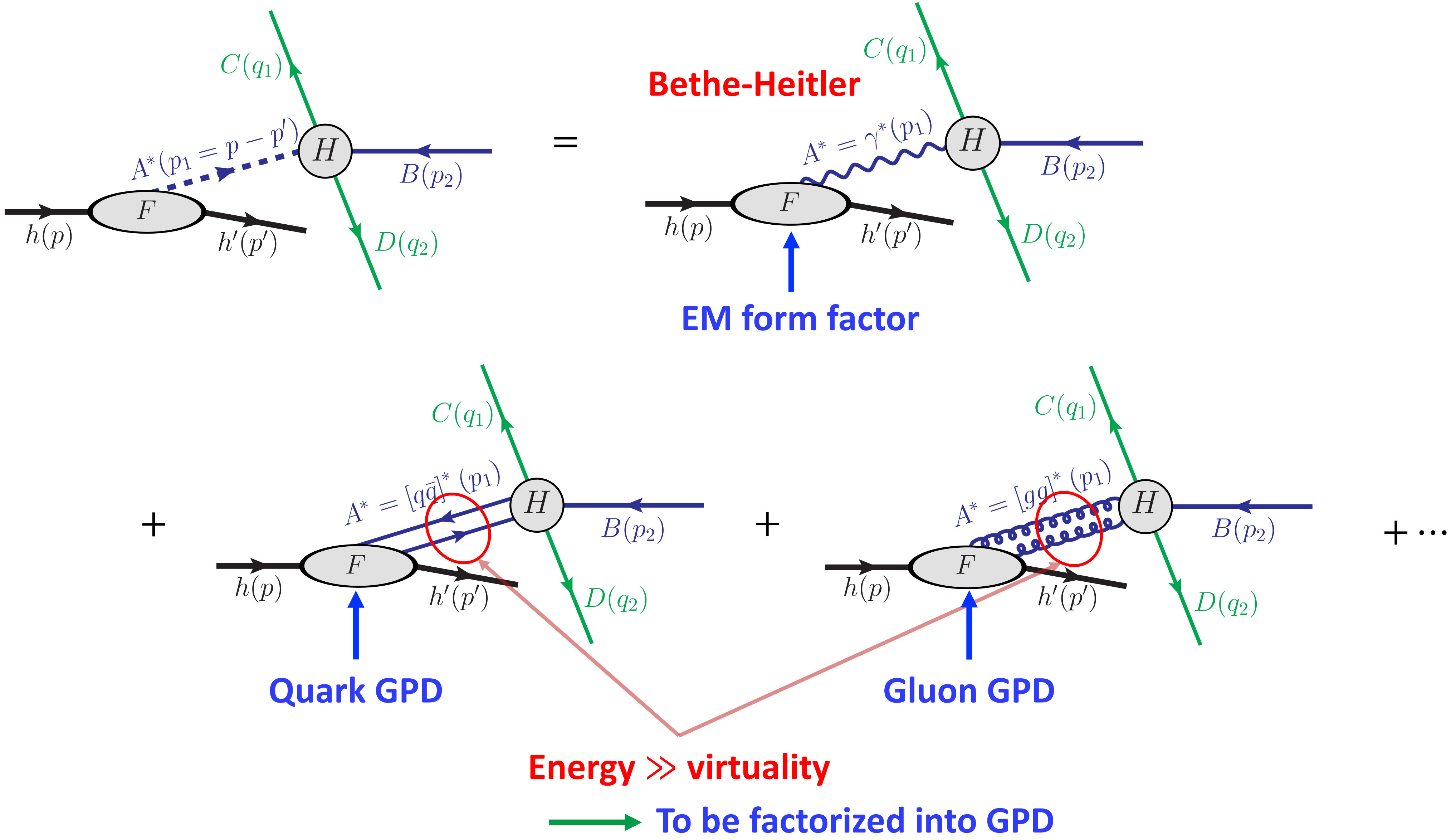
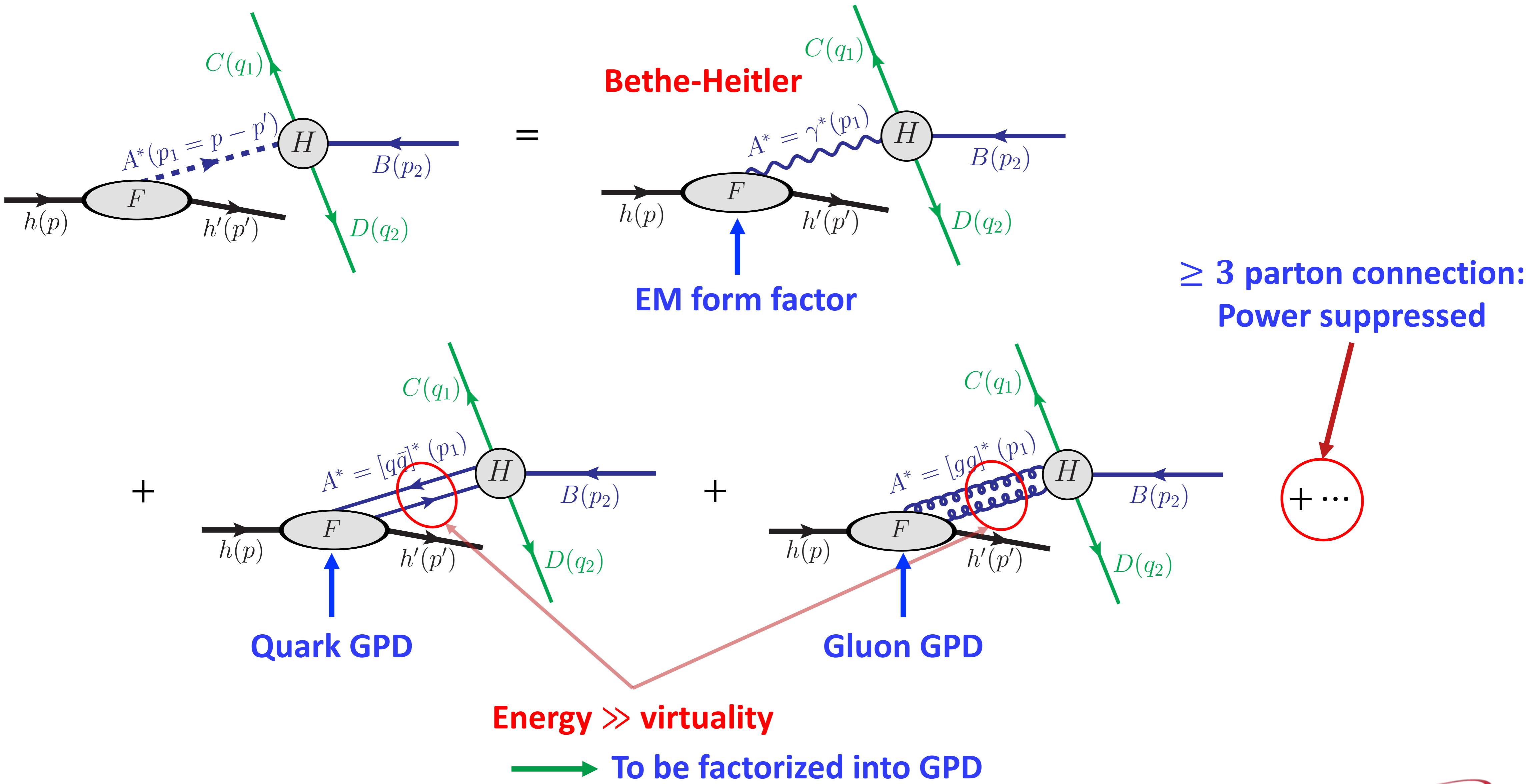


# SDHEP: Two-stage paradigm and channel expansion (twist expansion)



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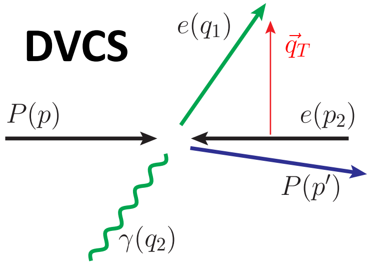
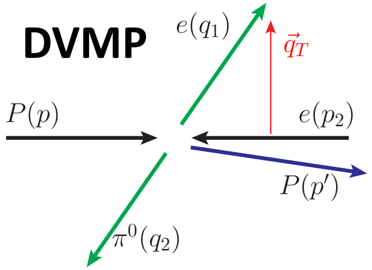
# Classification of SDHEPs

## ❑ Electro-production (JLab, EIC, …)

DVCS

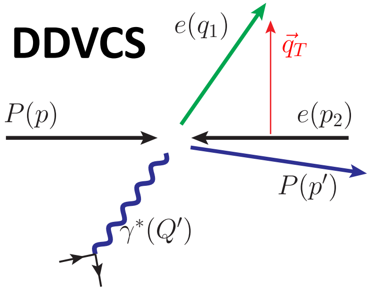



DVMP




add virtuality

DDVCS




DHQP…



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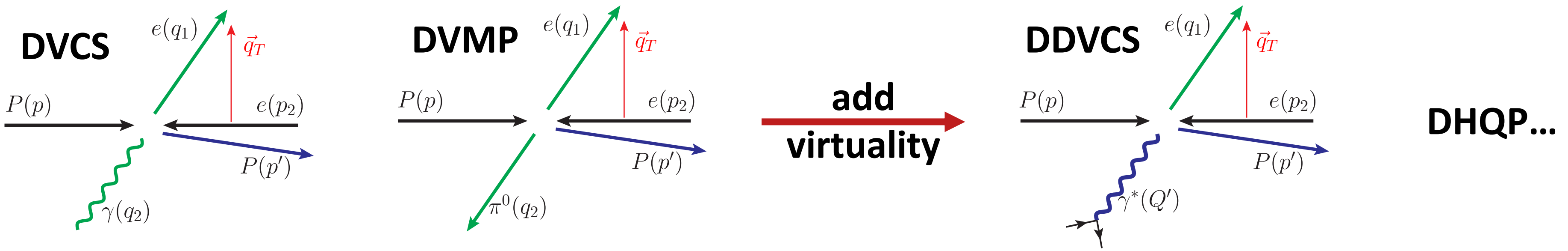
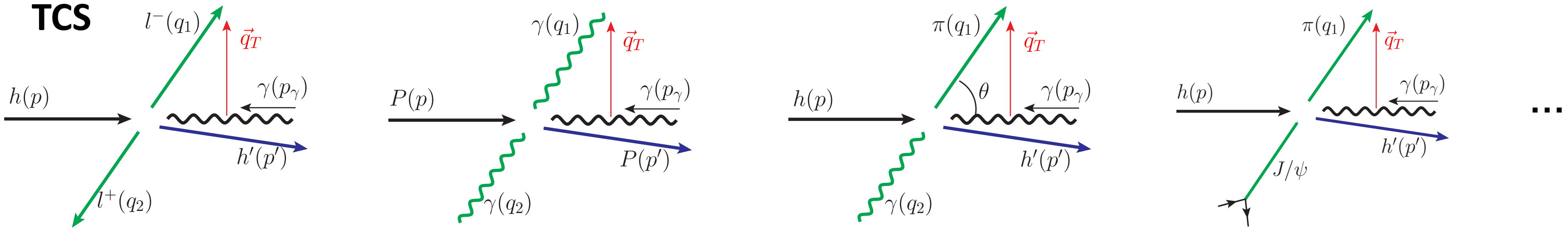



## ❑ Photo-production (JLab, EIC, …)



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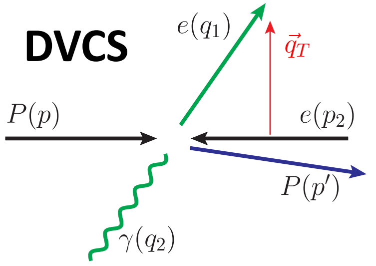
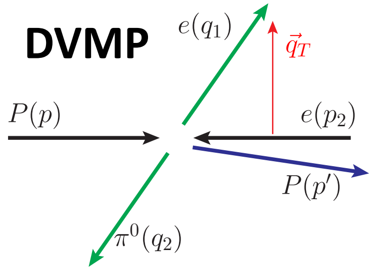
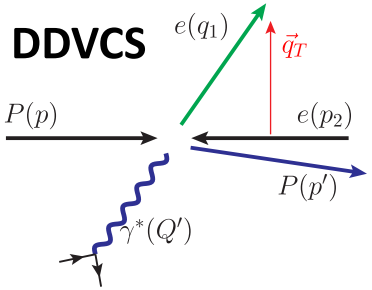



**DVMP**




**add virtuality**

**DDVCS**




**DHQP…**

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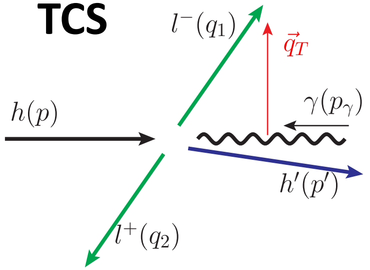
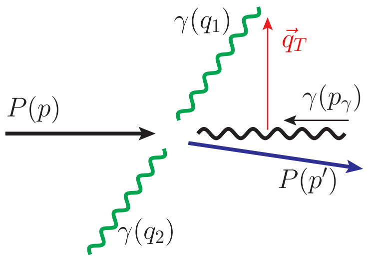
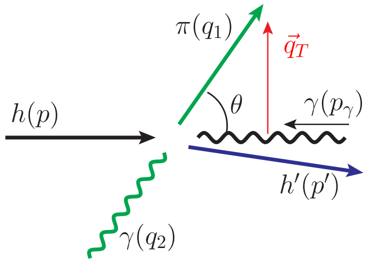
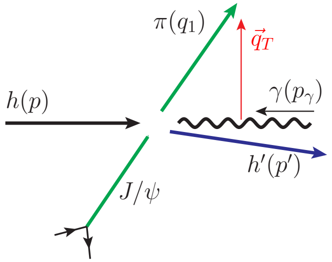
**TCS**



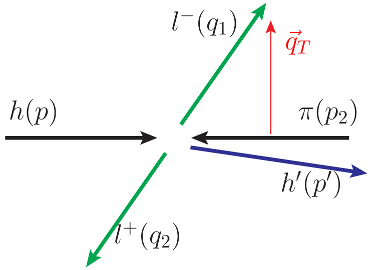



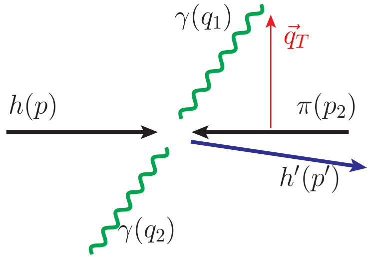




…

## ❑ Meso-production (AMBER, J-PARC, …)



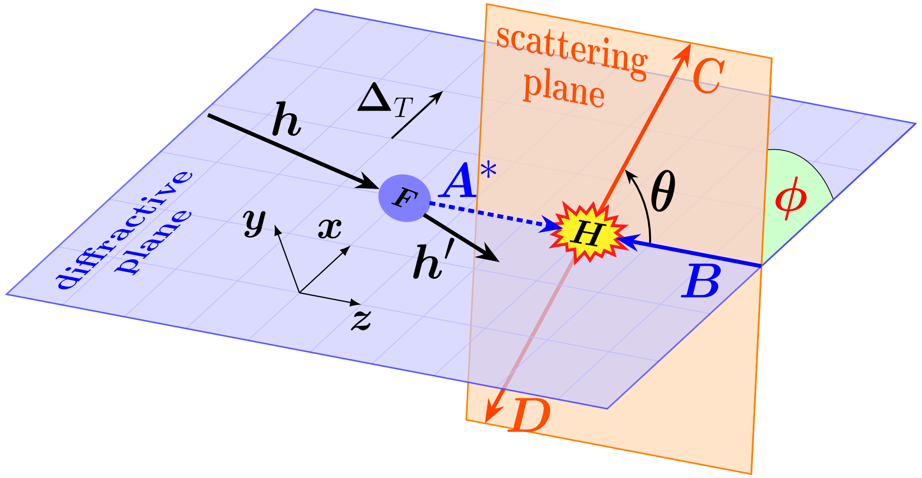




…

**Generic discussion**

[Qiu, Yu, PRD 107 (2023), 014007]

# Where does the sensitivity come from?




- $x$**-sensitivity ⇔ 2 → 2 hard scattering**

**Kinematics:**

1. $\hat{s} = 2\,\xi\,s\,/\,(1+\xi)$ ⟸ $\xi$
2. $\theta$ or $q_T = \sqrt{\hat{s}}\,\sin\theta/2$ ⟺ $x$
3. $\phi$ ⟸ $(A^*B)$ spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi}\cdot\int_{-1}^{1} dx\,F_A(x)\,C_A(x;Q) \qquad (Q=\theta \text{ or } q_T)$$

[suppressing $t$ and $\xi$ dependence]

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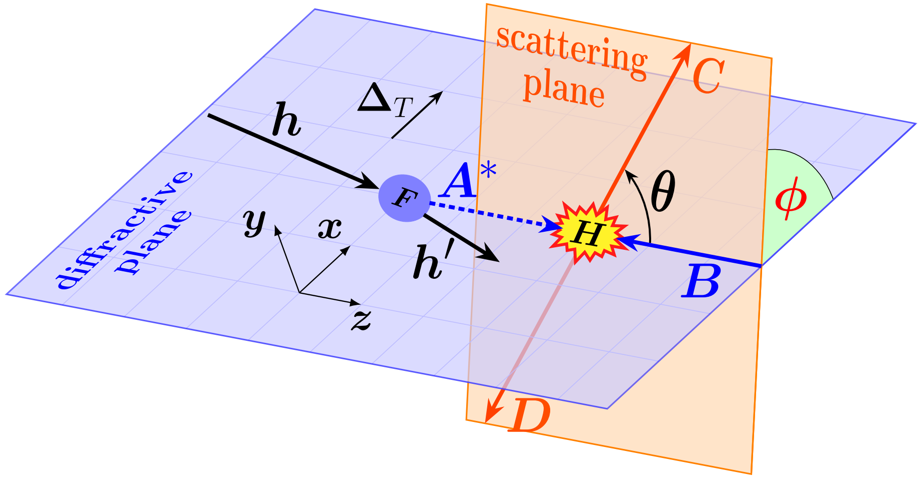


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➢ **Moment-type sensitivity** $C(x;Q) = G(x)\cdot T(Q)$ ⟹ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$

⟹ **Inversion problem: *shadow GPD*** $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

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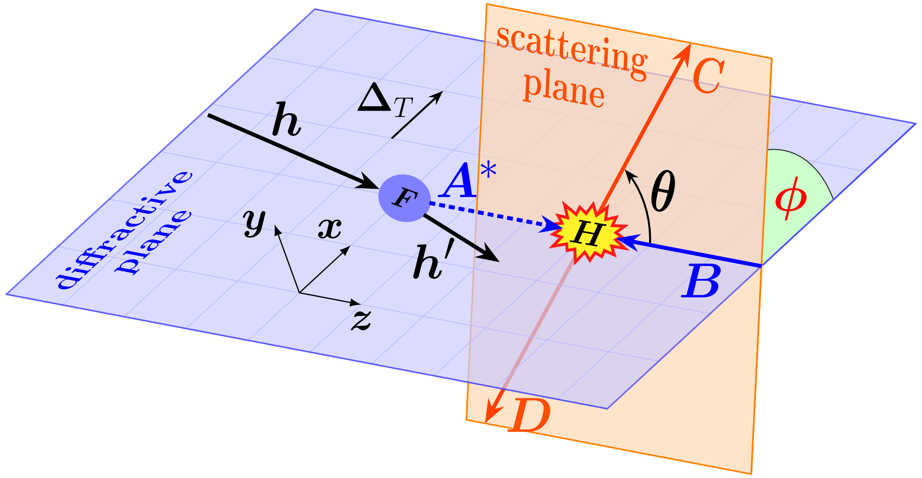


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⟶ **Inversion problem: *shadow GPD*** $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

➢ **Enhanced sensitivity** $C(x;Q) \neq G(x)\cdot T(Q)$ ⟶ $d\sigma/dQ \sim \left|C(x;Q)\otimes_x F(x,\xi,t)\right|^2$



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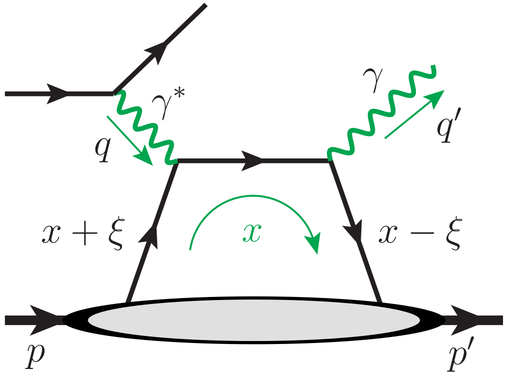
# Electroproduction processes: DVCS vs. DDVCS

## ❑ Moment sensitivity in DVCS




**DVCS**

$$i\mathcal{M} \supset \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi+i\epsilon} = F_0(\xi,t)$$

$q'^2 = 0$ ➡ **Lack of external scale to probe $x$**



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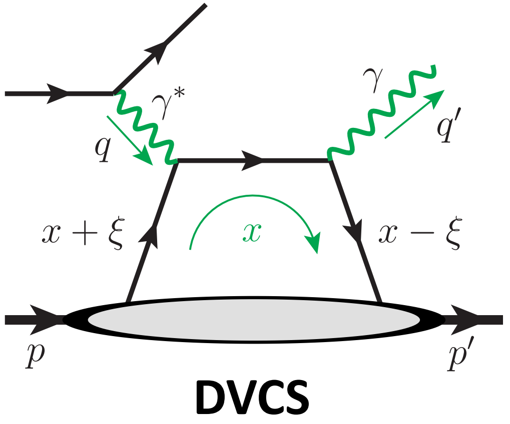
# Electroproduction processes: DVCS vs. DDVCS

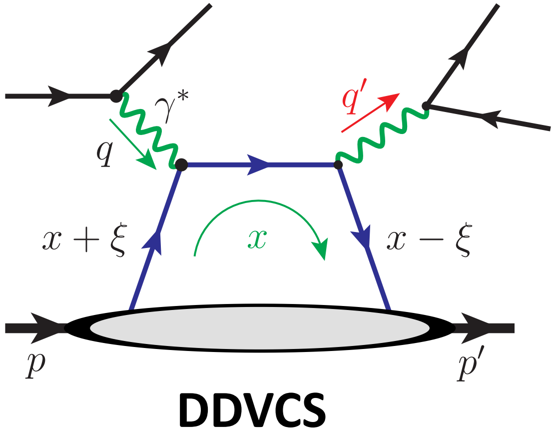
## ❑ Moment sensitivity in DVCS




$$i\mathcal{M} \supset \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi+i\epsilon} = F_0(\xi,t)$$

$q'^2 = 0$ ➡ **Lack of external scale to probe $x$**

## ❑ Enhanced sensitivity in DDVCS



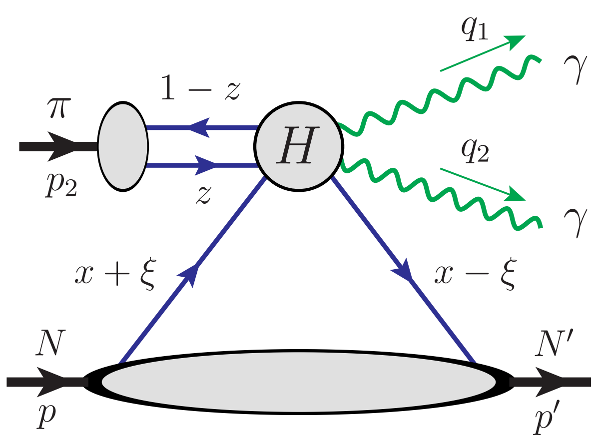

$$i\mathcal{M} \supset \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-x_p(\xi,Q'/Q)+i\epsilon}$$

$q'^2 = Q'^2$ ➡ **External scale to entangle with $x$**

$$x_p(\xi,Q'/Q) = \xi\left[\frac{1-(Q'/Q)^2}{1+(Q'/Q)^2}\right]$$

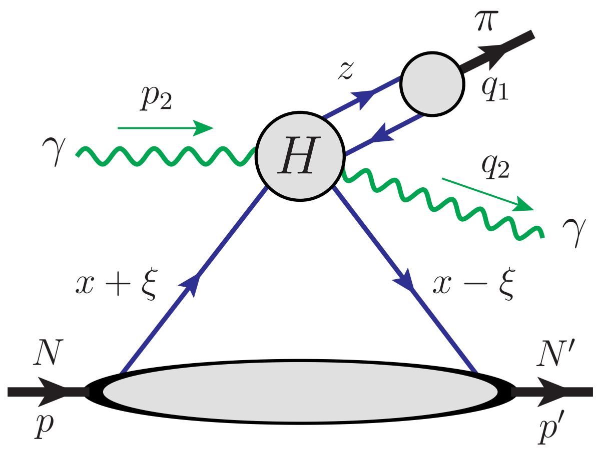
**Physically appealing, but experimentally challenging…**

# Two new example processes with enhanced $x$-sensitivity




**J-PARC, AMBER**

Qiu & Yu, JHEP 08 (2022) 103
Qiu & Yu, arXiv:2312.xxxxx

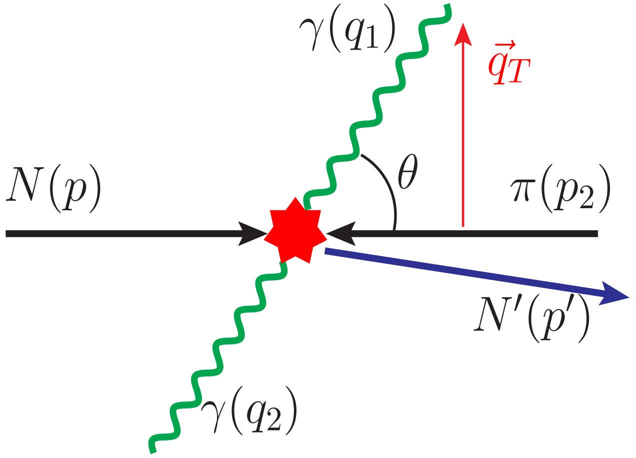


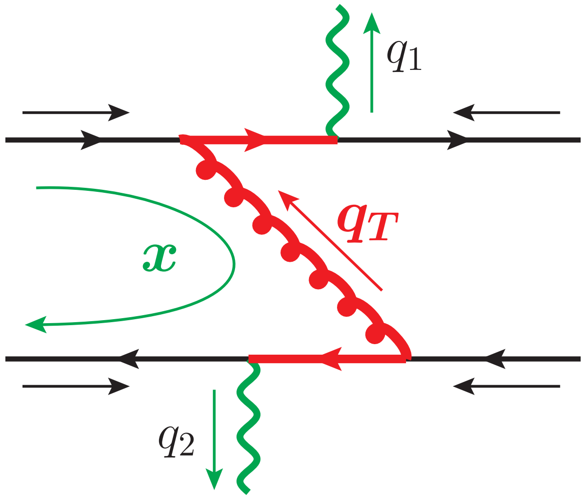

**JLab Hall D**

G. Duplancic et al., JHEP 11 (2018) 179
Qiu & Yu, PRD 107 (2023), 014007
Qiu & Yu, PRL 131 (2023), 161902

# *Enhanced x-sensitivity: (1) diphoton production*

[Qiu & Yu, JHEP 08 (2022) 103]



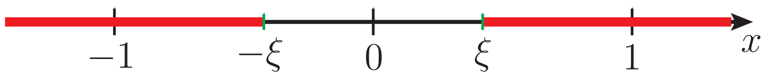

**In addition to**

$$F_0(\xi, t) = \int_{-1}^{1} \frac{dx\, F(x, \xi, t)}{x - \xi + i\epsilon}$$

$i\mathcal{M}$ **also contains**

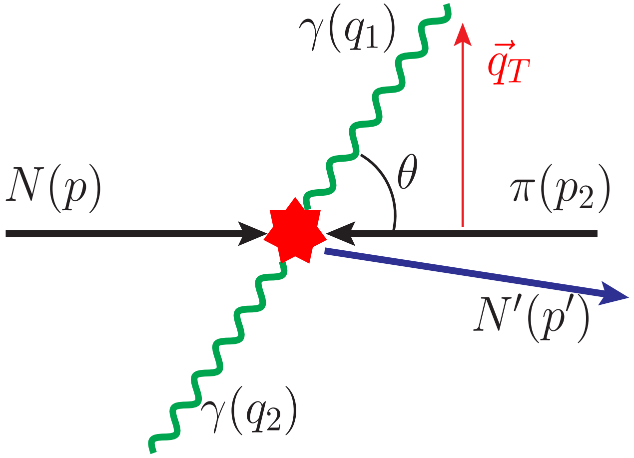
$$I(t, \xi; z, \theta) = \int_{-1}^{1} \frac{dx\, F(x, \xi, t)}{x - \rho(z;\theta) + i\epsilon\, \mathrm{sgn}\left[\cos^2(\theta/2) - z\right]}$$

$$\rho(z;\theta) = \xi \cdot \left[\frac{1 - z + \tan^2(\theta/2)\, z}{1 - z - \tan^2(\theta/2)\, z}\right] \in (-\infty, -\xi] \cup [\xi, \infty)$$

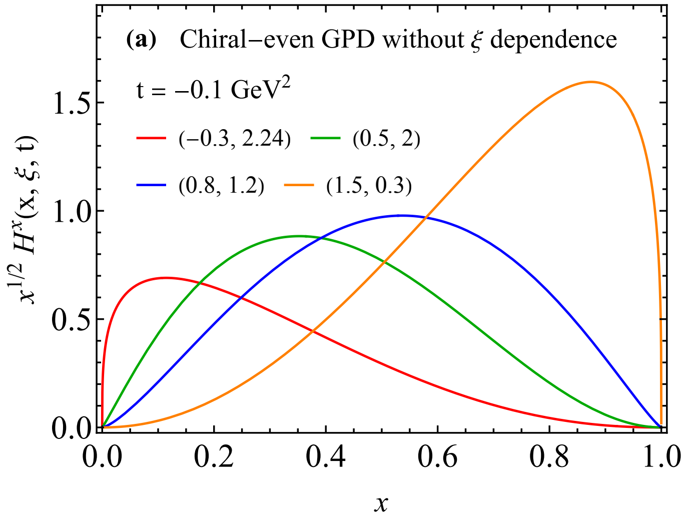


# *Enhanced $x$-sensitivity: (1) diphoton production*

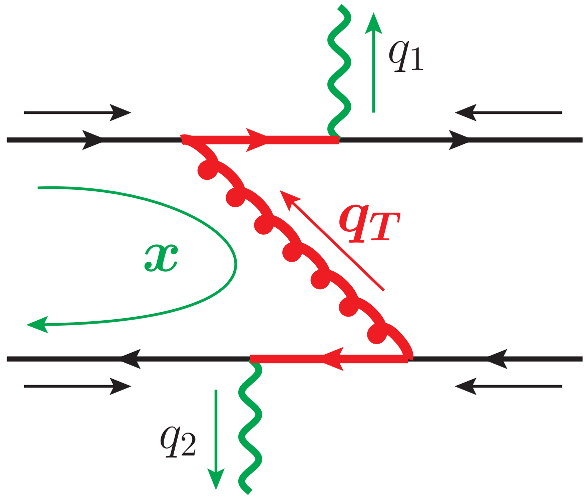
[Qiu & Yu, JHEP 08 (2022) 103]

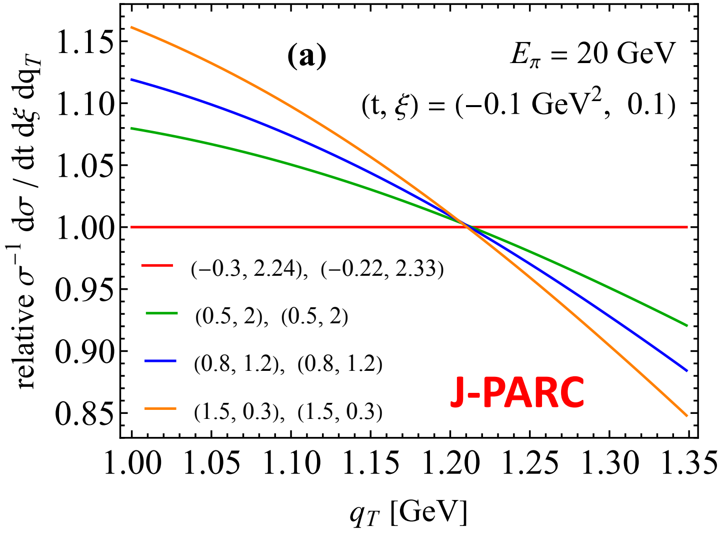


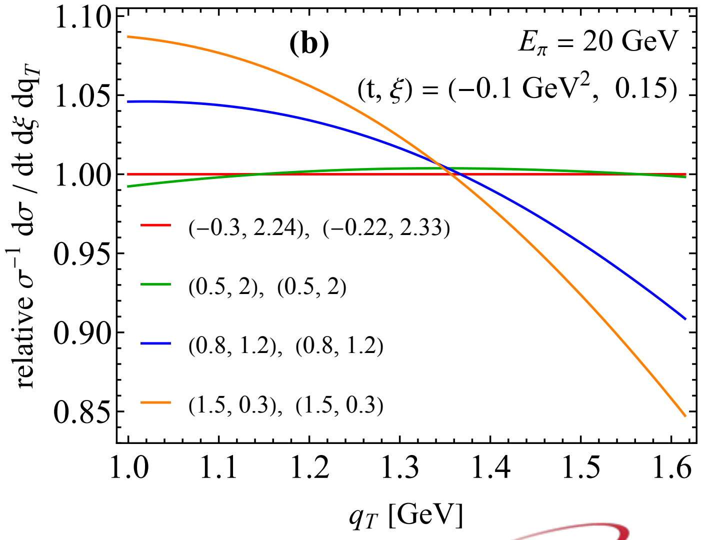

**Vary GPD $x$ shapes**




**Different $q_T$ shapes**



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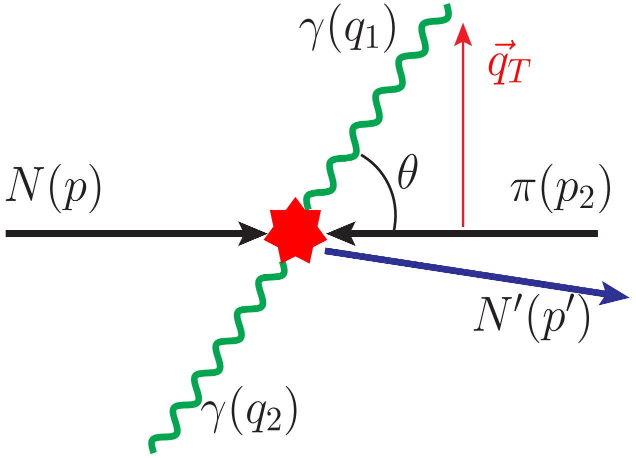


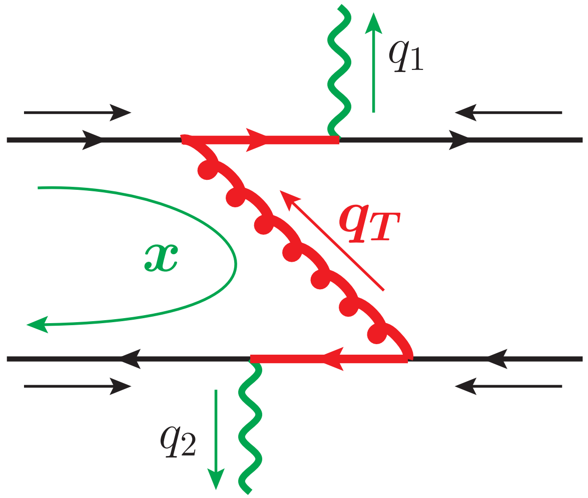


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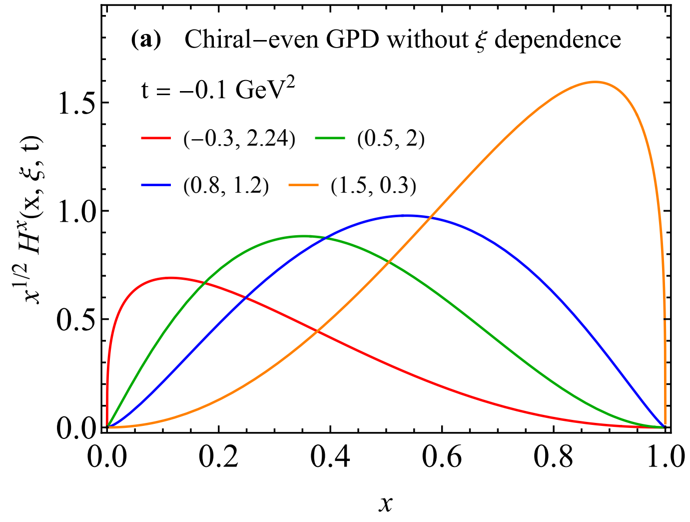
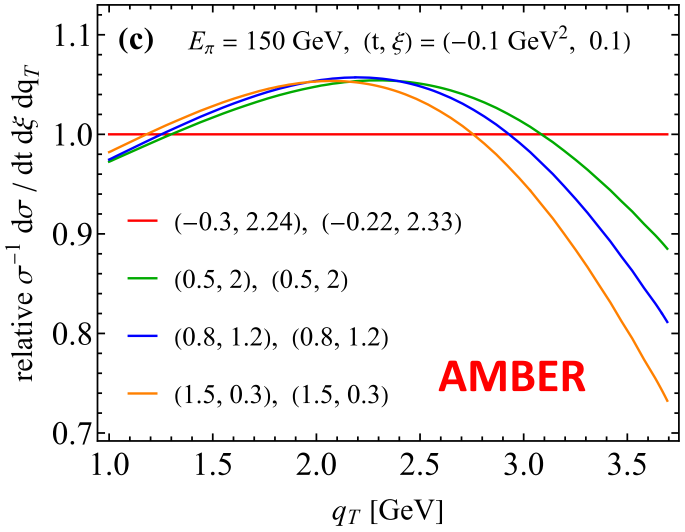
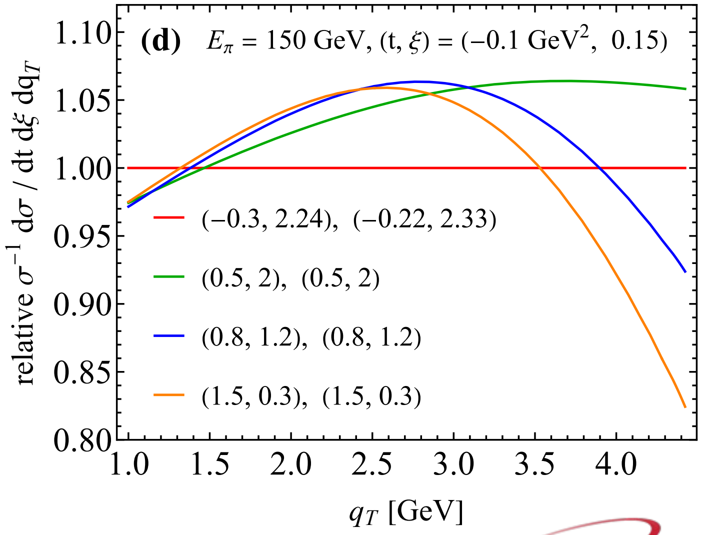
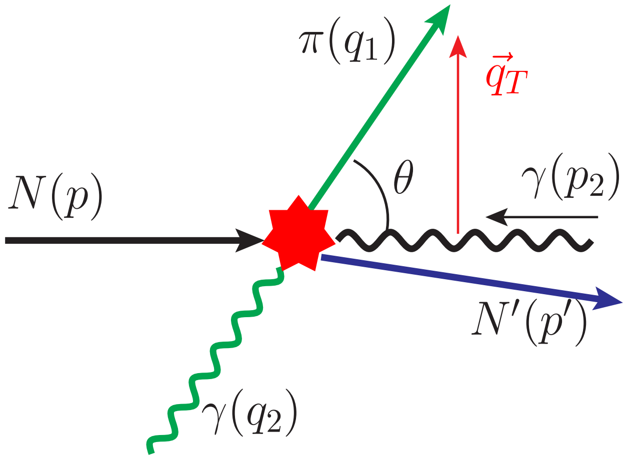


# *Enhanced $x$-sensitivity: (2) $\gamma$-$\pi$ pair photoproduction*

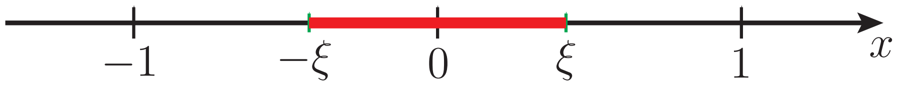
[Qiu & Yu, PRL 131 (2023) 161902]



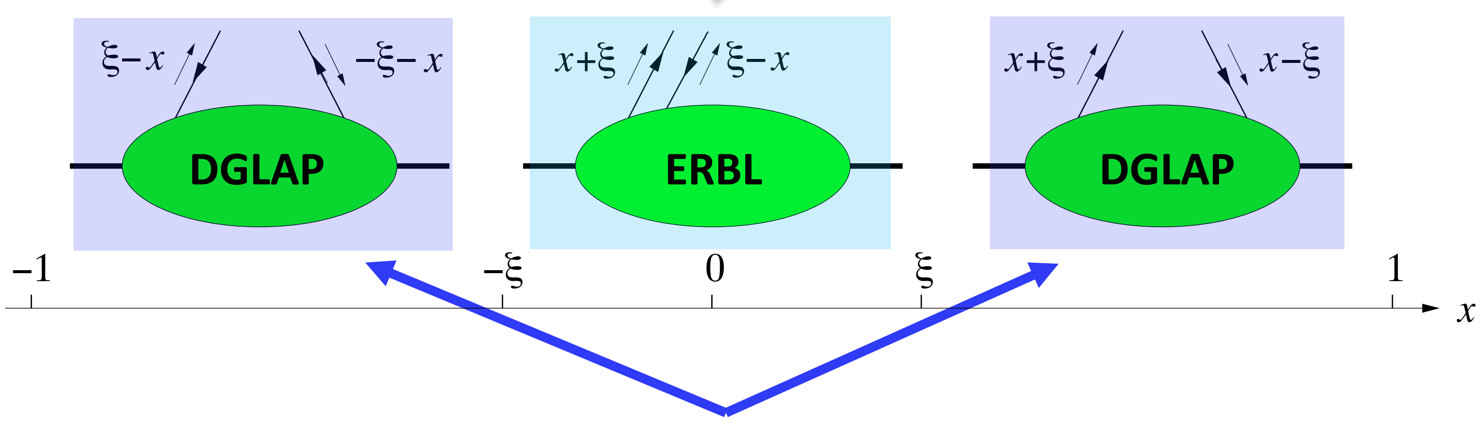

$i\mathcal{M}$ **also contains the special integral**

$$I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$$

$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$

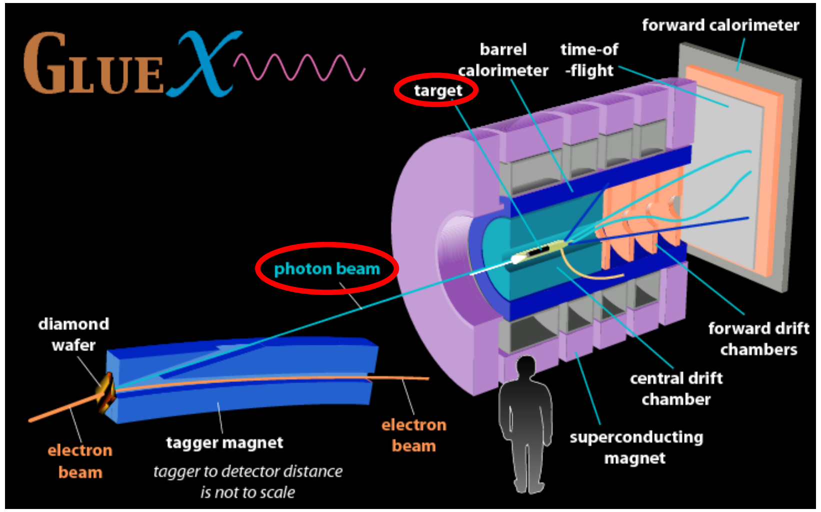


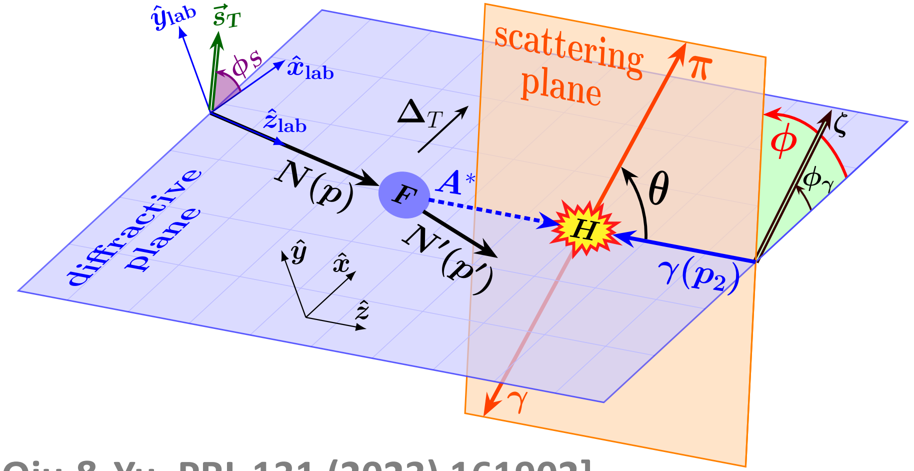

***Complementary sensitivity***




$N\,\pi \to N'\,\gamma\,\gamma$

# Enhanced $x$-sensitivity: (2) $\gamma$-$\pi$ pair photoproduction




[Qiu & Yu, PRL 131 (2023) 161902]

## Polarization asymmetries

$$\frac{d\sigma}{d|t|\, d\xi\, d\cos\theta\, d\phi} = \frac{1}{2\pi} \frac{d\sigma}{d|t| d\xi d\cos\theta} \cdot \left[1 + \lambda_N \lambda_\gamma A_{LL} + \zeta A_{UT} \cos 2(\phi - \phi_\gamma) + \lambda_N \zeta A_{LT} \sin 2(\phi - \phi_\gamma)\right]$$

$$\frac{d\sigma}{d|t|\, d\xi\, d\cos\theta} = \pi (\alpha_e \alpha_s)^2 \left(\frac{C_F}{N_c}\right)^2 \frac{1-\xi^2}{\xi^2 s^3} \Sigma_{UU}$$

$$\Sigma_{UU} = |\mathcal{M}_+^{[\widetilde{H}]}|^2 + |\mathcal{M}_-^{[\widetilde{H}]}|^2 + |\widetilde{\mathcal{M}}_+^{[H]}|^2 + |\widetilde{\mathcal{M}}_-^{[H]}|^2,$$

$$A_{LL} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*}\right],$$

$$A_{UT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\widetilde{\mathcal{M}}_+^{[H]}\,\widetilde{\mathcal{M}}_-^{[H]*} - \mathcal{M}_+^{[\widetilde{H}]}\,\mathcal{M}_-^{[\widetilde{H}]*}\right],$$

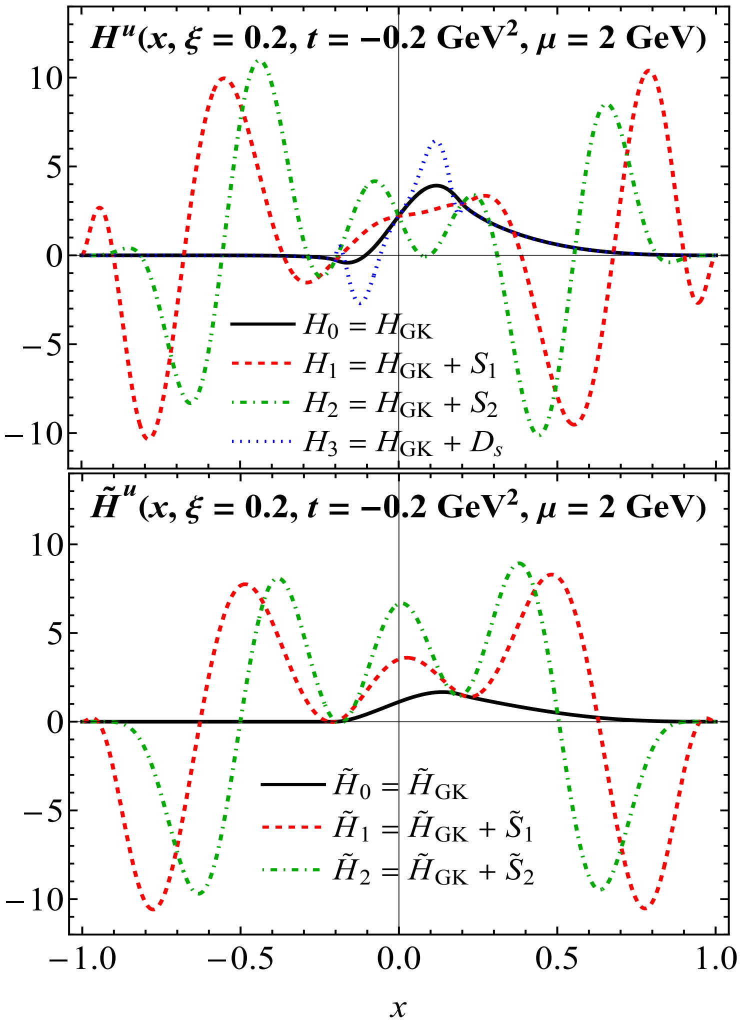
$$A_{LT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Im}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*}\right].$$

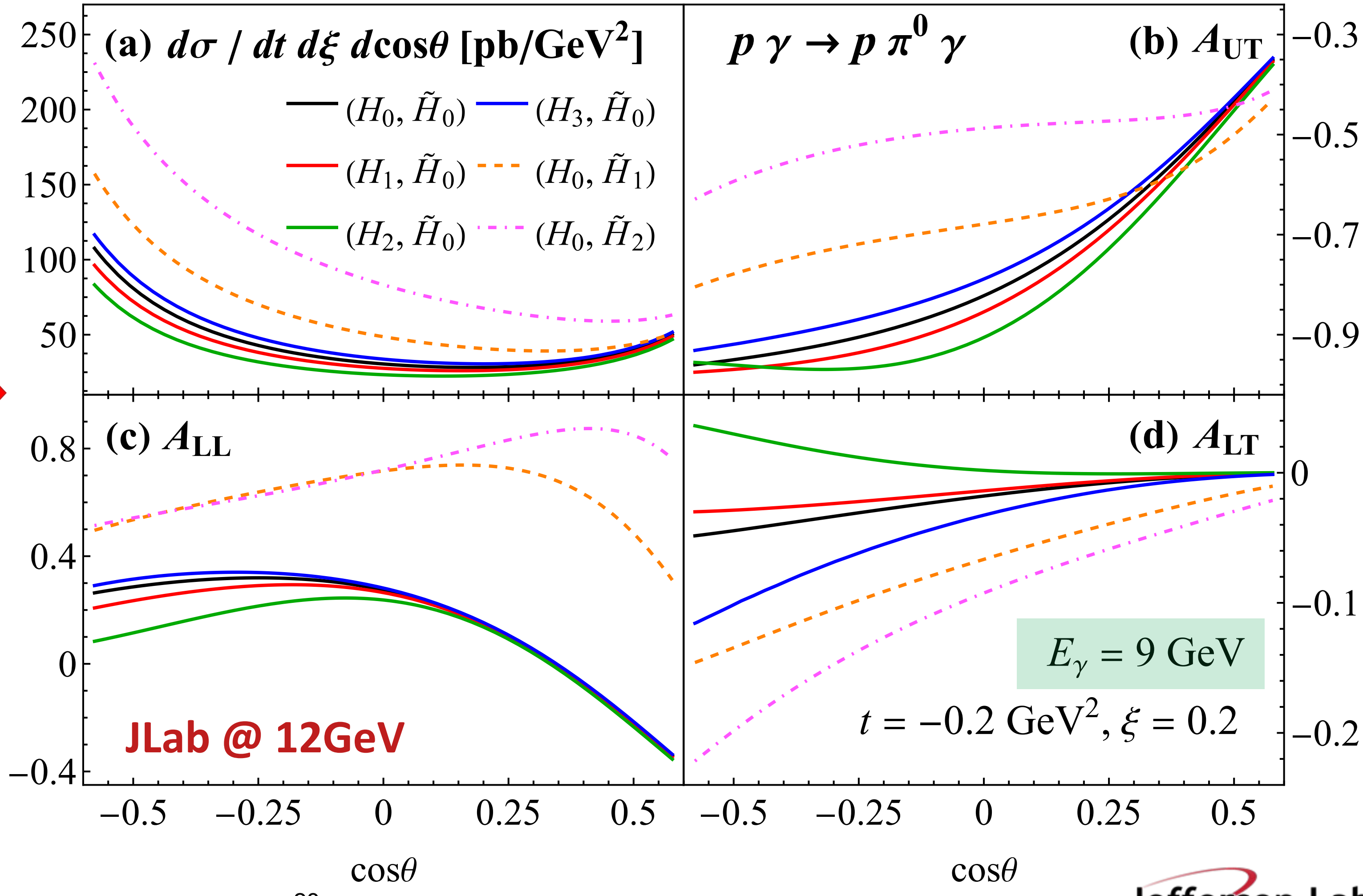
# Enhanced $x$-sensitivity: (2) $\gamma$-$\pi$ pair photoproduction

**GPD models = GK model + shadow GPDs**

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$

Goloskokov, Kroll, \`05, \`07, \`09
Bertone et al. \`21
Moffat et al. \`23



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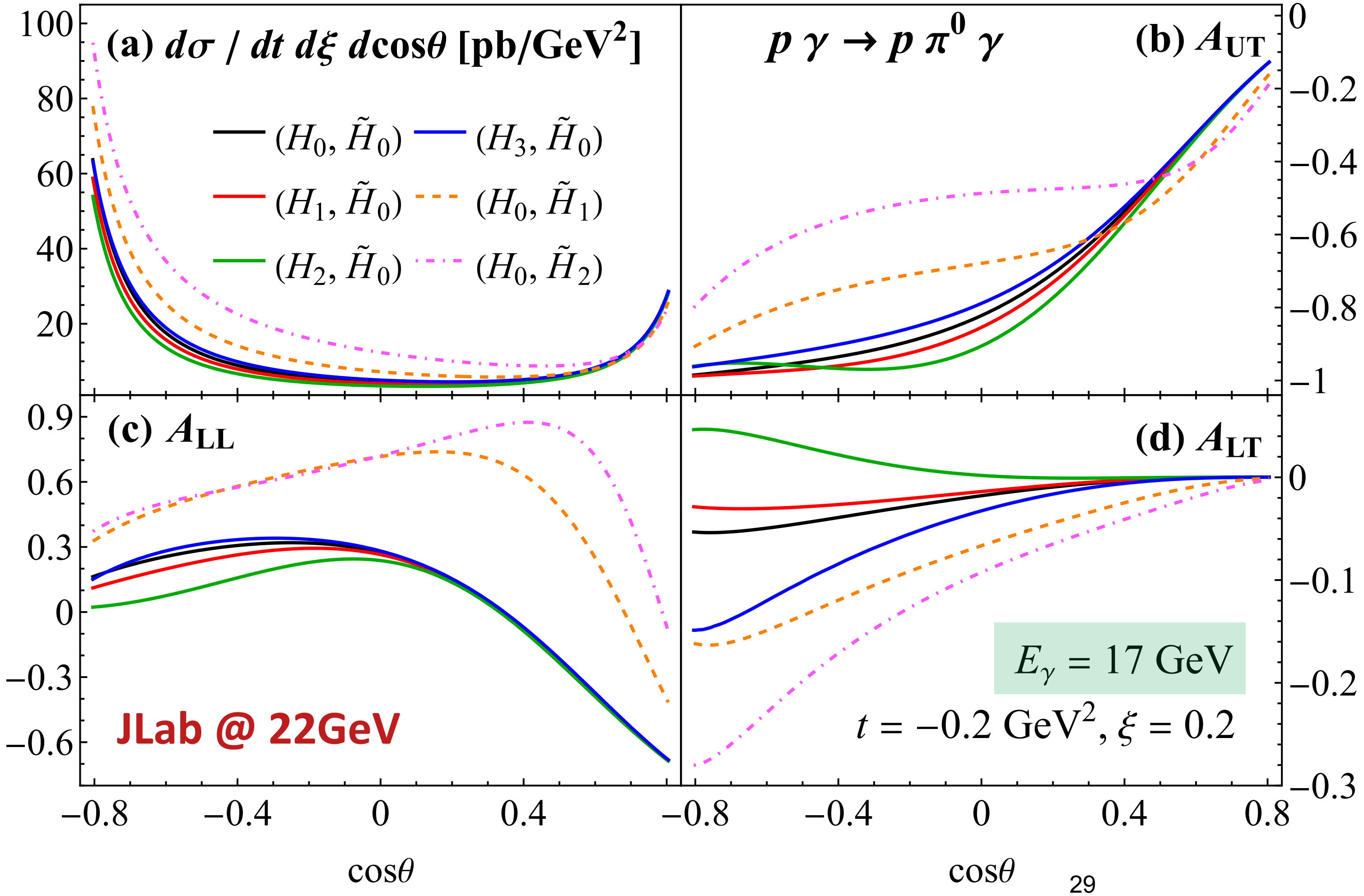
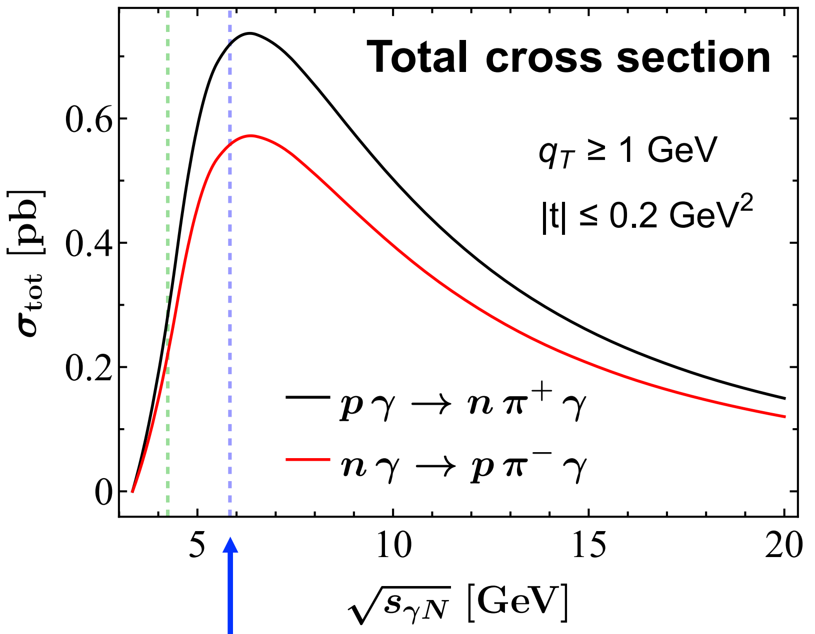


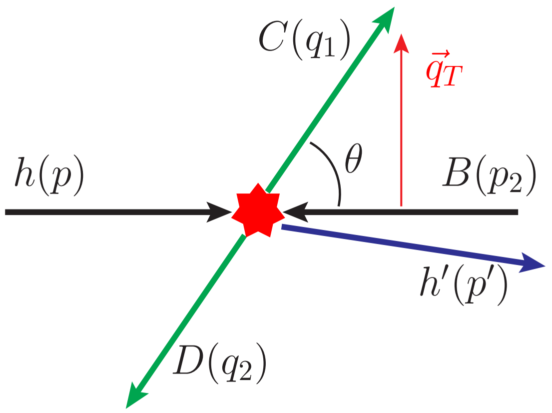

JLab @ 22GeV



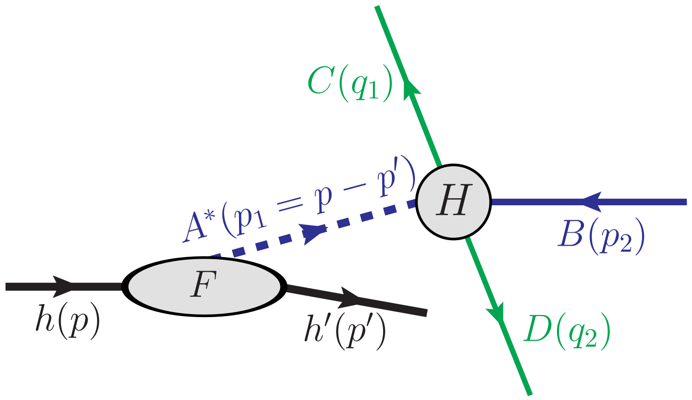
Jefferson Lab

# Summary

- ❑ **GPD and hadron 3D imaging**
- ❑ **Single Diffractive Hard Exclusive Processes (SDHEP)**
  - **Systematic factorization.**
  - **Roadmap for known and more new processes!**
- ❑ **GPD $x$ dependence is challenging**
  - **Multi-processes, multi-observables approach**
  - **Moment sensitivity is not sufficient**
  - **Enhanced sensitivity**
  - **JLab Hall D (also other halls and EIC, with good controls of quasi-real photon beams)**




SDHEP

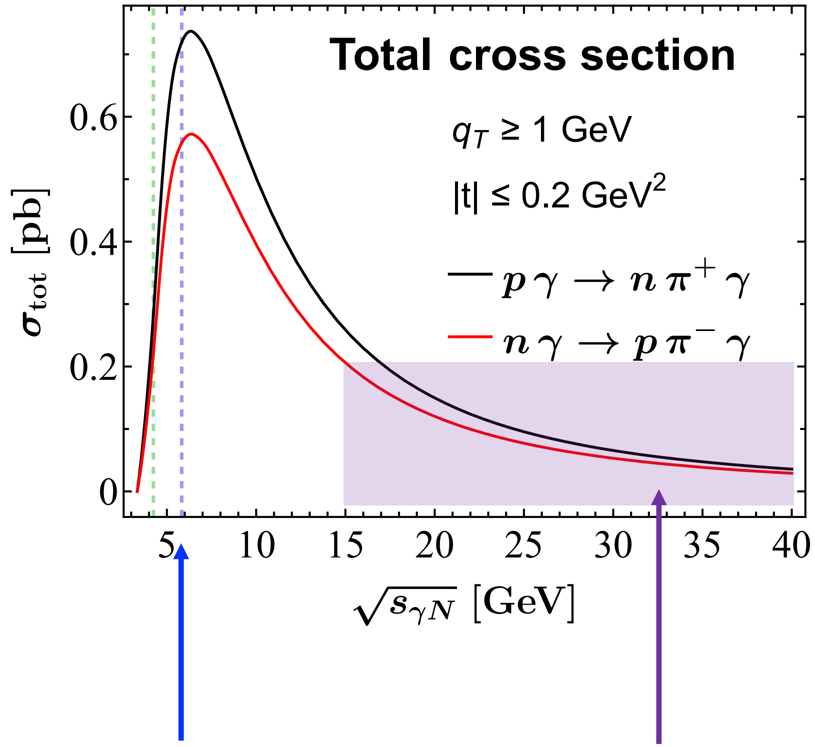



**A long but exciting way to go!**

**Thank you!**

# BACK UP

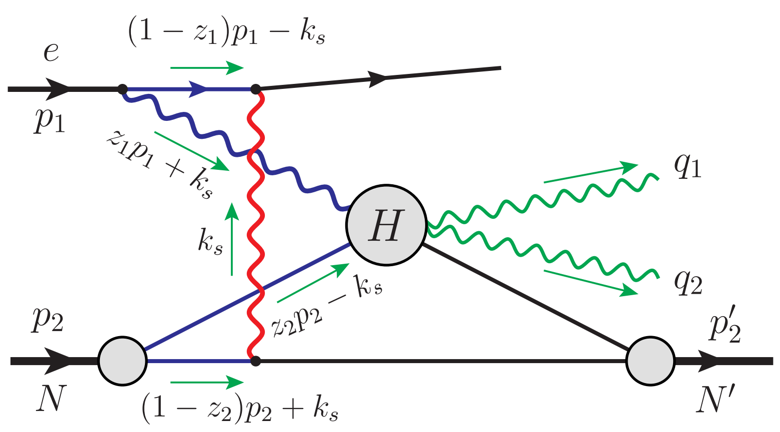
# *Photoproduction at EIC*




**JLab @ 22GeV** **EIC**

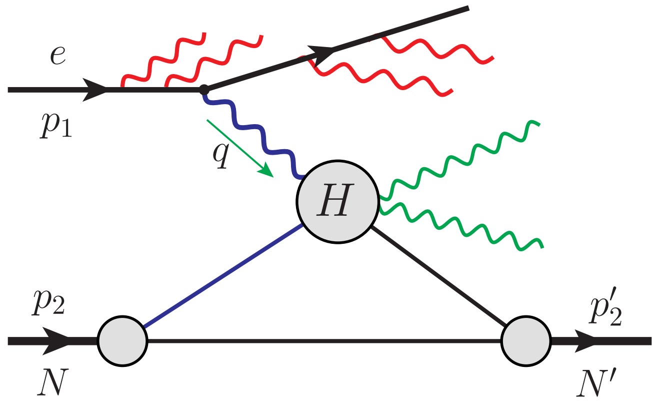
# Photoproduction at EIC?

## ❑ Theoretical challenge: double diffractive factorization?




**Both $k_s^+$ and $k_s^-$ are pinched in Glauber region!** ➡ **QED pinch: violates factorization**

## ❑ Experimental challenge: how "exclusive" can one go?

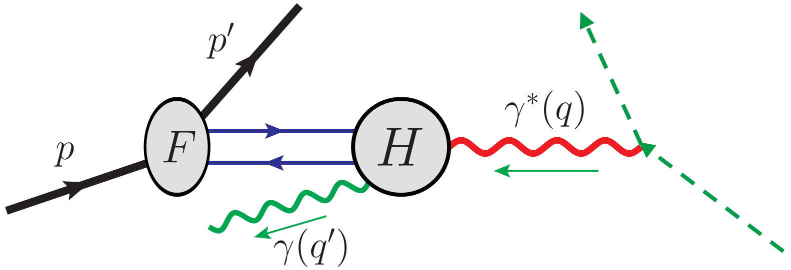



- **Electron scattering induces photon radiation.**
- **Exclusive processes could occur only in QCD due to color singlet nature of the hadron.**

**A consistent theoretical formalism (& applicable experimental approach) is still unclear now.**

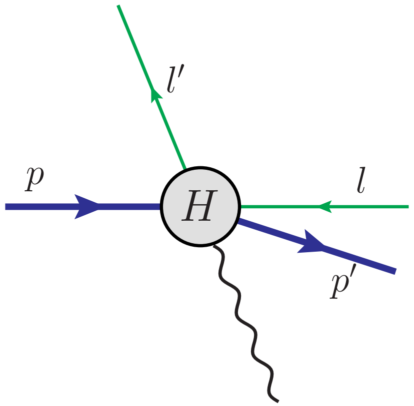
# How to *think* about the GPD processes?

**DVCS in Breit frame**




- **Hard scale $Q$ manifest**
- **Cannot put Bethe-Heitler process in a coherent framework**

**DVCS in lab frame**




- **Hard scale $q_T$**