Recent progress on nucleon form factors in lattice QCD at the physical point

Shoichi Sasaki for PACS Collaboration

TOHOKU
UNIVERSITY

In collaboration with: Y. Aoki, K.-I. Ishikawa, Y. Kuramashi, E. Shintani, R. Tsuji and T. Yamazaki

PACS Collaboration Members

PACS=**P**rocessor **A**rray for **C**ontinuum **S**imulation

N. Ishizuka, Y. Kuramashi, E. Shintani, N. Ukita, T. Yamazaki, T. Yoshie	Tsukuba Univ.
Y. Aoki, Y. Nakamura,	RIKEN-CCS
K.-I. Ishikawa, Y. Namekawa	Hiroshima Univ.
S. Sasaki, R. Tsuji,	Tohoku Univ.

Nucleon Structure Project

Our Physics Targets

Nucleon structure = properties of single nucleon

✓Elastic form factor: general properties of nucleon

✓Structure function → generalized parton distributions (GPD)

- Deep Inelastic scattering (proton spin puzzle)

Tsuji et al., PoS LATTICE 2021 (2022) 504 (2121.15276)

✓Experimentally inaccessible matrix elements: scalar and tensor charges

- Physics beyond the standard model

Tsuji et al., Phys. Rev. D106 (2022) 094505

Our Physics Targets

Nucleon structure = properties of single nucleon

✓ Elastic form factor: general properties of nucleon

✓ Structure function → generalized parton distributions (GPD)

- Deep Inelastic scattering (proton spin puzzle)

Tsuji et al., PoS LATTICE 2021 (2022) 504 (2121.15276)

✓ Experimentally inaccessible matrix elements: scalar and tensor charges

- Physics beyond the standard model

Tsuji et al., Phys. Rev. D106 (2022) 094505

Our Physics Targets

Nucleon structure = properties of single nucleon

✓Elastic form factor: general properties of nucleon

- K.I. Ishikawa et al., Phys. Rev. D98 (2018) 074510. (HPCI)
- E. Shintani et al., Phys. Rev. D99 (2019) 014510. (PACS10)
- K.I. Ishikawa et al., Phys. Rev. D104 (2021) 074514. (PACS10)
- R. Tsuji et al., arXiv:2311.10345. (PACS10)

Our Physics Targets

Nucleon structure = properties of single nucleon

✓ Elastic form factor

Vector $\langle p'|V^\mu(q)|p\rangle = \overline{u}(p')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2M}F_2(q^2)\right]u(p)$ weak and elemag

$$= \overline{u}(p')\left[\frac{(p'+p)^\mu}{2M}\frac{G_E(q^2) - \frac{q^2}{4M^2}G_M(q^2)}{1-\frac{q^2}{4M^2}} + i\sigma^{\mu\nu}\frac{q_\nu}{2M}G_M(q^2)\right]u(p)$$

Axial-vector $\langle p'|A^\mu(q)|p\rangle = \overline{u}(p')\left[\gamma^\mu\gamma_5 F_A(q^2) + iq^\mu\gamma_5 F_P(q^2)\right]u(p)$ only weak

➡ Five basic quantities: $g_A = F_A(0), \mu = G_M(0), G_{E,M}(q^2) = G_{E,M}(0)\left(1 - \frac{1}{6}r^2_{E,M}q^2 + \mathcal{O}(q^4)\right)$

axial charge (g_A), magnetic moment (μ), charge radius (r_E), magnetic radius (r_M), axial radius (r_A)

Our Physics Targets

Nucleon structure = properties of single nucleon

✓ Elastic form factor

Vector $\langle p'|V^\mu(q)|p\rangle = \overline{u}(p')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2M}F_2(q^2)\right]u(p)$ weak and elemag

$$= \overline{u}(p')\left[\frac{(p'+p)^\mu}{2M}\frac{G_E(q^2) - \frac{q^2}{4M^2}G_M(q^2)}{1-\frac{q^2}{4M^2}} + i\sigma^{\mu\nu}\frac{q_\nu}{2M}G_M(q^2)\right]u(p)$$

Axial-vector $\langle p'|A^\mu(q)|p\rangle = \overline{u}(p')\left[\gamma^\mu\gamma_5 F_A(q^2) + iq^\mu\gamma_5 F_P(q^2)\right]u(p)$ only weak

➡ Five basic quantities: $g_A = F_A(0), \mu = G_M(0), G_{E,M}(q^2) = G_{E,M}(0)\left(1 - \frac{1}{6}r^2_{E,M}q^2 + \mathcal{O}(q^4)\right)$

axial charge (g_A), magnetic moment (μ), charge radius (r_E), magnetic radius (r_M), axial radius (r_A)

Today's topics

Nucleon structure

- **Proton radius puzzle**

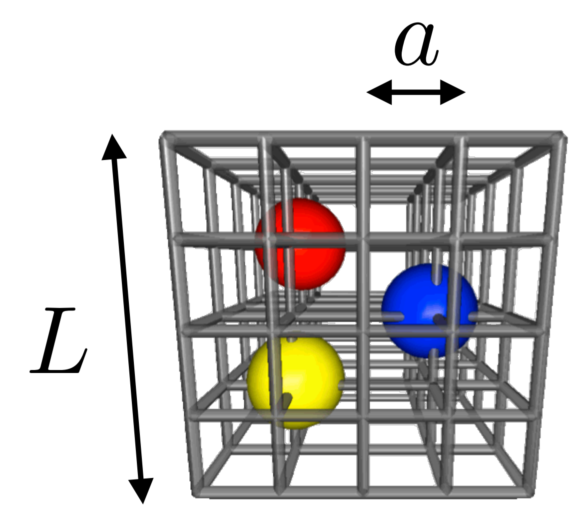
  ➡ Electric/magnetic form factor (rms radius)

- **Neutron lifetime puzzle & $\nu_\mu \rightarrow \nu_e$ oscillation**

  ➡ Axial-vector from factor (axial charge & axial radius)

An important opportunity to develop our understanding of nucleon structure using lattice QCD simulations

Our strategy



✓ Use **PACS10** gauge configurations

- **Physical point** → No chiral extrapolation
- **Very large spatial volume (L)** → No finite size effect & Low q^2 physics
- **3 different lattice cut-offs (a)** → Continuum limit (currently not available)

✓ **All-mode averaging technique** → High precision measurements

✓ **Highly tuned smearing** → Suppression of excited-state contributions

✓ **Model-independent Q^2 fit** by z-Expansion method

Status of PACS10 projects

Configuration	PACS10			HPCI
Resource	Oakforest-PACS → Fugaku			K-computer
N_f	2+1			2+1
m_π [MeV]	135			146
L [fm]	10 fm			8.1 fm
L^3 x T	128^4(64^4)	160^4	256^4	96^4
a [fm]	0.085	0.063	~0.04	0.085
Status	done	done	done	done
Nucleon FF	done	done	running	done
Renorm (SF, NPR)	done	partly done	planning	done

Iso-vector quantities

electromagnetic current

$$J_\mu^{\rm em} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d + \cdots$$

$$= \frac{1}{2}\boxed{(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)} + \frac{1}{6}\boxed{(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)} = J_\mu^V + \frac{1}{3}J_\mu^S$$

iso-vector   iso-scalar

matrix element (ME)

proton $\langle p|J_\mu^{\rm em}|p\rangle = \langle p|J_\mu^V|p\rangle + \frac{1}{3}\langle p|J_\mu^S|p\rangle$

neutron $\langle n|J_\mu^{\rm em}|n\rangle = \langle n|J_\mu^V|n\rangle + \frac{1}{3}\langle n|J_\mu^S|n\rangle$

iso-spin symmetry

$\langle p|J_\mu^S|p\rangle = \langle n|J_\mu^S|n\rangle$

$\langle p|J_\mu^V|p\rangle = -\langle n|J_\mu^V|n\rangle$

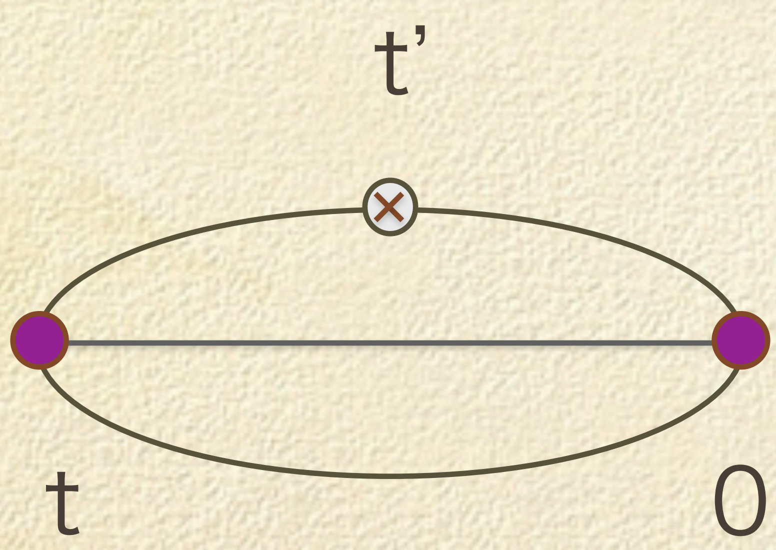
$$\langle p|J_\mu^{\rm em}|p\rangle - \langle n|J_\mu^{\rm em}|n\rangle = \boxed{\langle p|\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d|p\rangle} = \boxed{\langle p|\bar{u}\gamma_\mu d|n\rangle}$$

proton ME   **neutron ME**   **iso-vector**   Weak process

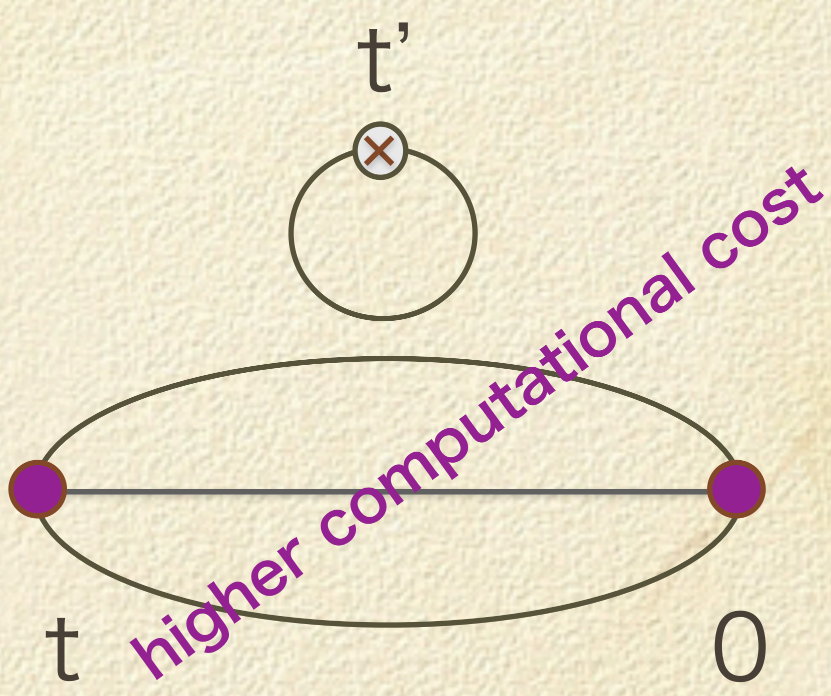
Iso-vector part receives NO disconnected contribution in 2+1 flavor QCD

Connected/disconnected diagrams

$\langle \mathcal{H}(t)\mathcal{O}(t')\mathcal{H}^\dagger(0)\rangle$ has **two** types of quark contraction diagrams (Wick contractions)



connected contribution



disconnected contribution

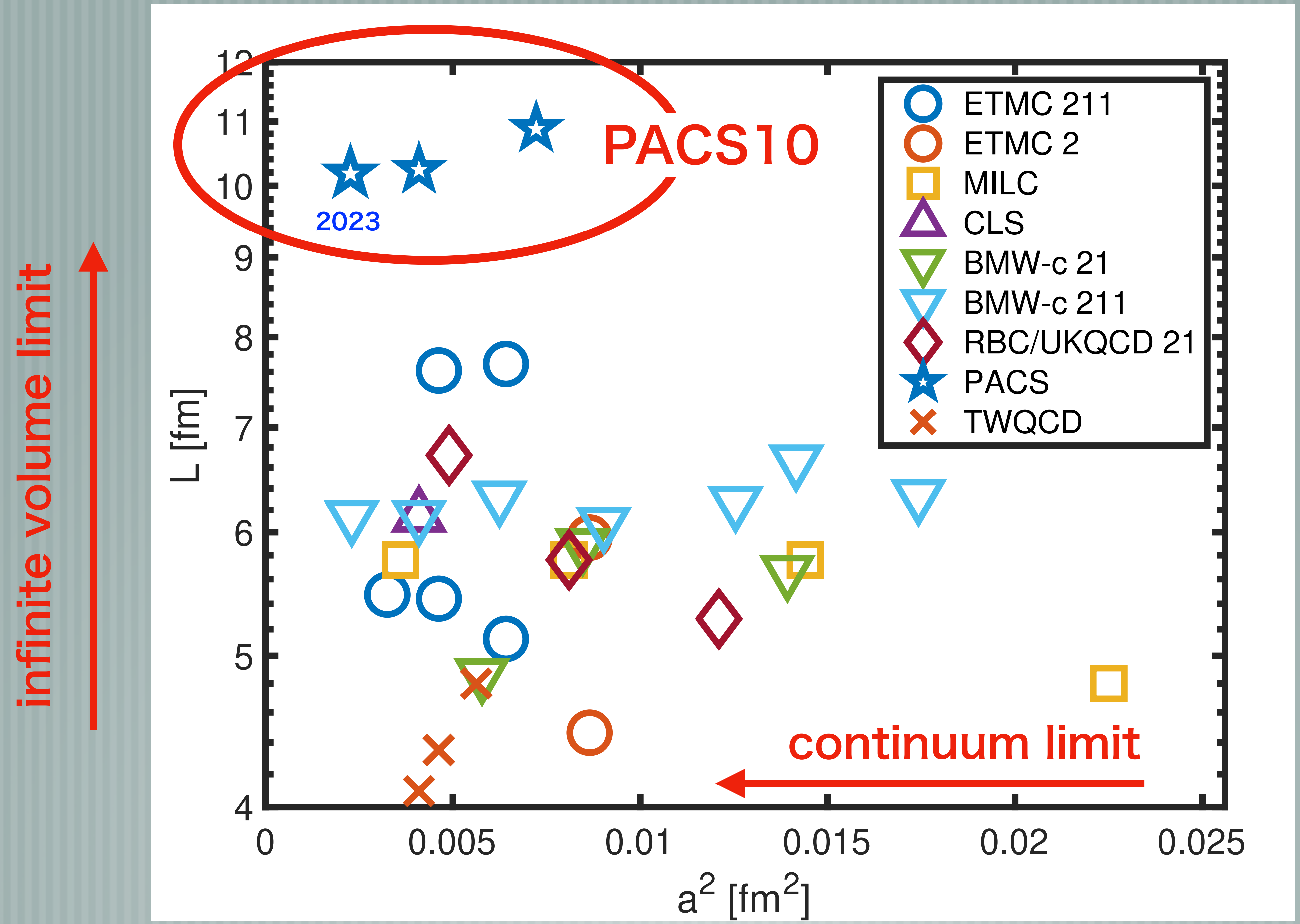
✓ **iso-vector** quantities

✓ β-decay (weak matrix elements)

✓ **flavor diagonal** quantities

✓ electro-magnetic matrix elements

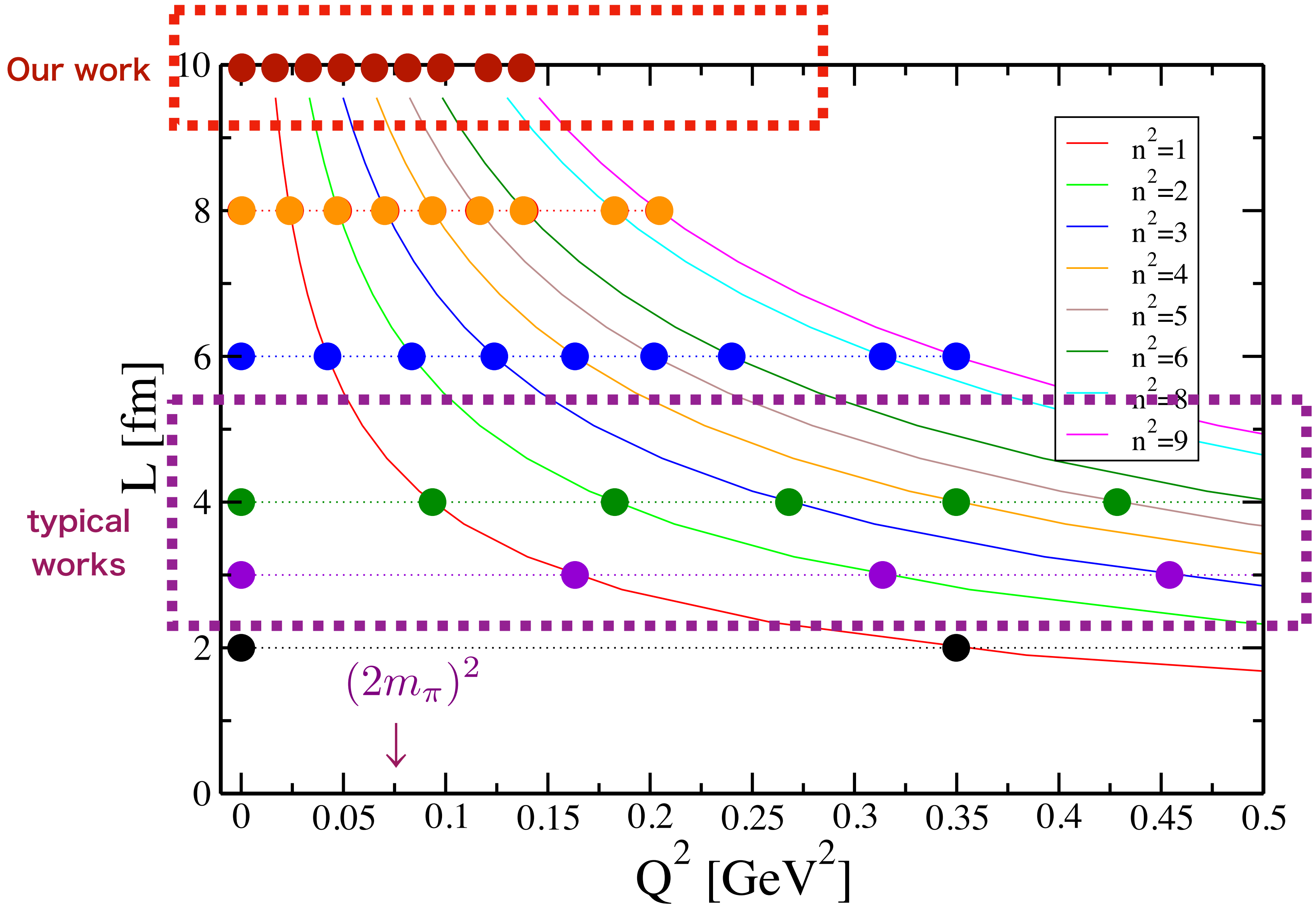
World status of lattice QCD projects near the physical point

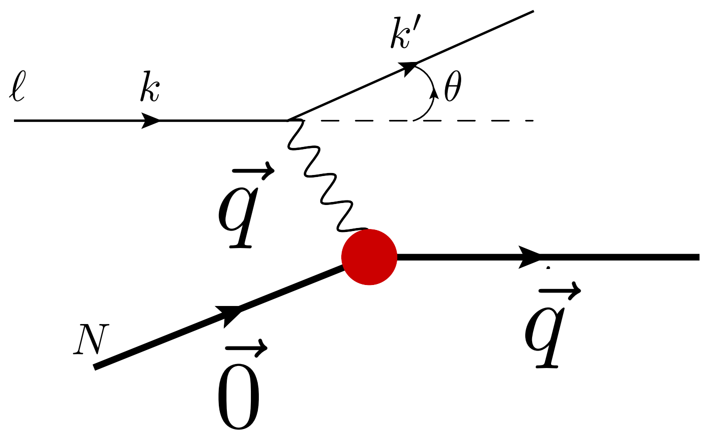


Plenary talk at Lattice 2022 given by Finkenrath

How Large Spatial Size is Necessary?

Discrete momenta on the lattice are related to the size of the spatial extent L





$$Q^2 = -q^2 = 2M_N(E(\vec{q}) - M_N)$$

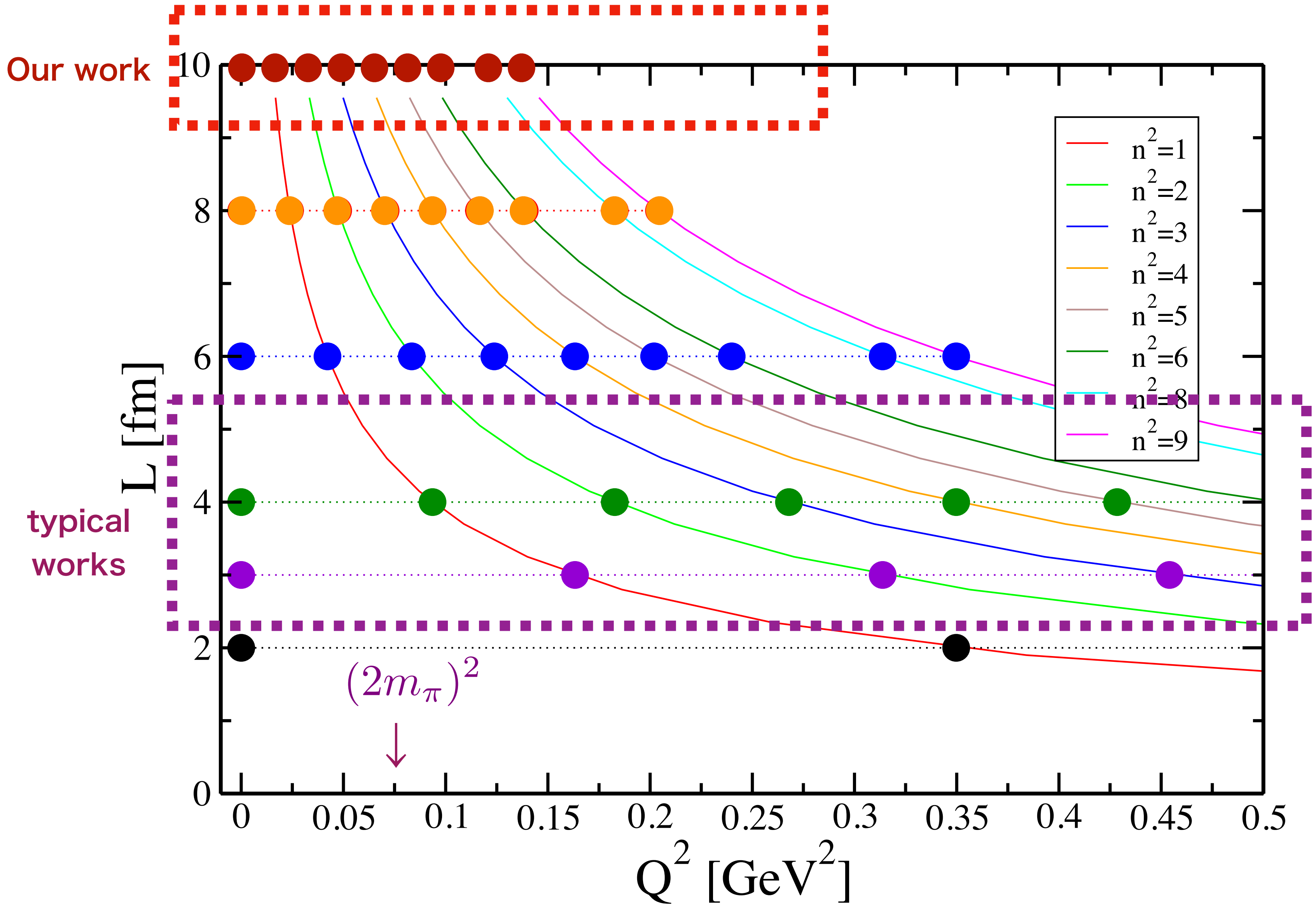
$$E(\vec{q}) = \sqrt{M_N^2 + \vec{q}^2}$$

$$\vec{q}^2 = \left(\frac{2\pi}{L}\right)^2 \vec{n}^2$$

✓can access the **small** momentum transfer **up to 114 MeV < 2m$_\pi$**

How Large Spatial Size is Necessary?

Discrete momenta on the lattice are related to the size of the spatial extent L



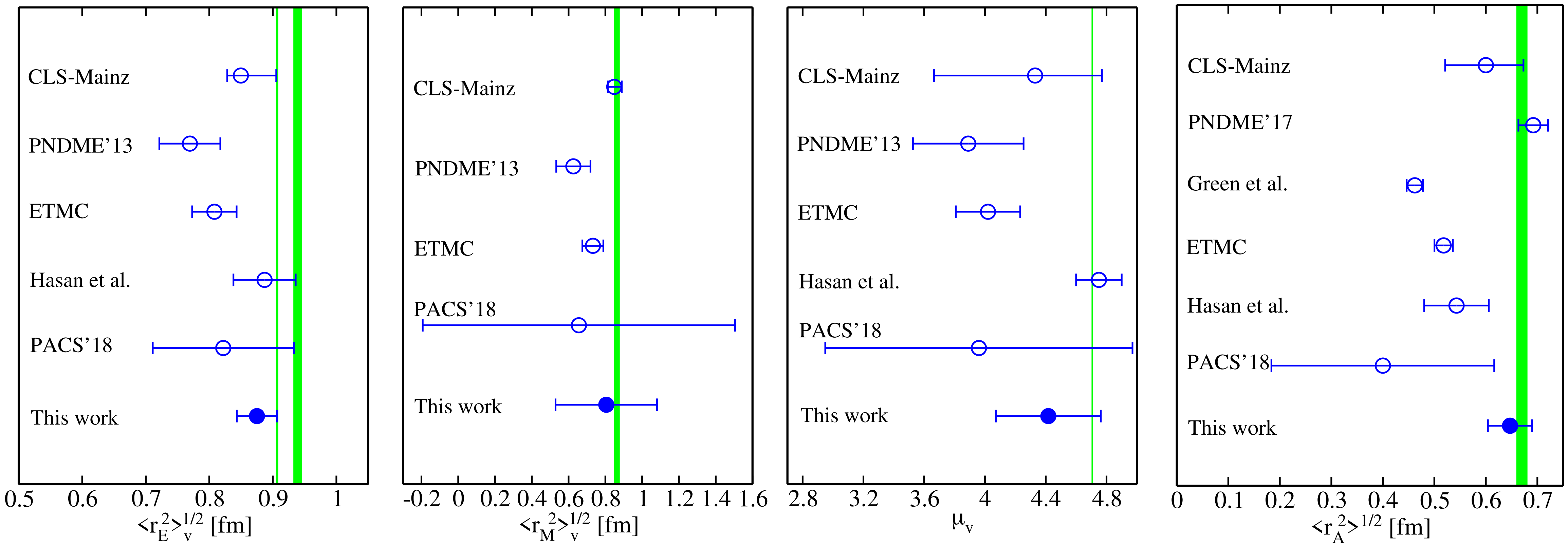
$$\langle r_E^2 \rangle = -\frac{6}{G_E(0)} \frac{dG_E(q^2)}{dq^2}\bigg|_{q^2=0}$$

Root-Mean-Square radius

$$R = \sqrt{\langle r_E^2 \rangle} = r_E$$

✓ can access the **small** momentum transfer **up to 114 MeV < 2m$_\pi$**

HAWAII 2018 or PRD99 (2019) 014510

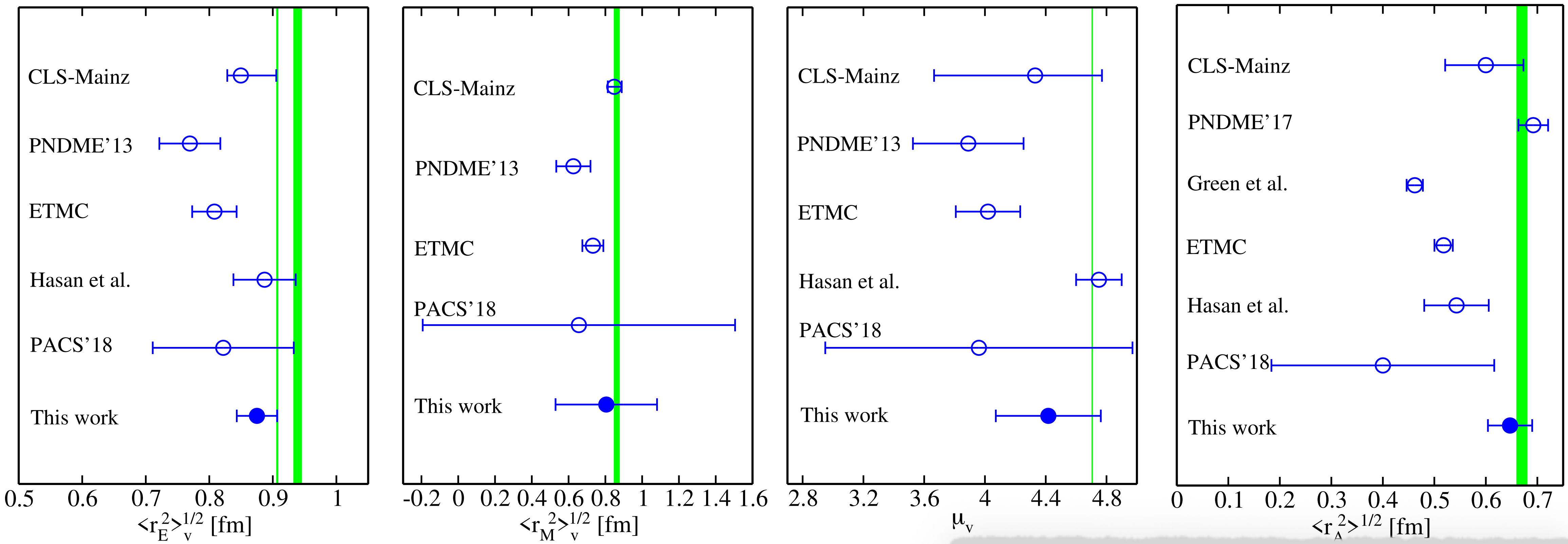


charge radius (r_E) magnetic radius (r_M) magnetic moment (μ) axial radius (r_A)

5 years ago, I concluded that

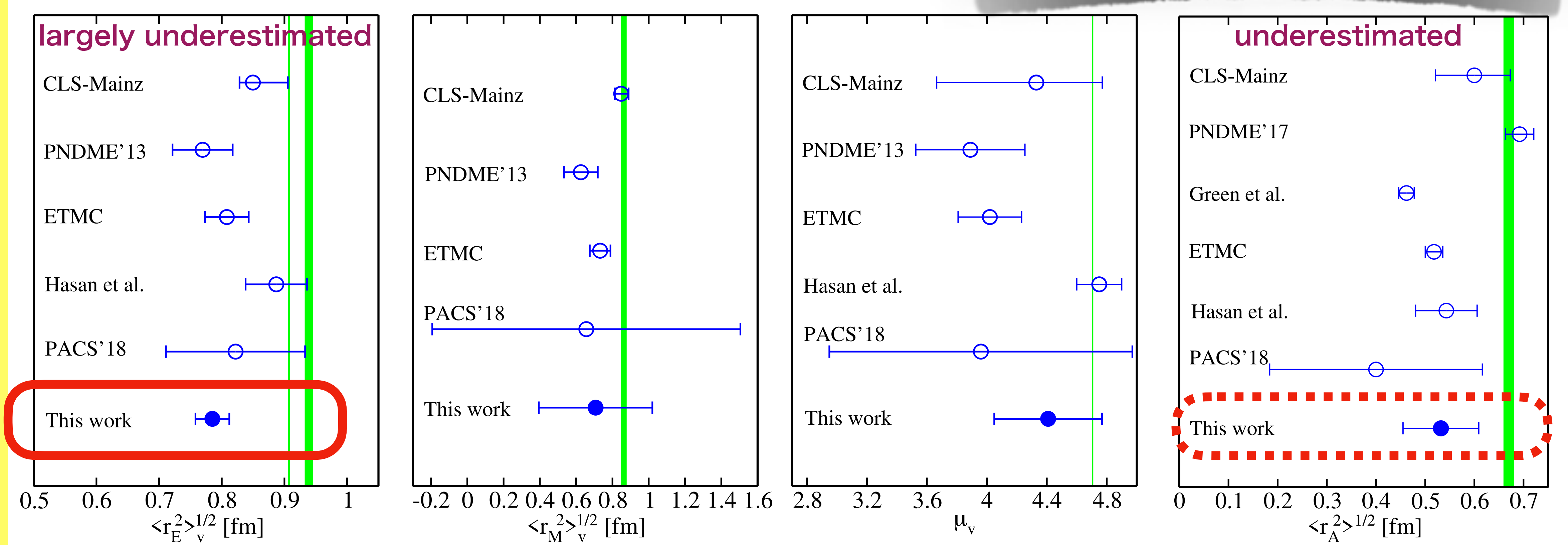
"All four quantities are consistent with experiments, as well as axial charge g_A "

HAWAII 2018 or PRD99 (2019) 014510



Erratum: PRD102 (2020) 019902

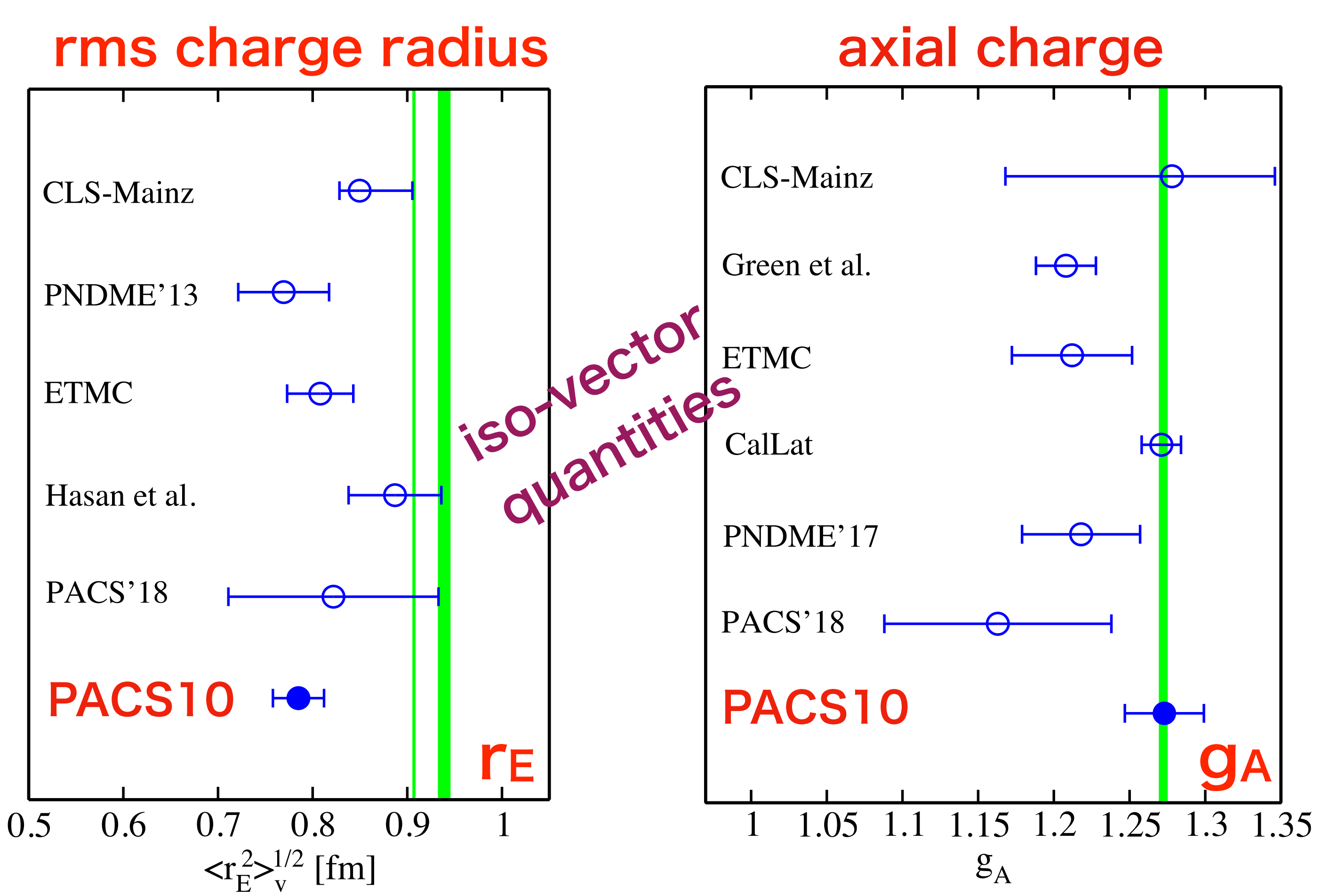
Using correct normalization



charge radius (r_E) magnetic radius (r_M) magnetic moment (μ) axial radius (r_A)

Topics covered in this talk

- Our **new PACS10** results at a=0.063 fm
  - (**10.1 fm**)^3 spatial volume with $m_\pi = \mathbf{138}$ MeV
  - **lattice discretization uncertainties** on r_E and g_A

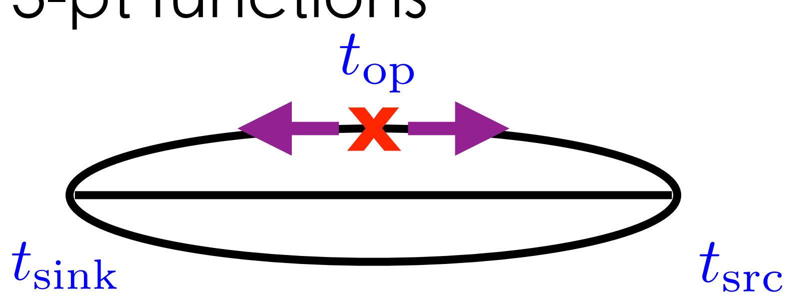


Status of PACS10 projects

Configuration	PACS10			HPCI
Resource	Oakforest-PACS → Fugaku			K-computer
N_f	2+1			2+1
m_π [MeV]	135	138	~135	146
L [fm]	10 fm			8.1 fm
L^3 x T	128^4(64^4)	160^4	256^4	96^4
a [fm]	0.085	0.063	~0.04	0.085
Status	done	done	done	done
Nucleon FF	done	done	running	done
SF, NPR	done	partly done	planning	done

Measurement Details for **160^4**

- Statistics: 19 configs (every 10 trajectories)
  - All-mode averaging technique (E. Shintani et al., PRD91 (2015) 114511)
    ➡ gain high statistical precision
  - O(100) measurements/config ⇒ O(10^3–10^4) measurements
- $L^3 \times T = 160^3 \times 160$ ⇒ (~10.1 fm)^3 spatial volume
- 7 choices for spatial momenta: $2\pi/L \times \vec{n}$
  - $\vec{n}$ = (1,0,0), (1,1,0), (1,1,1), (2,0,0), (2,1,0), (2,2,0)
  - minimum momentum=2π/L~122 MeV thanks to L~10.1 fm
  - allows to access FFs in the region of smaller Q^2
- Exponentially smeared src/sink operators for 2-pt and 3-pt functions
- 3 different src-sink separations: t_{sep}/a=13,16,19
  - fixed-sink in sequential source method
- Z_A and Z_V are determined in SF scheme

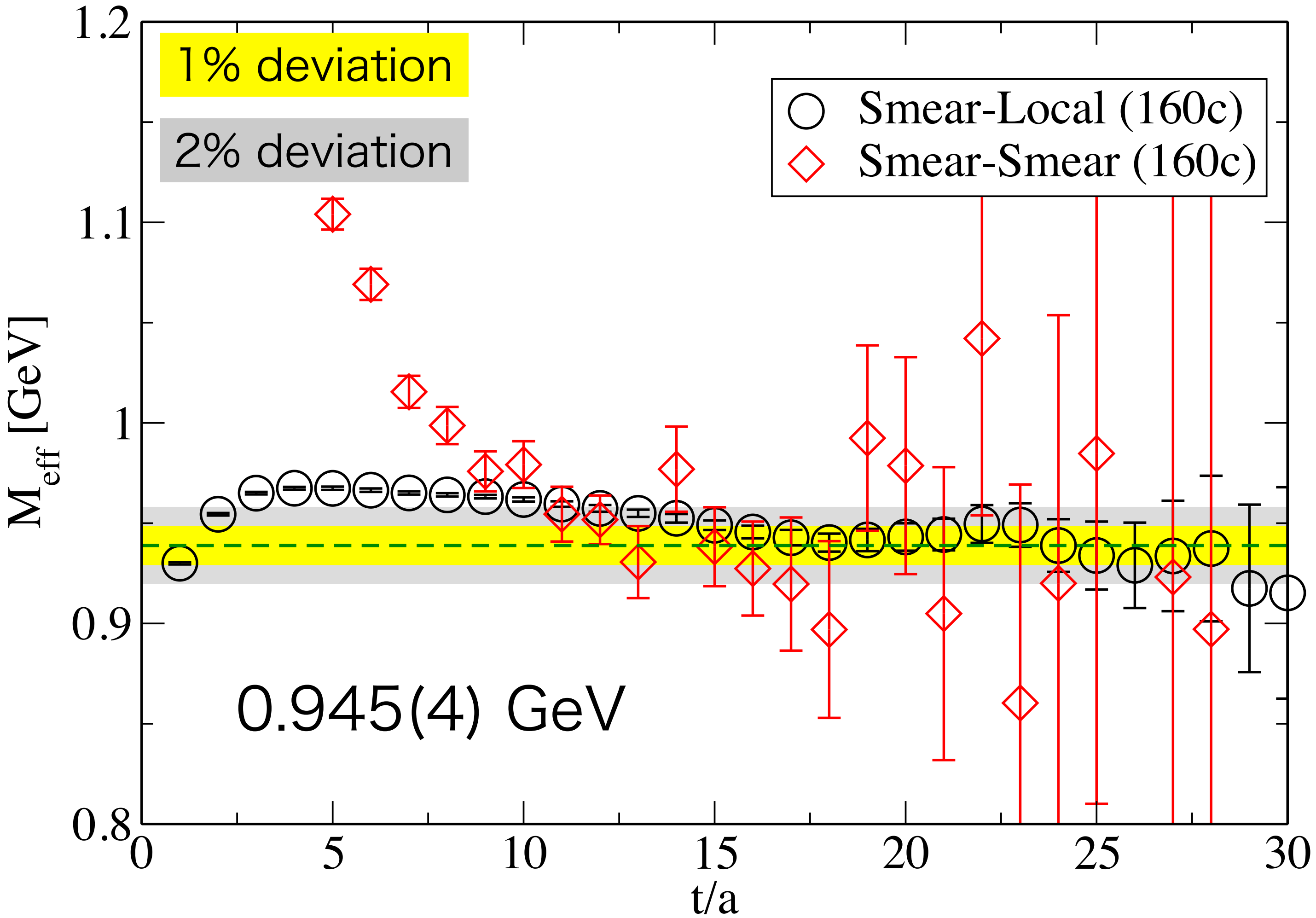


Nucleon mass

M_N

Effective mass plot for M_N

Smearing parameters are **highly tuned to maximize the ground-state dominance**.



$L^4=160^4$, a=0.063 fm

$$G(t) = \sum_i A_i \exp(-M_i t)$$

A sum of exponential func.

$$M_0 < M_1 < \cdots$$

$$M_{\text{eff}}(t) = \ln\{G(t)/G(t+1)\}$$

$$\xrightarrow[t\to\infty]{} M_0$$

$$A_0 \gg A_{i>0}$$

Achieving a percent level precision on the nucleon mass

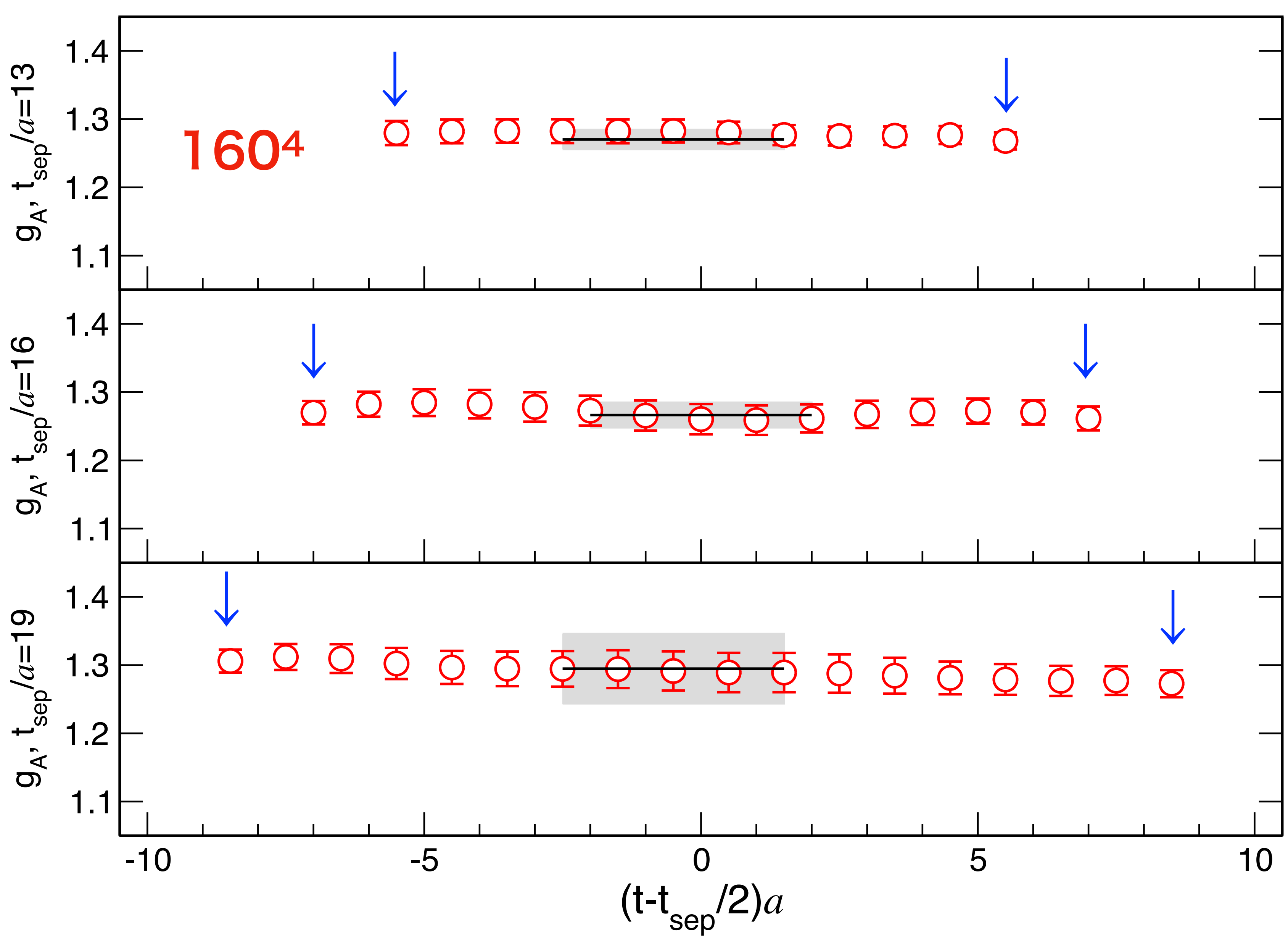
Axial charge

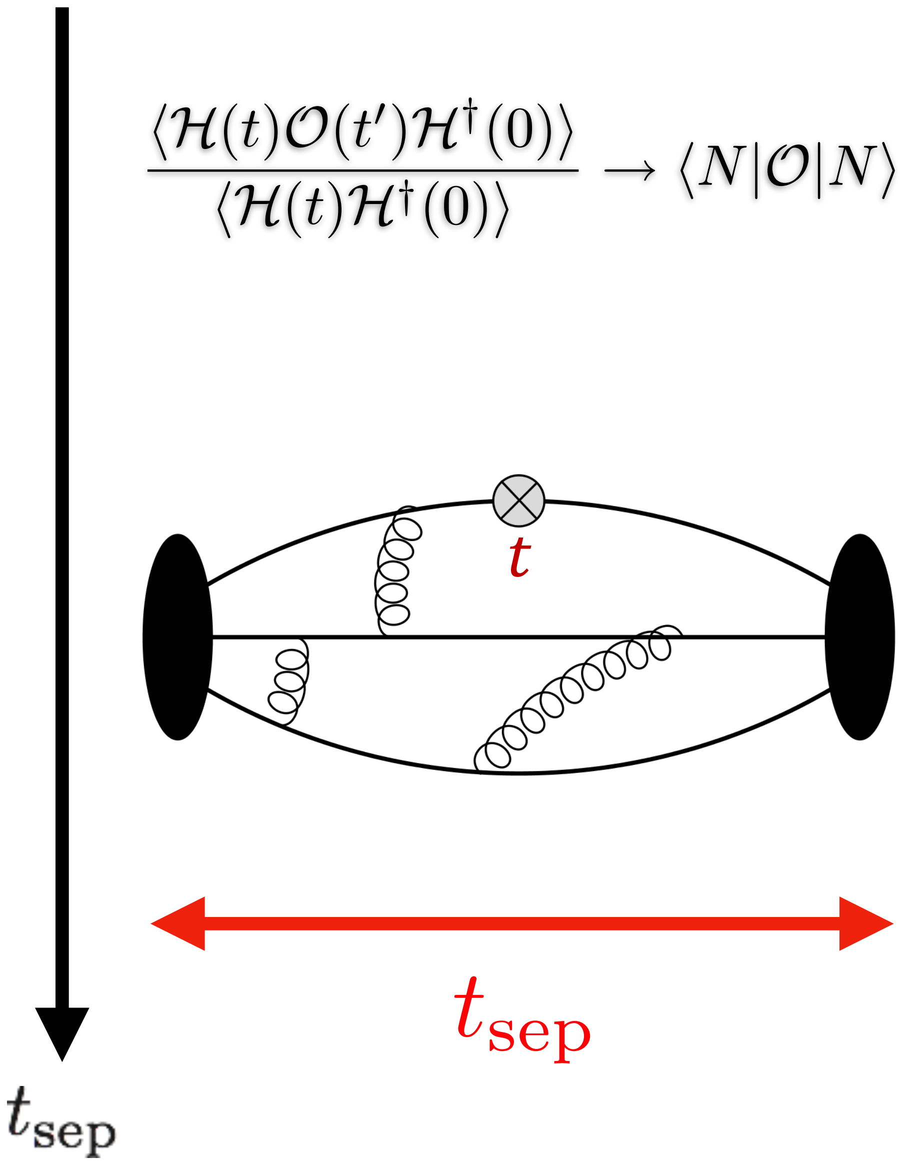
g_A

Ratio for iso-vector g_A

Tsuji et al. arXiv: 2311.10345

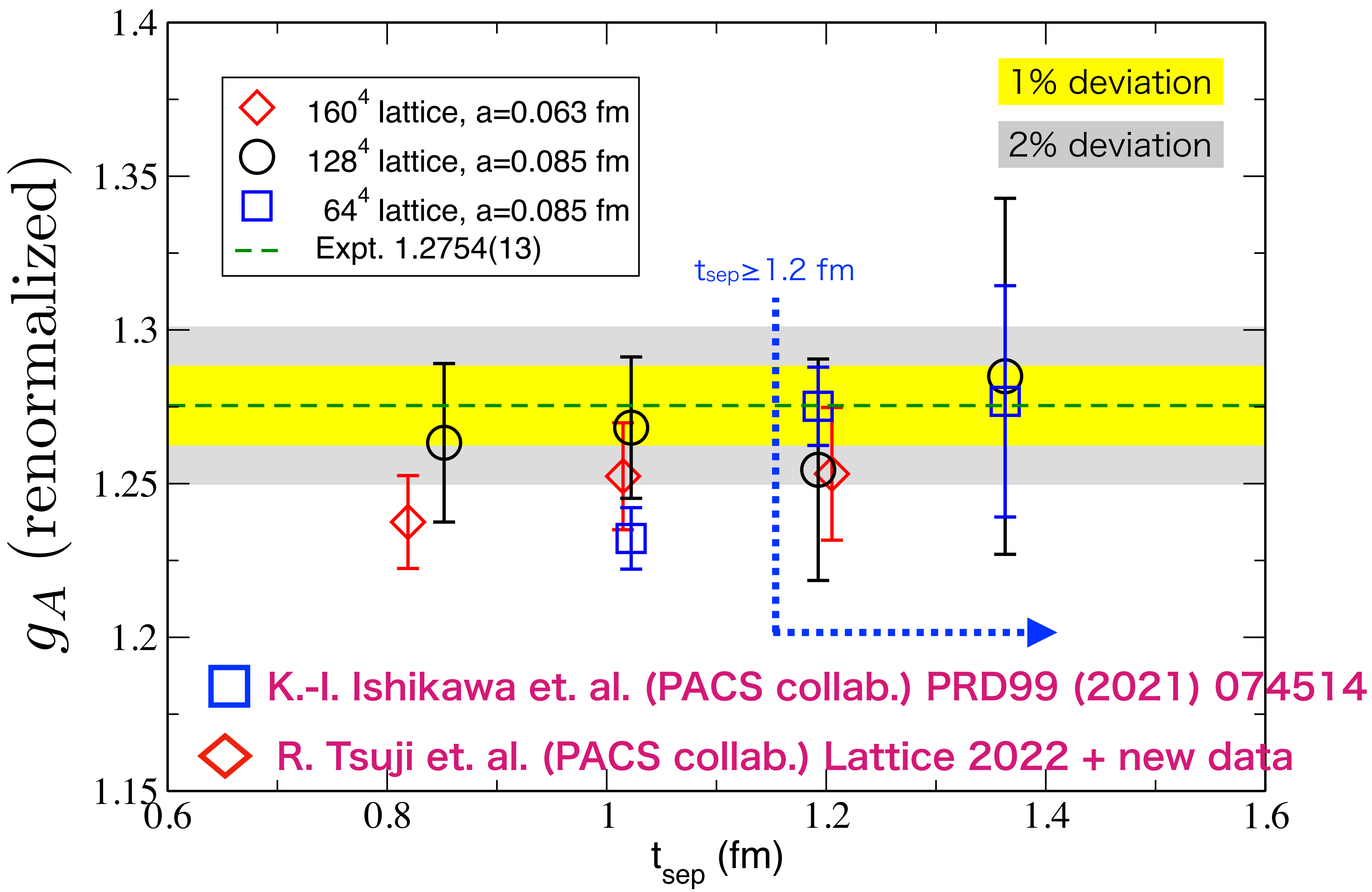
Effect of excited-state contamination is negligible





Good plateau for t_{sep}=13, 16, 19

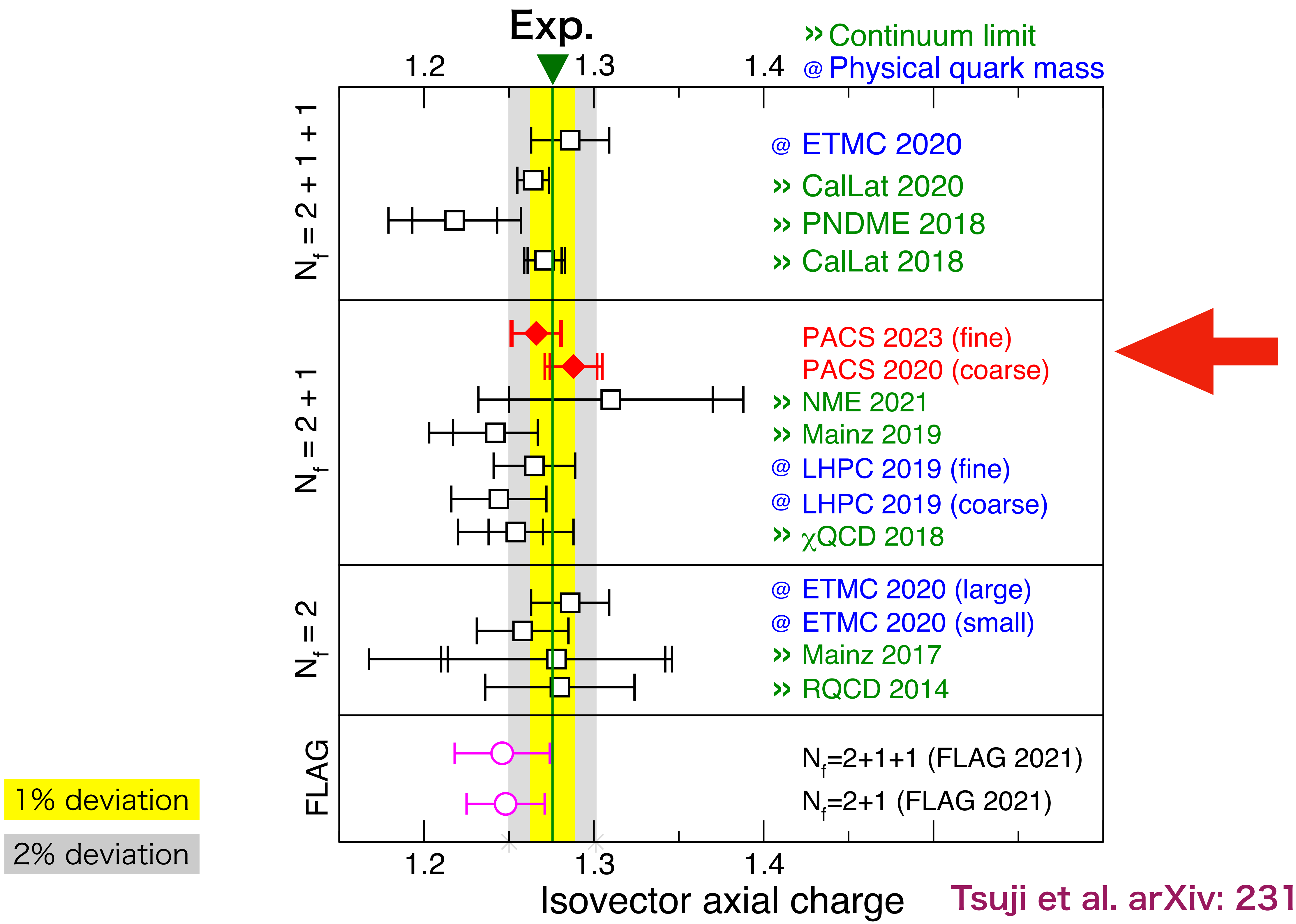
A precent-level determination of g_A



Effect of excited state contamination is negligible for $t_{sep} \geq 1.2$ fm.
Finite volume error is less than 1%.
Discretization error is less than 1%.

Tsuji et al. arXiv: 2311.10345

A percent-level determination of g_A



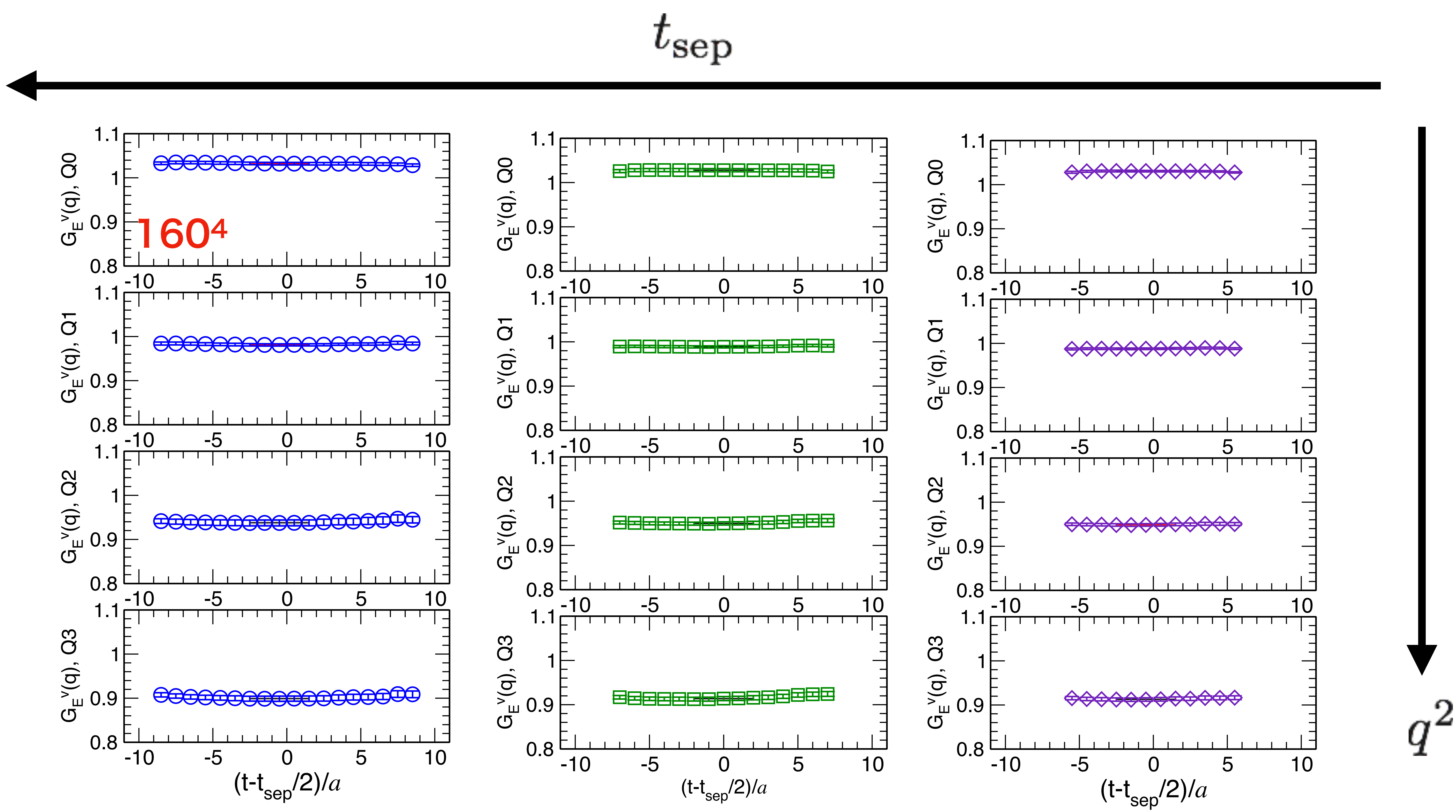
Tsuji et al. arXiv: 2311.10345

Lattice discretization error on g_A is negligibly small (< 1%)

Electric form factor

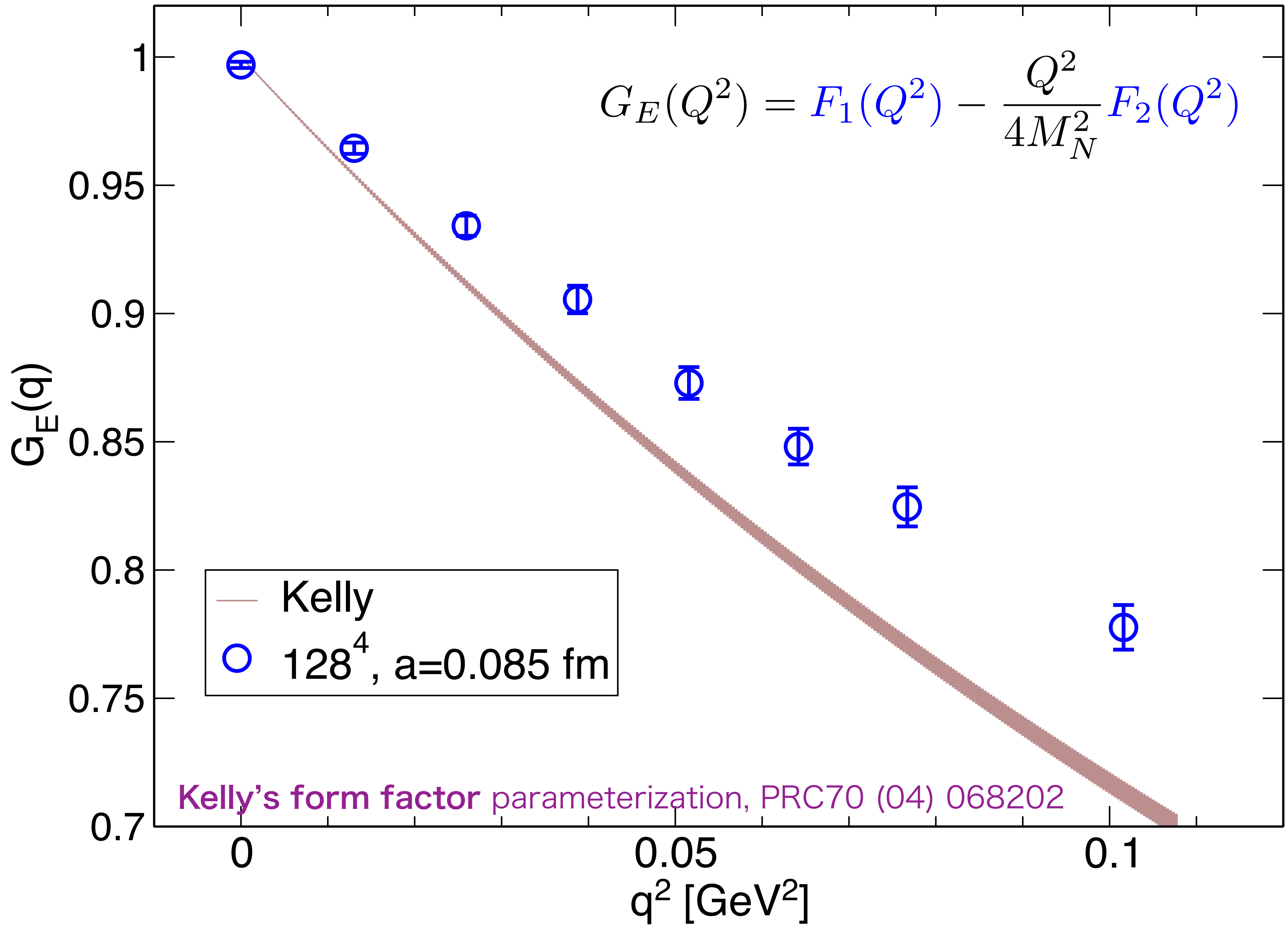
G_E

Ratio for iso-vector $G_E(q^2)$



Good plateau for t_{sep}=13, 16, 19

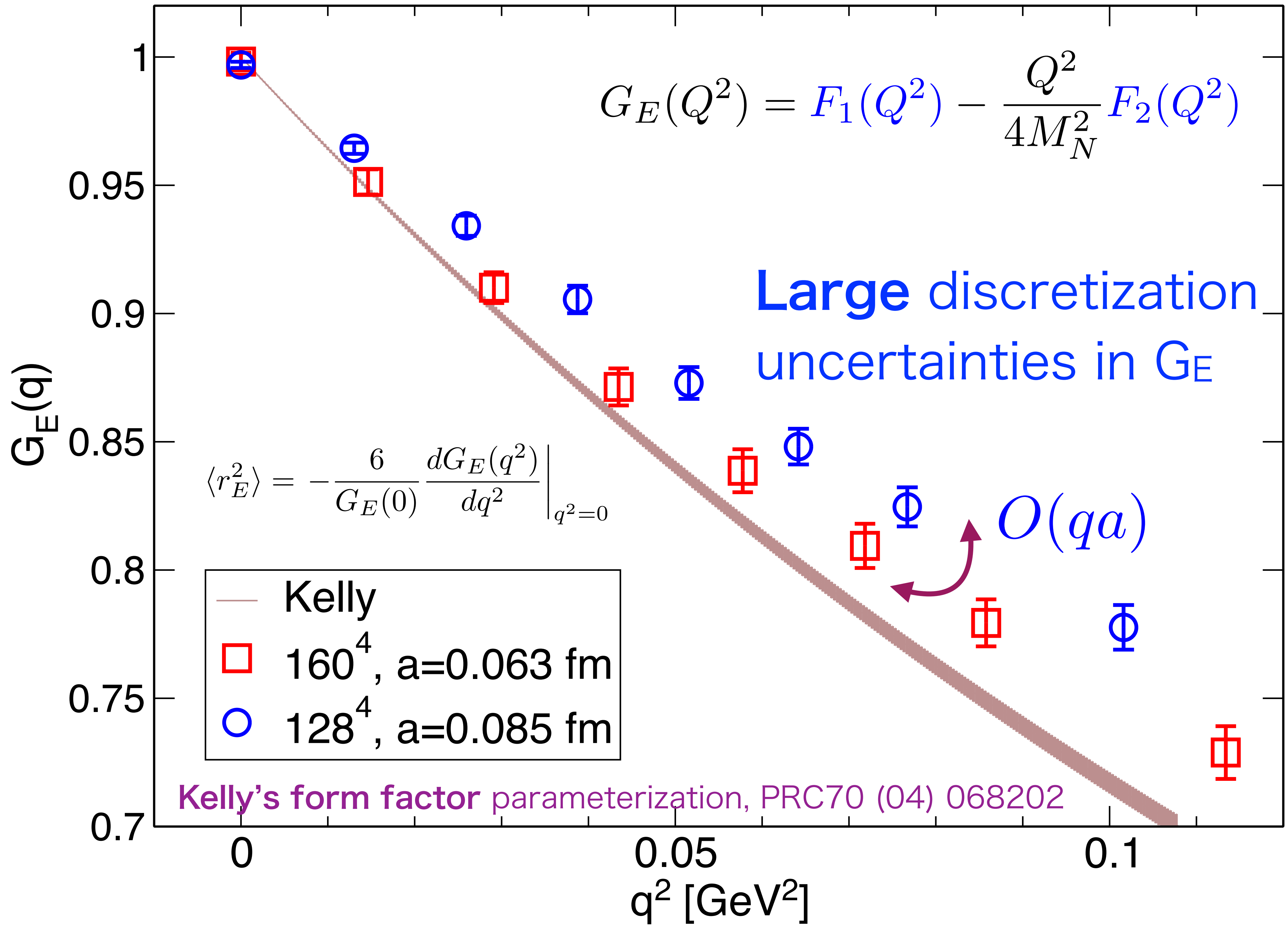
Iso-vector electric form factor G_E



The previous results disagree with the Kelly's curve

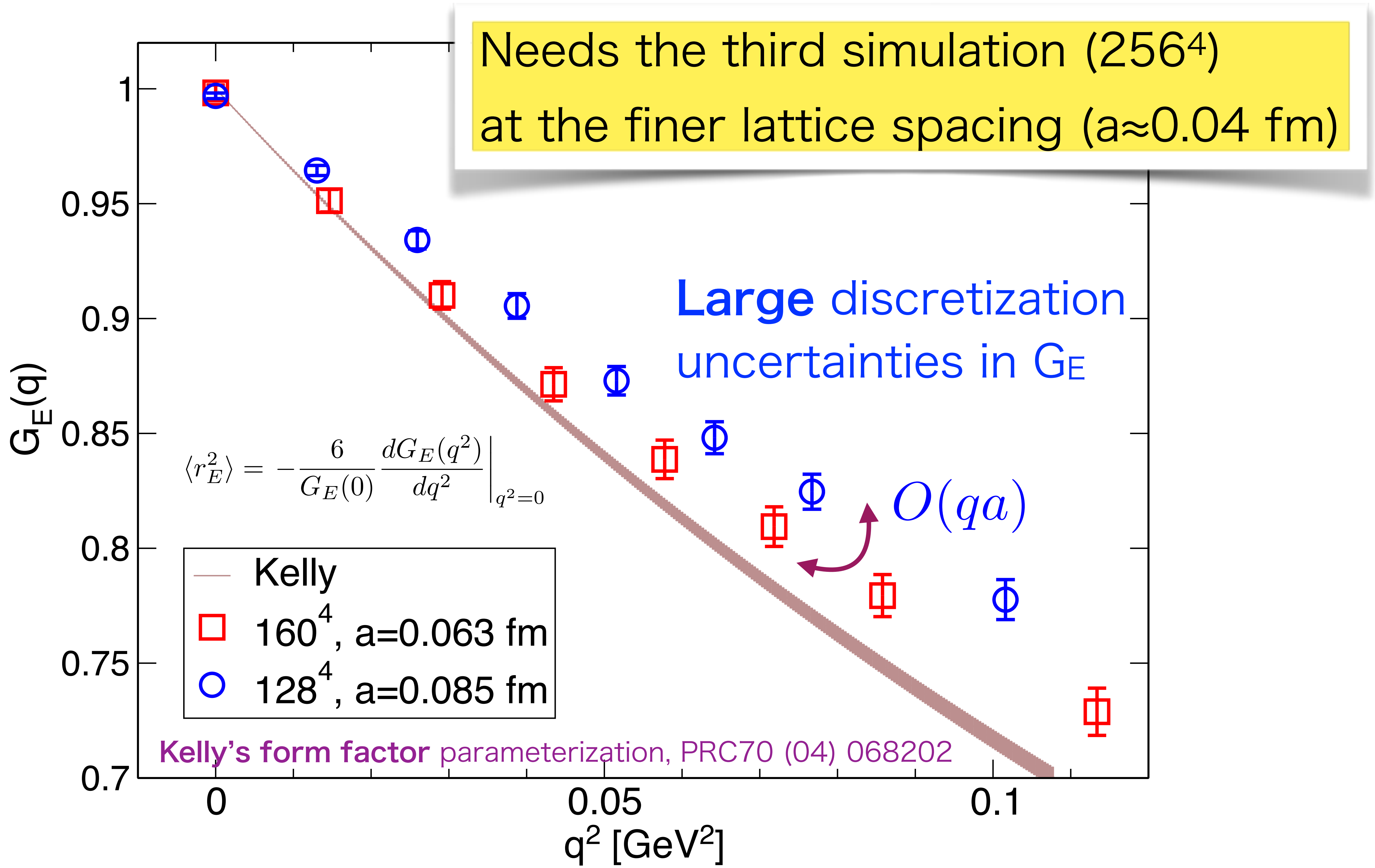
Iso-vector electric form factor G_E

Tsuji et al. arXiv: 2311.10345



The new results obtained with the fine lattice spacing (a≈0.06 fm) approaches to the Kelly's curve

Iso-vector electric form factor G_E

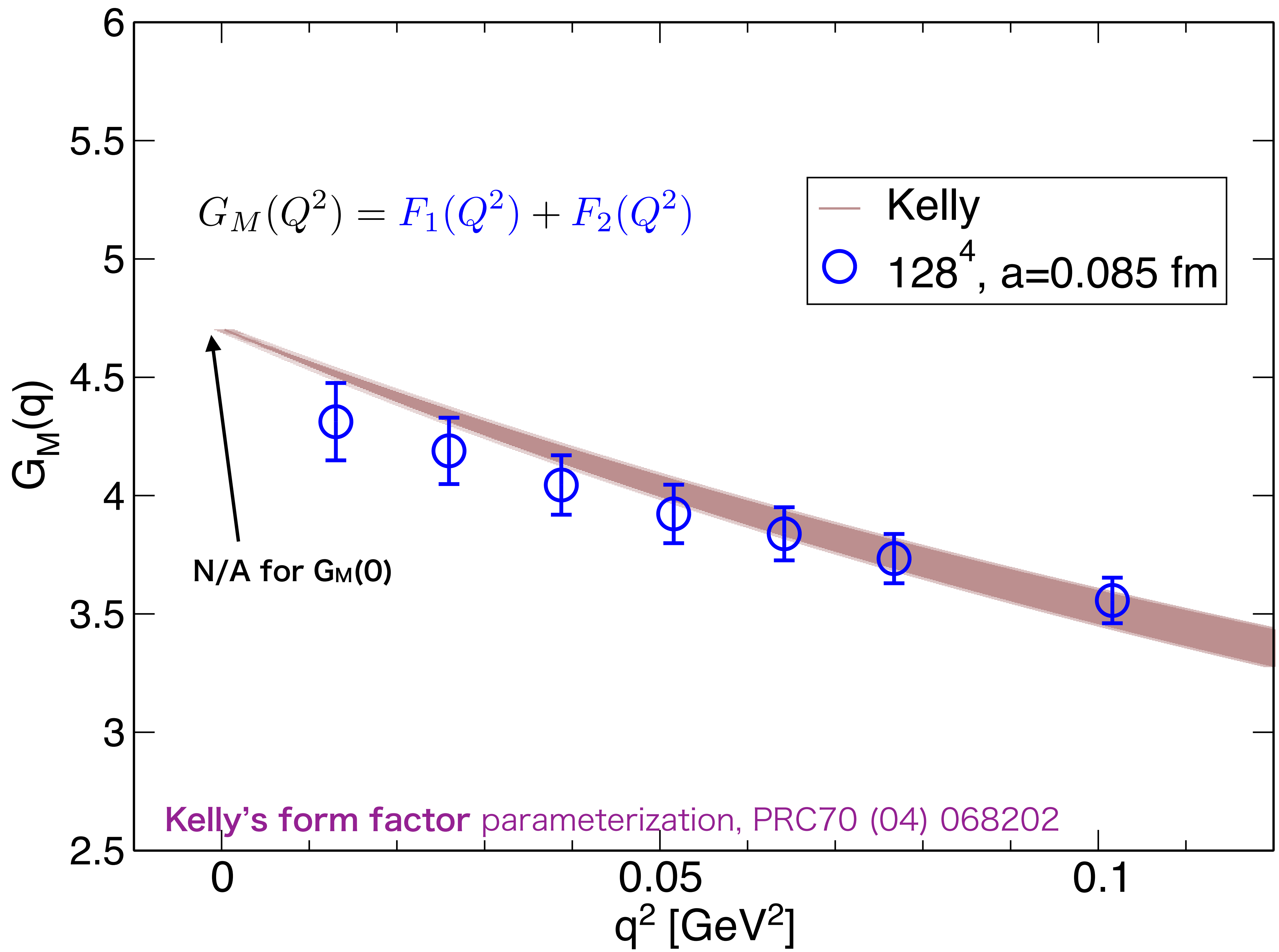


The results obtained with the fine lattice spacing (a≈0.06 fm) approaches to the Kelly's curve

Magnetic form factor

G_M

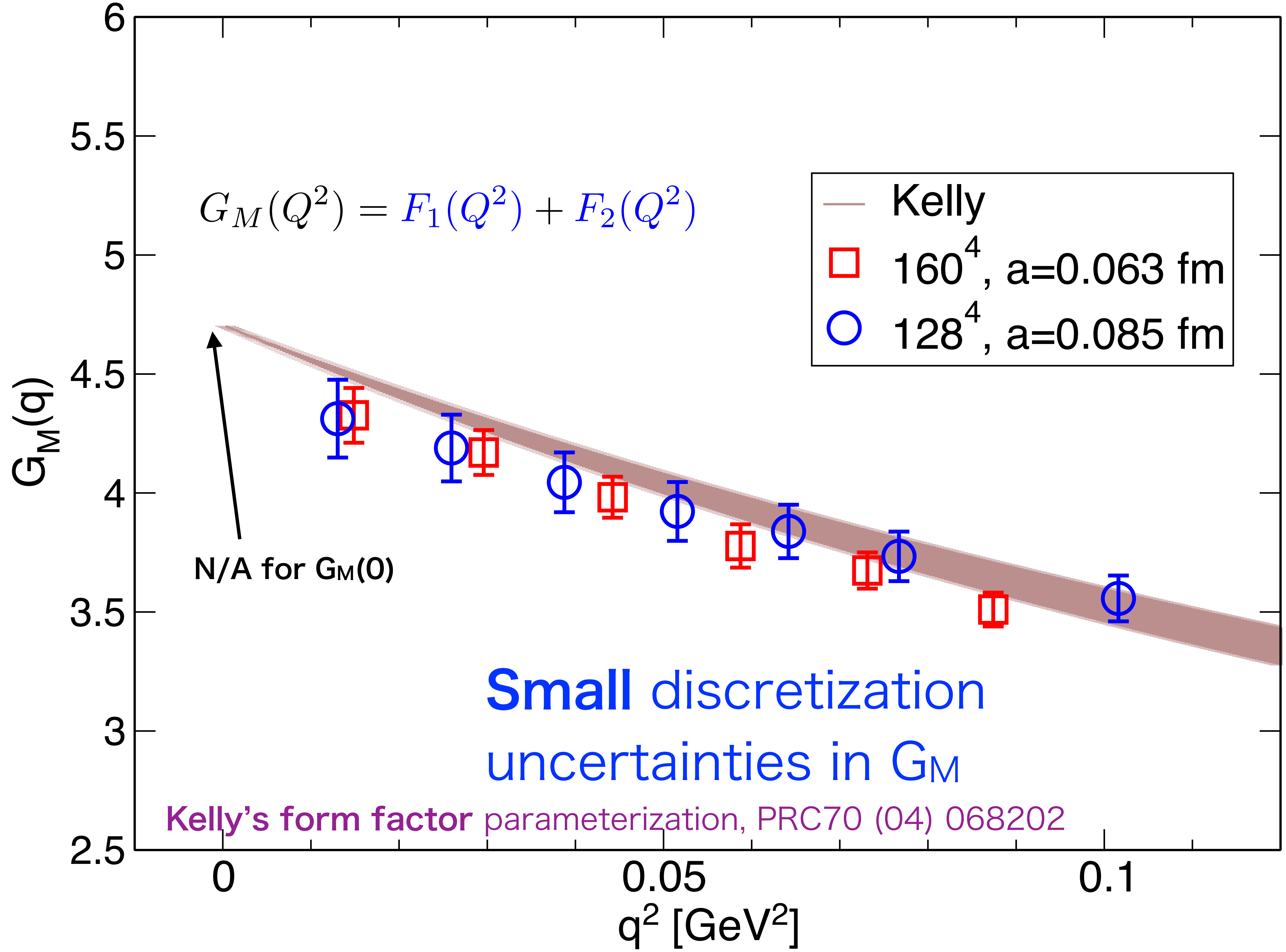
Iso-vector magnetic form factor G_M



The previous results barely agree with the Kelly's curve

Iso-vector magnetic form factor G_M

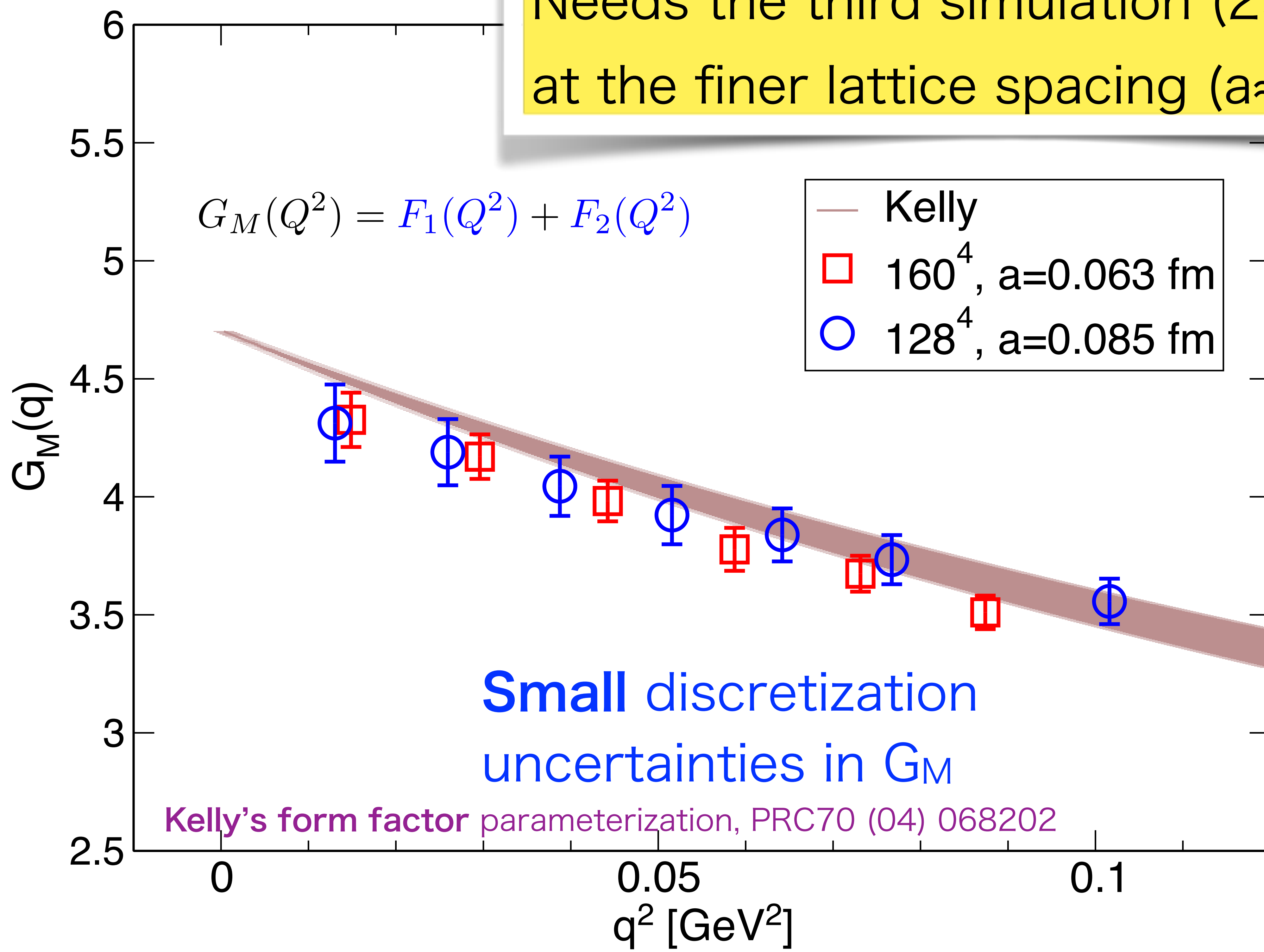
Tsuji et al. arXiv: 2311.10345



The results obtained with the fine lattice spacing (a≈0.06 fm) remain barely consistent with the Kelly's curve.

Iso-vector magnetic form factor G_M

Needs the third simulation (256^4)
at the finer lattice spacing (a≈0.04 fm)



The results obtained with the fine lattice spacing (a≈0.06 fm) remain barely consistent with the Kelly's curve.

Summary

- We have studied nucleon form factors (vector/axial-vector) calculated in 2+1 flavor QCD **at the physical point** on **(10 fm)^4** lattice at **two lattice spacings (a=0.085 and 0.063 fm)**

✓ Large spatial volume allows investigation in the **small momentum transfer region**, $q^2 < (2\, m_\pi)^2$

✓ **High statistical precision** is achieved by all-mode averaging technique

✓ t_{sep} dependence is systematically investigated

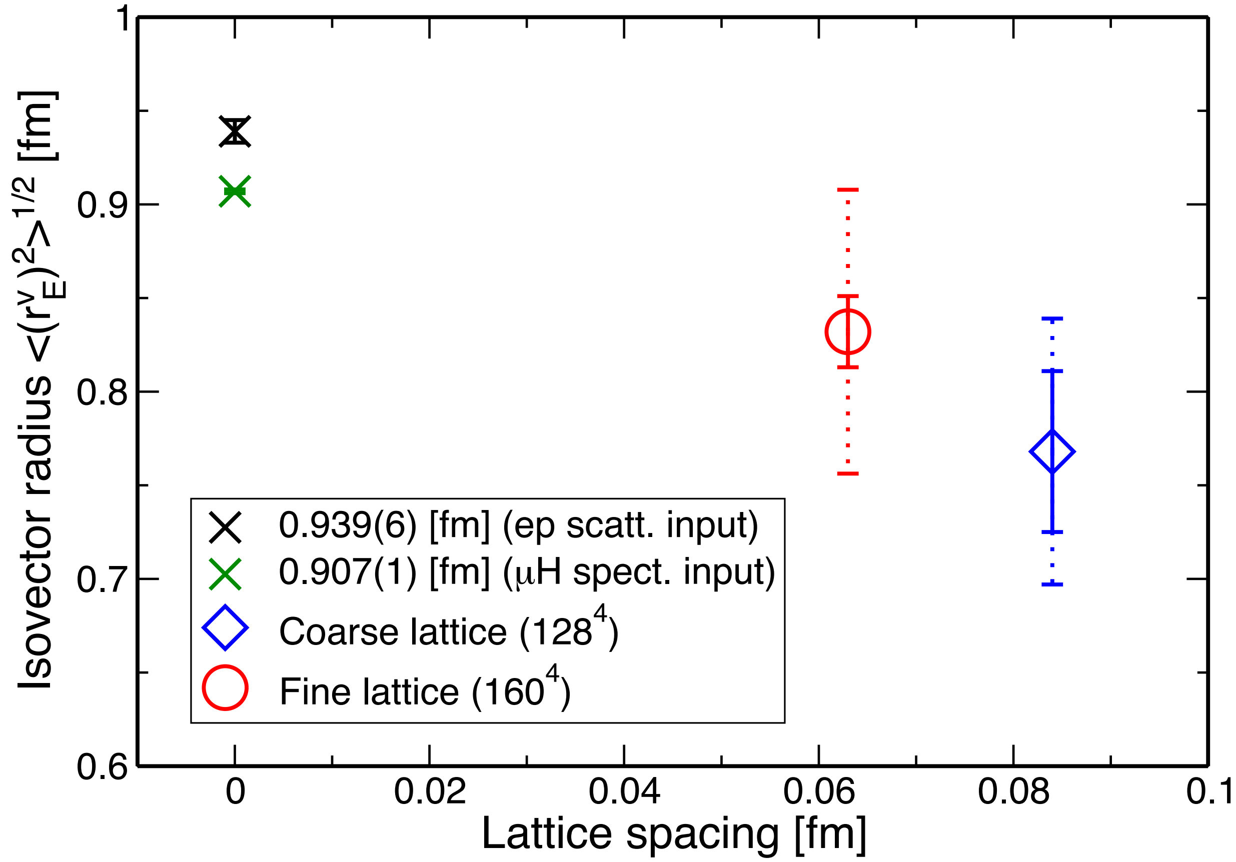
➡ g_A and G_E, G_M show **no t_{sep} dependence**

➡ excited-state contributions are negligible for $t_{sep} \geq 1.2$ fm

Summary (Cont.)

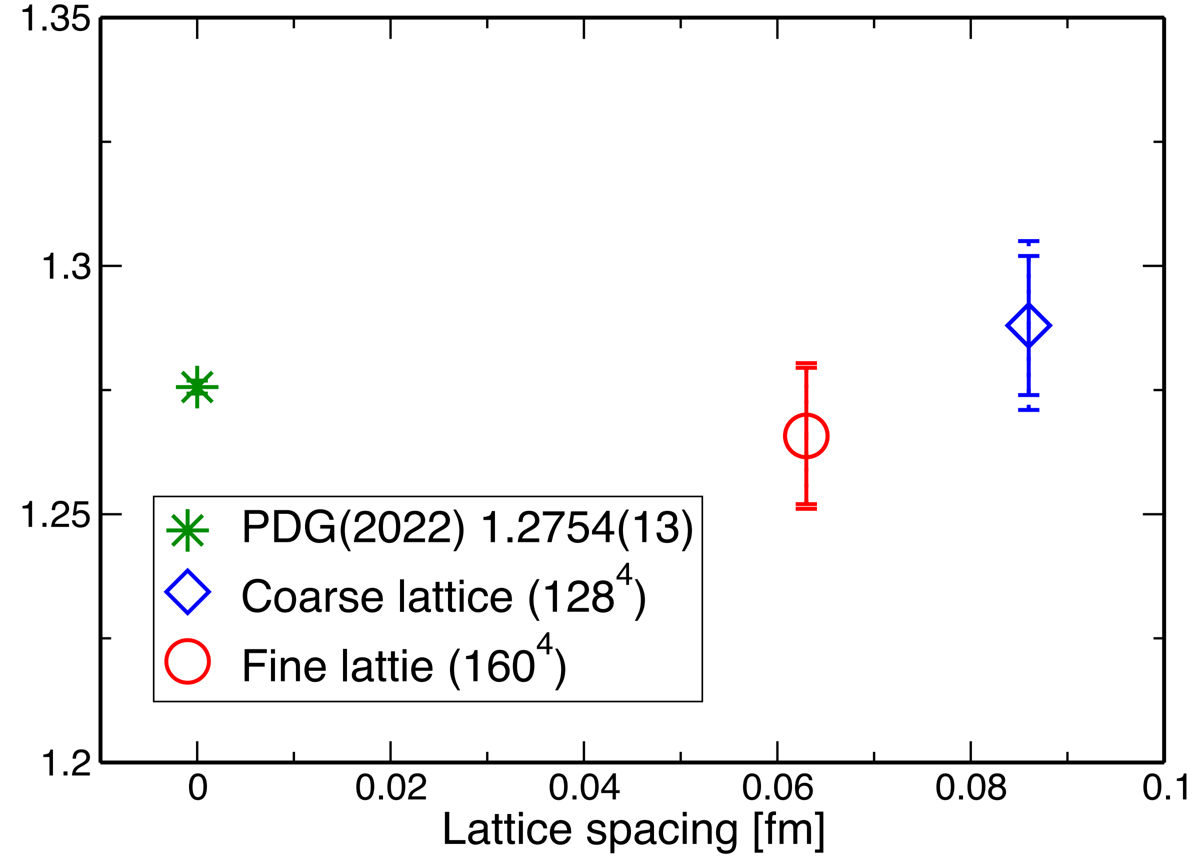
- **Lattice discretization uncertainties** on r_E and g_A

rms charge radius r_E



Significantly large(~10%)

axial charge g_A



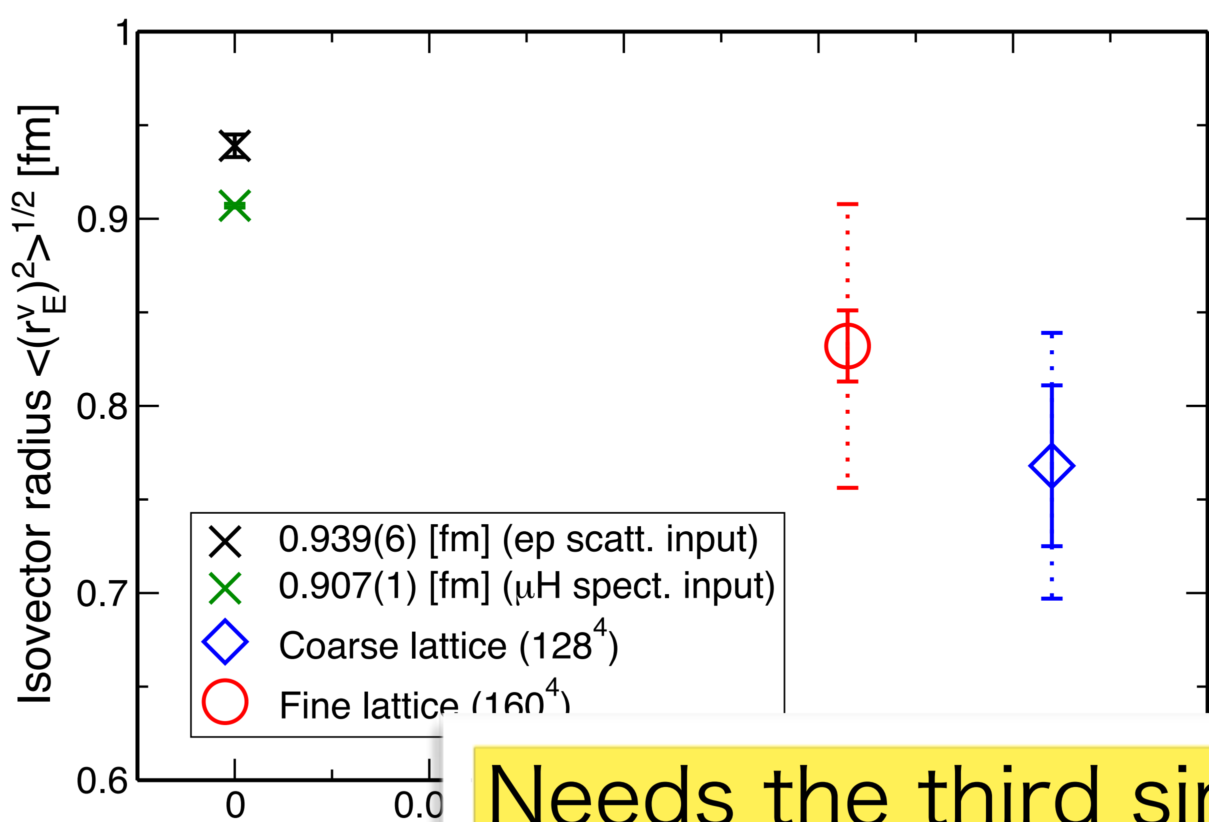
Negligibly small (< 1%)

Tsuji et al. arXiv: 2311.10345

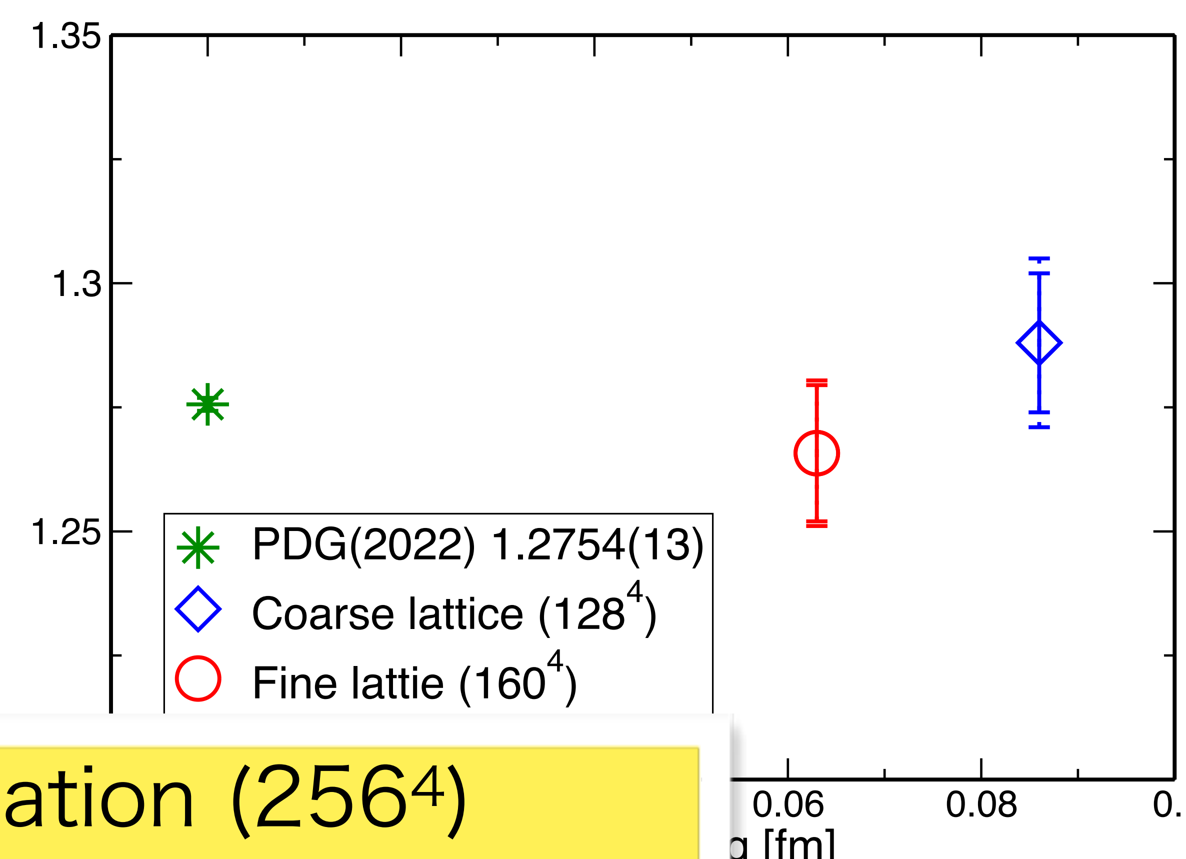
Summary (Cont.)

- **Lattice discretization uncertainties** on r_E and g_A

rms charge radius r_E



axial charge g_A

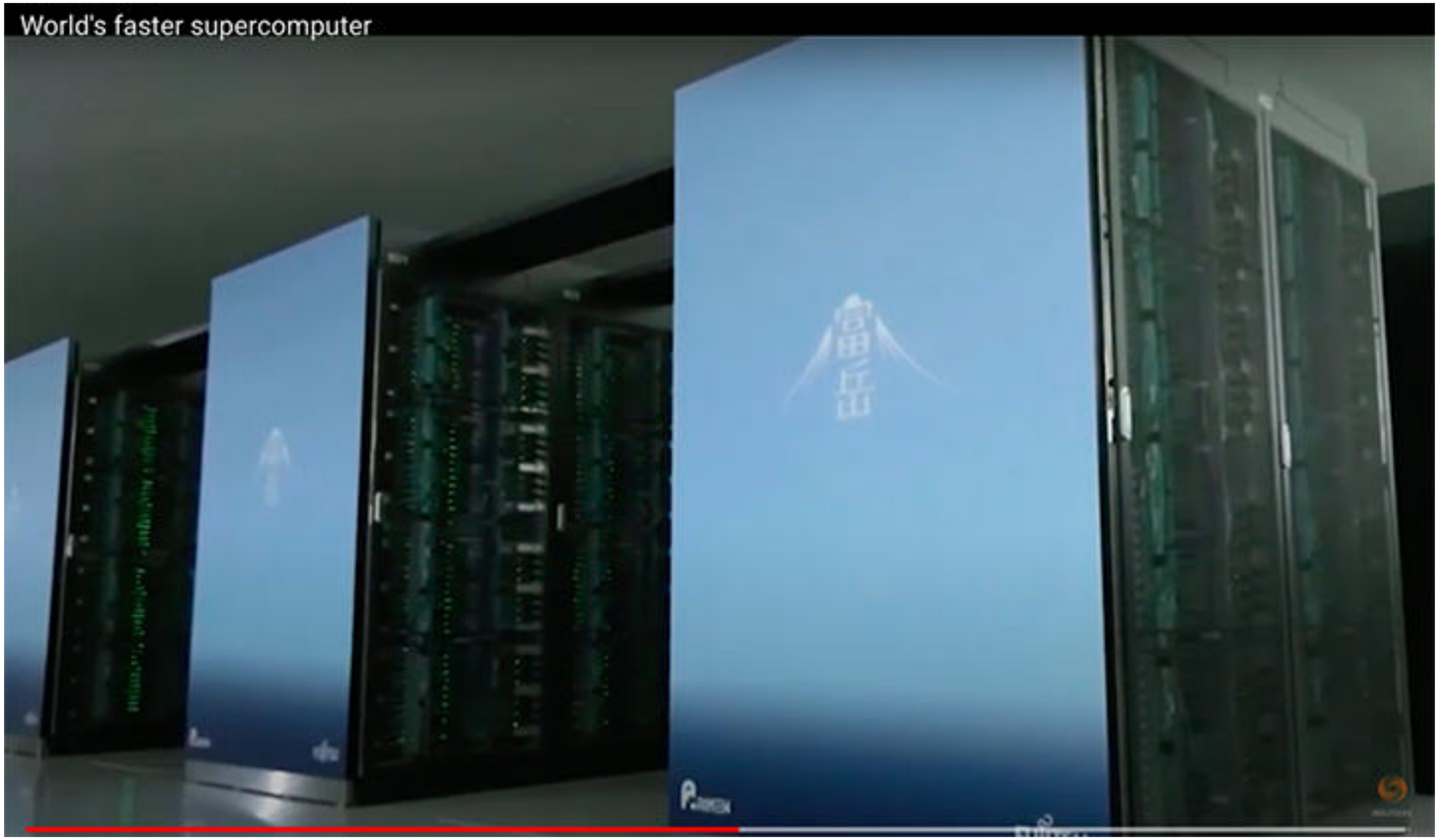


Needs the third simulation (256^4)
at the finer lattice spacing (a≈0.04 fm)

Signif … all (< 1%)

Tsuji et al. arXiv: 2311.10345

3rd simulation performed on Fugaku



The third simulation (256^4) at the finer lattice spacing (a≈0.04 fm)

☞ I hope to show you new results soon